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# An Example Usage of Graph Theory in Other Scientific Fields: On Graph Labeling, Possibilities and Role of Mind/Consciousness

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Additional information is available at the end of the chapter

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## Abstract

This paper provides insights into some aspects of the possibilities and role of mind, consciousness, and their relation to *mathematical logic* with the application of problem solving in the fields of psychology and graph theory. This work aims to dispel certain long-held notions of a severe psychological disorder and a well-known graph labeling conjecture. The applications of graph labelings of various types for various kinds of graphs are being discussed. Certain results in graph labelings using computer software are presented with a direction to discover more applications.

**Keywords:** mathematical logic, graph labeling, magic, antimagic, inner magic, inner antimagic, graceful, harmonious, felicitous, sequential, NP-Complete

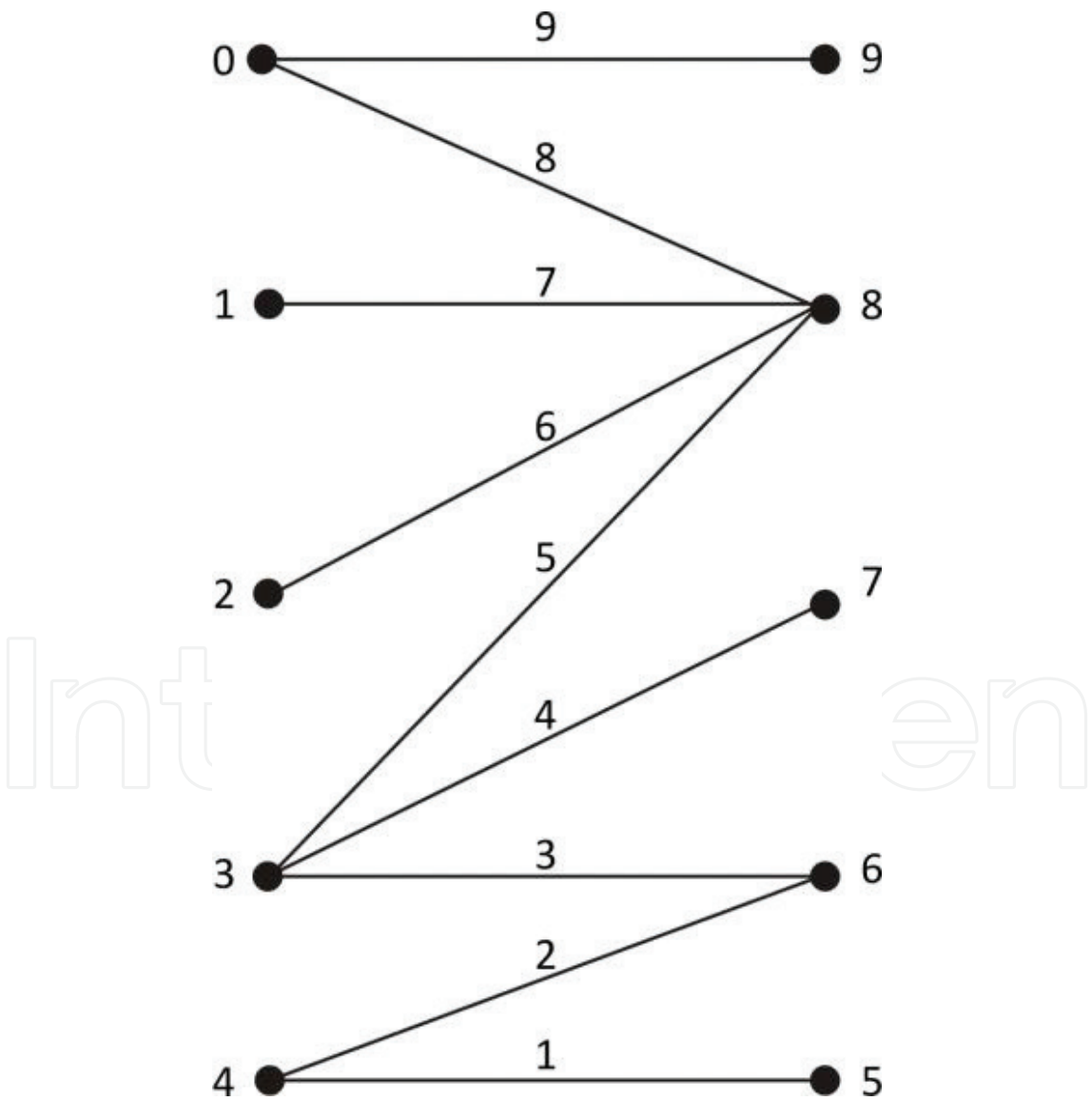
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## 1. Introduction

In this paper, we begin with looking at the problems and problem solving elements of *mathematical logic* of the field of Discrete Mathematics (of which graph theory is a subject) and its application in the realms of mind and consciousness. Section 2 focuses on dispelling certain notions about a severe psychological disorder namely *paranoid schizophrenia*. It is a much debated and often misunderstood illness affecting many people. Many remain untreated and undiagnosed not even able to adequately articulate their thoughts and feelings causing them a severe debility and an untold suffering.

Generally, the mind and consciousness aspects have been nearly left out from Mathematics for various reasons but mostly studied in other fields. Perhaps, the first instance (as per the available

literature) in solving a major problem in Mathematics providing an answer using *mathematical arguments* factoring in the role of mind/consciousness is given in Ref. [1] resolving the *graceful tree conjecture*. In Section 3, we look at some preset notions in assuming this conjecture to be true and some new insights into the work in Ref. [1]. This conjecture was widely believed to be true for about five decades that all trees are graceful. This conjecture was believed to be true by many researchers in the field of Graph Theory particularly in graph labelings even without proof. A *graceful labeling* in a graph with  $p$  vertices(points) and  $q$  edges(lines) is assigning the numbers (labels) to the vertices  $0,1,2,\dots,(q-1)$  such that the induced edge labels found by taking the absolute values (the positive value) are from  $1,2,\dots,q$  (**Figure 1**). This is an interesting area of Graph Theory as well as Psychology to be studied that why this conjecture was believed to be true even without proof for such a long time and that too in the field of Mathematics which is often considered by many to be so cut and dry in terms of proofs and truth of ideas.



**Figure 1.** Graceful bipartite tree where edges do not cross.

In Section 4, certain new aspects of applications of *antimagic*, *inner magic*, and *inner antimagic* labelings and other kinds of labelings given in Refs. [2, 3] are presented. The *inner magic* and *inner antimagic* are new kinds of labelings given in Ref. [4]. In Section 5, some results using computer software in the field of graph labelings are discussed. In Section 6, a discussion of the *lexicographic order* in graph labelings is presented with a direction to future study. At last, conclusion is given in Section 7.

## 2. Resolving aspects of paranoid schizophrenia

During the course of one's life, one comes across various psychological problems, sometimes of a severe nature of psychological disorder like *schizophrenia*. *Schizophrenia* has perplexed researchers, thinkers, doctors, philosophers for centuries, though the systematic study of *schizophrenia* is approximately 100 years old and has been called "arguably the worst disease affecting mankind, even AIDS not excepted (Nature 1988)" in Ref. [5]. In this section, we would focus on understanding *schizophrenia* particularly *paranoid schizophrenia*. The clinical symptoms are characterized by the following major symptoms as in [5–7].

1. Delusions of persecution like the person believing others conspiring against him/her or grandiosity.
2. Delusion of exalted birth like the belief that one is born with a messianic role.
3. Delusions of reference like believing that television, newspapers are referring to the patient in particular.
4. Delusions of jealousy/infidelity: hearing voices of threatening nature which give commands to patient.
5. In Ref. [7], "People with schizophrenia often have delusions, firmly held, unshakable beliefs with no basis in reality."
6. Hallucinations ("experience of perceiving things that do not actually exist" in [7]) of non-verbal auditory nature like laughing, humming, etc.
7. Hallucinations through other somatic sensations like smell, sight, taste, etc.
8. Withdrawal or hesitation from interacting with other people and feeling isolated.
9. In Ref. [5], "The delusions are usually well-systematized (i.e., thematically well connected with each other)."
10. In Ref. [5], under the first rank symptoms (SFRs) of *schizophrenia*.
  - (a) Voices heard arguing: two or more hallucinatory voices discussing the subject with third person.
  - (b) 'Made' volition of acts: In made effect, impulses and volitions, the person experiences feelings, impulses, or acts which are imposed by some external force. In "made"

volition, for example, one's own acts are experienced as being under control of some external forces.

(c) Somatic passivity: Bodily sensations, especially sensory symptoms are experienced as imposed on body by some external force.

(d) Delusional perception: Normal perception has a private and illogical meaning.

11. Ref. [6] gives three types of *thought alienation* under Schneiderian first rank symptoms of *schizophrenia* as follows:

(a) Patient believes that his/her thoughts are under control of some external agency and others participate in this thinking.

(b) External thoughts are being put in his/her mind by some external force.

(c) *Thought broadcasting*: patient feels that his/her thoughts are read by others and broadcast.

12. In Ref. [7], "schizophrenia is a form of regression to earlier experiences and stages of life."

13. Patient can become homeless, suicidal, unemployed and behavioral problems could be bizarre.

*Paranoid schizophrenia* is the most common type of *schizophrenia*, and the patient may not appear psychiatrically ill unless the symptoms of paranoia come to light.

Every person is conditioned by one's culture, educational training, ethics, environment, life events, etc. Often this illness is treated by the psychiatrist based on his/her medical training, experience, and often colored by the above mentioned conditioning with perhaps their own perception that the present knowledge about this illness is all there or just plain puzzled compounded by the fact that every patient is similar and different as well in many ways. There is much more to these hallucinations, delusions, and symptoms from the mathematical and scientific viewpoints and also there are certain *bases and reasons* of these symptoms which are often not understood adequately.

Often Nature's mysteries present themselves in form of problems to be resolved with deeper hidden truths contained in them. We discuss the above mentioned aspects of hallucinations and delusions using principles of *mathematical logic* followed by explanation and insights into these symptoms and this illness.

**Theorem 1:** Delusions and hallucinations exist from an objective point of view in *paranoid schizophrenia*. This is proven as follows:

Prove the validity of the argument:

If Neela is a patient of *paranoid schizophrenia*, then she has delusions and hallucinations. If delusions and hallucinations are experienced by Neela, then these are either considered "unreal" (imaginary) by Neela's doctors or delusions, hallucinations are considered "real"

(existent) by Neela. Either delusions or hallucinations are not considered “unreal” by her doctors (i.e., cause is found or insight into certain traumatic event(s) causing them and issue resolved) or no debility, behavioral problems caused by hallucinations and delusions remain (i.e., way of solution found to understand delusions and hallucinations of the *particular patient*) or delusions and hallucinations exist from an objective point of view in *paranoid schizophrenia*. Therefore, delusions and hallucinations exist from an objective point of view in *paranoid schizophrenia*.

**Proof:** First, we convert the given argument in symbolic form using the *propositional variables* *a,b,c,d,e,f* and *logical connectives* such as *and* ( $\wedge$ ), *or* ( $\vee$ ), *not* ( $\neg$ ), *implication if-then* ( $\rightarrow$ ):

- a:** Neela is a patient of *paranoid schizophrenia*.
- b:** She has delusions and hallucinations.
- c:** Delusions and hallucinations are considered “unreal” by Neela’s doctors.
- d:** Delusions and hallucinations are considered “real” by Neela.
- e:** Debility and behavioral problems are caused by delusions and hallucinations.
- f:** Delusions and hallucinations exist from an objective point of view in *paranoid schizophrenia*.

The given argument now becomes in symbolic form as follows:

$a \rightarrow b$  (If Neela is a patient of *paranoid schizophrenia* then she has hallucinations, delusions)  
 $b \rightarrow (c \wedge d)$  [If she has delusions, hallucinations then {(these are considered “unreal” by her doctors) and (these are considered “real” by Neela)}]  
 $\neg c \vee (\neg e \vee f)$  [delusions, hallucinations are *not* considered “unreal” by her doctors or {(no debility, behavioral problems caused by delusions, hallucinations remain) or (delusions, hallucinations exist from an objective point of view in *paranoid schizophrenia*)}]  
 $a \wedge e$  (Neela is a patient of *paranoid schizophrenia* and debility, behavioral problems are caused by delusions, hallucinations)

Therefore, *f* (Delusions and hallucinations exist from an objective point of view in *paranoid schizophrenia*).

Steps	Reasons
1. $a \rightarrow b$	Premise
2. $b \rightarrow (c \wedge d)$	Premise
3. $a \rightarrow (c \wedge d)$	Steps 1 and 2 and the law of syllogism
4. $a \wedge e$	Premise
5. $a$	Step 4 and rule of conjunctive simplification



Steps	Reasons
6. $c \wedge d$	Steps 5 and 3 and rule of detachment
7. $c$	Step 6 and rule of conjunctive simplification
8. $\neg c \vee (\neg e \vee f)$	Premise
9. $\neg (c \wedge e) \vee f$	Step 8, the associative law of 'v' and De'Morgan's law
10. $e$	Step 4 and rule of conjunctive simplification
11. $c \wedge e$	Steps 7 and 10 and rule of conjunction
12. Therefore, $f$	Steps 9 and 11 and rule of disjunctive syllogism

Thus, we see that the delusions and hallucinations exist from an objective point of view in *paranoid schizophrenia*. The rules of *mathematical logic* are in Ref.[8].

Often the hallucinations and delusions not being believed to be “real” by the doctors and the patient unable to verbalize the dreadful feelings contained in the delusions and hallucinations lead to the patient feeling helpless and hopeless. Everything which exists in nature has an element of truth in it and is very much “real” which can appear as something quite negative until not understood. Nature presents problems to uncover the hidden truth underlying the *causes* in a problem. We would look at the causes of development of hallucinations and delusions which are the major symptoms of *schizophrenia*.

There is often the basis of traumatic abusive events from distant past behind hearing the voices and delusions. Some abusive traumatic events by an older person in position of power and trust can get buried in the memory of the patient and get internalized as “just punishment” or even “love” thus making it seems like “normal.” This “normal” can create later on conflict in the personality between “right” and “wrong” accompanied by tremendous guilt, fear, anger, self-doubt, and paranoia. Oftentimes, the acts of abuse including rapes are carried out in childhood or during adulthood in a vulnerable situation of the victim by cajoling *suggestions* by the abuser with *cajoling* seeming like “love” to the victim or in a threatening manner causing terror in the victim. In either case such events create a lot of guilt, fear, and paranoia in the psyche of the victim. The power of a loving suggestion is not lost on anyone but these very suggestions to carry out a crime can be internalized very deeply by the victim and could be heard as “**voices**” later in life and being under “**control**” or “**persecuted**” by some external agent. Such cunning suggestions can also have the effect of *hypnosis* on the victim with the victim not realizing that he/she has fallen prey to a heinous crime. The whole incident(s) can lie buried in the consciousness of the victim for a long time like a volcano which can erupt anytime triggered by some sensory input from the outer environment, especially sound. For example, in distant past, some violent incidents were followed by the banging of utensils and food served to the patient in those utensils. In the present moment, the patient’s dormant memory can get horribly triggered by sound of some utensils or similar sound causing *auditory hallucinations*. Moreover, the victim can pick up the opinions and feelings of the abuser at the time of crime. For instance, picking up of *mocking attitude* of the abuser can result in hearing voices or hallucinations of “**nonverbal auditory nature**

like *laughing*" at a later date by the patient. The patient can feel that the abuser (the personality) has "**entered**" his/her mind-body system. This can cause confusion over time and an apparent incoherent jumbled up speech of the patient and having hallucinations. The patient can feel "**possessed**" by the person who committed the crime on him/her or in case of not remembering accurately, the patient can feel the same by projecting his/her experience to another person not responsible for the crime. In this way, the patient can feel that "**external thoughts are being put in his/her mind by some external force.**" Being termed the hallucinations as "unreal" or "*which actually do not exist*" by the doctor does not help the patient except increasing the terror felt by the patient and losing heart even further. Usually crimes of this nature have certain intimacy creating emotional dependence between the abuser and the victim. When such incidents are not discussed immediately or soon after the incidents, the thoughts and feelings tend to fester inside the mind-body of the patient and *not forgotten in the real sense* but deeply suppressed and internalized as part of the personality of the patient. The added guilt also comes from the victim being blamed for crimes of this nature for various socio-cultural reasons. The stigma and taboo associated with such crimes and mental disorders complicate the situation even further.

The abusive events and their memories keep on getting buried deeper and deeper in the consciousness of the victim with time, building up of *avoidance issues* and sucking up the psychic energy of the mind-body system causing debility. The victim tends to develop *thought patterns* of escaping the trauma and tends to get attracted to similar abusers inviting trouble for him/her and over long period of time, the thought patterns, feelings, events getting all mixed up developing hallucinations and delusions. *If* the hallucinations and delusions did not actually exist and were "**imaginary,**" **then** there would be *no debility*. The patient has an "**unshakable belief**" that the delusions and hallucinations are true through subconscious memory and knowledge but cannot articulate them due to the memories being overwhelming. Thus, the building up of trauma and thought patterns can go on till death and beyond. During intense emotional pain, the patient may realize that the psyche has very much a life of its own and certain past life event may **resonate** with similar event in the present lifetime. As in Ref. [7] "**schizophrenia is a form of regression to earlier experiences and stages of life.**" Whether one believes in past lives or not depends upon how convincingly one can recall certain events. Going back in one's life may be understood in this way that if one is an adult then he/she has been a child also and some traumatic event(s) must have caused the delusions and hallucinations. A particular patient may have the ability to regress to childhood trauma or could regress further to pre-birth times as well. The occurrence of past lives has got a lot to do with the patient's ability to recall correctly and can be understood from the **broader viewpoint of nature's perspective**. Nature creates, destroys, and maintains the creation through **dualities** like right-wrong, pleasure-pain, day-night, birth-death, etc. The body has a shelf-life of a few decades and, in general, has limitations of movement but the psyche is not limited like the body—its life is indefinite. Psyche is more powerful than the body and more subtle. It has more potential for evolution in terms of **self-knowledge**, though more complex and difficult than the physical. When the body is done with the death, the invisible psyche remains because mind/consciousness is a form of *invisible but palpable energy*. The psyche has a subconscious longing to resolve the pending issues of the trauma and is often



full of regret. This psyche assumes next body with mixed emotions with the already developed thought patterns and unresolved trauma. In the present lifetime of the patient, some memory may be triggered off by some stimuli or coming across the abuser in a different body and the patient may feel that the **“sensory symptoms are imposed on body by some external force.”** When looked at from a broader time-span, it can be understood that the psyche is a form of energy, only with various kinds of thoughts, feelings, and with some **unfinished business**. One can have restlessness due to some *unfinished business* for a few days in the same body then why not for a longer time-span? Thought acts as *chronological time* in the mind. It can be understood in this way that taking urgent action immediately which someone was thinking of taking after 2 years takes away those 2 years, and one feels unburdened thus bringing the “future” into the “present.” Some past memories can get triggered *now* which bring the “past” into the “present.” Therefore, the thoughts and feelings dictate to a great extent the interpretations of *past*, *present*, and *future*. So, the *psychological time* is different from the *chronological time*. Some gifted patients can recall the past lives in a “timeless” way i.e., by looking at his/her life events as events of one stretched out lifespan with events well connected and with insights into the thought patterns (as in no. 9 of symptoms: **“the delusions are usually well-systematized i.e., thematically well connected with each other”**). This is entirely possible if the patient, who has authentic paranormal abilities, is a conscientious, courageous warrior, and a lover of truth.

The intelligence of the mind-body complex records everything like a movie or even better. The dialogues between the abuser and the victim may appear as “voices” later on in life or if the person is sensitive may be constantly reminded of it. Based on some similar stimuli in the outside environment, this/these traumatic event(s) stored in the mind-body can get triggered off. Usually, the victim is blamed directly or indirectly thus causing a severe guilt and fear which when accumulated over time can result in **“delusion of reference, persecution”** or simply projecting his/her guilt and fear to everyone outside thus resulting in **“thought broadcasting.”** Guilt and low self-esteem can only intensify the feeling of **“others will read my thoughts and harm me.”** An abusive incident can breed the feelings of betrayal, infidelity, jealousy, insecurity, feelings of no sense of belonging to anyone or anywhere and broken-hearted leading to a solidified mental state of feeling orphaned, and **“homeless.”** This could translate to the patient actually running away from home, hospital, or refusing treatment. The patient may also “learn” infidelity attracting similar abusive people and “expecting” to be betrayed which may appear as “natural” in relationships and could cause major relationship troubles. Memories recorded in the consciousness of such abusive incidents are experienced as *hallucinations of a threatening nature giving commands to the patients*; these are very real to the patient but fear of not being believed and unable to talk about them only makes things worse because of guilt associated with infidelity and anger with jealousy.

A gifted individual can be subjected to actual events of ostracizing, persecution, and condemned for his/her free ideas not fitting in with the majority of people, so the *delusions of persecution* can be quite pronounced in this case with the patient feeling like a criminal burdened with fear, guilt, and a seething insane rage underneath. If he/she has certain fame due to excellence in work, then the **delusion of reference** can bother him/her like anything.

A patient can pick up certain disturbing *opinions* common to large number of people (*collective consciousness*) either through hearing, reading about them, watching a movie, etc. which caused much pain to the patient and he/she can feel “**attacked**” by several people triggering off hallucinations and delusions. In terms of picking up others’ thoughts and feelings the illness of *schizophrenia* can get as “**infectious**” or more than a communicable illness of the body.

In the case of an average patient or a gifted patient, abuse at a vulnerable situation can happen, and the insight into the truth of such events can involve strong emotions like betrayal, anger, and helplessness. The prospect of a daunting war to be fought within and with the abusers who could be far away coupled with the paralyzing stored up guilt, terror, anger, sorrow, debility, etc. may depress the patient like anything. This is the trial by fire and the test of inner strength of character which the patient may face. Moreover, an intelligent action of the medicine could try to release the suppressed traumatic memories (this may be understood as being akin to an operation on the body to take out a tumor, only in the psychological realm, it is much more complex and difficult). This can seem like worsening the situation thus the patient losing faith in the medicine, stopping it, and relapses taking place. An ardent wistful desire to avenge and revenge could be behind the *delusion of grandiosity and feelings of one’s messianic role in life*. Even when not triggered, the person may hear voices subconsciously of those stored events in the memory which are so old that the memory appears to be lost and cannot be recalled. If it could be recalled or some insight into the thoughts and feelings happen, then it could lead to recognizing and changing the *thought patterns* by the patient with suitable help. But the essential initiative must come from the patient and could lead to decrease in delusions and hallucinations and hence improvement in behavioral problems. As in Ref. [5], “**The delusions are usually well-systematized, i.e., thematically well connected with each other**” also points to the fact that delusions **stem from some events** and the need to start by believing the patient could be the first step instead of right away starting with the notion that voices or delusions are imaginary or “unreal.”

Even in the case of delusions and hallucinations being “unreal,” we have to see that **in nature or creation everything appears as dualities or pairs of opposites, and there is truth in both the extremes. Change or evolution which is continuous is brought about by these pairs of opposites by nature in creation**. For instance, like pleasure-pain, day-night, birth-death, true-false, for-against, etc., the “real”-“unreal” in this case is also another pair of opposites with truth in both, depending on whose viewpoint one is looking from, the puzzled doctor, or the troubled patient. In shaping of the consciousness of a person, the elements affecting the psychology like personal background, the kind of education one has had, culture, ethics, events in one’s life, etc. all play a role in shaping and conditioning the mind, thus the doctor is also conditioned by these factors as well as the patient. Moreover, these factors are *unique* for each person hence for each patient as well; therefore, the patient in order to get well has to take the initiative himself/herself to understand this conditioning with suitable help.

One more thing to be noted is the overemphasis in human society on the logical part and not emphasizing enough in the matters of heart. In climbing the social ladder, the calculative logical part plays a greater role disregarding the heart/conscience which speaks our truth to us, and this voice is ignored often creating transgressions against the conscience leading to dishonesty,

obscuring of the truth, and further complicating the disorders of the psyche. Another outcome of overemphasis on logic in terms of social conditioning is that when due to some input from the senses, some trauma is caused or triggered; the person is unable to feel and analyze the overwhelming emotions as emotions are far more powerful than thoughts and could lead to bizarre behavior. This conditioning is equivalent to total disregard of the heart leading to ignoring the heart's voice further leading to an *unbalanced personality*.

There is often deep depression, exhaustion, anxiety, panic, post traumatic stress leading to psychosis, obsession, and one, more, or all of these are experienced by the patient at some time or other in *schizophrenia* which makes it difficult to diagnose and understand this illness.

The problems in the psychological realm get complicated by the fact that the world is flooded with the advice of many experts on "positive thinking" which is an antithesis to the approach of *problem solving*. This *positive thinking* often translates into an avoidance, even denial, looking the other way and away from the problem. The **apparently negative approach to problem solving requires** hard work, paying attention to the problem and taking adequate actions in response to various psychological states, thoughts and feelings and is anything but escapist. Not working on *problem solving* only aggravates the problem.

About the correlating of thoughts and ideas, Ref. [5] says "Autistic thinking is one of the most classical features of schizophrenia. Here, thinking is governed by private and illogical rules. The patient may consider two things identical because they have identical predicates or properties (**von Domarus Law**).” For example, Jesus Christ was persecuted, I am persecuted; So I am Jesus Christ.

It is a matter of interest to see the similarity between the above law and the **Law of Detachment in mathematical logic** of Discrete Mathematics. Let us consider the same argument in light of the law of detachment as follows:

I feel persecuted. If I feel persecuted then I am Jesus Christ. Therefore, I am Jesus Christ.

In symbolic notation, the argument using the *propositional variables* is as follows:

p: I feel persecuted.

q: I am Jesus Christ.

Let us write the argument as per the Law of Detachment in symbolic notation as follows:

p (I feel persecuted)

$p \rightarrow q$  (If I feel persecuted then I am Jesus Christ)

---

Therefore, q (Therefore, I am Jesus Christ).

The Law of Detachment is a *valid rule of inference* in *mathematical logic*. This similarity points to the *common origin* of *logic/arguments* in Psychology and Mathematics. Thus, we observe that what appears as "**illogical**" to the doctor is a perfect valid argument mathematically and psychologically for the patient further establishing our findings in this section depending upon whose viewpoint one is looking from. In fact, the whole of the discussion in this section is of

*mathematical logic (reasoning)* applicable in psychological realm as well. The patient is obviously not Jesus Christ from the doctor's point of view but the patient unable to feel or understand the trauma **identifies** with the trauma he/she knows as what Jesus Christ faced and expresses in this manner.

Schizophrenia is an illness of facing the hard truths in one's life or lifetimes as the case may be depending on the patient's abilities, initiative, and sense of responsibility toward oneself preferably with suitable help.

### 3. Role of mind and consciousness in resolving the graceful tree conjecture

Sometimes holding on strongly to the binary logic of true/false in Graph Theory (a branch of Discrete Mathematics) fails to produce results and can even make mathematics look too rigid. This has been seen in the case of the well-known about five decades old *graceful tree conjecture* (all trees are graceful) being assumed to be true by a large number of researchers even without proof.

*Tree* is a kind of *graph*. A *graph labeling* is assigning numbers (labels) to vertices and edges such that the induced labels form a certain pattern. A *graceful labeling* in a graph with  $p$  vertices and  $q$  edges is assigning the numbers to the vertices  $0, 1, 2, \dots, (q-1)$  such that the induced edge labels found by taking the absolute value (the positive value) are from  $1, 2, \dots, q$  (see **Figure 1**). Ref. [9] was an attempt to solve the graceful tree conjecture. Ref. [9] was a *direct attempt* to solve the graceful tree conjecture as pointed out in Ref. [10].

Assuming this conjecture to be true led to no doubt a lot of work getting done and published but generally of specialized variety and not able to pinpoint why and how this conjecture was even formed in the first place. Zeroing in on answering this conjecture in Ref. [1] after attempting to solve it in Ref. [9] took years of delving into the nature of the mathematical problem as well as the nature of mind to discover that it seemed that the definition of tree being *connected and acyclic* was taken to be the controlling factor in determining that *all trees* would be *graceful*.

A *bipartite graph* is one whose set of vertices can be split into two subsets  $X$  and  $Y$  such that each edge of the graph joins a vertex in  $X$  and a vertex in  $Y$ . The following *algorithm G* in Ref. [9] gives *graceful labeling* for **bipartite trees in which the edges do not cross** (**Figure 1**).

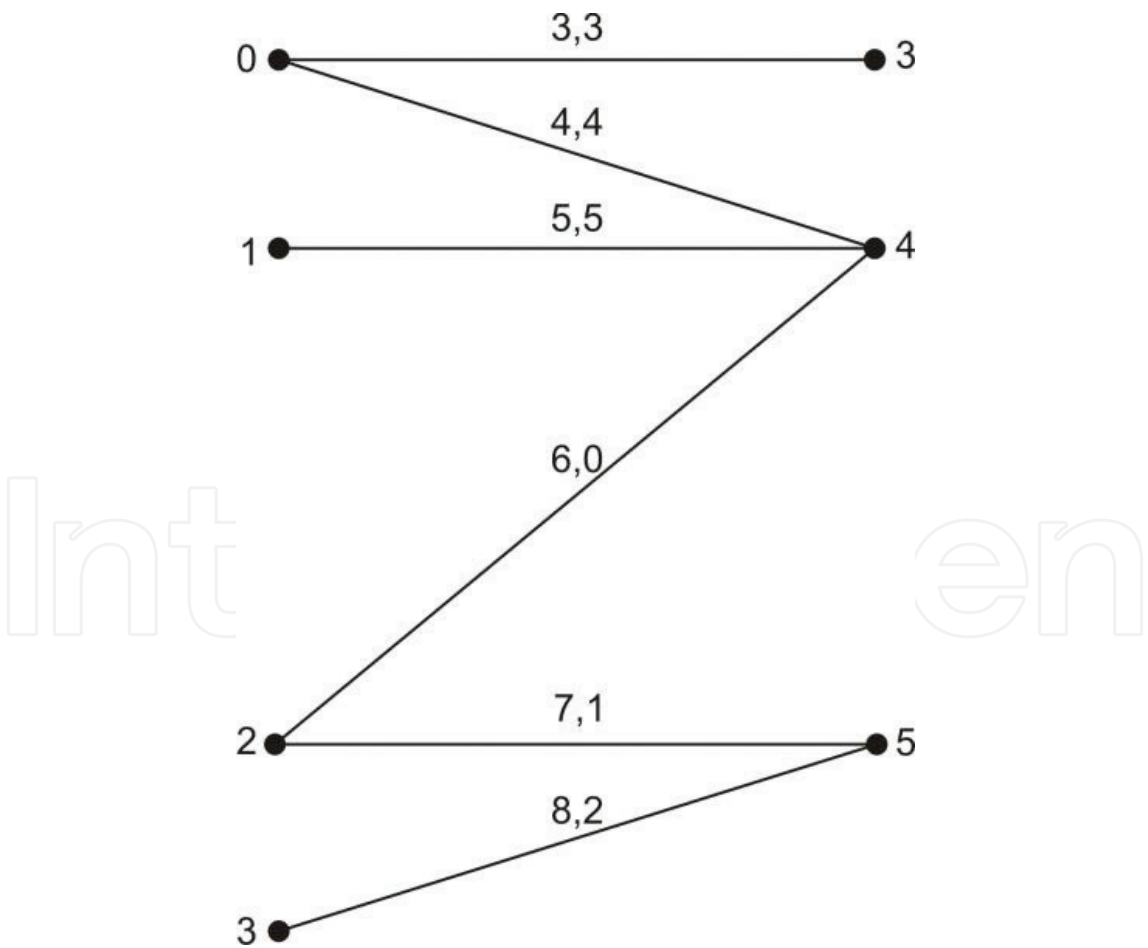
#### Algorithm G

1. Draw the tree as a bipartite graph in two partite sets denoted as Left(L) and Right(R). Let the number of vertices in L be  $x$ .
2. Number the vertices in L starting from top going to bottom consecutively as  $0, 1, \dots, (x-1)$ .
3. Number the vertices in R starting from bottom going to top consecutively as  $x, (x+1), (x+2), \dots, q$  ( $q$  is the no. of edges). Note that these numbers are the vertex labels.

- 4. Compute the edge labels by taking the absolute value of the difference of the incident vertex labels.
- 5. The resulting labeling is graceful.

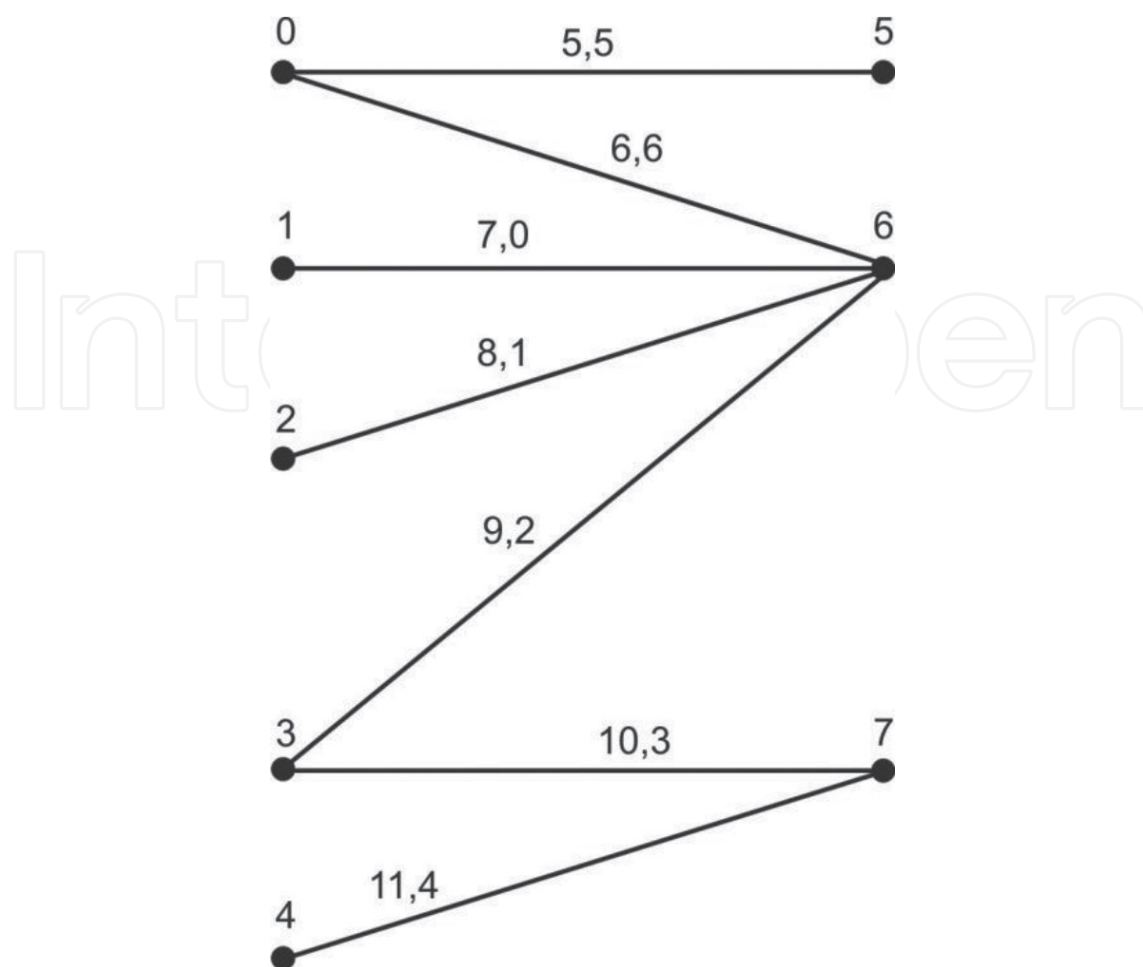
The trees which cannot be drawn as *bipartite trees* where the edges do not cross may not give *graceful labeling* with this algorithm. Other algorithms in [9] for labelings of such bipartite trees namely *harmonious* (vertex labels are added mod  $q$ ; one vertex label allowed to be repeated in trees: **Figure 2**), *sequential* (vertex labels are added with ordinary addition: **Figures 2 and 3**), *felicitous* (vertex labels are added mod  $q$ : **Figure 3**), and *antimagic labelings* (sum of all the incident edge labels at each vertex is distinct: **Figures 4 and 5**) are also based on this construction of trees.

In a result like this, the mind and its set perceptions play a role because the mind being conditioned by a long history of the traditional branches of mathematics where the mathematical structures are rigid, well defined and are controlled or predicted by the definitions, tends to **assume or project the same** in Graph Theory as well. In geometry also there are diagrams but these diagrams are defined by their formulas. For instance, ellipse, hyperbola, etc. are defined by their formulas or how many geometrical units will comprise a given



**Figure 2.** Sequential and harmonious bipartite tree where edges do not cross. Sequential and harmonious edge labels written on left and right, respectively, separated by a comma.





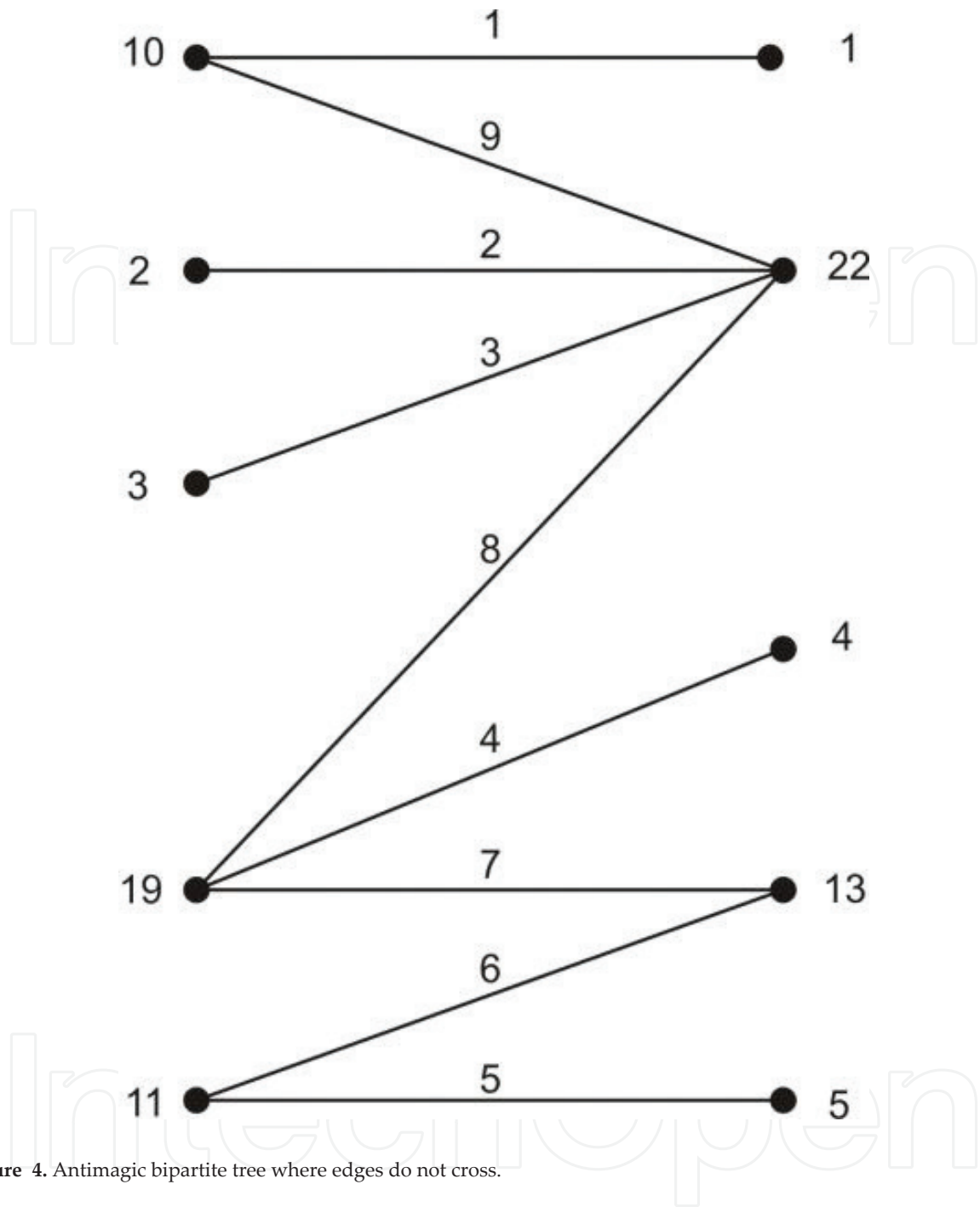
**Figure 3.** Sequential and felicitous bipartite tree where edges do not cross: Sequential and felicitous edge labels written on left and right, respectively, separated by a comma.

diagram is specified as in a triangle (three line segments and angles), square, etc. But *trees* are just defined as *connected and acyclic* and can be drawn in any way with ***no restrictions and nothing which can be measured or quantified resulting in infinite shapes*** rendering them suitable only for specialized results of particular kinds of *trees* and not generalized results. So far, the research in Mathematics has discounted how the mind influences the perceptions of looking at mathematical problems, formulating them, and going about solving them. How mind and its perceptions influence viewing a mathematical problem, its formulation and solution could be an area of research which can be quite subjective but could yield interesting results when done more consciously. Ref. [1] which answers the *graceful tree conjecture* and Ref. [11] which finds that the definition of tree alone can not be the controlling factor in similar conjectures, both factor in the psychological conditioning in looking at a mathematical problem.

From the rules of *mathematical logic* some of above mentioned ideas can be proved as follows:

**Theorem 2:** Definition of *tree* alone is not the controlling factor in assuming the *graceful tree conjecture* to be true.





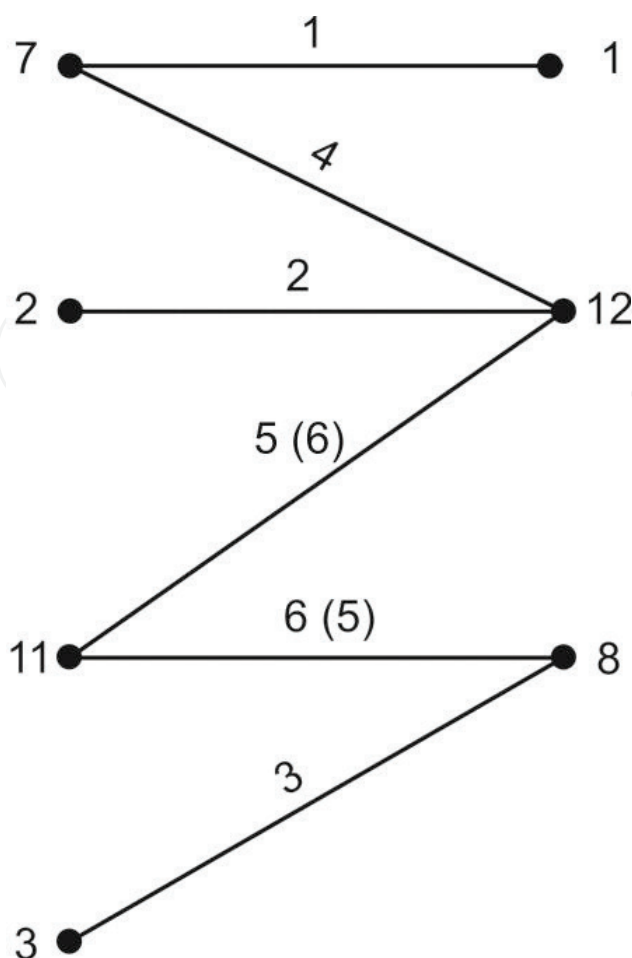
**Proof:** Let us write the argument using *propositional variables* and *logical connectives* as follows:

*p*: definition of *tree* alone is the controlling factor in assuming the *graceful tree conjecture* to be true.

*q*: *graceful tree conjecture* is assumed to be true By the Law of Modus tollens the argument is as follows:

$p \rightarrow q$  (if definition of *tree* alone is the controlling factor then the *graceful tree conjecture* is assumed to be true).

$\neg q$  (*graceful tree conjecture* is not assumed to be true).



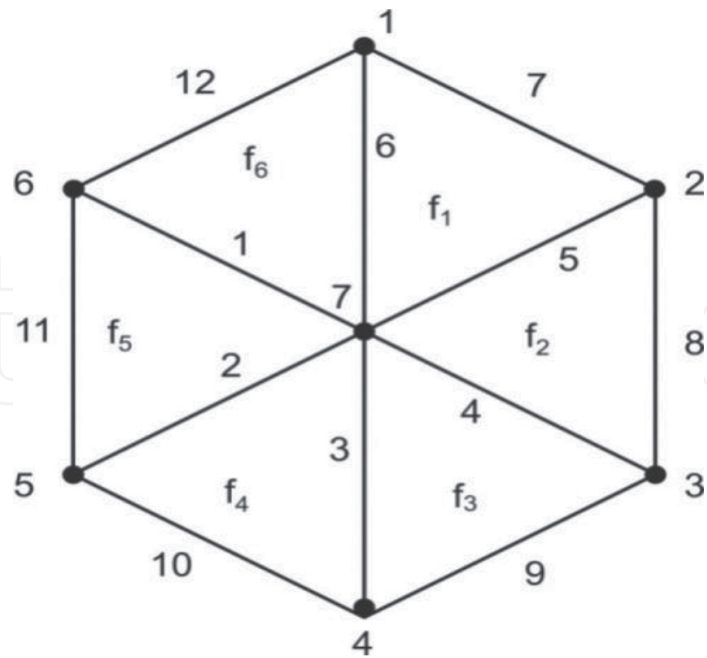
**Figure 5.** Antimagic bipartite tree where edges do not cross: the edge labels in parentheses show the interchanged edge labels as in step 4 of algorithm AM.

Therefore  $\neg p$  (definition of *tree* alone is *not* the controlling factor in assuming the *graceful tree conjecture* to be true).

Thus, it should be noted that pictorial representation of graphs gives rise to infinite number of shapes which makes impossible to classify them all and makes the definitions in Graph Theory particularly in *trees*, distinct from definitions in older, traditional branches of Mathematics, and may not be the basis of forming generalized conjectures like the *graceful tree conjecture*.

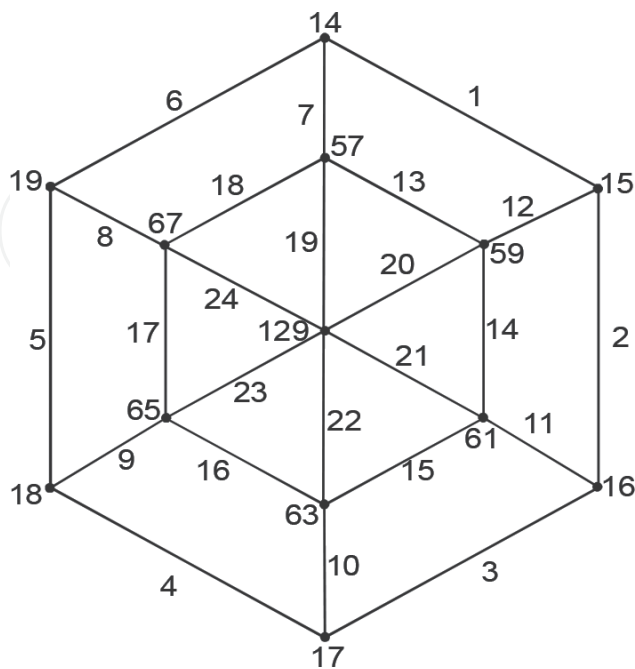
#### 4. Graph labelings and applications

As from the *antimagic*, *inner magic*, and *inner antimagic* graphs, applications have emerged and been presented in Ref. [2], and some new and modified interpretations are being given in this work. *Inner magic* and *inner antimagic* are new kinds of labelings given in Ref. [4]. In these labelings, the  $p$  vertices,  $q$  edges, and the  $f$  internal faces of a planar graph are labeled such that labels of the faces form an *arithmetic progression* with *common difference*  $d$ . If  $d = 0$ , then the graph is said to have an *inner magic* labeling, and if  $d \neq 0$ , then the graph is said to have *inner antimagic* labeling. **Figure 6** shows the *inner magic* and *inner antimagic* wheel.



**Figure 6.** Inner magic and inner antimagic wheel. Inner magic internal face labels:  $f_1 = 6, f_2 = 5, f_3 = 4, f_4 = 3, f_5 = 2, f_6 = 1$ . Inner magic weight number = 34. Inner antimagic internal face labels:  $f_1 = 1, f_2 = 2, f_3 = 3, f_4 = 4, f_5 = 5, f_6 = 6$ . Inner antimagic internal face weights: 29, 31, 33, 35, 37, 39.

An *antimagic* labeling of a graph with  $p$  vertices and  $q$  edges is one in which the  $q$  edges are labeled with numbers  $1, 2, \dots, q$  such that the sum of the incident edge labels on each vertex is distinct. In the *antimagic* graphs shown in **Figures 7–11**, the vertices could represent offices in a building and the *antimagic* labeled edges could represent codes/passwords to reach those offices. The *antimagic* label of the particular vertex representing office could be a password or code given



**Figure 7.** Antimagic double wheel  $D_6$ .

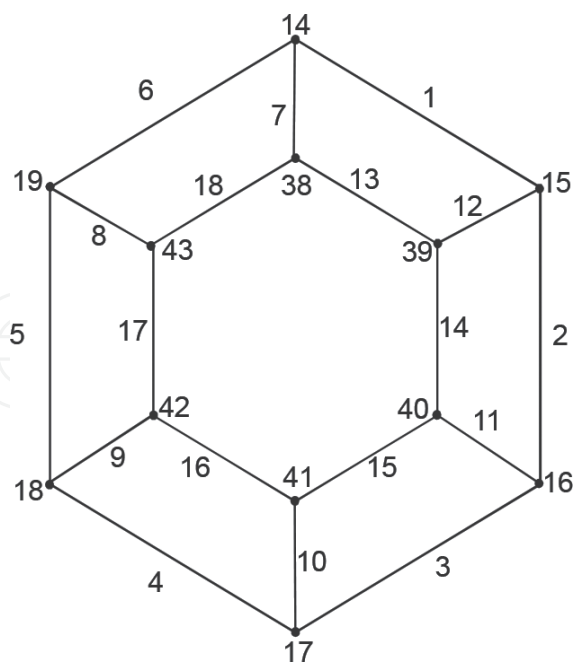


Figure 8. Antimagic centerless wheel-2  $CW2_6$ .

to the personnel in a security setup requiring confidentiality. Thus, these *antimagic* graphs could serve as a security model for various kinds of buildings in a scenario of urban planning

In *double wheel* and *helm* (Figures 7 and 10) of Refs. [12, 13], the central *antimagic* vertex could serve as the central headquarters for all the offices or the vertices and their passwords/codes or edges being the *antimagic* labeled edges. The *antimagic centerless wheels*  $CW2_n$ ,  $CW3_n$ , and

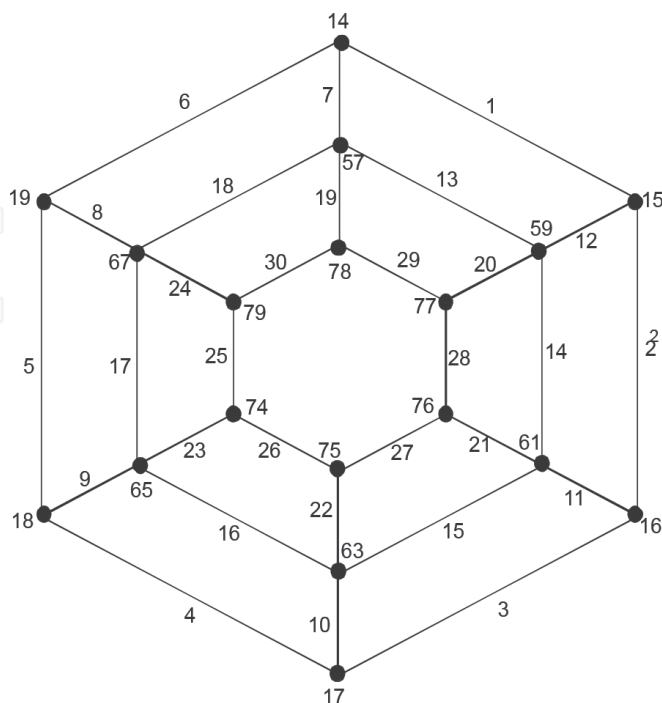


Figure 9. Antimagic centerless wheel-3  $CW3_6$ .

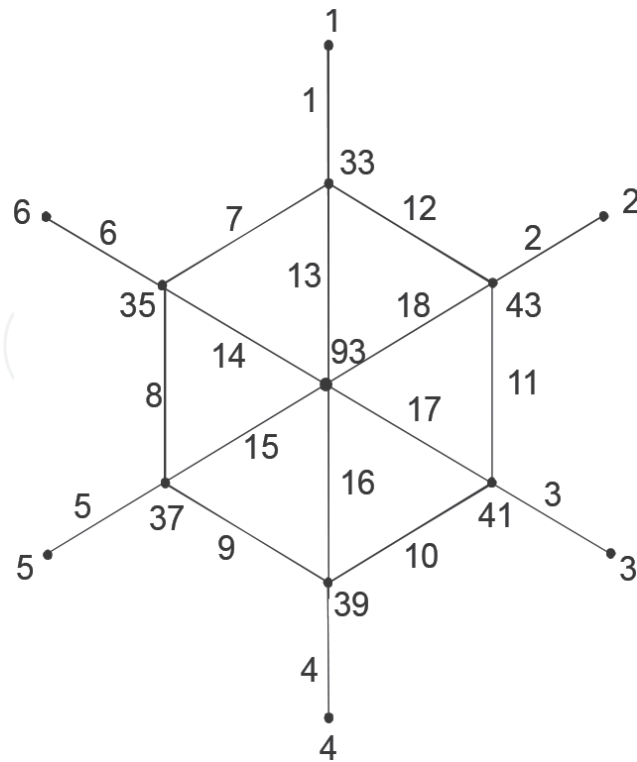


Figure 10. Antimagic helm.

*regular actinia* of Refs. [12, 13] in **Figures 8, 9, and 11** could serve as model for a security system without centralized control. The concepts of **cut-set**  $S$  in graph  $G$  (removal of all the edges in  $S$  disconnects  $G$ ; removal of some but not all of the edges in  $S$ , does not disconnect  $G$ ) and **vertex cut-set**  $S_1$  (analogous for vertices) could be used to disallow certain security personnel's

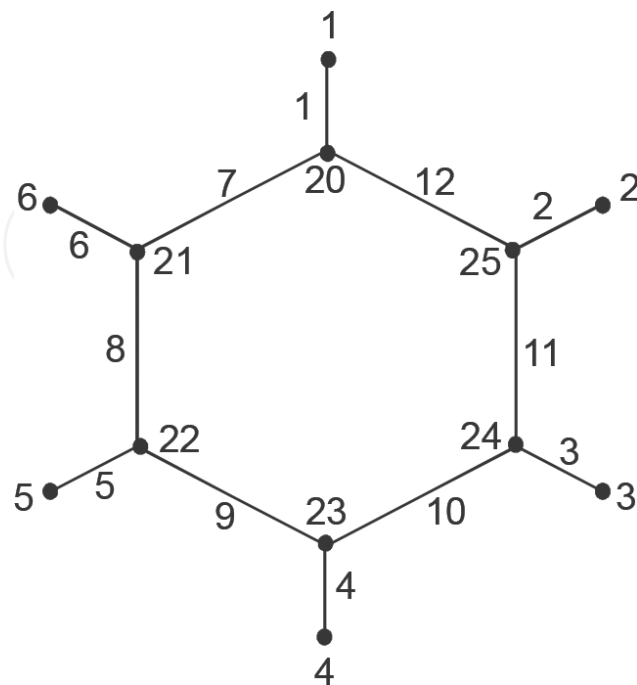
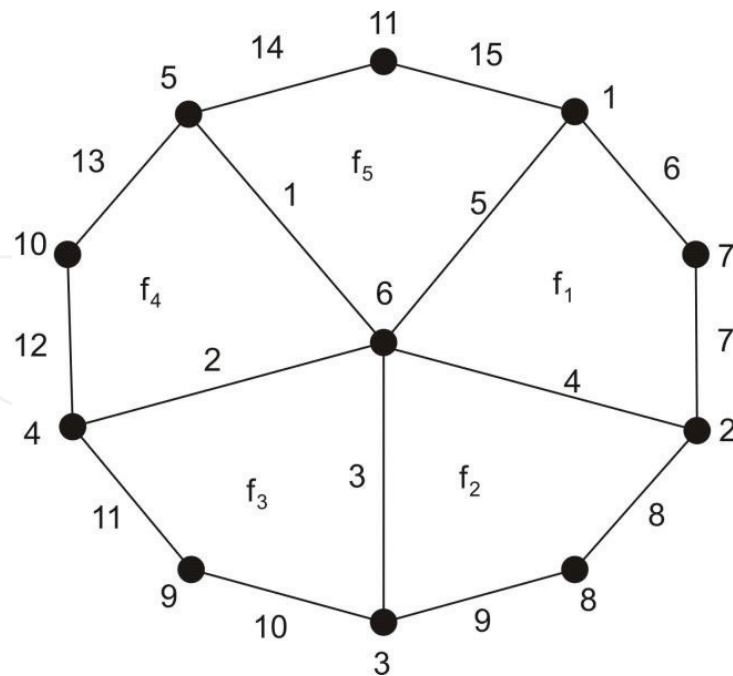


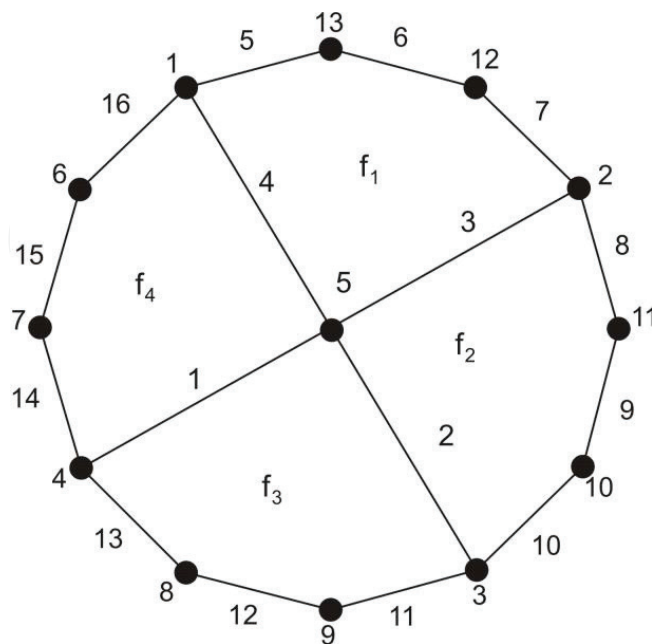
Figure 11. Antimagic regular actinia.



**Figure 12.** Inner antimagic flower-1. Inner antimagic inner antimagic labels and internal face labels are as follows: (1) 39, 45, 51, 57, 63.  $f_1 = 1, f_2 = 2, f_3 = 3, f_4 = 4, f_5 = 5$ . (2) 43, 47, 51, 55, 59.  $f_1 = 5, f_2 = 4, f_3 = 3, f_4 = 2, f_5 = 1$ .

legitimate access. The ideas of **vertex connectivity** (minimum number of vertices deleted to disconnect the graph) and **edge connectivity** (analogous for edges) could also be combined with these applications to check for the violation of codes/passwords.

The graphs studied in Ref. [4] are *wheels*, *flower-1*, and *flower-2* (**Figures 6, 12, and 13**). The planar graph *flower-1* has one central vertex and rest being outer vertices, and all the internal



**Figure 13.** Inner Antimagic Flower-2. Inner antimagic inner antimagic labels and infernal face labels are as follows : (1) 59, 65, 71, 77;  $f_1 = 1, f_2 = 2, f_3 = 3, f_4 = 4$ . (2) 62, 66, 70, 74;  $f_1 = 4, f_2 = 3, f_3 = 2, f_4 = 1$ .



faces are bound by four edges. *Flower-2* is a planar graph with one central vertex and rest being outer vertices and all internal vertices are bound by five edges. *Wheels* are found to have *inner magic* as well as *inner antimagic* labelings and *flower-1* and *flower-2* have *inner antimagic* labelings.

The larger of these graphs may have to be checked for existence of these labels. These graphs could also serve as models for surveillance or security systems, in designing building in an urban planning setup, network addressing, communication studies, etc.

*Inner magic* wheel could be used in a security system where matching of all the labels to the *inner magic* label could lead to decode or unlock the security system.

Above mentioned *antimagic*, *inner magic*, and *inner antimagic* graphs could have applications in cryptography also.

Ref. [14] mentions the need for efficient methods of graph labelings. Such algorithms are found in Ref. [9] for *graceful*, *harmonious*, *sequential*, *felicitous*, and *antimagic* labelings for ***bipartite trees where edges do not cross*** useful in communication networks and circuit design where the wires are such that there are **no crossing points**. The various algorithms developed for *bipartite trees*, where edges do not cross with  $p$  vertices and  $q$  edges for *graceful*, *harmonious*, *sequential*, *felicitous*, and *antimagic* labelings, could be explored further for applications where such *bipartite trees* serve as models.

## 5. Results in graph labelings using computer software

The software developed in Refs. [15, 16] to check the existence of major graph labeling methods namely *harmonious*, *sequential*, *graceful*, *felicitous*, *antimagic*, *magic* for an arbitrary graph has been used in certain studies. It was used in the studies done in Refs. [3, 17]. Statistical analysis of the antimagic graph labelings of *paths*, *cycles*, *wheels*, and *star* graphs in Ref. [3] shows that there is a relation between *degree* (*degree* is the number of edges incident on a vertex) of vertices and number of *antimagic* labelings obtained for these graphs. Presence of higher *degree* vertices increases the number of *antimagic* labelings obtained by the computer software. Moreover, various kinds of graphs were tested, and nearly all the graphs had *antimagic* labelings.

In the application of *antimagic* labelings to *complete graphs* (see **Figure 14**; in a *complete graph*, every vertex is connected to every other vertex), higher *degree* vertices imply more number of legitimate routes for persons who are allowed access to a particular office (where vertices represent offices of the graph and *antimagic* label for a vertex can represent the code/password). *Star* (see **Figure 15**) is a graph with a central vertex and the rest are *pendant vertices* (vertices of *degree* one). *Antimagic Stars* could represent one centralized control room with scattered single offices as the *pendant vertices*. *Antimagic Paths* could represent linear offices; for example, along a street.

Ref. [18] studies the effect of repeated vertex labels and shows that the labelings produced are faster and several in number on the computer when a vertex label is repeated. This also yields

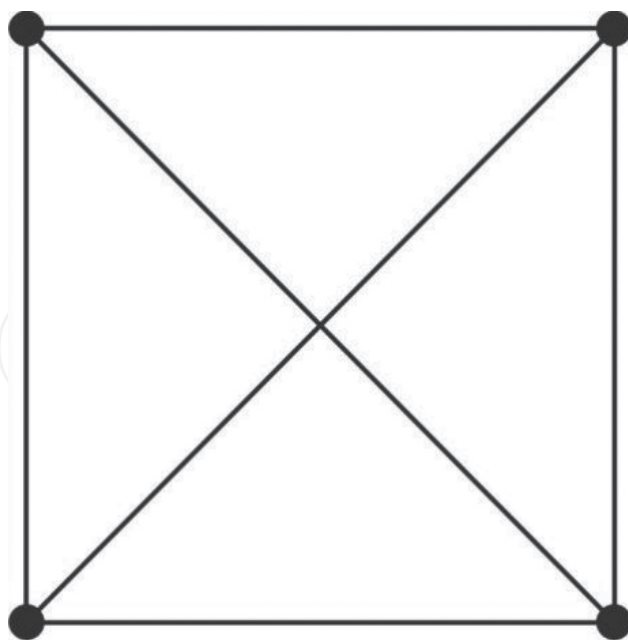


Figure 14. Complete graph.

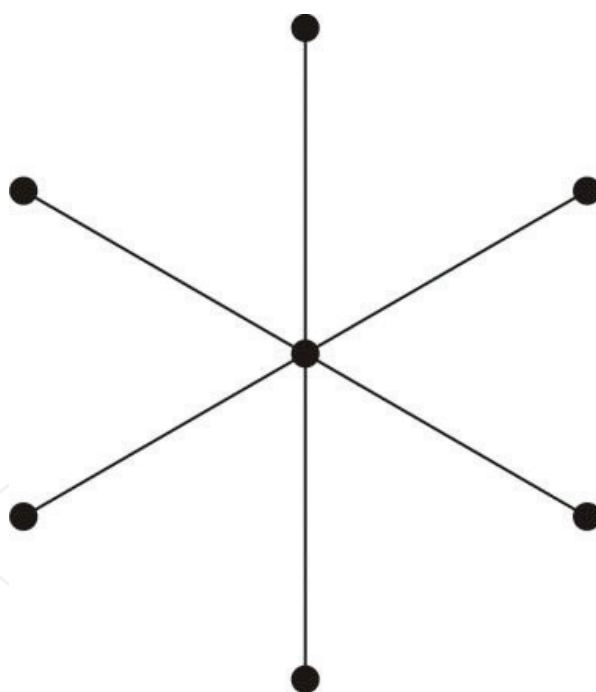


Figure 15. Star.

labeling where there is no labeling with the conventional definition. The repetition of a vertex label can have significance in applications where the repeated vertex label could represent some quantity which is repeated. This opens up possibilities for studies to obtain more labelings and can be explored further.

In Ref. [19], certain patterns in graph labelings have been discovered. It has been discovered that lower degree vertex can be manipulated more easily to yield labeling faster with the help of the software developed in Refs. [15, 16]. This approach could be followed while working manually also.

## 6. Lexicographic order in graph labelings

Applications so far undiscovered could be explored in the graph labelings in the following works which could be of further interest in research: *lexicographic order* of permutations and combinations is mentioned in Ref. [20] and made use of it to develop a generalized software to check existence of major graph labeling schemes namely *graceful, harmonious, felicitous, sequential, magic, and antimagic* for an arbitrary graph in Refs. [15, 16]. This software has been used to prove that harmonious graph is NP-Complete as given in Ref. [21]. Existence of *lexicographic order* in graph labelings has been studied in Refs. [22, 23].

In a *lexicographic order*, the word 'card' comes before 'cart' as in a dictionary or lexicon.

Permutations in *lexicographic order* of  $4! = 24$  permutations which are as follows: 1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 3412, 3421, 4123, 4132, 4213, 4231, 4312, 4321.

Combinations in *lexicographic order* of  ${}^6C_4$  are: 1234, 1235, 1236, 1245, 1246, 1345, 1346, 1356, 1456, 2345, 2356, 3456.

In *paths*, permutation 1 and permutation 2 give the *antimagic* labeling for *even* and *odd edges*, respectively, in [23] and is shown in **Figure 16**.

Let us look at the computer generated results for the *cycles* where the edges  $e_1, e_2, e_3, \dots$  and so on are labeled in the order of the permutations in Refs. [17] and is shown in **Figure 17**.

$C_4$ : Permutation 2: 1 2 4 3

Vertex labels (in ascending order): 3 4 6 7

$C_6$ : Permutation 2: 1 2 3 4 6 5

Vertex labels (in ascending order): 3 5 6 7 10 11

$C_8$ : Permutation 2: 1 2 3 4 5 6 8 7

Vertex labels (in ascending order): 3 5 7 8 9 11 14 15

$C_{10}$ : Permutation 2: 1 2 3 4 5 6 7 8 10 9

Vertex labels (in ascending order): 3 5 7 9 10 11 13 15 18 19

Now, let us look at the odd Cycles:

$C_5$ : Permutation 1: 1 2 3 4 5

Vertex labels (in ascending order): 3 5 6 7 9

$C_7$ : Permutation 1: 1 2 3 4 5 6 7

Vertex labels (in ascending order): 3 5 7 8 9 11 13

Permutation 3: 1 2 3 4 6 5 7

Vertex labels (in ascending order): 3 5 7 8 10 11 12

$C_9$ : Permutation 1: 1 2 3 4 5 6 7 8 9

Vertex labels (in ascending order): 3 5 7 9 10 11 13 15 17

Permutation 3: 1 2 3 4 5 6 8 7 9

Vertex labels (in ascending order): 3 5 7 9 10 11 14 15 16

$C_{11}$ : Permutation 1: 1 2 3 4 5 6 7 8 9 10 11

Vertex labels (in ascending order): 3 5 7 9 11 12 13 15 17 19 21

Permutation 3: 1 2 3 4 5 6 7 8 10 9 11

Vertex labels (in ascending order): 3 5 7 9 11 12 13 15 18 19 20

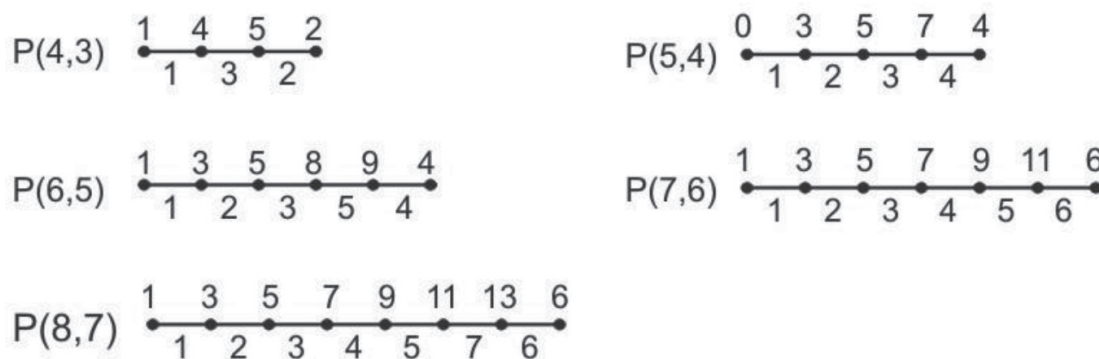


Figure 16. Antimagic paths.

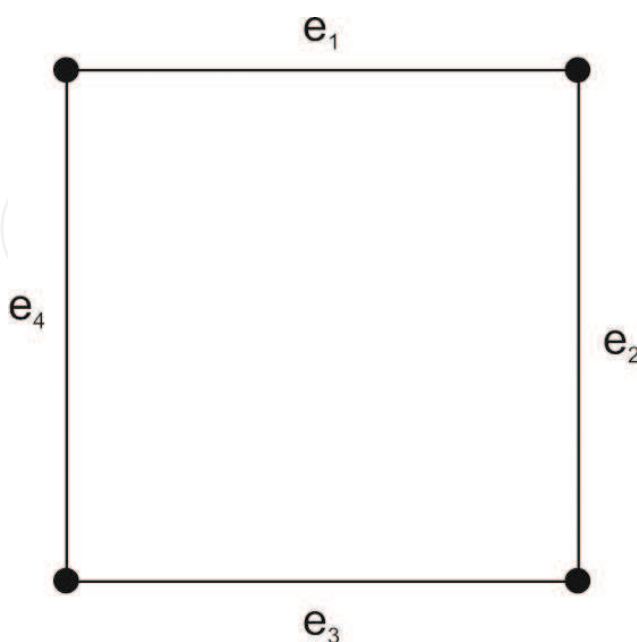


Figure 17. Cycle.

In cycles, permutation 1 gives *antimagic* labeling for *odd cycles* (*odd number of edges*), whereas permutation 2 gives *antimagic* labeling for *even cycle* (*even number of edges*). Permutation 3 also gives *antimagic* labeling for *odd cycles* except  $C_5$ . Permutation 1 results in induced vertex labels showing almost an *arithmetic progression* with difference 2.

*Lexicographic order* found in graph labelings could be studied for further applications.

## 7. Conclusion

Thus, we have seen that the various kinds of delusions and hallucinations often considered “unreal” of the severe psychological disorder of *paranoid schizophrenia*, a kind of *schizophrenia* have much truth and underlying inherent causes in them with insights provided in these factors in this work. Hallucinations and delusions exist in real terms have been proved by *mathematical logic* also.

From the viewpoint of things appearing in creation as dualities of nature, we can see that every pair of opposites like pleasure-pain, right-wrong, day-night, birth-death, stress-relief, for-against, etc. have truth in both the opposites. Thus, the “real”-“unreal” of hallucinations and delusions also have truth in them depending upon the viewpoint of the subject: patient and doctor, respectively. The duality of brain(logical)-heart(feeling) also brings about an unbalanced personality in an individual when there is an overemphasis on calculative logic in human society and overriding the voice of the heart/conscience. Nature brings evolution by these dualities in creation leading to an increase in understanding and knowledge of some unresolved problem.

The *graceful tree conjecture* has elements of psychological conditioning in it, being believed to be true even without a proof for about five decades. Conditioned by a long history of being used to the definitions in traditional Mathematics as the controlling factor in predicting behavior of *all* the defined mathematical structures will lead to projecting the same in the definition of *tree* also. The big difference in the definitions of traditional branches of Mathematics and a newer subject like Graph Theory is that the definition of tree is *unquantifiable and pictorial* thus *cannot* predict the behavior of *all* trees. A *tree* with four vertices is defined as “connected and acyclic” and a *tree* of two million vertices is also defined in the same way with no restrictions and *no measure* giving rise to infinite shapes for larger trees. In the case of a number being divisible by 2, 3, or 5, the definition is applicable to *any number no matter how large* thus *all* such numbers are predicted by this quantifiable definition unlike the *unquantified, pictorial representation* of a *tree*.

Applications of *antimagic*, *inner magic*, and *inner antimagic* graphs in urban planning and security setup, etc. have been given for various kinds of graphs. More applications for *bipartite trees where edges do not cross* for various kinds of labelings could be discovered where such trees with *graceful, harmonious, felicitous, sequential, and antimagic* labeling could serve as models. Some results of graph labelings using computer software developed to check the existence of major graph labeling schemes for an arbitrary graph have been discussed. Role of repeated vertex labels and low degree vertex has been presented. Finally, for further studies, the *lexicographic order* found in *antimagic* labelings of paths, cycles, etc. could be studied for discovering more applications.

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