Chapter 12

Information Transfer and Thermodynamic Point of View on Goedel Proof

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Additional information is available at the end of the chapter

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Abstract

Formula of an arithmetic theory based on Peano Arithmetics (including it) is a chain of symbols of its super-language (in which the theory is formulated). Such a chain is in convenience both with the syntax of the super-language and with the inferential rules of the theory (Modus Ponens, Generalization). Syntactic rules constructing formulas of the theory are not its inferential rules. Although the super-language syntax is defined recursively—by the recursive writing of mathematical-logical claims—only those recursively written super-language's chains which formulate mathematical-logical claims about finite sets of individual of the theory, computable totally (thus recursive) and always true are the formulas of the theory. Formulas of the theory are not those claims which are true as for the individual of the theory, but not inferable within the theory (Great Fermat's Theorem). They are provable but within another theory (with both Peano and further axioms). Also the chains expressing methodological claims, even being written recursively (Goedel Undecidable Formula) are not parts of the theory. The same applies to their negations. We show that the Goedel substitution function is not the total one and thus is not recursive. It is not defined for the Goedel Undecidable Formula's construction. For this case, the structure of which is visible clearly, we are adding the zero value. This correction is based on information, thermodynamic and computing considerations, simplifies the Goedel original proof, and is valid for the consistent arithmetic theories directly.

Keywords: arithmetic formula, inference, information transfer, information entropy, heat efficiency, infinite cycle

1. Introduction

The formula of an *arithmetic theory* based on *Peano Arithmetics* (including it) is a chain of symbols of its *metalanguage* in which the theory is formulated such that it is both in convenience with the *syntax* of the metalanguage and with the *inferential rules* of the theory [of the *inferential system* (*Modus Ponens, Generalization*)].

Syntactic rules constructing formulae of the theory (but not only!) *are not* its inferential rules. Although the metalanguage syntax is defined *recursively*—by the *recursive writing* of *mathematical-logical* claims, only those recursively written metalanguage's chains which formulate mathematical-logical claims about *finite* (precisely *recursive*) *sets of individual* of the theory, computable totally (thus *recursive*) and as *always true* are the formulae of the theory. Formulas of the theory are not those claims which are true as for the individual of the theory, but not inferable within the theory (*Great Fermat's Theorem*). They are provable but within *another* theory (with *further axioms* than only those of Peano). Also the chains expressing *methodological claims*, even being written recursively (*Goedel Undecidable Formula*), are not parts of the theory, and also they are not parts of the inferential system; the same is for their negations.

We show that the *Goedel substitution function* is not the total one and thus is not recursive. It is not defined for the Goedel Undecidable Formula's *construction*. For this construction, the structure of which is visible clearly, we are setting the zero value. This correction is based on *information, thermodynamic* and *computing* considerations, simplifies the Goedel original proof, and is valid for the *consistent* arithmetic theories directly.[1]

Remark: *Paradoxical claims* (paradoxes, *noetical* paradoxes, *contradictions* and *antinomia*) have two parts—both parts are true, but the truth of one part denies the truth of the second part.

They can arise by not respecting the *metalanguage* (*semantic*) *level*—which is the higher level of our thinking about problems and the *language* (*syntactic*) *level*—which is the lower level of formulations of our 'higher' thoughts. Also they arise by not respecting a *double-level organization and description of measuring*—by not respecting the need of a *'step-aside' of the observer from the observed*. And also they arise by not respecting various *time clicks* in time sequences. As for the latter case, they are in a contradiction with the *causality principle*. The common feature for all these cases is the *Auto-Reference* construction which itself, solved by itself, always states the requirement for ceasing the *II. Principle of Thermodynamics* and all its equivalents [10, 11, 12, 13].

Let us introduce the ***Russel's criterion*** *for removing paradoxes*[2]: **Within the flow of our thinking and speech we need and must distinguish between *two levels* of our thinking and expressing in order not to fall in a paradoxical claim *by mutual mixing and changing* them**.

These levels are the higher one, the metalanguage (semantic) level and the lower one, the language (syntactic) level. Being aware of the existence of these two levels, we prevent ourselves from their mutual mixing and changing, we prevent ourselves from *application* our *metalanguage claims on themselves* but now on the language level or vice versa.

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[1]**The reader of the paper should be familiar with the Goedel proof's way and terminology; *SMALL CAPITALS* in the whole text mean the Goedel numbers and working with them.** This chapter is based, mainly, on Ref. [17], which was improved as for certain misprints, and also, by a few more adequate formulations and by adding the part **Appendix** [14–16].
[2]B. Russel, L. Whitehead, **Principia Mathematica**, 1910, 1912, 1913 and 1927.

We must be aware that our claims about properties of considered objects are *created* on the higher level, rather richer both semantically and syntactically than the lower one on which we really express ourselves about these objects. The words and meanings of this lower (and 'narrower') level are common to both of them. Our speech is *formulated and performed* on the lower level describing here our 'higher' thoughts and on which the objects themselves have been described, defined yet too, of course from the higher level, but with the necessary (lower) limitations. (As such they are thought over on the higher level.) From this point of view, we understand the various meanings (levels) of the same words. Then, any mutual mixing and changing the metalanguage and language level or the auto-reference (paradox, noetical paradox, contradiction and antinomian) is excluded.

2. Goedel numbers, information and thermodynamics

Any *inference* within the system $\mathcal{P}$[3] sets the $\mathcal{T}_{\mathcal{PA}}$*-theoretical* relation[4] among its formulae $a_{[\cdot]}$. This relation is given by their gradually generated *special sequence* $\vec{a} = [a_1, \dots, a_q, \dots, a_p, \dots, a_k, a_{k+1}]$ which is the *proof* of the latest inferred formula a_{k+1}. By this, the *unique* arithmetic relation between their *Goedel numbers*, FORMULAE $x_{[\cdot]}$, $x_{[\cdot]} = \Phi(a_{[\cdot]})$, is set up, too. The gradually arising *SEQUENCE of FORMULAE* $x = \Phi(\vec{a})$ is the *PROOF* of its latest *FORMULA* x_{k+1}.

Let us assume that the given sequence $\vec{a} = [a_{o1}, a_{o2}, \dots, a_o, \dots, a_q, \dots, a_p, \dots, a_k, a_{k+1}]$ is a special one, and that, except of axioms (axiomatic schemes) $a_{01}, \dots, a_o$, it has been generated by the correct application of the rule *Modus Ponens only.*[5]

Within the process of the (*goedelian*) *arithmetic-syntactic analysis* of the latest formula a_{k+1} of the proof $\vec{a}$ we use, from the $\vec{a}$ *selected*, (special) subsequence $\overrightarrow{a_{q,p,k+1}}$ of the formulae a_q, a_p, a_{k+1}. The formulae a_q, a_p have already been derived, or they are axioms. It is valid that $q,p < k+1$, and we assume that $q < p$,

$$\begin{aligned}
&\overrightarrow{a_{q,p,k+1}} = [a_q, a_p, a_{k+1}], \quad a_p \cong a_q \supset a_{k+1}, \quad \overrightarrow{a_{q,p,k+1}} = [a_q, a_q \supset a_{k+1}, a_{k+1}], \\
&\quad x = \Phi(\vec{a}) = \Phi([\Phi(a_1), \Phi(a_2), \dots, \Phi(a_q), \dots, \Phi(a_p), \dots, \Phi(a_k), \Phi(a_{k+1})]]) \\
&\quad\quad = \Phi(\vec{x}) = \Phi(x_1) * \Phi(x_2) * \dots * \Phi(x_q) * \dots * \Phi(x_p) * \dots * \Phi(x_k) * \Phi(x_{k+1}) \\
&l(x) = l[\Phi(\vec{x})] = l[\Phi(\vec{a})] = k+1, \\
&x_{k+1} = \Phi(a_{k+1}) = l[\Phi(\vec{a})] Gl\ \Phi(\vec{a}) = (k+1) Gl\ x \\
&x_p = \Phi(a_p) = \Phi(a_q \supset a_{k+1}) = qGl\ \Phi(\vec{a}) * \Phi(\supset) * l[\Phi(\vec{a})] Gl\ \Phi(\vec{a}) \\
&\quad = qGl\ xImp\ [l(x)] Gl\ x \\
&x_q = \Phi(a_q) = qGl\ \Phi(\vec{a}) = qGl\ x
\end{aligned} \tag{1}$$

[3] *Formal arithmetic inferential system.*

[4] Peano Arithmetics Theory.

[5] For simplicity. The 'real' inference is applied to the formula a_{i+1} for $i = o$.

Checking the *syntactic and $\mathcal{T}_{\mathcal{PA}}$-theoretical correctness* of the analyzed chains a_i, as the formulae of the system $\mathcal{P}$ having been generated by inferring (*Modus Ponens*) within the system $\mathcal{P}$ (in the theory $\mathcal{T}_{\mathcal{PA}}$), and also the special sequence of the formulae $\vec{a}$ of the system $\mathcal{P}$ (theory $\mathcal{T}_{\mathcal{PA}}$), is realized by checking the *arithmetic-syntactic* correctness of the notation of their corresponding *FORMULAE* and *SEQUENCE of FORMULAE*, by means of the relations $Form(\cdot)$, $FR(\cdot)$, $Op(\cdot,\cdot,\cdot)$, $Fl(\cdot,\cdot,\cdot)$ 'called' from (the sequence of procedures) relations $Bew(\cdot)$, $(\cdot\cdot)B(\cdot)$, $Bw(\cdot)$[6]; the core of the whole (goedelian) arithmetic-syntactic analysis is the (procedure) relation of *Divisibility*,

$$
\begin{aligned}
&Form[\Phi(a_i)] = "1"/"0",\ FR[\Phi(\overrightarrow{a_1^{i+1}})] = "1"/"0",\ o \le i \le k \\
&Op[x_k, Neg(x_q), x_{k+1}] = Op[\Phi(a_p), \Phi[\sim (a_q)], \Phi(a_{k+1})] = "1"/"0" \\
&Fl[(k+1)Gl\ x, pGl\ x, qGl\ x] = "1"/"0" \\
&xB\ x_{k+1} = "1"/"0",\ Bew(x_{k+1}) = "1"/"0"; \\
&\underline{\Phi(a_p)||23^{3Gl\ \Phi(\overrightarrow{a_{q,p}k+1})}\ \&\ \Phi(a_p)||7^{1Gl\ \Phi(\overrightarrow{a_{q,p}k+1})} = "1"/"0"}
\end{aligned}
\tag{2}
$$

2.1. Inference in the system $\mathcal{P}$ and information transfer

The syntactic analysis of the special sequence of the formulae $\vec{a}$ of the system $\mathcal{P}$ in general, and therefore, also its arithmetic-syntactic version, that is the activity of (*goedelian*) *arithmetic-syntactic analyzer*, will be expressed by means of terms of *information transfer* through a certain *information transfer channel* $\mathcal{K}$.

As such, it is a sequence of successive *attempts i* to transfer information with *input, loss* and *output messages* $[a_{p_i}, a_{q_i}, a_{i+1}]$, $[a_{p_i}, a_{q_i}]$ *and* $[a_{i+1}]$ with their *information amounts* $J(\overrightarrow{a_{q_i,p_i,i+1}})$, $J(\overrightarrow{a_{q_i,p_i}})$ and $J(a_{i+1})$. Index i is a serial number of the *inferencing—analyzing—transferring* step, $0 < q_i < p_i < i+1 \le l[\Phi(\vec{a})] = k+1$. The Goedel numbering also enables us to consider the individual Goedel numbers x_i, $x_i|y_i$ and y_i of messages $[a_{p_i}, a_{q_i}, a_{i+1}]$, $[a_{p_i}, a_{q_i}]$ a $[a_{i+1}]$ as *messages too*, with their (and the same) information amounts $J(x_i)$, $J(x_i|y_i)$ a $J(y_i)$,

$$
\begin{aligned}
&[a_{p_i}, a_{q_i}, a_{i+1}] \triangleq \overrightarrow{a_{q_i,p_i,i+1}} \triangleq x_i = \Phi(\overrightarrow{a_{q_i,p_i,i+1}}),\ [a_{p_i}, a_{q_i}] \triangleq \overrightarrow{a_{q_i,p_i}} \triangleq x_i|y_i = \Phi(\overrightarrow{a_{q_i,p_i}}) \\
&[a_{i+1}] \triangleq a_{i+1} \triangleq y_i = \Phi(a_{i+1}) \\
&\Phi(\overrightarrow{a_{q_i,p_i,i+1}}) = \Phi(a_{q_i}) * \Phi(a_{p_i}) * \Phi(a_{i+1}) = \Phi(\overrightarrow{a_{q_i,p_i}}) * \Phi(a_{i+1}),\ \Phi(\overrightarrow{a_{q_i,p_i}}) = \Phi(a_{q_i}) * \Phi(a_{p_i}); \\
&J(x_i) = J[\Phi(\overrightarrow{a_{q_i,p_i,i+1}})],\ J(x_i|y_i) = J[\Phi(\overrightarrow{a_{q_i,p_i}})],\ J(y_i) = J[\Phi(a_{i+1})]
\end{aligned}
\tag{3}
$$

For each *i*th step of the goedelian syntactic analysis, we determine the values

[6]*Form*ula, *R*eihe von *F*ormeln, *Op*eration, *F*o*l*ge, *Gl*ied, *B*eweis, *Bew*eis, see Definition 1–46 in Refs. [3–5] and by means of all other, by them 'called', relations and functions (by their procedures).

$$
\begin{aligned}
J(x_i) &= \ln(x_i) = \ln[\Phi(\overrightarrow{a_{q_i,p_i,i+1}})] = J(\overrightarrow{a_{q_i,p_i,i+1}}) = J[2^{\Phi(a_{q_i})}\cdot 3^{\Phi(a_{p_i})}\cdot 5^{\Phi(a_{i+1})}] \\
&= \ln[2^{\Phi(a_{q_i})}\cdot 3^{\Phi(a_{p_i})}\cdot 5^{\Phi(a_{i+1})}] \\
J(x_i|y_i) &= \ln(x_i|y_i) = \ln[\Phi(\overrightarrow{a_{q_i,p_i}})] = J(\overrightarrow{a_{q_i,p_i}}) = J[2^{\Phi(a_{q_i})}\cdot 3^{\Phi(a_{p_i})}] = \ln[2^{\Phi(a_{p_i})}\cdot 3^{\Phi(a_{q_i})}] \\
J(y_i) &= \ln(y_i) = J(a_{i+1}) = J[5^{\Phi(a_{i+1})}] = \ln[5^{\Phi(a_{i+1})}]
\end{aligned}
\tag{4}
$$

We regard these values as *average* values $H(X)$, $H(X|Y)$ and $H(Y)$ of information amounts of *message sources* X, $X|Y$ and Y with *selective spaces* $\mathbb{X}$, $\mathbb{X}\times\mathbb{Y}$ and $\mathbb{Y}$, and with the *uniform probability distribution*,

$$
\begin{aligned}
& X \overset{\text{Def}}{=} [\mathbb{X}, \pi_X(x_i) = \text{const.}],\ \text{card}\ \mathbb{X} = 2^{\Phi(\overrightarrow{a_{q_i,p_i,i+1}})}, \qquad \pi_X(x_i) = \frac{1}{2^{\Phi(\overrightarrow{a_{q_i,p_i,i+1}})}} \\
& Y \overset{\text{Def}}{=} [\mathbb{Y}, \pi_Y(y_i) = \text{const.}],\quad \text{card}\ \mathbb{Y} = 5^{\Phi(a_{i+1})},\ \ \pi_Y(y_i) = \frac{1}{5^{\Phi(a_{i+1})}} \\
& \sum_{j=1}^{\text{card}\ \mathbb{X}} \frac{1}{2^{\Phi(\overrightarrow{a_{q_i,p_i,i+1}})}} = \frac{2^{\Phi(\overrightarrow{a_{q_i,p_i,i+1}})}}{2^{\Phi(\overrightarrow{a_{q_i,p_i,i+1}})}} = 1,\quad \sum_{j=1}^{\text{card}\ \mathbb{Y}} \frac{1}{5^{\Phi(a_{i+1})}} = \frac{5^{\Phi(a_{i+1})}}{5^{\Phi(a_{i+1})}} = 1
\end{aligned}
\tag{5}
$$

It is obvious that we consider a *direct* information transfer [11] through the channel $\mathcal{K}$ without *noise, disturbing* $y_i|x_i$, which means with the *zero noise* (*disturbing*) information $[J(y_i|x_i) = 0] \equiv [H(Y|X) = 0]$, $[y_i|x_i \cong \Phi\ (null)]$.

In each ith step of the activity of our *information model* $\mathcal{K}$ of the arithmetic-syntactic analysis, it is valid that $X := x_i = \Phi(\overrightarrow{a_{q_i,p_i,i+1}})$ and $Y := y_i = \Phi(a_{i+1}) = x_{i+1},$ and the *channel equation* is applicable [11],

$$
\begin{aligned}
T(X;Y) = H(X) - H(X|Y) \quad &= \quad H(Y) - H(Y|X) = T(Y;X) \\
T(X;Y) = J(x_i) - J(x_i|y_i) \quad &= \quad J(y_i) - J(y_i|x_i) = T(Y;X)\ \ \text{now in the form} \\
T(X;Y) = H(X) - H(X|Y) \quad &= \quad H(Y),\ \ T(X;Y) = J(x_i) - J(x_i|y_i) = J(y_i)
\end{aligned}
\tag{6}
$$

The relation $\Phi(\overrightarrow{a_{q_i,p_i,i+1}})B\ \Phi(a_{i+1})$ $(x_iB\ y_i)$ is evaluated by the relation of *Divisibility* and we identify its execution[7] with the actual direct information transfer in the channel $\mathcal{K}$. So, when our inference by *Modus Ponens* is done correctly, in each ith step, we have its *information interpretation*, in steps i,

$$
\begin{aligned}
& [x_iB\ y_i] \ \cong\ [J(x_i) - J(x_i|y_i) > 0] \ \equiv\ [T(x_i;y_i) > 0] \ \equiv\ [T(X;Y) > 0] \\
& \equiv [Fl(y_i,x_{p_i},x_{q_i})] \ \equiv\ Fl[\Phi(a_{i+1}),\Phi(a_{q_i}),\Phi(a_{p_i})] \ \equiv\ [\Phi(\overrightarrow{a_{q_i,p_i,i+1}})B\ \Phi(a_{i+1})] \\
& \equiv [\Phi(a_{p_i})||23^{3Gl\ x_i}\ \&\ \Phi(a_{p_i})||7^{x_i}] \ \equiv\ [\Phi(a_{p_i})||23^{3Gl\ \Phi(\overrightarrow{a_{q_i,p_i,i+1}})}\ \&\ \Phi(a_{p_i})||7^{1Gl\ \Phi(\overrightarrow{a_{q_i,p_i,i+1}})}]
\end{aligned}
\tag{7}
$$

Let us assume that, when inferring by *Modus Ponens*, $\frac{b,\ [(\sim b)\vee(c)]}{c}$, we make such an error that we write $\frac{b,\ [(\sim b)\vee(c)]}{d}$, $d \neq c$ where, however, the chain d (by chance) can also be (in the form

[7]And of the other relevant procedures too, see definitions 1–46 in Refs. [3–5].

of) a formula of the language $\mathcal{L}_{\mathcal{P}}$ of the system $\mathcal{P}$.[8] For the considered *NOT-INFERRABILITY* of y_i $[= d]$, being interpreted now from the point of information view, *we put* $J(\Phi(a_{i+1})) \stackrel{\text{Def}}{=} 0$, or better said, with regard of the properties of *INFERENCE*, we are forced to put $\Phi(a_{i+1}) \stackrel{\text{Def}}{=} 0$ within the framework of the theory $\mathcal{T}_{\mathcal{PA}}$ and then, informationally

$$\begin{aligned} H(Y) = T(X;Y) \stackrel{\text{Def}}{=} \ln[5^{\Phi(a_{i+1})}] = 0, &\qquad H(X) = H(X|Y) \\ J(x_i) - J(x_i|y_i) = J(y_i) = 0, &\qquad J(x_i) = J(x_i|y_i) \\ \eta_i \stackrel{\text{Def}}{=} \frac{J(y_i)}{J(x_i)} = \frac{H(Y)}{H(X)}, &\qquad 0 \le \eta_i \le 1 \end{aligned} \tag{8}$$

2.2. Thermodynamic consideration

The thermodynamic consideration of an information transfer [11] reveals that the input message $\overrightarrow{a_{q_i,p_i,i+1}}$ carries the *input heat energy* ΔQ_{W_i} transformed by the *reversible direct Carnot Cycle* (*Machine*) $\mathcal{C}$ into the output *mechanical work* ΔA_i corresponding to the output message a_{i+1}. The *heater* $\mathcal{A}$ of the Carnot Cycle (Machine) $\mathcal{C}$ has the temperature T_W and models the *source* of input messages (the message $\overrightarrow{a_{q_i,p_i,i+1}}$) of the channel $\mathcal{K}$. Its *cooler* $\mathcal{B}$ has the temperature T_0 determining the transfer efficiency η_i. By the value $\eta_i > 0$ the fact of inferrability of the chain a_{i+1} from the special sequence of formulae $\overrightarrow{a_{q_i,p_i,i+1}}$ as the formula of the theory $\mathcal{T}_{\mathcal{PA}}$ is stated.

Thus, the reversible direct Carnot Cycle $\mathcal{C}$ is the *thermodynamic model* of the direct information transfer through the channel $\mathcal{K}$ [11], and hereby of the inferring (*INFERRING*) itself, and also of the arithmetic-syntactic analysis of formulae of the language $\mathcal{L}_{\mathcal{T}\mathcal{P}_{\mathcal{A}}}$ of the theory $\mathcal{T}_{\mathcal{PA}}$.[9] Thus, we have

$$J(x_i) = \frac{\Delta Q_{W_i}}{\mathrm{k}T_W}, \qquad J(x_i|y_i) = \frac{\Delta Q_{0_i}}{\mathrm{k}T_W}, \qquad J(y_i) = \frac{\Delta A_i}{\mathrm{k}T_W} \tag{9}$$

Now we obtain the information formulation [11] of the *changes* of the *heat* (*thermodynamic*) *entropies* $\Delta\mathcal{S}_{\mathcal{C}}{}^{[i]}$, $\Delta\mathcal{S}_{\mathcal{AB}}{}^{[i]}$ and $\Delta\mathcal{S}_{\mathcal{A}}^{[i]}$ in the thermodynamic model $\mathcal{C}$ of our *information transfer—inferring* (*INFERRING*)—*arithmetic-syntactic analysis* within the (language of the) system $\mathcal{P}$,

$$\Delta\mathcal{S}_{\mathcal{C}}{}^{[i]} = \mathrm{k}H(X), \qquad \Delta\mathcal{S}_{\mathcal{AB}}{}^{[i]} = \mathrm{k}H(X|Y), \qquad \Delta\mathcal{S}_{A}{}^{[i]} = \mathrm{k}\cdot[H(X) - H(X|Y)] \tag{10}$$

In accordance with Ref. [11], it is valid that, within the *inferring—arithmetic-syntactic analysis—information transfer*, the thermodynamic entropy $\mathcal{S}_{\mathcal{C}}$ of an *isolated system*, in which the modeling reversible direct Carnot Cycle $\mathcal{C}$ is running parallelly, increases in every *i*th step by the value $\Delta\mathcal{S}_{\mathcal{C}}{}^{[i]}$,

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[8]We just think mistakenly that $d \triangleq a_{i+1}$ but $a_{i+1} = c$ is correct. Then the relation of *Divisibility is not* met. Neither is the relation of the *Immediate Consequence*.

[9]Formulated in the language $\mathcal{L}_{\mathcal{P}}$ of the system $\mathcal{P}$ in compliance with its (with the $\mathcal{T}_{\mathcal{PA}}$) inference rules.

$$\Delta\mathcal{S}_{\mathcal{C}}{}^{[i]} = \mathrm{k}J(a_{i+1}) = \mathrm{k}H(Y)),\ \ H(Y) \triangleq J(a_{i+1}) = \frac{\Delta A_{[i]}}{\mathrm{k}T_W} \geq 0 \tag{11}$$

Provided that the ith inferring step *has been done and written correctly* (*Modus Ponens*) the Goedel arithmetic-syntactic analyzer decides, correctly, for the obtained $\overrightarrow{a_1^{i+1}} \triangleq [\overrightarrow{a_1^i}, a_{i+1}]$, that the relations $\Phi(\overrightarrow{a_{q_i,p_i,i+1}}) B\, \Phi(a_{i+1})$ $[\Phi(\overrightarrow{a_1^{i+1}}) B\, \Phi(a_{i+1})]$ and $Bew[\Phi(a_{i+1})]$ are valid, and the information-thermodynamic model $(\mathcal{K}-\mathcal{C})$ generates the *non-zero, positive* output value $T(X;Y)$ for the inferring step i [for $X := x_i = \Phi(\overrightarrow{a_{q_i,p_i,i+1}})$ or $X := x_i = \Phi(\overrightarrow{a_1^i})$, respectively, and for $Y := y_i = \Phi(a_{i+1})$],

$$T(X;Y) = J(a_{i+1}) = H(Y) = \frac{\Delta\mathcal{S}_{\mathcal{C}}{}^{[i]}}{\mathrm{k}T_W} > 0 \tag{12}$$

The zero change of the whole heat entropy $\mathcal{S}_{\mathcal{C}}$ of the isolated system in which our model cycle $\mathcal{C}$ is running *occurs just when* in the inferential system $\mathcal{P}$, *from the perspective of the theory* $\mathcal{T}_{\mathcal{PA}}$, *nothing is being inferred* in the step i, $\Delta\mathcal{S}_{\mathcal{C}}{}^{[i]} = 0$. Now, particularly in that sense that we mistakenly apply the conclusion of the rule *Modus Ponens* and we declare it to be an inferring step. Then, from the point of view of the $\mathcal{T}_{\mathcal{PA}}$-inference, we do not exert any 'useful effort' or energy in order to derive a new $\mathcal{T}_{\mathcal{PA}}$-relation [formula a_{i+1}, *FORMULA* $\Phi(a_{i+1})$]. The previous 'effort' or energy associated with our inference (no matter that $\mathcal{T}_{\mathcal{PA}}$-correct) of the sequence of $\overrightarrow{a_i^i}$ is worthless. The formula a_{i+1} $[= d]$ is just arbitrarily added to the previous sequence $\overrightarrow{a_1^i}$ of formulae of the theory $\mathcal{T}_{\mathcal{PA}}$ in such a way that it does not include any such formulae a_{q_i} and a_{p_i} that it would be valid $\Phi(a_{p_i,q_i,i+1}) B\, \Phi(a_{i+1}) =$ "1". In the information-thermodynamic interpretation, we write (for $X := x_i$, $Y := y_i = d$)

$$\begin{array}{rcl} J(y_i) = H(Y) = 0 & \Rightarrow & J(x_i) = H(X) = H(X|Y) = J(x_i|y_i) \\ \eta_i = 0 & \Rightarrow & \Delta S_C{}^{[i]} = 0 \\ T_W = T_0 & \Rightarrow & \Delta Q_{W_i} = \Delta Q_{0_i} \\ \eta_i \cdot \Delta Q_{W_i} & = & \mathrm{k} \cdot J(y_i) = 0 \ \Rightarrow\ \eta_i = 0 \end{array} \tag{13}$$

We *have not exerted* any inferring energy within the framework of building up the theory $\mathcal{T}_{\mathcal{PA}}$, in order to create information $J(y_i) > 0$, and then we necessarily have $\eta_i = 0$, $J(y_i) = 0$ where $\eta_i = 0$ expresses this error. All before a_{i+1}, otherwise inferred correctly, is not related to it–the information transfer channel $\mathcal{K}$ is *interrupted*. The overall amount of our inference efforts exerted in vain up to a_i included can be evaluated by the whole heat energy[10]

$$\mathrm{k} \cdot H(X|Y) = \mathrm{k} \cdot \ln[\Phi(\overrightarrow{a_1^i})] = \ln[2^{\Phi(a_1)} \cdot 3^{\Phi(a_2)} \cdot\ \ldots\ \cdot \pi_i^{\Phi(a_i)}] \tag{14}$$

[10] π_i is the i-th prime number.

3. Goedel substitution function and FORMULA 17*Gen r*

Let us consider the *instance* of the relation $Q(X,Y)$ for the specific values x and y, $X := x$ and $Y := y$, which is the *constant* relation $Q(x,y)$, and let us define the Goedel numbers y and y' that the Goedel (variable) number (his *'CLASS' SIGN*) y arises from *Admissible Substitution* from the *FORMULA* $q(17,19)$ [the ARITHMETIZATION of $Q(X,Y)$],

$$y = Sb\begin{pmatrix} q(17,19) & \begin{matrix} 17 \\ \\ Z(x) \end{matrix} \end{pmatrix} = y(19)\ [= \Phi[Q(x,Y)]\ ] \qquad \text{and} \qquad y' = Sb\begin{pmatrix} y & \begin{matrix} 19 \\ \\ Z(y) \end{matrix} \end{pmatrix} \tag{15}$$

Any of the following notations can be used

$$\begin{aligned}
&q(u_1,u_2) = q(17,19) = \Phi[Q(X,Y)] = \Phi[q(u,v)] = \Phi[Q(X,Y)] \\
&q(u_1,u_2) = q(17,19) = \Phi[Q(X,Y)] \\
&q(u_1,u_2),\ q(17,19),\ q(u,v) \triangleq Q(X,Y),\ \ldots \\
&q[Z(x),u_2] = y(u_2) = q[Z(x),19] = y(19) = y = \Phi[Q(x,Y)] \triangleq Q(x,Y)
\end{aligned} \tag{16}$$

The following *Admissible Substitution* $Sb\begin{pmatrix} y & \begin{matrix} 19 \\ \\ Z(y) \end{matrix} \end{pmatrix}$ is carried out in the second step of the given *Double Substitution* $Sb\begin{pmatrix} q & \begin{matrix} 17 & 19 \\ \\ Z(x) & Z(y) \end{matrix} \end{pmatrix}$; in the Goedel variable number $q(17,19)$, we first put $17 := Z(x)$ and in the result $q[Z(x),19]$ we put $19 := Z(y)$. Then

$$y' = y[Z(y)] = [y(19)]_{19:=Z(y)} = q[Z(x),Z(y)] = \Phi[Q(x,y)] \triangleq Q(x,y) \tag{17}$$

The *CLAIM* y' only seems to be a constant $\mathcal{P}/\mathcal{T}_{\mathcal{PA}}$-*FORMULA*, which, as the *CLAIM* $y[Z(y)]$ speaks only about a common number y. But, by the *NUMERAL* $Z(y)]$ it is the y speaking about y and then, it is the *FORMULA* y speaking about itself.

Let us think of the goedelian arithmetic-syntactic *generator*, the job of which is to 'print' the Goedel numbers of the *constant FORMULAE* obtained by *Admissible Substitutions* of *NUMERALS* into their *FREE VARIABLES* (now of the *Type*-1). In case of the 'global' validity of the substitution $19 := Z(y)$[11] it creates from the given *FORMULA* y an infinite sequence of semantically identical *FORMULAE* y' $[= y[Z(y)]]$, $y[Z(y')]$ $[= y[Z[y[Z(y)]]]]$, … with the aim to end the process by 'printing' *just the value* y'. But it never reveals this outcome y'; however, we —metatheoretically—know it. It never gets as far as to print the natural number y' which it 'wants to reach' by creating the infinite sequence of outcomes of the permanently repeated substitution $19 := Z(y)$ which prevents it from this goal (y' marks the claim y about the claim y, the claim y about the claim y about the claim y etc.). It is even the first one, by which the

[11]Caused by the application of the (Cantor) diagonal argument.

analyzer is trying to calculate and 'print' y', that prevents it from this aim. We never obtain a constant Goedel number. The *FORMULA* $y[Z(y)]$ arises by applying the (*Cantor*) *diagonal argument,* which *is not any inference rule* of the theory $\mathcal{T}_{\mathcal{PA}}$ (and of the system $\mathcal{P}$), and thus, it is not an element of the language $\mathcal{L}_{\mathcal{TA}_{\mathcal{P}}}$ (and $\mathcal{L}_{\mathcal{P}}$). This is the reason for *not-recursivity* of the relations $Bew(\cdot)$; the *upper limit* of its computing process is missing. First, we have $q[Z(x), 19]_{19:=Z(y)} \cong q[Z(x),Z(y)] = y[Z(y)] = y'$ *and then 'try'* [12]

$$\begin{aligned}
y' &\cong q[Z(x),Z[q[Z(x),19]]]_{19:=Z(y)} \\
&\cong q[Z(x),Z[q[Z(x),Z(y)]]] = y[Z[y[Z(y)]]] \\
&\cong q[Z(x),Z[q[Z(x),Z[q[Z(x),19]]]]]_{19:=Z(y)} \\
&\cong q[Z(x),Z[q[Z(x),Z[q[Z(x),Z(y)]]]]] \\
&\cong q[Z(x),Z[q[Z(x),Z[q[Z(x),Z[q[Z(x),19]]]]]]]_{19:=Z(y)} \\
&\cong q[Z(x),Z[q[Z(x),Z[q[Z(x),Z[q[Z(x),Z(y)]]]]]]] \\
&\cong q[Z(x),Z[q[Z(x),Z[q[Z(x),Z[q[Z(x),Z[q[Z(x),19]]]]]]]]]_{19:=Z(y)} \;\cong\; \ldots \;\; \textit{ad lib.}
\end{aligned} \tag{18}$$

It is obvious that the *Substitution function*, no matter how much its execution complies with the recursive grammar, is not total and, therefore, nor recursive. For this reason, it is convenient to *redefine it as a total function and, therefore, also recursive one* and to put $[y[Z(y)]] = 0$ but, due to the inference properties, $Neg[y[Z(y)]] = 0$ too. Then

$$\begin{gathered}
\underline{\underline{Sb\begin{pmatrix} & 19 \\ y(19) & \\ & Z(y)\end{pmatrix} \overset{\text{Def}}{=} 0}} \;\&Sb\begin{pmatrix} & 19 \\ Neg[y(19)] & \\ & Z(y)\end{pmatrix} = \underline{\underline{\overline{Sb\begin{pmatrix} & 19 \\ y(19) & \\ & Z(y)\end{pmatrix}} \overset{\text{Def}}{=} 0}} \\
\underline{\underline{Bew[y[Z(y)]] = Bew(0) = 0}}\&\underline{\underline{Bew[Neg[y[Z(y)]]] = Bew(0) = 0}} \\
Q(x,y) \equiv \overline{xB\left[Sb\begin{pmatrix} & 19 \\ y & \\ & Z(y)\end{pmatrix}\right]} = q[Z(x), Z(y)] = y[Z(y)] = y' \triangleq \overline{xB\;y'}
\end{gathered} \tag{19}$$

Also see the Proposition V in Refs. [3–5]. The mere grammar *derivation*, writability convenient to the recursive grammar is quite different from the $\mathcal{T}_{\mathcal{PA}}$-provability. The Goedel number y', the *FORMULA* $y[Z(y)]$, is seemingly a *FORMULA* (and even constant) of the system $\mathcal{P}$ and thus it is not an element of the theory $\mathcal{T}_{\mathcal{PA}}$; is *not of an arithmetic type* (it is not recursive arithmetically, only as for its basic syntax, syntactically). As the *CLAIM* $y[Z(y)]$ it speaks about the number y only, but by that it is the number y itself, then as $y[Z(y)]$, it claims its own property, that from the Goedel number x it itself *IS NOT INFERRED* within the system $\mathcal{P}$ $[Bew(y') = 0]$. It is true for the given x and it 'says': 'I, *FORMULA* $y[Z(y)]$, am in the system $\mathcal{P}$

[12]By substitution $19 := Z(y)$ nothing changes in variability of *FORMULA* y' by the *VARIABLE* 19. The number y' should denote *infinite* and *not recursive* subset of natural numbers or to be equal to them.

(by it means) from the Goedel number x *UNPROVABLE*.' And, by this, it also states both the property of the system $\mathcal{P}$ and the theory $\mathcal{T}_{\mathcal{PA}}$.

3.1. *FORMULA* 17*Gen r* and information transfer

With regard of the fact that *FORMULA* y' is *constructed* by the diagonal argument, it is not *INFERRED* within the system $\mathcal{P}$—in the $\mathcal{T}_{\mathcal{PA}}$ and so, it is not provable for any x from $\mathbb{N}_0$. Then, within the framework of the theory $\mathcal{T}_{\mathcal{PA}}$, we put $\underline{\underline{17Gen\ y' \stackrel{\text{Def}}{=} 0}}$ and thus $\underline{\underline{J(17Gen\ y') \stackrel{\text{Def}}{=} 0}}$.[13] In the proof *we put* $p : 17Gen\ q$, $[17 \cong u_1 \triangleq X, 19 \triangleq u_2 \triangleq Y,\ q = q(17,19)]$, and then, in compliance with the Goedel notation,

$$p = 17Gen\ q(17,19) = \Phi[u_1 \Pi q(u_1,u_2)] \qquad [= \Phi[\forall_{x\in\mathbb{X}} | Q(x,Y)] \triangleq Q(\mathbb{X},Y)] \triangleq Q(\mathbb{N}_0,Y) \tag{20}$$

The *metalanguage symbol* $Q(\mathbb{X},Y)$ *in* (20) *or the symbol* $Q(\mathbb{N}_0,Y)$ *is read as follows*:

'None $x \in \mathbb{X}(\mathbb{N}_0)$ is in the relation *INFERENCE* to the content (to the selective space $\mathbb{Y}$) of the variable Y. From any given $x, x = \Phi(\vec{a}) = \Phi(\overrightarrow{[a_1^k, a_{k+1}]})$, $x \in \mathbb{X}(\mathbb{N}_0)$, any Goedel number $\Phi(a_{k+1}) \neq 0$, writable as the proposed outcome of the *INFERENCE* from the given x, is *NOT INFERRED* in reality.'

$$\textit{We put } r := Sb\begin{pmatrix} q & 19 \\ & Z(p) \end{pmatrix} = Sb\begin{pmatrix} q(17,19) & 19 \\ & Z(p) \end{pmatrix} = Sb\begin{pmatrix} q(17,19) & 19 \\ & Z[17Gen\ q(17,19)] \end{pmatrix}.$$

The Goedel number r, $r \triangleq r(17) = \Phi[Q(X,p)]$ is, by the substitution $Z(p)$, supposingly [3–5], the *CLASS SIGN* with the *FREE VARIABLE* 17, but also remains be the variable Goedel number in the *VARIABLE* 19. It contains the *FREE VARIABLE* 19 as hidden and 17 is both *FREE* and *BOUND* in it, $[q[17, Z[17Gen\ q(17,19)]]$,

$$\begin{aligned} r = r(17) &= q[17, Z[p(19)]] = q[17, Z[17Gen\ q(17,19)]] \triangleq q[u_1, Z(p)] \triangleq Q(X,p) \\ &= q[u_1, \Phi[u_1 \Pi q(u_1,u_2)]] = \Phi[Q[X, \Phi[\forall_{x\in X} | Q(x,Y)]]], \\ &\triangleq Q(X,p) = Q(X,Y)_{Y:=p} \triangleq Q[X, \Phi[Q(\mathbb{X},Y)]] \triangleq Q[X, \Phi[Q(\mathbb{N}_0,Y)]] \end{aligned} \tag{21}$$

Further[14] $Q(X,Y)_{X:=x} = Q(x,Y)$, $Q(x,Y)_{Y:=p} = Q(x,p)$ and then,

[13]From that y' is *NOT INFERRED* follows its *NOT-INFERRABILITY/NOT-PROVABILITY*.

[14]And similarly for $Q(X,Y)_{Y:=p} = Q(X,p)$, $Q(X,p)_{X:=x} = Q(x,p)$. It depends neither on the sequence of substitution steps nor on the sequence of operations $Sb\begin{pmatrix} \cdot & \cdot \\ & \cdot \end{pmatrix}$ and $[\cdot]$ Gen $[\cdot\cdot]$.

$$
\begin{aligned}
r[Z(\mathrm{x})] &= r(17)_{17:=Z(x)} = q[17, Z(p)]_{17:=Z(x)} \\
&= q[Z(\mathrm{x}), Z[17Gen\ q(17,19)]] = q[Z(x), Z(p)]] = q[Z(p)] = q' \\
&= q[Z(x), Z[\Phi[u_1 \Pi q(u_1, u_2)]]] = \Phi[Q[x, \Phi[\forall_{x\in X}|Q(x, Y)]]] \\
&= \Phi[Q[x, \Phi[Q(\mathbb{X}, Y)]]] = \Phi[Q[x, \Phi[Q(\mathbb{N}_0, Y)]]]
\end{aligned} \tag{22}
$$

With regard of quantification $r[Z(x)]$ over values $Z(x)$ of the variable u_1, we write

$$
\begin{aligned}
Z(x)Gen\ r[Z(x)] &= Z(x)Gen\ q[Z(x),Z(p)]] = Z(x)Gen\ p[Z(p)] = Z(x)Gen\ q' \\
&= Z(x)Gen\ q[Z(x),Z[17Gen\ q(17,19)]] = p[Z(p)] = p' \\
&\cong 17Gen\ q[17,Z[17Gen\ q(17,19)]] = 17Gen\ r(17) \\
&= 17Gen\ q[17,Z[p(19)]] = 17Gen\ q[17,Z[17Gen\ q(17,19)]] \\
&= \Phi[u_1\Pi[\Phi[q[u_1,\Phi[u_1\Pi q(u_1,u_2)]]]]] \\
&= \Phi[\forall_{x\in X}|\Phi[Q[x,\Phi[\forall_{x\in X}|Q(x,Y)]]]] = 17Gen\ r \\
&\triangleq Q(\mathbb{X},p) = Q(\mathbb{X},Y)_{Y:=p} \triangleq Q[\mathbb{X},\Phi[Q(\mathbb{X},Y)]] = Q[\mathbb{N}_0,\Phi[Q(\mathbb{N}_0,Y)]]
\end{aligned} \tag{23}
$$

The relation $Q(\mathbb{X},p)$, $Q(\mathbb{X},p) = \forall_{x\in\mathbb{X}}|Q[x,\Phi[\forall_{x\in\mathbb{X}}Q(x,p)]]$ and, therefore, the relation $\overline{T(\mathbb{X},p)}$ says that no such x exists to comply with the message transfer conditions of p from x; the *infinite cycle* is stipulated. Attempts to give the proof of the *FORMULA* 17*Gen r* within the framework of the inferential system $\mathcal{P}$, that is, attempts to 'decide' it *inside* the system $\mathcal{P}$ only by the means of the system $\mathcal{P}$ itself end up in the infinite cycle.

The claim 17*Gen r* does not belong to the theory $\mathcal{T}_{\mathcal{PA}}$ but gives a witness about it—about its property. It is so because it is formulated in a *wider/general formulative language* $\mathcal{L}^{\mathcal{P}*}$ than the language $\mathcal{L}_{\mathcal{P}}$ of the system $\mathcal{P}$ and so outside both of the language $\mathcal{L}_{\mathcal{P}}$ (and as such, outside of the language $\mathcal{L}_{\mathcal{TA}_{\mathcal{P}}}$ too). The *FORMULAE/CLAIMS* of both the theory $\mathcal{T}_{\mathcal{PA}}$ and the system $\mathcal{P}$ speak only about *finite* sets of arithmetic individuals but the theory $\mathcal{T}_{\mathcal{PA}}$ and the system $\mathcal{P}$ are the *countable*–$\mathcal{N}_0$-sets.[15] It seems only that 17*Gen r* is a part (of the *ARITHMETIZATION*) of the theory $\mathcal{T}_{\mathcal{PA}}$ and of the system $\mathcal{P}$ which is by it is written down (grammatically only) according to the common/general recursive syntax of the general formulative language $\mathcal{L}^{\mathcal{P}*}$ in which all the arithmetic relations are written (and, in addition, the $\mathcal{T}_{\mathcal{PA}}$-relations are inferred). On the other hand, there nothing special on its evaluation, *but from the point of view or position of the metalanguage only* (*!*). From the *formalistic* point of view, it is a number only. From the *semantic* point of view, it is an arithmetic *code* but of the not-arithmetic claim.[16]

Let the Goedel number $t[Z(x),Z(y)]$ be *DESCRIPTION* of the mechanism of the transfer y from x (on the level of the system $\mathcal{P}$ and the theory $\mathcal{T}_{\mathcal{PA}}$) in the channel $\mathcal{K}$,

[15]We have, inside of them, only $\mathcal{N}_0$ symbols for denoting their relations/formulae (or sets denoted by these relations/formulae). Thus, the *CLAIM* 17*Gen r* speaks about the element of the set with the cardinality $\mathcal{N}_1$ containing, as its elements, the $\mathcal{N}_0$-sets; thus it can speak about the theory T_{PA}, $\mathcal{N}_0 < \mathcal{N}_1$ and cannot be in it or in the system $\mathcal{P}$.
[16]Thus **it is not a common number** as the [3–5] claims and **neither is** *r*.

$$Sb\begin{pmatrix} & 17 & 19 \\ t & & \\ & Z(x) & Z(y) \end{pmatrix} \cong Subst\ t^{\mathcal{K}}(U_1, U_2)\begin{bmatrix} U_1 & U_2 \\ J(x) & J(y) \end{bmatrix} \equiv [J(x) - J(y) \neq J(x)] \tag{24}$$

But, when it is valid that $\underline{\underline{Sb\begin{pmatrix} & 19 \\ y & \\ & Z(y) \end{pmatrix}}} = 0 = Sb\begin{pmatrix} & 17 & 19 \\ q & & \\ & Z(x) & Z(y) \end{pmatrix}$ then the number $y$is not a FORMULA of the system $\mathcal{P}$ and *in the information interpretation* of inferring (*INFERRING*) within the system $\mathcal{P}$ it is valid that, $\underline{\underline{J(y) = 0}}$. Then we can consider the simultaneous validity of $\underline{\underline{[J(y) > 0]\&[J(y) < 0]}}$—also see the Proposition V in Refs. [3–5], which, from the *thermodynamic* point of view, means the equilibrium and, from the point of *computing*, the *infinite cycle* [14, 16]. For the information variant of the FORMULA 17*Gen r* and Goedel number $p' = p[Z(p)]$ is valid

$$\begin{aligned}
&p' \overset{\text{Def}}{=} Subst\ p(U_2)\begin{pmatrix} U_2 \\ J(p) \end{pmatrix} = Subst\ U_1 Gen\ q^{\mathcal{K}}(U_1, U_2)\begin{pmatrix} U_2 \\ J[U_1 Gen\ q^{\mathcal{K}}(U_1, U_2)] \end{pmatrix} \\
&p' = 17Gen\ q^{K}[U_1, J[U_1 Gen\ q^{K}(U_1, U_2)]] = U_1 Gen\ q^{K}[U_1, J(p)] = p[J(p)] \\
&= U_1 Gen\ r(U_1)\ = U_1 Gen\ r \\
&\cong u_1\Pi[q^{\mathcal{K}}[u_1, J[u_1\Pi q^{\mathcal{K}}(u_1, u_2)]]]] = \forall_{x\in X}|Q^{\mathcal{K}}[X, \Phi[\forall_{x\in X}|Q^{\mathcal{K}}(x, Y)]] \\
&= Q^{\mathcal{K}}(\mathbb{X}, p)] = Q^{\mathcal{K}}(\mathbb{X}, Y)_{Y:=p} = Q^{\mathcal{K}}[\mathbb{X}, \Phi[Q^{\mathcal{K}}(\mathbb{X}, Y)]] = Q^{\mathcal{K}}[\mathbb{N}_0, \Phi[Q^{\mathcal{K}}(\mathbb{N}_0, Y)]]
\end{aligned} \tag{25}$$

So, the message p' (the message p about itself) is not-transferrable from any message x,

$$[\overline{xB^{[\mathcal{K}]}\ p'} = \text{"1"}] \equiv [\overline{xB^{[\mathcal{K}]}\ p} = \text{"1"}] \equiv [\tau^{[\mathcal{K}]}(x,y) = \text{"0"}] \equiv [J(p) = 0] \equiv [J(p') = 0] \tag{26}$$

It is the attempt to transfer the message y ($y = 17Gen\ r$) through the channel $\mathcal{K}$, while this message itself causes its interruption and 'wants' to be transferred through this interrupted channel $\mathcal{K}$ as well.[17] Its 'errorness' is in our awaiting of the non-zero outcome $J(y) > 0$ when it is applied in the (direct) transfer scheme $\mathcal{K}$ because the information $J(y) > 0$, $y = 17Gen\ r$ (known from and valid in the metalanguage), from the point of transferrability through the channel $\mathcal{K}$ (from the point of inferrability in the theory $\mathcal{T}_{\mathcal{PA}}$) does not exist. *In the theory,* $\mathcal{T}_{\mathcal{PA}}$ *is* $J(y) = 0$ for the *CLAIM* 17*Gen r* is not arithmetic at all, it is the *metaarithmetic* one. From the point of the theory $\mathcal{T}_{\mathcal{PA}}$ and the system $\mathcal{P}$, it is not quite well to call *CLAIM* 17*Gen r* as the *SENTENTIAL FORMULA*; it has only such form. For this reason, we use the term *CLAIM* 17*Gen r* or *'SENTENTIAL FORMULA'/'PROPOSITION.'*

The message about that the channel $\mathcal{K}$ is for y interrupted *cannot* be transferred *through the same channel* $\mathcal{K}$ *interrupted for* y (however, through another one, uninterrupted for y, it can). Or we can say that the claim a_{k+1} [*CLAIM* y, $y = \Phi(a_{k+1}) = 17Gen\ r$] is not inferable (*INFERABLE*) in the given inferential system $\mathcal{P}$ (but in another one making its construction-*INFERENCE* possible, it is),

[17] In fact, it represents the very core of the sense of the **Halting Problem** task in the Computational Theory.

$$\overline{\exists_{x\in\mathbb{X}}|t^{\mathcal{K}}[J(x),J[\exists_{x\in\mathbb{X}}t^{\mathcal{K}}[J(x),J(y)]]] > 0} \equiv \overline{T(\mathbb{X},y) > 0} \equiv Q(\mathbb{X},y) \tag{27}$$

By constructing the *FORMULA* 17*Gen r* and from the point of information transfer, we have produced the claim 'the transfer channel $\mathcal{K}$ is from p' and on interrupted.' Or, we have made the interrupted transfer channel directly by this p' when we assumed it belonged to the set of messages transferrable from the source X. So, first we interrupt the channel $\mathcal{K}$ for p', and then, we want to transfer this p' from the input x which includes this p' (or is identical to it), and so the *internal* and *input* state of the channel $\mathcal{K}$ are (also from the point of the theory $\mathcal{T}_{\mathcal{PA}}$) equivalent informationally. It is valid that $\underline{\underline{J(p') = 0}}$ for any x, $x \in \mathbb{X}$ [so $\forall_{c\in\mathbb{X}}|[J(p') = 0]$],

$$\begin{gathered}
\forall_{x\in\mathbb{X}}|[J(x) = J(x|p')] \cong [J(\mathbb{X}|p') = J(\mathbb{X}) > 0] \ \text{ and for the simplicity is } \ J(p'|\mathbb{X}) = 0 \\
\left[\forall_{x\in\mathbb{X}}|\overline{\tau(x,p')}\right] \equiv \left[\forall_{x\in\mathbb{X}}|\overline{[J(x) - J(x|p') > 0]}\right] \equiv \left[\overline{\exists_{x\in\mathbb{N}_0}|[J(x) - J(x|p') > 0]}\right] = \text{"1"} \\
x = \Phi(\overrightarrow{a_1^k}) * 17Gen\ r = \Phi(\overrightarrow{a_1^k}) * Sb\begin{pmatrix} & 19 \\ p & \\ & Z(p)\end{pmatrix} = \Phi([a_1^k], p') \cong \Phi([a_1^k], null) \\
J(x) = J[\Phi(\overrightarrow{a_1^k}) * \Phi(0)] = J[\Phi([a_1^k]) \cdot 2^0] = J(x|p) = J[\Phi([a_1^k])], \quad x|p = \Phi([a_1^k])
\end{gathered} \tag{28}$$

The channel $\mathcal{K}$, however, always works only with the not zero and the positive difference of information amounts $J(x) - J(x|y)$ and in the theory $\mathcal{T}_{\mathcal{PA}}$ now it is valid that $J(y) = J(x) - J(x|p') = J(null) = 0$, $J(y) = J(p') = J(null) = 0$[18]. It means that our assumption about p' $[= r]$ is erroneous. No input message x having a relation to the output message p' exists. The *FORMULA* 17*Gen r* both creates and describes behavior of the not functioning (interrupted) information transfer, from p' on further. For the *efficiency* η of the information transfer, it is then valid [14, 16] that

$$\eta = \frac{J(p)}{J(\mathbb{X})} = 0 \tag{29}$$

The CLAIM ('SENTENTIAL PROPOSITION') 17*Gen r we interpret as follows:*

- No information transfer channel $\mathcal{K}$ transfers its (internal) state $x|y$ [the information $J(x|y)$] given as its input message x, it behaves as interrupted.
- There is no $x \in \mathbb{N}_0$ for which it is possible to generate the Goedel number $\Phi[Q(\mathbb{N}_0, Y)]$ which claims that there is no $x \in \mathbb{N}_0$ for which it is possible to generate the non-zero Goedel number y that we could write into the variable Y. This means that from any Goedel number x no *INFERENCE* is possible just for its latest part $y = \Phi(a_{k+1}) = 17Gen\ r$ has not been *INFERRED* either.

[18]Attention (!) but x contains the message p that $J(x) = J(x|p)$.

The metaarithmetic sense of the CLAIM ('SENTENTIAL PROPOSITION') 17*Gen r is*:

- Within the *general formulative language* $\mathcal{L}^{\mathcal{P}*}$ of the inconsistent metasystem $\mathcal{P}^*$ (containing the consistent subsystem $\mathcal{P}$ with the theory the $\mathcal{T}_{\mathcal{PA}}$) it is possible to construct [be the (Cantor) diagonal argument] such a claim (with the Goedel arithmetization code 17*Gen r*) which is true, but both this claim i and its negation are not provable/*PROVABLE* by the means of the system $\mathcal{P}$ (in the system $\mathcal{P}$) and thus, also in the theory $\mathcal{T}_{\mathcal{PA}}$–they are the meta-$\mathcal{T}_{\mathcal{PA}}$ and the meta-$\mathcal{P}$ claims not belonging to the system $\mathcal{P}$, but they belong to the inconsistent system $\mathcal{P}^*$, to its part $\mathcal{P}^* - \mathcal{P}$ ($\mathcal{P}^* \underset{\neq}{\supset} \mathcal{P}$).
- So, the Goedel Proposition $VI\ldots$ (1931) [3–5] should be, correctly, 'For the system exists …' (which Goedel also, but not uniquely says), 'For the theory exists, (nevertheless outside of them); by the author's conviction the error is to say.' In each consistent (?) system exists … or, even 'In the consistent (?) theory exists ….'

4. Conclusion

Peano arithmetic theory is generated by its inferential rules (rules of the inferential system in which it is formulated). It consists of parts bound mutually just by these rules but none of them is not identical with it nor with the system in their totality.

By information-thermodynamic and computing analysis of Peano arithmetic proving, we have showed why the Goedel formula and its negation are not provable and decidable within it. They are constructed, not inferred, by the (Cantor) diagonal argument which is not from the set of the inferential rules of the system. The attempt to prove them leads to awaiting of the end of the infinite cycle being generated by the application of the substitution function just by the diagonal argument. For this case, the substitution function is not countable, and for this, it is not recursive (although in the Goedel original definition is claimed that it is). We redefine it to be total by the zero value for this case. This new substitution function generates the Goedel numbers of chains which are not only satisfying the recursive grammar of formulae but it itself is recursive. The option of the zero value follows also from the vision of the inferential process as it would be the information transfer. The attempt to prove the Goedel Undecidable Formula is the attempt of the transfer of that information which is equal to the information expressing the inner structure of the information transfer channel. In the thermodynamic point of view we achieve the equilibrium status which is an equivalent to the inconsistent theory. So, we can see that the Goedel Undecidable Formula is not a formula of the Peano Arithmetics and, also, that it is not an arithmetical claim at all. From the thermodynamic consideration follows that even we need a certain effort or energy to construct it, within the frame of the theory this is irrelevant. It is the error in the inference and cannot be part of the theory and also it is not the system. Its information value in it (as in the system of the information transfer) is zero. But it is the true claim about inferential properties of the theory (of the information transfer).

We have shown that the *CLAIM/'FORMULA'* 17*Gen r*, no matter how much it complies with the grammar of recursive writing of $\mathcal{T}_{\mathcal{PA}}$-arithmetic *FORMULAE*, is not such a *FORMULA*; it is not an element of the theory $\mathcal{T}_{\mathcal{PA}}$ and in convenience with [1, 2, 6, 7, 18, 19] nor an element of the system $\mathcal{P}$[19] and neither is r. The same is for $Neg(17Gen\ r)$ (it cannot be inferred in $\mathcal{P}$ for is not inferable in $\mathcal{P}$.) Nevertheless, we are in accordance with the *intuitive and obviously intended sense* of the Goedel Proposition VI[20] which we, as the metalanguage one, have proved by metalanguage (information-thermodynamic-computing) means. We see, with our correction, that the *CLAIMS* (the Goedel *'SENTENTIAL PROPOSITIONS'/'FORMULAE'*) $17Gen\ r$, $Neg(17Gen\ r)$ and the Proposition VI as the claim about them are metaarithmetic (methodological) statements.

5. Appendix

5.1. Auto-reference in information transfer, self-observation

In any *information transfer channel* $\mathcal{K}$ the *channel equation*

$$H(X) - H(X|Y) = H(Y) - H(Y|X) \tag{30}$$

it is valid [?]. This equation describes the mutual relations among *information entropies [(average) information amounts]* in the channel $\mathcal{K}$.

The quantities $H(X)$, $H(Y)$, $H(X|Y)$ and $H(Y|X)$ are the *input*, the *output*, the *loss* and the *noise* entropy.

The difference $H(X) - H(X|Y)$ or the difference $H(Y) - H(Y|X)$ defines the *transinformation* $T(X;Y)$ or the transinformation $T(Y;X)$, respectively,

$$H(X) - H(X|Y) \triangleq T(X;Y) \quad = \quad T(Y;X) \triangleq H(Y) - H(Y|X) \tag{31}$$

When the channel $\mathcal{K}$ transfers the information (entropy) $H(X)$, but now just at the value of the entropy $H(X|Y)$, $H(X) = H(X|Y)$, then, necessarily, must be valid

$$T(X;Y) = 0 \quad [= H(Y) - H(Y|X)] \tag{32}$$

- For $H(Y|X) = 0$, we have $T(X;Y) = H(Y) = 0$.
- For $H(Y|X) \neq 0$ we have $H(Y) = H(Y|X) \neq 0$

In both these two cases, the channel $\mathcal{K}$ operates as the *interrupted* (*with the absolute noise*) and the output $H(Y)$ is without any relation to the input $H(X)$ and, also, it does not relate to the structure of $\mathcal{K}$. This structure is expressed by the value of the quantity $H(X|Y)$. We assume, for simplicity, that $H(Y|X) = 0$.

[19]In the contrast to Refs. [3–5].

[20]Because, on the other hand, Goedel 1931 [3–5] also says, correctly, '**For** the system exists …,' '**For** the theory exists …, (**nevertheless outside of them** - the author's remark); the **error** is to say **in** the system exists …, **in** the theory exists ….'

From Eqs. (30) to (32) follows that the channel $\mathcal{K}$ cannot transfer (within the same step p of its *transfer process*) such an information which describes its inner structure and, thus, it cannot transfer—observe (copy, measure) itself. It is valid both for the concrete information value and for the average information value, as well.

Any channel $\mathcal{K}$ cannot transfer its own states considered as the input messages (within the same steps p). Such an attempt is the information analogy for the Auto-Reference known from Logics and Computing Theory. Thus, a certain 'step-aside' leading to a non-zero transfer output, $H(Y) = H(X) - H(X|Y) > 0$, is needed. (For more information see [14, 15, 16].

5.2. Auto-reference and thermodynamic stationarity

The transfer process running in an information transfer channel $\mathcal{K}$ is possible to be comprehended (modeled or, even, constructed) as the *direct* Carnot Cycle $\mathcal{O}$ [8, 10]. The relation $\mathcal{O} \cong \mathcal{K}$ is postulated. Further, we can imagine its observing method, equivalent to its 'mirror' $\mathcal{O}'' \cong \mathcal{K}''$. This *mirror* $\mathcal{O}''$ is, at this case, the direct Carnot Cycle $\mathcal{O}$ as for its structure, but functioning in the *indirect, reverse* mode [8, 10].

Let us connect them together to a *combined heat cycle $\mathcal{O}\mathcal{O}''$ in such a way that the mirror* (*the reverse cycle $\mathcal{O}''$*) is gaining the message about the structure of the direct cycle $\mathcal{O}$. This message is (carrying) the information $H(X|Y)$ about the structure of the transformation (transfer) process ($\mathcal{O} \cong \mathcal{K}$) being 'observed.' *The mirror $\mathcal{O}'' \cong \mathcal{K}''$ is gaining this information $H(X|Y)$ on its noise 'input' $H(Y''|X'')$ [while $H(X'') = H(Y)$ is its input entropy].*

The quantities ΔQ_W, ΔA and ΔQ_0 or the quantities $\Delta Q''_W$, $\Delta A''$ and $\Delta Q''_0$, respectively, define the information entropies of the information transfer realized (thermodynamically) by the *direct* Carnot Cycle $\mathcal{O}$ or by the *reverse* Carnot Cycle $\mathcal{O}''$ (the mirror), respectively, (the *combined* cycle $\mathcal{O}\mathcal{O}''$ is created),

$$\begin{aligned} H(X) &= \frac{\Delta Q_W}{\mathrm{k}T_W},\ resp.\ \ H(Y'') = \frac{\Delta Q''_W}{\mathrm{k}T''_W} \\ H(Y) &= \frac{\Delta A}{\mathrm{k}T_W},\ resp.\ H(X'') = \frac{\Delta A''}{\mathrm{k}T''_W} \\ H(X|Y) &= \frac{\Delta Q_0}{\mathrm{k}T_W},\ resp.\ \ H(Y''|X'') = \frac{\Delta Q''_0}{\mathrm{k}T''_W} \end{aligned} \tag{33}$$

Our aim is to gain the *non-zero* output mechanical work ΔA^* of the combined heat cycle $\mathcal{O}\mathcal{O}''$, $\Delta A^* > 0$. We want to gain non-zero information $H^*(Y^*) = \frac{\Delta A^*}{\mathrm{k}T_W} > 0$.

To achieve this aim, for the efficiencies η_{max} and η''_{max} of the both connected cycles $\mathcal{O}$ and $\mathcal{O}''$ (with the working temperatures $T_W = T''_W$ and $T_0 = T''_0$, $T_W \geq T_0 > 0$), it must be valid that $\eta_{max} > \eta''_{max}$; we want the validity of the relation[21]

[21]We follow the proof of *physical and thus logical impossibility* of the construction and functionality of the *Perpetuum Mobile* of the *II*. and, equivalently [10], of the *I*. type.

$$\Delta^* A = \Delta A - \Delta A'' > 0 \ \left[\Delta A'' = \Delta Q''_W - \Delta Q''_0\right] \tag{34}$$

When $\Delta Q_0 = \Delta Q''_0$ should be valid, then must be that $\Delta Q''_W < \Delta Q_W$ $[\Leftarrow (\eta_{max} > \eta''_{max})]$, and thus, it should be valid that

$$\begin{aligned} \Delta A^* = \Delta Q_W \cdot \eta_{max} - \Delta Q''_W \cdot \eta''_{max} > 0 \qquad \text{but} \\ \Delta Q_W \cdot \eta_{max} - \Delta Q''_W \cdot \eta''_{max} = \Delta Q_0 - \Delta Q''_0 = 0 \end{aligned} \tag{35}$$

Thus, the output work $\Delta A^* > 0$ *should be generated* without any lost heat and by the *direct change* of the whole heat $\Delta Q_W - \Delta Q''_W$ but within the cycle $\mathcal{O}\mathcal{O}''$. For $\eta_{max} < \eta''_{max}$ the same heat $\Delta Q_W - \Delta Q''_W$ should be pumped from the cooler with the temperature T_0 to the heater with the temperature T_W *directly*, without any compensation by a mechanical work. We see that $\Delta A^* = 0$ is the reality.

Our combined machine $\mathcal{O}\mathcal{O}''$ should be the *II*. *Perpetuum Mobile* in both two cases. Thus, $\eta_{max} = \eta''_{max}$ must be valid (the heater with the temperature T_W and the cooler with the temperature T_0 are common) that

$$\eta_{max} = \eta''_{max} < 1 \ \text{ and then } \ \Delta Q_W = \Delta Q''_W \tag{36}$$

We must be aware that for $\eta_{max} = \eta''_{max} < 1$ the whole information entropy of the environment in which our (reversible) combined cycle $\mathcal{O}\mathcal{O}''$ is running changes on one hand *by the value*

$$H(X) \cdot \eta_{max} = \frac{\Delta Q_W}{\mathrm{k}T_W} \cdot (1 - \beta) > 0, \qquad \beta = 1 - \eta_{max} = \frac{T_0}{T_W} \tag{37}$$

and on the other hand it is also changed *by the value* $-H(X) \cdot \eta_{max} = -\frac{\Delta Q_W}{\mathrm{k}T_W} \cdot (1 - \beta)$ Thus, it must be changed by the *zero* value

$$H^*(Y^*) = \frac{= \Delta A^*}{\mathrm{k}T_W} H(X) \cdot \eta_{max} - H(Y'') \cdot \eta''_{max} = H(X) \cdot \left(\eta_{max} - \eta_{max}\right) = 0 \tag{38}$$

The whole combined machine or the thermodynamic system with the cycle $\mathcal{O}\mathcal{O}''$ is, when the cycle $\mathcal{O}\mathcal{O}''$ is seen, as a whole, in the *thermodynamic equilibrium*. (It can be seen as an unit, analogous to an interruptable operation in computing.)

Thus, the observation of the observed process $\mathcal{O}$ by the observing reverse process $\mathcal{O}''$ with the same structure (by itself), or the Self-Observation, is impossible in a physical sense, and, consequently, in a logical sense, too (see the Auto-Reference in computing).

Nevertheless, the construction of the Auto-Reference is describable and, as such, is recognizable, *decidable* just as a construction *sui generis*. It leads, necessarily, to the requirement of the *II*. Perpetuum Mobile functionality when the requirements (34) and (35) are sustained.

(Note that the Carnot Machine itself is, by its definition, a construction of the infinite cycle of the states of its working medium and as such is identifiable and recognizable.) For the methodological step demonstrating the *Information Thermodynamic Concept Removing* see [14, 15, 16].

5.3. Gibbs paradox - auto-reference in observation

Only just by a (thought) 'dividing' of an equilibrium system $\mathcal{A}$ by *diaphragms* [9, 10, 11, 13], without any influence on its thermodynamic (macroscopic) properties, a non-zero difference of its entropy, before and after its 'dividing,' is evidenced.

Let us consider a thermodynamic system $\mathcal{A}$ in volume V and with n matter units of ideal gas in the thermodynamic equilibrium. The *state equation* of $\mathcal{A}$ is $pV = nR\Theta$. For an elementary change of the *internal* energy U of $\mathcal{A}$, we have $\mathrm{d}U = nc_v\mathrm{d}\Theta$.

From the state equation of $\mathcal{A}$, and from the general *law of energy conservation* [for a (substitute) reversible exchange of heat δq between the system and its environment], we formulate the *I. Principle of Thermodynamics*, $\delta q = \mathrm{d}U + p\mathrm{d}V$

From this principle, and from *Clausius equation* $\Delta S \overset{\mathrm{Def}}{=} \frac{\Delta q}{\Theta}$, $\Delta q = c_v \Delta\Theta + \frac{R\Theta\Delta V}{V}$, $\Theta > 0$, follows that

$$S = n\int\left(c_v\frac{\mathrm{d}\Theta}{\Theta} + R\frac{\mathrm{d}V}{V}\right) = n(c_v \ln\Theta + R\ln V) + S_0(n) = \sigma(\Theta,V) + S_0(n) \qquad (39)$$

Let us 'divide' the equilibrial system $\mathcal{A}$ in a volume V and at a temperature Θ, or, better said, the whole volume V (or, its whole state space) occupiable, and just occupied now by all its constituents (particles, matter units), with diaphragms (thin infinitely, or, 'thought' only), not affecting thermodynamic properties of $\mathcal{A}$ supposingly, to m parts $\mathcal{A}_i$, $i \in \{1, \ldots, m\}$, $m \geq 1$ with volumes V_i with matter units n_i. Evidently $n = \sum_{i=1}^{m} n_i$ and $V = \sum_{i=1}^{m} V_i$.

Let now $S_0(n) = 0$ and $S_{0i}(n_i) = 0$ for all i. For the entropies S_i of $\mathcal{A}_i$ considered individually, and for the change ΔS, when volumes V, V_i are expressed from the state equations, and for $p = p_i$, $\Theta = \Theta_i$ it will be gained that $\sigma_{[i]} = Rn_{[i]}\ln n_{[i]}$. Then, for $S_i = \sigma_i = n_i(c_v\ln\Theta + R\ln V_i)$ is valid, we have that

$$\begin{aligned} &\sum_{i=1}^{m} S_i = \sum_{i=1}^{m}\sigma_i = nc_v\ln\Theta + R\ln\left(\prod_{i=1}^{m} {V_i}^{n_i}\right), \\ &\Delta S = S - \sum_{i=1}^{m} S_i = \sigma - \sum_{i=1}^{m}\sigma_i = \Delta\sigma = R\ln\frac{V^n}{\prod_{i=1}^{m} {V_i}^{n_i}} = -nR\sum_{i=1}^{m}\frac{n_i}{n}\ln\frac{n_i}{n} > 0 \end{aligned} \qquad (40)$$

Let us denote the last sum as B further on, $B < 0$. The quantity $-B$ expressed in (40) is information entropy of a source of messages with an alphabet $[n_1, n_2, \ldots, n_m]$ and probability distribution $\left[\frac{n_i}{n}\right]_{i=1}^{m}$. Such a division of the system to m parts defines an information source with the information entropy with its maximum $\ln m$.

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The result (37), $\Delta S = -nRB$, is a *paradox*, a contradiction with our presumption of not influencing a thermodynamic state of $\mathcal{A}$ by diaphragms, and, leads to that result that the heat entropy S (of a system in equilibrium) *is not* an extensive quantity. But, by the definition of the differential $\mathrm{d}S$, this *is not* true.

Due to this contradiction, we must consider a non-zero integrating constants $S_0(n)$, $S_{0i}(n_i)$, in such a way, that the equation $\Delta S = (\sigma + S_0) - \sum_{i=1}^{m}(\sigma_i + S_{0i}) = 0$ is solvable for the system $\mathcal{A}$ and all its parts $\mathcal{A}_i$ by solutions $S_{0[i]}(n_{[i]}) = -n_{[i]}R\ln\frac{n_{[i]}}{\gamma_{[i]}}$.

Then, $S_{[i]} \triangleq S_{[i]}^{Claus}$, and we write and derive that

$$S^{Claus} = \sum_{i=1}^{m} S_i^{Claus} = \sum_{i=1}^{m} n_i R\ln\gamma_i = nR\ln\gamma \Rightarrow \gamma = \gamma_i; \qquad \Delta S = 0. \tag{41}$$

Now let us observe an equilibrium, $S^* = S^{Claus} = S^{Boltz} = -kNB* = -kN\ln N$.

Let, in compliance with the *solution* of Gibbs Paradox, the integration constant S_0 be the (change of) entropy ΔS which is added to the entropy σ *to figure out* the measured entropy S^{Claus} of the equilibrium state of the system $\mathcal{A}$ (the final state of Gay-Lussac experiment) at a temperature Θ. We have shown that without such correction, the less entropy σ is evidenced, $\sigma = S^{Claus} - \Delta S,\ \Delta S = S_0$.

Following the previous definitions and results, we have

$$\begin{aligned} &\Delta S = \frac{\Delta Q_0}{\Theta} = -nR\ln\frac{n}{\gamma}, \\ &\ln\gamma = \frac{\Delta S}{knN_A} + \ln n = \frac{\Delta S}{kN} + \ln N - \ln N_A,\ \gamma = N \ \Rightarrow\ \frac{\Delta S}{kN} = \ln N_A. \end{aligned} \tag{42}$$

By the entropy ΔS the 'lost' heat ΔQ_0 (at the temperature Θ) is defined.

Thus, our observation can be understood as an information transfer $\mathcal{T}$ in an information channel $\mathcal{K}$ with entropies $H(X)$, $H(Y)$, $H(X|Y)$ and $H(Y|X)$ in (33) but now bound physically; we have these information entropies per one particle of the observed system $\mathcal{A}$:

$$\begin{aligned} &\textit{input} \quad H(X) \overset{\text{Def}}{=} \frac{S^*}{kN} = \ln\gamma = -B* = \ln N = -rB(r) \\ &\textit{output} \quad H(Y) \overset{\text{Def}}{=} \frac{\sigma}{kN} \triangleq -B^{Gibbs} = -B^{Boltz} = -B(r), \\ &\textit{loss} \quad H(X|Y) \overset{\text{Def}}{=} \frac{S_0}{kN}, \\ &\textit{noise} \quad H(Y|X) \overset{\text{Def}}{=} 0 \quad \text{for the simplicity ;} \\ &H(X|Y) = -rB(r) - [-B(r)] = -B(r)\cdot(r-1) = (-B*)\cdot\frac{r-1}{r},\ r \geq 1;\quad \frac{1}{r} = \eta_{max}. \end{aligned} \tag{43}$$

For a number m of cells of our railings in the volume V with $\mathcal{A}$, $m \leq N$ or for the accuracy r of this description of the 'inner structure' of $\mathcal{A}$ (a thought structure of V with $\mathcal{A}$) and for the number q of diaphragms creating our railings of cells and constructed in such a way that $q \in <1, m-1>$, we have that $r = \frac{N-1}{q}$.

Our observation of the equilibrium system $\mathcal{A}$, including the *mathematical correction* for Gibbs Paradox, is then describable by the Shannon transfer scheme $[X, \mathcal{K}, Y]$, where

$$H(X) = \frac{S^{Claus}}{kN}, \qquad H(X|Y) = \frac{S_0}{kN}, \qquad H(Y) = \frac{S^{Claus}}{kN}, \qquad H(Y|X) = \frac{\Delta S}{kN}. \tag{44}$$

However, a real observation process described in (44), equivalent to that one with $r = 1$, is impossible.

We conclude by that, the diminishing of the measured entropy value about ΔS against S^* awaited, evidenced by Gibbs Paradox, *does not originate in a watched system itself*. Understood this way, it is a contradiction of a *gnozeologic character* based on not respecting *real* properties of any observation [8–10].

With our sustaining on the 'fact' of the Gibbs Paradox reality also mean the circulating value of ΔS (in our brain) just depending on our starting point of thinking about the observed system with or without the (thought) railings. Simultaneously (&) and in the cycle our brain would have $[\Delta S < 0]$ & $[\Delta S > 0]$–see the validity of the Goedel Proposition V [3–5] for the inconsistent system $\mathcal{P}^*$.

This and, also, **Figure 1**, is the thermodynamic equivalent to the paradoxical understanding to the Goedel Incompleteness Theorems, also known as the Goedel Paradox. In fact, both

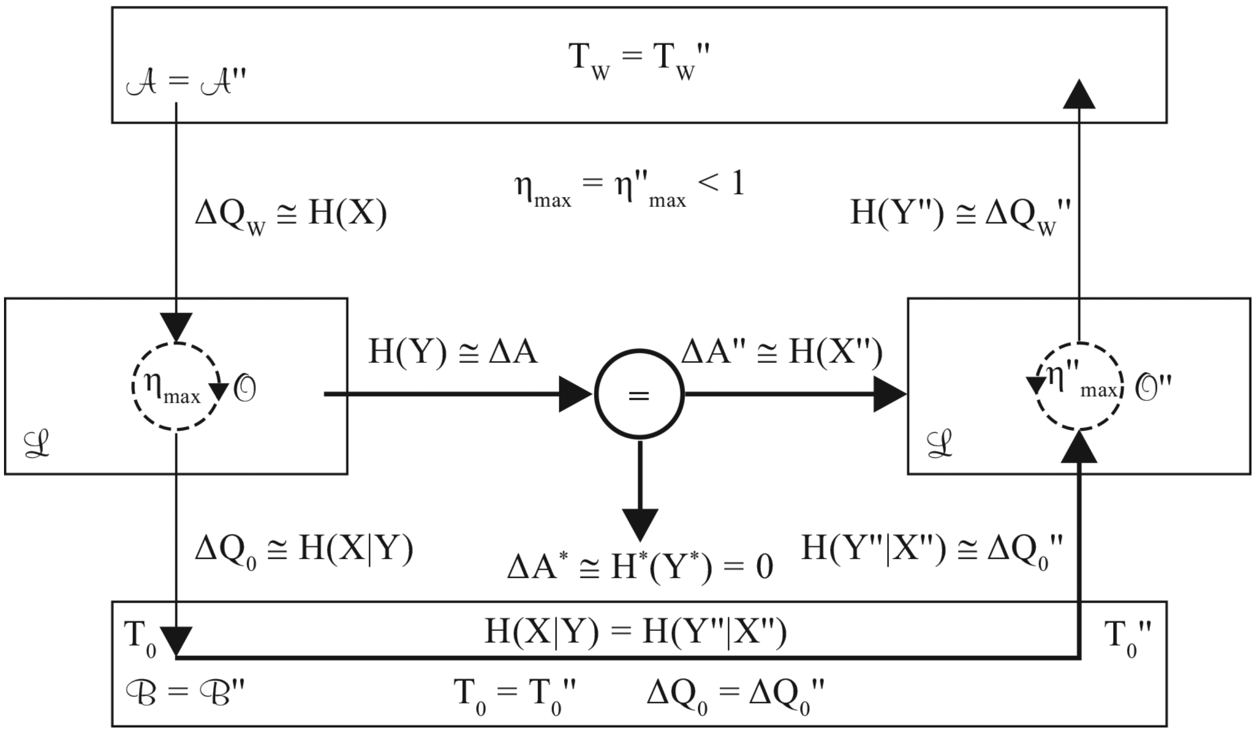

Figure 1. Stationarity of the double cycle $\mathcal{O}\mathcal{O}''$.

paradoxes do not exist in the described reality—they are in our brain, caused by the mixing of (our) consideration levels (the higher or methodology level and the lower or object/theoretical level) and, also, reveal themselves as the contradictions (on the lower level).

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