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Planning Hydropower Production of Small Reservoirs Under Resources and System Knowledge Uncertainty

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Additional information is available at the end of the chapter

Abstract

Available energy from water varies widely from season to season, depending on precipitation and streamflows, especially in small catchments. In addition, the reservoir operation problem is associated with the inability of operators to formulate crisp boundary conditions, due to uncertainty in knowledge. In this chapter, an approach for planning the operation of small multipurpose reservoir systems for hydropower generation and flood control under consideration of the stochastic nature of inflows and initial storage levels and allowed formulation of constraints with some range of uncertainty will be presented. The approach is based on joint chance constrained and fuzzy programming, which addresses the problem of including risk directly in the optimization. Therefore, the stochastic nature of inputs is incorporated directly in the model through the use of convolution of random variables. Furthermore, probabilistic/vague constraints and preassigned tolerance levels are used to transform the stochastic optimization problem into its deterministic equivalent. The approach searches for a control strategy, which maximizes the benefits acquired from hydropower generation and minimizes the economic losses incurred due to not meeting the required reliability levels from the various purposes served by the reservoir system. Besides the optimal reservoir release strategy, this approach also determines the optimal reliabilities of satisfying hydropower demand and flood control storage requirements. Therefore, this tool has some advantages in planning the operations of reservoirs in extreme hydrological events such as floods and droughts. The system is applied to the Wuyang small hydropower plants cascade in the People's Republic of China.

Keywords: stochastic optimization, fuzzy programming, uncertainty, hydropower

1. Problem description

Operating small hydropower plant reservoirs is a very difficult task. The managers should make future plans of releasing the water in the reservoir in order to achieve all stakeholders' requirements under consideration of the water availability. For scheduling reservoirs using optimization methods, information of the water coming from the catchment should be forecasted, the initial reservoir water levels are predefined, and the decisions are made on the amount of water to be released. It is prevailing to use past information to deterministically forecast the future, but this is quite erratic due the variability of climate and runoff. Supplementary to this, reservoir managers as humans introduce uncertainty in the interpretation of constraints into the reservoir operation system. This calls upon the consideration of robustness into the optimization system. Constraints can be generally classified into two categories: (1) physical limits and (2) operating limits. A schedule violating physical limit or constraint would not be acceptable. However, operating limits are often introduced to enhance system security, but do not represent physical bounds. Such operating limits can be temporarily violated to a certain extend if necessary, and therefore, they are fuzzy in nature, and crisp treatment of them may lead to over conservative solutions. Crisp constraints are required for the implementation of traditional deterministic optimization models. Therefore, the goal of this work is to take into account the hydrologic variability and allow formulation of constraints with some range of uncertainty.

2. Introduction

Linear programming has been used to solve many real-world problems. This method assumes that the data are definitely known, the constraints are crisp, and the objectives are well defined and can be easily formalized. However, this is not realistic in many situations. Imprecise and vague data make solving many optimization problems difficult. There are different types of uncertainty: (1) uncertainty caused by scarcity of information or (2) that the future state of the system under consideration might not be completely known. This type of uncertainty has been handled by probability theory [1–3].

Archibald et al. [1] use inflow scenarios instead of inflow probability distributions to solve the stochastic optimization problem. Faber and Stedinger [3] also apply this method as well as Schwanenberg et al. [4] who applied the approach in a real-time reservoir operation setup. With streamflow probability distributions, it may take as many state variables to represent the streamflows as there are reservoirs in the cascade. This is the case when there is little or no correlation among concurrent reservoir inflows. This type of problem is difficult to solve with SDP in an amount of time which is reasonable, if several reservoirs reside on the river. It has been shown that the number of state variables in SDP problems can be reduced by applying transformations such as principal component analysis [5–7].

Stochastic linear programming (SLP), in which the inflows are represented by first-order Markov chains, has been developed for optimizing operating strategy of a reservoir [8]. Theoretically, SLP presented in Ref. [8] can be extended to any cascade of reservoirs. However, in practice, the number of reservoirs in the cascade should be small, due to the fact that the computation time increases exponentially with the number of projects in series, as in the case of SDP. Birge [9] showed that a large stochastic multistage linear programming problem can be decomposed into one-stage linear programming problems by applying the Benders' decomposition. Pereira and Pinto [10] used the same method to determine monthly operating policy over half a year for a hydropower system of 37 reservoirs in Brazil. They represented streamflows by scenario trees with two branches in month one, four in month two, and so on. Their method is known as stochastic dual dynamic programming (SDDP).

Linear programming (LP) has been applied to solve implicit stochastic optimization problems. In this case, implicit states that a deterministic problem is solved several times, each time with a different streamflow scenario. To obtain a closed-loop solution, the results achieved from the optimizations are fed to a regression model. This method was also applied by Karamouz et al. [11] to determine a reservoir operating strategy. However, Seifi and Hipel [12] showed that there is no guarantee that the strategy determined will be feasible and efficient enough.

Chance constrained programming (CP) is often applied in reservoir management to reduce the risk of violating the boundary conditions, for example, water level and discharge. But it was shown by Loucks and Dorfman [13] that CP models are very conservative and generate operating policies that exceed the desired reliability levels. However, to solve this problem, Simonovic and Marino [14] developed in their paper a two-step method to determine the best reliability levels. The reliability levels are set in step 1, while the optimal open-loop strategy for operation of the reservoirs is determined in step 2 with linear programming (LP). However, Strycharczyk and Stedinger [15] mentioned in their paper one of the problems with this method that the reservoir discharge in period t is constant, although the problem is stochastic. In stochastic reservoir management, the content of the reservoir is a random variable since it is fed by a streamflow, which is random. The content of the reservoir at start of period t can take any value between the dead water level and the maximum reservoir volume. The optimal reservoir release for a certain period is a function of the reservoir content. Therefore, the open-loop operating strategy described in Ref. [14] not quite acceptable for listing as a stochastic reservoir management problem.

However, CP is applicable to stochastic reservoir management if there is "enough" observed data, so that probability distribution function can be formulated. In some cases, information is deposited in form of expert knowledge. This requires the introduction of the fuzzy model. Bellman and Zadeh [16] introduced the notion of a fuzzy decision making. Recently, a large number of papers have been devoted to the application of fuzzy linear programming (FLP) in modeling and solving problems of real life. Further, Zimmermann [17] and Tanaka et al. [18] applied fuzzy optimization to LP problems with multiple conflicting objectives. Zhang et al. [19] formulated a FLP problem as a four-term objective constrained optimization problem, where the cost coefficients were not crisp.

3. Methodology

3.1. Dealing with uncertainty

Hydrologic processes are random, and thus, the uncertainty as a result of variability may be appropriately quantified using the probabilistic approach. Unfortunately, this approach may lead to unreliable results due to its sole dependency on amount of the available data, the choice of the applied PDFs, inability to deal with knowledge, and human bias. Hence, other methods should be applied in these cases, where the probabilistic approach is partly applicable. The fuzzy method has proven to be very applicable to map systems, which are uncertain and/or include vague expert knowledge. According to the previous information, it is clear that the two methods can produce promising results if they are applied in conjunction. In this chapter, the power of these two approaches is integrated together as illustrated in **Figure 1**.
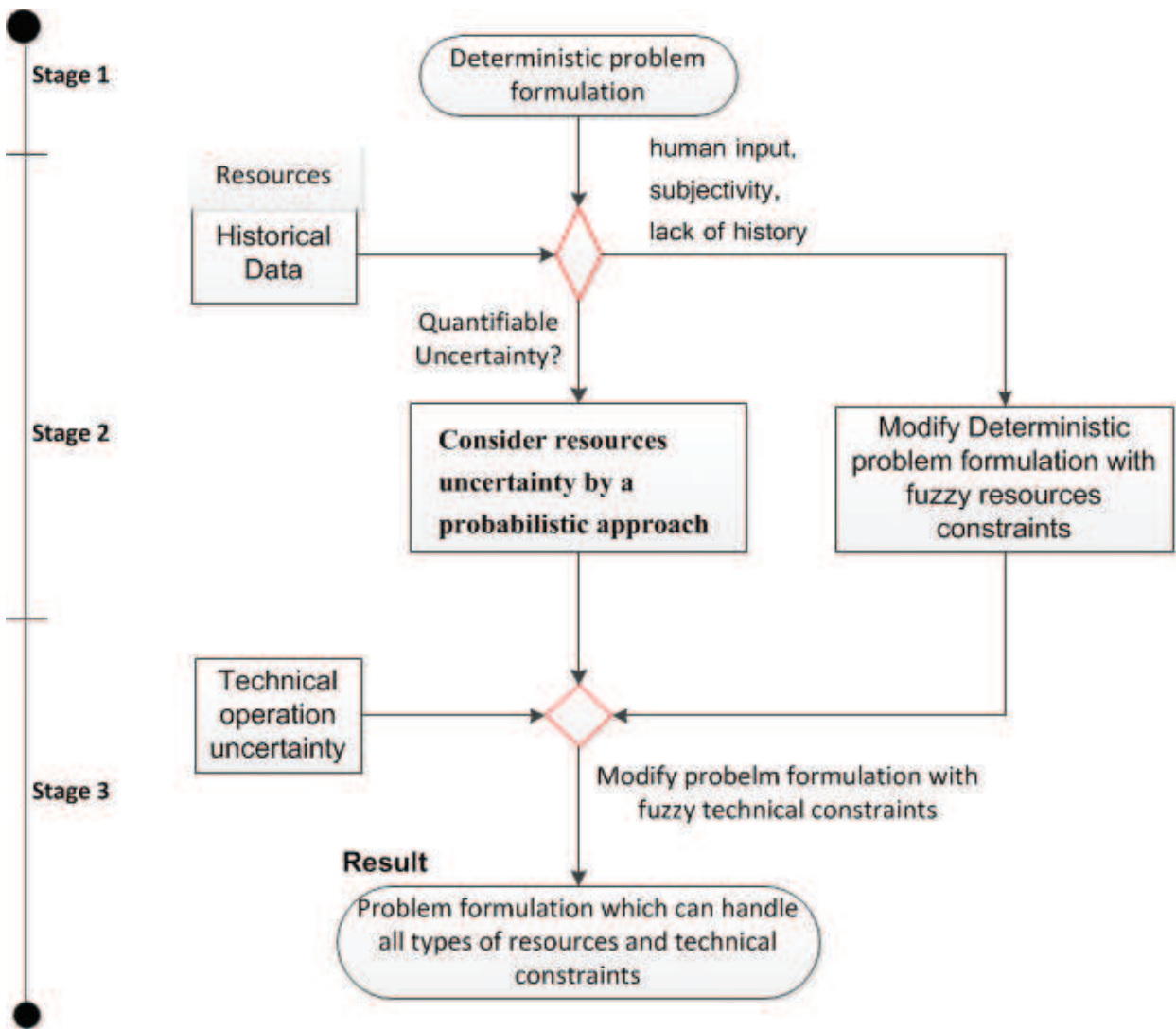

Figure 1. Approach to dealing with different types of uncertainty.

In the first stage, a deterministic optimization of the reservoir cascade is formulated to under-
stand the system. The deterministic formulation is extended in two ways in the second stage
depending on the data availability to consider random variables, for example, variability in the
inflows and demand. An extension with chance constraints is applied if data are available as
option one, an approach that has been extensively used in water resources [20]. In case of
historical data scarcity, option 2 applies, whereby the variable resources are considered fuzzy.
In addition, in the third stage, the problem will be addressed using a fuzzy optimization
approach to include vagueness in the constraints.

3.2. Mathematical formulation

3.2.1. Stage one: understanding the system using a deterministic approach

In **Figure 2**, a flux diagram of a cascade composed of several reservoirs is shown. The deter-
ministic modeling technique enables us to describe all water fluxes as shown in **Figure 3**
during every simulation stage.

Note that for each month the storage is calculated for each reservoir taking the difference of
total inflows and total outflows. Total outflow is equal to the summation of discharged and
spilled water, which is the release of the dam and will flow through to a downstream dam.
Total inflow is equal to the summation of the released flow from an upstream reservoir and
intermediate flows.

The storage equation is defined in a loop, where the storage at the end of time step "k" is
dependent on the storage at the end of time step "k-1$_L$" the inflow during time step "k$_L$" and
the turbine and spilled flow during "k$_L$" This is written in the following format:

$$S_i^k = S_i^{k-1} + I_i^k + Q_{i-1}^{k-1} + SP_{i-1}^{k-1} - Q_i^k - SP_i^k \tag{1}$$

for $k = 1, \ldots, N$ and $i = 1, \ldots, M$, S_i^0 is given, $Q_i^0 = 0$ and $SP_i^0 = 0$
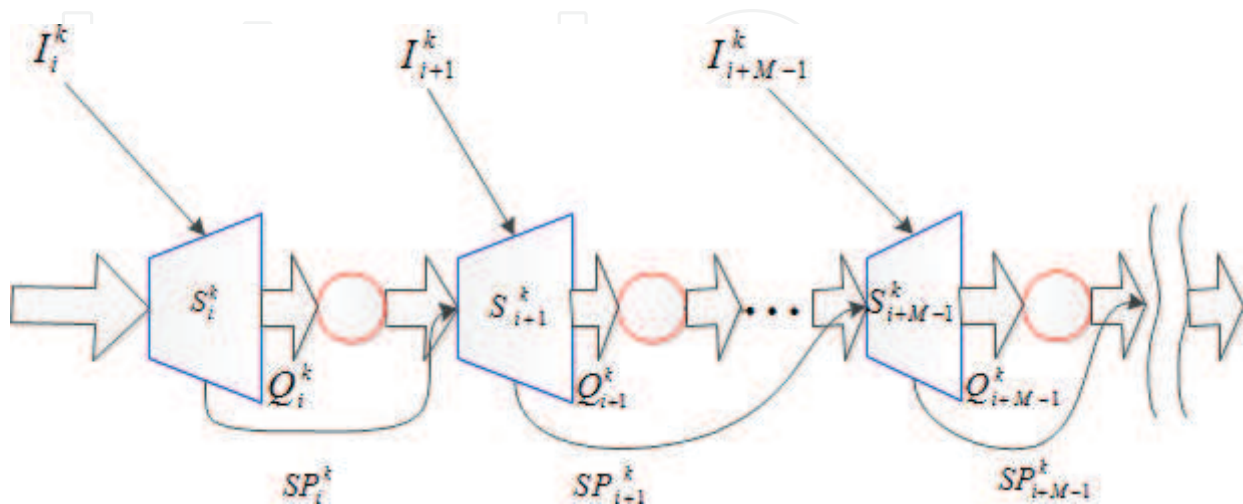

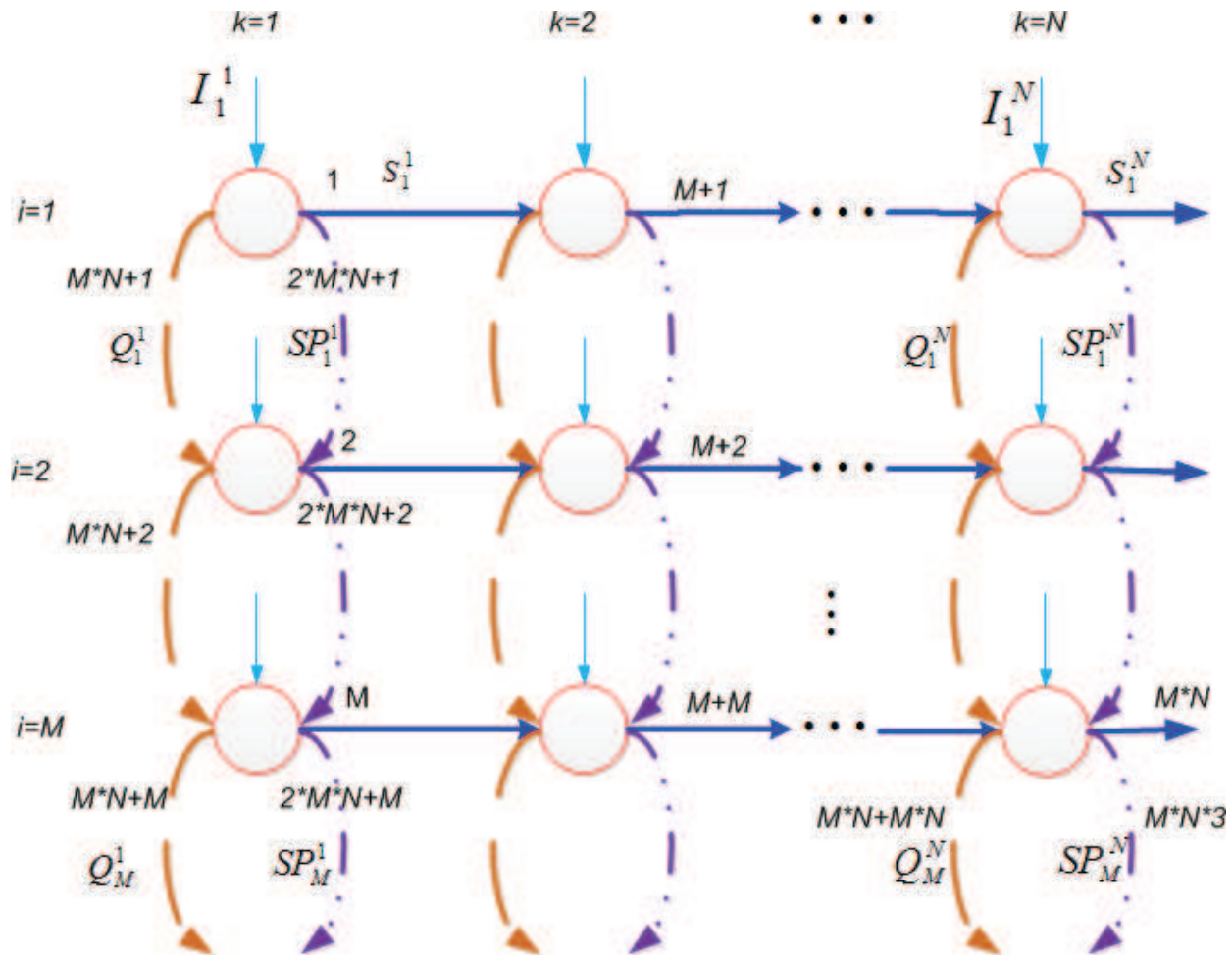**Figure 2.** Representation of a cascade with M reservoirs.

Figure 3. Water fluxes in a N time period.

where S_i^k is reservoir storage volume of reservoir i at the end of period k, I_i^k is intermediate flow (inflow to the reservoir i during period k, apart from the release from an upstream reservoir). Accordingly, it can be seen in **Figure 3** that every stage is composed of three decision nodes. Therefore, to completely determine a given stage, a set of M equations is sufficient:

$$S_1^1 = S_1^0 + I_1^1 - Q_1^1 - SP_1^1$$

$$S_2^1 = S_2^0 + I_2^1 + Q_1^1 + SP_1^1 - Q_2^1 - SP_2^1$$

$$S_3^1 = S_3^0 + I_3^1 + Q_2^1 + SP_2^1 - Q_3^1 - SP_3^1$$

$$\cdots \qquad\qquad (2)$$

$$S_1^N = S_1^{N-1} + I_1^N - Q_1^N - SP_1^N$$

$$S_2^N = S_2^{N-1} + I_2^N + Q_1^{N-1} + SP_1^{N-1} - Q_2^N - SP_2^N$$

$$S_3^N = S_3^{N-1} + I_3^N + Q_2^{N-1} + SP_2^{N-1} - Q_3^N - SP_3^N$$

Parameters determined in these equations must be within real system design ranges. This means that each variable should respect the following constraints:

$$\underline{S}_i \leq S_i^k \leq \overline{S}_i$$

$$\underline{Q}_i \leq Q_i^k \leq \overline{Q}_i$$
$$SP_i^k \geq 0 \qquad , k = 1, \ldots, N \text{ and } i = 1, \ldots, M \qquad (3)$$

$$S_i^0 \leq S_i^N$$

where $\underline{S}_i$ and $\overline{S}_i$ are lower and upper bounds of stored water of reservoir i, $\underline{Q}_i$ and $\overline{Q}_i$ are discharged water lower and upper bounds of reservoir i. Even though the volume is not forced to follow a certain reference, the last constraint ensures that the final volume of the reservoir is not below the initial value [21].

Equation systems (1) to (2) are sufficient to linearly describe the operational strategy of any cascade system with M projects. However, this problem can be simplified further by considering that every branch is represented by one state variable. This suggests a transformation of variable that still represents water fluxes dynamics. The next set of Eqs. (4)–(6) shows how to attain this for the first stage for a three-reservoir system, for example:

$$z(1) = S_1^0 + I_1^1 - z(M*N + 1) - z(2*M*N + 1) \qquad (4)$$

$$z(2) = S_2^0 + I_2^1 + z(M*N + 1) + z(2*M*N + 1) \\ -z(M*N + 2) - z(2*M*N + 2) \qquad (5)$$

$$z(3) = S_3^0 + I_3^1 + z(M*N + 2) + z(2*M*N + 2) \\ -z(M*N + 3) - z(2*M*N + 3) \qquad (6)$$

Eqs. (7), (8), and (9) are the equivalent for all remaining decision points:

$$z(j) = z(j - i) + I_i^k - z(M*N + j) - z(2*M*N + j) \qquad (7)$$

$$z(j + 1) = z(j + 1 - i) + I_2^k + z(M*N + j - 1) + z(2*M*N + j - 1) \\ -z(M*N + j) - z(2*M*N + j) \qquad (8)$$

$$z(j + 2) = z(j + 2 - i) + I_3^k + z(M*N + j - 1) + z(2*M*N + j - 1) \\ -z(M*N + j) - z(2*M*N + j). \qquad (9)$$

The initial volume (S_i^0) in each reservoir i and its inflows (I_i^k) are known, and together they build the initial data set for this problem. In order to apply these data, in an iterative optimization algorithm, they are represented in vector form as follows:

$$
B = \begin{bmatrix} S_1^0 + I_1^1 \\ S_2^0 + I_2^1 \\ S_3^0 + I_3^1 \\ \vdots \\ I_1^N \\ I_2^N \\ I_3^N \end{bmatrix} \tag{10}
$$

Consequently, equations are represented in matrices form; therefore, A represents equality constraints for state variables, and finally, Z is the vector of variables:

$$
A = \begin{bmatrix} 1 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & & 0 & 0 & 0 \\ \vdots & & \ddots & & & \vdots & \\ 0 & 0 & 0 & & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 1 & 0 \\ 0 & 0 & 0 & & 0 & -1 & 1 \end{bmatrix} ; \quad Z = \begin{matrix} z(1) \\ z(2) \\ z(3) \\ \vdots \\ z(N_m - 2) \\ z(N_m - 1) \\ z(N_m) \end{matrix} \tag{11}
$$

The problem formulation is completed by designing the optimization criteria (objective function). The objective function for hydropower energy maximization can be expressed as a product of the head for hydropower generation and the release. Therefore, nonlinear programming (NLP) may be considered for solving this problem. However, this research takes a different approach by applying linear programming to the linear operation model.

Theoretically, hydropower capacity of a storage plant installed at a reservoir can be expressed as

$$
P = \eta \rho g Q(t) H_n(t) = \gamma Q(t) H_n(t) \tag{12}
$$

where $\eta \rho g$, γ can be a constant value, η is the overall, ρ is the water density, and g is the gravity acceleration; $Q(t)$ is the release for the power generation of at time tL and the net head $H_n(t)$ can then be written as

$$
H_{n(t)} = H(t) - H_{tail}(t) - H_{loss}(t) \tag{13}
$$

where $H(t)$ denotes reservoir water level, $H_{tail}(t)$ is the water level of the downstream, and $H_{loss}(t)$ denotes the head loss at a given instant t. If $H_{tail}(t)$ and $H_{loss}(t)$ are negligible in comparison with $H(t)$, the approximation $H_n(t) \cong H(t)$ is applied, and the objective function for maximizing hydropower energy can be formulated as

$$E = \gamma \sum_{k=1}^{N} Q(k)H(k) = \gamma \, \mathbf{Q} \cdot \mathbf{H} \qquad (14)$$

where k is the discrete time and $Q(k)$ and $H(k)$ are the releases for hydropower generation and reservoir water level at time k. N is the total time. Eq. (14) is the nonlinear product of the vectors $\mathbf{Q}$ and $\mathbf{H}$. However, in this paper, a linear function is used based on the fact that Eq. (15) is maximized by maximizing each member of $\mathbf{Q}[Q(1), Q(2), \dots, Q(J)]$ and $\mathbf{H}[H(1), H(2), \dots, H(N)]$.

$$\mathbf{Q} \cdot \mathbf{H} = Q(1)H(1) + Q(2)H(2) + \cdots + Q(N)H(N) \qquad (15)$$

In turn, the nonlinear Eq. (14) can be replaced by the linear Eq. (16) as an objective function Z for maximizing hydropower energy. Eq. (16) linearly sums up each element of $\mathbf{Q}$ and $\mathbf{H}$.

$$\max_{Q,H} w_H \sum_{k=1}^{N} H(k) + w_Q \sum_{k=1}^{N} Q(k) \qquad (16)$$

where w_H and w_Q are penalty factors on water level and the releases, respectively. Eq. (17) can replace Eq. (16) due to the fact that $H(j)$ is directly proportional to $S(j)$, that is, w_H can be substituted by w_S as:

$$\max_{Q,S} \sum_{i=1}^{M} \left(w_S \sum_{k=1}^{N} Q(k) + w_Q \sum_{k=1}^{N} S(k) \right) \qquad (17)$$

Eq. (17) is the linear combination of reservoir storage $S(j)$ and release $Q(j)$, and it can be represented as an alternative objective function for maximizing hydropower energy.

Subject to

$$A \cdot Z = B$$
$$\underline{S}_i \leq S_i^k \leq \overline{S}_i$$
$$\underline{Q}_i \leq Q_i^k \leq \overline{Q}_i \qquad (18)$$
$$SP_i^k \geq 0$$
$$S_i^0 \leq S_i^N$$

The constraints are linear, the state equation is linear, and the objective function is chosen in linear form. The optimal solution can be obtained using various software tools readily available. Time lag or routing of flows between reservoirs is neglected in this formulation, which is reasonable for monthly time steps. Rainfall and net losses due to seepage, evaporation, and other reservoir losses are subtracted directly from the river inflows.

After formulating the problem deterministically, stage 2 follows which considers the uncertainty in the available resources. As shown in **Figure 2**, stage 2 is performed in two options according to the quantifiability of the available data.

3.2.2. Consideration of resources uncertainty by a probabilistic approach

To deal with LP under uncertainty, the chance constrained programming was introduced by Charnes and Cooper [22]. It extends the LP to enable the violation of the constraints to a certain extend. The reliability $\alpha \in [0, 1]$ of not violating a constraint is specified by the decision maker, and thus, it allows for decision maker to directly control the level of risk he/she finds acceptable.

The deterministic reservoir model developed previously will be transformed here in the probabilistic form to deal with some uncertain inputs. The transformation to stochastic optimization is done through the introduction of an additional probabilistic constraint, which is shown below

$$P\{\tilde{S}_i^k \le S_{i,target}\} \ge \alpha, \ k = 1, \ldots, N \text{ and } i = 1, \ldots, M \tag{19}$$

where $\tilde{S}_i k$ is the random equivalent of S_i^k, the storage at the end of period k, $S_{i,\ target}$ is the known decision maker specified target storage level of the reservoir, and α is the decision maker specified reliability of not violating constraint Eq. (19). It takes values between 0 and 1. This formulation is the realistic representation of the cascade control problem since in practice, bounds on storage are often violated by expansion of the conservation pool into the flood control pool. Adopting chance constraints simplifies the stochastic problem to deterministic. The random variables $\tilde{I}_i^k$ and $\tilde{S}_i 0$ are additive in consecutive periods, which makes the application of the convolution method feasible, whether they are linearly correlated or not [23]. The following procedure is used to do the transformation of the stochastic constraint.

Step 1. Eq. (1) is substituted into Eq. (19) to form:

$$P\{\tilde{S}_i^k = S_i^{k-1} + \tilde{I}_i k + Q_{i-1}^k + SP_{i-1}^k - Q_i^k - SP_i^k \le S_{i,target}\} \ge \alpha \tag{20}$$

where $\tilde{I}_i^k$ is the random equivalent of I_i^k, the inflow during $k = 1, \ldots, N$ and $i = 1, \ldots, M$.

Step 2. A deterministic equivalent of the Eq. (20) is found by inversion and rearrangement leading to:

$$S_{i,target} - S_i^{k-1} + \tilde{I}_i k + Q_{i-1}^k + SP_{i-1}^k - Q_i^k - SP_i^k \le F_{x_i^k}^{-1}(1 - \alpha) \tag{21}$$

where for $k = 1, \ldots, N$ and $i = 1, \ldots, M$, $F_{x_i^k}^{-1}(1 - \alpha)$ is the inverse value of the cumulative distribution function of the convoluted $\tilde{I}_i^k$, evaluated at (1-α). Henceforth, it will be replaced by $x_i^{k,1-\alpha}$.

Step 3. The expression for two deterministic chance constraint time steps is given below,

for $k = 1$ and $i = 1, \ldots, M$,

$$P\left\{\tilde{S}_i^1 \leq S_{i,target}\right\} \geq \alpha$$

$$P\left\{\tilde{S}_i^0 + \tilde{I}_i 1 + Q_{i-1}^1 + SP_{i-1}^1 - Q_i^1 - SP_i^1 \leq S_{i,target}\right\} \geq \alpha$$

$$P\left\{\tilde{I}_i^1 \leq Q_i^1 + SP_i^1 + S_{i,target} - \tilde{S}_i 0 - Q_{i-1}^1 - SP_{i-1}^1\right\} \geq \alpha \tag{22}$$

$$x_i^{1,1-\alpha} \geq Q_i^1 + SP_i^1 + S_{i,target} - \tilde{S}_i^0 - Q_{i-1}^1 - SP_{i-1}^1$$

for $k = 2$ and $i = 1, \dots, M$,

$$P\left\{\tilde{S}_i^2 \leq S_{i,target}\right\} \geq \alpha$$

$$P\{S_i^1 + \tilde{I}_i^2 + Q_{i-1}^2 + SP_{i-1}^2 - Q_i^2 - SP_i^2 \leq S_{i,target}\} \geq \alpha \tag{23}$$

Substituting for S_i^1

$$P\left\{\begin{array}{c}(\tilde{S}_i^0 + \tilde{I}_i^1 + Q_{i-1}^1 + SP_{i-1}^1 - Q_i^1 - SP_i^1) \\ +\tilde{I}_i^2 + Q_{i-1}^2 + SP_{i-1}^2 - Q_i^2 - SP_i^2 \leq S_{i,target}\end{array}\right\} \geq \alpha$$

$$P\left\{\begin{array}{c}\tilde{I}_i^1 + \tilde{I}_i^2 \leq Q_i^1 + SP_i^1 - Q_{i-1}^1 - SP_{i-1}^1 \\ -Q_{i-1}^2 - SP_{i-1}^2 + Q_i^2 + SP_i^2 + S_{i,target} - \tilde{S}_i^0\end{array}\right\} \geq \alpha \tag{24}$$

$$x_i^{2,1-\alpha} \geq Q_i^1 + SP_i^1 - Q_{i-1}^1 - SP_{i-1}^1 - Q_{i-1}^2$$

$$-SP_{i-1}^2 + Q_i^2 + SP_i^2 + S_{i,target} - \tilde{S}_i^0$$

Eq. (21) can thus be expressed in final simplified chance constraint deterministic form as:

$$S_{i,target} - \tilde{S}_i 0 + \sum_{k=1}^{N}(Q_i^k + SP_i^k) - \sum_{k=1}^{N}(Q_{i-1}^k + SP_{i-1}^k) \leq x_i^{k,1-\alpha} \tag{25}$$

for $k = 1, \dots, N$ and $i = 1, \dots, M$.

It is important to note that the random variables inflow and initial storage are summed here. For the time interval $k = 1$, the sum is $\tilde{I}_i 1 + \tilde{S}_i 0$, for $k = 2$ it is $\tilde{S}_i 0 + \tilde{I}_i 1 + \tilde{I}_i 2, \dots$, and for $k = N$ it is $\tilde{S}_i 0 + \tilde{I}_i 1 + \tilde{I}_i 2 + \cdots + \tilde{I}_N$. In the two-step algorithm developed by Simonovic to determine the best reliability levels for chance constraints, reliability levels are set in the first step, while the optimal open-loop reservoir operating strategy is determined by linear programming in step 2 [14]. This operating strategy sets the discharge from reservoir i in period t equal to x_t^i. The discharges x_t^i (for all i and t) are then used to transform the probabilistic constraints into deterministic ones. Strycharczyk and Stedinger [15] in their paper stressed out the problem with Simonovic and Marino [14] method that the discharge from a given reservoir i in a given period t is a constant, although the problem is stochastic. In stochastic reservoir management, the content of the reservoir is a random variable since it is fed by a streamflow, which is random. The content of the reservoir at start of period t can take any value between the dead water level and the maximum reservoir volume. The optimal reservoir release for a certain period is a function of the reservoir content. Therefore, the open-loop operating strategy

described in Ref. [14] is not quite acceptable for listing as a stochastic reservoir management problem. Therefore, in this paper we consider the initial reservoir storage level as a random variable also.

The random variables inflow and initial storage have known marginal probability distribution functions (PDF), $f(I_k)$ and $f(S_{0,\ i})$, respectively, as a result of fitting a distribution to available historical data. However, the distributions of the sums have to be found. This is accomplished through a step-by-step iterative convolution method:

For $k = 1$, iterative convolution is used to reduce the two random variables $\tilde{I}_i^k$ and $\tilde{S}_i 0$ appearing in the equation of state to a single random variable. The new random variable is obtained as the sum of the random inflow and the random initial storage. Since their probability density functions are known, that of the new variable is obtained by convolution, which is expressed in discrete form by

$$P\{S = r\} = \sum_{i-j=r} p_i^I p_j^S \tag{26}$$

where $r_{\min} \leq r \leq r_{\max}$, p_i^I is the probability distribution of inflow, p_j^S is the probability distribution of the initial storage. The summation is carried out over the variable I, and hence, expression (26) yields:

$$P\{S = r\} = \sum_{i=a}^{b} P_i^I P_{i-r}^S. \tag{27}$$

The magnitudes of r_{min} and r_{max} are found from $r_{min} = i_{min} - j_{max} = a - d$ and $r_{max} = i_{max} - j_{min} = b - c$ under the constraints $a \leq i \leq b$ and $c \leq j \leq d$.

From $k = 2$ to $k = N$, the recursive equation for convolution can be expressed in genral form as in [24] as follows:

$$p_r(k) = \sum_{i-j=r} P\{i_k = i\} p_j(k-1) \tag{28}$$

$$r_{\min} \leq r \leq r_{max} \tag{29}$$

where $r_{min} = i_{min} + j_{min}$, $r_{max} = i_{max} + j_{max}$, under the constraints: $a \leq i \leq b$ and $c \leq j \leq d$.

The problem formulation becomes similar to linear formulation in the deterministic approach; Eq. (6) with the addition of the deterministic chance constraint and Eq. (25) can be solved with the same linear programming approach as the deterministic model formulation.

3.2.3. Stage two and three: consideration of resources and technical uncertainty using a fuzzy approach

If in stage 2 historical data are scarce, we can apply a fuzzy approach for both resources and technical uncertainty. The reservoir operation optimization model formulation will be expanded to utilize the fuzzy linear optimization approach, and in doing so, it will depart

from the classical assumptions that all coefficients of the constraints need to be crisp numbers [25]. In the present section, a FLP formulation based on the work of [25] and further by Tanaka et al. [18] that considers both technological coefficients and resources characterized by uncertainty is presented in brief as follows:

The fuzzy version of the traditional linear programming optimization problem presented in Eq. (29) is:

$$
\begin{aligned}
cx &\leq z_0 \\
Ax &\leq b \\
x &\geq 0
\end{aligned}
\tag{30}
$$

The manager's targets and system boundaries are the inequalities in the fuzzy system. The equation expressed that the manager's targets can be lower than his/her desired level z_0. The same applies to the boundaries that they should be in the tolerance level b. The importance of targets and the constraints is set at the same level. For fully symmetric objective and constraints, Zimmermann [26] formulated the problem in simplified form as follows:

$$
\begin{aligned}
Bx &\leq d \\
x &\geq 0
\end{aligned}
\tag{31}
$$

where $B = \begin{bmatrix} c \\ A \end{bmatrix}, d = \begin{bmatrix} z_0 \\ b \end{bmatrix}$.

The following expression for the (monotonically decreasing) linear membership function was proposed by Zimmerman for the jth fuzzy inequality $(Bx)_j \leq d_j$.

$$
\mu_j\big((Bx)_j\big) = \begin{cases} 1 & if\ (Bx)_j < d_j \\ 1 - \big((Bx)_j - d_j\big)/p_j & if\ d_j \leq (Bx)_j \leq d_j + p_j \\ 0 & if\ (Bx)_j \geq b_j + p_j \end{cases}
\tag{32}
$$

where d_j and p_j are the desired level and the tolerance for violation of the jth inequality, respectively. A jth membership function value of 1 denotes that the targets and constraints are fully satisfied. Violating the constraint outside the tolerance band, p_j gives a membership value of 0 and in-between values are linear. The membership function of the fuzzy set "decision" of model in Eq. (6) including the linear membership functions is shown below. The problem of finding the maximum decision is to choose x^* such that

$$
\mu_D(x^*) = \max_{x \geq 0} \min_{j=0,\dots,m} \big\{ \mu_j\big((Bx)_j\big) \big\}.
\tag{33}
$$

In other words, the problem is to find the $x^* \geq 0$ which maximizes the minimum membership function value. This value satisfies the fuzzy inequalities, $(Bx)_j \leq d_j$ with the degree of x* [18].

Substituting the expression (31) for linear membership function into Eq. (32) yields

$$\mu_D(x^*) = \max_{x \geq 0} \min_{i=0,\dots,m} \left\{ 1 + \frac{d_j}{p_j} - \frac{(Bx)_j}{p_j} \right\}.$$ (34)

The fuzzy set for decision can be transformed to an equivalent conventional linear programming problem by introducing the auxiliary variable λ:

$$\text{Maximize } \lambda$$ (35)

subject to $\lambda \leq 1 + \frac{d_j}{p_j} - \frac{(Bx)_j}{p_j}$

$$x \geq 0$$

It should be emphasized that the above formulation is for a minimization of the objective function and less than constraints and thus should be modified appropriately for other conditions.

Using the fuzzy optimization approach just described, and using the deterministic model given by Eq. (6) with modification for considering linear membership function for "greater than" constraints, the fuzzy formulation becomes:

$$\text{Maximize } \lambda$$ (36)

Subject to

$$\frac{(Bx)_j}{p_i} + \leq 1 + \frac{d_j}{p_j} \quad i = 0, \dots, 36$$ (37)

$$\frac{(Bx)_j}{p_i} - \leq \frac{d_j}{p_j} \quad i = 37, \dots, 60$$ (38)

$$x \geq 0$$

Expanding by substituting for $(Bx)_j$

$$\frac{\sum_{i=1}^{M} \left(w_S \sum_{k=1}^{N} Q(k) + w_Q \sum_{k=1}^{N} S(k) \right)}{p_j} + \leq 1 + \frac{d_j}{p_j} \quad j = 0$$ (39)

all other constraints for $k = 1, \dots, N$ and $i = 1, \dots, M$

$$\frac{S_i^k - S_i^{k-1} + Q_i^k + SP_i^k - Q_{i-1}^k - SP_{i-1}^k}{p_j} + \leq 1 + \frac{d_j}{p_j}$$ (40)

$$j = 1, \dots, M * N$$

$$\frac{S_i^k}{p_j} + \, \leq 1 + \frac{d_j}{p_j}, \; j = M*N+1, ..., 2*M*N \tag{41}$$

$$\frac{Q_i^k}{p_j} + \, \leq 1 + \frac{d_j}{p_j}, \; j = 2*M*N+1, ..., 3*M*N \tag{42}$$

$$\frac{S_i^k}{p_j} - \, \geq \frac{d_j}{p_j}, \; j = 3*M*N+1, ..., 4*M*N \tag{43}$$

$$\frac{S_i^k - S_i^{k-1} + Q_i^k + SP_i^k - Q_{i-1}^k - SP_{i-1}^k}{p_j} - \, \geq \frac{d_j}{p_j}, \; j = 4*M*N+1, ..., 5*M*N \tag{44}$$

$$Q_i^k \geq 0 \tag{45}$$

$$SP_i^k \geq 0 \tag{46}$$

4. Numerical example

The following demonstrates the application of the methodology for a reservoir cascade composed of three projects. All stages will be shown, from deterministic to its modification for the implementation of the probabilistic and fuzzy domains.

4.1. Case study

The reservoir cascade optimization case study is the Wuyang cascade system in the People's Republic of China. An optimization problem is formulated for 12-month time period ($k = 12$) as discussed in preceding sections and solved using data provided from the Institute of Hydraulic and Water Research. The available data are listed in **Table 1**. It is from 1953 to 2009 and includes the constraints in storage capacity and the stream flows.

	Reservoir 1	Reservoir 2	Reservoir 3
Maximum reservoir capacity (m^3)	1.373E8	6.572E8	7.1E7
Dead or minimum reservoir storage, S_{min} (m^3)	500000.0	727000.0	792000.0
Sill of dam elevation operator goal storage, S_{target} (m^3)	1.012E8	4.872E8	5.34E7
Initial storage, S_0 (m^3)	5.12E7	1.206E8	4.48E7
Maximum possible release for non-flooding condition, R_{max} (m^3/s)	50	50	50

Table 1. Hydropower projects data.

5. Results

The cumulative distribution values for $f(I_k)$ and $f(S_{0,\,i})$, obtained by the summation of PDFs with reliability tolerance of 0.9, are shown in **Figure 4**.
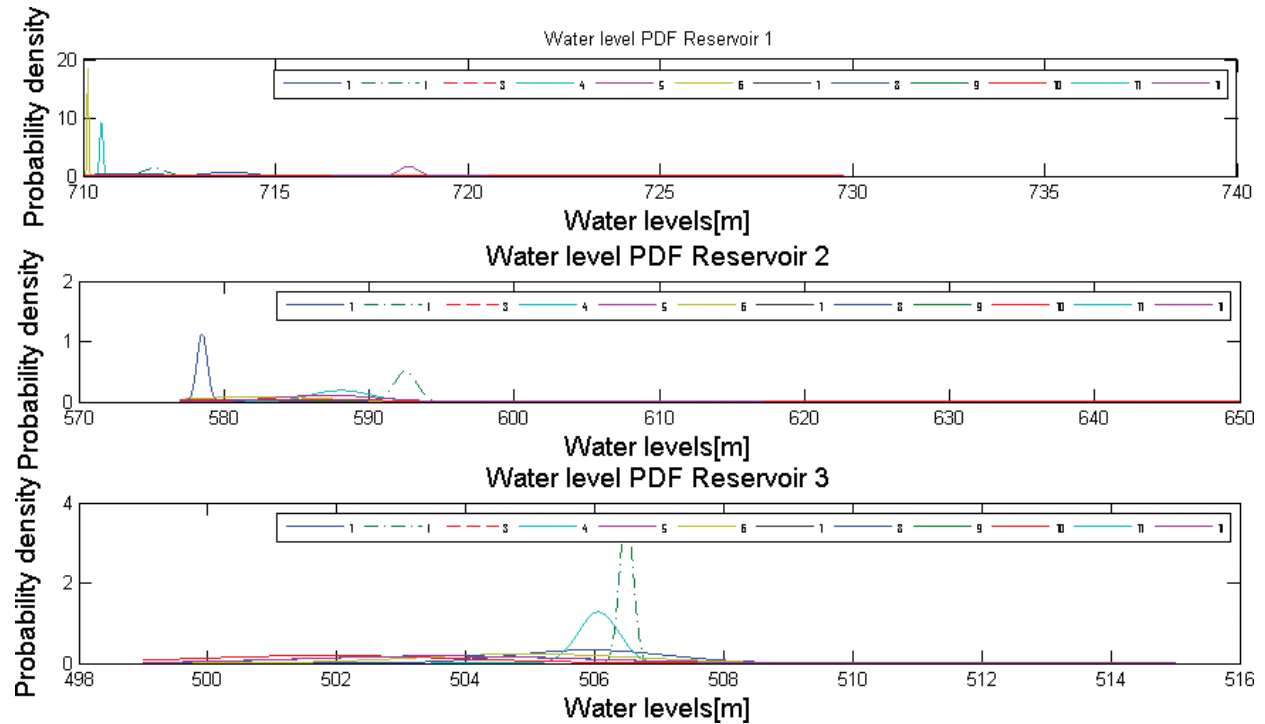

Figure 4. Probability distribution functions of the sum of and .

It is also considered that the water managers wanted some flexibility in the constraint to account for the uncertainty in knowledge, which is not available with the sharp constraint requirements of the deterministic model. Further, the water managers assessed that the annual maximum acceptable storage to mitigate damage due to flood should not exceed S_{flood}. The managers formulated there constraints according to their experience and comfort. Therefore, the FLP approach was chosen to facilitate this insight, and LB and UB of d_j and p_j were determined.

The results in **Figure 5**–**Figure 7** include a series of release rules for operating period of the 12 months that reservoir operators can follow in order to fulfill the defined objective.

In stage 1, using the deterministic optimization approach and substituting the given data, the above problem with 144 balance equations and 145 constraints becomes readily solvable using linear programming. The optimal solution is shown in **Figure 5**.

In stage 2, the probabilistic optimization approach was applied. Firstly, a PDF was selected from experiments and experience from the structure of the stream flows. Stream flows are positive, and there variance changes proportionally to their mean, which is characteristic of lognormal functions. α was set to 0.9, and the addition of the stream flows and possible initial water levels was determined using a convolution process as described previously in this chapter. Once the convolution process is complete and inflow convoluted values
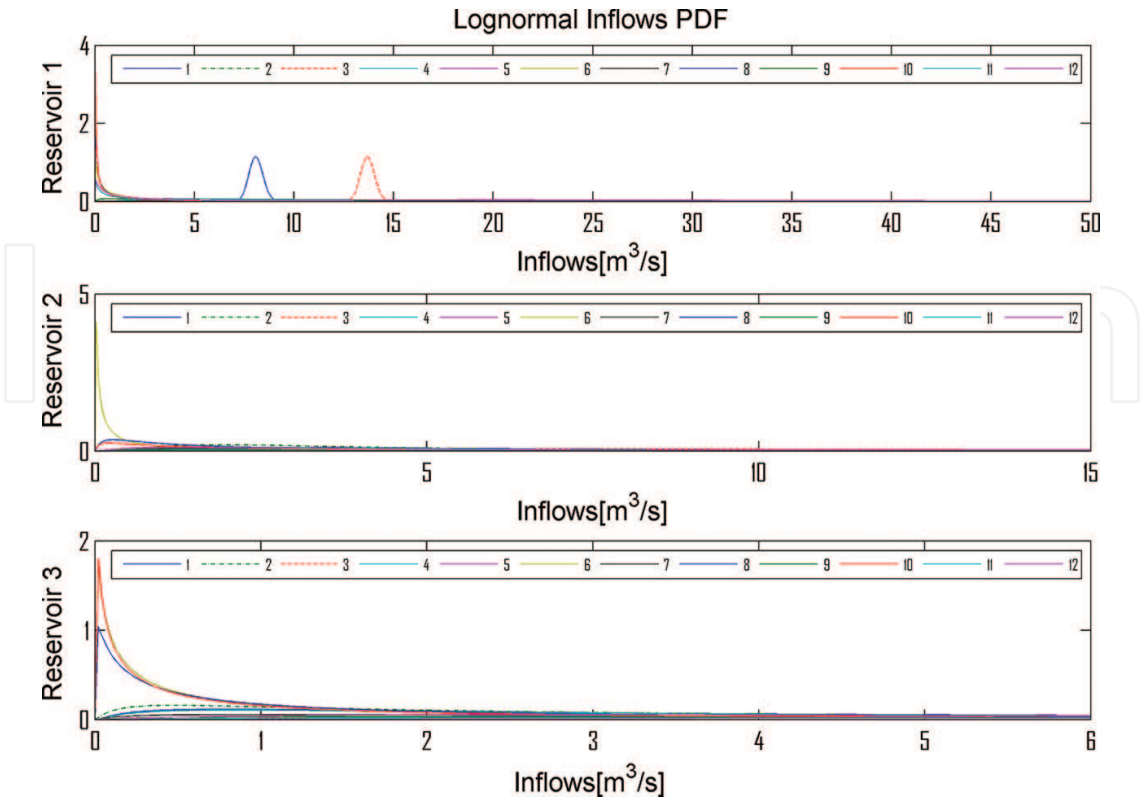
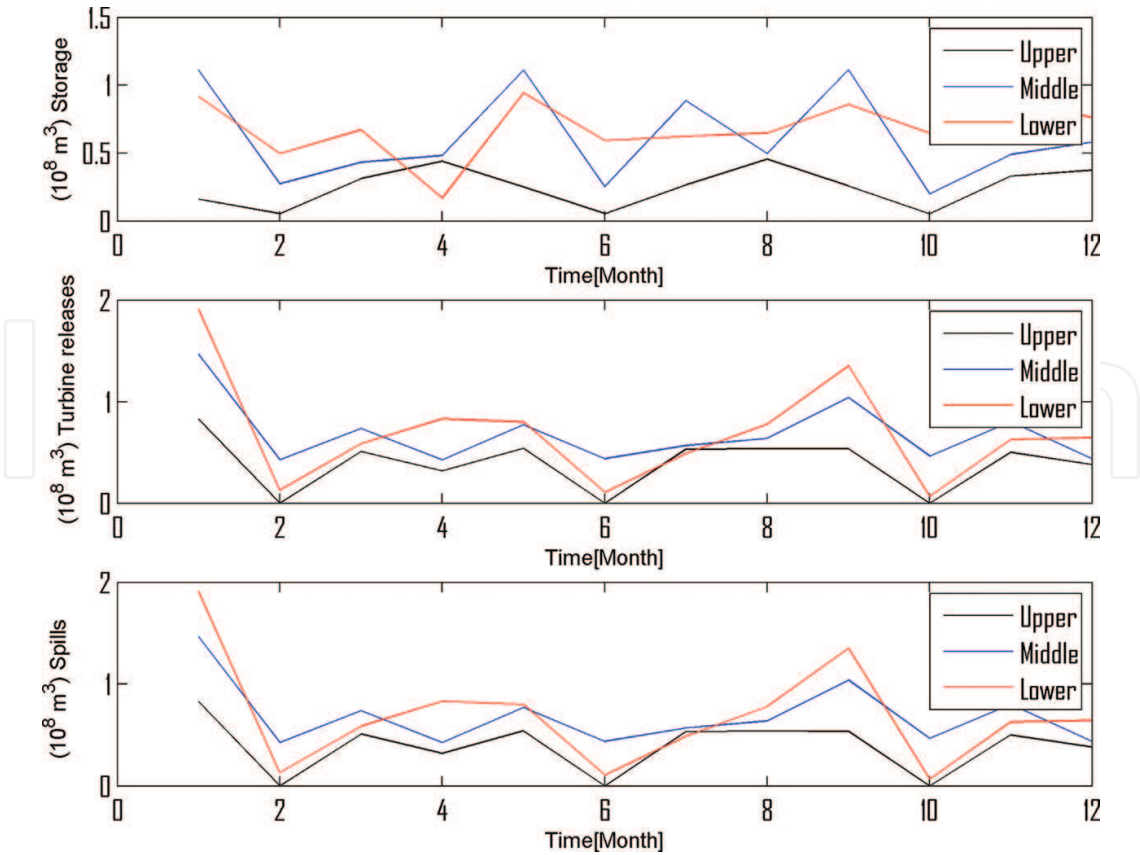**Figure 5.** Results of the deterministic method (objective function Z = 0.3857 m^3).

Figure 6. Chance constrained method (objective function Z = 1.4643 m^3).

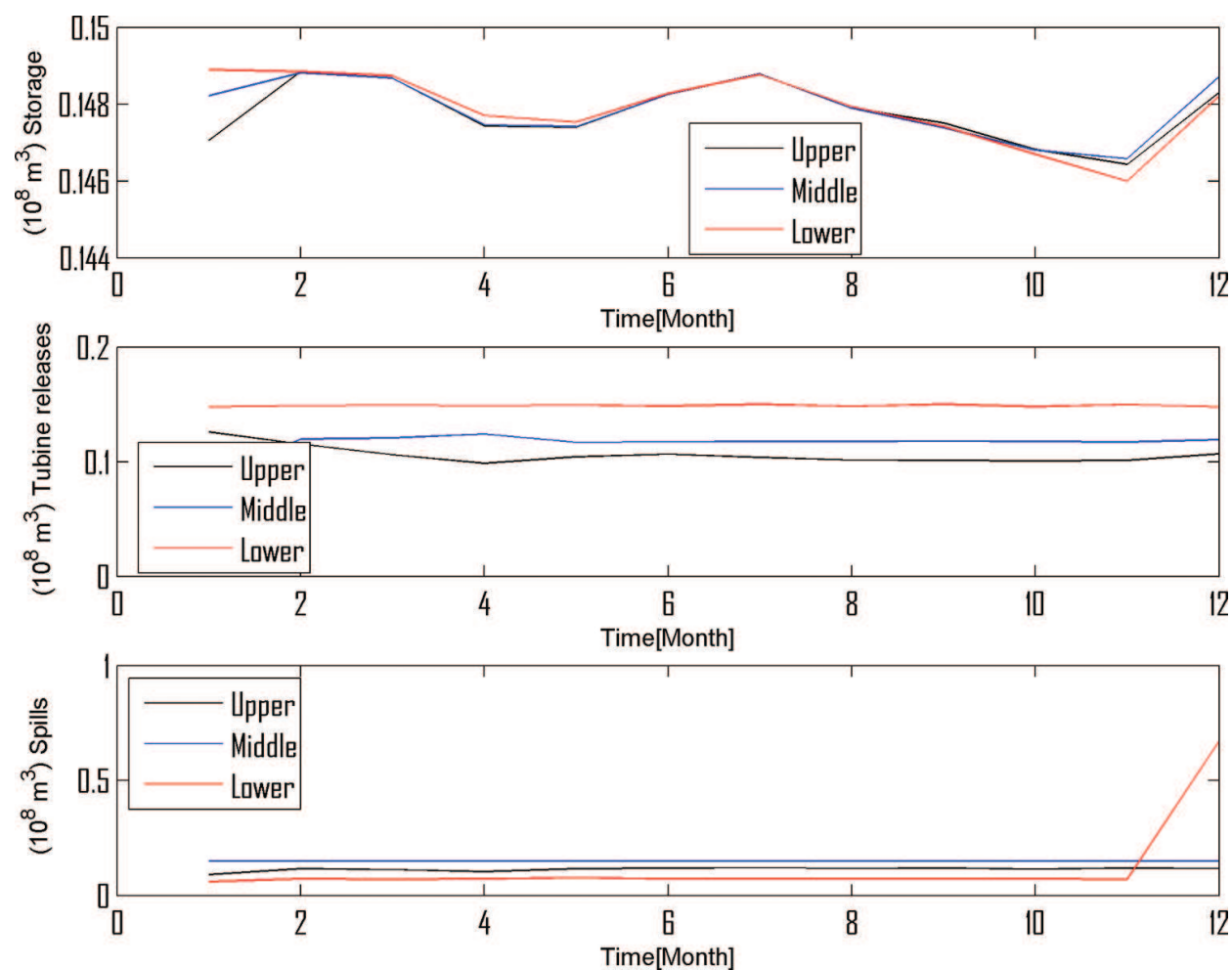**Figure 7.** Fuzzy chance constrained method (Z = 4.4515 m³).

corresponding to the reliability index selected are found, the problem was solved using linear optimization as in the case of the deterministic formulation. The optimal solution is shown in **Figure 6** for reliability level $\alpha = 0.9$.

In the final stage, using fuzzy optimization approach, the values of the tolerance interval d_j and spread of tolerance p_j are substituted into the constraints and the problem is solved using a linear programming solver. The resulting $\lambda = 0.0626$ with corresponding storage and release is shown in **Figure 7**.

6. Discussion

It could be seen that the problem could be solved by all the three methods with different results. However, the linear approach uses water spillage to achieve optimal solution. The chance constraint method offers a way to include reliability in formation of the optimization problem. The requirements of the water managers to have some flexibility in the constraint to account for the uncertainty in knowledge could be realized satisfactory with the fuzzy method.

7. Conclusions

Crisp-defined boundaries do not often resemble real-life situations. It is a fact that uncertainty in decision-making processes occurs at every stage. Therefore, it is necessary to understand uncertainties, which in consequence require understanding of their sources. In water resource management (WRM), uncertainty can be put into two categories: (1) uncertainty due to inherent variability in hydrology and uncertainty contributed by scarcity of data and knowledge. The transformation of deterministic problems into the probabilistic and fuzzy domains was presented in this paper. The fuzzy approach integrates the managers indirectly into the optimization process through their expert information. As shown in this chapter, the probabilistic method is very good in dealing with quantifiable uncertainty. However, its robustness to handle different sources of uncertainties is not sufficient to justify its use under all circumstances. Caution must be taken, pending on the level of precision desired the stochastic or even the deterministic approach may be the better alternative. But the probabilistic approach can be implemented only if uncertainties are quantifiable and sufficient historical data are available. The holic view of integrating the two approaches gives good opportunities to solve problems of reservoir optimization in case of vast data availability, data scarcity, and availability of information from expert knowledge and experience, which cannot be quantified. To show the robustness of the approaches, they were applied to a real case of the Wuyang river cascade in PR China. Here, these data are scarce, and most of the decisions are made subjectively. The obtained results are quite promising. In further sensitivity tests performed (not part of this paper), it could be shown that the combination of the CP and the FLP method is very robust to changes in stream flows, initial reservoir levels, and formulated targets.

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References

[1] Archibald, T. W., McKinnon, K. I. M., and Thomas, L. C. (1997). An aggregate stochastic dynamic programming model of multireservoir systems. Water Resources Research, 33 (2), 333–340. doi:10.1029/96WR02859

[2] Simonovic, C. P., and Peck, A. (2009). Updated rainfall intensity duration frequency curves for the city of London under changing climate. Water Resources Research Report

No. 065, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 64 p. ISBN: (print) 978-0-7714-2819-7; (online) 987-0-7714-2820–3.

[3]   Faber, B. A., and Stedinger, J. R. (2001). Reservoir optimization using sampling SDP with ensemble streamflow prediction (ESP) forecasts. Journal of Hydrology, 249(1–4), 113–133.

[4]   Schwanenberg, D., Xu, M., Ochterbeck, T., Allen, C., and Karimanzira, D. (2014). Short-term management of hydropower assets of the Federal Columbia River power system. Journal of Applied Water Engineering and Research, 2(1), 25–32.

[5]   Saad, M., Turgeon, A., Bigras, P., and Duquette, R. (1994). Learning disaggregation techniques for the operation of long-term hydroelectric power systems. Water Resources Research, 30(11), 3203–3295.

[6]   Saad, M., Turgeon, A., Bigras, P., and Duquette, R. (1996). Fuzzy learning decomposition for the scheduling of hydroelectric power systems. Water Resources Research, 32(1), 179–186.

[7]   Saad, M., and Turgeon, A. (1988). Application of principal component analysis to long-term reservoir management. Water Resources Research, 24(7), 907–912.

[8]   Loucks, D. P. (1968). Computer models for reservoir regulations. Journal of the Sanitary Engineering Division. American Society of Civil Engineers, 94(SA4), 657–669.

[9]   Birge, J. R. (1985). Decomposition and partitioning methods for multistage stochastic linear programs. Operations Research, 33(5), 989–1007.

[10]   Pereira, M. V. F., and Pinto, L. M. V. G. (1985). Stochastic optimization of a multireservoir hydroelectric system: a decomposition approach. Water Resources Research, 21(6), 779–792.

[11]   Karamouz, M., Houck, M., and Delleur, J. (1992). Optimization and simulation of multiple reservoir systems. Journal of Water Resources and Planning Management, 118(1), 71–81.

[12]   Seifi, A., and Hipel, K. (2001). Interior-point method for reservoir operation with stochastic inflows. Journal of Water Resources and Planning Management, 127(1), 48–57.

[13]   Loucks, D. P., and Dorfman, P. (1975). An evaluation of some linear decision rules in chance-constrained models for reservoir planning and operation. Water Resources Research, 11(6), 777–782.

[14]   Simonovic, S., and Marino, M. (1982). Reliability programming in reservoir management: 3. Systems of multipurpose reservoirs. Water Resources Research, 18(4), 735–743.

[15]   Strycharczyk, J. B., and Stedinger, J. R. (1987). Evaluation of a "reliability programming" reservoir model. Water Resources Research, 23(2), 225–229.

[16]   Bellman, R., and Zadeh, L. (1970). Decision making in a fuzzy environment. Management Science, 17, 141–164.

[17] Zimmermann, H. J. (1976). Description and optimization of fuzzy systems. International Journal of General Systems, 2(4), 209–215.

[18] Tanaka, H., Ichihashi, H., and Asai, K. (1991). Formulation of fuzzy linear programming problem based on comparison of fuzzy numbers. Control Cybernetics, 3(3), 185–194.

[19] Zhang, G., Yong-Hong, W. U., Remias, M., and Lu, J. (2003). Formulation of fuzzy linear programming problems as four-objective constrained optimization problems. Applied Mathematics and Computation, 139, 383–399.

[20] Simonović, S. P. (2002). Understanding risk management. In: Proceedings of the Annual Conference of the Canadian Society for Civil Engineering, Montréal, Québec, Canada (pp. 1–12). http://pedago.cegepoutaouais.qc.ca/media/0260309/0378334/SCGC BON/Documents/GE108-Simonovic.pdf.

[21] ReVelle, C. (1999). Optimizing Reservoir Resources: Including a New Model for Reservoir Reliability. New York: Wiley.

[22] Charnes, A., and Cooper, W. (1959). Chance-constrained programming. Management Science, 6(1), 73–79.

[23] Curry, L. G., Helm, J. C., and Clark, R. A. (1973). Chance–constrained model of system of reservoirs. Journal of the Hydraulics Division, 99(12), 2353–2366 (ASCE, 12, HY).

[24] Simonović, S. P. (1979). Two-step algorithm for design-stage long-term control of a multipurpose reservoir. Advanced Water Resources, 2, 47–49 (118).

[25] Zimmermann, H. J. (1996). Fuzzy Set Theory and Its Applications. 3rd edition. Boston, MA: Kluwer Academic Publishers.

[26] Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Systems, 1, 45–55.