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# Acoustic Wave Monitoring of Fluid Dynamics in the Rock Massif with Anomaly Density, Stressed and Plastic Hierarchic Inclusions

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Additional information is available at the end of the chapter

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## Abstract

The geological environment is an open system, on which external and internal factors act. They lead it to an unstable state, which, as a rule, manifests itself locally in the form of zones, called dynamically active elements, which are indicators of potential catastrophic sources. These objects differ from the host geological environment by structural forms, which are often forming of a hierarchical type. The process of their activation can be observed using monitoring with wave fields, for mathematical support of which new modeling algorithms have been developed using the method of integral and integral-differential equations. A new approach to the interpretation of wave fields has been developed, to determine contours or surfaces of locally stressed hierarchical objects. An iterative process of solving the theoretical inverse problem for the case of determining configurations of 2D hierarchical inclusions of the  $k$ -th rank is developed. When interpreting monitoring results, it is necessary to use data from such monitoring systems that are tuned to study the hierarchical structure of the environment.

**Keywords:** hierarchical structures, acoustic wave fields monitoring, anomaly density, stressed, plastic, fluid saturated hierarchic inclusions, equation of theoretical inverse problem

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## 1. Introduction

It is known that the geological environment is an open dynamic system that is influenced by natural and artificial effects at various scale levels that change its state, resulting to a complex multi-ranked hierarchical evolution. That is one of the subjects of geosynergetic study [1], see the bibliography. Using the synergetic approach, it is necessary to clear distinguish the scale of natural phenomena. The paradigm of physical mesomechanics introduced by academician

Panin V. and his school, which includes the synergetic approach, is a constructive tool for studying and changing the state of heterogeneous materials.

Determining the state and its dynamics of the massif is a more complex problem than mapping its structure. Individual parts of the massif can be in a different stress state, and the corresponding deformations can be either elastic or plastic. The medium can be multiphase. A sharp change of the state of the blocks can lead to loss of stability of the whole massif and to a rock shock. The change of the state is determined both by natural and technogenic influence on the massif and manifests itself by the formation of man-made cavities and pumping by mechanical energy during mass explosions provided for mining technologies. The phenomenon of non-stationary state of the rock massif for today is a well-known fact [2, 3]. Manifestations of it can be from insignificant ones in the form of increase in permeability due to increase in fracturing of rocks, which are already registered in the form of shocks, micro-impacts, rock impacts, mountain-tectonic impacts [4]. The latter refer already to catastrophic phenomena, which are initiated by both internal and external technogenic causes. In our studies of the non-stationary geological environment in the framework of natural experiments in real rock massive under strong man-made influence, it was shown that the dynamics of the state can be detected using synergetic effect in hierarchical environments. An important role for the study of the state of dynamic geological systems consists on a combination of active and passive geophysical monitoring, which can be carried out using electromagnetic and seismic fields. The change of the system state on the investigated spatial bases and times is manifested in parameters related to the structural features of the medium of the second and higher ranks. Thus, the study of the state dynamics, its structure and the phenomenon of self-organization of the massif should be studied by geophysical methods tuned to the multi-rank hierarchical nonstationary model of the environment.

In the work [1] the results of the method of studying synergetic manifestations of a geological environment under active external influence were generalized using the method of phase portraits for researching the problem of reflecting the synergetic properties of a geological medium in the data of active electromagnetic and seismic monitoring. The results, obtained from the analysis of the detailed seismological catalog from the point of view of the mathematical foundations of synergetics and open dynamical systems possessing the properties of nonlinearity and dissipativity lead us to the necessity of formulating a new mathematical modeling problem different from the one previously performed [5, 6].

The processes of development of oil and gas fields are associated with the motion of multiphase multicomponent media that are characterized by no equilibrium and non-linear rheological properties. The real behavior of reservoir systems is determined by the complexity of the rheology of moving fluids and the morphological structure of the porous medium, as well as by the variety of processes of interaction between the liquid and the porous medium [7]. Accounting for these factors is necessary for a meaningful description of filtration processes due to nonlinearity, non-equilibrium and in homogeneity inherent in real systems. In this case, new synergetic effects are revealed (loss of stability with the appearance of oscillations, the formation of ordered structures). This allows us to propose new methods for monitoring and managing complex natural systems that are tuned to account for these phenomena. Thus, the reservoir system from which to extract oil is a complex dynamic hierarchical system.

A major result of studies of the last century was the conclusion about the fundamental role of the block-hierarchical structure of rocks and massive for explaining the existence of a wide range of nonlinear geomechanical effects and the emergence of complex self-organizing geosystems in the analysis of the formation of large and super-large deposits. The hierarchical structure is characteristic for many systems, especially for the Earth's lithosphere, where more than 30 hierarchical levels from tectonic plates with a length of thousands of kilometers to individual mineral grains of millimeter size were identified by geophysical studies. Thus, the Earth's crust is a continuous medium that includes a discrete block system and, like any synergetic discrete ensemble, has hierarchical and self-similar properties. This must be taken into account when creating new complex geophysical systems for studying the Earth's lithosphere. Iterative algorithms of 2-D modeling for sound diffraction and a linearly polarized transverse elastic wave on inclusion with a hierarchical elastic structure located in the  $J$ -th layer of the  $N$ -layer elastic medium are constructed.

In the present chapter we consider the case where the inclusion density of each rank differs from the density of the enclosing medium, and the elastic parameters coincide with the elastic parameters of the enclosing layer. We consider also the case when the inclusion density of each rank coincides with the density of the host medium, and the elastic parameters of the inclusion of each rank differ from the elastic parameters of the enclosing layer. We used the method of integral and integral-differential equations for the spatial-frequency representation of the distribution of wave fields. It follows from the constructed theory that when combining acoustic, geomechanical and gravitational fields, it is necessary to use such data that are obtained within the framework of observation systems that are tuned to the study of the hierarchical structure of the medium. The use of density values obtained from the correlation between the values of the longitudinal wave velocity determined from the kinematic interpretation of seismic data and the density to construct a density model for gravity data may lead to a mismatch between this model and the real composition of the geological medium studied. The use of values of elastic parameters without taking into account density anomalies can lead to a discrepancy between the geomechanical model and the acting stresses in the geological environment.

The algorithm developed in [8] is based on Hooke's law [9]. Equations of motion are obtained by equating the elastic forces of the products of masses with accelerations, and the action of the other forces is not assumed. This assumption is completely justified for small deformations and quite often agrees well with the experimental data. However, if vibrations occur in the medium, then part of the elastic energy passes into heat due to internal friction. At the present time, the theory of internal friction in solids is developing [9]. There are several indirect methods of determining internal friction that arise in the samples, which are associated with the assumption that the restoring forces are proportional to the amplitude of the oscillation, and the dissipative forces are proportional to the velocity.

The present work is devoted to the developing an algorithm for the propagation of the seismic field in the acoustic approximation in a layer-block elastic medium with a hierarchical plastic inclusion (the case of taking into account internal friction in viscoelastic inclusion), with anomaly density, anomaly stressed and fluid-saturated hierarchical inclusion.

## 2. Algorithms of mathematical modeling of acoustic wave distribution in block layered hierarchical structure with different physical properties

### 2.1. Modeling of sound diffraction on 2D plastic hierarchic heterogeneity, located in $N$ -layered elastic medium

In the book [9], considering the motion equation:

$$P = M\ddot{\xi} + \eta\dot{\xi} + E\xi \quad (1)$$

for oscillating body it was made an assumption, that the elastic reducing force  $E$  is proportional to the displacement  $\xi$ , and the dissipative force is proportional to the velocity  $\dot{\xi}$ , by that  $E$  in (1) depends from the elastic constants, and  $\eta$  depends from the dissipative forces, the nature of these in [9] did not been discussed. Let us consider the Voigt model [9] for an elastic-viscous environment that in contrary of the Hookers model introduces in the relations for the elastic constants of Lamé such expressions:

$$\lambda + \lambda' \omega_1; \mu + \mu' \omega_2 \quad (2)$$

In the paper [10] it is described the algorithm of sound diffraction modeling on elastic 2D homogeneous inclusion, located in the  $J$ -th layer of the  $N$ -layered medium.

$G_{Sp,j}(M, M^0)$  – a source function of seismic field, the boundary problem of it is formulated in the paper [10],  $k_{1ji}^2 = \omega^2(\sigma_{ji}/\lambda_{ji})$  – wave number for the longitudinal wave, in the cited expressions index  $ji$  identifies the medium properties inside the inclusion,  $ja$  – outside the inclusion,  $\lambda$  – constant of Lamé,  $\sigma$  – medium density,  $\omega$  – cycle frequency,  $\vec{u} = \text{grad}\varphi$  – displacement vector,  $\varphi^0$  – potential of the normal seismic field inside the layered medium without the inclusion:  $\varphi_{ji}^0 = \varphi_{ja}^0$ .

$$\begin{aligned} & \frac{(k_{1ji}^2 - k_{1j}^2)}{2\pi} \iint_{S_c} \varphi(M) G_{Sp,j}(M, M^0) d\tau_M + \frac{\sigma_{ja}}{\sigma_{ji}} \varphi^0(M^0) - \\ & - \frac{(\sigma_{ja} - \sigma_{ji})}{\sigma_{ji} 2\pi} \oint_c G_{Sp,j} \frac{\partial \varphi}{\partial n} dc = \varphi(M^0), M^0 \in S_c \\ & \frac{\sigma_{ji}(k_{1ji}^2 - k_{1j}^2)}{\sigma(M^0) 2\pi} \iint_{S_c} \varphi(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi^0(M^0) - \\ & - \frac{(\sigma_{ja} - \sigma_{ji})}{\sigma(M^0) 2\pi} \oint_c G_{Sp,j} \frac{\partial \varphi}{\partial n} dc = \varphi(M^0), M^0 \notin S_c. \end{aligned} \quad (3)$$

Let us use the expression (2),  $k_{1ji}^2 = \omega^2(\sigma_{ji}/(\lambda_{ji} + \lambda'_{ji}\omega_1))$ , where  $\omega \neq \omega_{1ji}$  and  $\lambda_{ji} \neq \lambda'_{ji}$ , that identifies the influence of the inner friction in the inclusion in the Voigt model. If the inclusion

has a structure of the  $l$ -th rank then according to Ref. [10] and (2) the system (3) can be rewritten as follows:

$$\begin{aligned} & \frac{(k_{1jil}^2 - k_{1ji}^2)}{2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \frac{\sigma_{ja}}{\sigma_{jil}} \varphi_{l-1}^0(M^0) - \\ & - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma_{jil} 2\pi} \oint_{cl} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi_l(M^0), M^0 \in S_{cl} \\ & \frac{\sigma_{jil} (k_{1jil}^2 - k_{1ji}^2)}{\sigma(M^0) 2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) - \\ & - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma(M^0) 2\pi} \oint_{cl} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi_l(M^0), M^0 \notin S_{cl}, \end{aligned} \quad (4)$$

where  $G_{Sp,j}(M, M^0)$  – source function of the seismic field, it coincides with the function of the expressions (3),  $k_{1ji}^2 = \omega^2 \left( \sigma_{ji} / \left( \lambda_{ji} + \lambda'_{ji} \omega_{1ji} \right) \right)$  – wave number for the longitudinal wave,  $ji$  and  $ja$  coincides with expressions from (3),  $l = 1 \dots L$  – number of the hierarchic level,  $\vec{u}_l = \text{grad} \varphi_l$ ,  $\varphi_l^0$  – potential of the normal seismic field in the layered medium by absent of heterogeneity of previous rank, if  $l = 2 \dots L$   $\varphi_l^0 = \varphi_{l-1}$ , if  $l = 1$ ,  $\varphi_l^0 = \varphi^0$ , that coincides with the corresponding case (3).

If by transition on the next hierarchic level the axis of two-dimensionality does not change and only the geometry of the sections of included structures change, then similarly (4) we can develop the iteration process of seismic field modeling (the case of distribution only longitudinal wave). The iteration process relates to modeling of displacement vector by transition from previous (rank) level to a next (rank). Inside each hierarchic rank the integral-differential equation and the integral-differential expression is calculated using the algorithm (4). If on some hierarchical rank the structure of the local heterogeneity divides on some heterogeneities, then the integrals in the expression (4) are calculated over all heterogeneities. In that algorithm we consider the case when the physical properties for one and the same rank are equal, the boundaries can be only different.

It must be noted, that the structure of the integral-differential equations remain the same also for the case of elastic inclusion, but the vector  $\vec{u}_l = \text{grad} \varphi_l$  now depends from two additional parameters for each rank:  $\lambda'_{jil}, \omega_{1jil}$ , that can lead the system to a resonant state.

## 2.2. Modeling of elastic transversal wave diffraction on 2D plastic hierarchic heterogeneity located in the $N$ -layered medium

Similarly to (4) it can be developed the same process for modeling an elastic transversal wave in  $N$ -layered medium with 2D hierarchic structure with arbitrary section morphology with use the integral equations from the paper [10] and (2), where  $\mu_{jil} = \mu_{jil}^e + \mu_{jil}' \omega_{2jil}$ .



$$\begin{aligned}
& \frac{(k_{2jil}^2 - k_{2j}^2)}{2\pi} \iint_{S_{Cl_l}} u_{xl}(M) G_{Ss,j}(M, M^0) d\tau_M + \frac{\mu_{ja}}{\mu_{jil}} u_{x(l-1)}^0(M^0) + \\
& + \frac{(\mu_{ja} - \mu_{jil})}{\mu_{jil} 2\pi} \oint_{Cl} u_{xl}(M) \frac{\partial G_{Ss,j}}{\partial n} dc = u_{xl}(M^0), M^0 \in S_{Cl} \\
& \frac{\mu_{jil}(k_{2jil}^2 - k_{2j}^2)}{\mu(M^0) 2\pi} \iint_{S_{Cl_l}} u_{xl}(M) G_{Ss,j}(M, M^0) d\tau_M + u_{x(l-1)}^0(M^0) + \\
& + \frac{(\mu_{ja} - \mu_{jil})}{\mu(M^0) 2\pi} \oint_{Cl} u_{xl}(M) \frac{\partial G_{Ss,j}}{\partial n} dc = u_{xl}(M^0), M^0 \in S_{Cl}
\end{aligned} \tag{5}$$

where  $G_{Ss,j}(M, M^0)$  – source function of the seismic field for the considering problem, it coincides with the Green function formulated in the paper [10] for the appropriate problem,  $k_{2jil}^2 = \omega^2(\sigma_{jil}/\mu_{jil})$  – wave number of the transversal wave,  $\mu_{jil} = \mu_{jil}^e + \mu_{jil}^i \omega_{2jil}$ ,  $\mu$  – constant of Lamé,  $u_{xl}$  – component of the displacement vector of the rank  $l$ ,  $l = 1 \dots L$  – number of the hierarchic rank,  $u_{xl}^0$  – component of the displacement vector of the previous rank in the layered medium when the heterogeneity of the previous rank is absent, if  $l = 2 \dots L$ ,  $u_{xl}^0 = u_{x(l-1)}$ , if  $l = 1$ ,  $u_{xl}^0 = u_x^0$ , that coincides with the corresponding expression for the normal field in the paper [10].

Thus the iteration processes (4) and (5) allow to define by given modules of elasticity inside the layered medium that include the hierarchic heterogeneity with additional plastic parameters which depend from the frequency of inner oscillations of the medium inclusion the space frequency distribution of the components of the acoustic field on each hierarchic level. Then, using the known formula of the book [11], for each hierarchic level we can define the distribution of the components of the deformation tensor and stress tensor using the distribution of the components of the displacement vector, that depends not only from the influenced frequency, but from the frequency that is defined by the inner friction. On each hierarchic level it can be itself. Interacting with the influenced frequency medium creep state or resonant excitations can be occurred. That information plays a significant role for estimation of medium state, depending from the hierarchic structure and degree of its change.

### 2.3. Modeling sound diffraction on 2D anomaly dense hierarchical heterogeneity, located in a $N$ -layered elastic medium

In the paper [6] it was described the algorithm of modeling of sound diffraction on elastic hierarchic inclusion, located in the  $J$ -th layer of the  $N$ -layered medium.  $G_{Sp,j}(M, M^0)$  – is a source function of the seismic field, for which is formulated the boundary problem in the paper [6],  $k_{1ji}^2 = \omega^2(\sigma_{ji}/\lambda_{ji})$  – wave number for the longitudinal wave, indexes  $ji$  and  $ja$ ,  $\lambda$ ,  $\sigma$ ,  $\omega$ ,  $\vec{u} = \text{grad}\varphi$ ,  $\varphi^0$ ,  $\varphi_{ji}^0 = \varphi_{ja}^0$  are the same, that are described in the paragraph 1. Let us consider that the elastic parameters of the hierarchic inclusion for all ranks and the layer, where it is located are equal, but the density of the hierarchic inclusion for all ranks differs from the

density of the layer where the inclusion is located. Then the system of equations [6] can be rewritten as follows:

$$\begin{aligned} & \frac{(k_{1ji}^2 - k_{1j}^2)}{2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \frac{\sigma_{ja}}{\sigma_{ji}} \varphi_{l-1}^0(M^0) - \\ & - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma_{jil} 2\pi} \oint_{Cl} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi(M^0), M^0 \in S_{Cl} \\ & \frac{\sigma_{jil} (k_{1jil}^2 - k_{1j}^2)}{\sigma(M^0) 2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) - \\ & - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma(M^0) 2\pi} \oint_{Cl} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi_l(M^0), M^0 \notin S_{Cl}, \end{aligned} \quad (6)$$

where  $G_{Sp,j}(M, M^0)$  – a source function of seismic field, the boundary problem of it is formulated in the paper [6],  $k_{1jil}^2 = \omega^2(\sigma_{jil}/(\lambda_{jil}))$ ;  $\lambda_{jil} = \lambda_{ja}$ ; – wave number for the longitudinal wave,  $l = 1 \dots L$  – number of the hierarchical level,  $\varphi_l^0$  – potential of the acoustic field in the layered medium when the inclusion of the previous rank is absent, if  $l = 2 \dots L$ ,  $\varphi_l^0 = \varphi_{l-1}$ , if  $l = 1$ ,  $\varphi_l^0 = \varphi^0$ , that coincides with the corresponding expression (3).

If by transition on the next hierarchic level the axis of two-dimensionality does not change and only the geometry of the sections of included structures change, then similarly (4) we can develop the iteration process of seismic field modeling (the case of distribution only longitudinal wave). The iteration process relates to modeling of displacement vector by transition from previous (rank) level to a next (rank). Inside each hierarchic rank the integral-differential equation and the integral-differential expression is calculated using the algorithm (6). If on some hierarchical rank the structure of the local heterogeneity divides on some heterogeneities, then the integrals in the expression (6) are calculated over all heterogeneities. In that algorithm we consider the case when the physical properties for one and the same rank are equal; the boundaries can be only different.

#### 2.4. Modeling of elastic transversal wave diffraction on 2D anomaly dense hierarchical heterogeneity, located in a N-layered elastic medium

Similarly to (6) it can be developed the same process for modeling distribution elastic transversal wave in N-layered medium with 2D hierarchic structure with arbitrary section morphology with use the integral equations from the paper [6].

$$\begin{aligned} & \frac{(k_{2jil}^2 - k_{2j}^2)}{2\pi} \iint_{S_{Cl_i}} u_{xl}(M) G_{Ss,j}(M, M^0) d\tau_M + u_{x(l-1)}^0(M^0) = u_{xl}(M^0), M^0 \in S_{Cl} \\ & \frac{\mu_{jil} (k_{2jil}^2 - k_{2j}^2)}{\mu(M^0) 2\pi} \iint_{S_{Cl_i}} u_{xl}(M) G_{Ss,j}(M, M^0) d\tau_M + u_{x(l-1)}^0(M^0) = u_{xl}(M^0), M^0 \notin S_{Cl} \end{aligned} \quad (7)$$



where  $G_{Ss,j}(M, M^0)$  – source function of the seismic field for the considering problem, it coincides with the Green function formulated in the paper [6] for the appropriate problem,  $k_{2jil}^2 = \omega^2(\sigma_{jil} / \mu_{jil})$  – wave number of the transversal wave,  $\mu_{jil} = \mu_{ja}$ ,  $\mu$  – constant of Lamé,  $u_{xl}$  – component of the displacement vector of the rank  $l$ ,  $l = 1 \dots L$  – number of the hierarchic rank,  $u_{xl}^0$  – component of the displacement vector of the previous rank in the layered medium when the heterogeneity of the previous rank is absent, if  $l = 2 \dots L$ ,  $u_{xl}^0 = u_{x(l-1)}$ , if  $l = 1$ ,  $u_{xl}^0 = u_x^0$ , that coincides with the corresponding expression for the normal field in the paper [10].

It must be noted that the structure of the system (6) coincides with the general case, when the hierarchical heterogeneity had not only density parameters that differ from the density of its included layer, but also the elastic parameters for all ranks differ from the elastic parameters of the included layer. The difference of the system (6) consists only in the values of the wave number. Thus more sensitive to the hierarchical inclusions of anomaly density in the massif is the medium response, linked with the longitudinal wave that is also sensitive to the form of the hierarchical inclusion, than the transversal wave. That must be taken into account by mapping and monitoring such complicated geological medium.

## 2.5. Modeling sound diffraction on 2D anomaly stressed hierarchical heterogeneity, located in an $N$ -layered elastic medium

In the paper [6] was described the algorithm of modeling of sound diffraction on elastic hierarchic inclusion, located in the  $J$ -th layer of the  $N$ -layered medium.  $G_{Sp,j}(M, M^0)$  – source function of the seismic field, for which is formulated the boundary problem in the paper [6],  $k_{1ji}^2 = \omega^2(\sigma_{ji} / \lambda_{ji})$  – wave number for the longitudinal wave, indexes  $ji$  and  $ja$ ,  $\lambda$ ,  $\sigma$ ,  $\omega$ ,  $\vec{u} = \text{grad} \varphi$ ,  $\varphi^0$ ,  $\varphi_{ji}^0 = \varphi_{ja}^0$  are the same, that are described in the paragraph 1. Let us consider that the density parameters of the hierarchic inclusion for all ranks and the layer, where it is located are equal, but the elastic parameters in the hierarchic inclusion for all ranks differ from the elastic parameters of the layer where the inclusion is located. Then the system of equations [6] can be rewritten as follows:

$$\begin{aligned} \frac{(k_{2jil}^2 - k_{1j}^2)}{2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) &= \varphi_l(M^0), M^0 \in S_{cl} \\ \frac{\sigma_{jil}(k_{2jil}^2 - k_{1j}^2)}{\sigma(M^0)2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) &= \varphi_l(M^0), M^0 \notin S_{cl} \end{aligned} \quad (8)$$

The designations in (8) are the same as for the system of equations (6).

## 2.6. Modeling of elastic transversal wave diffraction on 2D anomaly stressed hierarchical heterogeneity, located in an $N$ -layered elastic medium

Similarly to (4) it can be developed the same process for modeling distribution elastic transversal wave in  $N$ -layered medium with 2D hierarchic structure with arbitrary section morphology with use the integral equations from the paper [6].

$$\begin{aligned}
 & \frac{(k_{2jil}^2 - k_{2j}^2)}{2\pi} \iint_{S_{Cl_l}} u_{xl}(M) G_{Ss,j}(M, M^0) d\tau_M + \frac{\mu_{ja}}{\mu_{jil}} u_{x(l-1)}^0(M^0) + \\
 & + \frac{(\mu_{ja} - \mu_{jil})}{\mu_{jil} 2\pi} \oint_{Cl} u_{xl}(M) \frac{\partial G_{Ss,j}}{\partial n} dc = u_{xl}(M^0), M^0 \in S_{Cl} \\
 & \frac{\mu_{jil}(k_{2jil}^2 - k_{2j}^2)}{\mu(M^0) 2\pi} \iint_{S_{Cl_l}} u_{xl}(M) G_{Ss,j}(M, M^0) d\tau_M + u_{x(l-1)}^0(M^0) + \\
 & + \frac{(\mu_{ja} - \mu_{jil})}{\mu(M^0) 2\pi} \oint_{Cl} u_{xl}(M) \frac{\partial G_{Ss,j}}{\partial n} dc = u_{xl}(M^0), M^0 \in S_{Cl}
 \end{aligned} \tag{9}$$

where  $G_{Ss,j}(M, M^0)$  – source function of the seismic field for the considering problem, it coincides with the Green function formulated in the paper [6] for the appropriate problem,  $k_{2jil}^2 = \omega^2(\sigma_{jil}/\mu_{jil})$  – wave number of the transversal wave,  $\mu_{jil} \neq \mu_{ja}$ ,  $\sigma_{jil} = \sigma_{ja}$ ,  $\mu$  – constant of Lamé,  $u_{xl}$  – component of the displacement vector of the rank  $l$ ,  $l = 1 \dots L$  – number of the hierarchic rank,  $u_{xl}^0$  – component of the displacement vector of the previous rank in the layered medium when the heterogeneity of the previous rank is absent, if  $l = 2 \dots L$ ,  $u_{xl}^0 = u_{x(l-1)}$ , if  $l = 1$ ,  $u_{xl}^0 = u_x^0$ , that coincides with the corresponding expression for the normal field in the paper [6].

It must be noted that the structure of the system (9) coincides with the general case, when the hierarchical heterogeneity had not only density parameters that differ from the density of its included layer, but also the elastic parameters for all ranks differ from the elastic parameters of the included layer. The difference of the system (9) consists only in the values of the wave number. Thus more sensitive to the hierarchical inclusions of anomaly density in the massif is the medium response, linked with the transversal wave. That must be taken into account by mapping and monitoring such complicated geological medium.

## 2.7. Algorithm of modeling the distribution of the longitudinal wave in the layered medium with fluid saturated hierarchic inclusions

The idea, written in the paper [6] for solution of the direct problem for 2D case of longitudinal wave distribution through the local elastic heterogeneity with hierarchic structure, located in the  $J$ -the layer of  $N$ -layered medium, let us spread on the case when on the  $L$ -th hierarchic layer a fluid saturated porous inclusion will occur.  $G_{Sp,i}(M, M^0)$  – source function of the seismic field, it coincides with the function from the paper [6].  $k_{1jil}^2 = \omega^2(\sigma_{jil}/\lambda_{jil})$  – wave number for the longitudinal wave. Indexes  $ji$  and  $ja$ ,  $\lambda$ ,  $\sigma$ ,  $\omega$ ,  $\vec{u} = \text{grad } \varphi$ ,  $\varphi^0$ ,  $\varphi_{ji}^0 = \varphi_{ja}^0$  are the same, that are described in the paragraph 1.  $l = 1 \dots L-1$  – number of the hierarchic level.

If by transition on the next hierarchic level the axis of two-dimensionality does not change and only the geometry of the sections of included structures change, then similarly (4) we can develop the iteration process of seismic field modeling (the case of distribution only longitudinal wave). The iteration process relates to modeling of displacement vector by transition from

previous (rank) level to a next (rank). Inside each hierarchic rank the integral-differential equation and the integral-differential expression is calculated using the algorithm (4). If on some hierarchical rank the structure of the local heterogeneity divides on some heterogeneities, then the integrals in the expression (4) are calculated over all heterogeneities. In that algorithm we consider the case when the physical properties for one and the same rank are equal; the boundaries can be only different. If  $l = L$ , inside these hierarchic level the porous fluid saturated inclusion occurred. Then the system (4) with account [12] will be rewritten:

$$\begin{aligned}
 & \frac{(k_{1jil}^2 - k_{1j}^2)}{2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \frac{\sigma_{ja}}{\sigma_{jil}} \varphi_{l-1}^0(M^0) - \\
 & - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma_{jil} 2\pi} \oint_{cl} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = (\varphi_l(M^0) + \alpha p_2), M^0 \in S_{cl} \\
 & \frac{\sigma_{jil}(k_{1jil}^2 - k_{1j}^2)}{\sigma(M^0) 2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) - \\
 & - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma(M^0) 2\pi} \oint_{cl} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi_l(M^0), M^0 \notin S_{cl},
 \end{aligned} \tag{9a}$$

where  $\alpha = 1 - \chi - \frac{K}{K_0}$ ,  $K = \lambda$  – module of all-around compression,  $\chi$  – porosity,  $K_0$  – true modulus of phase compressibility, pore hydrostatic pressure  $p_2$ . If  $l = L + 1$  and on the next level the hierarchic heterogeneity is again elastic, for continuing the iteration process we can use again the equations (4).

### 3. Defining of the 2-D surface of the anomaly stressed hierarchical object, located in the layered blocked geological medium using the data of acoustic monitoring

In Ref. [13], the concept of a step-by-step interpretation of a variable electromagnetic field was proposed. At the first stage, the parameters of the normal section, or the parameters of the enclosing one-dimensional non-magnetic medium, are anomalous conductive or magnetic inclusions. In the second stage, an anomalous alternating electromagnetic field is selected by a system of singular sources placed in a horizontally layered medium with geoelectrical parameters determined at the first stage. At the third stage, the theoretical inverse problem is solved, i.e. At given geoelectrical parameters of the host environment for a set of parameters of non-homogeneities, the contours of this heterogeneity are determined. We obtain explicit integral-differential equations of the theoretical inverse scattering problem for two-dimensional and three-dimensional alternating and three-dimensional stationary electromagnetic fields in the framework of models: a conductive or a magnetic body in the  $v$ -th layer of a conductive  $N$ -layered half-space.

In this chapter, using the approach presented [14, 15], an algorithm is developed for obtaining the equation of the theoretical inverse problem for an acoustic field (transverse acoustic wave)

for the elastic anomaly stressed hierarchical heterogeneity of the  $k$ -th rank, whose density coincides with the density of the host medium for all hierarchical ranks, located in the  $\nu$ -th layer of the elastic  $N$ -layered half-space.

Let a simply connected domain  $D$  of the Euclidean space  $R^2$ , bounded by a continuously differentiable closed curve is located in the  $\nu$ -th layer of the  $N$ -layered half-space. Suppose that this domain contains inside  $K$  simply connected hierarchical inclusions, bounded by continuously differentiable closed curves  $\partial D_k$  and extending parallel to the axis  $OX$ . The boundaries  $l_j$  of the layers  $P_j$  ( $j = 1, \dots, N$ ) are parallel to the  $OY$  axis of the  $XOY$  plane of the Cartesian coordinate system. The axis  $OZ$  is directed vertically downward. We place the origin of the coordinate system on the upper boundary of the surface of the first layer and match it with the point that is the projection onto  $OY$  of the point, with respect to which the domain  $D$  is stellar. Let  $U(y, z)$  be complex-valued twice continuously differentiable function that satisfy the two-dimensional scalar Helmholtz equation:

$$\Delta U + c(M)U = -f(M), \quad (10)$$

where  $\Delta = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ ;

$$c(M) = \begin{cases} c_j; M \in \Pi_j / \overline{D} \ (j = 0, \dots, n) \\ c_{ak}; M \in D_k \ (k = 1, \dots, K) \end{cases}. \quad (11)$$

Let the function  $U^1(y, z)$  satisfies the equation:

$$\Delta U^1 + p(M)U^1 = -f(M), \quad (12)$$

$$p(M) = \begin{cases} c_j; M \in \Pi_j / \overline{D} \ (j = 0, \dots, n) \\ c_\nu; M \in D_k \ (k = 1, \dots, K) \end{cases}. \quad (13)$$

Let us first consider the case, when  $k = 1$ . For  $M \in R^2 \setminus \overline{D} \ (j = 0, \dots, N)$ . We shall define:

$$U^+(M) = U(M) - U^1(M) \quad (14)$$

Function  $U^+(M)$  satisfies the Eq. (10). On the boundaries  $l_j$  of the layers  $P_j$  the following boundary conditions are fulfilled:

$$\begin{aligned} U_j &= U_{j+1}; \\ U_j^+ &= U_{j+1}^+; M \in l_j \ (j = 1, \dots, n-1); \\ U_j^1 &= U_{j+1}^1; \end{aligned} \quad (15)$$

$$b_j \frac{\partial U_j}{\partial n} = b_{j+1} \frac{\partial U_{j+1}}{\partial n}; b_j \frac{\partial U_j^+}{\partial n} = b_{j+1} \frac{\partial U_{j+1}^+}{\partial n}; b_j \frac{\partial U_j^1}{\partial n} = b_{j+1} \frac{\partial U_{j+1}^1}{\partial n}; M \in l_j, \quad (16)$$

$b_j$  are complex coefficients ( $j = 0, \dots, N$ ) and in general case:  $b_j \neq b_{j+1}$ ; on the contour  $\partial D_k$ : for  $k = 1$ :

$$U_\nu = U_\nu^+ + U_\nu^1. \quad (17)$$

Function  $U_\nu$  satisfies the equation:

$$\Delta U_v + c_v(M)U_v = -f(M). \quad (18)$$

$U_v^+$  – is a function  $U^+$  in the layer  $P_v \notin D$ ;  $U_v^1$  – is a function  $U^1$  in the layer  $P_v \notin D$ ; in the domain  $D$  for  $k = 1$ :

$$U_a = U_a^+ + U_a^1; M \in \bar{D}; \Delta U_a + c_a U_a = 0. \quad (19)$$

Boundary conditions on  $\partial D$  ( $k = 1$ ):

$$U_a^+ = U_v^+, \quad b_a \frac{\partial U_a}{\partial n} - b_v \left( \frac{\partial U_v^+}{\partial n} + \frac{\partial U_v^1}{\partial n} \right) = 0, \quad (20)$$

By  $M \rightarrow \infty$  the functions  $U(M), U^+(M), U^1(M)$  satisfy the radiation condition [16]. The algorithm of calculation of function  $U^1$  for the electromagnetic case is written [13].

Let us introduce the function  $G(M, M_0)$ , that satisfies the following equation:

$$\Delta G + p(M)G = -\delta(M, M_0), \quad (21)$$

and the boundary conditions (15, 16), by  $M \rightarrow \infty$  the function  $G$  satisfies the radiation condition [16], by  $M \rightarrow M_0$  the function  $G$  has a singularity as:  $\ln 1/\rho(M, M_0)$ :

$$\rho(M, M_0) = \sqrt{(y - y_0)^2 + (z - z_0)^2}. \quad (22)$$

Algorithm of calculation of function  $G$  for the case, when the domain  $D$  is located in the  $v$  – th layer, is written [13]. Let us introduce the function  $G^a$ , that coincides with the fundamental solution of the Eq. (11) for  $k = 1$ . Let us use the Green formula [16] for two functions  $U^+, G$ ; ( $M \in R^2 \setminus \bar{D}, M_0 \in P_i$ ) in each layer  $P_j (j=0, \dots, N)$ . Let us fulfill the procedure similarly [17]: let us multiply the defined expressions for each layer on  $b_j$  reciprocally,  $j=0, \dots, N$  and add them term by term with account (11–14), (16) and (17). As a result we receive:

$$2\pi U^+(M_0) = -(b_v/b_i) \int_{\partial D} \left( U_v^+(M) \frac{\partial G(M, M_0)}{\partial n} - G(M, M_0) \frac{\partial U_v^+}{\partial n} \right); M \in \Pi_v; M_0 \in \Pi_i. \quad (23)$$

In the domain  $D$  let us use the Greens formula for the two functions  $U_a(M), G^a(M, M_0)$ . As a result we receive:

$$0 = \int_{\partial D} \left( U_a(M) \frac{\partial G^a(M, M_0)}{\partial n} - G^a(M, M_0) \frac{\partial U_a}{\partial n} \right) dl. \quad (24)$$

Let us add expressions (23) and (24), taking into account (20) and (21), and also the expression (13):

$$0 = \left( -b_v/b_i \right) \int_{\partial D} \left( U_v^1(M) \frac{\partial G(M, M_0)}{\partial n} - G(M, M_0) \frac{\partial U_v^1}{\partial n} \right) dl; M \in \bar{D}; M_0 \in \Pi_i. \quad (25)$$

Then we shall receive:

$$2\pi U^+(M_0) = \int_{\partial D} \left( (U_v^+(M) + U_v^1(M)) \left( \frac{\partial G^a(M, M_0)}{\partial n} - \left( b_v / b_i \right) \frac{\partial G(M, M_0)}{\partial n} \right) - \right. \\ \left. - b_v \left( \frac{\partial U_v^+}{\partial n} + \frac{\partial U_v^1}{\partial n} \right) \left( (1/b_a) G^a(M, M_0) - (1/b_i) G(M, M_0) \right) \right) dl. \quad (26)$$

Eq. (26) is the explicit equation of the theoretical inverse problem for the two-dimensional scalar Helmholtz equation in a layered medium with homogeneous inclusion for given values of the boundary conditions [13–15]. As a result of the solution of the integral-differential equation (26) related to the function  $r(\varphi)$ , that describes the contour of the sought homogeneous object, it is possible to determine it for known values of the physical parameters of the host medium and the desired object, and also for given values of the functions.

According to [5, 8], the problem of diffraction of a linear polarized elastic transverse wave by a two-dimensional elastic heterogeneity of a hierarchical type located in a layer of an N-layer medium within the framework of the described model reduces to solving a similar problem with the following changes. The equation of the theoretical inverse problem (26) for the scalar Helmholtz equation, to which our problem reduces, remains valid by that:

$$b_v = \xi_v; b_i = \xi_i; b_a = \xi_a; \quad (27)$$

$\xi_v, \xi_i, \xi_a, \rho_v, \rho_i, \rho_a$  – the values of the parameter Lamé and the density in the  $v$ -th layer, in the layer where the point  $M_0$  is located and inside the heterogeneity at  $k = 1$ . An important difference between the present problem and the one considered above is that  $\rho_a = \rho_v$  for all  $k$ , physically, this means that the anomaly in the acoustic field is created by an anomaly of the stressed state of the medium and can be associated with a focus of either a rock shock or earthquakes.

$$U^+ = u_x^+; U_v^+ = u_{xv}^+; U_v^1 = u_{xv}^1, \quad (28)$$

where  $u_x$  is the component of the displacement vector, different from zero for the selected model.

$$G(M, M_0) = G_{SS}(M, M_0); G^a(M, M) = G_{SS}^a(M, M_0); \partial D, dl, \\ k_{2a}^2 = \omega^2 \frac{\rho_v}{\xi_a}; k_{2v}^2 = \omega^2 \frac{\rho_v}{\xi_v}. \quad (29)$$

The algorithm for calculating the Green's function  $G_{SS}(M, M_0)$  was written in Ref. [18]. Thus, the equation of the theoretical inverse problem for  $k = 1$  is written in the form:

$$2\pi u_x^+(M_0) = \int_{\partial D1} \left( (u_{xv}^+(M) + u_{xv}^1(M)) \left( \frac{\partial G_{SS}^{a1}(M, M_0)}{\partial n} - \left( \xi_v / \xi_i \right) \frac{\partial G_{SS}(M, M_0)}{\partial n} \right) - \right. \\ \left. - \xi_v \left( \frac{\partial u_x^+}{\partial n} + \frac{\partial u_{xv}^1}{\partial n} \right) \left( (1/\xi_a) G_{SS}^{a1}(M, M_0) - (1/\xi_i) G_{SS}(M, M_0) \right) \right) dl. \quad (30)$$



Let  $k = 2$ , that is the sought object is a hierarchic inclusion with elastic parameter Lamé  $\xi_1$  and density  $\rho_v$ ,  $\partial D_1$  – contour of the external inclusion and with elastic parameter Lamé  $\xi_2$ , density  $\rho_v$ ,  $\partial D_2$  – contour of the inner inclusion. The inclusions are uncoordinated. It is needed to define the two contours. For solution of our problem in the expression (27) we substitute  $\xi_a = \xi_1$ , and in the expression (29):

$$\partial D = \partial D_1; dl = dl_1; G_{SS}^a = G_{SS}^{a1}; \frac{\partial}{\partial n} (G_{SS}^a) = \frac{\partial}{\partial n} (G_{SS}^{a1}); k_{2a}^2 = \frac{2a12}{k} = \omega^2 \frac{\rho_v}{\xi_{a1}}.$$

Solving Eq. (30) related to the function  $r_1(\varphi)$ , that describes the contour  $\partial D_1$ , we calculate the functions:  $u_x; u_x^+; u_x^1$  by the algorithm for solving the direct problem (9), (31) and (32) inside and outside the heterogeneity placed in a layered medium,  $u_x^1$  the elastic field component in a layered medium in the absence of a heterogeneity.

$$\begin{aligned} u_x(M_0) &= \frac{\xi_v}{\xi_a} u_x^1(M_0) + \frac{k_{2a}^2 - k_{2v}^2}{2\pi} \iint_{S_1} u_x(M) G_{SS}(M, M_0) dS_1 + \\ &+ \frac{\xi_v - \xi_a}{2\pi \xi_a} \int_{\partial D_1} u_x(M) \frac{\partial G_{SS}}{\partial n} dl; M_0 \in S_1, \end{aligned} \quad (31)$$

$$\begin{aligned} u_x(M_0) &= u_x^1(M_0) + \frac{\xi_a(k_{2a}^2 - k_{2v}^2)}{2\pi \xi(M_0)} \iint_{S_1} u_x(M) G_{SS}(M, M_0) dS_1 + \\ &+ \frac{(\xi_v - \xi_a)}{2\pi \xi(M_0)} \int_{\partial D_1} u_x(M) \frac{\partial G_{SS}}{\partial n} dl; M_0 \notin S_1. \end{aligned} \quad (32)$$

This completes the first iteration cycle, and we proceed to the second iterative cycle  $k = 2$ . The calculated function  $u_x(M_0)$  (32) is denoted as:  $u_x^{1(k-1)}$  (33), in the expression (27)  $\xi_a = \xi_2$ , in the expression (29):

$$\partial D = \partial D_2; dl = dl_2; G_{SS}^a = G_{SS}^{a2}; \frac{\partial}{\partial n} (G_{SS}^a) = \frac{\partial}{\partial n} (G_{SS}^{a2}); k_{2a}^2 = k_{2a2}^2 = \omega^2 \frac{\rho_v}{\xi_{a2}}. \quad (33)$$

The Eq. (30) can be rewritten as following:

$$\begin{aligned} 2\pi u_x^+(M_0) &= \int_{\partial D_2} \left( \left( u_{xv}^+(M) + u_{xv}^{1(k-1)}(M) \right) \left( \frac{\partial G_{SS}^a(M, M_0)}{\partial n} - \left( \frac{\xi_v}{\xi_i} \right) \frac{\partial G_{SS}(M, M_0)}{\partial n} \right) - \right. \\ &\left. - \xi_v \left( \frac{\partial u_x^+}{\partial n} + \frac{\partial u_{xv}^{1(k-1)}}{\partial n} \right) \left( (1/\xi_{a2}) G_{SS}^a(M, M_0) - (1/\xi_i) G_{SS}(M, M_0) \right) \right) dl_2; \end{aligned} \quad (34)$$

We solve the Eq. (34) relatively the function  $r_2(\varphi)$  that describes the contour  $\partial D_2$ . If  $K = 2$ , then the problem is solved, if  $k > 2$ ,  $k = k + 1$ , the iteration process is continued.

We solve the functions:

$$u_x^{k-1}; u_x^{+(k-1)}, \quad (35)$$

using the algorithm of the direct problem inside and outside the hierarchical heterogeneity of the rank  $k - 1$ , located into the layered medium (the physical parameters of the layered medium are not changed) (9), (36) and (37).

$$u_x^{k-1}(M_0) = \frac{\xi_v}{\xi_{a(k-1)}} u_x^{1(k-2)}(M_0) + \frac{k_{2a(k-1)}^2 - k_{2v}^2}{2\pi} \iint_{S(k-1)} u_x^{k-1}(M) G_{SS}(M, M_0) dS_{(k-1)} +$$

$$+ \frac{\xi_v - \xi_{a(k-1)}}{2\pi \xi_{a(k-1)}} \int_{\partial D(k-1)_1} u_x^{k-1}(M) \frac{\partial G_{SS}}{\partial n} dl_{k-1}; M_0 \in S_{(k-1)}, \quad (36)$$

$$u_x^{k-1}(M_0) = u_x^{1(k-2)}(M_0) + \frac{\xi_a(k_{2a}^2 - k_{2v}^2)}{2\pi \xi(M_0)} \iint_{S(k-1)} u_x^{k-1}(M) G_{SS}(M, M_0) dS_1 +$$

$$+ \frac{(\xi_v - \xi_a)}{2\pi \xi(M_0)} \int_{\partial D(k-1)_1} u_x^{k-1}(M) \frac{\partial G_{SS}}{\partial n} dl_{k-1}; M_0 \notin S_{(k-1)}. \quad (37)$$

The calculated function  $u_x^{k-1}(M_0)$  (28) we denote as:

$$u_x^{1(k-1)} \quad (38)$$

In the expression (27)  $\xi_a = \xi_k$ , in the expression (29):

$$\partial D = \partial D_k; dl = dl_k; G_{SS}^a = G_{SS}^{ak}; \frac{\partial}{\partial n} (G_{SS}^a) = \frac{\partial}{\partial n} (G_{SS}^{ak}); k_{2a}^2 = k_{2ak}^2 = \omega^2 \frac{\rho_v}{\xi_{ak}} \dots$$

Eq. (30) is rewritten as following:

$$2\pi u_x^+(M_0) = \int_{\partial D} \left( \left( u_{xv}^+(M) + u_{xv}^{1(k-1)}(M) \right) \left( \frac{\partial G_{SS}^a(M, M_0)}{\partial n} - \left( \xi_v / \xi_i \right) \frac{\partial G_{SS}(M, M_0)}{\partial n} \right) - \right.$$

$$\left. - \xi_v \left( \frac{\partial u_x^+}{\partial n} + \frac{\partial u_{xv}^{1(k-1)}}{\partial n} \right) \left( (1/\xi_a) G_{SS}^a(M, M_0) - (1/\xi_i) G_{SS}(M, M_0) \right) \right) dl_2; \quad (39)$$

We solve the Eq. (39) relatively the function  $r_k(\varphi)$  that describes the contour  $\partial D$ .  $k = k + 1$ . Iteration process (35–39) continues up to  $k = K$ .

## 4. Conclusions

The chapter considers the problem of modeling a seismic field acoustic approximation in a layered medium with inclusions of a hierarchical structure. Algorithms of modeling in the seismic case in the acoustic approximation for 2-D plastic heterogeneity are constructed.

Comparing expressions (6) and (7), (8) and (9), we can draw the following conclusions. When constructing a complex seismic gravity model without taking into account the anomalous effect of a stress–strain state inside the inclusion, an analysis of the anomalous acoustic effect

using data on the propagation of a longitudinal wave shows that it is more sensitive also to the inclusion form, in comparison with the acoustic effect of the propagation of a transverse wave. However, it follows from these expressions that elastic parameters in the seismic model cannot be neglected in the massif, and they influence the interpretation of the values of the unknown anomalous densities. If these values are used in the construction of a density gravitational model, then these density values will not reflect the material composition of the analyzed medium. In the construction of an anomalously stressed geomechanical model without taking into account the anomalous effect of density heterogeneities within the inclusion, an analysis of the anomalous acoustic effect using data on the propagation of a transverse wave shows that it is more sensitive also to the inclusion form, in comparison with the acoustic effect of the propagation of a longitudinal wave. However, it follows from these expressions that the influence of density parameters in the massif in the seismic model cannot be neglected, and they influence the interpretation of the values of the unknown anomalous elastic parameters that cause the anomalous stress state. If these values are used in the construction of a geomechanical model, these values of the elastic parameters will not reflect the stress state of the analyzed medium.

The proposed simulation algorithm, the mapping and monitoring method for a heterogeneous, complex two-phase medium, can be used to control the production of viscous oil in mine conditions and light oil in sub horizontal wells. The requirements of efficient for the most economic indicators and the most complete extraction of hydrocarbons in the fields dictate the need for the creation of new geotechnology for the development of oil and gas fields based on fundamental advances in geophysics and geomechanics [19].

Additionally it is of interest with the use of the obtained algorithms to investigate the question of studying the connection between the strain and strain tensors at each hierarchical level and the possible deviation from the generalized Hooke's law.

In the third part of our chapter we considered the problem of constructing an algorithm for solving the inverse problem using the equation of the theoretical inverse problem for the 2-D Helmholtz equation. An explicit equation is obtained for the theoretical inverse problem for the scattering of a linearly polarized elastic wave in a layered elastic medium with a hierarchical elastic anomal stressed inclusion, whose density for all ranks is equal to the density of the enclosing layer. An iterative algorithm for determining the contours of non-axial inclusions of the  $k$ -th rank in a hierarchical structure with successive use of the solution of the direct problem of calculating the elastic field of  $k-1$  rank is constructed. With the increase in the degree of hierarchy of the structure of the medium, the degree of spatial nonlinearity of the distribution of the components of the acoustic field increases, which involves the elimination of methods for linearizing the problem when creating interpretation methods. This problem is inextricably linked with the solution of the inverse problem for the propagation of the acoustic field in such complex media using explicit equations of the theoretical inverse problem. For the first time an equation for determining the surface of anomalously stressed inclusion in a hierarchical layered block environment was derived from the data of acoustic monitoring. In practice, using this algorithm, we can localize from the acoustic monitoring data the area of a potential hotbed or a forthcoming earthquake and estimate the degree of anomalous elastic stresses.

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