Chapter 4

Comparison of CAPM, Three-Factor Fama-French Model and Five-Factor Fama-French Model for the Turkish Stock Market

Yaşar Erdinç

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/intechopen.70867

Abstract

In this study, I try to test the capital asset pricing model (CAPM), three-factor Fama-French (3F-FF) model and five-factor Fama-French (5F-FF) model for the Turkish stock market. The sample is from June 2000 to May 2017. My results show that the five-factor model explains better the common variation in stock returns than the three-factor model and capital asset pricing model. Moreover, the CAPM has no power in explaining monthly excess returns of sorted portfolios. Although three-factor model seems to have significant coefficients, intercepts in this model have significant *t*-values indicating that the model has problems in explaining the portfolio returns. I use equal weight market portfolio for all the models in order to explain the cross-sectional variations in the stock returns.

Keywords: CAPM, three-factor Fama-French model, five-factor Fama-French model, Turkish stock market, size, book to market, profitability, investment

1. Introduction

Borsa Istanbul (BIST) stock exchange was established in 1985 and commenced stock trading on 3 January 1986. Acceptance of BIST as a full member to the World Federation of Exchanges (WFE) was in 1992. As elsewhere, obviously for all investors (institutional or individual), the main goal is to get the highest possible return in a stock market.

This study tests the capital asset pricing model (CAPM hereafter), the three-factor Fama-French model (3F-FF hereafter) and the five-factor Fama-French model (5F-FF hereafter) in the case of the Turkish stock market. This study extends the asset pricing tests in three ways: (a) this study is the first application of the 5F-FF Fama-French model for the Turkish stock market. (b) It expands the test of the 3F-FF model to the Turkish market for a longer period, and this is the first study that covers 17 years of the Turkish data. The main result is that the

5F-FF model explains better the common variation and the cross section of stock returns than the 3F-FF model and the CAPM. (c) I test all the models (CAPM, 3F-FF and 5F-FF) with 48 different market portfolios. 5F-FF model portfolios capture the common variation in stock returns and can explain the cross section in returns.

In Section 2, I give a literature review on asset pricing models and applications. In Section 3, data selection, variable definitions, return periods and filtering data issues are given. Section 4 explains the methodology to apply the CAPM and Fama-French factor models to the Turkish stock market. In this section, explanatory variables and dependent variables are defined. First of all, Fama-French factors are constructed. Then, regression portfolios are constructed by sorting the stocks by their size, book-to-market ratios (B/M), profitability (OP) and investment (INV). In Section 5, I give regression portfolio statistics to see patterns in the behaviour of portfolios. Section 6 defines and estimates factor spanning regressions, which are important to see if an explanatory factor can be explained by a combination of other factors. Having estimated and tested factor spanning regressions, I go on testing of hypothesis of joint significance of portfolio regressions' alphas in Section 7. In this section, I conduct Gibbons-Ross-Shanken (GRS) test for the regression portfolios and give preliminary results of the performance of the CAPM, the 3F-FF and the 5F-FF models for the Turkish case. Section 8 is devoted to the detailed analysis of regressions, and the main messages of these regressions are presented. Section 9 concludes and presents the main findings of the study.

2. Literature review

What kinds of factors determine the price of an asset? Since Markowitz formulated a model of asset pricing [1], the debate on this question continues. The main determinants of asset prices and risk factors that affect the demand for assets and asset prices have been an important issue in finance theory and practice. One can find enormous number of studies on this issue. Earlier studies in this area are by Markowitz, Sharpe, Ross, Fama and French [1–4].

Since the literature on asset pricing model (APM) is very well known and can be reached easily in finance textbooks, I do not go into a detailed explanation of evolution of APM. However, I would like to briefly state that all the asset pricing models developed so far have included risk as the most important determinant. For example, [1] defines the expected return and variance of returns on a portfolio as the basic criteria for portfolio selection.

Markowitz's model requires large data inputs. Because of input drawback, new models have been developed to simplify the inputs to portfolio analysis. William Sharpe's market model [2] is as follows:

$$R_{it} = \alpha_i + \beta_i^* R_{Mt} + e_{it}, \tag{1}$$

where R_{it} is the return of stock i in period t, α_i is the unique expected return of security i, β_i is the sensitivity of stock i to market movements, R_{mt} is the return on the market in period t and e_{it} is the unique risky return of security i in period t and has a mean of zero and finite variance

http://dx.doi.org/10.5772/intechopen.70867

σ^2_{ei}, uncorrelated with the market return, pairwise and serially uncorrelated. This equation explains the return on asset i by the return on a stock market index. β in Eq. (1) is a risk measure arising from the relationship between the return on a stock and the market's return.

Later on, the equilibrium models have been developed. The difference between the market model and the equilibrium model was that asset returns are related to excess market return rather than market return. The first and basic form of the general equilibrium model, which was developed by Sharpe, Lintner and Mossin [2, 4, 5] called capital asset pricing model (CAPM), is given in Eq. (2):

$$R_{it} = R_f + \beta_i{}^* \left(R_{Mt} - R_f\right) + e_{it} \tag{2}$$

where R_{it} is the return of stock i in period t, R_f is risk free rate, β_i is the sensitivity of stock i to excess return on a market portfolio, R_{mt} is the return on the market in period t and e_{it} is the unique risky return of security i in period t and has a mean of zero and variance σ^2_{ei}.

Black et al. [6] derived a new model of the CAPM by relaxing the assumption of risk-free lending and borrowing. Basu [7] considers a different time series model, which is written in terms of returns in excess of the risk-free rate R_f and shows that returns are positively and linearly related to β, as follows:

$$R_{it} - R_{ft} = \alpha_i + \beta_i \left(R_{Mt} - R_{ft}\right) + e_{it} \tag{3}$$

While the CAPM is still the most widely accepted description for security pricing, empirical studies found contradicting evidence (see [7–14]). Therefore, researchers concentrated on finding better models for the behaviour of stock returns and added more explanatory variables into CAPM.

In the early 1990s, one of the most influential researches was by Fama and French [15, 16]. Fama and French [15] reject the market beta associated with the CAPM and instead find that stock size and book-to-market (B/M) ratio better capture the cross-sectional variation in average stock returns. One year later the same researchers proposed that a 3F-FF asset pricing model augmenting the CAPM with size and book-to-market proxies for risk might be a superior description of average returns [16]. After these two influential studies, along with earlier evidence against the CAPM drove the finance community into investigating the reasons behind the anomalies found in [10–14].

Recent studies have found additional factors that seem to exhibit a strong relationship with average returns. Novy-Marx [17] finds that firms with high profitability generate significantly higher returns than unprofitable firms. Aharoni et al. [18] find that a statistically significant relation exists between an investment proxy and average returns. In the wake of these findings, Fama and French [19] expanded the 3F-FF model with *profitability* and *investment*. They reveal that the 5F-FF model performs better than the 3F-FF model in explaining average returns for their sample. The same model was tested using international data [20, 21], and they have found similar results.

This study adds to research conducted on CAPM, three-factor model and the new 5F-FF model by testing all these models on the Turkish stock market.

3. Data selection and issues

3.1. The sample

The data sample used in the analysis consists of monthly price, total return and accounting data downloaded from 'Finnet Data Yayıncılık'. The data set contains nonfinancial 263 firms listed in BIST (Borsa Istanbul or Istanbul stock exchange) for the period between 31.12.1999 and 30.05.2017. The collected accounting data includes total assets, total liabilities, outstanding shares, owner's equity and operating income, where operating income is defined as 'net sales minus operating expenses' and operating expenses is defined as the 'sum of all expenses related to operations'. Data was collected for all available active and dead stocks in Istanbul stock exchange totalling 204 observations. The data was quoted in the Turkish Lira (TRY hereafter).

The downloaded sample included a large amount of stocks, which were already dead at the beginning of the research period, as well as some missing data types and data errors, which ought to be removed.

3.2. Variable definitions

This subsection defines the variables needed in the factor creation process. Market capitalisation or market cap was used as a measure of size for each stock and was calculated by multiplying the price (P) at the 31st of December each year with outstanding shares at the 31st of December for the same year. The price data was obtained from FDY. Book equity was calculated as yearly total assets minus total liabilities from FDY. Book-to-market ratio (B/M) was calculated from the previous two variables by dividing book equity by market cap. Operating profitability (OP) was calculated by dividing operating income by book equity following [22]. Finally, investment (Inv) was calculated as in Eq. (4):

$$\frac{Total\ Assets_{t-1} - Total\ Assets_{t-2}}{Total\ Assets_{t-2}} \tag{4}$$

TRY 3-month Libor rate is used as a proxy of risk-free rate (R_f), while market return (R_m) is approximated by natural log difference of BIST-100 Index of the Istanbul stock exchange. It consists of 100 stocks, which are selected among the stocks of companies listed on the national market (excluding list C companies). Monthly returns for stocks are all calculated as natural log difference of monthly stock data.

3.3. The return period

Fama and French use 6-month gap between the ends of the fiscal year and the portfolio formation date can be considered as convenient and conservative. Since all the accounting data in BIST is available by the end of May of each year, I use 5-month gap. Hence, to ensure that all accounting variables are known by investors, I assume that all accounting information is made public by the end of May, and I use monthly returns from the beginning of June to the end of the following year in May. And, each year at the end of May, I sort the portfolios.

Fama and French [15, 16] used value-weighted returns in their study; however, they also stressed that equal-weighted returns do a better job than value-weighted returns in explaining returns by 3F-FF model. Lakonishok et al. [23] also suggest to use equal-weighted portfolios to investigate the relationship between risk factors and stock returns. Hence, the equal-weighted monthly returns on each portfolio were used in this study.

3.4. Filtering data

At the end of each year, I eliminated firms that have the following specifics: (1) negative book-to-market values were removed, and (2) the companies with yearly increase in their investment, as defined in Eq. (4), which is either less than −50% or higher than 100% in a certain year were eliminated. This would imply that the company in question lost half of its assets, or more than doubled its assets in the given year, which seems very unlikely during normal recurring circumstances.

4. Methodology

4.1. Model definitions

In this study, I test three models, namely, CAPM, 3F-FF model and 5F-FF model for BIST. The model definitions are given below:

$$\mathrm{R}_{it} - \mathrm{R}_{ft} = a_i + \left(R_{Mt} - R_{ft}\right) + e_{it-\mathrm{CAPM}} \tag{5}$$

$$\mathrm{R}_{it} - \mathrm{R}_{ft} = a_i + \left(R_{Mt} - R_{ft}\right) + s_i SMB_t + h_i HML_t + e_{it}(\text{3F-FF model}) \tag{6}$$

$$\mathrm{R}_{it} - \mathrm{R}_{ft} = a_i + \left(R_{Mt} - R_{ft}\right) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}(\text{5F-FF model}) \tag{7}$$

where R_{it} is the return of portfolio i at time t (portfolio creation procedure is given in the following topic); R_{ft} is the risk-free rate approximated by 3-month TRY Libor rate at time t; $R_{it}–R_{ft}$ is the excess return of portfolio i at time t; R_{Mt} is the monthly market return approximated by natural log difference of BIST-100 Index at time t; SMB_t, HML_t, RMW_t and CMA_t are defined in details in the following topic; a_i is the intercept; β_i is the coefficient of $R_{Mt}–R_{ft}$ for portfolio i; s_i is the coefficient of SMB_t for portfolio i; h_i is the coefficient of HML_t for portfolio i; r_i is the coefficient of RMW_t for portfolio i; and c_i is the coefficient of RMW_t for portfolio i.

4.2. Construction of Fama-French factors

The next step in the analysis is to *create sorted portfolios* from which the *Fama and French factor return series* could be calculated. The factors used in the analysis were constructed in a manner similar to the process described in [19], relying solely on 2 × 3 sorts for creating the factors. Other sorting choices might have been used; however, [19] finds no differences in model performance when testing differing sorting methods.

My first data point is at the 31st of December 1999, and the investment variable is calculated as in Eq. (4); the first available year of accounting data used in the sorting process is at the end of fiscal year 2000. The portfolios were sorted at the end of May each year, and therefore the first available return observation in the final analysis is the return of June 2000, sorted according to accounting data at the end of fiscal year 1999. Thus, the time period for the actual analysis is June 2000 to May 2017 or 204 months of return data.

The sorting process is as follows:

(1) First of all, stocks are sorted according to their market cap to define small-sized and big-sized stocks. Fifty percent of the market cap was used as the breakpoint for size. (2) For all other factors, yearly sample 30th and 70th percentiles were used as breakpoints in the sorting method. (3) After the determination of the breakpoints, the stocks in the sample were independently distributed for every year into six size-B/M (where B/M is book-to-market ratio) portfolios, six size-OP (where OP is operational profits divided by book equity showing profitability) portfolios and six size-Inv (where Inv is yearly increase in total assets) portfolios created from the intersections of the yearly breakpoints. (4) All portfolios are value-weighted according to their market cap. (5) Monthly returns were calculated for each of the 18 portfolios. (6) After calculating the sorted portfolio returns, the actual factor returns were calculated.

There are two size groups and three book-to-market (B/M), three operating profitability (OP) and three investment (Inv) groups. The resulting groups are labelled with two letters. The first letter describes the size group, small (S) or big (B). The second letter describes the B/M group, high (H), neutral (N) or low (L); the OP group, robust (R), neutral (N) or weak (W); or the Inv group, conservative (C), neutral (N) or aggressive (A). Stocks in each component are value-weighted to calculate the component's monthly returns (**Figure 1**).

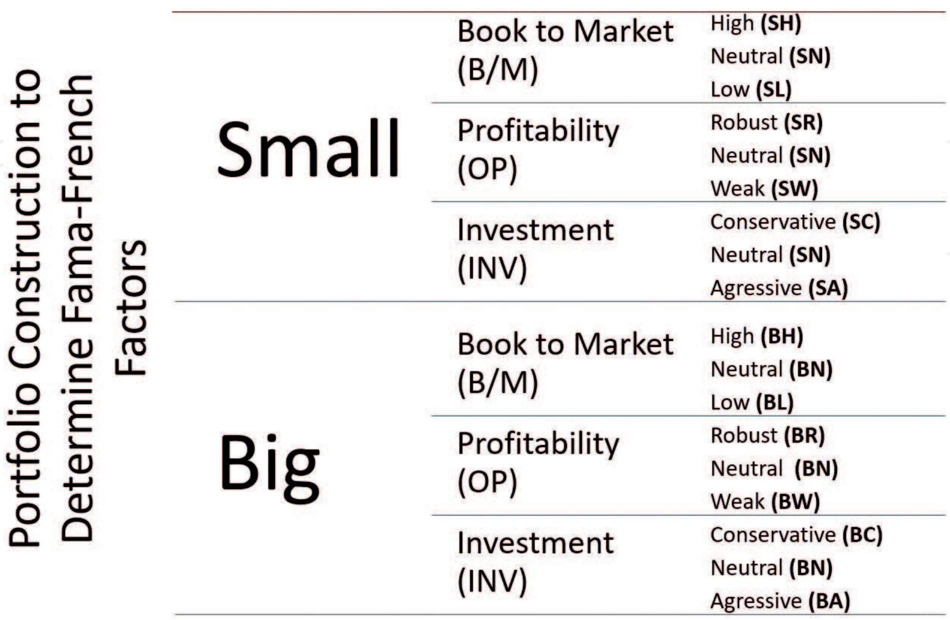

Figure 1. Sorted portfolio groups to construct Fama-French factors.

http://dx.doi.org/10.5772/intechopen.70867

Eqs. (8)–(14) define calculations of the Fama-French factors:

$$SMB_{B/M} = (SH + SN + SL)/3 - (BH + BN + BL)/3 \tag{8}$$

$$SMB_{OP} = (SR + SN + SW)/3 - (BR + BN + BW)/3 \tag{9}$$

$$SMB_{Inv} = (SC + SN + SA)/3 - (BC + BN + BA)/3 \tag{10}$$

$$SMB = \left(SMB_{B/M} + SMB_{OP} + SMB_{Inv}\right)/3 \tag{11}$$

$$HML = (SH + BH)/2 - (SL + BL)/2 \tag{12}$$

$$RMW = (SR + BR)/2 - (SW + BW)/2 \tag{13}$$

$$CMA = (SC + BC)/2 - (SA + BA)/2 \tag{14}$$

4.3. Regression portfolios and other data used in the regression analysis

Regressions were run on three sets of 16 left-hand-side regression portfolios. Monthly returns for the portfolios were calculated in a manner similar to constructing the factor portfolios;

Book to market (sorted from low to high (BM1 (lowest); BM4 (highest))				
	BM1	**BM2**	**BM3**	**BM4**
S1	S1BM1	S1BM2	S1BM3	S1BM4
S2	S2BM1	S2BM2	S2BM3	S2BM4
S3	S3BM1	S3BM2	S3BM3	S3BM4
S4	S4BM1	S4BM2	S4BM3	S4BM4
Profitability (sorted from robust to weak: PO1 (robust profitability); PO4 (weak profitability))				
	OP1	**OP2**	**OP3**	**OP4**
S1	S1OP1	S1OP2	S1OP3	S1OP4
S2	S2OP1	S2OP2	S2OP3	S2OP4
S3	S3OP1	S3OP2	S3OP3	S3OP4
S4	S4OP1	S4OP2	S4OP3	S4OP4
Investment (sorted from conservative to aggressive: INV1 (conservative); INV4 (aggressive))				
	INV1	**INV2**	**INV3**	**INV4**
S1	S1INV1	S1INV2	S1INV3	S1INV4
S2	S2INV1	S2INV2	S2INV3	S2INV4
S3	S3INV1	S3INV2	S3INV3	S3INV4
S4	S4INV1	S4INV2	S4INV3	S4INV4

For example, S1BM1 shows average of the monthly excess returns of stocks included in the smallest size and lowest book-to-market portfolio.

Table 1. Construction of dependent variables: each notation of a dependent variable in this table shows the monthly excess return of the corresponding sorted portfolio.

Panel A	BM1 (low)	BM2	BM3	BM4 (high)
S1 (small)	7	12	14	19
S2	10	11	14	19
S3	8	8	8	7
S4 (big)	21	15	12	6
Panel B	**OP1 (robust)**	**OP2**	**OP3**	**OP4 (weak)**
S1 (small)	10	9	13	22
S2	11	12	16	15
S3	12	15	12	11
S4 (big)	14	15	11	3
Panel C	**INV1 (conservative)**	**INV2**	**INV3**	**INV4 (aggressive)**
S1 (small)	17	12	12	12
S2	15	15	12	12
S3	14	14	13	12
S4 (big)	8	14	17	14

Table 2. Average number of stocks in regression portfolios.

equal-weighted portfolios were created from independent 4×4 sorts with 25th, 50th and 75th yearly sample percentiles as breakpoints for both sorting variables. **Table 1** shows the constructed dependent variables.

Panels A through C in **Table 2** show the average number of stocks in each of the regression portfolios. It is evident from Panel A that high B/M companies are often smaller companies, while low B/M is tilted towards bigger companies. A similar phenomenon can be observed in Panel B, where low operating profitability is a feature of smaller companies and high operating profitability is more often found in stocks with higher market capitalisations.

5. Regression portfolio statistics

In this section, I give descriptive statistics for the regression portfolios and explanatory factors used in the regressions.

The main aim of this research is to see if well-targeted regression models can explain average monthly excess returns on portfolios with large differences in constituent size, B/M, profitability and investment. In **Table 3**, the standard deviations of monthly excess return of portfolios seem to be very high. One of the explanations is that portfolio groups include small numbers of stocks. The second explanation could be the economic crisis experienced in 2001 in Turkey. This crisis created very high volatility in the financial markets, and the daily change in stock market index (viz. BIST-100) reached to 30%. When I exclude data covering the years from 2000 to 2003, standard deviations decrease by 35% on average. On the other hand, it should also be noted that global crisis in 2008 and Greece's haircut in 2010–2011 created very high

http://dx.doi.org/10.5772/intechopen.70867

Excess returns (%)					Standard deviation (%)				
Panel A: size/BM portfolios									
	BM1 (low)	**BM2**	**BM3**	**BM4 (high)**		**BM1 (low)**	**BM2**	**BM3**	**BM4 (high)**
S1 (small)	0.40	0.35	0.35	0.66	**S1 (small)**	10.34	11.64	10.57	10.93
S2	−0.03	0.69	0.32	0.67	**S2**	9.69	9.64	10.19	11.42
S3	0.46	0.75	0.28	1.12	**S3**	10.52	10.94	9.71	9.21
S4 (big)	0.43	0.89	0.76	1.09	**S4 (big)**	10.12	11.43	10.55	10.84
Panel B: size/OP portfolios									
	OP1 (robust)	**OP2**	**OP3**	**OP4 (weak)**		**OP1 (robust)**	**OP2**	**OP3**	**OP4 (weak)**
S1 (small)	1.40	0.48	0.89	−0.43	**S1 (small)**	10.66	11.25	10.38	10.45
S2	0.28	0.34	1.24	0.67	**S2**	9.52	9.71	10.53	11.05
S3	1.17	−0.32	0.72	0.49	**S3**	10.50	10.40	9.23	9.52
S4 (big)	1.27	1.10	1.55	−0.01	**S4 (big)**	10.54	11.09	10.42	10.41
Panel C: size/Inv portfolios									
	Inv1 (consv.)	**Inv2**	**Inv3**	**Inv4 (aggr.)**		**Inv1 (consv.)**	**Inv2**	**Inv3**	**Inv4 (aggr.)**
S1 (small)	0.22	0.44	0.42	0.52	**S1 (small)**	10.97	10.67	9.96	10.55
S2	0.63	0.92	0.43	0.46	**S2**	9.33	9.63	10.91	10.70
S3	0.72	0.59	0.80	1.13	**S3**	9.94	10.42	9.34	9.71
S4 (big)	0.73	0.73	0.93	0.93	**S4 (big)**	10.69	10.56	9.98	10.42

Table 3. Average monthly excess returns and standard deviations (for definitions of variables, see Section 4.3).

volatility in many stock markets. In normal circumstances, I would expect the standard deviations to be half of the figures in **Table 3**.

Table 3 shows that big-sized portfolios tend to benefit from high book-to-market ratios. On the other hand, small portfolios tend to benefit from high profitability, while the effect is weak on big portfolios. In Panel A, we see size and book-to-market (B/M hereafter) pairs of portfolios. The highest average monthly excess return portfolios are S3BM4 and S4BM4 with 1.12% and 1.09%, respectively. The Turkish data reveals that big-sized companies (in groups S3 and S4) with high B/M ratios have the highest monthly excess returns. Worst performers are small-sized companies with low B/M ratios.

In Panel B, returns of portfolios sorted and grouped according to their size and profitability are given. The highest returns in the table belong to S1OP1, S4OP1 and S4OP3. In the Turkish stock market, the highest monthly returns coincide with the high profitability. As you will see in the remainder of the study, I think the most important factor determining the returns of stocks is profitability. The main message is that, even if a company is grouped in the smallest size, if its profitability is high, its return is expected to be high. But one peculiar result is that S4OP3, which represents a portfolio with low profitability and the biggest size, has a monthly excess return of 1.55%. One explanation could be that even if the biggest-sized companies have low profitability, if it is an aggressive investing company and the expectation of the market participants is positive, then a monthly average return of 1.55% would be justified. As you may see in Panel C of **Table 3**, big companies with aggressive investing have the highest returns.

6. Factor spanning regressions

Factor spanning regressions are a means to test if an explanatory factor can be explained by a combination of other explanatory factors. Spanning tests are performed by regressing returns of one factor against the returns of all other factors and analysing the intercepts from that regression.

Table 4 shows regressions for the 5F-FF model's explanatory variables, where four factors explain returns on the fifth. In the RM-Rf regressions, the intercept is not statistically significant (t = −0.55). Regressions to explain HML, RMW and CMA factors are strongly positive. However, regressions to explain SMB show insignificant intercept, with intercept of −0.27% (t = −1.34). These results suggest that removing either the RM-Rf or SMB factor would not hurt the mean-variance-efficient tangency portfolio produced by combining the remaining four factors.

	Intercept	RM-Rf	SMB	HML	RMW	CMA	R^2
RM-Rf							
Coefficient	−0.45		0.34	−0.41	−0.29	−0.21	0.20
t-Stat	−0.55		1.17	−1.81	−1.33	−1.02	
SMB							
Coefficient	−0.27	0.02		0.00	0.07	0.01	0.12
t-Stat	−1.34	1.17		0.07	1.32	0.29	
HML							
Coefficient	−0.55	−0.04	0.01		0.06	0.10	0.19
t-Stat	**−2.19**	−1.81	0.07		0.82	1.50	
RMW							
Coefficient	−1.06	−0.03	0.12	0.06		0.07	0.17
t-Stat	**−4.18**	−1.33	1.32	0.82		1.01	
CMA							
Coefficient	−1.04	−0.02	0.03	0.11	0.07		0.16
t-Stat	**−3.95**	−1.02	0.29	1.50	1.01		

RM-Rf is the equal-weighted return on BIST-100 Index minus the 3-month Tryribor rate. SMB is the size factor, HML is the value factor, RMW is the profitability factor, and CMA is the investment factor. The factors are constructed using individual sorts of stocks into two size groups and three B/M groups, three OP groups or three Inv groups. Bolded and shaded t-statistics indicate the significance at the 5% level.

Table 4. Factor spanning regressions on five factors: spanning regressions using four factors to explain average returns on the fifth (June 2000–May 2017, 204 months).

7. Hypothesis tests of joint significance of the regressions' alphas and regressions

Having presented the methodology and statistical results, in this part, I present an answer to important question if the estimated models can completely capture expected returns. To obtain a more absolute answer to this question GRS f-tests were conducted on results obtained from the first hypothesis' portfolio regressions. The GRS statistic is used to test if the alpha values from regressions are jointly indistinguishable from zero.

If a model completely captures expected returns, the intercept should be indistinguishable from zero. Hence, the first hypotheses are:

H_0: $\hat{\alpha}1 = \hat{\alpha}2 = \ldots = \hat{\alpha}16 = 0$ (*the regression alpha is not significantly different from zero*).

H_1: $\hat{\alpha}1, \hat{\alpha}2, \ldots, \hat{\alpha}16 \neq 0$ (*the regression alpha is significantly different from zero*).

The second hypotheses for all equations are:

H_0: $\hat{\alpha}1 = \hat{\alpha}2 = \ldots = \hat{\alpha}48 = 0$ (*the regression alphas are jointly indistinguishable from zero*).

H_1: $\hat{\alpha}1, \hat{\alpha}2, \ldots, \hat{\alpha}48 \neq 0$ (*the regression alphas are jointly distinguishable from zero*).

Finally, average individual regression alphas and joint GRS regression f-values were used together in order to compare the performance between the tested models.

7.1. The GRS regression equation

The GRS test was developed by Gibbons et al. [24] and serves as a test of mean-variance efficiency between a left-hand-side collection of assets or portfolios and a right-hand-side model or portfolio. The following regression defines the GRS test:

$$fGRS = \frac{T}{N} \times \frac{T-N-L}{T-L-1} \times \frac{\widehat{\alpha}' \times \widehat{\Sigma}^{-1} \times \widehat{\alpha}}{1 + \overline{\mu}' \times \widehat{\Omega}^{-1} \times \overline{\mu}} \sim F(N, T-N-L) \tag{15}$$

where $\hat{\alpha}$ is a $N \times 1$ vector of estimated intercepts, $\widehat{\Sigma}$ an unbiased estimate of the residual covariance matrix, $\overline{\mu}$ a $L \times 1$ vector of the factor portfolios' sample means and $\widehat{\Omega}$ an unbiased estimate of the factor portfolios' covariance matrix.

The GRS test is used in this study to determine whether the alpha values from individual model regressions are jointly non-significant and hence to find out if a model completely captures the sample return variation. As intercepts from individual regressions approach zero, the GRS statistic will also approach zero. However, since the GRS statistic derives its results from comparing the optimal LHS and RHS portfolios, the resulting statistic is not strictly comparable between models.

7.2. Model performance

A set of several summary metrics were deployed in order to compare the performance of the asset pricing models. GRS statistics and average alpha values were used as the main two statistics in order to determine how good the different asset pricing models performed in explaining portfolio returns. In addition to these statistics, average absolute alpha spread was added for a more complete picture of the alpha results. Furthermore, different models' explanatory power was measured using R^2 values.

If a capital asset pricing model (CAPM, three-factor model or five-factor model) completely captures expected returns, the intercept (alphas) is indistinguishable from zero in a regression of an asset's excess returns on the model's factor returns.

Table 5 shows the GRS statistics of [24] that tests this hypothesis for combinations of LHS portfolios and factors. The GRS test easily rejects all models considered for all LHS portfolios and RHS factors. The probability, or p-value, of getting a GRS statistic larger than the one observed if the true intercepts are all zero is shown in column 'pGRS'. One can see from **Table 5** that except CAPM in Panel A and Panel C, sets of left-hand-side returns, the p-values round to zero to at least three decimals. Only five-factor model in Panel C has a p-value of 0.30, and it is still significant at the 5% level.

	fGRS	**pGRS**	**Avg $\lvert\alpha\rvert$**	**Avg $\lvert\alpha - \overline{\alpha}\rvert$**	**Avg R^2**
Panel A: size-B/M portfolios					
CAPM	0.95	2.74	0.57	0.25	0.06
Three-factor (S/BM)	**2.13**	0.00	1.32	0.31	0.34
Five-factor (S/BM)	**2.47**	0.00	0.73	0.72	0.49
Panel B: size-OP portfolios					
CAPM	**4.23**	0.00	0.77	0.49	0.07
Three-factor (S/OP)	**5.63**	0.00	1.41	0.44	0.33
Five-factor (S/OP)	**6.15**	0.00	0.91	0.87	0.50
Panel C: size-Inv portfolios					
CAPM	0.94	2.66	0.66	0.18	0.06
Three-factor (S/Inv)	**2.93**	0.00	1.38	0.19	0.33
Five-factor (S/Inv)	1.33	0.30	0.73	0.74	0.50
Panel D: all portfolios					
CAPM (all)	**3.20**	0.00	0.67	0.30	0.06
Three-factor (all)	**13.37**	0.00	1.15	0.57	0.39
Five-factor (all)	**2.66**	0.00	0.79	0.78	0.49

The table tests the ability of CAPM, 3F-FF and 5F-FF models to explain monthly excess returns on 16 size-B/M portfolios (Panel A), 16 size-OP portfolios (Panel B), 16 size-Inv portfolios (Panel C) and a joint sample of all 48 portfolios (Panel D). For each panel, the table shows the tested model; the GRS statistic testing whether the expected values of all 16 or 48 intercept estimates are zero; the p-value for the GRS statistic; the average absolute value of the intercepts, Avg $\lvert\alpha\rvert$; the average absolute deviation from the average intercept; and the average R^2. Bolded and shaded GRS statistics indicate the significance at the 5% level.

Table 5. Summary of statistics for model comparison tests: summary of statistics for tests of CAPM, 3F-FF and 5F-FF models (June 2000–May 2017, 204 months).

Fama and French [19] state that they want to identify the model that is the best (but imperfect) story for average returns on portfolios formed in different ways, and they use absolute value of average alphas ($|\alpha|$) and two more statistics (for a detailed description of these statistics, see ([21], pp. 10). Following [22], in this study I use only average absolute values of alphas ($|\alpha|$) and the average absolute deviation from the average intercept (Avg $|\alpha - \overline{\alpha}|$). Both [19, 22] pay specific importance of the absolute values of alphas. Relatively, small values of Avg$|\alpha|$ or Avg $|\alpha - \overline{\alpha}|$ in equations are regarded as better in identifying the best model. However, as is seen in **Table 5**, Avg$|\alpha|$ of CAPM model in Panel A has the lowest value, but fGRS and pGRS show that we cannot reject the hypothesis that the regression alpha is significantly different from zero. The same case is valid for the CAPM in Panel C in **Table 5**. Therefore, I think concentrating on the magnitude of the alpha values or absolute deviation from the average intercept (Avg $|\alpha - \overline{\alpha}|$) in the equations to decide on the best performing model may not be a healthy approach. Keeping this in mind, my main conclusion from the model comparison tests follows.

The GRS test in Panel D clearly rejects the second hypothesis for all models tested. Furthermore, the GRS test in Panel D suggests that the 5F-FF model is the model closest to a complete description of asset returns. Beware that, as discussed in [19], fGRS values between models cannot be strictly compared. Instead, the fGRS is mainly interpreted as a test to reject or accept a model's explanation of returns on a set of left-hand-side portfolios. The fGRS in [21, 22] is used in comparisons between models only in combination with a comparison of average alpha values. Fama and French [19] instead use the numerator of a GRS regression as a comparison value when choosing which factors to include in a model. In every panel of **Table 5**, explanatory power measured by average R^2 clearly improves with the inclusion of more factors. For this reason, metrics for explanatory power are important in providing additional help to compare these types of asset pricing models.

The main conclusions from the performance comparison tests can be summarised as follows. A GRS test on the joint set of all tested portfolios clearly rejects all tested models as complete descriptions of average returns. The CAPM model elicits the lowest average absolute alpha values of the three tested models throughout all tests but shows a statistically insignificant fGRS value compared to other models. Considering all the evidence in **Table 5**, it is clear that the 5F-FF model shows the strongest performance out of the three models for the sample. In the following section, I provide alpha values from individual left-hand-side portfolio regressions, and I re-examine the alpha values from a different perspective.

8. Regression details

In this part, I give individual regression alphas, the coefficients that were defined in Eqs. (5)–(7), their corresponding t-values and R-squared values in order to provide a more detailed picture of model performance. I concentrate on the significance of alphas. Significant alpha patterns between models are compared and further analysed by looking at regression slopes in **Tables 6–8**.

Dep. variables		**Five-factor size-B/M models**							**Three-factor size-B/M models**					**CAPM**		
		α	**Rm-Rf**	**SMB**	**HML**	**RMW**	**CMA**	R^2	α	**Rm-Rf**	**SMB**	**HML**	R^2	α	**Rm-Rf**	R^2
S1BM1	Coeff.	1.26	−0.07	1.73	0.39	−0.24	0.15	0.45	1.18	−0.06	1.20	0.58	0.44	0.42	−0.06	0.06
	t-Stat	1.69	−1.15	**6.58**	1.88	−1.18	0.79		1.74	−0.88	**6.60**	**2.79**		0.58	−0.84	
S1BM2	Coeff.	1.42	−0.06	2.13	0.78	−0.33	0.06	0.51	1.50	−0.03	1.46	0.99	0.51	0.36	−0.05	0.04
	t-Stat	1.76	−0.85	**7.50**	**3.48**	−1.50	0.28		**2.05**	−0.46	**7.46**	**4.43**		0.45	−0.61	
S1BM3	Coeff.	1.46	−0.05	1.45	0.99	−0.30	0.15	0.47	1.51	−0.04	1.05	1.15	0.48	0.37	−0.07	0.07
	t-Stat	1.94	−0.83	**5.47**	**4.72**	−1.50	0.77		**2.23**	−0.58	**5.81**	**5.54**		0.50	−0.94	
S1BM4	Coeff.	1.76	−0.04	1.55	0.86	−0.31	0.20	0.45	1.75	−0.03	1.13	1.04	0.46	0.68	−0.05	0.05
	t-Stat	**2.23**	−0.61	**5.57**	**3.91**	−1.46	0.97		**2.47**	−0.37	**5.95**	**4.77**		0.88	−0.66	
S2BM1	Coeff.	0.37	−0.07	1.51	0.19	−0.33	0.07	0.41	0.55	−0.05	1.12	0.35	0.43	−0.02	−0.05	0.05
	t-Stat	0.52	−1.20	**6.00**	0.97	−1.74	0.37		0.86	−0.89	**6.55**	1.80		−0.02	−0.73	
S2BM2	Coeff.	1.40	−0.07	1.54	0.48	−0.19	−0.01	0.44	1.51	−0.05	1.18	0.65	0.48	0.71	−0.06	0.06
	t-STAT	**2.00**	−1.14	**6.24**	**2.45**	−0.99	−0.04		**2.43**	−0.88	**7.13**	**3.41**		1.04	−0.88	
S2BM3	Coeff.	0.35	−0.04	1.57	0.73	−0.65	−0.25	0.48	1.02	0.01	0.51	0.75	0.28	0.33	−0.02	0.02
	t-Stat	0.49	−0.64	**6.19**	**3.65**	**−3.38**	−1.32		1.43	0.11	**2.68**	**3.41**		0.46	−0.23	
S2BM4	Coeff.	0.41	−0.03	1.98	1.00	−1.09	−0.33	0.58	1.59	0.04	0.65	1.00	0.33	0.67	0.01	0.00
	t-Stat	0.55	−0.48	**7.51**	**4.81**	**−5.44**	−1.71		**2.02**	0.50	**3.10**	**4.15**		0.83	0.07	
S3BM1	Coeff.	−0.14	0.00	1.43	0.84	−1.19	−0.23	0.54	1.21	0.06	0.54	0.82	0.30	0.44	0.03	0.03
	t-Stat	−0.19	0.00	**5.67**	**4.24**	**−6.18**	−1.26		1.64	0.86	**2.75**	**3.67**		0.60	0.47	
S3BM2	Coeff.	0.24	0.02	1.44	0.84	−1.07	−0.27	0.50	1.48	0.08	0.52	0.82	0.28	0.73	0.05	0.05
	t-Stat	0.32	0.29	**5.37**	**3.97**	**−5.24**	−1.37		1.94	1.06	**2.55**	**3.52**		0.95	0.68	
S3BM3	Coeff.	0.22	0.02	1.32	0.73	−0.51	−0.37	0.45	0.86	0.07	0.29	0.70	0.25	0.26	0.04	0.04
	t-Stat	0.32	0.34	**5.37**	**3.73**	**−2.71**	**−2.02**		1.26	1.01	1.61	**3.35**		0.38	0.60	
S3BM4	Coeff.	1.20	−0.01	1.48	0.88	−0.76	−0.16	0.55	1.95	0.04	0.59	0.91	0.37	1.11	0.01	0.01

Dep. variables		Five-factor size-B/M models							Three-factor size-B/M models					CAPM		
		α	Rm-Rf	SMB	HML	RMW	CMA	R^2	α	Rm-Rf	SMB	HML	R^2	α	Rm-Rf	R^2
	t-Stat	1.93	−0.10	**6.76**	**5.11**	**−4.57**	−0.98		**3.12**	0.68	**3.52**	**4.76**		1.72	0.18	
S4BM1	Coeff.	−0.46	0.05	1.14	0.57	−0.74	−0.65	0.45	0.75	0.11	0.03	0.45	0.17	0.40	0.09	0.09
	t-Stat	−0.64	0.78	**4.46**	**2.79**	**−3.80**	**−3.46**		1.03	1.57	0.15	**2.03**		0.57	1.30	
S4BM2	Coeff.	−0.39	0.08	1.40	0.91	−0.99	−1.00	0.56	1.46	0.16	0.15	0.76	0.24	0.85	0.13	0.11
	t-Stat	−0.51	1.18	**5.20**	**4.28**	**−4.82**	**−5.02**		1.80	**2.05**	0.71	**3.07**		1.06	1.64	
S4BM3	Coeff.	−0.51	0.10	1.06	0.73	−0.66	−1.10	0.53	1.19	0.17	0.18	0.59	0.23	0.70	0.14	0.14
	t-Stat	−0.70	1.63	**4.18**	**3.63**	**−3.38**	**−5.86**		1.59	**2.36**	0.91	**2.58**		0.96	**2.04**	
S4BM4	Coeff.	−0.15	0.03	1.08	0.79	−0.80	−1.01	0.51	1.58	0.10	0.13	0.65	0.20	1.07	0.07	0.07
	t-Stat	−0.20	0.46	**4.04**	**3.78**	**−3.93**	**−5.15**		**2.04**	1.33	0.61	**2.74**		1.40	0.97	

At the end of May each year, stocks are distributed into four size groups using sample quartile breakpoints. Stocks are independently allocated to four B/M groups, using sample quartile breakpoints. The RHS variables are RM-Rf for the CAPM; RM-Rf, SMB and HML for the 3F-FF model; and RM-Rf, SMB, HML, RMW and CMA for the 5F-FF model. The intersections of the two sorts produce 16 size-B/M portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 size-B/M portfolios. The factors are constructed using independent 2 × 3 sorts on size and each of the B/M, OP and Inv portfolios. Shaded t-statistics indicate the significance of coefficients at the 5% level of significance. (For the definitions of dependent variables, see **Table 1**).

Table 6. CAPM, 3F-FF and 5F-FF regressions for 16 value-weighted size-B/M portfolios (June 2000–May 2017, 204 months).

Dep. variables		Five-factor size-OP models							Three-factor size-OP models					CAPM		
		α	Rm-Rf	SMB	HML	RMW	CMA	R^2	α	Rm-Rf	SMB	HML	R^2	α	Rm-Rf	R^2
S1OP1	Coeff.	2.85	−0.06	1.83	0.79	0.03	0.09	0.50	2.51	−0.04	1.26	1.00	0.50	1.43	−0.06	0.06
	t-Stat	**3.83**	−0.89	**6.99**	**3.82**	0.13	0.49		**3.72**	−0.69	**6.98**	**4.84**		1.91	−0.89	
S1OP2	Coeff.	1.81	−0.11	1.86	1.04	−0.41	0.27	0.53	1.80	−0.09	1.30	1.24	0.53	0.52	−0.12	0.11
	t-Stat	**2.36**	−1.64	**6.86**	**4.85**	−1.98	1.34		**2.58**	−1.33	**6.98**	**5.81**		0.66	−1.55	
S1OP3	Coeff.	1.81	−0.04	1.21	0.70	−0.39	0.33	0.40	1.76	−0.03	0.81	0.84	0.36	0.91	−0.05	0.05
	t-Stat	**2.36**	−0.66	**4.47**	**3.28**	−1.87	1.64		**2.49**	−0.45	**4.28**	**3.89**		1.25	−0.73	
S1OP4	Coeff.	0.38	−0.03	1.69	0.74	−0.40	0.09	0.47	0.55	−0.01	1.13	0.89	0.45	−0.42	−0.03	0.03
	t-Stat	0.51	−0.53	**6.46**	**3.56**	−1.99	0.45		0.81	−0.14	**6.19**	**4.29**		−0.57	−0.38	
S2OP1	Coeff.	1.13	−0.08	1.42	0.55	−0.09	0.00	0.42	1.12	−0.06	1.08	0.71	0.46	0.30	−0.07	0.08
	t-Stat	1.62	−1.25	**5.80**	**2.82**	−0.49	0.00		1.81	−1.05	**6.53**	**3.74**		0.45	−1.12	
S2OP2	Coeff.	1.33	−0.11	1.42	0.88	−0.20	0.00	0.48	1.44	−0.09	1.07	1.03	0.51	0.38	−0.11	0.12
	t-Stat	1.93	−1.79	**5.88**	**4.60**	−1.07	−0.01		**2.35**	−1.54	**6.54**	**5.51**		0.56	−1.74	
S2OP3	Coeff.	1.09	−0.03	1.59	0.80	−1.09	0.01	0.54	1.96	0.02	0.45	0.80	0.27	1.24	0.00	0.00
	t-Stat	1.52	−0.49	**6.29**	**3.99**	**−5.66**	0.03		**2.65**	0.35	**2.28**	**3.55**		1.67	−0.03	
S2OP4	Coeff.	0.08	−0.02	1.76	0.73	−1.24	−0.23	0.56	1.38	0.04	0.64	0.73	0.28	0.67	0.02	0.02
	t-Stat	0.10	−0.36	**6.76**	**3.57**	**−6.21**	−1.18		1.79	0.58	**3.10**	**3.09**		0.86	0.30	
S3OP1	Coeff.	0.53	−0.03	1.06	0.84	−0.93	−0.43	0.46	1.88	0.02	0.42	0.79	0.27	1.18	−0.01	0.01
	t-Stat	0.71	−0.53	**4.00**	**4.00**	**−4.60**	**−2.17**		**2.55**	0.25	**2.12**	**3.51**		1.59	−0.13	
S3OP2	Coeff.	−0.74	−0.01	1.44	0.93	−0.94	−0.40	0.53	0.43	0.05	0.36	0.89	0.29	−0.33	0.02	0.02
	t-Stat	−1.03	−0.18	**5.74**	**4.70**	**−4.89**	**−2.16**		0.59	0.70	1.85	**3.98**		−0.45	0.23	
S3OP3	Coeff.	0.68	0.03	1.32	0.62	−0.80	0.00	0.48	1.30	0.08	0.46	0.64	0.28	0.70	0.06	0.06
	t-Stat	1.04	0.60	**5.76**	**3.40**	**−4.56**	0.01		**2.01**	1.23	**2.68**	**3.23**		1.08	0.89	
S3OP4	Coeff.	0.07	0.00	1.39	0.82	−1.09	−0.16	0.56	1.26	0.06	0.63	0.83	0.35	0.48	0.03	0.04

Dep. variables		Five-factor size-OP models							Three-factor size-OP models					CAPM		
		α	Rm-Rf	SMB	HML	RMW	CMA	R^2	α	Rm-Rf	SMB	HML	R^2	α	Rm-Rf	R^2
	t-Stat	0.11	0.07	**6.22**	**4.67**	**−6.39**	−0.99		1.94	0.93	**3.59**	**4.17**		0.71	0.50	
S4OP1	Coeff.	−0.07	0.06	1.11	0.68	−0.80	−1.01	0.52	1.60	0.13	−0.04	0.51	0.19	1.23	0.10	0.10
	t-Stat	−0.09	0.88	**4.32**	**3.33**	**−4.09**	**−5.34**		**2.12**	1.76	−0.19	**2.19**		1.67	1.44	
S4OP2	Coeff.	−0.29	0.14	1.30	0.69	−0.72	−1.16	0.56	1.42	0.22	0.02	0.52	0.23	1.03	0.20	0.18
	t-Stat	−0.38	**2.21**	**4.94**	**3.33**	**−3.63**	**−6.02**		1.81	**2.92**	0.10	**2.15**		1.34	**2.63**	
S4OP3	Coeff.	0.40	0.07	0.88	0.54	−0.68	−0.83	0.43	1.85	0.12	0.11	0.42	0.17	1.51	0.11	0.11
	t-Stat	0.52	1.03	**3.29**	**2.57**	**−3.32**	**−4.21**		**2.47**	1.74	0.53	1.83		**2.07**	1.52	
S4OP4	Coeff.	−1.35	0.01	1.03	0.58	−0.86	−0.88	0.48	0.33	0.08	0.11	0.45	0.15	−0.03	0.06	0.06
	t-Stat	−1.83	0.15	**3.98**	**2.86**	**−4.38**	**−4.61**		0.45	1.06	0.56	1.94		−0.04	0.81	

At the end of May each year, stocks are distributed into four size groups using sample quartile breakpoints. Stocks are independently allocated to four OP groups, using sample quartile breakpoints. The RHS variables are RM-Rf for the CAPM; RM-Rf, SMB and HML for the 3F-FF model; and RM-Rf, SMB, HML, RMW and CMA for the 5F-FF model. The intersections of the two sorts produce 16 size-OP portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 size-OP portfolios. The factors are constructed using independent 2 × 3 sorts on size and each of B/M, OP and Inv portfolios. Shaded t-statistics indicate the significance of coefficients at the 5% level of significance (for the definitions of dependent variables, see **Table 1**).

Table 7. CAPM, 3F-FF and 5F-FF regressions for 16 value-weighted size-OP portfolios (June 2000–May 2017, 204 months).

Dep. variables		Five-factor size-Inv models							Three-factor size-Inv models					CAPM		
		α	Rm-Rf	SMB	HML	RMW	CMA	R^2	α	Rm-Rf	SMB	HML	R^2	α	Rm-Rf	R^2
S1INV1	Coeff.	1.28	−0.06	1.79	0.62	−0.43	0.36	0.47	1.20	−0.04	1.27	0.83	0.46	0.24	−0.05	0.05
	t-Stat	1.64	−0.83	**6.50**	**2.84**	**−2.07**	1.77		1.69	−0.55	**6.67**	**3.80**		0.31	−0.66	
S1INV2	Coeff.	1.31	−0.05	1.76	0.76	−0.53	0.22	0.49	1.46	−0.02	1.13	0.93	0.45	0.46	−0.04	0.04
	t-Stat	1.75	−0.73	**6.65**	**3.67**	**−2.63**	1.14		**2.10**	−0.31	**6.06**	**4.37**		0.61	−0.55	
S1INV3	Coeff.	1.70	−0.08	1.45	0.78	−0.08	0.18	0.46	1.46	−0.07	1.07	0.97	0.48	0.45	−0.09	0.10
	t-Stat	**2.38**	−1.33	**5.78**	**3.96**	−0.43	0.95		**2.29**	−1.20	**6.27**	**4.94**		0.65	−1.41	
S1INV4	Coeff.	1.42	−0.10	1.47	0.89	−0.19	−0.10	0.45	1.59	−0.08	0.99	1.02	0.44	0.56	−0.10	0.10
	t-Stat	1.88	−1.49	**5.49**	**4.20**	−0.92	−0.49		**2.30**	−1.18	**5.36**	**4.82**		0.76	−1.44	
S2INV1	Coeff.	1.20	−0.08	1.39	0.67	−0.47	0.09	0.46	1.53	−0.05	1.02	0.81	0.46	0.65	−0.07	0.08
	t-Stat	1.80	−1.31	**5.91**	**3.63**	**−2.63**	0.49		**2.54**	−0.92	**6.32**	**4.39**		1.00	−1.09	
S2INV2	Coeff.	1.62	−0.06	1.56	0.57	−0.47	0.21	0.47	1.79	−0.04	1.13	0.73	0.47	0.93	−0.05	0.05
	t-Stat	**2.36**	−1.01	**6.46**	**2.97**	**−2.58**	1.19		**2.87**	−0.65	**6.77**	**3.85**		1.38	−0.75	
S2INV3	Coeff.	0.01	−0.02	1.64	0.75	−0.88	−0.42	0.50	1.17	0.04	0.67	0.75	0.30	0.42	0.02	0.02
	t-Stat	0.01	−0.25	**6.12**	**3.56**	**−4.29**	**−2.11**		1.53	0.57	**3.29**	**3.24**		0.55	0.28	
S2INV4	Coeff.	−0.19	−0.05	1.56	0.80	−1.02	−0.47	0.53	1.22	0.01	0.63	0.78	0.30	0.46	−0.01	0.01
	t-Stat	−0.26	−0.78	**6.04**	**3.95**	**−5.19**	**−2.49**		1.63	0.19	**3.18**	**3.43**		0.61	−0.13	
S3INV1	Coeff.	0.37	−0.02	1.42	0.78	−1.14	−0.04	0.55	1.40	0.04	0.44	0.77	0.28	0.71	0.01	0.01
	t-Stat	0.55	−0.29	**6.02**	**4.16**	**−6.34**	−0.21		**2.02**	0.57	**2.36**	**3.61**		1.02	0.18	
S3INV2	Coeff.	0.14	−0.01	1.26	0.77	−1.05	−0.17	0.48	1.22	0.04	0.31	0.74	0.24	0.59	0.02	0.02
	t-Stat	0.19	−0.16	**4.85**	**3.78**	**−5.29**	−0.88		1.65	0.63	1.55	**3.27**		0.80	0.24	
S3INV3	Coeff.	0.19	0.01	1.31	0.85	−0.94	−0.46	0.56	1.52	0.07	0.49	0.82	0.32	0.78	0.04	0.05
	t-Stat	0.30	0.25	**5.97**	**4.90**	**−5.60**	**−2.85**		**2.36**	1.15	**2.83**	**4.12**		1.19	0.69	
S3INV4	Coeff.	0.67	−0.02	1.50	0.66	−0.77	−0.45	0.51	1.76	0.03	0.58	0.64	0.28	1.13	0.02	0.02

Dep. variables		Five-factor size-Inv models							Three-factor size-Inv models					CAPM		
		α	Rm-Rf	SMB	HML	RMW	CMA	R^2	α	Rm-Rf	SMB	HML	R^2	α	Rm-Rf	R^2
	t-Stat	1.00	−0.35	**6.35**	**3.50**	**−4.29**	**−2.59**		**2.60**	0.53	**3.18**	**3.10**		1.65	0.25	
S4INV1	Coeff.	−0.31	0.08	1.22	0.66	−0.72	−0.87	0.49	1.12	0.15	0.12	0.53	0.20	0.69	0.13	0.12
	t-Stat	−0.41	1.27	**4.59**	**3.14**	**−3.55**	**−4.42**		1.46	**2.03**	0.56	**2.28**		0.92	1.74	
S4INV2	Coeff.	−0.41	0.09	1.12	0.64	−0.73	−0.89	0.49	1.07	0.16	0.06	0.50	0.20	0.68	0.14	0.13
	t-Stat	−0.55	1.44	**4.27**	**3.09**	**−3.65**	**−4.60**		1.42	**2.20**	0.29	**2.18**		0.92	1.91	
S4INV3	Coeff.	−0.49	0.02	1.00	0.67	−0.78	−1.07	0.54	1.32	0.09	0.12	0.52	0.18	0.90	0.07	0.07
	t-Stat	−0.72	0.35	**4.16**	**3.54**	**−4.26**	**−6.08**		1.84	1.29	0.63	**2.35**		1.29	0.99	
S4INV4	Coeff.	−0.38	0.11	0.85	0.59	−0.69	−0.97	0.48	1.20	0.17	−0.03	0.44	0.20	0.88	0.15	0.15
	t-Stat	−0.52	1.65	**3.26**	**2.89**	**−3.45**	**−5.02**		1.61	**2.37**	−0.14	1.92		1.22	**2.10**	

At the end of May each year, stocks are distributed into four size groups using sample quartile breakpoints. Stocks are independently allocated to four Inv groups, using sample quartile breakpoints. The RHS variables are RM-Rf for the CAPM; RM-Rf, SMB and HML for the 3F-FF model; and RM-Rf, SMB, HML, RMW and CMA for the 5F-FF model. The intersections of the two sorts produce 16 size-Inv portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 size-Inv portfolios. The factors are constructed using independent 2 × 3 sorts on size and each of the B/M, OP and Inv portfolios. Shaded t-statistics indicate the significance of coefficients at the 5% level of significance (for the definitions of dependent variables, see **Table 1**).

Table 8. CAPM, 3F-FF and 5F-FF regressions for 16 value-weighted size-Inv portfolios (June 2000–May 2017, 204 months).

Looking explicitly at the number of significant alpha values in **Tables 6–8** (at the 5% level of confidence), both the CAPM and the 3F-FF model perform poorly, while 5F-FF model's performance although is not the best but much better. The highest number of significant alpha values belongs to 3F-FF model. This shows that 3F-FF model leaves a high proportion of unexplained part in the behaviour of LHS variables. From the regression tables, it seems that CAPM model has no any significant values for alphas; interestingly enough, none of the coefficients of RM-Rf is significant at the 5% level of confidence, and R^2 terms are too low in CAPMs. In addition to this observation, in none of the models, RM-Rf has no statistically significant coefficient except in a few models (see rows 13, 14 and 15 in **Tables 6–8**).

On the other hand, inspection of alphas in 5F-FF models for size-B/M, size-OP and size-Inv portfolios reveals that very few of alpha values are significant at the 5% level confidence and I think the best performing model (but imperfect) for the Turkish data is the 5F-FF Fama-French model.

There is an interesting pattern in the results of the regression tables (**Tables 6–8**). As the size of the companies in portfolios increases, more factors become statistically significant, and the explanatory power of the 5F-FF model rises. This makes me to think that monthly excess returns of portfolios, covering high-market-cap companies, are more sensitive to SMB, HML, RMA and CMA factors.

Before I present the regression details, readers should be aware that in an OLS regression, R-squared values will always increase with the inclusion of more factors. Another point is that R^2 increases and become much stronger for almost any explanatory power metric when the regressions' explanatory factors are created from return differences in the data itself. In other words, correlation between explanatory variables, which are created from return differences, and dependent variables has to be highly correlated. Hence, R^2s in the equations become stronger and true.

Sundqvist in [22] states that:

> *'The method to construct the factor will hence automatically bring the augmented model's explanatory power in small portfolio regressions closer to the R-squared values found in big portfolio regressions, boosting the average explanatory power more than it would for regressions of randomly picked portfolio sets. This same phenomenon is apparent in all other risk factors, which are included as sorting variables and simultaneously used as explanatory variables created from return differences in the sample data'.*

When the t-values in **Table 6** are analysed, CAPM's results (column labelled with CAPM) are far away from being reliable and informative. None of the t-statistic of RM-Rf (except for S4BM3) is significant at the 5% level of confidence. This might have been due to the fact that BIST-100 Index, which approximates the monthly excess return of the market, is heavily financial stock weighted. However, using different market indices in all models did not improve the results (more information on this issue is given in the discussion of the results in **Table 7**).

In this study, I include only nonfinancial sector stocks registered with the Istanbul stock exchange since the profitability and investment variables' definition is not comparable to the financial stocks. Therefore, I think the coefficient of RM-Rf variable, namely, β_i does not reflect the market risk properly, and CAPM performs poorly.

The 3F-FF regressions find larger *intercept t-values* on most portfolios compared to the 5F-FF regressions, of which three are more than five standard errors from zero. I can find no exceptional characteristics that might shed light into the reasons behind the significant alpha value.

The 5F-FF regressions show high SMB slopes for the portfolio when compared to 3F-FF regressions' slopes. All of the coefficients of SMB variable are statistically significant at the 5% level of confidence in 5F-FF model, while the same metrics have 11 significant coefficients in 3F-FF models. It is somewhat unusual that none of the coefficients of RM-Rf in 5F-FF model regressions are significant at the 5% level of significance. However, as I already discussed above, this may be due to the fact that BIST-100 Index is heavily financially sector stock weighted.

From **Table 6** it is easily seen that (last four rows in the table) excess monthly returns of the biggest-sized company portfolios are best explained by the 5F-FF model. However, as can be seen from the t-statistics, when the company size gets smaller (portfolios starting with the letter S1 and S2), RMW and CMA variables have no effect on excess monthly returns of small-sized portfolios, while SMB and HML variables have significant t-statistics. Assuming that the 5F-FF model is the true definition of the monthly excess returns of portfolios sorted by size and B/M, this result may imply that, when constructing portfolios, the market does not take into account the profitability or investing factors for relatively small-sized companies.

Fama and French [19] note however that one should not expect univariate characteristics and multivariate regression slopes connected to the characteristic to line up. The slopes estimate marginal effects holding constantly all other explanatory variables, and the characteristics are measured with lags relative to returns when pricing should be forward-looking.

The dependent variables are the monthly excess returns of the portfolios sorted by size and profitability (for the definitions of the variables, see **Table 1**). The results in **Table 7** are not much different from the ones that are in **Table 6**. The best performing model is the 5F-FF model. The CAPM has no power to explain the cross-sectional variance of expected returns for the size-B/M, size-OP and size-Inv portfolios I examine. Peculiarly enough, as in the case of **Table 6**, none of the alphas in CAPM are significant at the 5% level of significance. This result could have been due to market return data (RM, represented by BIST-100 Index), which is composed of mainly financial sector companies' price data. To save space not presented in this study, I inform the reader that all of the models (CAPM, three-factor Fama-French model and five-factor Fama-French model) using BIST-TUM Index, which covers all stocks registered with the Turkish stock market, instead of BIST-100 Index, the power of the models in explaining the sorted portfolio monthly excess returns, did not improve. This shows that, at least in the Turkish case, market risk is not a preliminary and strong factor in determining monthly excess returns of portfolios.

When one concentrates on alpha coefficients of the 5F-FF model, it is seen that only 3 of the 16 alphas are statistically significant. But in the case of the 3F-FF model, 9 of the 16 alpha coefficients are significant at the 5% level, showing that three-factor model leaves a high percentage of unexplained part of cross-sectional variance of expected returns.

One of the main messages of the results of the 5F-FF model is that as the size of the companies under investigation increases, the explanatory power of the model rises. It seems from the results of both **Tables 6** and **7** that the best explanatory factor for the monthly excess returns of portfolios is the size factor.

As the size of the companies in portfolios that have been sorted by size and profitability gets smaller, RMW and CMA become ineffective in determining the monthly excess returns of these portfolios (see the coefficients of RMW and CMA in the first six rows of **Table 7**). However, bigger size portfolios have significant RMW and CMA coefficients.

Table 8 gives the results of CAPM, 3F-FF model and 5F-FF model for portfolios that are sorted by size and investment. The CAPM has no power in explaining the returns of portfolios, while the coefficients of the factors in 3F-FF model seem powerful, 8 of the 16 alpha values are significant at the 5% level. This result indicates that the CAPM and 3F-FF model are not the true definition of a model to explain the variations in monthly excess returns of portfolios. However, as in the results of the previous tables (**Tables 6** and **7**), the 5F-FF model in general has statistically significant coefficients for the Fama-French factors, namely, SMB, HML, RMW and CMA. Also, only 2 of the 16 coefficients of intercepts (alphas) are significant at the 5% level of significance.

Variations of the monthly returns of portfolios constructed by big-sized companies are best explained by the 5F-FF model. However, CMA factor turns out to be insignificant when estimating the model for portfolios with relatively small-sized companies.

9. Conclusion

This study adds to the asset pricing literature using the Turkish data. One of the main findings is that CAPM and 3F-FF model cannot explain cross-sectional variations in portfolio returns properly. The best suited model (but not perfect) for the Turkish case is 5F-FF model.

As seen elsewhere [17, 19–23], as the number of explanatory variables increases in the regression portfolios, explanatory power of the equation increases, and the R^2 rises. In the Turkish case, although 3F-FF model has high R^2 compared to the CAPM, more than half of the intercepts of the 3F-FF model are significant at the 5% level, and this shows that SMB and HML factors alone do not explain the cross-sectional variations of portfolio returns.

Besides testing all intercepts individually, we also tested whether all the pricing errors were jointly equal to zero. Gibbons et al. [24] suggested GRS test statistic to test whether all pricing errors are zero. A GRS test on the joint set of all tested portfolios clearly rejects all tested models as complete descriptions of average returns. The CAPM model elicits the lowest average absolute alpha values of the three tested models throughout all tests but shows a statistically insignificant fGRS value compared to other models. The 5F-FF model shows the strongest performance out of the three models for the sample.

When I compare the R^2s of all my models (the highest 56%) to the earlier studies done for other countries [15, 16, 19–22] (the R^2s change between 70 and 90%), in general, it is the case that R^2s found in my study are much lower than other R^2s found in other studies. This might be due to the fact that RMW and CMA factors add little in explaining the cross-sectional variations of portfolio returns in Turkey. It is clear that, in the Turkish case, both SMB and HML factors have significant effect on monthly excess returns of portfolios, but further research may be carried on by employing new factors other than RMW and CMA.

Readers should be aware that correlation between explanatory variables, which are created from return differences (as in the case of definition of Fama-French factors), and dependent variables has to be highly correlated. Hence, R^2s in the equations become stronger and true.

Consequently, our empirical results are reasonably consistent with [19–22] findings, and the 5F-FF model was found viable and superior to CAPM and 3F-FF model for the Turkish stock market.

Author details

Yaşar Erdinç

Address all correspondence to: yerdinc65@gmail.com

Bosphorus University, Istanbul, Turkey

References

[1] Markowitz H. Portfolio selection. The Journal of Finance. 1952;**7**(1):77-99

[2] Sharpe WF. Capital asset prices: A theory of market equilibrium under conditions of risk. The Journal of Finance. 1964;**19**(3):425-442

[3] Ross SA. The arbitrage theory of capital asset pricing. Journal of Economic Theory. 1976;**13**(3):341-360

[4] Lintner J. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Review of Economics and Statistics. 1965;**47**(1):13-37

[5] Mossin J. Equilibrium in a capital asset market. Econometrica. 1966;**34**:68-783

[6] Black F, Jensen MC, Scholes M. The capital asset pricing model: Some empirical tests. In: Black F, Jensen MC, Scholes M, editors. Studies in Theory of Capital Markets. 1st ed. New York, NY: Praeger; 1972. p. 79-121

[7] Basu S. Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis. The Journal of Finance. 1977;**32**(3):663-682

[8] Bhandari LC. Debt/equity ratio and expected common stock returns: Empirical evidence. The Journal of Finance. 1988;**43**(2):507-528

[9] Jegadeesh N. Evidence of predictable behavior of security returns. The Journal of Finance. 1990;**45**(3):881-898

[10] DeBont WFM, Thaler R. Does the stock market overreact? The Journal of Finance. 1985; **40**(3):793-805

[11] Singleton KJ, Hansen LP. Generalized instrumental variables estimation of nonlinear rational expectations models. Econometrica. 1982;**50**:1269-1286

[12] Singleton KJ, Hansen LP. Stochastic consumption, risk aversion, and the temporal behavior of asset returns. Journal of Political Economy. 1983;**91**:249-265

[13] Parker JA, Julliard C. Consumption risk and the cross-section of expected returns. Journal of Political Economy. 2005;**113**:185-222. DOI: 10.1086/426042

[14] Campbell JY, Cochrane JH. Explaining the poor performance of consumption-based asset pricing models. The Journal of Finance. 2000;**55**:2863-2878. DOI: 10.1111/0022-1082.00310

[15] Fama EF, French KR. The cross-section of expected stock returns. The Journal of Finance. 1992;**47**(2):427-465

[16] Fama EF, French KR. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics. 1993;**33**:3-56

[17] Novy-Marx R. The other side of value: The gross profitability premium. Journal of Financial Economics. 2013;**108**(1):1-28

[18] Aharoni G, Grundy B, Zeng Q. Stock returns and the Miller Modigliani valuation formula: Revisiting the Fama French analysis. Journal of Financial Economics. 2013;**110**(2): 347-357

[19] Fama EF, French KR. A 5F-FF asset pricing model. Journal of Financial Economics. 2015;**116**(1):1-22. DOI: 10.1016/j.jfineco.2014.10.010

[20] Fama EF, French KR. Choosing Factors. Working paper; 2016

[21] Fama EF, French KR. International tests of a 5F-FF asset pricing model. Journal of Financial Economics. 2016;**123**(3):441-463. DOI: 10.1111/j.1540-6261.2008.01368

[22] Sundqvist T. Tests of a Fama-French 5F-FF asset pricing model in the Nordic stock markets [thesis]. Helsinki: Hanken School of Economics; 2017. p. 82 Available from: https://helda.helsinki.fi/dhanken/handle/123456789/167972

[23] Lakonishok J, Schleifer A, Vishny RW. Contrarian investment, extrapolation and risk. The Journal of Finance. 1994;**49**:1541-1578

[24] Gibbons MR, Ross SA, Shanken J. A test of the efficiency of a given portfolio. Econometrica. 1989;**57**:1121-1152