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Modeling Default Probability via Structural Models of Credit Risk in Context of Emerging Markets

Maria Kovacova and Boris Kollar

Additional information is available at the end of the chapter

Abstract

The chapter is focused on the structural models of credit risk introducing basic concepts of risk-neutral world, as well as models and different options for the credit risk quantification. An important part is also the introduction of structural approach for credit risk modeling. Furthermore, the chapter presents basic division of structural models and then presents mathematical derivation of individual apparatuses of models. Among tested models are Merton model, KMV model, Black-Cox model, and Credit Grades model. The practical part is focused on the application of these models under the conditions of local emerging market—Slovakia. Additionally, it pointed out the connection between default probability and credit spreads generated with the use of default mode credit risk models. The main objective is to adjust credit risk model to real market data.

Keywords: credit risk, financial modeling, probability of default, credit spread, structural models

1. Introduction

The chapter is focused on the structural models of credit risk introducing basic concepts of risk-neutral world, as well as models and different options for the credit risk quantification. An important part is also the introduction of structural approach for credit risk modeling. Furthermore, the chapter presents basic division of structural models and then presents mathematical derivation of individual apparatuses of models. Among tested models are Merton model, KMV model, Black-Cox model, and Credit Grades model. The practical part is focused on the application of these models under the conditions of local emerging market—Slovakia. Additionally, it pointed out the connection between default probability and credit spreads generated with the use of default mode credit risk models. The main objective is to adjust credit risk model to real market data.

2. Structural models of credit risk

The main goal of structural models is the objective quantification of credit risk. Objective in the sense is that the process of quantification is not an individual decision of any individual or group of people. On the contrary, the estimation of this risk is the outcome of model, which attempts to describe the causality between the attributes of a particular company applying for a loan (or a company that has already acquired the loan) and the threat that the company will fall into default. We consider market value of its assets to be the main attribute of this company.

The causality mentioned above does not only mean an empirical analysis based on the big set of data and on the choice of proper variables for these data. Correlation does not have to mean causality and prediction. The objective of structural models is to capture the relationship between company fundamentals and the probability of default from time point of view, based on the current market information. The model can then provide a warning of increasing level of credit risk (increasing probability of default) based on changes in company fundamentals. This fact reflects the fundamental and the most important difference between structural and reduced form credit risk models.

Structural models are based on modern financial theory, more precisely on the option pricing theory. The initials of these models are therefore associated with the names of Myron Scholes, Fischer Black, and Robot Merton. These authors contributed significantly to the valuation of options and also laid the foundations for structural models of credit risk. The idea of Black and Scholes [1] to understand corporate liabilities and shares as a derivate written for some corporate assets is considered by Jones et al. [2] to be an even more valuable asset to academic field than the derivation of European option pricing formula itself. It cannot be argued that their approach is the first attempt to quantify credit risk, in particular, the risk of default.

The work of Merton is followed, for example, by Geske [3], who predicts the heterogeneous structure of the company's obligations. This means that he puts both coupon and noncoupon bonds on the side of liabilities. Credit risk is then subsequently valued with the help of barrier option on the company's assets.

Later, the so-called FPT models raised into prominence. Unlike Merton and Geske, they do not assume absolute priority of creditors in the case default, but they work with the idea that part of the value of assets is spent on the costs of bankruptcy or other charge [4]. These assumptions are expressed with the use of coefficients obtained by the empirical observations of defaulted companies and their recovery rates. FPT models assume either a deterministic or stochastic default barrier.

The deterministic barrier is not a constant and it is a function of time, resp. interest rate is a function of time. This type of barrier is used, for example, by Black and Cox [5].

According to Cisko and Klieštik [6], we can divide the structural models into:

- Mark-to-market models are based on the assumption that at the end of the observed risk period, the financial instrument or company will be in one of the n-predefined states.

- Default mode models assume that at the end of the observed risk period, the financial instrument can only be in one of the two states, either default or survival.

3. Default mode models

In this part of this paper, we focus on selected models, which were characterized in the previous sections. In particular, we focus on predicting probability of defaults and credit spreads on empirical data. Because of the fact that the capital market in the Slovak Republic is significantly limited, and it is necessary to know the market price of shares for the purpose of calculation, we have selected company that operates on our market and its shares are traded on the stock exchange, and at the same time, this company is rated by Moody's rating agency.

We use the synthesis of theory and empirical studies of structural models of credit risk. Knowledge from these empirical studies is used, for example, to determine the default barrier height in FPT models. We also limit ourselves only to models with a constant interest rate, as empirical studies show that the stochastic character of the interest rate does not have a significant impact on the credit spread.

4. Calculation of ČEZ, a.s.

The ČEZ Group is the largest energy group operating in Central and Southeastern Europe. Besides the headquarters in Bohemia, it has representation in most of the countries in the region, including Slovakia. To date, ČEZ ranks among the top 10 European energy companies in terms of market capitalization and is the largest company among the new European Union countries. The activities of the ČEZ Group include, in addition to the sale and production of electricity, also telecommunications, informatics, nuclear research, design, construction and maintenance of energy equipment, extraction of raw materials, and processing of various secondary energy intermediates and products. ČEZ is the most profitable and also the least indebted energy company in the region, which reflects its high credit rating. Its stable growth is also achieved due to a balanced portfolio of resources.

The first step is the construction of yield curves for ČEZ, a.s. For the 3, 6, 9, and 12 months horizon, we use the interbank interest rates PRIBOR and for horizons from 2 to 20 years, we use known values of government bonds of Czech Republic. Subsequently, we use the Nelson-Siegel-Svensson method. We construct the yield curves for each year of the prediction, as is shown in **Figure 1**.

Yield curves do not have a traditional shape, except the curve constructed for 2011. The most specific shape is in the case of curve for 2014. In this case, short-term interest rates were higher than government bond rates over 1 year. Changes and natural growth in interest rates occur only in the fifth year. However, the interest rates described by this yield curve follow very low values. Similarly, the low values are also observable in the case of curve constructed for 2015.

In order to determine the market value of assets, we have to stabilize the timeline first, by weighing down the yield on the shares. Consequently, we calculate the market value of assets using the iterative procedures based on Black-Scholes formula [7]. From **Figure 2**, we can see that the shares value of ČEZ, a.s. has a long-lasting character. Shares as the main input of market valuation of assets reflect market reactions to published annual reports and hence
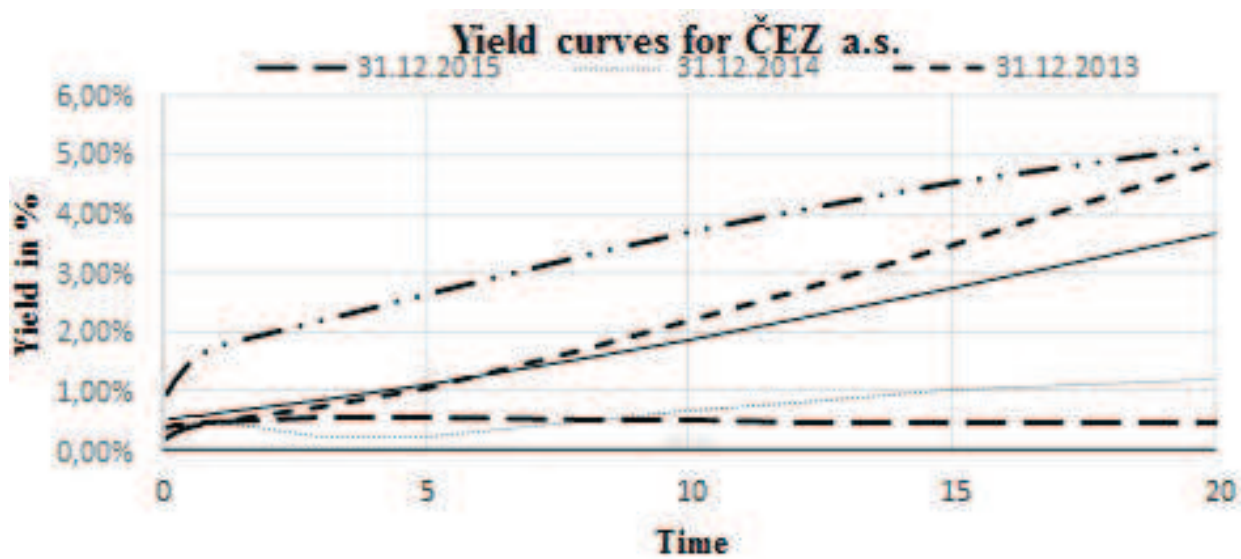
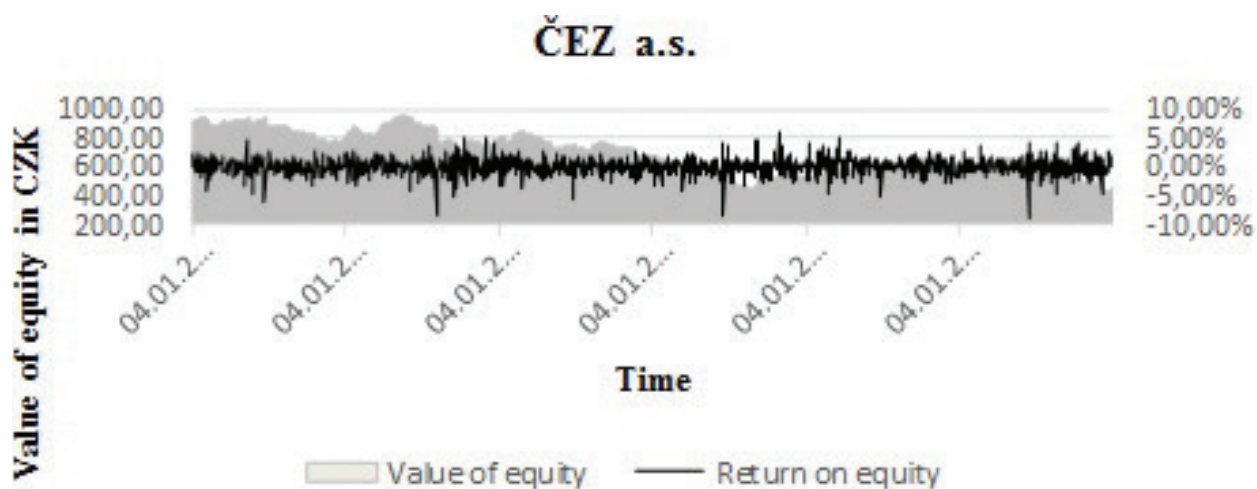**Figure 1.** Yield curves for ČEZ a.s.

Figure 2. Value of equity for ČEZ, a.s.

reaction to company results. Moody's rating has a worsening tendency just like the value of assets, which has always deteriorated in the past 2 years due to the expected debt growth in the coming years.

Figure 3 shows the development of the equity volatility between 2010 and 2015. We chose the moving average method for the volatility calculation, EWMA, and GARCH (1.1) methods. The highest volatility levels in each of observed periods were the ones generated by GARCH (1.1) model. **Table 1** shows the input data for the calculation of each model. The book value of assets is higher than their market value in the first 3 years of the forecast, which means that the company's assets are underestimated. In the last 2 years, the situation has changed and the market value is higher than the accounting value.
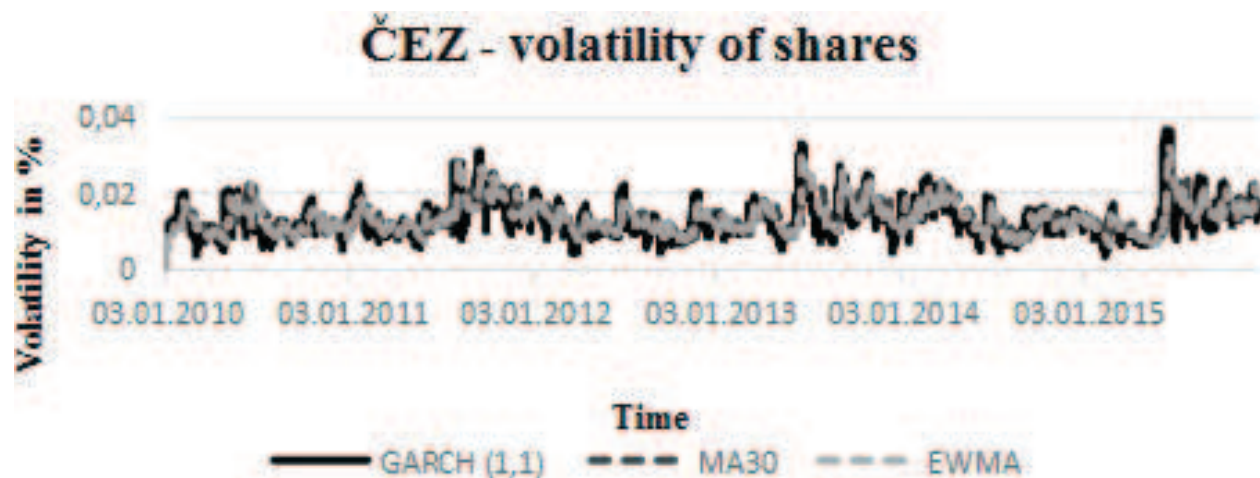
Figure 3. Volatility of shares for ČEZ, a.s.

ČEZ, a.s.		31-12-2015	31-12-2014	31-12-2013	31-12-2012	31-12-2011
Market value of equity		566,369	655,335	632,778	720,103	762,852
Book value of equity		602,686	627,870	641,136	636,070	598,107
Volatility of equity value	MA30	11.57%	12.69%	12.31%	13.86%	14.36%
	EWMA	11.41%	12.52%	12.14%	13.46%	14.35%
	GARCH (1.1)	12.09%	14.07%	17.85%	14.97%	14.95%
Value of debt		330.531	362.019	358.787	360.023	349.083
Market value of shares		237.355	315.662	276.138	363.198	419.756
Volatility of shares value	MA30	22.67%	21.56%	24.24%	19.61%	23.84%
	EWMA	22.53%	21.37%	23.79%	19.29%	23.85%
	GARCH (1.1)	24.11%	22.57%	26.33%	19.71%	25.39%
Default barrier		212,115	235,925	239,252	239,483	237,238

Table 1. Input data for ČEZ, a.s.

We will work with a 20-year time horizon, which is identical to how the individual yield curves were constructed. In the case of the Credit Grades model, we will also need an average rate of return on all debts. Here, we will use the value that the studies and the Credit Grades technical document recommend, and we will work with $\bar{\Lambda} = 0.5$ [8]. We also choose the volatility of the barrier on the basis of these sources at $\lambda = 0.3$.

Figure 4 shows the probability of default in all models for all calculated volatility types by the end of 2015. The KMV model achieves a significantly lower probabilities of default. In the first half of the predicted time horizon, the probability of default is approaching zero. In short time horizons, the probability of defaults is at minimum values close to zero. This is due to fact
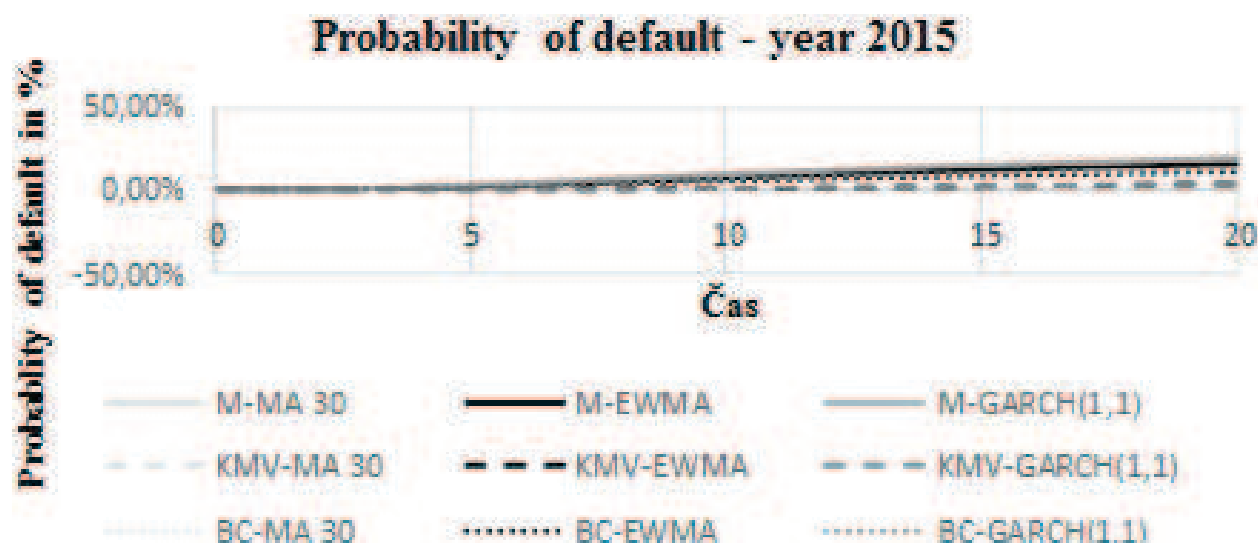
Figure 4. Probability of default in all models for 2015.

that ČEZ a.s. had relatively low volatility of shares during the observed period. The highest Merton probabilities were generated by original Merton model (**Figures 5–12**).

Default curves of ČEZ, a.s. have similar pattern in all models. However, the predicted probability values differ considerably. In a time horizon of up to 1 year, predictions in each of the selected models are close to zero. In the forecast for 2013, there is a sharp increase in the probability of default. This is particularly influenced by the volatility and value of leverage in 2013, which is high in that year in comparison with the other monitored years. In the Black-Cox model, there is an interesting situation when the curves representing the first three seasons are nearing zero. In the long run, they have the highest probability of default in all models for the years 2014 and 2015, which is also influenced by the shape of the yield curves for 2014 and 2015, based on very low interest rates.

The Credit Grades model generates interesting types of curves. Within a short horizon of up to 1 year, the company is virtually safe for all applied models. In the long run, the forecasts vary considerably from 5 to 33%. This is due to both the different design and the ability of models to sensitively react to changes in the input parameters in such a long horizon. Interesting in terms of progress is 2013, which is close to zero in most models, with the exception of the Credit Grades model. The model works with a stochastic barrier, but even then, none of the applied models were able to generate curves with decreasing character. This may be due to the nonrepresentativeness of the sample size.

Credit spreads generated by each model have very similar patterns in all cases. In other applications of these models, we obtained results with negative spreads in the case of KMV model, but not this time. Looking at the reason why this effect does not occur in this case, it is necessary to note the probability of defaults. Over a longer time horizon at higher probability of defaults, the pattern tends to show negative spreads. Therefore, it is better to apply this model to predictions with a shorter time horizon. Another weakness in all three Merton based models is low spreads in short time horizons within 1 year. Highest credit spreads again were however generated by Credit Grades model, because of stochastic barrier.
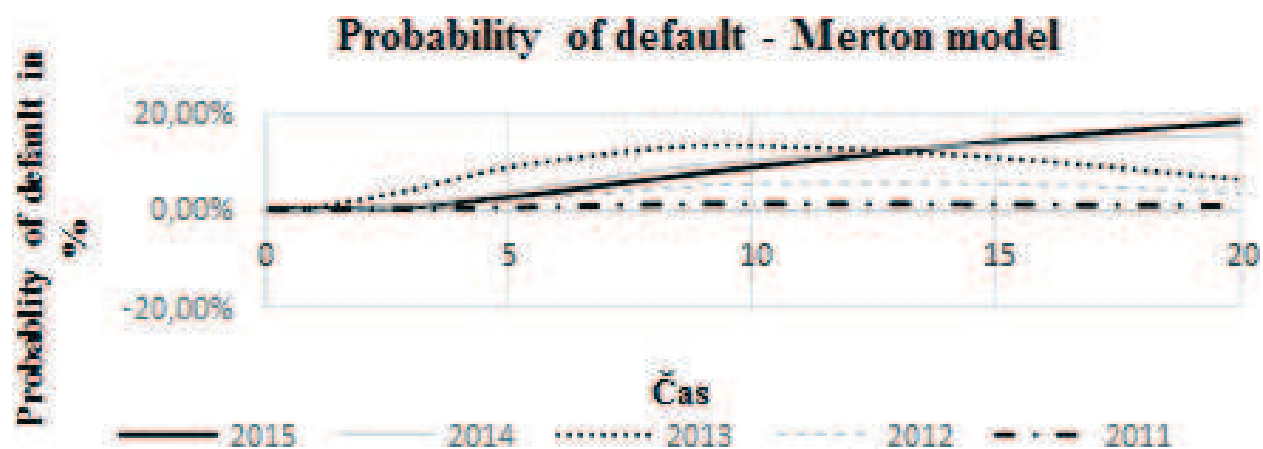
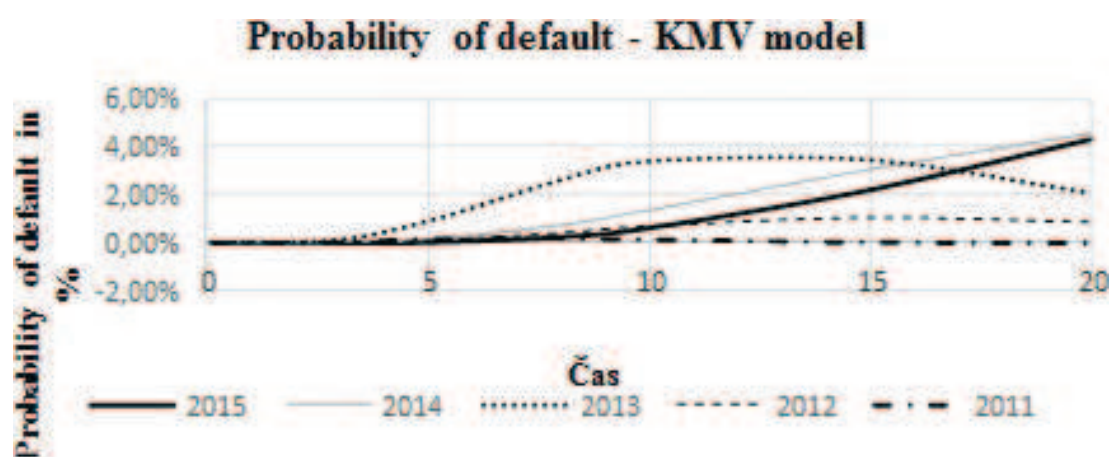**Figure 5.** Probability of default for ČEZ, a.s. between years 2011 and 2015—Merton model.

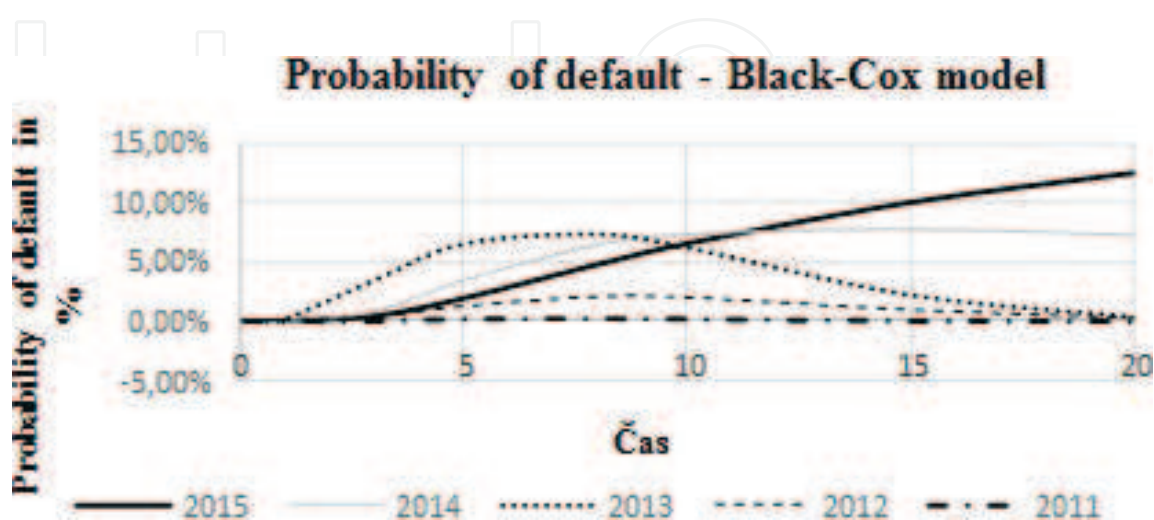**Figure 6.** Probability of default for ČEZ, a.s. between years 2011 and 2015—KMV model.

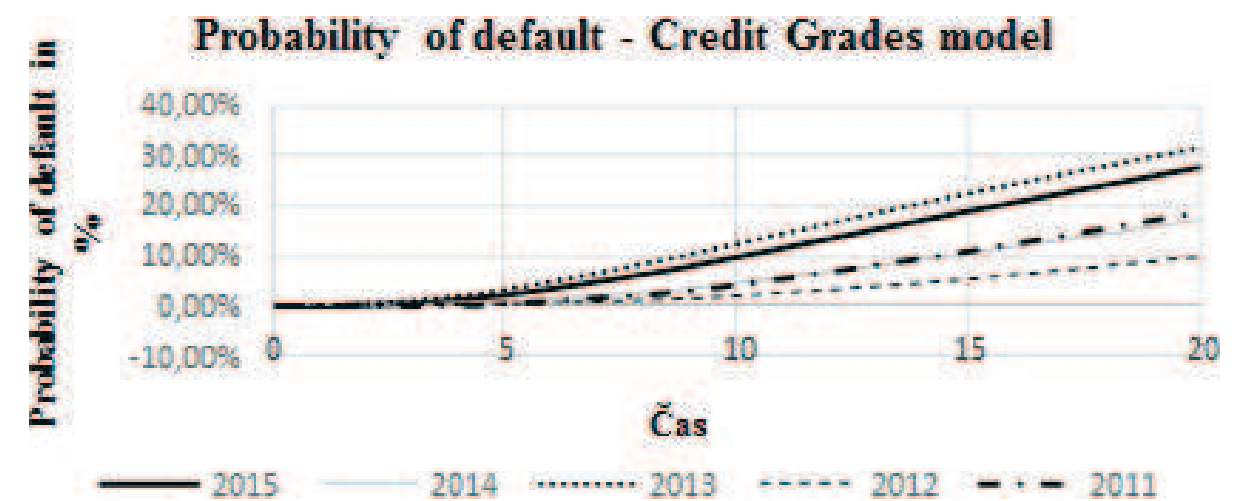**Figure 7.** Probability of default for ČEZ, a.s. between years 2011 and 2015—Black-Cox model.

Figure 8. Probability of default for ČEZ, a.s. between years 2011 and 2015—Credit Grades model.
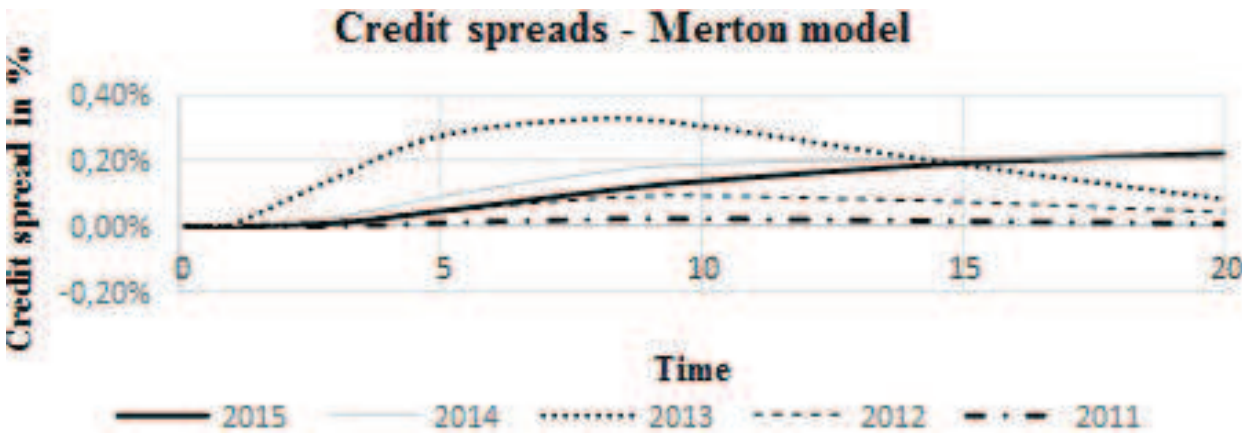

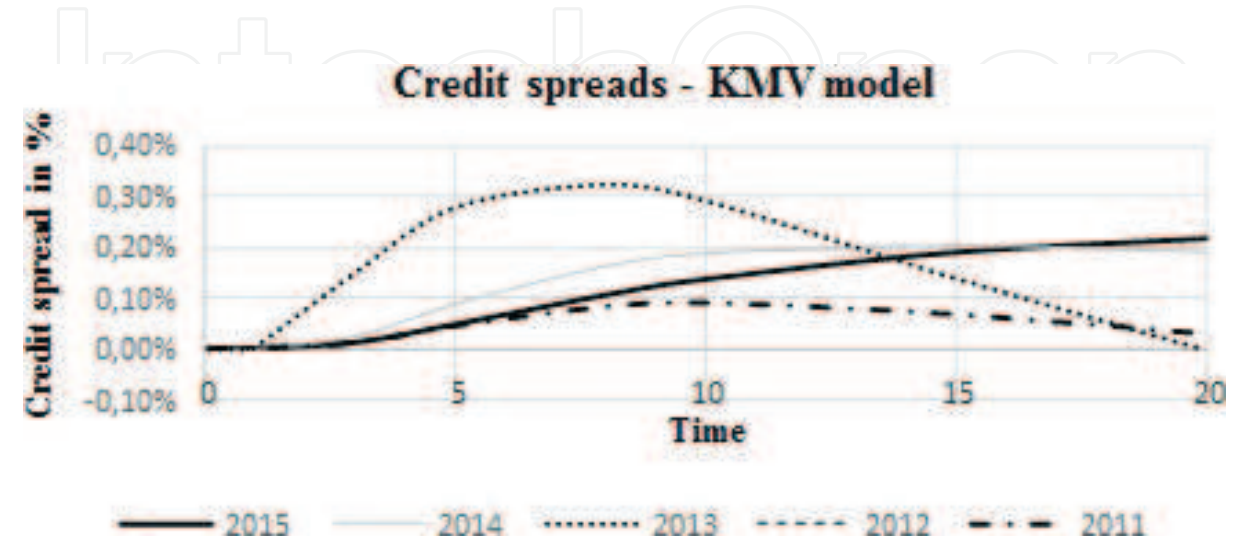**Figure 9.** Credit spreads for ČEZ, a.s. between years 2011 and 2015—Merton model.

Figure 10. Credit spreads for ČEZ, a.s. between years 2011 and 2015—KMV model.
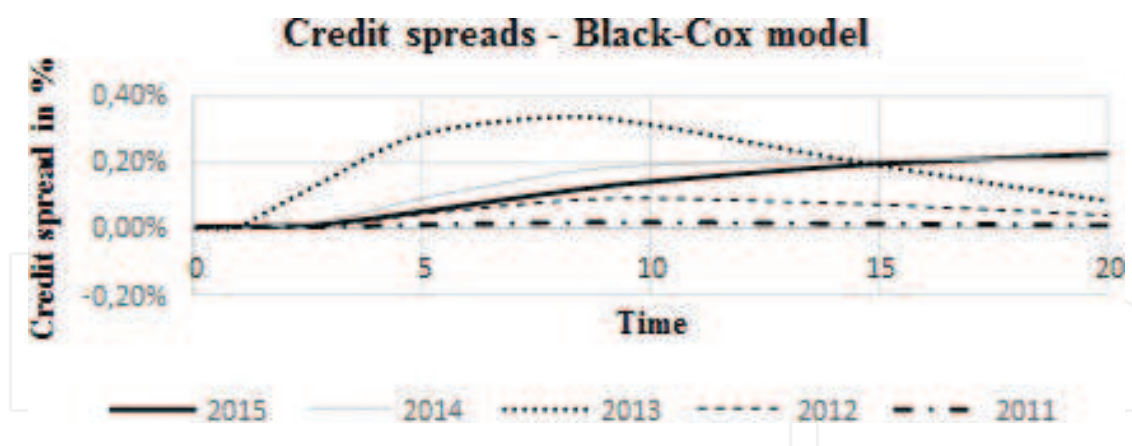
Figure 11. Credit spreads for ČEZ, a.s. between years 2011 and 2015—Black-Cox model.
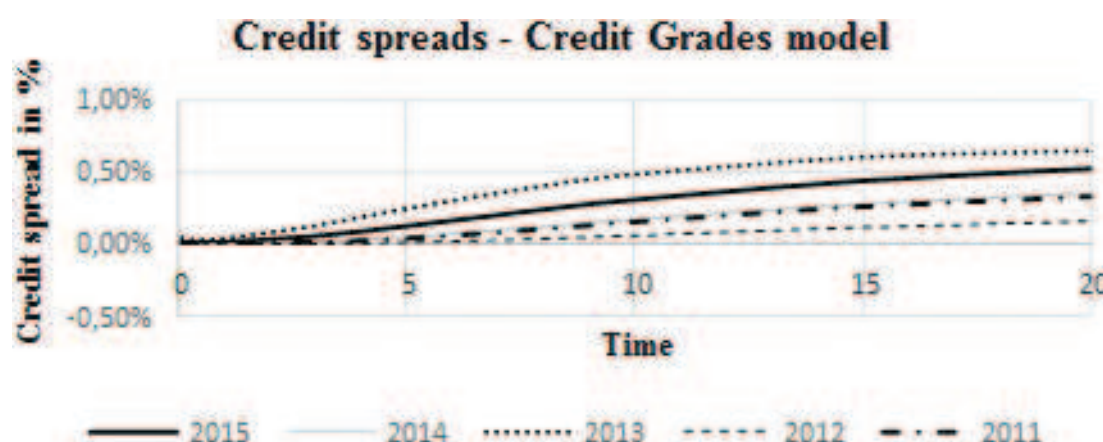

Figure 12. Credit spreads for ČEZ, a.s. between years 2011 and 2015—Credit Grades model.

5. Credit risk quantification with the use of default mode models

Credit spreads generated by selected models have very similar patterns in all cases. Over a longer period at higher probability of defaults, the pattern tends to show negative spreads. Therefore, it is advisable to apply this model to predictions with a shorter time period. Another weakness in all models is low value of spreads in short time horizons within 1 year. Highest credit spreads are generated by Credit Grades model.

While applying the default mode models to companies under the conditions of emerging marker, the problem is to decide whether companies are suitable. We have come across the first barrier of research, as there are only a very small number of companies that have their shares publicly traded on the stock exchange. However, many of these companies are traded on local stock exchanges and their shares are traded only in minimum volumes; and therefore the share price cannot be the basis for calculation of market value. Market value of company and its volatility is one of the basic input parameters of the default mode models.

The next step was the systematization of historical data, in particular the historical values of stocks, buffered indices, as well as interest rates relevant to the stock exchange on which the

company is traded. These data were inputs to calculate the market value of assets with the use of iterative procedures. An important source of information is also the analysis and collection of necessary input data from the annual reports of the surveyed companies.

After analyzing the company's historical data, we have identified the input parameters of selected structural credit models and their subsequent quantification. In the process of quantification, not only the calculations but also their synthesis with the studies of credit rating agencies and the works of other authors played an important role. We used the procedure to determine the default barrier height, where we relied on Moody's approach to their commercial KMV model based on Black-Scholes equation. In the case of the Black-Cox model, based on the studies, we chose a default barrier discount rate of 7%. The values recommended by the technical document as well as by other authors have also been used in the Credit Grades model when we worked with the recommended yield rate and barrier volatility.

Finally, we went on to quantify credit risk by using the probability of default and credit spreads within each model. The results of the individual models differ in some cases. Credit Grades modeled in all cases different curves of default probability and credit spreads compared to other models based on the original Merton model. He alone worked with a stochastic barrier, with its volatility having a significant effect on the calculated values.

6. Own approach to determine the probability of default

One of the most criticized assumptions of Merton model, which is the starting model of structural models, is the assumption of a normal distribution of distance from defaults. This critique is also supported by empirical observations. It is for this reason that we have decided to leave the assumption of a normal distribution. The aim was to find a suitable functional relationship between the distance to default and its probability. An extensive statistical sample would be needed to identify such a functional relationship. Since, in pure region, it is not possible to obtain such a sample in view of the underdeveloped capital market, we have attempted to obtain a similar functional relationship that Moody's uses in its commercially successful KMV model. As we mentioned before, KMV is a commercial implementation of Merton's original model.

In 2003, an article was published in the CFO magazine, which contains hundreds of the largest bond issuers on the US capital market. In the article, the probability of default was quantified for companies based on Moody's internal methodology; asset volatility and the leverage of these companies were also presented [9]. Moody's predicts default probability with a 1-year horizon. In addition to this source, Moody's regularly publishes research on defaulted companies on their site in the research section and they have become another source of information for sample augmentation. Due to these studies, we also acquired companies with higher business risk.

From these collected data, we have subsequently calculated distance to default. In the case of risk-free interest rate, which is meant to express the growth of assets in rated company in a risk-neutral world, we have used US dollar yields with a maturity of up to 1 year for 2003 to match the sample of surveyed companies as much as possible. Based on this, it was then possible to assign the Moody's distance to default values. This relationship is shown in **Figure 13**.
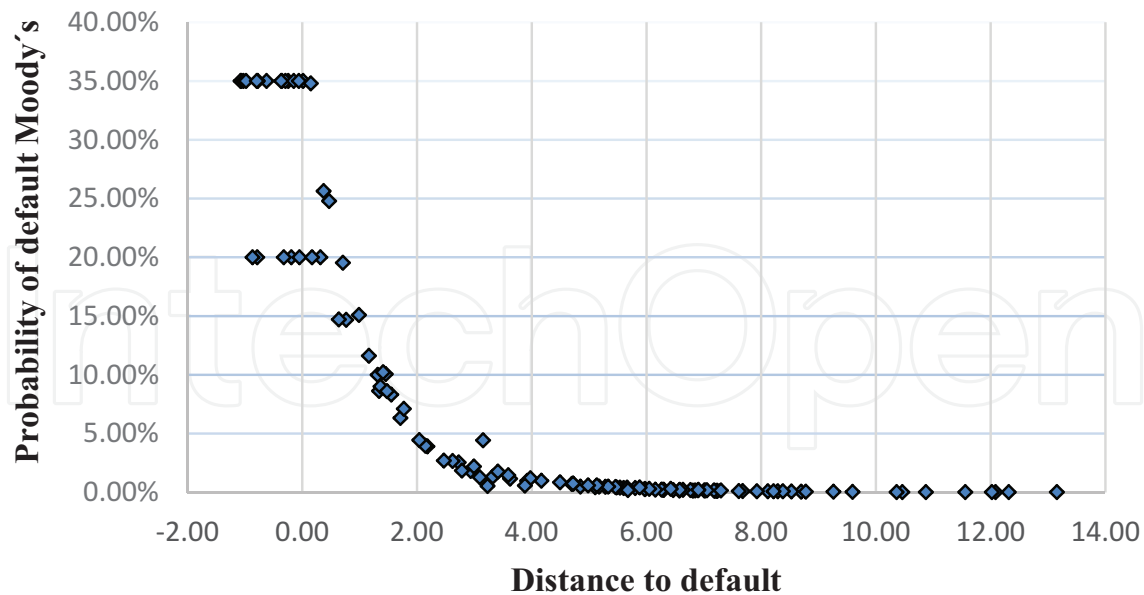
Figure 13. Functionality between distance to default and Moody's probability of default.

Figure 13 shows two areas of anomalies. The first is the area around the probability of defaults at 20%. It is obvious that the negative dependence of distance from defaults and their probability is not ensured. The following conclusions can also be drawn from **Figure 13**:

- The Moody's model assigns probability of default at the level of 35% for defaulted companies and companies with high degree of business risk. Higher values were not even observed in this the model.

- With increasing value of distance to default, its probability decreases exponentially.

- For companies with low degree of business risk and good financial stability, the model assigns a probability of 0.02%. The lower values were not observed for this set of data.

We used these findings to calibrate our own approach to determine the probability of default. In the case of companies that have lower market value of assets than the default point or if their distance to defaults is negative, we can assume default in the near future. Such companies therefore should not be able to meet their obligations and the owners of the capital would thus not apply the purchase option that was written out on the assets of such company. We will associate these companies with the probability of default at 35%. On the contrary, those with a very good financial position, financial stability with a low degree of creditor risk assigned probability at 0.02%. The probability of default for other companies moves between these two extremes of function. Depending on the distance to default based on Moody's (EDF_M) data, the probability of default is described by the following exponential function:

$$EDF_M = 0.1797\, e^{-0.652*DD} \tag{1}$$

The selected functional relationship showed the determination index at 0.9683. The synthesis of the acquired knowledge has determined the resulting function describing the relationship between EDF_M as follows:

$$EDF_M = \begin{cases} 35\% & pre\ DD \leq 0 \\ 0.1797\ e^{-0.652*DD} & pre\ DD > 0 \end{cases} \qquad (2)$$

At the same time, $EDF_M \in \langle 0.02\%; 35\% \rangle$. EDF_M expressed in this way, defines the probability of default in a risk-neutral environment. **Figure 14** compares the probability of a default calculated on the base of normal distribution with the probability of default on the base of Eq. (2).

Based on **Figure 14**, we can say that the probability of default—EDF_M calculated with the use of relationship differs from EDF with normal distribution. This is particularly the case for companies with a very low distance to default values. In this extreme, EDF_M values are systematically overestimated. With the increase in distance to default, the opposite situation occurs, and thus the systematic underestimation of probability of default for the normal distribution. For companies with high values of distance to default, these differences are negligible. EDF with normal distribution is asymptotically approaching zero and EDF_M has a minimum of 2%.

Quantification of the probability of default with the use of KMV model takes several forms [10]. Basic and generally known works with a normal distribution of distance to default. However, empirical studies show that this distance varies depending on the probability of default, taking into account several different factors like business sector, its geographical situation as well as other relevant factors. The calibrated relationship between these two variables offered different results in comparison with other models presented in this paper as we can see in **Figure 15**:

To offer better overview, we offer only comparison of default probability for 2 years. From the results, we can see that modeled probabilities of default have an expected progression. During these two curves, there should be two intersections at specific points as we can see in **Figure 15**.

In practice, credit ratings established by renowned agencies are the most commonly used, because they offer sufficient reporting ability in terms of the financial stability of the analyzed companies. However, most of these ratings are in the form of paid service. Moody's officially
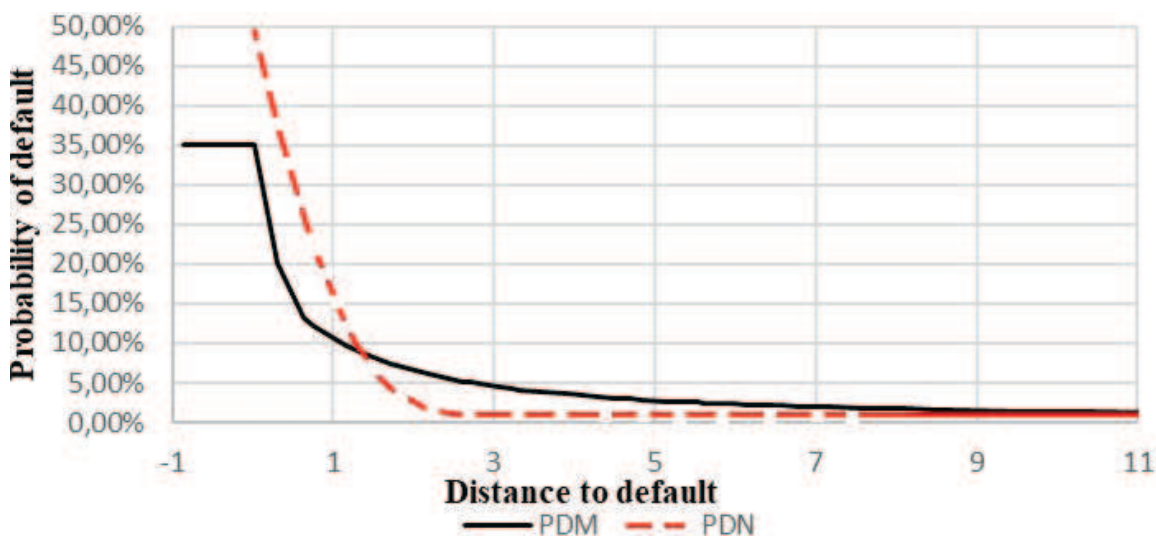
Figure 14. Comparison of functionalities between distance to default and probability of default for normal and calculated distribution.
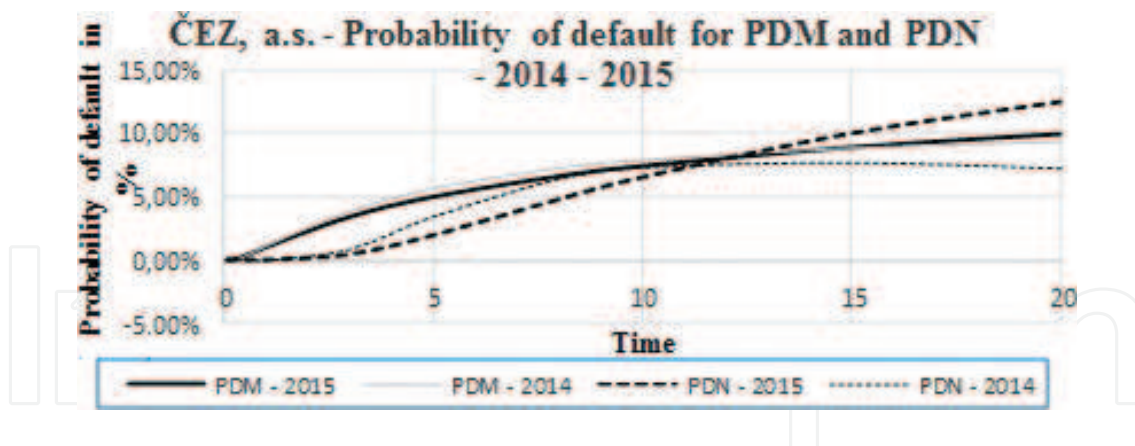
Figure 15. Comparison of probabilities of default with normal and calculated distribution for ČEZ, a.s.

assigns ČEZ, a.s. rating at Baa1 level. The 1-year probability of default level is 1.18%. This value is in the group also assigned by RMA study to this rating as we can see in **Table 2**. Therefore, we can say that our approach to probability of default is relatively close to that one of Moody's agency, also more we would need more data to verify this even further.

EDF_M	Moody's rating scale
0.00–0.27%	Aaa
0.28–0.39%	Aa1
0.40–0.49%	Aa2
0.50–0.57%	Aa3
0.58–0.60%	A1
0.61–0.68%	A2
0.69–1.04%	A3
1.05–1.38%	Baa1
1.39–2.08%	Baa2
2.09–4.34%	Baa3
4.35–6.55%	Ba1
6.56–10.20%	Ba2
10.21–14.77%	Ba3
14.78–17.49%	B1
17.50–21.51%	B2
21.52–26.00%	B3
>26.00%/	Caa1

Table 2. Relationship between EDF_M and Moody's rating scale.

7. Conclusion

Despite the fact that numerous models for credit risk quantification have been developed, they differ not only in the way of their construction but also in the amount of input data, in the difficulty of their calculation, and also in the complexity of their usage. In this chapter, we focused on the structural models of credit risk introducing basic concepts of risk-neutral world, as well as models and different options for credit risk quantification. Furthermore, we have tested selected structural models namely Merton model, KMV model, Black-Cox model, and Credit Grades model under the conditions of local emerging market—Slovakia. These calculations were provided on the company data of the ČEZ Group, which is the largest energy group operating in Central and Southeastern Europe. Besides the headquarters in Bohemia, it has its representation in most of the countries in the region, including Slovakia.

The main goal of the chapter was to adjust credit risk model to real market data. Calculation of default curves of ČEZ, a.s. has shown similar pattern in all models. However, the predicted probability values differ considerably. On the other side, the Credit Grades model generated interesting types of curves. Within a short horizon of up to 1 year, the company is virtually safe for all applied models, but in the long run, the forecasts vary considerably from 5 to 33%. This is given by the different design and also by the ability of models to sensitively react to changes in the input parameters in such a long horizon.

Similarly, to default curves also credit spreads generated by each model have similar patterns in all cases. Over a longer period at higher probability of defaults, the pattern tends to show negative spreads. Therefore, it is advisable to apply this model to predictions with a shorter time period. Another weakness in all models is low value of spreads in short time horizons within 1 year. Highest credit spreads are generated by Credit Grades model.

Since one of the most criticized assumptions of Merton model, is the assumption of a normal distribution of distance from defaults, we have decided to leave this assumption and to find a suitable functional relationship between the distance to default and its probability. Based on the collected data, we have calculated distance to default and found out interesting findings. In practice, credit ratings established by renowned agencies are the most commonly used, because they offer sufficient reporting ability in terms of the financial stability of the analyzed companies. Moody's official assigns ČEZ, a.s. rating at Baa1 level. The one-year probability of default level is 1.18%. This value is in the group also assigned by RMA study to this rating.

Therefore, we can summarize that our approach to probability of default is relatively close to that one of Moody's agency, also more we would need more data to verify this even further, but can be successfully applied in the condition of emerging markets.

Acknowledgements

Author details

Maria Kovacova* and Boris Kollar

*Address all correspondence to: maria.misankova@fpedas.uniza.sk

Department of Economics, Faculty of Operation and Economics of Transport and Communications, University of Zilina, Zilina, Slovak Republic

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