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Fourier Analysis for Harmonic Signals in Electrical Power Systems

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Additional information is available at the end of the chapter

Abstract

The harmonic content in electrical power systems is an increasingly worrying issue since the proliferation of nonlinear loads results in power quality problems as the harmonics is more apparent. In this paper, we analyze the behavior of the harmonics in the electrical power systems such as cables, transmission lines, capacitors, transformers, and rotating machines, the induction machine being the object of our study when it is excited to nonsinusoidal operating conditions in the stator winding. For this, a model is proposed for the harmonic analysis of the induction machine in steady-state regimen applying the Fourier transform. The results of the proposed model are validated by experimental tests which gave good results for each case study concluding in a model proper for harmonic and nonharmonic analysis of the induction machine and for "harmonic" analysis in an electrical power system.

Keywords: harmonic, power quality, induction machine, nonsinusoidal, power systems

1. Introduction

Electrical power systems are an area that is receiving a great deal of attention recently. The issues and considerations associated with electrical power systems are often misunderstood. With the growth and expansion of power electronics and proliferation of nonlinear loads in

electrical power system applications, the harmonics and their effects on power quality are a topic of concern. Currently in the United States, only 15–20% of the utility distribution loading consists of nonlinear loads.

Nowadays, the recommendation from IEEE Std. 519 imposed by utilities is becoming stricter due to the increase in proportion of nonlinear load. The problems of harmonic in the electrical power systems are low, but their analysis can help to increase plant power system reliability. The harmonics are a problem when their magnitude produces an electrical power system resonance.

The analysis and modeling of the harmonics are supported for the Fourier analysis. In the eighteenth and nineteenth century, J. B. Joseph Fourier (1768–1830) and other mathematicians performed basic calculations of harmonics. In the 1920s and 1930s, the distortion in voltage waveforms caused by power converters was noticed and studied. In the 1950s and 1960s, the study of harmonics in power converters extended to the transmission of voltage in the electrical power system. Currently, the electrical power systems have a large number of nonlinear elements that generate other waves at different frequencies. They generate these waves from sinusoidal waveforms to network frequency. This causes a phenomenon known as harmonics. Harmonics are phenomena that cause problems for both the users and the electricity suppliers. They have various harmful effects on the equipment in the electrical network.

2. Definition of harmonics

The term *harmonic* comes from acoustics. It refers to the vibration of a column of air at a frequency which is a multiple of the basic frequency of repetition.

In electric signals, a harmonic is defined as the signal content at a specific frequency, which is a multiple integral of the current frequency system or main frequency produced by the generators. With an oscilloscope, it is possible to observe a complex signal in the domain of time. At any moment in the given time, the amplitude of the waveform is displayed. If the same signal is applied to a high-fidelity amplifier, the result in sounds is a mix of frequencies. The phase relationship does not affect the audible effects, which is acceptable in acoustics. But this is not the case with electric signals. The position of harmonics and the phase relationship in the harmonic from a different source can considerably alter the effects in electric signals. To define harmonic, it is important to first define the quality of the voltage wave, which must have a constant amplitude and frequency, as well as the sinusoidal form. **Figure 1** depicts the waveform without any content of harmonics, with a constant frequency of 60 Hz and a constant amplitude of 1 pu.

When a periodic wave does not have a sinusoidal form, it is said to have a harmonic content. This can alter its peak value and/or its RMS value causing alterations in the normal functioning of any equipment that undergoes this voltage. The frequency of the periodic wave is known as the fundamental frequency and the harmonics are the signals whose frequency is an integer multiple of this frequency. **Figure 2** shows a voltage wave with a content of 30% of the fifth harmonic.
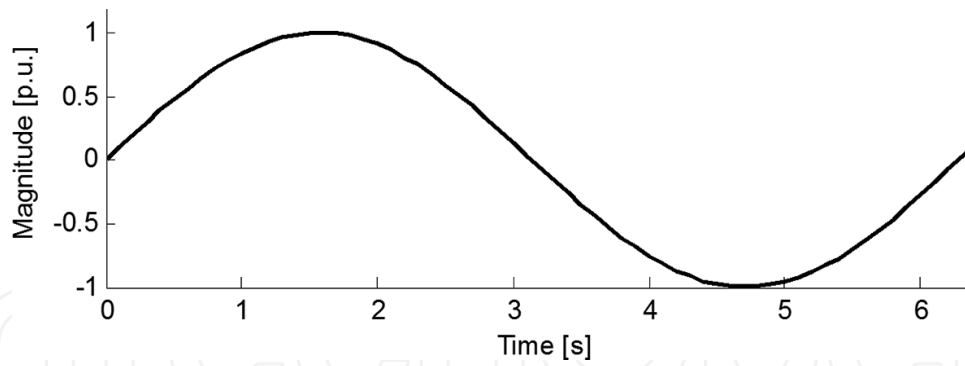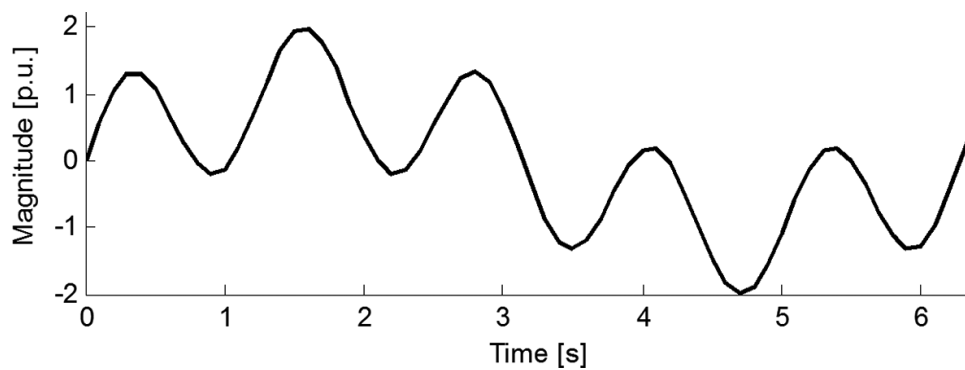
Figure 1. Wave without harmonic content.

Figure 2. Voltage waveform with harmonic content.

3. Fourier analysis

The analysis of harmonics is the process of calculating the magnitudes and phases of the fundamental and high order harmonics of the periodic waveforms. The resulting series is known as Fourier series. It establishes a relation between a function in the domain of time and a function in the domain of frequency.

The Fourier's theorem states that every nonsinusoidal periodic wave can be decomposed as the sum of sine waves through the application of the Fourier series, given the following conditions:

- The integral over one period of the function is a finite value.

- The function possesses a finite number of discontinuities in a period.

- The function possesses a finite number of maxima and minima in a period.

Coefficients and Fourier series. The Fourier series of a periodic function $x(t)$ is expressed as:

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right) \right) \tag{1}$$

This constitutes a representation of periodic function in the domain of the frequency.

In this expression, a_0 is the average value of the function $x(t)$, where a_n and b_n are the coefficients of the series besides being the rectangular components of the n harmonic. For the corresponding n harmonic its vector is:

$$A_n \varnothing_n = an + jbm \tag{2}$$

With a magnitude and an angle of phase:

$$A_n = \sqrt{(a^2 n + b^2 n)}, \varnothing_n = \tan^{-1} b_n \tag{3}$$

4. Harmonic sources

Harmonics are the result of nonlinear loads which give a nonsinusoidal response to a sinusoidal signal. The main sources of harmonics are:

- Arc furnaces and other elements of arc discharge, such as fluorescent lamps. Arc furnaces are considered as voltage harmonic generators more than current generators. Typically all harmonics (2nd, 3rd, 4th, 5th,...) appear but the odd harmonics are predominant with typical values with regard to the fundamental harmonic:

- The third harmonic represents 20%, and the fifth harmonic represents 10%.

- The seventh harmonic represents 6%, and the ninth harmonic represents 3%.

- Magnetic cores in transformers and rotating machines require third harmonic current to excite the iron.

- The inrush current of transformers produces second and fourth harmonics.

- Adjustable speed controllers used in fans, pumps, and process controllers.

- Solid-state switches which modulate control currents, light intensity, heat, etc.

- Controlled sources for electronic equipment.

- Rectifiers based on diodes and thyristors for welding equipment, battery chargers, etc.

- Static reactive power compensators.

- DC high voltage transmission stations.

- AC to DC converters (inverters).

The AC electrical power system harmonic issues are mainly due to the substantial increase of nonlinear loads due to technological advances, such as the use of power electronics circuits and devices, in AC/DC transmission links, or loads in the control of power systems using power electronic or microprocessor controllers. Such equipment creates load-generated harmonics throughout the electrical power system.
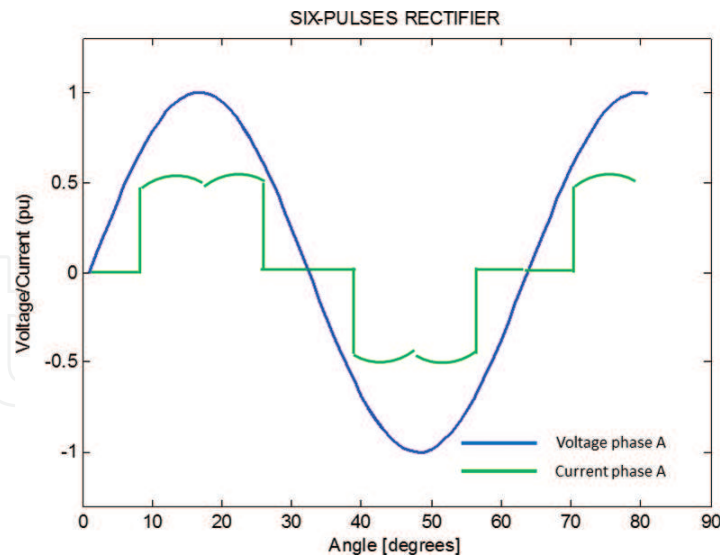
Figure 3. Current and voltage of a typical six-pulse rectifier input.

In the case of a motor drive, the AC current at the input to the rectifier looks more like a square wave than a sine wave (see **Figure 3**).

The rectifier can be thought of as a harmonic current source and produces roughly the same amount of harmonic current over a wide range of electrical power system impedances. The characteristic current harmonics that are produced by a rectifier are determined by the pulse number. The following equation allows determination of the characteristic harmonics for a given pulse number:

$$h = kq \pm 1 \tag{4}$$

where:

h is the harmonic number (integer multiple of the fundamental),

k is any positive integer, and

q is the pulse number of the converter.

The harmonics 5th, 7th, 11th, 13th, 17th, 19th, 23rd, 25th, etc., are the harmonics that a 6-pulse rectifier will exhibit and which are multiples of the fundamental. The quotient of the fundamental current and the harmonic number will result in the magnitudes of the harmonic currents (e.g., the magnitude of the 5th harmonic would be about 1/5th of the fundamental current). When it comes to a 12-pulse systems, a small amount of the 5th, 7th, 17th, and 19th harmonics will be present (the magnitudes will be approximately a 10 percent of those for a 6-pulse drive). The induction machines are quite affected by the harmonic currents produced by inverters. Most of these harmonics produced are integer multiples of the inverter frequency and their magnitude will depend on the algorithm switching power semiconductors of the inverter. It is common that there are "interharmonics" currents at the input or the output of the inverter but they do not necessarily occur at integer multiples of the power

supply or inverter fundamental frequency. For this reason, it needs a good design of DC link to minimize the presence of interharmonics.

Some authors [1] agree to classify the sources of harmonic distortion in three groups: small and predictable (harmonics generated by residential consumers), large and transient (voltage fluctuations produced by arc furnaces), and large and predictable (SVC and HVDC transmission causing characteristic and uncharacteristic harmonics). Now, if at a point common coupling harmonic currents are not within the permissible limits, it is necessary to take appropriate measures to comply with regulations. For example the IEEE 519-1981, "*IEEE Guide for Harmonic Control and Reactive Compensation of Static Power Converters*," originally established levels of voltage distortion acceptable to the distribution system for individual nonlinear loads. This distortion is a steady-state deviation from a sine wave of power frequency called waveform distortion [2]. Fourier series is generally used to analyze this nonsinusoidal waveform.

5. Effects of harmonics

Normally, the presence of harmonic signals in the electrical power system is rare but it is possible that a large number of undesirable effects occur. High levels of harmonic distortion can cause undesirable effects in the transformer, capacitor, motor or generator heating, disoperation of electronic equipment, interference with telephone circuits, etc., and it gets worse if a resonant condition is presented. Resonance occurs when a harmonic frequency produced by a nonlinear load closely coincides with the natural frequency of the electrical power system. There are two forms of resonance which can occur: parallel resonance and series resonance.

Parallel resonance: The parallel resonance occurs when the natural frequency of the inductive components of the system, connected in parallel with capacitive reactive impedance components are too close to the harmonic frequency of the system. If this frequency coincides with a frequency generated by the harmonic source, it causes severe complications leading to excessive voltages and currents, causing damage to capacitors or overheating transformer and other electrical equipment (see **Figure 4**).

Series resonance: It occurs when the source harmonic current is connected in series with the combination, also in series, of the inductive impedance of the system and the capacitive reactance of a capacitor bank (usually connected to the end of a branch supply), its impedance being very low.

The effect of a series resonance can be a high-voltage distortion between the inductive impedance and the capacitive reactance (see **Figure 5**).

5.1. Effects on cables

The current distribution through the cross section of a conductor is uniform only when the current is a direct one. In alternating current as the frequency increases, the nonuniformity of the current distribution becomes steeper.
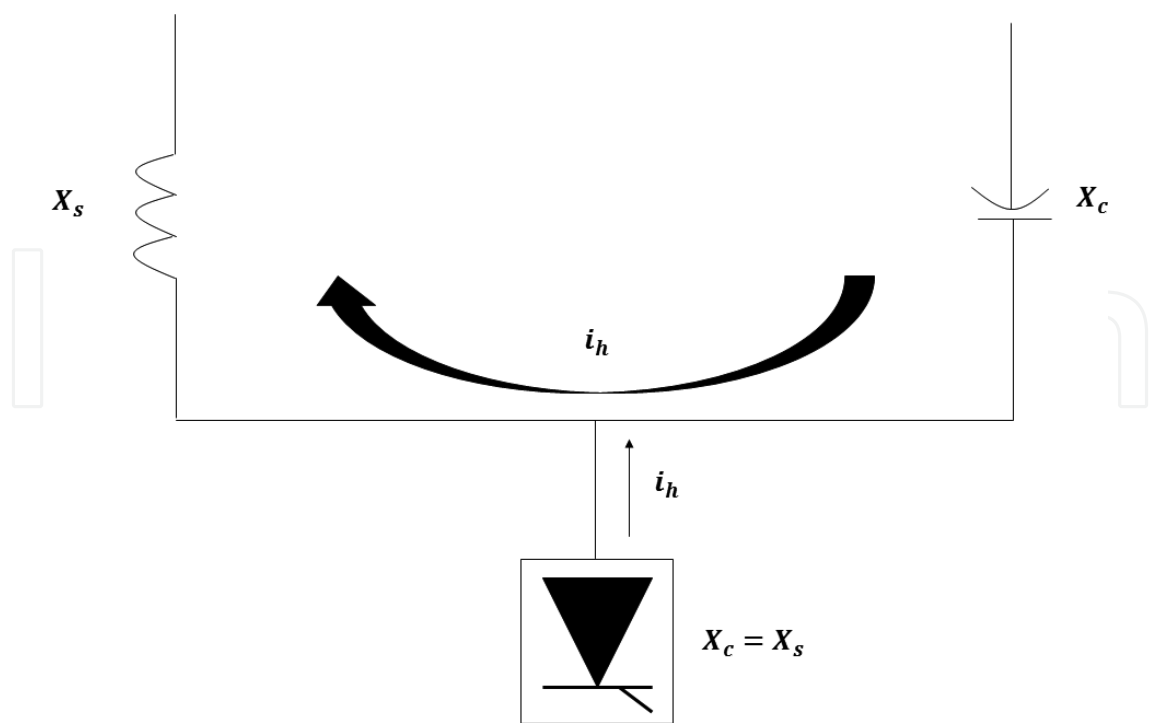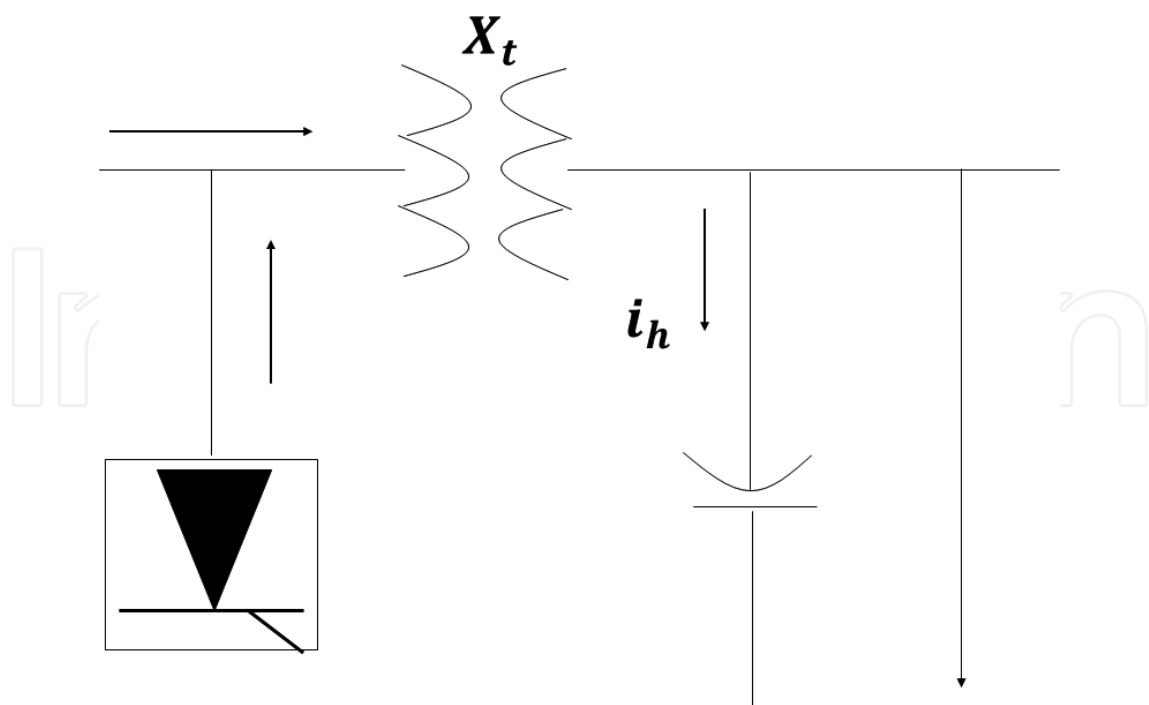
Figure 4. Parallel resonance.

Figure 5. Series resonance.

In circular conductors, the current density increases from the center to the surface. The outer layers are less bounded by the magnetic flow than the inner layers. This means that more voltage is longitudinally induced with alternating current inside the conductor than on the surface. Therefore the current density is increasing from the interior to the outer layers of the conductor. This phenomenon is called the skin effect.

Figure 6 shows the variation of the ratio r_{ac}/r_{dc} with regard to frequency for some wire sizes used in electrical installations. The figure shows how the skin effect becomes more pronounced with a higher caliber (less r_{dc}). If a conductor with a cross section a_{cond} conducts a DC current I_{DC}, the current density $j_{DC} = I_{DC}/a_{cond}$ is uniform within the conductor and a resistance R_{DC} can be assigned to the conductor representing the radio between the applied voltage V_{DC} and the resulting current I_{DC}, that is, $R_{DC} = V_{DC}/I_{DC}$. For (periodic) AC currents, $i_{ACh}(t)$, the current flows mostly near the surface on the conductor and the current density j_{ACh} is nonuniform within the conductor (**Figure 6**). In general, R_{DC}/R_{ACh}, the higher the order h of the harmonic current $i_{ACh}(t)$ the larger is the skin effect.
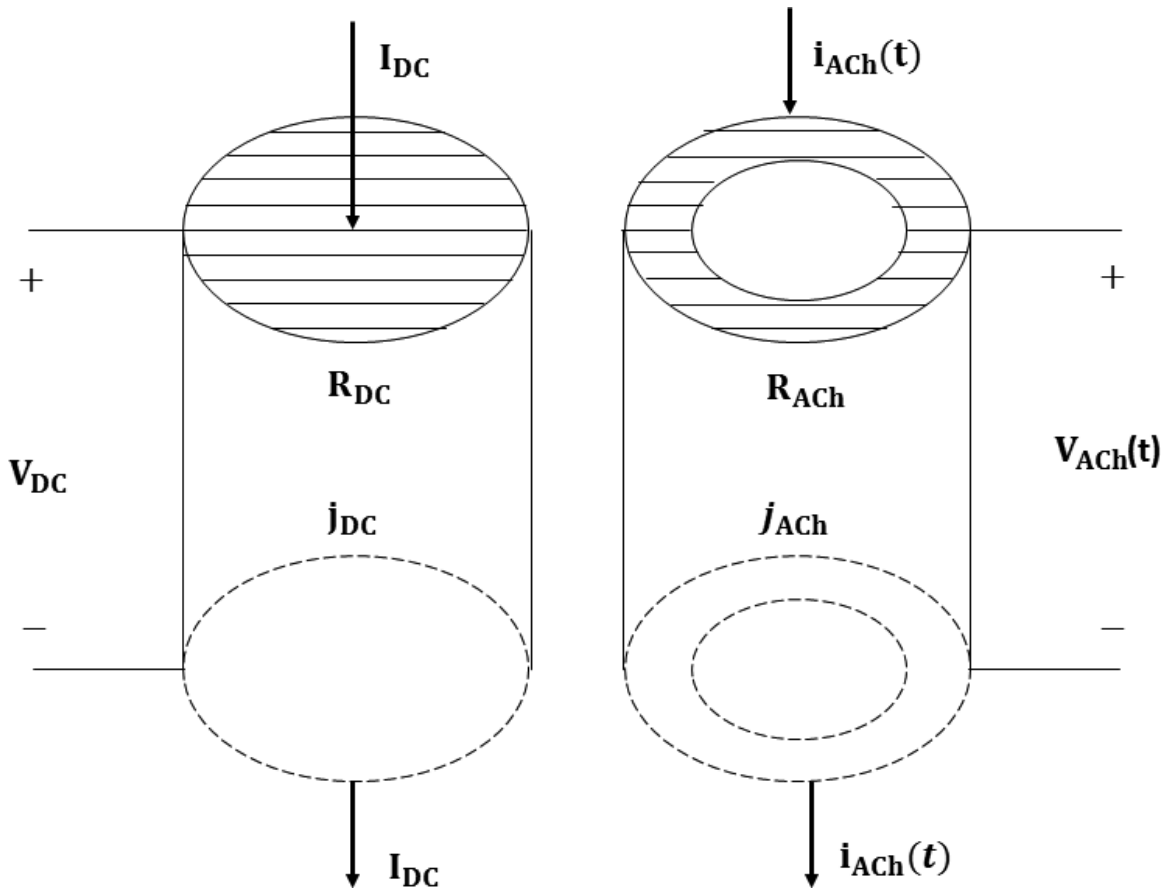

Figure 6. DC resistance R DC versus AC resistance R ACh.

5.2. Effects on transformers

The normal operating conditions of the transformer is a well-researched subject. In fact, many steady-state and transient models are available. The transformer can be modeled into

two-state regime: transient model and steady-state model. The transient model needs much computation time, while the steady-state model requires less amount of computation time as it takes place in phasor analysis in the frequency domain to analyze the behavior of the transformer.

However, the material with which the core of the transformers is built has nonlinear characteristics. These nonlinear characteristics are neglected by the transformer models that use linear techniques. They exhibit three types of nonlinearities that make their analysis difficult: saturation effect, hysteresis (major and minor) loops, and eddy currents. The factors that influence additional losses and the generation of harmonic signals in the transformer are the temperature and possible resonance between transformer winding inductance and supply capacitance. In addition, if the losses in the transformer are considered, then the modeling becomes complicated, for that reason those losses are neglected according to the following expression,

$$P_{fe} = P_{hys} + P_{eddy} = K_{hys}\left(B_{max}\right)^{s}f + K_{eddy}\left(B_{max}\right)^{2}f^{2} \tag{5}$$

Where $P_{hys}, P_{eddy}, B_{max}$ and f are hysteresis losses, eddy-current losses, flux density, and fundamental frequency system, respectively. K_{hys} is the constant of the type of iron used and K_{eddy} is the eddy-current constant for the conductive material. S is the Steinmetz exponent ranging from 1.5 to 2.5 depending on the operating point of transformer core. **Figure 7** illustrates a relatively simple and accurate frequency-based linear model.

In **Figure 7**, R_c is the core loss resistance, L_m is the magnetizing inductance, and R_p, R'_s, L_p and L'_s are the resistances and inductances of the primary and secondary windings of the transformer,
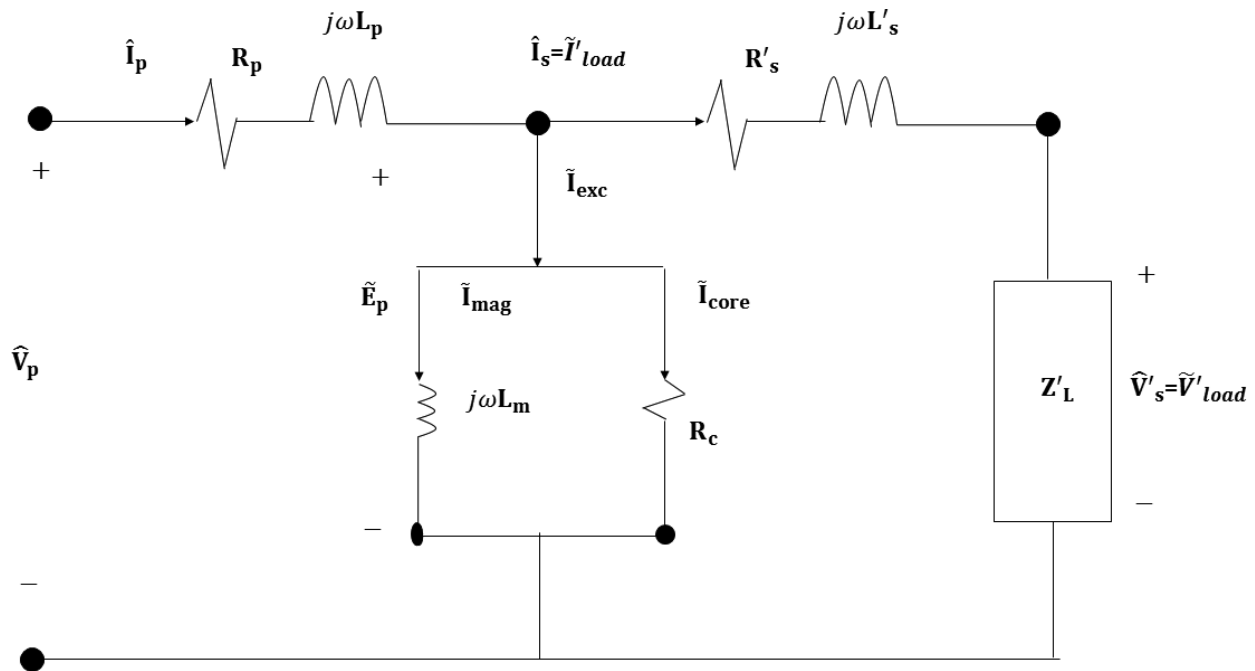

Figure 7. Linear single-phase, steady-state transformer model for sinusoidal analysis.

respectively. Superscript ′ is used for quantities referred from the secondary winding to the primary winding of the transformer. Losses in transformers consist of losses with no-load or core and load losses, which include I^2R losses, eddy current losses, and additional losses in the tank, fasteners, or other iron parts. The effect of the harmonics on each type of loss is explained below:

- No-load or core losses: they are produced by the excitation voltage in the core. The voltage waveform in the primary winding is considered as sinusoidal independently of the load current. Thus, the losses are not expected to increase when load currents are nonsinusoidal. Although, the magnetizing current contains very weak harmonics compared with the harmonics current load, so their effects on the total losses are minimal.

- Joule losses: if the load current contains harmonics, these losses will also increase due to the skin effect.

- Eddy current losses: these losses at fundamental frequency are proportional to the square of the current load and the square of the frequency. Then, there might be an excessive increase of losses in the windings conducing nonsinusoidal current loads (and thus also in its temperature).

- Additional losses: these losses cause the temperature to increase in the structural parts of the transformer and, depending on the type of transformer, they will or will not contribute to the hottest temperature in the winding.

The generation of harmonic signals in the transformer plays an important role in the model of such electrical machines. The methodology for the harmonic design of a transformer is as follows: First is the construction and design where mainly the nonlinearity of the core is analyzed that causes nonsinusoidal magnetizing and core-loss currents. The relationship between the parameters and variables of the model of the transformer with respect to the generated harmonic frequencies would be the next step. In the following references, several harmonic models for transformers have been proposed and implemented with respect to time-domain simulation [3–8], frequency-domain simulation [9–12], combined frequency- and time-domain simulation [13, 14], and numerical (e.g., finite-difference, finite-element) simulation [15–21]. Most previous references considered the influence of skin effects and proximity effects in the harmonic model. The problem with this model is the determination of the magnetizing currents and losses in the core, as these are the main harmonic sources in the transformer (see **Figure 8**).

In the previous figure, R_p, i_p, and V_p are the resistance, current, and voltage of the primary winding, L_{pl} is the leakage inductance, i_{exc}, i_{core} and i_{mag} is the excitation, core, and magnetization currents and e_p is the potential difference in the primary. For the second winding of transformer correspond to the variables: R_s, i_s and V_s are the resistance, current, and voltage of the secondary winding and L_{sl} is the leakage inductance. The triplen (i.e., 3rd, 9th, 15th…) harmonic currents cannot propagate in distribution transformers downstream but circulate in the primary delta winding of the transformer causing localized overheating. With linear loading, the three-phase currents will cancel out in the neutral conductor called homopolar currents. However, when nonlinear loads are being supplied, the triplen harmonics in the phase cur-
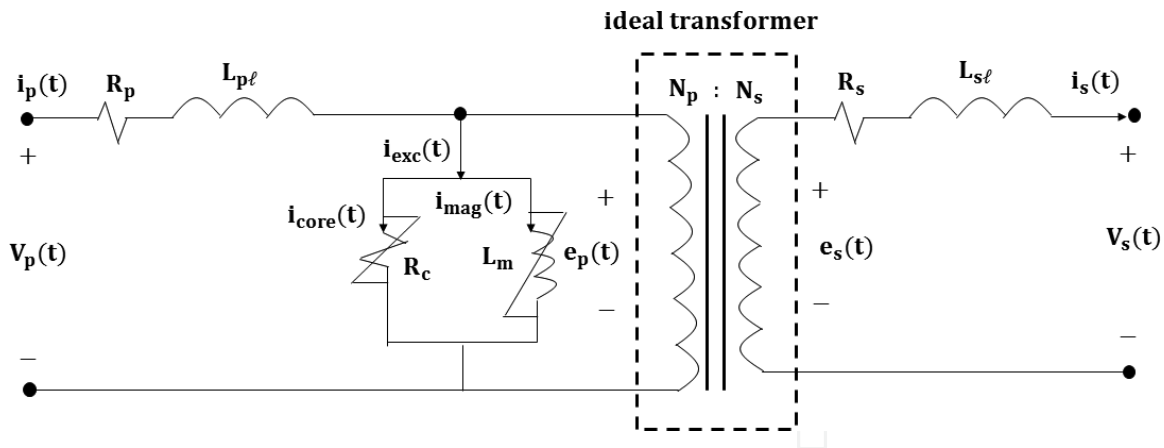
ideal transformer

Figure 8. General harmonic model of a transformer.

rents do not cancel out, but instead add cumulatively in the neutral conductor at a frequency of predominately 180 Hz (3rd harmonic), overheating the transformers and occasionally causing overheating and burning of neutral conductors. Typically, the uses of appropriate "K factor" rated units are recommended for nonlinear loads.

5.3. Effects on the capacitors

The capacitors are used in the electrical power systems for voltage control, reactive power compensation, filtering of signals, and in many cases power-factor correction. For this latter topic, there are two different types of power factor that must be considered in the case when voltage and current waveforms are nonsinusoidal. The first type of power factor is the input displacement factor (IDF), which refers to the cosine of the angle between the fundamental frequency of the voltage and current waveforms. If the harmonic content increases, then the distortion factor will decrease as the total power factor (PF) being the product of the input displacement factor and the distortion factor.

The use of systems and control equipment have increased considerably since the 1990s including electronic loads fed by residential feeders, arc furnaces in industrial networks, etc, resulting in a power quality poor of electrical power systems and an increase in the harmonic disturbances operating to low power-factors, which causes increases line losses, poor voltage regulation, and other factors. The capacitor is very important in the harmonic analysis because it provides the response system at fundamental and harmonic frequencies and it is in the capacitor banks where the issues with harmonics often occur resulting in fuse blowing and/or capacitor failure.

For this reason, it is important to know whether capacitors form either series or parallel resonant circuits, which increase and distort their electrical variables. There are many solutions to these problems: changing location capacitors as well as its size, producing an alteration in the frequency of system response, also altering source characteristics, and designing harmonic filters. The presence of series/parallel resonances can result in unacceptable stresses regarding the equipment installation so it is recommended to use joint capacitor banks for power-factor correction and reactive power compensation, although excessive use of capacitors in

the power networks causes problems that affect power quality, especially in the presence of harmonics.

In summary, the capacitors are important components within an electrical power system because they offer power-factor correction, voltage control/regulation, and filters with special design although its use continue may cause problems associated with capacitor switching and series resonance. In most cases, triplen (multiples of 3) and even harmonics do not exist in a three-phase system because they are uncoupled (see **Figure 9**). There are some cases in which harmonic triplen of zero sequence may exist within the three-phase power systems because the triplen harmonics are very dominant in single-phase systems, unlike even harmonics because these are mostly negligibly small within single- and three-phase systems. Both factors are equal when harmonic is not present.
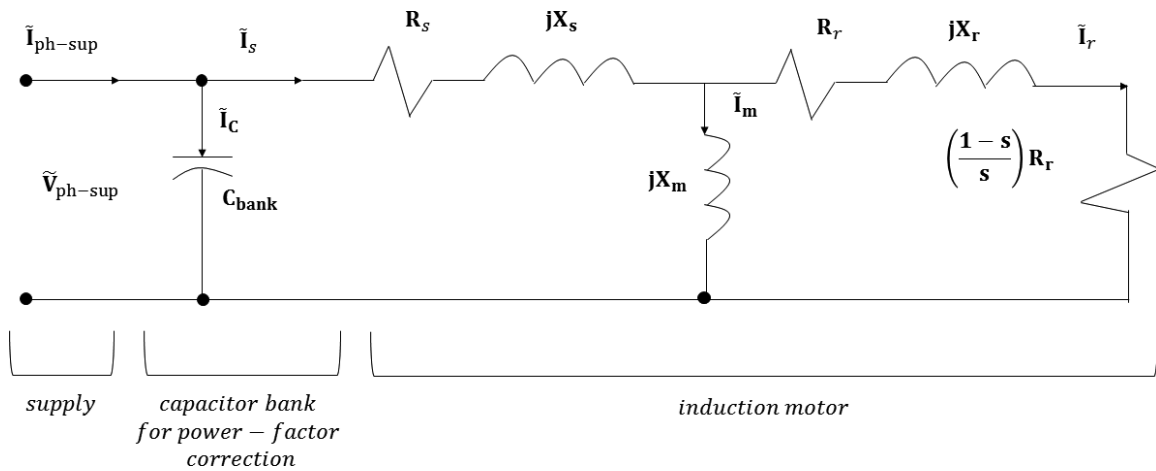

Figure 9. Equivalent circuit of induction motor with displacement FP correction capacitor bank.

5.4. Effects on rotating machines

Fourier Transform offers a method that allows the expression of the nonsinusoidal periodic input signals as a sum of the sinusoids. Each one of these sinusoidal components is supposed to be applied to a linear system. Their particular response as a sinusoid is determined by means of phasors and $H(j\omega)$. If there is a unique pulse instead of a stream of periodic repetitive waves, the phasors and the Fourier series cannot be used to express such pulses. In order to express them, the Fourier series needs to be generalized in the Fourier Transform. In this way, the series can operate not only with all periodic input signals, but also with many other types of nonperiodic pulses.

The Fourier Transform is the analytical tool that finds the way in which such functions of time, for example the sinusoids, the impulses, etc., can be expressed in the domain of frequency.

This Fourier Transform can be used for the analysis and the detection of failure in induction machines. The most likely faults in induction machines are broken rotor bars, bearing damage, short circuits, and eccentricity. Most failures in induction machines can be sorted in two groups: isolation failures and mechanical failures. The coil short-circuits in the stator winding are characteristic in isolation failures, while mechanical faults has to do with the rotor. Among

the most significant rotor failures are the bearing damage, rotor broken bars and rings, static and dynamics eccentricities, voltage unbalances, etc. The electrical faults in machines are dominated by failures in bearings and stator coils. These failures are summarized in **Figure 10**.
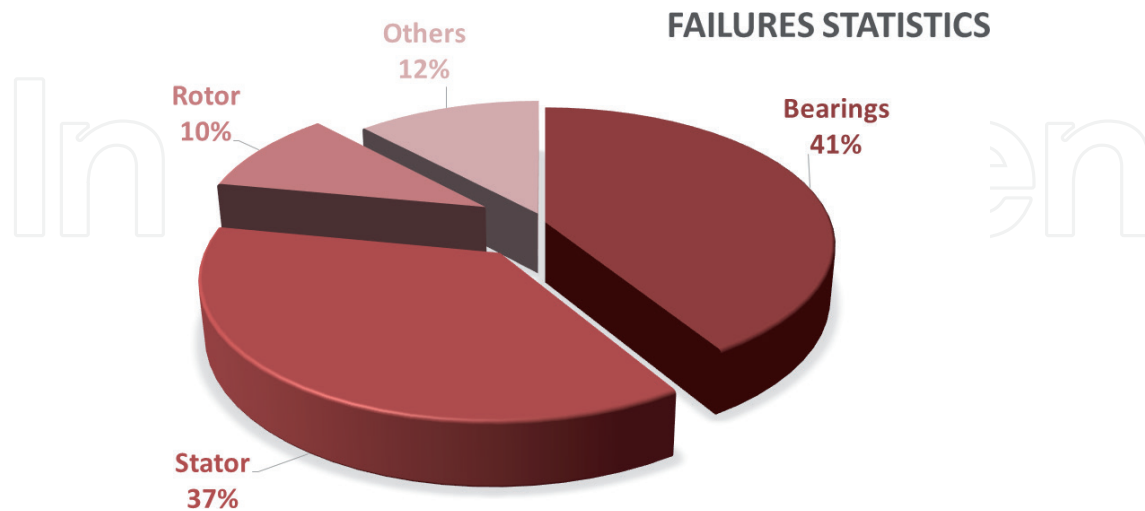
FAILURES STATISTICS

Others
12%

Rotor
10%

Bearings
41%

Stator
37%

Figure 10. Failure statistics in induction machines.

To establish the level of failures in the induction machines it is necessary to develop a methodology that consists of finding the machine slip using only the stator current. This parameter could be used for many applications, but in this case the focus is on fault detection based on the fact that an unbalanced machine, when supplied with a three-phase balanced voltage, produces specific components in the stator current whose magnitude and frequency depends on the asymmetry level and the nature of the fault. This is based on the current signal spectrum decomposition, analyzed via the Fourier Transform. Another very important aspect of induction machines to establish the level of failures is the detection monitoring of the mechanical faults [22–24]. Vibration monitoring is the most reliable method for assessing the overall health of a rotor system. The spectral analysis of vibrations has been used in rotating machines fault diagnosis for decades as this method, in time domain, is more effective for calculating some simple quantities as root mean square (RMS), kurtosis, crest factor, etc., but the problem is that they often do not offer enough information on the vibrations for a thorough diagnosis [25].

To the analysis of the systems in the harmonic domain of the polyphase AC, the concept was presented by Nikola Tesla [26] in 1888, there was a competition between AC and DC systems. Steinmetz [27], Richter [28], Kron [29], Veinott [30], Schuisky [31], Bodefeld [32], Alger [33], Umans et al. [34], Lyon [35], and Say [36] were the pioneers in the study of single- and three-phase induction machines which published in this area of expertise, being the most recent Matsch [37], Chapman [38], and Fuchs et al. [39, 40].

The studies were carried out under transient and steady-state conditions. Currently, electrical power systems are affected by the insertion of nonlinear components and loads and the three-phase machines are subjected to nonsinusoidal operating conditions not taking into account the harmonic signals generated in voltage and/or current on three-phase induction

machines causing a poor power quality and these in turn, abnormal operation, static and dynamic rotor eccentricities, excessive saturation of iron cores, one-sided magnetic pull due to DC currents, shaft fluxes and associated bearing currents, mechanical vibrations, dynamic instability when connected to weak systems, increasing copper losses, reduction of overall efficiency, generation of inter- and subharmonic torques, production of (harmonic) resonance and ferroresonance conditions, failure of insulation due to high voltage stress caused by quick changes in supply current and lightning surges, unbalanced operation due to an imbalance of power systems voltage caused by harmonics, etc. For that reason, it is necessary to analyze the machine and get a harmonic model of induction machine for loss calculations, harmonic torque calculations, and harmonic power flow studies.

5.5. Three-phase induction machine model

Figure 11 illustrates an equivalent circuit simple and accurate frequency-based linear model to fundamental frequency and **Figure 12** shows a complete linear of a three-phase induction machine for harmonic analysis. The nomenclature is the following: ω_{es} is the fundamental angular frequency (or velocity) and s is the fundamental slip. The core-loss resistance are neglected, L_M is the (linear) magnetizing inductance, r_s, L_{ls}, $r'_r \wedge L_{lr}$ are the stator and the rotor (reflected to the stator) resistances and leakage inductances, respectively [41].
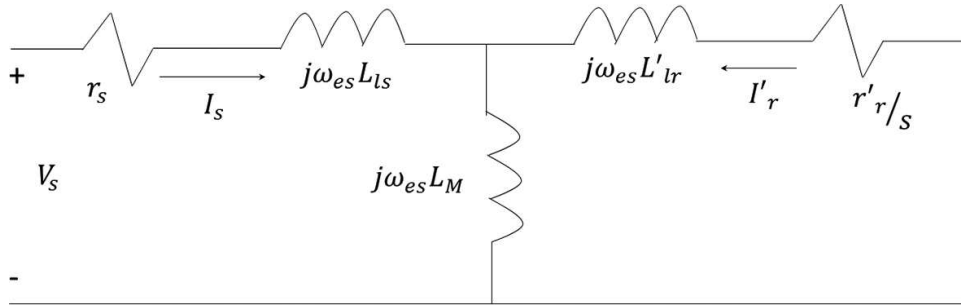

Figure 11. Complete linearity of a three-phase induction machine for sinusoidal analysis.
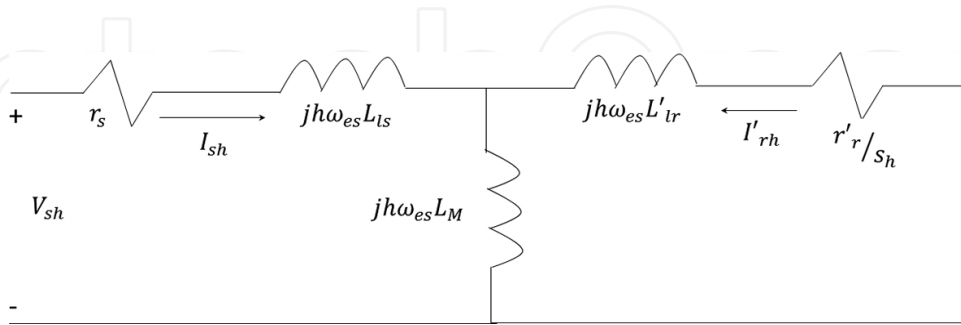

Figure 12. Complete linearity of a three-phase induction machine for harmonic analysis.

When the concept changes, i.e., when it is a doubly fed induction machine, the harmonics can be generated by both windings of the machine: harmonics generated in the stator winding voltage source with frequencies $f_{sh} = hf_{es}$ and harmonics generated in the rotor winding voltage source with frequencies $f_{rh} = hf_{er}$, where h is an integer number. However, it is necessary to know

that the harmonics induced in the rotor winding, due to harmonics in the stator winding, are not harmonics of the rotor fundamental frequency and therefore they cannot be called harmonics but subharmonic or interharmonic.

When a harmonic voltage source with frequency hf_{es} fed to the stator winding of the induction machine the rotor is short circuited. This machine's model is a well-accepted steady-state, with all the parameters seen from the stator, as showed in the circuit in **Figure 12**. Then the equation that represents the circuit is:

$$
\begin{bmatrix} V_{sh} \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + jh\,\omega_{es}(L_{ls} + L_M) & jh\,\omega_{es}\,L_M \\ jh\,\omega_{es}\,L_M & r_r^{'}/s_h + jh\,\omega_{es}(L_{lr}^{'} + L_M) \end{bmatrix} \begin{bmatrix} I_{sh} \\ I_{rh}^{'} \end{bmatrix}
\tag{6}
$$

$$
s_h = \frac{\pm h\,\omega_{es} - \omega_r}{\pm h\,\omega_{es}}
\tag{7}
$$

where:

The sign – is used for negative and + for positive sequence, respectively. Harmonics have different behavior for each sign, i.e., for negative is $h = 3k - 1$ for $k = 1, 2, 3, \ldots$ and the positive sequence behavior are $h = 3k + 1$ where the most common harmonics are the 5, 7, 11, 13, 15, 17… known as the characteristic harmonics. Solving the equation of the voltage equation we obtain the harmonic phasors current of a harmonic voltage source in the stator winding. If it analyzed it in its own winding with an analysis in the time domain, we have,

$$
i_{sh} = |I_{sh}| \cos(h\,\omega_{es}\,t + \varphi_{sh})
\tag{8}
$$

$$
i_{rh}^{'} = I_{rh}^{'} \vee \cos(s_h\,h\,\omega_{es}\,t + \varphi_{rh}^{'} \mp \theta_{ef})
\tag{9}
$$

If zero sequence corresponding to the harmonic $h = 3k$, then the circuit of **Figure 12** is invalid since the induction machine zero sequence works with two uncoupled windings observed in **Figure 13**, where the voltages are given by,
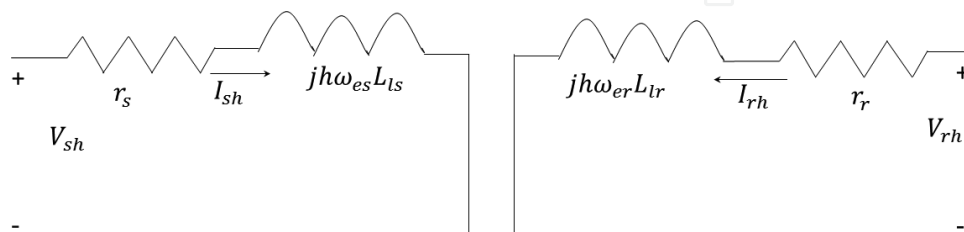

Figure 13. Induction machine triplex harmonic model seen from stator.

$$
V_{sh} = (r_s + jh\,\omega_{es}\,L_{ls})I_{sh}
\tag{10}
$$

$$V_{rh} = \left(r_r + jh\,\omega_{er}\,L_{1r} \right) I_{rh} \tag{11}$$

The solution of the last equation gives the harmonic current phasors because of their respective voltage sources: i.e., $I_{sh} = |I_{sh}| \angle\phi_{sh}$ and $I_{rh} = |I_{rh}| \angle\phi_{rh}$. Their representations in the time domain in their respective windings are:

$$i_{sh} = |I_{sh}| \cos\left(h\,\omega_{es}\,t + \varphi_{sh} \right) \tag{12}$$

$$i_{rh} = |I_{rh}| \cos\left(h\,\omega_{er}\,t + \varphi_{rh} \right) \tag{13}$$

The general solution for balance conditions including voltage sources at fundamental and harmonic frequencies in the stator and rotor are:

$$
\begin{aligned}
i_s &= \sum_{h=1}^{H} |I_{sh}| \cos\left(h\,\omega_{es}\,t + \phi_{sh} \right) + \sum_{h=1,3k+1}^{H} |I'_{sh}| \cos\left(s_{rh}\,h\,\omega_{er}\,t + \varphi'_{sh} + \theta_{ef} \right) \\
&\quad + \sum_{h=3k-1}^{H} |I'_{sh}| \cos\left(s_{rh}\,h\,\omega_{er}\,t + \varphi'_{sh} - \theta_{ef} \right)
\end{aligned}
\tag{14}
$$

The first summation includes all harmonic current generated in the nonsinusoidal source voltage in the stator winding, which contains positive, negative, and zero sequence harmonics. The second summation includes all the current harmonics generated for the induction effect of the positive sequence voltage source harmonics in the rotor winding. The third summation includes all the current harmonics generated for induction effect of the negative sequence voltage source harmonics in the rotor winding [42]. This procedure is the same for the rotor winding:

$$
\begin{aligned}
i_r &= \sum_{h=1}^{H} |I_{rh}| \cos\left(h\,\omega_{er}\,t + \varphi_{rh} \right) + \sum_{H}^{h=1,3k+1} |I'_{rh}| \cos\left(s_h\,h\,\omega_{es}\,t + \varphi'_{rh} - \theta_{ef} \right) \\
&\quad + \sum_{H}^{h=3k-1} |I'_{rh}| \cos\left(s_h\,h\,\omega_{es}\,t + \varphi'_{rh} + \theta_{ef} \right)
\end{aligned}
\tag{15}
$$

To validate the proposed model, a three-phase induction machine of ¼ H.P., 208 V, and 1.3 A is utilized for experimental validation. It is important to mention that a three-phase programmable voltage source of 200/208V at 50/60Hz and 24A capable of generating harmonic signals is used as the main voltage source to supply induction machine.

Table 1 shows the parameters of the induction machine. The proposed model in steady-state model is compared with the dynamic equations of the induction machine and the results of experimentation. It should be considered that for all the study cases a mechanical torque of 0.3 N·m was used.

The results from the proposed model (steady-state) are compared with those obtained in the laboratory (measurement) and compared with those obtained from the simulated complete model (dynamic), once the steady-state has been attained.

5.5.1. Case I. Stator-fed induction machine and rotor short-circuited

In this case, a sinusoidal three-phase balanced voltage source of 80 V at 60 Hz in the stator winding excites the induction machine with the connections in the rotor in short-circuit. The

results of the waveforms of the stator and rotor harmonic currents of both the simulation and the experimentation are shown in **Figures 14** and **15**, respectively and we can see that the results match in the analysis.
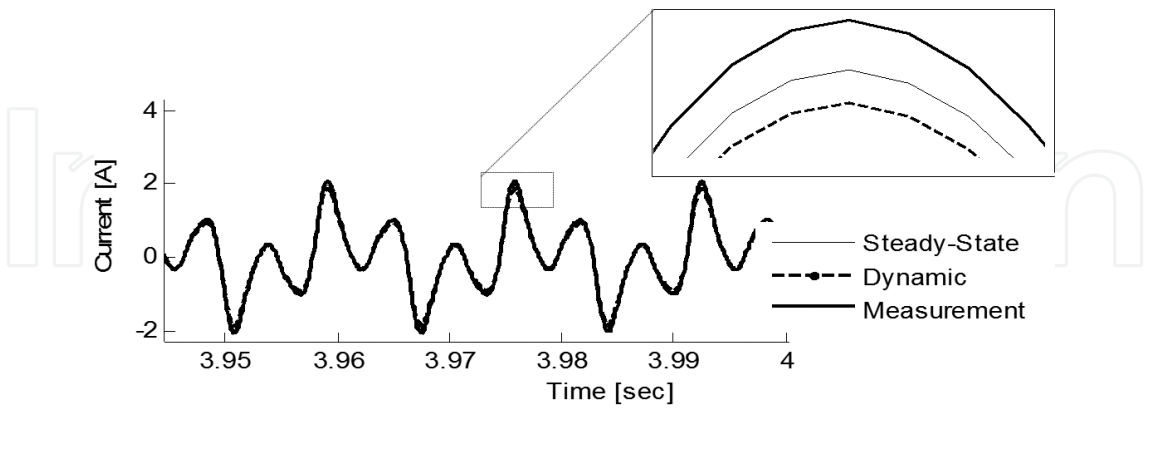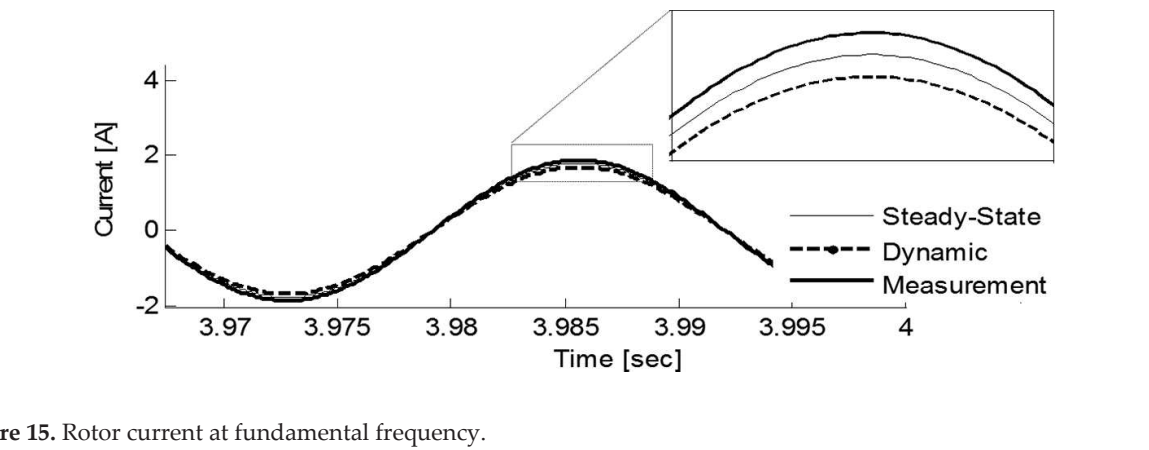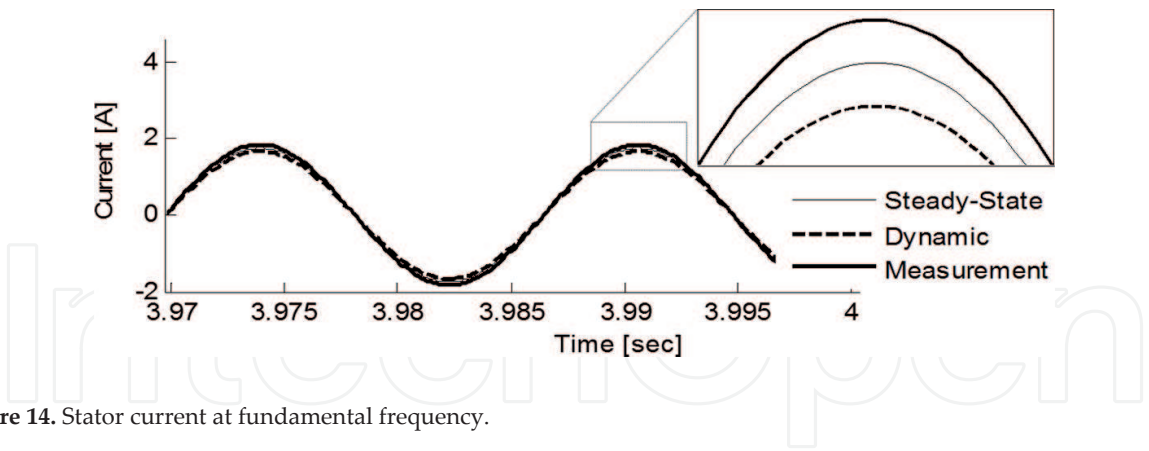
Parameters	0.23 H.P/175 W
Number of poles	4
Inertia	0.0068 kg·m²
Nominal line current	1.3 Amps
Nominal line-to-line voltage	120 Vrms
Nominal torque	2.481 N·m
Nominal frequency	60 Hz
Stator resistance, r_s	14 Ω
Stator inductance, L_{ls}	9 H
Rotor resistance, r_r	7.7 Ω
Rotor inductance, L_{lr}	9 H
Magnetizing inductance, $L_{mr} = L_{ms}$	155 H
Rotor speed	1500 rpm

Table 1. Induction machine parameters.

5.5.2. Case II. Stator-fed induction machine at harmonic frequencies

For this case, the nonsinusoidal voltage source at 120 V at 60 Hz which excites the stator winding contains harmonic of the third, fifth and seventh order while the rotor winding is short-circuited. The voltage harmonic components magnitude and angle are 40 ∠ 113° V, 24 ∠ 42.85° V, and 17.1428 ∠ 137.15° V for the third, fifth, and seventh harmonic, respectively. **Figures 16** and **17** show the resulting currents in the stator and rotor of the induction machine. The results in steady-state clearly match those obtained by measurement and with the dynamic model.

The harmonic slips for each harmonic component are s0.3342, s_5 = 1.1332 and s_7 = 0.9047. The induced frequencies in the rotor are obtained with $\left(s_h \times h \times \omega_{es}\right)/2\pi$: the fundamental frequency in the stator induces $(0.3342 \times 377)/2\pi$ = 20 Hz in the rotor; the fifth harmonic in the stator induces $(1.1332 \times 5 \times 377)/2\pi$ = 340 Hz in the rotor; and the seventh harmonic in the stator induces $(0.9047 \times 7 \times 377)/2\pi$ = 380 Hz in the rotor. These frequencies are not harmonics of the fundamental frequency in the stator (integer multiples of 60 Hz), but the seventeenth and nineteenth harmonic of the rotor fundamental frequency (integer multiples of 20 Hz). These frequencies induced in the rotor cannot be called as harmonic frequencies are not integer multiples of the fundamental frequency but are to be noted that the harmonic seventeenth and nineteenth of the fundamental frequency of the rotor.

Figure 14. Stator current at fundamental frequency.

Figure 15. Rotor current at fundamental frequency.

Figure 16. Stator current at harmonic frequencies.

5.5.3. Case III. Stator-fed induction machine with six-pulse voltage source

It is considered that a nonsinusoidal three-phase balanced voltage source of 120 V at 60 Hz excites to the induction machine in the stator winding with the rotor windings in short-circuit. The voltage source is six-pulse as shown in **Figure 18** with harmonics components in **Table 2**.

Figures 19 and **20** show the current waveforms obtained from measurement and from simulation.

Table 3 summarizes the harmonic currents in the induction machine for the case studies. Note that the waveform current has been attained from the current shown in this table, which have been obtained from the solution of the equations mentioned in the previous section.
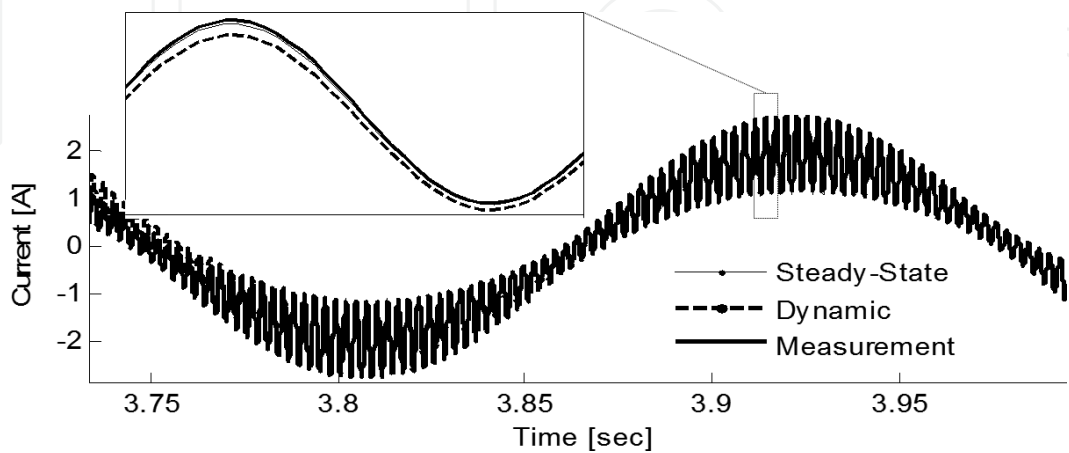

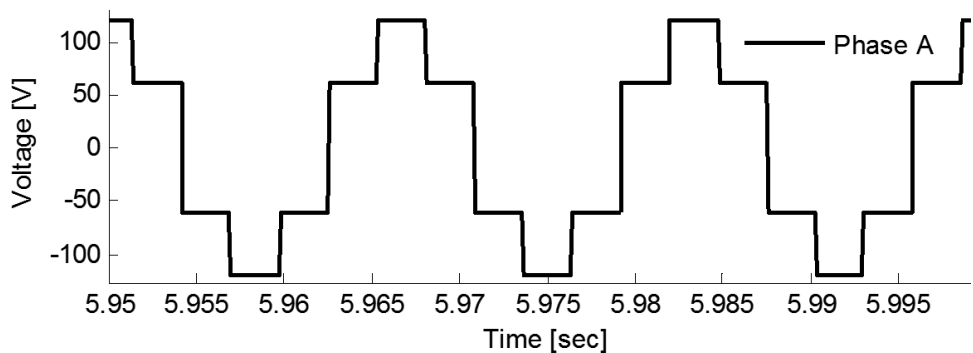**Figure 17.** Rotor current at harmonic frequencies.

Figure 18. Three-phase voltage source.

	Harmonic components							
	5th	7th	11th	13th	17th	19th	23th	25th
Voltage (%)	20.3	13.8	9.35	7.17	6.3	4.6	4.4	2.3
Degrees	163	88.1	175	83.7	-17	88	-166	101

Table 2. Harmonic components.

Harmonic analysis in the electrical power systems becomes increasingly necessary since by the proliferation of nonlinear loads the problems of power quality and especially the harmonics signals are more apparent. The proposed model analyzes the behavior of the induction machine under nonsinusoidal operating conditions for the inclusion of harmonics signals in the stator winding voltage source. In the end, the results of the steady-state proposed model
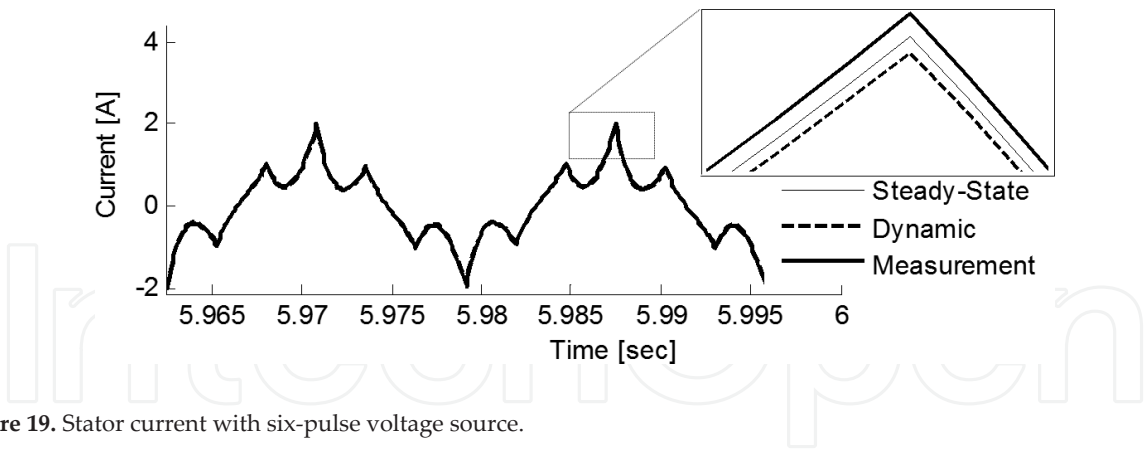
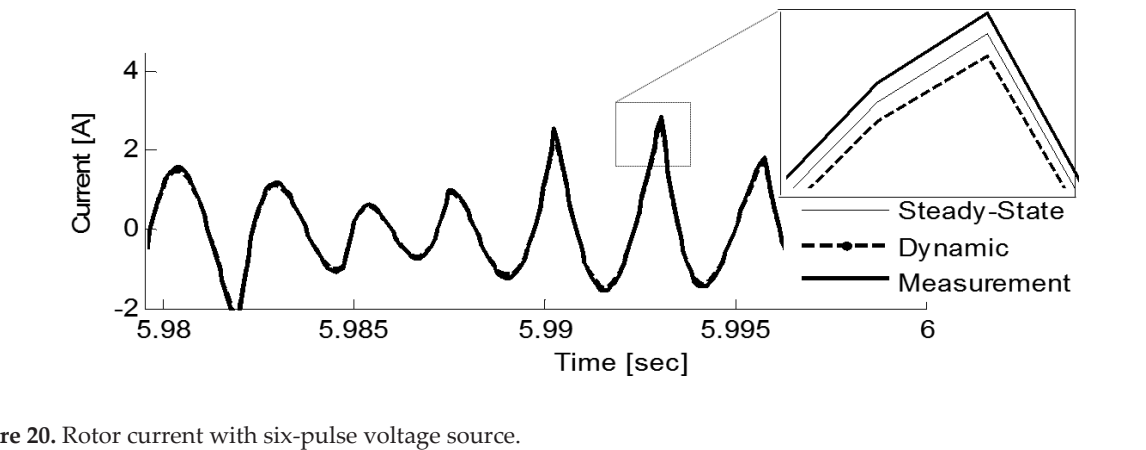**Figure 19.** Stator current with six-pulse voltage source.

Figure 20. Rotor current with six-pulse voltage source.

Case study	Stator current				Rotor current			
	Seq. (+,−,0)	Magnitude	Angle	Frequency (Hz)	Sequence (+,−,0)	Magnitude	Angle	Frequency (Hz)
Case I	+	0.588	-0.72	60	+	3	2.57	19.8
Case II	+	0.588	0.72	60	+	3	2.57	19.8
	+	0.111	-1.39	420	+	0.2	1.8	322.9
	−	0.216	-1.34	300	−	0.17	1.74	397
	0	1.07	-1.09	180				
Case III	+	2.1	14.1	60	+	1.8	40.9	9.6
	−	0.79	4.2	300	−	1.32	222.9	48
	+	0.38	178	420	+	2.81	-42.1	67.2
	−	0.152	166.9	660	−	0.80	28.6	105.6
	+	0.115	-16.4	780	+	0.56	117.5	124.8
	−	0.067	-24.3	1020	−	0.15	204	163.2
	+	0.057	150.1	1140	+	0.24	-63.3	182.4
	−	0.032	142.1	1380	−	0.12	14.1	220.8
	+	0.029	-38.6	1500	+	0.13	92.2	240

Table 3. Summary of the harmonic currents for the cases studies.

are compared with the results obtained in transient-state and both models are validated by experimental tests in the laboratory getting the same results for each case validating the precision and accuracy of the proposed model besides that this model is proper for harmonic and nonharmonic analysis of the induction machine exciting only to the stator winding. This model also can be used for "harmonic" analysis in an electrical power system.

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References

[1] Arrillaga, J.; Watson, N. R.; and Chen, S. Power System Quality Assessment. 2nd ed. University of Michigan: John Wiley & Sons, 2000; 2007. 300 p. DOI: 0471988650.

[2] Gonen T. Electrical Power Transmission System Engineering: Analysis and Design, 3rd ed. California State University, Sacramento: CRC Press; 2015. 1093 p. DOI: 1482232235.

[3] Dommel, H. Digital computer solution of electromagnetic transients in single and multiphase network. IEEE Transactions on Power Apparatus and Systems. 2007;**PAS-88**(4):388–399. DOI: 10.1109/TPAS.1969.292459.

[4] Dommel, H. Transformer models in the simulation of electromagnetic transient. In: IEEE, editor. 5th Power Systems Computation Conference; September 1–5; Cambridge. England: Paper 3.1/4; 1975. pp. 1–16.

[5] Hatziantoniu, C.; Galanos, G. D.; and Milias-Argitis, J. An incremental transformer model for the study of harmonic overvoltages in weak AC/DC systems. IEEE Transactions on Power Delivery. 1988;**3**(3):1111–1121.

[6] Dolinar, D.; Pihler, J.; and Grcar, B. Dynamic model of a three-phase power transformer. IEEE Transactions on Power Delivery. 1993;**8**(4):1811–1819.

[7] Leon, F.; and Semlyen, A. Complete transformer model for electromagnetic transients. IEEE Transactions on Power Delivery. 1994;**9**(1):231–239. DOI: 10.1109/61.277694.

[8] García, S.; Medina, A.; and Pérez, C. A state space single-phase transformer model incorporating nonlinear phenomena of magnetic saturation and hysteresis for transient and periodic steady-state analysis. IEEE Power Engineering Society Summer Meeting, 2000. 2000;**4**:2417–2421. DOI: 10.1109/PESS.2000.867369.

[9] Huang, S.-R.; Chung, S. C.; Chen, B.-N.; and Chen, Y.-H. A harmonic model for the nonlinearities of single-phase transformer with describing functions. IEEE Transactions on Power Delivery. 2003;**18**(3):815–820. DOI: 10.1109/TPWRD.2003.813878A.

[10] Medina, A.; and Arrillaga, J. Generalised modelling of power transformers in the harmonic domain. IEEE Transactions on Power Delivery. 2002;**7**(3):1458–1465. DOI: 10.1109/61.141865.

[11] Semlyen, A.; Acha, E.; and Arrillaga, J. Newton-type algorithms for the harmonic phasor analysis of nonlinear power circuits in periodical steady state with special reference to magnetic nonlinearities. IEEE Transactions on Power Delivery. 2002;**3**(3):1090–1098. DOI: 10.1109/61.193891.

[12] Pedra, J.; Corcoles, F.; Sainz, L.; and López R. Harmonic nonlinear transformer modelling. IEEE Transactions on Power Delivery. 2004;**19**(2):884–890. DOI: 10.1109/TPWRD.2004.825254.

[13] Dugui, W.; and Zheng, X. Harmonic model of power transformer. In: POWERCON '98. 1998 International Conference on, editor. Power System Technology, 1998. Proceedings; 18–21 Aug; Hangzhou, China. IEEE; 1998. DOI: 10.1109/ICPST.1998.729244.

[14] Greene, J. D.; and Gross, C. A. Nonlinear modelling of transformers. IEEE Transactions on Industry Applications. 1988; **24**(3):434–438. DOI: 10.1109/28.2892.

[15] Fuchs, E. F.; Masoum, M. A. S.;. and Roesler, D. J. Large signal nonlinear model of anisotropic transformers for nonsinusoidal operation. I. λ-i characteristics. IEEE Transactions on Power Delivery. 1991;**6**(1):1874–1886. DOI: 10.1109/61.103773.

[16] Masoum, M. A. S.; Fuchs, E. F.; and Roesler, D. J. Large signal nonlinear model of anisotropic transformers for nonsinusoidal operation. II. Magnetizing and core-loss currents. IEEE Transactions on Power Delivery. 1991; **6**(4):1509–1516. DOI: 10.1109/61.97686.

[17] Stensland, T.; Fuchs, E. F.;. Grady, W. M; and. Doyle, M. T. Modelling of magnetizing and core-loss currents in single-phase transformers with voltage harmonics for use in power flow. IEEE Transactions on Power Delivery. 1997; **12**(2):768–774. DOI: 10.1109/61.584366.

[18] Yamada, S.; Bessho, K.; and Lu, J. Harmonic balance finite element method applied to nonlinear AC magnetic analysis. IEEE Transactions on Magnetics. 1989; **25**(4):2971–2973. DOI: 10.1109/20.34341.

[19] Silvester, P.; and Chari, M. V. K. Finite element solution of saturable magnetic field problems. IEEE Transactions on Power Apparatus and Systems. 1970; **PAS-89**(7):1642–1651. DOI: 10.1109/TPAS.1970.292812.

[20] Enokizono, M. and Soda, N. Finite element analysis of transformer model core with measured reluctivity tensor. IEEE Transactions on Magnetics. 1997; **33**(5):4110–4112. DOI: 10.1109/20.619679.

[21] Wu, J.-C.; Jou, H.-L.; Wu, K.-D.; and Shen, N. C. Power converter-based method for protecting three-phase power capacitor from harmonic destruction. IEEE Transactions on Power Delivery. 2004;**19**(3):1434–1441. DOI: 10.1109/TPWRD.2004.829926

[22] Albrecht, P. F.; Appiarius, J. C.; McCoy, R. M.; Owen, E. L.; and Sharma, D. K. Assessment of the reliability of motors in utility applications – updated. IEEE Transactions on Energy Conversion. 1986;**EC-1**(1):39–46. DOI: 10.1109/TEC.1986.4765668.

[23] Thompson, W. T. (1999). A Review of On-Line Condition Monitoring Techniques for Three-Phase Squirrel-Cage Induction Motors – Past, Present and Future, The Robert Gordon University, Schoolhill, Aberdeen, Scotland.

[24] Eschmann, P. editor. Ball and Roller Bearings: Their Theory, Design, and Application. 1st ed. University of Michigan: Heyden; 2007. 375 p.

[25] Harris, T. A. editor. Rolling Bearing Analysis. 4th ed. University of Michigan: Wiley; 2001. 1086 p.

[26] Neidhoefer, G; "Early three-phase power," IEEE Power & Energy, vol. 5, no. 5, September/October 2007, pp. 88-112.

[27] Steinmetz, C. P. Steinmetz on Induction-Motor. New York: AIEE Trans; 1898. 300 p.

[28] Richter, R. Elektrische Maschinen: Zweiter Band: Synchronmaschinen und Einankerumformer. 3rd ed. Germany: Springer-Verlag; 2013. 707 p. DOI: 3034841396.

[29] Kron, G. The generalized theory of electrical machinery. AIEE Transactions. 1930;**49**(4):666-683.

[30] Veinott, C. G; "Performance calculation of induction motors," AIEE Transactions, 1932, pp. 743-755.

[31] Schuisky, W.; Calculation of electrical machines, Springer-Publishing, Wien, 1960.

[32] Bödefeld, T.; and Sequenz, H.; Electrical machines, Springer-Publishing, Wien, 1962.

[33] Alger, P. L. The Nature of Polyphase Induction Machines. 1st ed. University of Michigan: John Wiley & Sons; 2006. 397 p.

[34] Umans, S.; Fitzgerald, A.; and Kingsley, C. Electric Machinery. 5th ed. New York: McGraw-Hill Higher Education; 2013. 631 p.

[35] Lyon, W. V. Transient Analysis of Alternating-current Machinery: An Application of Method of Symmetrical Components. 1st ed. New York: Technology Press of Massachusetts Institute of Technology, and Wiley; 2006. 310 p.

[36] Say, M. G. Alternating Current Machines. 5th ed. Michigan: Wiley; 1983. 632 p.

[37] Matsch, L. W.; and Morgan, J. D. Electromagnetic and Electromechanical Machines. 3rd ed. Michigan: Wiley; 1986. 592 p.

[38] Chapman, S. Electric Machinery Fundamentals. 3rd ed. Massachusetts: McGraw-Hill Companies, Incorporated; 2005. 746 p.

[39] Fuchs, E. F. Optimization of Induction Motor Efficiency: Single-Phase Induction Motors. 4th ed. Colorado: Electric Power Research Institute; 1985. 385 p.

[40] Blanusa, B. New Trends in Efficiency Optimization of Induction Motor Drives. 1st ed. Spain: INTECH Open Access; 2010. 953 p.

[41] Krause, P. C.; Wasynczuk, O.; and Sudhoff, S. D. Analysis of Electric Machinery. 2nd ed. Michigan: IEEE Press; 1995. 564 p.

[42] Hernandez , E.; and Madrigal, M. A step forward in the modeling of the doubly-fed induction machine for harmonic analysis. IEEE Transactions on Energy Conversion. 2014;**29**(1):149–157. DOI: 10.1109/TEC.2013.2290297.