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15

Modelling of Magnetosensitive Elastomers

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1. Introduction

Magnetorheological elastomers (MREs), also referred to as magnetosensitive (MS) elastomers, are smart materials composed of micron-sized ferrous particles dispersed in a polymer matrix. Commonly, magnetic fields are applied to the polymer composite during cross-linking so that chainlike columnar particle structures can be formed and fixed in the matrix after curing. The unique characteristic of MRE is that its shear modulus can be continuously controlled by the external magnetic field (Gong et al., 2005). Shearing of the cured composite in the presence of the magnetic field causes particle displacement from their low energy state, thereby requiring additional work. This work rises monotonically with applied magnetic field, thus resulting in a field dependent shear modulus. It has been reported (Jolly and all., 1996) that the maximum increase in the shear modulus due to the MR effect is about 50–60% of the zero-field modulus, depending on the matrix elastomer. For hard elastomers like natural rubber the relative increase has typically been 30–40%. The field-induced modulus increase is substantial even at kilohertz mechanical frequencies (Ginder et al., 2002). Such properties make MREs promising in many applications in automotive industry as variable stiffness suspension systems and active damping components (Carlson & Jolly, 2000, Lerner & Cunefare, 2007, Kalio et al., 2007, Deng & Gong, 2007).
To provide basic guidelines for designing and optimization of MR devices it is necessary to simulate the static and dynamic behaviour of magnetosensitive materials submitted simultaneously to the action of the mechanical loading and magnetic field. Modern design practices in the rubber industry are largely based on finite element simulations and the accuracy of these analyses relies on the ability of the used constitutive model to predict the mechanical response of the MS material.
Nowadays the so–called magnetoelastic coupling is widely used for simulations of the reciprocal effect between the magnetic and the elastic field. In particular, the linear elastic behaviour is usually considered along with the linear or non linear magnetic properties. Most of the simulations are based on very simple linear magnetoelastic models defined by using the magnetic forces as loads. In these weak-coupling models the magnetic equations and the mechanical equations are solved separately. More accurate linear elastic models, based on the strong coupling, solve simultaneously the governing equations of the problem (Belahcen, 2004, Hasebe et al., 2007, Zhou & Wang, 2006a,b).
However, elastomers exhibit strongly nonlinear elastic behaviour and undergo large deformations. The truthful magnetoelastic models of MS elastomers should incorporate the

nonlinear response at finite strains in order to enable the simulation of basic features and important aspects of the experimentally observed behaviour. The mathematical theory of nonlinear elasticity, the application of this theory to the solution of boundary-value problems and the analysis of the mechanical properties of solid materials capable of large elastic deformations are available in the books by Ogden (1984) and Holzapfel (2000).

A rigorous phenomenological theory of magnetoelastic interaction in elastic solids based on finite strain theory and the classical theory of ferromagnetism was developed by Brown (1966). His concepts of magnetic force and stress were used by Pao and Yeh (1973) who deduced the nonlinear field equations, boundary conditions and constitutive equations for soft ferromagnetic elastic materials and applied them to the magnetoelastic buckling of a plate. The comprehensive review of the basic principles, theories and equations in mechanics and electromagnetism can be found in the paper of Pao (1978) where the corresponding constitutive equations for solids and boundary conditions are discussed as well. Modern consistent electromagnetic theory is presented in the recent book by Kovetz (2000) where the complete set of laws of electromagnetism, mechanics and thermodynamics is treated.

While the theory of magnetoelasticity is well-known and advanced, the theoretical basis of most current research in the field of MS elastomers is very recent. The full system of equations suitable for deformable MS solids in an electro-magnetic field was first considered by Brigadnov and Dorfmann (2003, 2004) who suggested a simple energy function for isotropic MS elastomers and derived the basic system of constitutive relations. The strain tensor and the magnetic induction vector are chosen as the basic variables. They presented also a numerical simulation of the simple shear of an incompressible MS elastomer between two infinite parallel plates subjected to a magnetic field perpendicular to shear direction.

In the recent series of papers of Dorfmann and Ogden (2003-2005) a theory of nonlinear magnetoelasticity for MS elastomers was developed and applied to a number of simple boundary-value problems. Other recent related theoretical works are those by Steigmann (2004, 2007) and Kankanala and Triantafyllidis (2004, 2007). Important guidelines for the experimentalists and for people involved in numerical simulations of MS elastomer response can be found in further papers of Dorfmann et al. (2005) and Bustamante et al. (2006). In the recent paper of Ottenio et al. (2008) the linearized equations governing incremental effects in a magnetoelastic solid subject to finite deformation in the presence of a magnetic field are derived and the tensors of magnetoelastic moduli are defined. These equations are then used to examine the problem of surface stability of a homogeneously pre-strained half-space.

Mentioned papers also contain solutions to representative boundary-value problems for which exact solutions can be found. For the most part such solutions are idealized in the sense that they apply only to bodies of infinite extent in one or more directions so that edge effects are not present. The first numerical simulation of a boundary-value problem involving finite geometry is reported by Bustamante et al. (2007). In their paper the problem of a circular cylindrical tube of finite length that is deformed and then subjected to an axial magnetic field is examined. The magnetic field vector is chosen as the independent magnetic variable in the constitutive law and the resulting boundary-value problem is solved using a finite-difference method.

The purpose of the present paper is to outline a possibility of the magnetoelastic problem implementation in the finite element code Comsol Multiphysics which is suitable for the simulation of coupled-field problems.

We adopt the formulation of Dorfmann and Ogden (2004) as the starting point. The relevant magnetic and mechanical balance equations and boundary conditions are summarized in Sect. 2.

Then, in Sect. 3, the general constitutive equations for magnetoelastic interactions are summarized following Dorfmann and Ogden (2005) for both compressible and incompressible magnetoelastic materials and then specialized for specific application to incompressible, isotropic magnetoelastic materials.

In Sect. 4, we present the simulation of the simple shear of a rectangular block of finite size subjected to a magnetic field, which, in the far field, is uniform and perpendicular to the shear direction. The constitutive equations are based on a modified free-energy function that depends, in addition to the deformation gradient, on the magnetic flux density vector as the independent magnetic variable.

2. Governing equations

In this section the equations for nonlinear magnetoelastic deformations, as developed by Dorfmann and Ogden (2003-2005), are summarized. We suppose that a body made of MS elastomer deforms under the common action of mechanical loading and magnetic field. The deformation is characterized by the deformation gradient tensor **F** and the strain tensors **C** or **b**. We describe the magnetic field by the magnetic flux density vector **B**, the magnetic field vector **H** and the magnetization vector **M.**

2.1 Basic kinematics of finite deformations

We consider a continuum body with fixed reference configuration $\mathcal{B}_0$ and reference time *t=0*. Then an assumed motion χ maps the reference configuration $\mathcal{B}_0$ to the current configuration $\mathcal{B}$ at time *t*. Hence a point $\mathbf{X} \in \mathcal{B}_0$ transforms to a point $\mathbf{x} \in \mathcal{B}$, where $\mathbf{X}$ and **x** (with material and spatial coordinates X_1, X_2, X_3 and x_1, x_2, x_3) characterize positions of a particle in the reference and current configurations. The deformation gradient tensor **F** and its determinant J are

$$\mathbf{F} = \mathrm{Grad}\chi, \quad F_{ia} = \frac{\partial x_i}{\partial X_a}, \quad J = \det \mathbf{F} > 0, \tag{1}$$

where Grad is the gradient operation with respect to **X** and the Jacobian of the deformation gradient J is the volume ratio. If we consider a geometric setting relative to the reference configuration (Lagrangian description) it is appropriate to introduce a strain measure in terms of material coordinates which is the symmetric and positive definite right Cauchy-Green tensor $\mathbf{C}=\mathbf{F}^T\mathbf{F}$. The strain measure in terms of spatial coordinates (Eulerian description) is the symmetric and positive definite left Cauchy-Green tensor $\mathbf{b}=\mathbf{F}\mathbf{F}^T$.

2.2 Basic equations of magnetostatics

When a magnetizable elastic solid is placed in the magnetic field, magnetic moments are induced in the body. The magnetic moment per unit volume of the deformed body is called the magnetization and it is denoted by **M** with units [A/m]. Inside the deformed body, the magnetic induction vector is **B** with units [T] or [Wb/m^2]. The induced magnetization is related to **B** by the relation

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}), \tag{2}$$

where **H** is the magnetic intensity vector and $\mu_0=4\pi \cdot 10^{-7}$ [H/m] is the permeability of vacuum. In a vacuum **M=0** and $\mathbf{B}=\mu_0\mathbf{H}$. In a ponderable body, two of these three quantities are related by a constitutive law which depends on the properties of the solid under investigation. For example, once **M** is given in terms of **B** by an appropriate constitutive law Eq. (2) determines **H**.

The **B** field satisfies Gauss' law for magnetism and is solenoidal. The field **H** is governed by Ampère's law. If we suppose the quasistatic deformation, the stationary magnetic field and non-conducting MS elastomeric material, we may omit the induced current effects and the statical magnetic field equations thus can be written in the global form

$$\int_{\partial\mathcal{B}} \mathbf{B}\cdot\mathbf{n}ds = 0, \quad \oint_{c} \mathbf{H}\cdot d\mathbf{x} = 0. \tag{3}$$

After application of the divergence theorem to Eq. $(3)_1$ and Stokes' theorem to $(3)_2$ we can deduce the local form

$$\operatorname{div}\mathbf{B} = 0, \quad \operatorname{curl}\mathbf{H} = \mathbf{0}, \tag{4}$$

where div and curl relate to the spatial coordinates. Across a boundary surface of deformed body $\partial\mathcal{B}$ the boundary conditions for **B** and **H** are

$$\mathbf{n}\cdot\left[\mathbf{B}^{+} - \mathbf{B}^{-}\right] = 0, \quad \mathbf{n}\times\left[\mathbf{H}^{+} - \mathbf{H}^{-}\right] = \mathbf{0}, \tag{5}$$

where the square brackets indicate a discontinuity across the surface and **n** is the unit normal to the surface, which, at the material boundary, is taken as the outward pointing normal, thus the (−) subscript indicates interior of the body while (+) subscript is used to indicate the surrounding space.

The fields **B** and **H** may be pulled back from $\mathcal{B}$ to $\mathcal{B}_0$ to their Lagrangian forms, denoted $\mathbf{B}_l$ and $\mathbf{H}_l$ respectively. By using Nanson formula (Ogden, 1984) and relation $d\mathbf{x}=\mathbf{F}d\mathbf{X}$ in (3) we receive the pull-back of the field variables

$$\mathbf{B}_l = J\mathbf{F}^{-1}\mathbf{B}, \quad \mathbf{H}_l = \mathbf{F}^{T}\mathbf{H}. \tag{6}$$

In terms of these quantities the analogues of (4) in Lagrangian coordinates become

$$\operatorname{Div}\mathbf{B}_l = 0, \quad \operatorname{Curl}\mathbf{H}_l = \mathbf{0}. \tag{7}$$

Since **H** and **M** occur as a sum in (2), it is appropriate to define, similarly to $(6)_2$, a Lagrangian form of **M**, denoted $\mathbf{M}_l$, by $\mathbf{M}_l = \mathbf{F}^{T}\mathbf{M}$. The Lagrangian counterpart of (2) reads

$$J^{-1}\mathbf{C}\mathbf{B}_l = \mu_0(\mathbf{H}_l + \mathbf{M}_l), \tag{8}$$

where **C** is the right Cauchy-Green deformation tensor. The boundary conditions (5) can also be expressed in Lagrangian form, namely

$$\mathbf{N}\cdot\left[\mathbf{B}_l^{+} - J\mathbf{F}^{-1}\mathbf{B}^{-}\right] = 0, \quad \mathbf{N}\times\left[\mathbf{H}_l^{+} - \mathbf{F}^{T}\mathbf{H}^{-}\right] = \mathbf{0}, \tag{9}$$

where **N** is the outward unit normal to the surface $\partial\mathcal{B}_0$ in the reference configuration. Under certain circumstances it can be helpful to formulate the problem in term of the magnetic vector potential **A** that is given by the equality **B**=curl**A** (Perez-Apparicio, 2004) or in term of a scalar magnetic potential function $\mathbf{H} = -\mathrm{grad}\varphi$ (Bustamante, 2007) with appropriate boundary conditions. Such option ensures that the expression (41) or (42) will be fulfilled identically.

2.2 Basic equations of mechanics

From the mass balance we have the following relation between the mass density ρ_0 in the material configuration $\mathcal{B}_0$ and the mass density ρ in the spatial configuration $\mathcal{B}$

$$\rho = J^{-1}\rho_0. \tag{10}$$

Cauchy's equations of motion of elastostatics resulting from the basic balance principles of mechanics such as the momentum balance principle and the angular momentum balance principle and Cauchy's stress theorem (Holzapfel, 2000) are

$$\mathrm{div}\boldsymbol{\sigma} + \rho\mathbf{f} = \rho\dot{\mathbf{v}}, \quad \boldsymbol{\sigma} = \boldsymbol{\sigma}^{\mathrm{T}}, \quad \mathbf{t} = \boldsymbol{\sigma}\cdot\mathbf{n}, \tag{11}$$

where $\boldsymbol{\sigma}$ is the symetric Cauchy's stress tensor, **f** is the body force per unit mass and **t** represents the Cauchy's traction vector (force per unit surface area).

For the case of continuum magnetoelasticity there are several different ways of defining stresses, tractions and body forces. The influence of the magnetic field on the deforming continuum may be incorporated either through magnetic force per unit volume $\mathbf{f}_{\mathrm{m}}$

$$\mathrm{div}\boldsymbol{\sigma} + \rho\mathbf{f} + \mathbf{f}_{\mathrm{m}} = \mathbf{0}, \tag{12}$$

or through the total Cauchy stress measure $\boldsymbol{\tau}$ which is symetric and includes both mechanical and electromagnetic contributions (Kovetz, 2000)

$$\mathrm{div}\boldsymbol{\tau} + \rho\mathbf{f} = \mathbf{0}. \tag{13}$$

The expressions for the magnetic force and for the total stress tensor depend on the type of the particular constitutive model. Summary of possible forms of constitutive equations, body forces and traction conditions can be found in Kankanala & Triantafyllidis (2004) and in Dorfmann et al. (2005).

At the external boundary of the body the boundary condition for the total stress tensor is

$$\boldsymbol{\tau}\cdot\mathbf{n} = \mathbf{t}, \tag{14}$$

where the traction vector **t** includes applied mechanical tractions and the contribution of Maxwell stress in the surrounding. The Maxwell stress in the absence of material is

$$\boldsymbol{\tau} = \mu_0^{-1}\left[\mathbf{B}\otimes\mathbf{B} - \frac{1}{2}(\mathbf{B}\cdot\mathbf{B})\mathbf{I}\right], \tag{15}$$

where **I** is the unit second order tensor and $\otimes$ signifies the tensor product.

3. Constitutive equations

Constitutive models for nondissipative MS elastic materials are based on modified free-energy function Ω per unit volume (Dorfmann & Ogden, 2004, Bustamante et al., 2006) which depends on a deformation measure in addition to a magnetic field variable. This modified free-energy function incorporates the magnetic energy contribution.
If the deformation gradient tensor **F** and the magnetic induction $\mathbf{B}_l$ are selected as the basic variables, the free-energy function reads Ω= Ω(**F**,$\mathbf{B}_l$). The total nominal stress tensor **T** and the magnetic field vector $\mathbf{H}_l$ in the Lagrangian configuration, the total stress tensor **τ** and the magnetic field **H** in the current configuration are by sequel

$$\mathbf{T} = \frac{\partial \Omega}{\partial \mathbf{F}}, \qquad \mathbf{H}_l = \frac{\partial \Omega}{\partial \mathbf{B}_l}, \qquad \boldsymbol{\tau} = \mathrm{J}^{-1}\mathbf{F}\frac{\partial \Omega}{\partial \mathbf{F}}, \qquad \mathbf{H} = \mathbf{F}^{-\mathrm{T}}\frac{\partial \Omega}{\partial \mathbf{B}_l}. \tag{16}$$

Note that nominal stress tensor **T** corresponds to the transpose of first Piola-Kirchhoff stress tensor. Analogical relations in which the independent variable is the magnetic field $\mathbf{H}_l$ or the magnetization $\mathbf{M}_l$ are presented by Bustamante et al. (2006).
For a volume preserving (isochoric) deformation we have J=0. It is the internal constraint for so called incompressible material which is a common idealization for rubberlike materials. The constitutive relations for stress in such materials include the Lagrange multiplier p as in

$$\mathbf{T} = \frac{\partial \Omega}{\partial \mathbf{F}} - \mathrm{p}\mathbf{F}^{-1}, \qquad \boldsymbol{\tau} = \mathbf{F}\frac{\partial \Omega}{\partial \mathbf{F}} - \mathrm{p}\mathbf{I}. \tag{17}$$

In order to be objective, the free-energy function is the scalar function of the invariants of an objective strain measure such is the right (or left) Cauchy-Green strain tensor **C** (or **b**) and its first three invariants. The magnetic field induces a preferred direction in the initially isotropic MS material. The constitutive expressions are similar to those of a transversely isotropic material and depend on the additional invariants involving the vector $\mathbf{B}_l$

$$\begin{gathered} \mathrm{I}_1 = \mathrm{tr}(\mathbf{C}),\ \mathrm{I}_2 = \frac{1}{2}\left[\left(\mathrm{tr}(\mathbf{C})\right)^2 - \mathrm{tr}\left(\mathbf{C}^2\right)\right],\ \mathrm{I}_3 = \det\mathbf{C}, \\ \mathrm{I}_4 = \left|\mathbf{B}_l\right|^2,\ \mathrm{I}_5 = \left(\mathbf{C}\mathbf{B}_l\right)\cdot\mathbf{B}_l,\ \mathrm{I}_6 = \left(\mathbf{C}^2\mathbf{B}_l\right)\cdot\mathbf{B}_l\,. \end{gathered} \tag{18}$$

The explicit forms of the total stress tensor **τ** and the magnetic field vector **H** obtained from Eqs (6, 15-17) are

$$\begin{gathered} \boldsymbol{\tau} = 2\Omega_1\mathbf{b} + 2\Omega_2(\mathrm{I}_1\mathbf{b} - \mathbf{b}^2) - \mathrm{p}\mathbf{I} + 2\Omega_5\mathbf{B}\otimes\mathbf{B} + 2\Omega_6(\mathbf{B}\otimes\mathbf{b}\mathbf{B} + \mathbf{b}\mathbf{B}\otimes\mathbf{B}), \\ \mathbf{H} = 2\left(\Omega_4\mathbf{b}^{-1}\mathbf{B} + \Omega_5\mathbf{B} + \Omega_6\mathbf{b}\mathbf{B}\right), \\ \Omega_\mathrm{i} = \frac{\partial\Omega}{\partial\mathrm{I}_\mathrm{i}}, \quad \mathbf{b} = \mathbf{F}\mathbf{F}^\mathrm{T}. \end{gathered} \tag{19}$$

The constitutive theory developed by Dorfmann, Ogden and coworkers captures the basic features of the MS elastomer magnetoelasticity and constitutes comprehensive guidelines for experimentalists in the design of tests and loading conditions necessary for practical evaluations of the material response. The relatively simple formulation of the constitutive equations is suitable for the implementation into the finite element code.

4. Modelling of magnetoelastic coupling in COMSOL Multiphysics

COMSOL Multiphysics is the interactive software for modelling and solving scientific and engineering problems based on partial differential equations (PDEs). This environment runs finite element analysis together with adaptive meshing and error control using a variety of numerical solvers. COMSOL Multiphysics converts all application mode and PDE mode equation formulations and systems to the weak form before solving them with the finite element method.

The software includes many built-in application modes which can be combined into a single multiphysics model capable of simultaneous solving of the coupled physical problem. The application modes consist of predefined templates and user interfaces already set up with equations and variables for specific areas of physics.

The application mode interfaces consist of dialog boxes for the physics in subdomains and on boundaries, edges, and points along with PDEs. The equation system view provides the possibility to modify the underlying PDEs in the case where a predefined application mode does not exactly match the application. The predefined constitutive relations can be easily modified or replaced as well.

We focused our FE simulation on the simple structure and loading as the highly nonlinear character of the coupled problem impedes so far the modelling of complicated boundary value problems. We present the FE solution of a simple quasistatic boundary value problem representing a feasible experimental set-up as well. We chose a block of MS elastomer in plain strain subjected to the simple shear and to the action of the magnetic field which, due to the chosen magnetic far-field boundary conditions, is homogeneous and perpendicular to the shear direction. We simplify the problem to two-dimensional due to the calculation time requirement however the extend to three dimensions is straightforward.

4.1 Implementation

We use the Structural Mechanics Module in combination with Magnetostatics Application Mode which allow the magnetic field and strain coupling in both directions. With regard to large displacements we adopt Moving Mesh Application Mode for the calculation of magnetic field.

We utilize the free-energy function $\Omega = \Omega(\mathbf{F},\mathbf{B}_l)$ which is the combination of Mooney-Rivlin free-energy function for hyperelastic materials implemented in Comsol and two additional terms involving the invariant I_4 and I_5 mentioned in (18)

$$\Omega(\mathbf{F},\mathbf{B}_l) = \frac{1}{2}\kappa(J-1)^2 + C_{10}\left(\overline{I}_1 - 3\right) + C_{01}\left(\overline{I}_2 - 3\right) + \mu_0^{-1}(\alpha I_4 + \beta I_5), \tag{20}$$

where $\overline{I}_1$ and $\overline{I}_2$ are the modified invariants (18) and the coefficients α and β are dimensionless material parameters characterizing the magnetoelastic coupling. Similar function was proposed by Ottenio et al. (2008). From the Eqs (19) we see that the constant α contained in Ω_4 does not affect the stress but provides the coupling between the magnetic properties of material and the deformation. On the other hand the stress is influenced by means of the parameter β contained in Ω_5. The positive value of β produces the stiffening of material in the direction of magnetic field i.e. a larger stress is required to achieve the given deformation. The parameter β simultaneously furnish the linear term in the constitutive relation (19_2) of the magnetic field $\mathbf{H}$.

Instead of the common approach, using the conjugate pairs second Piola-Kirchhoff stress and Green-Lagrange strain, Comsol uses the first Piola-Kirchhoff stress $\mathbf{P}$ and its conjugate strain measure the displacement gradient, Grad$\mathbf{u}$. The first Piola-Kirchhoff stress $\mathbf{P}$ is then calculated as

$$\mathbf{P} = \frac{\partial \Omega}{\partial \mathrm{Grad}\mathbf{u}} \tag{21}$$

and the variation of the corresponding part of the internal energy is

$$\delta \mathbf{W}_{\mathrm{int}_e} = \int_{\mathcal{B}_0} \mathbf{P} : \mathrm{Grad}\delta\mathbf{u}\mathrm{d}V. \tag{22}$$

The Magnetostatics Application Mode uses the magnetic vector potential $\mathbf{A}$ defined by the relation $\mathbf{B}$=curl$\mathbf{A}$ together with the Coulomb gauge transformation defined by div$\mathbf{A}$=0. The variation of the magnetic part of the internal energy is

$$\delta \mathbf{W}_{\mathrm{int}_{mg}} = \int_{\mathcal{B}} \mathbf{H} \cdot \delta \mathrm{curl}\mathbf{A}\mathrm{d}v. \tag{23}$$

As we use the moving mesh for the magnetostatics and the magnetic variables are in the spatial configuration, the coupled part of the stress $2\Omega_5 \mathbf{B} \otimes \mathbf{B}$ must be pulled-back into material coordinates as well as the Maxwell stress (14) used for determination of the traction vector at boundaries where the displacements are not prescribed.

4.2 Example-Simple shear

In two recent papers (Brigadnov & Dorfmann, 2003; Dorfmann & Brigadnov, 2004) the closed-form solution of the problem of unidirectional shear of an incompressible MS elastomer between two infinite parallel plates subjected to a magnetic field perpendicular to shear direction was presented. We reconsider here this problem but for a finite geometry where the magnetic field is no longer homogeneous. The problem is considered as two-dimensional restricted to x-z plane as illustrated in Fig.1.

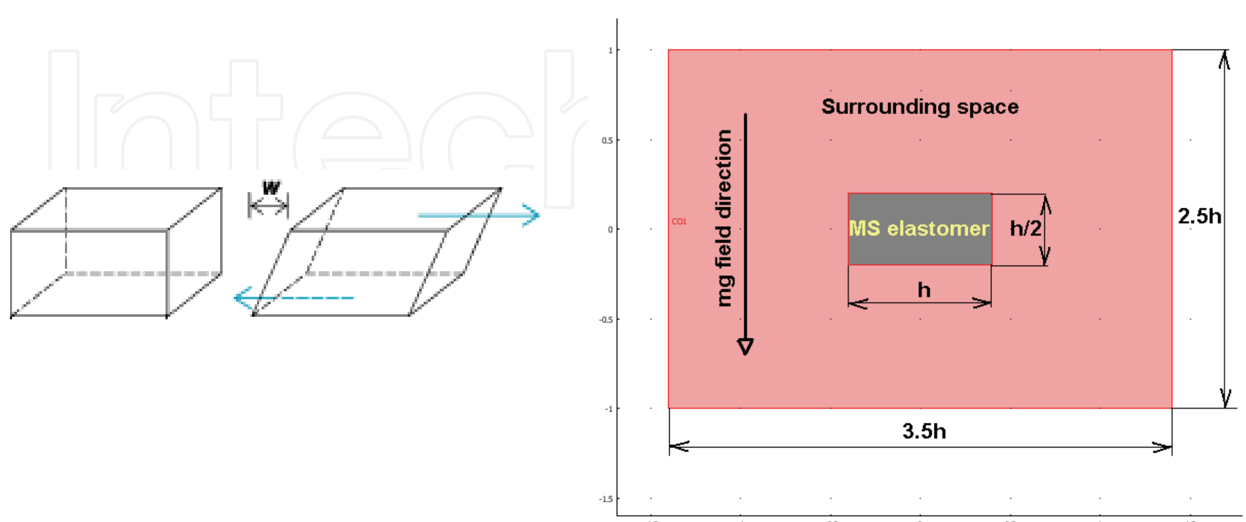

Figure 1. Block of MS elastomer in simple shear and its surrounding in Comsol geometry

The model consists of two parts – of a MS block in plain strain and of the surrounding representing the free space.
The lower face of the MS block is fixed and the upper face is subjected to the prescribed displacement in the x-direction. The lateral faces of the MS elastomeric block are free to move.
The magnetic field in the surrounding depends on the vector potential **A** prescribed to be the linear function of x-coordinate at the external boundaries. The direction of the resulting magnetic field in the surrounding space is then parallel with the y-axis. At the internal boundaries the conditions (5) for **B** and **H** are prescribed.
The moving mesh is used for the calculation of the magnetic field values. The mesh movement inside the internal subdomain and at its boundaries is determined by the displacements of the deformed block. The deformation of the mesh is smoothed in the surrounding domain toward the external boundaries where the mesh is fixed.
The loading proceeds in three stages:

- the upper face of the block is displaced gradually up to the final x-distance w=h/8, in this stage the magnetic field is inactive
- the block displacement w is constant and the magnetic field start to act gradually from 0 to its final value
- the magnetic field is constant and the upper face of the block returns gradually to its starting position

The numerical values of the parameters used in the free energy function are C_{10}=0.2 MPa, C_{01}=0.2 MPa, κ=104 MPa, α=0.5, β=0.5.

4.3 Results of the simulation

The shear stress increases with the magnetic field intensity and rearranges simultaneously namely at free boundaries in consequence of the applied traction vector due to Maxwell stress at the surrounding as shown at Fig. 2 and 3.

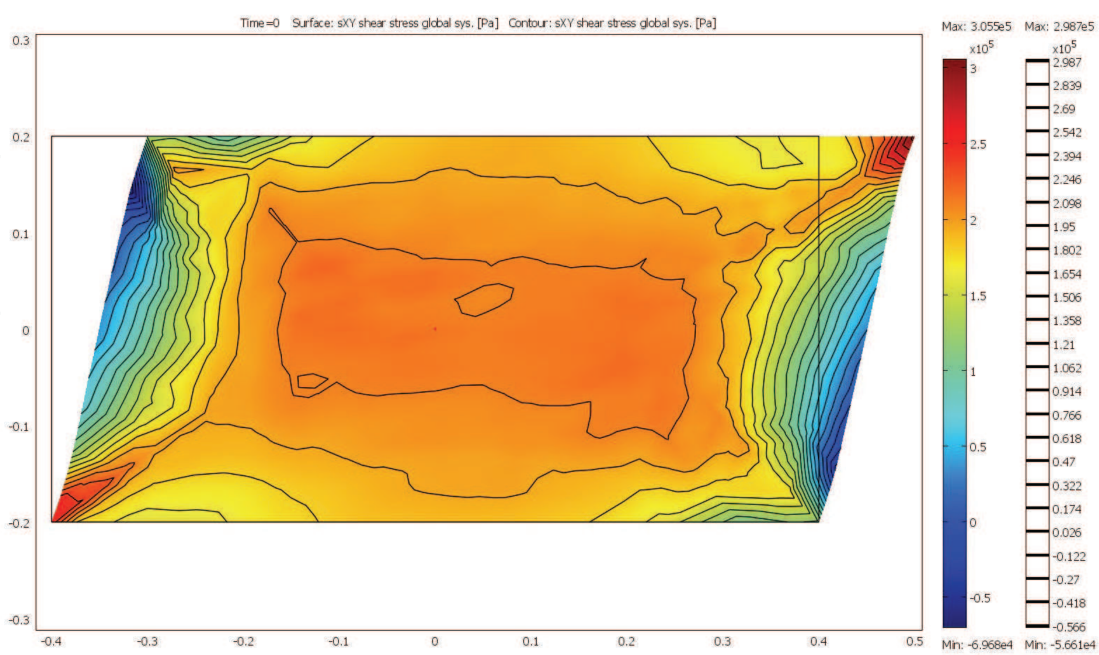

Figure 2. Shear stress σ_{xy} in the deformed block without the magnetic field

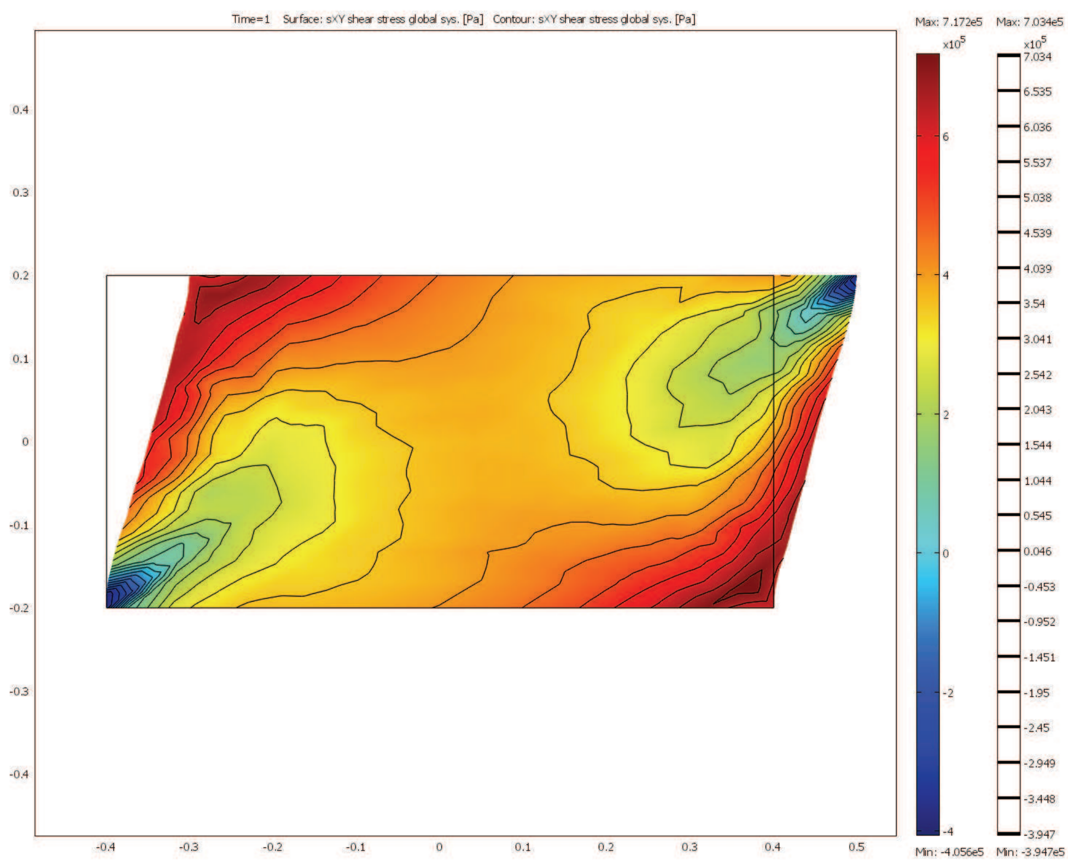

Figure 3. Shear stress σ_{xy} in the deformed block with the magnetic field action

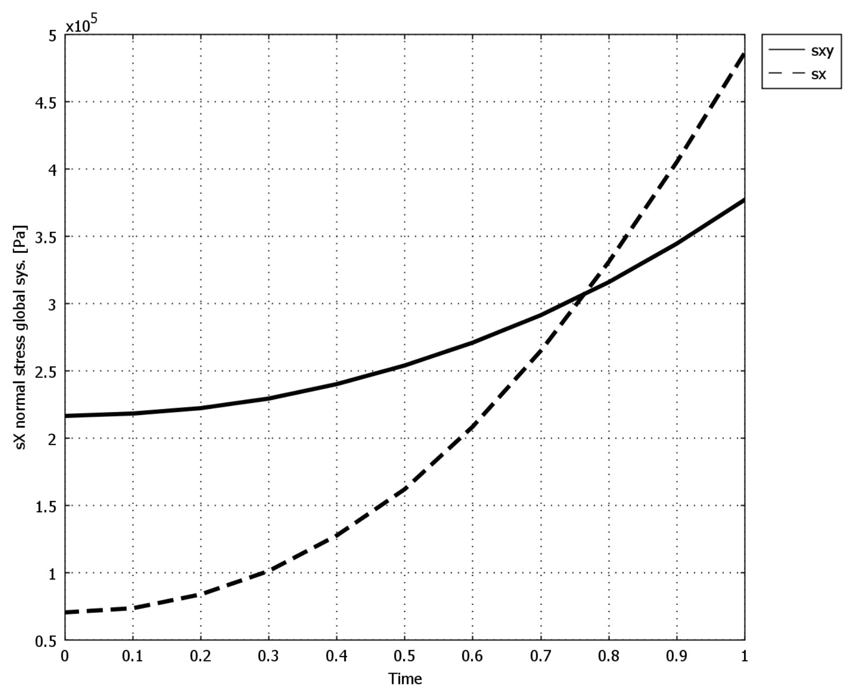

Figure 4. Variation of the shear stress σ_{xy} (solid line) and of normal stress σ_x (dashed) in the center of the block - constant deformation of the block, growing intensity of the magnetic field

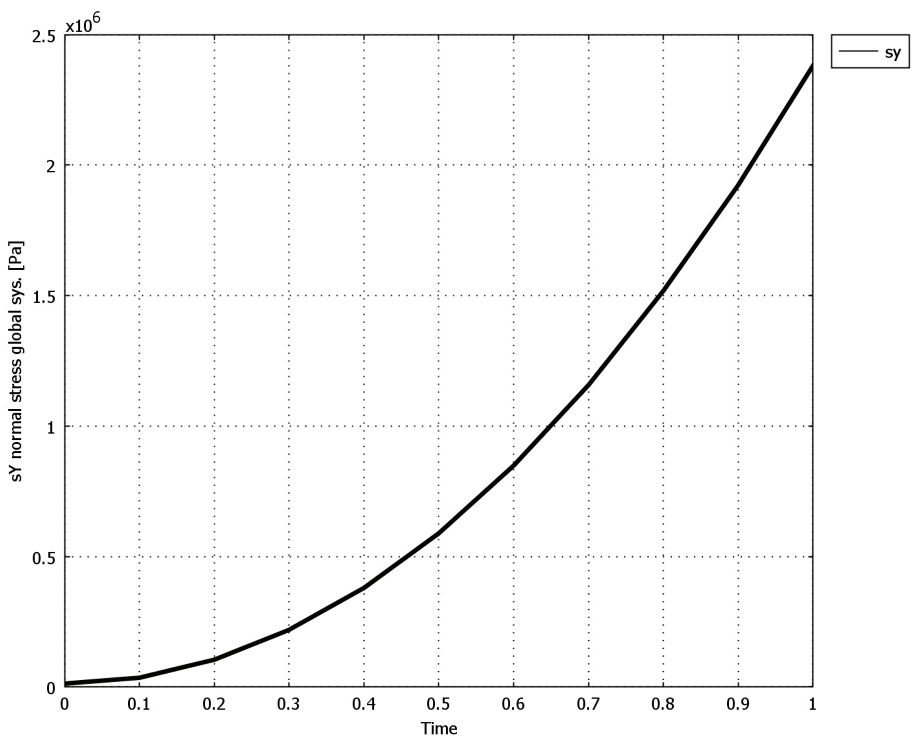

Figure 5. Rise of the normal stress σ_y in the center of the block - constant deformation of the block, growing intensity of the magnetic field

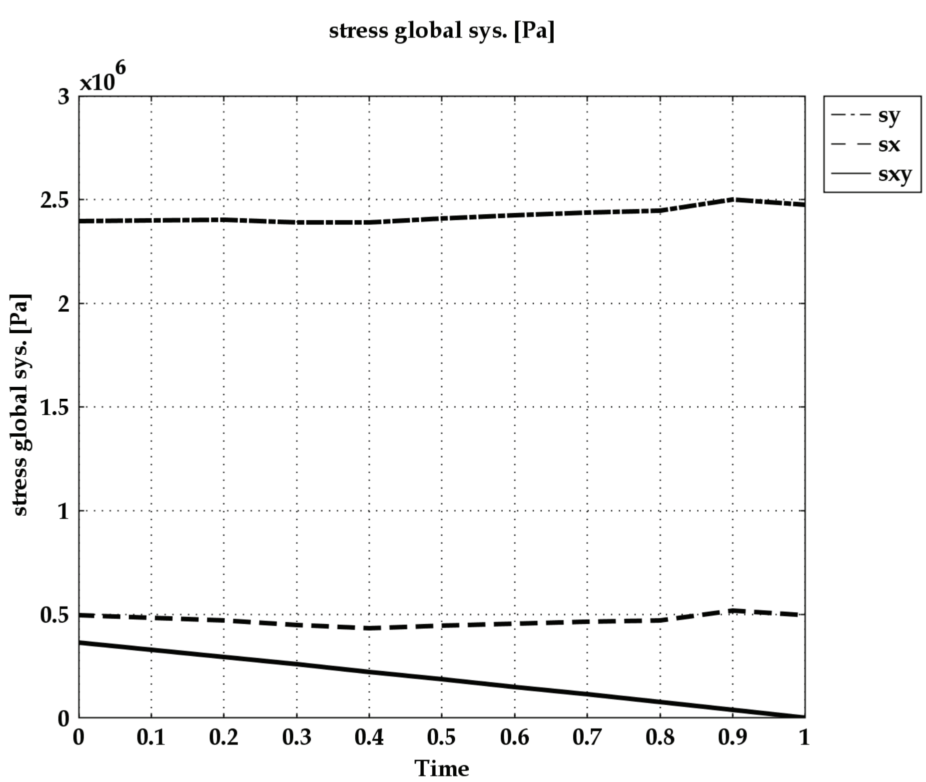

Figure 6. Variation of the stresses in the center of the block - constant intensity of the magnetic field, decreasing deformation of the block

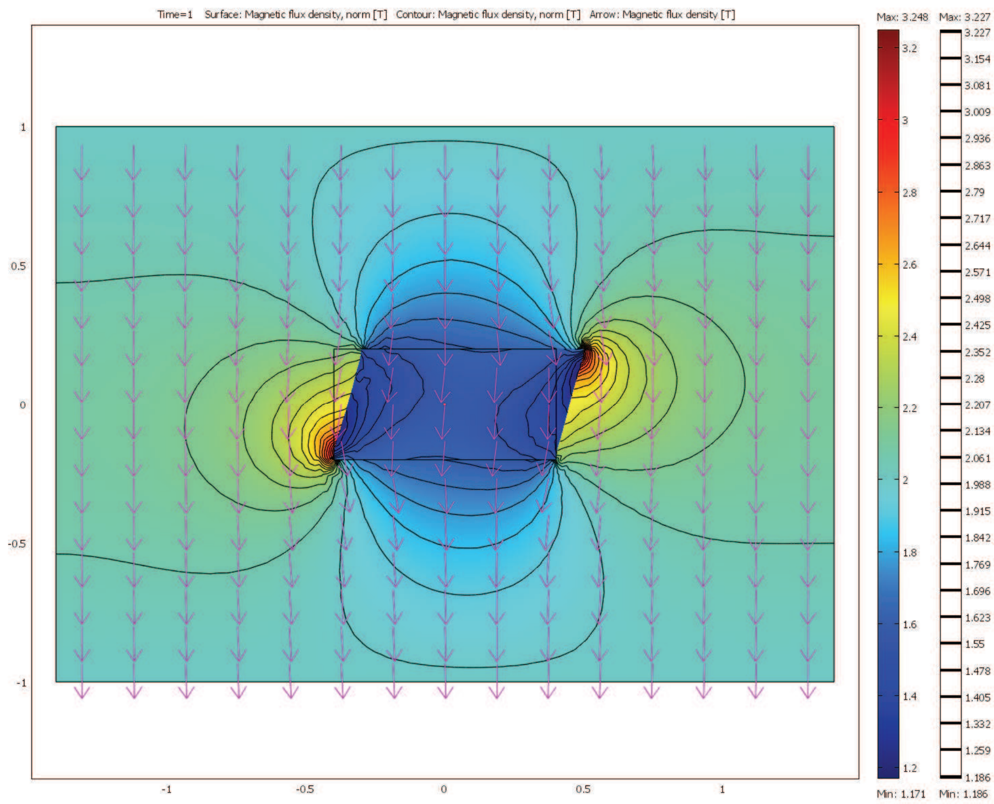

Figure 7. Norm of density of magnetic flux B – deformed block

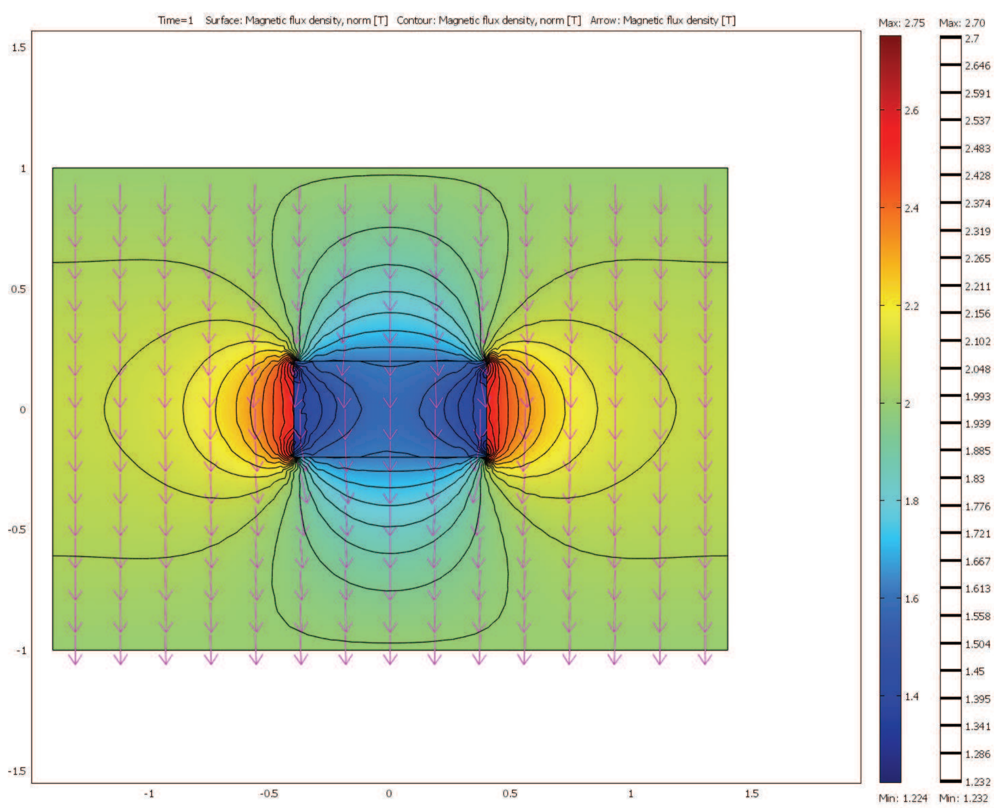

Figure 8. Norm of density of magnetic flux B – undeformed block

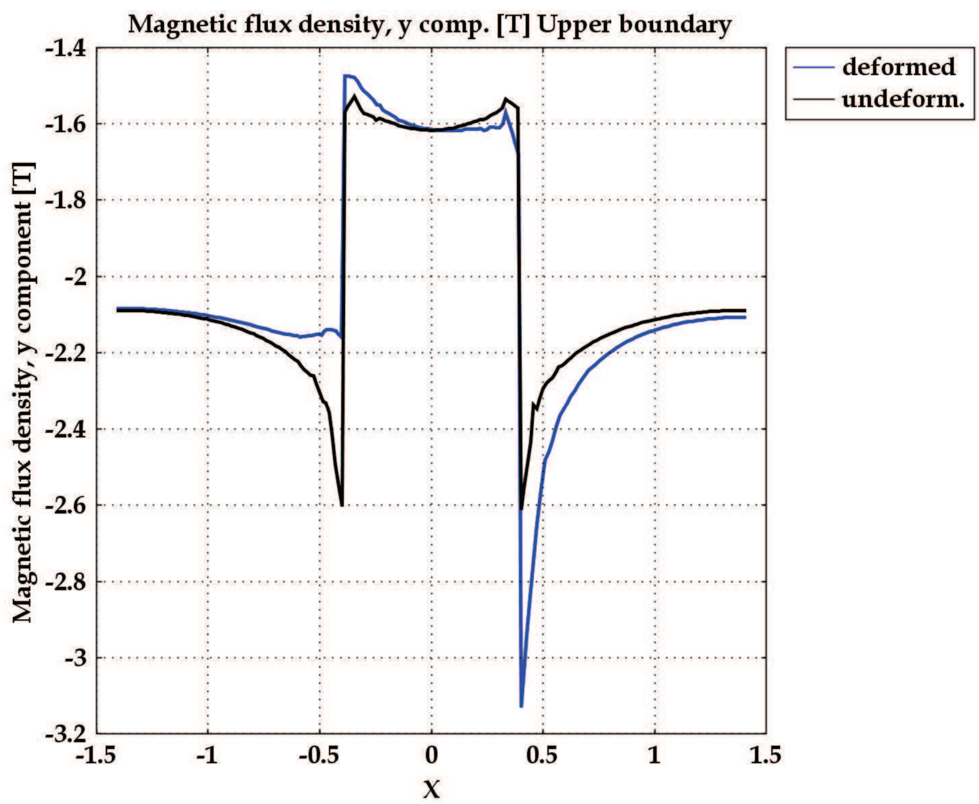

Figure 9. Magnetic flux density component Bx near the upper boundary level of the block

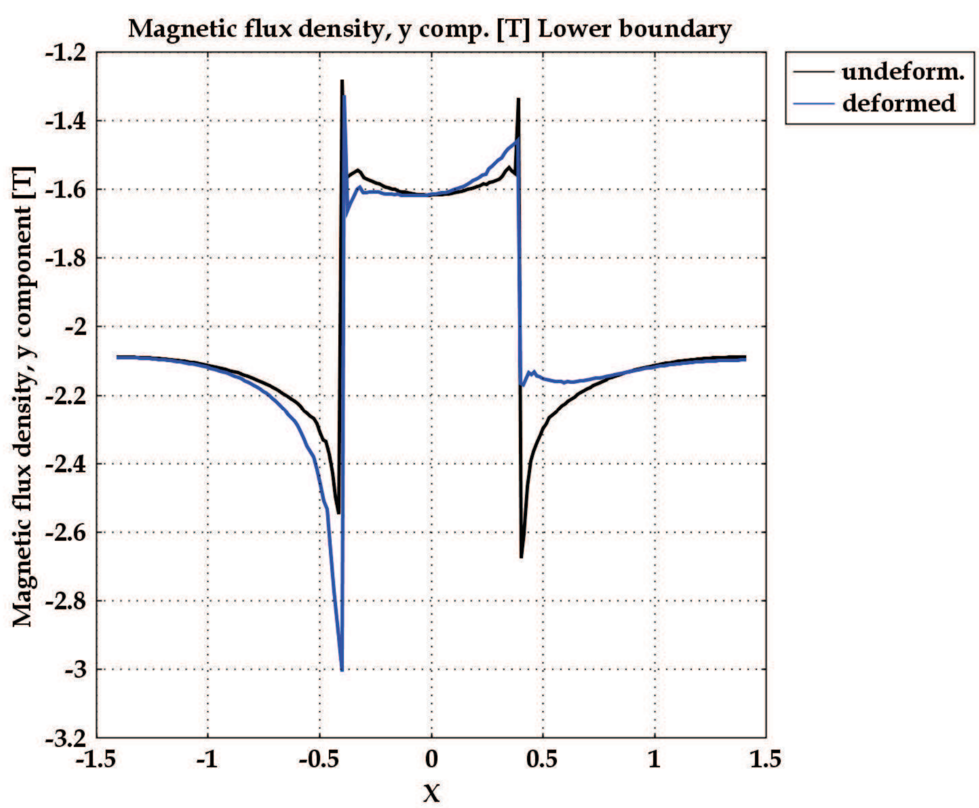

Figure 10. Magnetic flux density component Bx near the lower boundary level of the block

At the second stage of the loading, when the displacement of the upper boundary of the block is held constant and the magnetic field is switched on, the stress components start to grow cf. Fig. 4. Not only the shear stress increase with the increasing stiffness caused by the magnetic field, the component of stress σ_y spring up and grows also due to the magnetostriction, see Fig. 5. During the third stage of loading, when the magnetic field holds its maximum value and the block returns to its initial position, the component σ_y remains almost constant and even grows slightly in the final phase when the block becomes straight as can be seen at Fig. 6. The change of the norm of the magnetic flux density is obvious from the Figs 7. and 8. and from the graphs at Figs 9. and 10. where the component B_y near the block boundaries is displayed. We can conclude that the results of the simulation are in the qualitative accordance with the reality.

5. Conclusion

We present an illustrative simulation of a simple nonlinear boundary value problem of magnetoelastic interaction at finite strains. The finite element software used proved a flexibility and ability of an easy implementation of fairly complicated coupled problem. Our FE simulation involved not only the edge effects due to the finite geometry of the body but also the influence of the large displacement of the boundaries. The free energy function that we have used is of a very simple form and represents only a first approach towards a valuable constitutive model. Appropriate experiments which are in preparation will allow the elaboration of the constitutive relations. The constitutive model should involve also the anisotropy of MS elastomers cured in the presence of the magnetic field and the complex dissipative behaviour of the material.
In the thirties the renowned imaginative Czech artist Josef Sima painted a series of enigmatic pictures entitled “Mlno” (Czech archaic term for electromagnetism). Possibly, only artists can contemplate deep mysteries of this abstract science.

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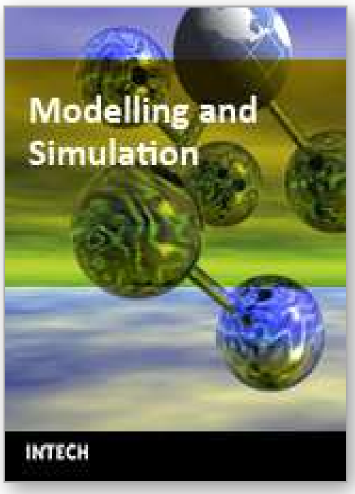

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