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# Localized Bessel Beams: Basic Properties and Emerging Communication Applications

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## Abstract

Relevant properties of Bessel beams in terms of nondiffracting propagation over ideally infinite range, with unchanged transverse profile and self-healing capability, are revised and discussed in the present chapter. Promising applications in the framework of new-generation communication systems are also outlined.

**Keywords:** focused systems, nondiffracting fields, Bessel beam, propagation, slow waves

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## 1. Introduction

Diffractive phenomena are strictly related to the wave nature of light: any field of wavelength  $\lambda$ , initially confined to a finite area having radius  $r$ , will be subjected to a diffractive spreading beyond a characteristic distance equal to  $\frac{r^2}{\lambda}$ , known as the Rayleigh range [1]. In the late 1980s, a class of diffraction-free mode solutions has been introduced [2] to describe well-defined nonspreading beams with extremely narrow central spot. They are called Bessel beams, due to the particular shape in the transverse plane, which is mathematically described by a Bessel function [3]. A qualitative comparison of Gaussian (diffractive) and ideal Bessel (localized) beams is illustrated in **Figures 1** and **2**: it is evident that Gaussian beam (**Figure 1**) soon diffracts for very short distance range, while the Bessel beam (**Figure 2**) ideally maintains a stationary transverse pattern along the propagation distance.

As deeply addressed in Section 2, Bessel beams are solutions to the Helmholtz equation in circular cylindrical coordinates, revealing many appealing features, namely

- they theoretically guarantee an indefinite extension along the axial (propagation) direction;
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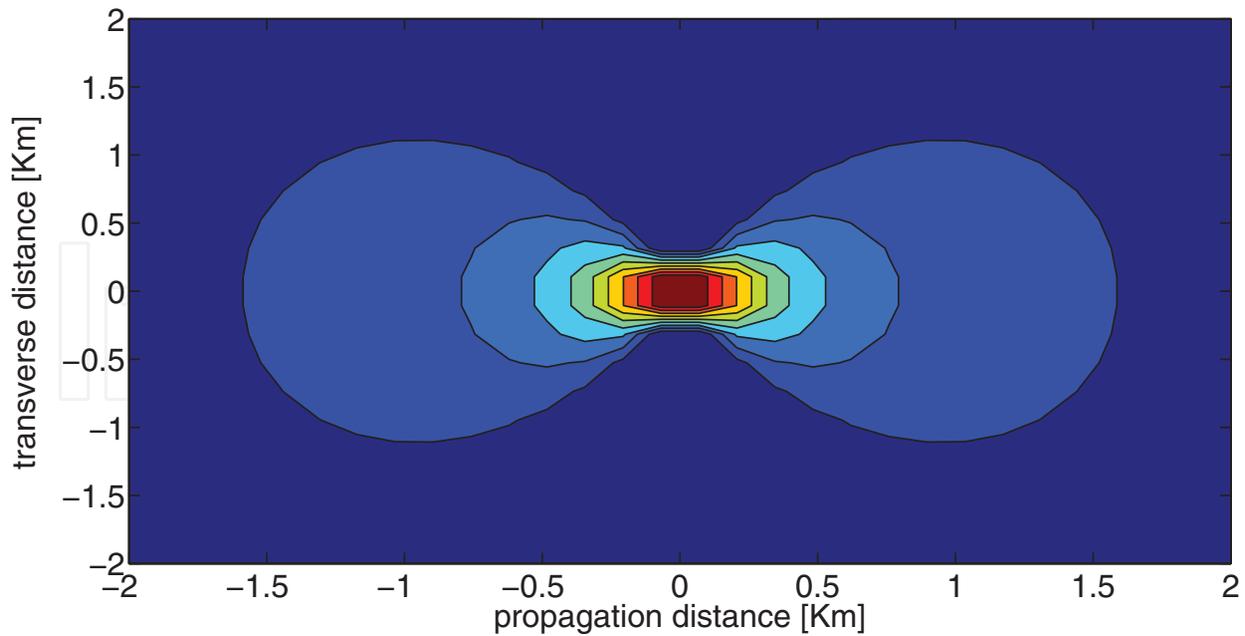


Figure 1. Gaussian beam propagating into a homogeneous medium.

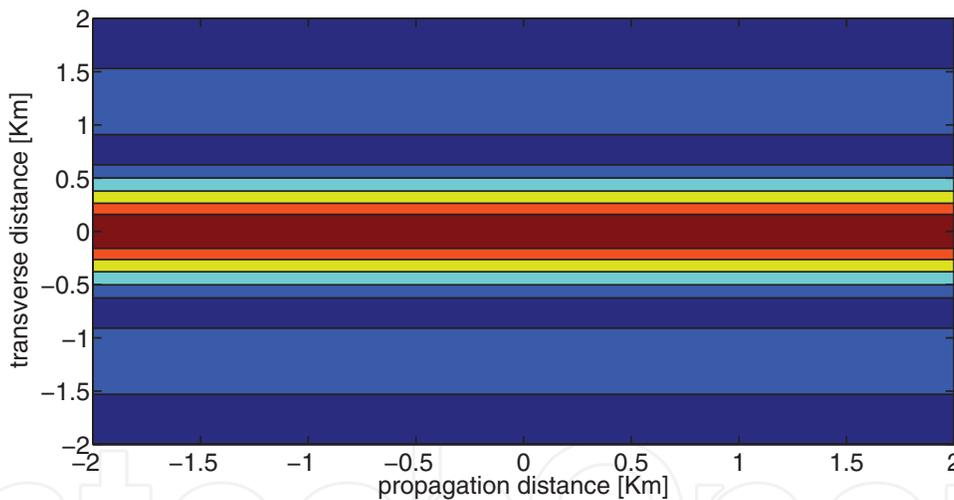


Figure 2. Bessel beam propagating into a homogeneous medium.

- they own the property of reconstruction after obstruction (“self-healing”), that is, if an obstacle occurs at the center of the beam, it will not block the rays, as they will interfere each other to reform the beam.

The ideal Bessel beam, having an infinite number of rings and covering an infinite distance, cannot physically exist, as it requires an infinite energy. Thus, physically meaningful beams are the apertured Bessel beams, possessing nearly diffracting properties within a limited axial distance. In this chapter, an overview of the basic properties of localized Bessel beams is provided, and some specific applications in the framework of emerging communication technologies are discussed. In particular, mathematical details on Bessel beams solutions are presented in Section 2, where the possibility to express the Bessel beam in terms of plane waves traveling on a

cone with the same phase velocity is demonstrated. In Section 3.1, a practical realization example of microwave Bessel beam source to be applied for focused high-penetration applications is described, while in Sections 3.2 and 3.3, a discussion about potential applications in the framework of high-performance communication systems is outlined.

## 2. Localized Bessel beam solutions

Let us consider the scalar wave equation:

$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0 \quad (1)$$

where  $c$  is the free-space velocity.

In any system of cylindrical coordinates  $(u_1, u_2, z)$ , the wave equation (1) is satisfied by a solution of the form [4]:

$$\psi(\rho, z, t) = f(\rho)e^{j(k_z z - \omega t)} \quad (2)$$

where

$$\rho = \sqrt{u_1^2 + u_2^2} \quad (3)$$

is the transverse distance, and the radial function shape  $f(\rho)$  is assumed to be preserved along the propagation axis  $z$ .

When replacing the solution (2) into Eq. (1), the following equation is obtained:

$$\rho^2 \frac{d^2 f(\rho)}{d\rho^2} + \rho \frac{df(\rho)}{d\rho} + \rho^2 (k^2 - k_z^2) f(\rho) = 0 \quad (4)$$

where  $k^2 = \omega^2/c^2$ .

Eq. (4) satisfied by the radial function  $f(\rho)$  can be easily recognized as Bessel's equation, namely

$$x^2 \frac{d^2 J_p(x)}{dx^2} + x \frac{dJ_p(x)}{dx} + (x^2 - p^2) J_p(x) = 0 \quad (5)$$

where  $J_p(\dots)$  is the cylindrical Bessel function of order  $p$ .

Thus, a solution of the wave equation (1) which maintains unchanged its radial shape can be written as

$$\psi(\rho, z, t) = J_0(k_\rho \rho) e^{j(k_z z - \omega t)} \quad (6)$$

where

$$k_\rho = k^2 - k_z^2$$

$$k^2 = \frac{\omega^2}{c^2} = k_1^2 + k_2^2 + k_z^2 = k_\rho^2 + k_z^2 \quad (7)$$

Recalling that

$$k_1 = k \sin \theta \cos \phi, \quad k_2 = k \sin \theta \sin \phi, \quad k_z = k \cos \theta \quad (8)$$

we have

$$k_\rho^2 = k_1^2 + k_2^2 = k^2 \sin^2 \theta \quad (9)$$

Replacing expression (9) into the solution (6), we can write

$$\psi(\rho, z, t) = J_0(\rho k \sin \theta) e^{j(kz \cos \theta - \omega t)} \quad (10)$$

In particular, for  $\theta = 0^\circ$ , the solution (10) reduces to a plane wave propagating along the  $z$ -direction.

Now, let us recall the Bessel function can be written as

$$J_0(\rho) = \frac{1}{2\pi} \int_0^{2\pi} e^{j\rho \cos \alpha} d\alpha \quad (11)$$

When imposing

$$\alpha = \phi - \phi', \quad u_1 = \rho \cos \phi', \quad u_2 = \rho \sin \phi' \quad (12)$$

Eq. (11) can be written as

$$J_0(\rho) = \frac{1}{2\pi} \int_0^{2\pi} e^{j(u_1 \cos \phi + u_2 \sin \phi)} d\phi \quad (13)$$

Then, if replacing expression (13) into solution (10), we have

$$\Psi(\rho, z, t) = \frac{1}{2\pi} \int_0^{2\pi} e^{jk(u_1 \sin \theta \cos \phi + u_2 \sin \theta \sin \phi + z \cos \theta)} e^{-j\omega t} d\phi = \frac{1}{2\pi} \int_0^{2\pi} e^{j(k \cdot \mathbf{u} - \omega t)} d\phi \quad (14)$$

The latter equation demonstrates that a nondiffracting Bessel beam can be expressed in terms of plane waves, as it describes a cone of plane waves having the same inclination angle  $\theta$  with respect to the propagation axis  $z$ .

Let us consider the solution expressed by Eq. (10). For  $0 < k_\rho \leq \frac{\omega}{c}$ , it represents a nondiffracting beam whose intensity profile decays at a rate proportional to the product  $k_\rho \rho$ . The maximum value of the radial wavenumber  $k_\rho$  to have a nonevanescant field is given by  $k_\rho = \frac{\omega}{c}$ . From this condition, the spot width  $2\rho$  of the Bessel beam can be easily derived to be equal to  $\frac{3\lambda}{4}$  (**Figure 3**).

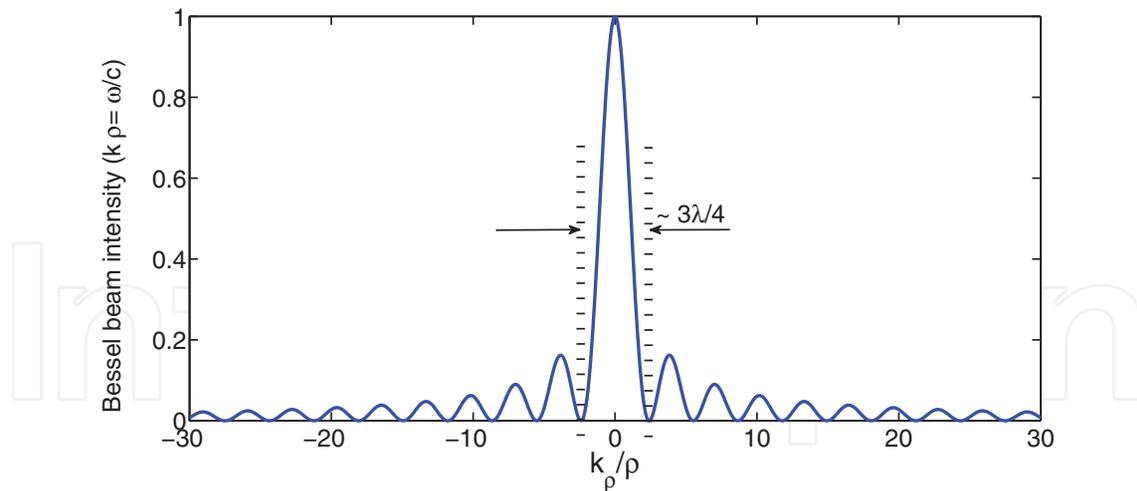


Figure 3. Spot width of Bessel beam.

As a matter of fact, the first null of the Bessel beam is equal to

$$k_{\rho}\rho = 2.325 \tag{15}$$

Thus, taking into account that  $k_{\rho} = \frac{2\pi}{\lambda}$ , it results

$$\rho = 0.37\lambda \tag{16}$$

As yet outlined in Section 1, the solution (10) describes an ideal Bessel beam, with extremely narrow intensity profile and infinite propagation distance. However, from simple energy argumentations, it can be demonstrated that such a Bessel beam is not physically realizable. The intensity field distribution given by Eq. (10) is described by the Bessel function  $J_0(\dots)$ , which is not square integrable (it decays as  $\frac{1}{\rho}$ ). This means that each ring contains the same amount of energy as the central spot. An infinite energy would be required to create the Bessel beam over the entire plane, due to the presence of an infinite number of rings. Thus, we should conclude that only an approximated Bessel beam can be practically realized over a finite area. It however maintains all appealing features of the ideal Bessel beam in terms of nondiffracting propagation over a finite distance  $z_{\max}$ , as illustrated in Figure 4.

To derive the expression of the finite range  $z_{\max}$ , let us consider the Bessel beam as given by the superimposition of plane waves, all having the same amplitude and traveling at the same angle  $\theta$  relative to the propagation  $z$ -axis.

From Eq. (9), we have

$$\theta = \sin^{-1}\left(\frac{k_{\rho}\lambda}{2\pi}\right) \tag{17}$$

Then, applying geometrical optics considerations, the following expression can be derived:

$$z_{\max} = \frac{r}{\operatorname{tg}\theta} = r\sqrt{\frac{1}{\sin^2\theta} - 1} = r\sqrt{\left(\frac{2\pi}{k_\rho\lambda}\right)^2 - 1} \quad (18)$$

$r$  being the aperture radius (**Figure 4**).

From Eq. (18) we can observe that, in order to produce a Bessel beam propagating on a long distance, either the radius  $r$  should be taken large, or the radial wavenumber  $k_\rho$  should be small, or alternately we should increase the operating frequency.

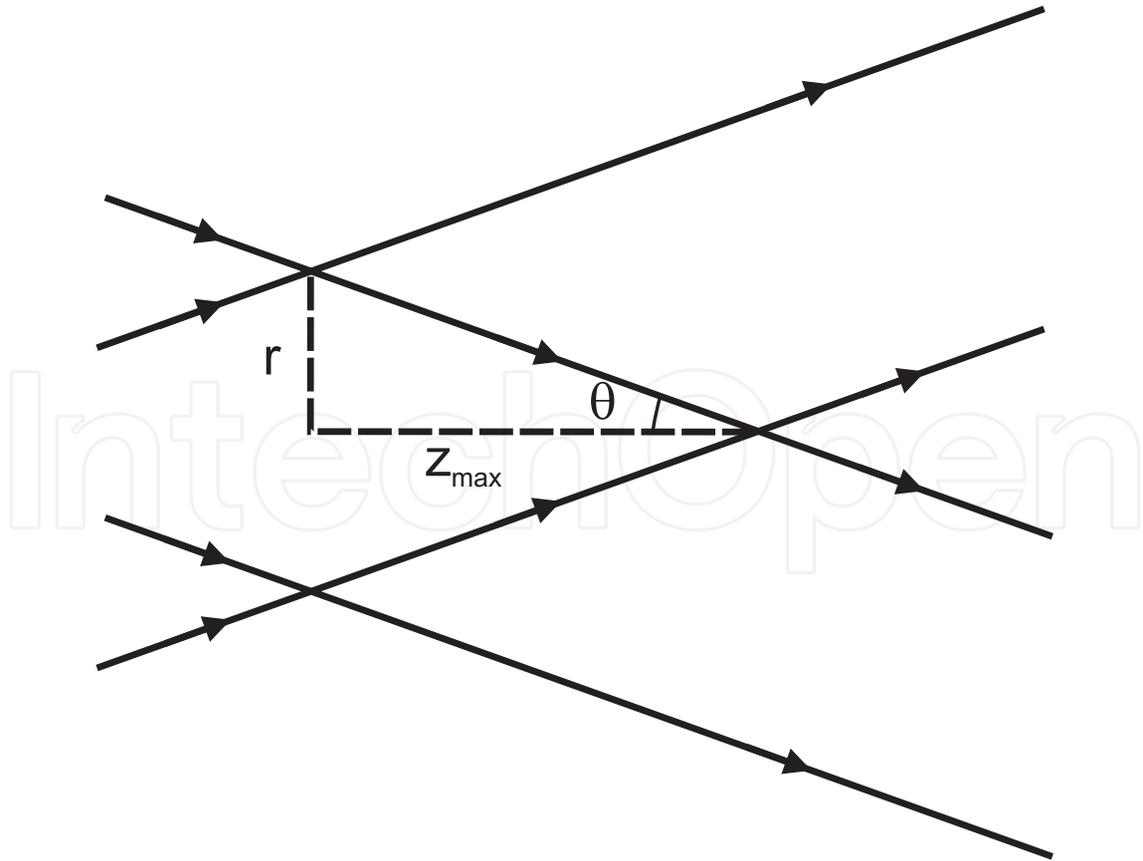
Let us consider the power  $P_r$  contained in a Bessel beam up to a radius  $r$ :

$$P_r = \int_0^{2\pi} \int_0^r |\psi(\rho)|^2 \rho d\rho d\phi = P_r = \int_0^{2\pi} \int_0^r |J_0(k_\rho\rho)|^2 \rho d\rho d\phi = r^2 [J_0^2(k_\rho r) + J_1^2(k_\rho r)] \quad (19)$$

When imposing relation (15), we have

$$P_r = \pi r^2 J_1^2(2.325) \quad (20)$$

If comparing the above result with that relative to a Gaussian beam produced by the same aperture, an increased power delivery can be demonstrated.



**Figure 4.** Finite propagation range of Bessel beam from finite aperture of radius  $r$ .

### 3. Potential applications of localized Bessel beams

The unique properties of Bessel beams in terms of self-reconstruction and profile stability over large distances make them ideal candidates in a variety of applications, requiring highly localized energy and/or diffraction mitigation. In this section, a few specific application examples to be adopted in the framework of new-generation telecommunication systems are reviewed. They include a microwave realization of Bessel beam launcher and the potential use of Bessel beams as an efficient transmission medium to increase data rate and overcome diffraction limits in long distance communications.

#### 3.1. Waveguide-based microwave Bessel beam launcher

Most of existing works in the literature are mainly focused on the generation of Bessel beams in the optical regime, through the adoption of annular slit, axicons, or lasing devices [5–9], while much less results exist in the microwave regime.

In a recent ESA (European Space Agency) research study on “Microwave Drilling” [10], a practical X-band realization of Bessel beam launcher to be adopted as a focused near-field source has been performed. Following the theoretical approach outlined in [11], the microwave Bessel beam is generated as the aperture field at the open end of a metallic circular waveguide. In particular, a transverse electric (TE) representation is adopted for the zero-order Bessel beam, with an expansion in terms of a finite number of propagating  $TE_{0n}$  modes of the circular waveguide. These modes are produced by a set of elementary loop antennas, whose excitation coefficients are properly chosen to obtain the prescribed beam intensity and spot size. By imposing the Bessel beam to have  $n$  distinct annular sections, and choosing the radial wavenumber  $k_\rho$  to match the relative cutoff wavenumber  $\chi_{cn}$  of the modes inside the circular waveguide [12], the Bessel beam can be fully represented by the single  $TE_{0n}$  mode.

The schematic configuration of X-band Bessel beam launcher developed in [10] is illustrated in **Figure 5**. A single-loop antenna is considered for the proper field generation inside the circular waveguide of radius  $r = a$ . The excitation antenna is located at a distance equal to  $\frac{\lambda_0}{2}$  (at the central operating frequency  $f_0$ ) from the short-circuited side of the circular waveguide, in order to match with the maximum value of the current, so to realize the maximum coupling with the magnetic field.

A Bessel beam with  $n = 3$  distinct annular rings is chosen, so deriving the value of radius  $a$  from the relation [13]:

$$k_\rho a = \chi_{03} = 10.1735 \quad (21)$$

which gives  $a \cong 1.5\lambda_0$ .

The realized prototype, for a design frequency  $f_0 = 8.74\text{GHz}$ , is illustrated in **Figure 6**, where the microstrip loop antenna, designed on a standard substrate Arlon Diclاد 880 ( $\epsilon_r = 2.2$ , thickness  $t = 0.762\text{ mm}$ ) is also reported.

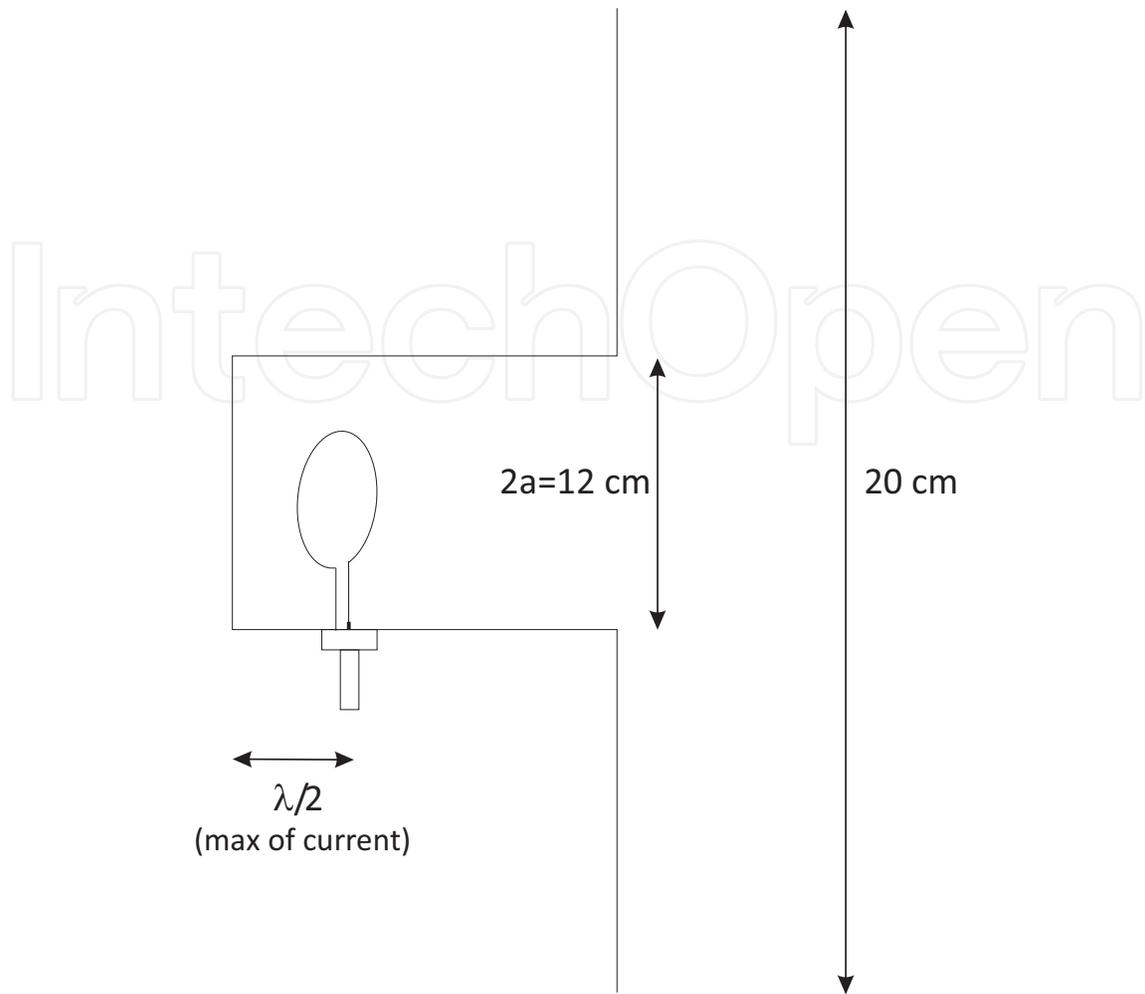


Figure 5. Schematic configuration of microwave Bessel beam launcher (taken from [10]).

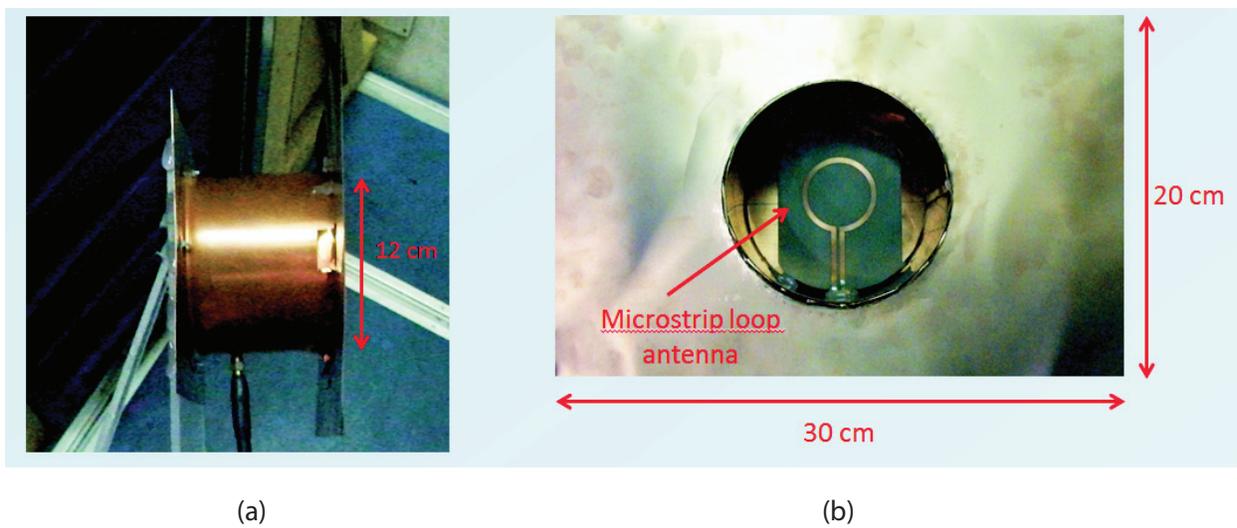


Figure 6. Photograph of realized microwave Bessel beam launcher (taken from [10]): (a) side view and (b) front view.

In order to verify the nondiffracting feature along the propagation range, near-field measurements are performed on various planes placed at different distances from the Bessel beam launcher. All near-field acquisitions are realized on a square grid of  $43 \times 43$  points, with uniform spacing equal to  $\frac{\lambda_0}{2}$ , to satisfy the Shannon-Nyquist criterion. Near-field tests are executed into the anechoic chamber at the Microwave Laboratory of the University of Calabria, by adopting a standard X-band rectangular waveguide as a measuring probe. Contours profiles reported in **Figure 7**, referring to the acquisition planes at distances equal to 8 and 10 cm, respectively, reveal that Bessel beam produced by the designed microwave launcher properly maintains its shape on a long propagation distance.

### 3.2. Bessel beam as information carrier in telecommunication systems

The impressive property of unchanged shape over extended propagation distances makes Bessel beams appealing also in communication systems as information carriers. High-order Bessel beams expressed by

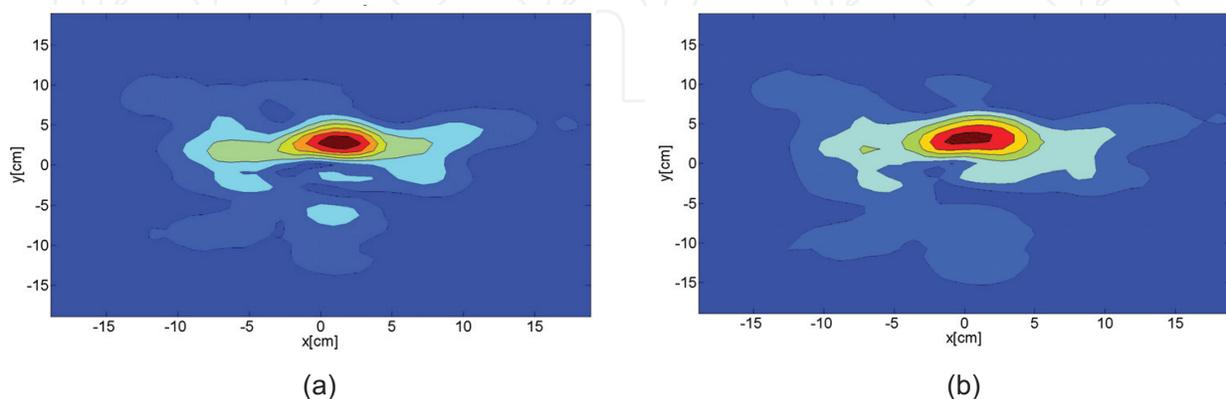
$$J_p(k_\rho \rho) e^{jp\phi} \tag{22}$$

have azimuthal index  $p$  which gives an additional degree of freedom to create custom fields [14–16] for carrying encoded information, by adopting the expansion:

$$\sum_p a_p(\rho) e^{jp\phi} \tag{23}$$

whose harmonics are independent of spatial scale and orthonormal over the azimuthal plane.

Researchers are currently looking at this interesting application of Bessel beam as a mean of transferring data, with a special focus on the development of efficient techniques to perform modal decomposition, but avoiding false detections due to cross-talk effects between neighboring modes.



**Figure 7.** Intensity profile of measured Bessel beam at distances equal to (a) 8 cm and (b) 10 cm (taken from [13]): (a) side view and (b) front view.

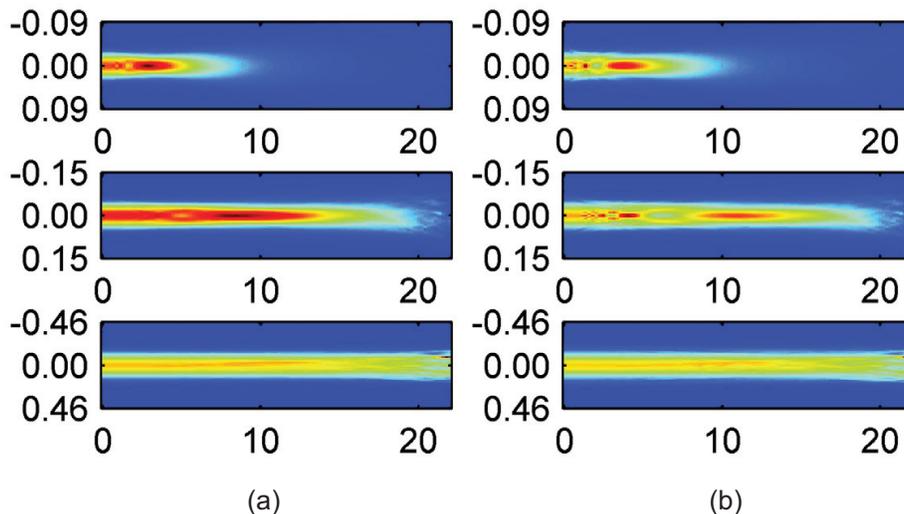
### 3.3. Bessel beam application in free-space optical communication

Free-space optical (FSO) communication is a robust method to transmit information with high capacity, high speed, and security [17]. Gaussian beams are typically adopted to realize the propagation; however, they suffer from limitations caused by diffraction, leading to the spread of the beam's energy, thus lowering the signal-to-noise ratio (SNR) at the receiver and increasing the bit error rate (BER).

In order to investigate the above effects, nondiffracting Bessel beams can be successfully adopted as alternative to Gaussian beams. An efficient FSO communication system should have a transmission beam as small as possible, with high peak intensity and high power. As deeply discussed in [18], these criteria are fully satisfied by Bessel beams. First of all, thanks to relation (15), the aperture radius required for a Bessel beam to transmit the half power is smaller than that required by a Gaussian beam, with a reduction of about 25% [18]. Furthermore, when comparing Bessel and Gaussian beams generated with the same aperture, the peak intensity of Bessel beam results to be about 1.2 times greater than that of the Gaussian beam.

As a validation example, the intensity cross-sections for long range propagation at a distance of 22 km through atmosphere are simulated in **Figure 8** for Bessel beams (left hand) and Gaussian beams (right hand) and three different aperture radii. The presented results are taken from [18], vertical axis giving the radial distance in [m] and horizontal axis representing the propagation distance in [km]. The outperformances of Bessel beams are clearly visible.

The actual challenge to really achieve FSO communication systems with improved power delivery features for long propagation distances still remains the realizations of launchers less complex than standard axicons but able to produce Bessel beams efficiently.



**Figure 8.** Intensity cross-sections for long-range propagation (taken from [18]): (a) Bessel beams and (b) Gaussian beams.

## 4. Conclusions

The basic features of nondiffracting Bessel beams have been reviewed in the present chapter, and mathematical discussions have been outlined to derive the relevant properties, such as the spot size, the maximum nondiffracting propagation range, and the delivered power. Assuming a waveguide-based structure as a beam launcher, it has been shown that three degrees of freedom exist to maximize the propagation distance of a Bessel beam without spreading, namely the radius of the aperture from which the beam is generated, the radial wavenumber (in turn depending on the launcher geometry and the dielectric medium properties), and the operating frequency (higher propagation ranges can be achieved when increasing frequency). Finally, potential applications of Bessel beam as an efficient information carrier for long range communication systems have been outlined.

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