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LQ and H_2 Tuning of Fixed-Structure Controller for Continuous Time Invariant System with H_∞ Constraints

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1. Introduction

The problem of fixed-order and fixed-structure controller tuning has been known for more than half a century and is a one of the classic problems of the control theory. Great number of papers and several monographs are devoted to this problem (e.g., Rotach et al., 1984; Datta, 1998; Datta et al., 2000; Astrom & Hagglund, 2006). Analytic methods based on information on structure and form of plant mathematical model play the main role among the methods for solving this problem. These include:

- tuning methods based on single-stage solution of controller parameters synthesis problem (Rotach et al., 1984; Astrom & Hagglund, 2006);
- automatic tuning methods based on application of relay feedback (Rotach et al., 1984; Datta, 1998; Datta et al., 2000; Hjalmarsson, 2002; Astrom & Hagglund, 2006);
- methods based on indirect adaptive control, or implicit reference model (internal model control) (Petrov & Rutkovskiy, 1965; Datta, 1998; Datta et al., 2000; Astrom & Hagglund, 2006).

For recent two decades, many papers devoted to application of powerful \mathcal{H}_2 and \mathcal{H}_∞ optimization tools to design and tuning problems for fixed-structure controllers have been presented (McFarlane & Glover, 1992; Zhou et al., 1996; Balandin & Kogan, 2007). Moreover, the concepts of robust design have brought to a new view of known controller tuning methods.

In (McFarlane & Glover, 1992), a practically effective solution for fixed-order controller tuning problem was obtained. It is based on shaping frequency responses of open control loop by means of pre- and post-filters (loop shaping) in conjunction with minimizing \mathcal{H}_∞ norm of closed-loop system. The main advantage of this approach consists in that the resulting controller is not only stabilizing, but possesses assured performance characteristics in conditions of uncertainty. The method has been successfully applied for synthesis of PID (Proportional-Integrating-Derivative) controller for SISO (Single-Input Single-Output) plant, as well as multiloop PID controller for MIMO (Multi-Input Multi-Output) plant. The controller tuning problem is close to the plant identification problem that implies using of constrained and unconstrained optimization technique for finding optimal controller tuning algorithms in model matching problem (Poznyak, 1991) and, in particular, in internal model

control. In (Tan et al., 2002), for solving the problem of PID controller design for MIMO plant the authors use BMI (bilinear matrix inequality) technique and minimization of \mathcal{H}_∞ norm of adjusted system transfer function introduced in (McFarlane & Glover, 1992). In (Balandin & Kogan, 2007), the authors present the synthesis method for adjusted system with fixed-order controller based on LMI (linear matrix inequality) technique guaranteeing boundedness of \mathcal{H}_2 norm of the adjusted system transfer matrix together with its stability.

In (Bao et al., 1999), the authors introduce a technique for multiloop PID controller tuning based on Bounded Real Lemma (BRL) allowing to obtain the numerical solution via semi-definite programming. This method of controller tuning based on direct synthesis algorithms with application of LMI technique has certain advantages, namely:

- This is the first LMI-based controller tuning method that has shown its validity and effectiveness in solving a number of applied problems.
- There is standard software tools (e.g., Matlab) for implementation of this method.

But this tuning method also has a number of drawbacks:

- This approach poorly fits for synthesis from viewpoint of required control performance.
- The synthesis problem solution results in controller of general full-order observer form. It requires solving additional approximation problem in frequency domain for PID controller tuning.
- For fixed-structure controller, the method requires use of pre- and post-filters and, in general case, results in solving BMIs.
- The solution depends on chosen initial conditions.

The problem of fixed-order and fixed-structure controller tuning formulated in terms of quadratic optimization was solved in (Yadykin, 1985). It results in classic least-squares method of controller tuning algorithm synthesis. This approach is based on application of indirect adaptive control with implicit reference model of linear plant (also called internal model control). The principal distinction between this approach and other methods mentioned before consists in that the adjusted system performance is given directly by fixed parameters of the implicit reference model. Criterion of proximity for dynamic characteristics of the adjusted control system and its reference model can be expressed in terms of Frobenius norm for coefficients of polynomials generated by transfer functions of the control system and its reference model. The main idea of new approach introduced in this Chapter consists in replacement of the aforementioned tuning functional by \mathcal{H}_2 norm of difference between transfer functions of closed-loop adjusted and reference systems and switching from unconstrained optimization to optimization under constraints in form of LMIs guaranteeing bounded \mathcal{H}_∞ norm of transfer function of closed-loop system. By virtue of Parseval's Theorem, it is the \mathcal{H}_2 norm of difference between transfer functions of closed-loop adjusted and reference systems gives direct estimation of difference between transients in the closed-loop adjusted and reference systems. Thus, the tuning objective consists in providing the adjusted system with transient performance of the reference model.

2. Problem Statement

Consider linear continuous time invariant control system consisting of the dynamic plant and fixed-structure controller

$$P(s): \quad \begin{bmatrix} \dot{x}_p(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A_p & B_p \\ C_p & 0 \end{bmatrix} \begin{bmatrix} x_p(t) \\ u(t) \end{bmatrix}, \quad x_p(0) = x_{p0}, \quad (1)$$

$$K(s): \quad \begin{bmatrix} \dot{x}_c(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_{cm} & B_c \\ C_{cm} & D_c \end{bmatrix} \begin{bmatrix} x_c(t) \\ g(t) - y(t) \end{bmatrix}, \quad x_c(0) = x_{c0}, \quad (2)$$

where $x_p(t) \in \mathbb{R}^{n_p}$ is the plant state, $y(t) \in \mathbb{R}^1$ is the plant output, $u(t) \in \mathbb{R}^1$ is the control, $x_c(t) \in \mathbb{R}^{n_c}$ is the controller state, $g(t) \in \mathbb{R}^1$ is the reference signal, and the matrices A_p , B_p , C_p , A_{cm} , B_c , C_{cm} , and D_c have compatible dimensions. Assume that plant (1) is completely controllable and observable, the state-space realizations (A_p, B_p, C_p) and $(A_{cm}, B_c, C_{cm}, D_c)$ are minimal, and the matrices A_p , B_p , and C_p are known or can be defined at the earlier stage of parametric identification. Also assume $g(t) \in \mathcal{L}_2[0, +\infty)$. We are interested in tracking the reference input $g(t)$ for an arbitrary set of plant parameters inside of some bounded region Σ . It is assumed that controller (2) has fixed structure. The feature of this controller tuning problem is in that the controller structure does not change in tuning process, i.e. the matrices A_{cm} and C_{cm} are fixed, and only elements of the vector B_c and scalar value D_c are to be adjusted. Such situation appears, for instance, when controller (2) is a PID controller. Denote the generalized tuning vector

$$G = [B_c^T \quad D_c]^T. \quad (3)$$

The goal of controller tuning on the base of principle of internal model of control loop consists in reaching the identity

$$y(t) \equiv y_m(t), \quad (4)$$

where $y_m(t) \in \mathbb{R}^1$ is the output of implicit (virtual) reference model of system (1)–(2) under assumption that the plant input is fed by the test signal $g(t)$ and the plant parameters belong to some admissible and bounded set

$$\Sigma = \{A_p, B_p, C_p : \underline{a}_{pij} \leq a_{pij} \leq \bar{a}_{pij}, \underline{b}_{pij} \leq b_{pij} \leq \bar{b}_{pij}, \underline{c}_{pij} \leq c_{pij} \leq \bar{c}_{pij}\}.$$

The implicit reference model can be described by the following state-space equation system

$$P_m(s): \quad \begin{bmatrix} \dot{x}_{pm}(t) \\ y_m(t) \end{bmatrix} = \begin{bmatrix} A_{pm} & B_{pm} \\ C_{pm} & 0 \end{bmatrix} \begin{bmatrix} x_{pm}(t) \\ u_m(t) \end{bmatrix}, \quad x_{pm}(0) = x_{pm0}, \quad (5)$$

$$K_m(s): \quad \begin{bmatrix} \dot{x}_{cm}(t) \\ u_m(t) \end{bmatrix} = \begin{bmatrix} A_{cm} & B_{cm} \\ C_{cm} & D_{cm} \end{bmatrix} \begin{bmatrix} x_{cm}(t) \\ g(t) - y_m(t) \end{bmatrix}, \quad x_{cm}(0) = x_{cm0}, \quad (6)$$

where the state vectors of reference plant and controller, as well as the reference plant output and control have the same dimensions as their counterparts in system (1)–(2). Naturally, reference closed-loop system (5), (6) is assumed to be stable. The standard controller tuning procedure after plant identification consists of two stages (Astrom & Hagglund, 2006):

- synthesis of the controller parameters in nominal mode;
- optimal controller tuning according to given tuning criterion.

At that, it is assumed that the plant parameters at zero time take on any constant values from the admissible set Σ .

Tuning objective (4) in frequency domain under assumption of zero initial conditions is equivalent to the identities

$$\Phi(j\omega) \equiv \Phi_m(j\omega) \quad \forall \omega \in (-\infty, +\infty), \quad (7)$$

$$W(j\omega) \equiv W_m(j\omega) \quad \forall \omega \in (-\infty, +\infty), \quad (8)$$

where $W(s)$ and $\Phi(s) = W(s)/(1 + W(s))$ are the transfer functions of open- and closed-loop systems, respectively. Denote in advance that identities (7) and (8) are equivalent if some conditions, namely, full adaptability conditions hold true. The conditions (criteria) of weak, full, and partial adaptability of a control system (Yadykin, 1981) are some generalizations of controllability and observability criteria. Similar to the latter criteria, adaptability of a system can be determined in terms of ranks of some special adaptability matrices. The notion of system adaptability will be considered in the next section.

Condition (8) expresses the requirement of proximity of the dynamic operators of the adjusted and reference open-loop systems along the whole set of admissible plant parameters Σ . This is equivalent to proximity of transient responses of these systems when their inputs are fed with the unit step. Condition (7) expresses the same requirement for the closed-loop systems. In nominal mode we obviously have $W(j\omega) = W_m(j\omega)$.

Let us pass from the identity of transfer functions to the identity of polynomials generated by these transfer functions. The transfer functions of plant (1) and controller (2), as well as transfer functions of reference plant (5) and controller (6) are given by

$$P(s) = C_p(sI - A_p)^{-1}B_p, \quad (9)$$

$$K(s) = C_{cm}(sI - A_{cm})^{-1}B_c + D_c, \quad (10)$$

$$P_m(s) = C_{pm}(sI - A_{pm})^{-1}B_{pm}, \quad (11)$$

$$K_m(s) = C_{cm}(sI - A_{cm})^{-1}B_{cm} + D_{cm}, \quad (12)$$

respectively. Substituting expressions (9)–(12) into identity (8), we obtain the following polynomial controller tuning equation:

$$\begin{aligned} & C_p(sI - A_p)^{-1}B_pC_{cm}(sI - A_{cm})^{-1}B_c + C_p(sI - A_p)^{-1}B_pD_c \\ & = C_{pm}(sI - A_{pm})^{-1}B_{pm}C_{cm}(sI - A_{cm})^{-1}B_{cm} + C_{pm}(sI - A_{pm})^{-1}B_{pm}D_{cm}. \end{aligned} \quad (13)$$

Applying series expansion of resolvents in left-hand and right-hand parts of the last equality and multiplying its both parts to the product of characteristic polynomials of the plant, controller, and implicit reference plant and controller models, we obtain the following equation for the controller tuning polynomial (Datta, 1998):

$$(P_1 - N_1)s^{2n_c+n_p-1} + \dots + (P_{2n_c+n_p-1} - N_{2n_c+n_p-1})s + (P_{2n_c+n_p} - N_{2n_c+n_p}) = 0.$$

Define the adaptability matrices

$$L = \begin{bmatrix} L_{\mu 1} & L_{\mu 2} \end{bmatrix}, \quad N^T = \begin{bmatrix} N_{\mu 1}^T & N_{\mu 2}^T \end{bmatrix} \quad (14)$$

and the linear-quadratic (*LQ*) tuning functional J_1 as follows

$$J_1 = \sum_{\mu=0}^{2n_c+n_p-1} \text{tr}(P_{\mu} - N_{\mu})^T (P_{\mu} - N_{\mu}), \quad P_{\mu} = L_{\mu 1} B_c + L_{\mu 2} D_c, \quad N_{\mu} = N_{\mu 1} + N_{\mu 2}, \quad (15)$$

$$L_{\mu 1} = \sum_{\sigma=0}^{n_c} \sum_{i=j}^{n_c-1} \sum_{\eta=v}^{n_p-1} a_{\sigma} a_{cm\eta+1} a_{i+1} C_p A_p^{i-j} B_p C_{cm} A_{cm}, \quad L_{\mu 2} = \sum_{\sigma=0}^{n_c} \sum_{i=j}^{n_c-1} \sum_{v=0}^{n_p} a_{\sigma} a_{cmv} a_{i+1} C_p A_p^{i-j} B_p,$$

$$N_{\mu 1} = \sum_{\sigma=0}^{n_c} \sum_{i=j}^{n_c-1} \sum_{\eta=v}^{n_p-1} a_{\sigma} a_{cm\eta+1} a_{i+1} C_{pm} A_{pm}^{i-j} B_{pm} C_{cm} A_{cm}^{\eta-v} B_{cm}, \quad N_{\mu 2} = \sum_{\sigma=0}^{n_c} \sum_{i=j}^{n_c-1} \sum_{v=0}^{n_p} a_{\sigma} a_{cmv} a_{i+1} C_{pm} A_{pm}^{i-j} B_{pm} D_{cm},$$

$$\forall \sigma, v, j: \sigma + v + j = \mu,$$

where a_i , a_{mi} , a_{cmi} are the coefficients of the characteristic polynomials of the plant, as well as implicit reference model of plant and controller, correspondingly.

Identity (8) can be rewritten as

$$\frac{M_c(j\omega)M_p(j\omega)}{Q_c(j\omega)Q_p(j\omega)} = \frac{M_o(j\omega)}{Q_o(j\omega)} \equiv \frac{M_{om}(j\omega)}{Q_{om}(j\omega)} = \frac{M_{cm}(j\omega)M_{pm}(j\omega)}{Q_{cm}(j\omega)Q_{pm}(j\omega)} \quad \forall \omega \in (-\infty, +\infty),$$

where $M_o(s)$, $M_c(s)$, $M_p(s)$ are the numerator polynomials of transfer functions of the open-loop system, controller, and plant, $Q_o(s)$, $Q_c(s)$, $Q_p(s)$ are the respective denominator polynomials of these transfer functions. Let us denote

$$P_o(s) = M_o(s)Q_{cm}(s), \quad N_o(s) = M_{om}(s)Q_o(s), \quad F_o(s) = P_o(s) - N_o(s).$$

Then

$$F_o(s) = \sum_{i=0}^{2n_c+n_p-1} (P_i - N_i) s^i.$$

Let us also consider another one tuning functional

$$J_2 = \|\Phi(s) - \Phi_m(s)\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (\Phi(-j\omega) - \Phi_m(-j\omega))(\Phi(j\omega) - \Phi_m(j\omega)) d\omega \quad (16)$$

as a criterion of proximity of the adjusted and reference closed-loop systems.

Having introduced the tuning functionals J_1 and J_2 , let us formulate the following two tuning problems for given plant (1), the controller matrices A_{cm} , C_{cm} , and reference model (5), (6).

Problem 1 (*LQ* Optimal Controller Tuning): Find $G = \begin{bmatrix} B_c^T & D_c \end{bmatrix}^T$ such that

$$J_1 \rightarrow \min_G. \quad (17)$$

Problem 2 (\mathcal{H}_2 Optimal Controller Tuning): Find G such that

$$J_2 \rightarrow \min_G. \quad (18)$$

Before giving solutions to the established problems, we need consider the notice of control system adaptability and properties of the adaptability matrices in some more details.

3. Adaptability of Control System and Properties of Adaptability Matrices

Adaptability is a structural property of a control system. It characterizes the potential ability of the control system to retain its dynamic characteristics when adjusting the parameters of the system toward its given reference model in the situation where the parameter set of the plant scatters around the parameter set of the nominal (reference) operating conditions of the control system (Yadykin, 1999).

Let us consider a control system consisting of plant (1) and controller (2) given stable closed-loop reference model (5), (6). It is assumed that plant (1) is completely controllable and observable, and the state-space realizations $(A_p, B_p, C_p) \in \Sigma$ and $(A_{cm}, B_c, C_{cm}, D_c)$ are minimal. Define the output error of system (1), (2) with respect to reference model (5), (6) as

$$e(t) = y(t) - y_m(t). \quad (19)$$

Definition 1 (Complete Adaptability): Control system (1), (2) is said to be completely adaptable with respect to the output $y(t)$ if for any triple of matrices $(A_p, B_p, C_p) \in \Sigma$ there exists a unique vector $G^* = [B_c^{T*} \ D_c^*]^T$ such that

$$\inf_{(A_p, B_p, C_p) \in \Sigma} \|e(t, g, x_{p0}, x_{c0}, x_{pm0}, x_{cm0}, A_p, B_p, C_p, G^*)\|_2 = 0$$

$$\forall t \in [0, +\infty), g(t) \in \mathcal{L}_2[0, +\infty), x_{p0}, x_{c0}, x_{pm0}, x_{cm0}.$$

Definition 2 (Partial Adaptability): Control system (1), (2) is said to be partially adaptable with respect to the output $y(t)$ if for any triple of matrices $(A_p, B_p, C_p) \in \Sigma$ and any vectors G there exists a unique vector G^* such that

$$\inf_{(A_p, B_p, C_p) \in \Sigma} \|e(t, g, x_{p0}, x_{c0}, x_{pm0}, x_{cm0}, A_p, B_p, C_p, G)\|_2 = \|e(t, g, x_{p0}, x_{c0}, x_{pm0}, x_{cm0}, A_p, B_p, C_p, G^*)\|_2$$

$$\forall t \in [0, +\infty), g(t) \in \mathcal{L}_2[0, +\infty), x_{p0}, x_{c0}, x_{pm0}, x_{cm0}.$$

Definition 3 (Weak Adaptability): Control system (1), (2) is said to be weakly adaptable with respect to the output $y(t)$ if for any triple of matrices $(A_p, B_p, C_p) \in \Sigma$ and any vectors G there exists a set of vectors G^* such that

$$\inf_{(A_p, B_p, C_p) \in \Sigma} \|e(t, g, x_{p0}, x_{c0}, x_{pm0}, x_{cm0}, A_p, B_p, C_p, G)\|_2 = \|e(t, g, x_{p0}, x_{c0}, x_{pm0}, x_{cm0}, A_p, B_p, C_p, G^*)\|_2$$

$$\forall t \in [0, +\infty), g(t) \in \mathcal{L}_2[0, +\infty), x_{p0}, x_{c0}, x_{pm0}, x_{cm0}.$$

Notice that all three kinds of adaptability characterize structural properties of the control system but not of the plant characterized by the invariant properties called controllability, observability, stabilizability, and detectability. Also denote that the adaptability property can be verified experimentally.

The above adaptability definitions can be extended onto linear discrete time invariant systems, dynamic systems with static nonlinearities, bilinear control systems, as well as onto MIMO linear and bilinear control systems (Yadykin, 1981, 1983, 1985, 1999; Morozov & Yadykin, 2004; Yadykin & Tchaikovsky, 2007).

Adaptability matrices (14) possess the following properties (Yadykin, 1999):

1. The adaptability matrix L is the block Toeplitz matrix for MIMO systems. For SISO systems L is the Toeplitz matrix.
2. The adaptability matrix L has maximal column rank if and only if

$$\det(C_p B_p) \neq 0. \quad (20)$$

Condition (20) is the necessary and sufficient condition of partial adaptability of control system (1), (2), as well as the necessary condition of its complete adaptability.

3. Each block N_μ of the block adaptability matrix N equals to (block) scalar product of the (block) row of the matrix L and column vector G where all variables subscripts are added with subscript m in the cases when it is absent, and vice versa.
4. Each block of the matrix L is a linear combination of block products of the plant matrices $C_p A_p^{i-j} B_p$, controller matrices $C_{cm} A_{cm}^{\eta-v}$, B_c , D_c , and products of the coefficients of the characteristic equations of the plant, controller, and their reference models.
5. Upper and lower square blocks of the adaptability matrix L have upper and lower triangle form, respectively.

4. Solutions to LQ and H₂ Tuning Problems

In this section we consider the solutions of LQ and \mathcal{H}_2 optimal tuning problems (17) and (18) for fixed-structure controllers formulated in Section 2 and briefly outline an approach to LQ optimal multiloop PID controller tuning for bilinear MIMO control system.

4.1 LQ Optimal Tuning of Fixed-Structure Controller

Let us determine the gradient of the tuning functional J_1 given by (15) with respect to vector argument using formula

$$\frac{\partial \text{tr}(Ax)}{\partial x} = A^T.$$

Applying this formula to expression (15), we obtain

$$\frac{\partial J_1}{\partial G} = 2 \sum_{\mu=0}^{2n_p+n_c-1} (P_\mu - N_\mu) \frac{\partial P_\mu}{\partial G}, \quad \frac{\partial P_\mu}{\partial G} = L_\mu^T, \quad \frac{\partial L_\mu}{\partial G} = L_\mu^T, \quad \mu = \overline{0, 2n_p + n_c - 1}.$$

Thus, the necessary minimum condition for the tuning functional J_1 is

$$\frac{\partial J_1}{\partial G} = L^T(LG - N) = 0. \quad (21)$$

In paper (Yadykin, 2008) it has been shown that necessary minimum condition (21) holds true in the following two cases:

1. If $LG - N = 0$ then system (1), (2) is completely adaptable.
2. If $LG - N \neq 0$ but $L^T(LG - N) = 0$ then system (1), (2) is partially or weakly adaptable.

In the first case (complete adaptability), the equation

$$LG - N = 0 \quad (22)$$

has a unique exact solution. In this case, necessary minimum condition (21) is also sufficient. In the second case (partial or weak adaptability), equation (22) does not have an exact solution, but the equation

$$L^T(LG - N) = 0 \quad (23)$$

has a unique approximate solution or a set of approximate solutions. Thus, if the matrix L has maximal column rank, then the vector (matrix)

$$G^* = (L^T L)^{-1} L^T N = L^+ N \quad (24)$$

is the solution to equation (23). In expression (24), L^+ denotes Moore-Penrose generalized inverse of the matrix L (Bernstein, 2005).

The following Theorem establishing the necessary and sufficient conditions of complete and partial adaptability of system (1), (2) follows from the theory of matrix algebraic equations (Gantmacher, 1959).

Theorem 1: Let plant (1) be completely controllable and observable, and the state-space realizations (A_p, B_p, C_p) and $(A_{cm}, B_c, C_{cm}, D_c)$ be minimal. Control system (1), (2) is completely adaptable with respect to the output $y(t)$ if and only if

$$\text{Im } N \subseteq \text{Im } L, \quad (25)$$

$$\text{Ker } L = 0, \quad (26)$$

where Im denotes the matrix image and Ker denotes the matrix kernel. Control system (1), (2) is partially adaptable with respect to the output $y(t)$ if and only if condition (26) holds.

To illustrate LQ optimal tuning algorithm (24), let us consider a simple example.

Example 1: Let control system (1), (2) consists of a linear oscillator and PI (Proportional-Integrating) controller in forward loop closed by the negative unitary feedback. The state-space realizations of the plant and controller are given by

$$\left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & 0 \end{array} \right] = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -1 & -1/(2\zeta_p T_p) & b \\ \hline 1 & 0 & 0 \end{array} \right], \quad \left[\begin{array}{c|c} A_{cm} & B_c \\ \hline C_{cm} & D_c \end{array} \right] = \left[\begin{array}{c|c} 0 & k_p k_I \\ \hline 1 & k_p \end{array} \right].$$

We suppose that $\Sigma = \{b : \underline{b} \leq b \leq \bar{b}, b \neq 0\}$. The transfer functions of the plant and controller, as well as the reference plant and controller are as follows:

$$P(s) = \frac{b}{T_p^2 s^2 + 2T_p \zeta_p s + 1}, \quad K(s) = k_p + \frac{k_p k_I}{s} = k_p k_{Im} \frac{k_{Im}^{-1} s + 1}{s},$$

$$P_m(s) = \frac{b_m}{T_{pm}^2 s^2 + 2T_{pm} \zeta_{pm} s + 1}, \quad K_m(s) = k_{pm} k_{Im} \frac{k_{Im}^{-1} s + 1}{s}.$$

Substituting these expressions into identity (8) and eliminating equal factors, we obtain

$$b k_p = b_m k_{pm}$$

from which it follows that LQ optimal tuning of the controller parameters is given by

$$k_p^* = b^{-1} b_m k_{pm}. \quad (27)$$

Thus, for any values of the plant coefficient b from the admissible set Σ tuning algorithm (27) provides identical coincidence of the transfer functions of the open-loop adjusted system and its reference model. This means that the considered system is completely adaptable with respect to the output in terms of Definition 1 in the class of the linear oscillators with a single variable parameter (coefficient b).

Let us now assume that the plant is characterized by three variable parameters:

$$\Sigma = \{b, T_p, \zeta_p : \underline{b} \leq b \leq \bar{b}, \underline{T}_p \leq T_p \leq \bar{T}_p, \underline{\zeta}_p \leq \zeta_p \leq \bar{\zeta}_p, b \neq 0\}.$$

We are interested in tuning of two parameters of PI controller, k_p and k_I , or, equivalently, the scalars B_c and D_c . Applying formulas (15), one can easily obtain the following expressions for the adaptability matrices:

$$L = \begin{bmatrix} b & 0 \\ 2bT_{pm}\zeta_{pm} & b \\ bT_p^2 & 2bT_{pm}\zeta_{pm} \\ 0 & bT_p^2 \end{bmatrix}, \quad N = \begin{bmatrix} b_m B_{cm} \\ 2b_m B_{cm} \zeta_p T_p + b_m D_{cm} \\ b_m B_{cm} T_p^2 + 2b_m D_{cm} \zeta_p T_p \\ b_m D_{cm} T_p^3 \end{bmatrix},$$

where $B_{cm} = k_{pm} k_{Im}$, $D_{cm} = k_{pm}$. Denote that the elements of the matrix L are periodic:

$$l_{11} = l_{22}, \quad l_{21} = l_{32}, \quad l_{31} = l_{42}, \quad l_{41} = l_{12}.$$

According to LQ tuning algorithm (24), the optimal controller parameters are defined as

$$\begin{bmatrix} B_c^* \\ D_c^* \end{bmatrix} = \frac{b_m}{b} \begin{bmatrix} 1 + 4T_{pm}^2 \zeta_{pm}^2 + T_p^4 & 2T_{pm} \zeta_{pm} (1 + T_p^2) \\ 2T_{pm} \zeta_{pm} (1 + T_p^2) & 1 + 4T_{pm}^2 \zeta_{pm}^2 + T_p^4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 2T_{pm} \zeta_{pm} & 1 \\ T_p^2 & 2T_{pm} \zeta_{pm} \\ 0 & T_p^2 \end{bmatrix}^T \begin{bmatrix} B_{cm} \\ 2B_{cm} \zeta_p T_p + D_{cm} \\ B_{cm} T_p^2 + 2D_{cm} \zeta_p T_p \\ D_{cm} T_p^3 \end{bmatrix}.$$

4.2 LQ Optimal PID Controller Tuning for Bilinear MIMO System

Let us outline an approach to extension of LQ optimal fixed-structure (PID) controller tuning algorithm presented in Subsection 4.1 onto the class of bilinear continuous time invariant MIMO systems with piecewise constant input signals. This approach can be found in more details in papers (Morozov & Yadykin, 2004; Yadykin & Tchaikovsky, 2007).

Let us consider the bilinear continuous time-invariant plant described by the equations

$$\left. \begin{aligned} \dot{x}(t) &= A_p x(t) + B_p u(t) + \sum_{i=1}^r N_{pi} x(t) u_i(t), \\ y(t) &= C_p x(t), \end{aligned} \right\} \quad (28)$$

where $x_p(t) \in \mathbb{R}^n$ is the plant state, $u(t) = [u_1(t) \ \cdots \ u_r(t)]^T \in \mathbb{R}^r$ is the control, $y(t) \in \mathbb{R}^r$ is the plant output, and the matrices A_p , B_p , C_p , N_{pi} , $i = \overline{1, r}$, have compatible dimensions. Also consider the fixed-structure controller, namely, multiloop PID controller for plant (28) with transfer matrix

$$K(s) = \text{diag}\{K_1(s), \dots, K_r(s)\}, \quad (29)$$

where

$$K_i(s) = k_i \left(1 + \frac{1}{TS_i s} + TD_i s \right) \frac{1}{TL_i s + 1}.$$

The state-space equations for PID controller (29) are given by (2) with

$$\begin{aligned} A_c &= \text{diag}\{A_{c1}, \dots, A_{cr}\}, \quad A_{ci} = \begin{bmatrix} -k_{1i} & 0 \\ 0 & 0 \end{bmatrix}, \quad B_c = \text{diag}\left\{ \begin{bmatrix} k_{21} \\ k_1 \end{bmatrix}, \dots, \begin{bmatrix} k_{2r} \\ k_r \end{bmatrix} \right\}, \\ C_c &= \text{diag}\{[1 \ 1], \dots, [1 \ 1]\}, \quad D_c = \text{diag}\{k_{31}, \dots, k_{3r}\}, \\ k_{1i} &= (TL_i)^{-1}, \quad k_{3i} = k_i TD_i / L_i, \quad k_{2i} = k_i / TL_i - (k_i / TS_i + k_i TD_i / TL_i^2). \end{aligned}$$

The reference plant model is given by

$$\left. \begin{aligned} \dot{x}_m(t) &= A_{pm} x_m(t) + B_{pm} u_m(t) + \sum_{i=1}^r N_{pmi} x_m(t) u_{mi}(t), \\ y_m(t) &= C_{pm} x_m(t), \end{aligned} \right\} \quad (30)$$

where all vectors and matrices have the same dimensions as their counterparts in actual plant (28). The reference controller has the same structure as controller (29):

$$K_m(s) = \text{diag}\{K_{m1}(s), \dots, K_{mr}(s)\}, \quad (31)$$

where

$$K_{mi}(s) = k_{mi} \left(1 + \frac{1}{T_m S_{mi} s} + T_m D_{mi} s \right) \frac{1}{T_m L_{mi} s + 1},$$

and its state-space equations are given by (6) with corresponding structure of the realization matrices.

We are interested in tuning the parameters k_i , TD_i , TS_i , TL_i , $i = \overline{1, r}$, of controller (29) such that to ensure the identity

$$y(t) \equiv y_m(t)$$

in steady-state mode provided that the parameters of plant (28) and control signal vary as step functions of time within some bounded regions Σ , Ω .

The main idea of applying approach described in Subsection 4.1 for solving this problem consists in linearization of bilinear plant (28) and reference plant (30) with respect to the deviations from the steady-state values. In this case we obtain the linearized model of the actual plant

$$\begin{bmatrix} \frac{\Delta \dot{x}_p(t)}{\Delta y(t)} \end{bmatrix} = \begin{bmatrix} \bar{A}_p & \bar{B}_p \\ \bar{C}_p & 0 \end{bmatrix} \begin{bmatrix} \frac{\Delta x_p(t)}{\Delta u(t)} \end{bmatrix}, \quad (32)$$

$$\bar{A}_p = A_p + \sum_{i=1}^r N_{pi}(u_i^0 + \Delta u_i^0), \quad \bar{B}_p = B_p, \quad \bar{C}_p = C_p,$$

and the reference plant

$$\begin{bmatrix} \frac{\Delta \dot{x}_{pm}(t)}{\Delta y_m(t)} \end{bmatrix} = \begin{bmatrix} \bar{A}_{pm} & \bar{B}_{pm} \\ \bar{C}_{pm} & 0 \end{bmatrix} \begin{bmatrix} \frac{\Delta x_{pm}(t)}{\Delta u_m(t)} \end{bmatrix}, \quad (33)$$

$$\bar{A}_{pm} = A_{pm} + \sum_{i=1}^r N_{pmi}(u_i^0 + \Delta u_i^0), \quad \bar{B}_{pm} = B_{pm}, \quad \bar{C}_{pm} = C_{pm}.$$

Then, the problem of PID controller tuning for bilinear plant (28) reduces to Problem 1, and we can apply LQ optimal controller tuning algorithm described in Subsection 4.1 to solve it.

4.3 H_2 Optimal Tuning of Fixed-Structure Controller

To evaluate the squared \mathcal{H}_2 norm of difference between the transfer functions of the adjusted and reference closed-loop systems, we need the following result.

Lemma 1: Let $W(s) = (A, B, C)$ be the strictly proper transfer function of a stable dynamic system of order n without multiple poles. Let (A, B, C) -realization of the transfer function $W(s)$ be the minimal realization. Then the following relations hold

$$\|W(s)\|_2^2 = \sum_{i=1}^n W^+(s_{i-}) \Re s W^-(s_{i-}) = \sum_{i=1}^n \frac{M^+(s)M^-(s)}{Q^+(s) \frac{d}{ds} Q^-(s)} \Big|_{s_i=s_{i-}}, \quad (34)$$

$$\|W(s)\|_2^2 = \sum_{i=0}^n \frac{\sum_{\lambda=0}^n \left\{ s_{i-}^\lambda \sum_{j=\lambda+1}^n a_j C A^{j-\lambda-1} B \right\} \sum_{\lambda=0}^n \left\{ (-1)^\lambda s_{i-}^\lambda \sum_{j=\lambda+1}^n a_j C A^{j-\lambda-1} B \right\}}{\sum_{j=0}^n \left\{ a_j s_{i-}^j \sum_{j=0}^n \left\{ j a_j s_{i-}^j \sum_{j=0}^n (-1)^j a_j s_{i-}^j \right\} \right\}}, \quad (35)$$

where s_{i+} are the poles of the main system, s_{i-} are the poles of the adjoint system, that is, $s_{i+} = (-1) \cdot s_{i-}$, a_j are the coefficients of the characteristic polynomial of the matrix A ,

$$\begin{aligned} W^+(s) &= \frac{M^+(s)}{Q^+(s)}, & W^-(s) &= \frac{M^-(s)}{Q^-(s)}, \\ M^+(s) &= M(s), \quad Q^+(s) = Q(s), & M^-(s) &= M(s)|_{s=-s}, \quad Q^-(s) = Q(s)|_{s=-s}, \\ Q^+(s_{i+}) &= 0, & Q^-(s_{i-}) &= 0. \end{aligned}$$

Proof: When the Lemma 1 assumptions hold true, we have for the main and adjoint systems

$$W^+(s) = C(sI - A)^{-1}B = \frac{M^+(s)}{Q^+(s)}, \quad W^-(s) = C(-sI - A)^{-1}B = \frac{M^-(s)}{Q^-(s)}. \quad (36)$$

As is well known, the resolvent of the matrix A has the following series expansion (Strejc, 1981):

$$(sI - A)^{-1} = \frac{1}{\sum_{i=0}^n a_i s^i} \sum_{j=0}^{n-1} s^j \sum_{i=j+1}^n a_i A^{i-j-1}. \quad (37)$$

Substitution of (37) into (36) gives

$$M^+(s) = \sum_{j=0}^{n-1} s^j \sum_{i=j+1}^n a_i C A^{i-j-1} B, \quad Q^+(s) = \sum_{i=0}^n a_i s^i, \quad (38)$$

$$M^-(s) = \sum_{j=0}^{n-1} (-1)^j s^j \sum_{i=j+1}^n a_i C A^{i-j-1} B, \quad Q^-(s) = \sum_{i=0}^n (-1)^i a_i s^i. \quad (39)$$

By definition of \mathcal{H}_2 norm,

$$\|W(s)\|_2^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} W(-j\omega)W(j\omega)d\omega.$$

Since by assumption the integration element in the last integral is strictly proper rational function, let us apply the Theorem of Residues forming closed contour \mathfrak{C} consisting of the imaginary axis and semicircle with infinitely big radius and center at the origin at the right half of the complex plain. Inside of this contour, there are only isolated singularities defined by the roots of the characteristic equation $Q^-(s) = 0$ of the adjoint system. It follows that

$$\begin{aligned} \frac{1}{2\pi} \int_{-\infty}^{+\infty} W(-j\omega)W(j\omega)d\omega &= \sum_{i=1}^n \frac{M^+(s)M^-(s)}{Q^-(s) \frac{d}{ds} Q^+(s) + Q^+(s) \frac{d}{ds} Q^-(s)} \Big|_{s_i=s_{i-}} \\ &= \sum_{i=1}^n \frac{M^+(s)M^-(s)}{Q^+(s) \frac{d}{ds} Q^-(s)} \Big|_{s_i=s_{i-}} = \sum_{i=1}^n W^+(s_{i-}) \mathfrak{Res} W^-(s_{i-}). \end{aligned}$$

Applying (38), (39), we obtain expression (35).

Correctness of the following equalities in notation of Section 2 can be proved by direct substitution:

$$W(s) - W_m(s) = \frac{M_o(s)Q_{om}(s) - M_{om}(s)Q_o(s)}{Q_o(s)Q_{om}(s)} = \frac{F_o(s)}{Q_o(s)Q_{om}(s)}, \quad (40)$$

$$\Phi(s) - \Phi_m(s) = \frac{F_o(s)}{(Q_o(s) + M_o(s))(Q_{om}(s) + M_{om}(s))}. \quad (41)$$

It is obvious that if the adjusted system is completely adaptable then $F_o(s) \equiv 0$ and

$$\text{Arg min}_G J_1 = \text{Arg min}_G J_2.$$

The following Theorem answer the question: Whether this equality retains when the system is not completely adaptable?

Theorem 2: Let plant (1) be completely controllable and observable, the transfer functions $P(s) = (A_p, B_p, C_p)$ and $K(s) = (A_c, B_c, C_c, D_c)$ be strictly proper rational functions with no multiple and right poles. Then the following statements hold true:

1. The necessary minimum conditions for functionals J_1 and J_2 coincides and are given by either

$$LG - N = 0 \quad (42)$$

or $LG - N \neq 0$, but

$$L^T(LG - N) = 0. \quad (43)$$

2. If equation (42) has a unique solution, then the necessary minimum condition is also sufficient.
3. The optimal controller tuning algorithms for functionals J_1 and J_2 coincide and are given by

$$G^* = L^+ N. \quad (44)$$

Proof: Applying Lemma 1 and equality (41), we obtain

$$J_2 = \sum_{i=1}^{n_p+n_c} \left. \frac{F_o^+(s)}{R_o^+(s)R_{om}^+(s)} \frac{F_o^-(s)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} \right|_{s=s_{i-}^c} + \sum_{i=1}^{n_p+n_c} \left. \frac{F_o^+(s)}{R_o^+(s)R_{om}^+(s)} \frac{F_o^-(s)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} \right|_{s=s_{mi-}^c}, \quad (45)$$

where $R_o(s) = Q_o(s) + M_o(s)$ and $R_{om}(s) = Q_{om}(s) + M_{om}(s)$ are the characteristic polynomials of closed-loop system and its implicit reference model (superscripts “+” and “-” are used for the main and adjoint systems, respectively), s_{i-}^c and s_{mi-}^c are the poles of the adjoint system and its reference model. Denoting

$$S^+(s) = \begin{bmatrix} 1 & s & s^2 & \dots & s^{2n_c+n_p-1} \end{bmatrix}, \quad S^-(s) = \begin{bmatrix} 1 & -s & s^2 & \dots & (-1)^{2n_c+n_p-1} s^{2n_c+n_p-1} \end{bmatrix},$$

one can put down

$$\frac{\partial}{\partial G}(W(s) - W_m(s)) = \frac{1}{N_o(s)N_{om}(s)} \frac{\partial}{\partial G} F_o(s) = \frac{1}{N_o(s)N_{om}(s)} \frac{\partial \text{tr}\{S(s)(LG - N)\}}{\partial G} = \frac{L^T S^T(s)}{N_o(s)N_{om}(s)}. \quad (46)$$

Applying expressions (40), (45), and (46) to the transfer functions and characteristic polynomials of the main and adjoint systems, we have

$$\frac{\partial J_2}{\partial G} = \left(\frac{\partial J_2}{\partial G} \right)_I + \left(\frac{\partial J_2}{\partial G} \right)_{II}, \quad (47)$$

where

$$\begin{aligned} \left(\frac{\partial J_2}{\partial G} \right)_I = & \sum_{i=1}^{n_p+n_c} \left\{ \frac{\frac{\partial}{\partial G} F_o^+(s)}{R_o^+(s)R_{om}^+(s)} \frac{F_o^-(s)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} + \frac{F_o^+(s)}{R_o^+(s)R_{om}^+(s)} \frac{\frac{\partial}{\partial G} F_o^-(s)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} \right\} \Big|_{s=s_{i-}^c} \\ & + \sum_{i=1}^{n_p+n_c} \left\{ \frac{\frac{\partial}{\partial G} F_o^+(s)}{R_o^+(s)R_{om}^+(s)} \frac{F_o^-(s)}{R_o^-(s) \frac{d}{ds} R_{om}^-(s)} + \frac{F_o^+(s)}{R_o^+(s)R_{om}^+(s)} \frac{\frac{\partial}{\partial G} F_o^-(s)}{R_o^-(s) \frac{d}{ds} R_{om}^-(s)} \right\} \Big|_{s=s_{mi-}^c}, \end{aligned} \quad (48)$$

$$\begin{aligned} \left(\frac{\partial J_2}{\partial G} \right)_{II} = & \sum_{i=1}^{n_p+n_c} \left\{ \frac{F_o^+(s)F_o^-(s)}{(R_o^+(s))^2 R_{om}^+(s)} \frac{\frac{\partial}{\partial G} R_o^+(s)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} - \frac{F_o^+(s)F_o^-(s)}{R_o^+(s)R_{om}^+(s)} \frac{\frac{\partial}{\partial G} \frac{d}{ds} R_o^-(s)}{R_{om}^-(s) \left(\frac{d}{ds} R_o^-(s) \right)^2} \right\} \Big|_{s=s_{i-}^c} \\ & - \sum_{i=1}^{n_p+n_c} \left\{ \frac{\frac{\partial}{\partial G} R_o^+(s)}{(R_o^+(s))^2 R_{om}^+(s)} \frac{F_o^+(s)F_o^-(s)}{R_o^-(s) \frac{d}{ds} R_{om}^-(s)} - \frac{F_o^+(s)F_o^-(s)}{R_o^+(s)R_{om}^+(s)} \frac{\frac{\partial}{\partial G} \frac{d}{ds} R_o^-(s)}{R_o^-(s) \left(\frac{d}{ds} R_{om}^-(s) \right)^2} \right\} \Big|_{s=s_{mi-}^c}. \end{aligned} \quad (49)$$

With (45) and (46) in mind, denoting

$$H(s) = \text{diag} \{ (-1)^{j-1} s^{2(j-1)} \},$$

let us transform expressions (48), (49) into

$$\begin{aligned} \left(\frac{\partial J_2}{\partial G} \right)_I = & \sum_{i=1}^{n_p+n_c} \left\{ \frac{1}{R_o^+(s)R_{om}^+(s)} \frac{H(s)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} + \frac{1}{R_o^+(s)R_{om}^+(s)} \frac{H(s)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} \right\} \Big|_{s=s_{i-}^c} \\ & + \sum_{i=1}^{n_p+n_c} \left\{ \frac{1}{R_o^+(s)R_{om}^+(s)} \frac{H(s)}{R_o^-(s) \frac{d}{ds} R_{om}^-(s)} + \frac{1}{R_o^+(s)R_{om}^+(s)} \frac{H(s)}{R_o^-(s) \frac{d}{ds} R_{om}^-(s)} \right\} \Big|_{s=s_{mi-}^c} \cdot 2L^T(LG - N), \end{aligned} \quad (50)$$

$$\begin{aligned} \left(\frac{\partial J_2}{\partial G} \right)_{II} = & \sum_{i=1}^{n_p+n_c} \left\{ \frac{(LG - N)^T}{(R_o^+(s))^2 R_{om}^+(s)} \frac{H(s) \frac{\partial}{\partial G} R_o^+(s) (LG - N)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} \right\} \Big|_{s=s_{i-}^c} \\ & - \sum_{i=1}^{n_p+n_c} \left\{ \frac{(LG - N)^T}{R_o^+(s)R_{om}^+(s)} \frac{H(s) \frac{\partial}{\partial G} \frac{d}{ds} R_o^-(s) (LG - N)}{R_{om}^-(s) \left(\frac{d}{ds} R_o^-(s) \right)^2} \right\} \Big|_{s=s_{i-}^c} \\ & - \sum_{i=1}^{n_p+n_c} \left\{ \frac{(LG - N)^T}{(R_o^+(s))^2 R_{om}^+(s)} \frac{H(s) \frac{\partial}{\partial G} R_o^+(s) (LG - N)}{R_o^-(s) \frac{d}{ds} R_{om}^-(s)} \right\} \Big|_{s=s_{mi-}^c} \\ & - \sum_{i=1}^{n_p+n_c} \left\{ \frac{(LG - N)^T}{R_o^+(s)R_{om}^+(s)} \frac{H(s) \frac{\partial}{\partial G} \frac{d}{ds} R_o^-(s) (LG - N)}{R_o^-(s) \left(\frac{d}{ds} R_{om}^-(s) \right)^2} \right\} \Big|_{s=s_{mi-}^c}. \end{aligned} \quad (51)$$

For the numerator polynomial of the open-loop system we have

$$M_o(s) = \frac{LG}{\sum_{i=0}^{n_c} a_{mi} s^i}.$$

Differentiating the last expression, we obtain

$$\frac{\partial}{\partial G} M_o^+(s) = S_1^+(s) L^T, \quad \frac{\partial}{\partial G} M_o^-(s) = S_1^-(s) L^T, \quad \frac{\partial}{\partial G} \frac{d}{ds} M_o^+(s) = T^+(s) L^T, \quad \frac{\partial}{\partial G} \frac{d}{ds} M_o^-(s) = T^-(s) L^T,$$

where

$$\begin{aligned} S_1^+(s) &= \text{diag} \left\{ \frac{s^{j-1}}{\sum_{i=0}^{n_c} a_{mi} s^i} \right\}, \quad S_1^-(s) = \text{diag} \left\{ \frac{(-1)^{j-1} s^{j-1}}{\sum_{i=0}^{n_c} a_{mi} s^i} \right\}, \\ T^+(s) &= \frac{d}{ds} S_1^+(s) = \text{diag} \left\{ (j-1)s^{j-2} - \frac{s^{j-1} \sum_{i=0}^{n_c} i a_{mi} s^{i-1}}{\left\{ \sum_{i=0}^{n_c} a_{mi} s^i \right\}^2} \right\}, \\ T^-(s) &= \frac{d}{ds} S_1^-(s) = \text{diag} \left\{ (-1)^{j-1} (j-1)s^{j-2} - \frac{(-1)^{j-1} s^{j-1} \sum_{i=0}^{n_c} i a_{mi} s^{i-1}}{\left\{ \sum_{i=0}^{n_c} a_{mi} s^i \right\}^2} \right\}. \end{aligned}$$

Using these formulas, it is not hard to obtain

$$\begin{aligned} \left(\frac{\partial J_2}{\partial G} \right)_{II} &= \sum_{i=1}^{n_p+n_c} \left\{ \frac{(LG-N)^T}{(R_o^+(s))^2 R_{om}^+(s)} \frac{H(s)S(s)L^T(LG-N)}{R_{om}^-(s) \frac{d}{ds} R_o^-(s)} \right\} \Big|_{s=s_{i-}^c} \\ &\quad - \sum_{i=1}^{n_p+n_c} \left\{ \frac{(LG-N)^T}{R_o^+(s) R_{om}^+(s)} \frac{H(s)T^-(s)L^T(LG-N)}{R_{om}^-(s) \left(\frac{d}{ds} R_o^-(s) \right)^2} \right\} \Big|_{s=s_{i-}^c} \\ &\quad - \sum_{i=1}^{n_p+n_c} \left\{ \frac{(LG-N)^T}{(R_o^+(s))^2 R_{om}^+(s)} \frac{H(s)S(s)L^T(LG-N)}{R_o^-(s) \frac{d}{ds} R_{om}^-(s)} \right\} \Big|_{s=s_{mi-}^c} \\ &\quad - \sum_{i=1}^{n_p+n_c} \left\{ \frac{(LG-N)^T}{R_o^+(s) R_{om}^+(s)} \frac{H(s)S_1^-(s)L^T(LG-N)}{R_o^-(s) \left(\frac{d}{ds} R_{om}^-(s) \right)^2} \right\} \Big|_{s=s_{mi-}^c}. \end{aligned} \tag{52}$$

From (50) and (52) it follows that all terms of sum (47) are the products of the complex matrices being the values of the complex-valued diagonal matrices with compatible dimensions in the poles of the adjoint closed-loop system and its reference model and the matrix factors of the form $L^T(LG-N)$ and $(LG-N)^T$. Since the complex-valued matrix factors cannot be identically zero on the set Σ , the necessary conditions for minimum of the functional J_2 are given by (42) or (43) and coincide with the necessary minimum conditions for the functional J_1 . Thus, the first statement of the Theorem is proved.

Let equation (43) have a unique solution for any given point of the plant parameter set Σ . Then this solution is given by (44) and determines one of the local minimums of the functionals J_1 and J_2 . The analytic expressions for the functionals J_1 and J_2 include as factors the polynomials $F_o^+(s)$ and $F_o^-(s)$ that equal to zero according to (7). Since equality (42) holds true, conditions (21) hold and, consequently, the mentioned minimums must be global and coinciding. This proves the second and third statements of the Theorem.

The tuning procedure determined by (44) gives the solution to unconstrained minimization problem for the criteria J_1 and J_2 . But it does not guarantee stability of the adjusted system for the whole set Σ .

The main drawback of this tuning algorithm consists in that the direct control of stability margin of the adjusted system is impossible. This drawback can be partially weakened by evaluating the characteristic polynomial of the closed-loop system or its roots. Let us consider another approach to managing the mentioned drawback.

5. H_2 Tuning of Fixed-Structure Controller with H_∞ Constraints

The most well-known and, perhaps, the most efficient approach to solving this problem is the direct minimization of \mathcal{H}_∞ norm of transfer function of the adjusted system on the base of loop-shaping (McFarlane & Glover, 1992; Tan et al., 2002). The main advantages of this approach consist in the direct solution to the controller tuning problem via synthesis, simplicity of the design procedure subject to internally contradictory criteria of stability and performance, as well as good interpretation of engineering design methods.

Drawbacks consist in need for design of pre- and post-filters complicating the controller structure, as well as in optimization result dependence on chosen initial approach. Bounded Real Lemma allows expressing boundedness condition for \mathcal{H}_∞ norm of transfer function of the adjusted system in terms of linear matrix inequality for rather common assumptions on the control system properties (Scherer, 1990). Consider application of Bounded Real Lemma to forming linear constraint for the constrained optimization problem.

The feature of mixed tuning problem statement is that the linear constraints guarantee some stability margin, but not performance, since it is assumed that performance can be provided by proper choice of matrices of the implicit reference model, and then performance can only be maintained by means of adaptive controller tuning.

The problem statement is as follows. Let us consider the closed-loop system consisting of plant (1) and fixed-structure controller (2)

$$\Phi(s): \begin{bmatrix} \dot{x}_{cl}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & 0 \end{bmatrix} \begin{bmatrix} x_{cl}(t) \\ g(t) \end{bmatrix} \quad (53)$$

with

$$\begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & 0 \end{bmatrix} = \begin{bmatrix} A_p - B_p D_c C_p & B_p C_{cm} & B_p D_c \\ -B_c C_p & A_{cm} & B_c \\ C_p & 0 & 0 \end{bmatrix},$$

and the closed-loop reference model

$$\Phi_m(s): \begin{bmatrix} \dot{x}_{clm}(t) \\ y_m(t) \end{bmatrix} = \begin{bmatrix} A_{clm} & B_{clm} \\ C_{clm} & 0 \end{bmatrix} \begin{bmatrix} x_{clm}(t) \\ g(t) \end{bmatrix}, \quad (54)$$

$$\|\Phi_m(s)\|_\infty < \gamma_m. \quad (55)$$

We are interested in finding the controller parameters B_c and D_c such that

$$J_2 = \|\Phi(s) - \Phi_m(s)\|_2 \rightarrow \min_{B_c, D_c}, \quad (56)$$

$$\|\Phi(s)\|_\infty < \gamma \quad (57)$$

$\forall A_p, B_p, C_p \in \Sigma$ and the matrix A_{cl} be Hurwitz.

By virtue of Theorem 2, the necessary condition for minimum of functional (56) is

$$L^T L \begin{bmatrix} B_c^T & D_c \end{bmatrix}^T - L^T N = 0 \quad (58)$$

$\forall A_p, B_p, C_p \in \Sigma$. According to Bounded Real Lemma (Scherer, 1990), condition (57) holds true if and only if there exists a solution $X = X^T > 0$ to matrix inequality

$$\begin{bmatrix} XA_{cl} + A_{cl}^T X & XB_{cl} & C_{cl}^T \\ B_{cl}^T X & -\gamma I & 0 \\ C_{cl} & 0 & -\gamma I \end{bmatrix} < 0. \quad (59)$$

Matrix inequality (59) is not linear and jointly convex in variables X , B_c , and D_c . In order to pass from inequality (59) to LMI constraints, let us use a technique similar to (Gahinet & Apkarian, 1994; Balandin & Kogan, 2007). Define the matrix of the controller parameters

$$\Theta = \begin{bmatrix} A_{cm} & B_c \\ C_{cm} & D_c \end{bmatrix}$$

and represent the closed-loop system matrices as

$$A_{cl} = A_0 + B\Theta C, \quad B_{cl} = B_0 + B\Theta D_1, \quad C_{cl} = C_0 + D_2\Theta C,$$

$$A_0 = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix}, \quad B_0 = 0, \quad C_0 = \begin{bmatrix} C_p & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & B_p \\ I & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & I \\ -C_p & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad D_2 = 0.$$

Substitute these expressions into (59) and represent the resulting inequality as linear matrix inequality with respect to Θ :

$$\Psi + P^T \Theta^T Q + Q^T \Theta P < 0, \quad (60)$$

$$\Psi = \begin{bmatrix} A_0^T X + XA_0 & 0 & C_0^T \\ 0 & -\gamma I & 0 \\ C_0 & 0 & -\gamma I \end{bmatrix}, \quad P = \begin{bmatrix} C & D_1 & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} B^T X & 0 & 0 \end{bmatrix}.$$

According to Projection Lemma (Gahinet & Apkarian, 1994), inequality (60) is solvable with respect to the matrix Θ if and only if

$$W_P^T \begin{bmatrix} A_0^T X + X A_0 & 0 & C_0^T \\ 0 & -\gamma I & 0 \\ C_0 & 0 & -\gamma I \end{bmatrix} W_P < 0, \quad W_Q^T \begin{bmatrix} A_0^T X + X A_0 & 0 & C_0^T \\ 0 & -\gamma I & 0 \\ C_0 & 0 & -\gamma I \end{bmatrix} W_Q < 0, \quad (61)$$

where the columns of the matrices W_P and W_Q form the respective bases of $\text{Ker } P$ and $\text{Ker } Q$. To eliminate the unknown matrix X from the matrix Q , let us represent

$$Q = R \begin{bmatrix} X & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad R = \begin{bmatrix} B^T & 0 & 0 \end{bmatrix},$$

from which it follows that

$$W_Q = \begin{bmatrix} X^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} W_R.$$

Substituting this expression into (61) and denoting $Y = X^{-1}$, we obtain the following result.

Theorem 3: Given $\gamma > 0$, fixed-structure controller (2) providing minimum for the tuning functional J_2 and ensuring condition (57) exists if and only if there exist the inverse matrices $X = X^T > 0$ and $Y = Y^T > 0$ such that

$$W_P^T \begin{bmatrix} A_0^T X + X A_0 & 0 & C_0^T \\ 0 & -\gamma I & 0 \\ C_0 & 0 & -\gamma I \end{bmatrix} W_P < 0, \quad W_R^T \begin{bmatrix} A_0 Y + Y A_0^T & 0 & Y C_0^T \\ 0 & -\gamma I & 0 \\ C_0 Y & 0 & -\gamma I \end{bmatrix} W_R < 0, \quad XY = I. \quad (62)$$

If conditions (62) hold true, and the matrices X and Y are found, the controller parameters B_c and D_c are defined from solution of linear matrix inequality (60) subject to equality constraint (58).

Denote that further simplification of (62) via respective choice of the matrices W_P and W_R is possible (see, e.g., Gahinet & Apkarian, 1994), but this is not required by the numerical algorithm for solving linear matrix inequalities with respect to inverse matrices presented in (Balandin & Kogan, 2005).

Taking into account the block structure of the controller matrix Θ that includes constant and variable blocks, let us consider some aspects of solving inequality (60). Let the matrix X satisfying (62) be found. Partition it into the blocks

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}$$

in accordance with the orders of plant and controller. Then

$$\Psi = \begin{bmatrix} A_p^T X_{11} + X_{11} A_p & A_p^T X_{12} & 0 & C_p^T \\ X_{12}^T A_p & 0 & 0 & 0 \\ 0 & 0 & -\gamma I & 0 \\ C_p & 0 & 0 & -\gamma I \end{bmatrix}, \quad (63)$$

$$P = \begin{bmatrix} 0 & I & 0 & 0 \\ -C_p & 0 & I & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} X_{12}^T & X_{22} & 0 & 0 \\ B_p^T X_{11} & B_p^T X_{12} & 0 & 0 \end{bmatrix},$$

$$Q^T \Theta P = \begin{bmatrix} -(X_{12} B_c + X_{11} B_p D_c) C_p & X_{12} A_{cm} + X_{11} B_p C_{cm} & X_{12} B_c + X_{11} B_p D_c & 0 \\ -(X_{22} B_c + X_{12}^T B_p D_c) C_p & X_{22} A_{cm} + X_{12}^T B_p C_{cm} & X_{22} B_c + X_{12}^T B_p D_c & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (64)$$

Substituting (63) and (64) into (60), one can obtain linear matrix inequality with respect to the unknown controller parameters B_c and D_c .

Thus, the procedure of \mathcal{H}_2 optimal controller tuning with \mathcal{H}_∞ constraints consists of two stages. At the first stage, one need find two inverse positive-definite matrices X and Y satisfying (62) with $\gamma = \gamma_m$. At the second stage, when the matrices X and Y are obtained, the controller parameters B_c and D_c can be found from linear matrix inequality (60), (63), (64) subject to equality constraint (58). Numerical solution to linear matrix inequality subject to linear equality constraints can be obtained using Matlab software toolbox SeDuMi Interface (Peaucelle, 2002).

For the purpose of numerical illustration, let us give a simple numerical example.

Example 2: Consider the problem of a first-order controller tuning for a second-order unstable linear oscillator. The reference model is given by (5), (6) with

$$\left[\begin{array}{c|c|c} A_{pm} & B_{pm} & 1 \\ \hline C_{pm} & 0 & 0 \end{array} \right] = \left[\begin{array}{cc|c} 0 & 1 & 1 \\ -100 & 0.3 & 0 \\ \hline 1 & 0 & 0 \end{array} \right], \quad \left[\begin{array}{c|c|c} A_{cm} & B_{cm} & 1 \\ \hline C_{cm} & D_{cm} & 0 \end{array} \right] = \left[\begin{array}{cc|c} -23.33604 & -2.54 \cdot 10^{-5} & 1 \\ \hline -9.09 \cdot 10^{-7} & 31.62046 & 0 \end{array} \right], \quad (65)$$

at that $\lambda_{1,2}(A_{pm}) = 0.15 \pm 9.9989j$. The reference model controller $K_m(s)$ is a solution to the following \mathcal{H}_∞ suboptimal problem: find fixed-order controller (6) for plant (5) guaranteeing internal stability of reference closed-loop system (54) and fulfilment of condition (55) with $\gamma_m = 1.02$ (Balandin & Kogan, 2007). In this example, we consider the actual plant given by (1) with two sets of parameters:

$$\left[\begin{array}{c|c|c} A_p & B_p & 0.6 \\ \hline C_p & 0 & 0 \end{array} \right] = \left[\begin{array}{cc|c} 0 & 1 & 0.6 \\ -140 & 0.5 & 0 \\ \hline 1.4 & 0 & 0 \end{array} \right], \quad \lambda_{1,2}(A_p) = 0.25 \pm 11.8295j, \quad (66)$$

and

$$\left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & 0 \end{array} \right] = \left[\begin{array}{cc|c} 0 & 1 & 1.4 \\ -60 & 0.1 & 0 \\ \hline 0.6 & 0 & 0 \end{array} \right], \quad \lambda_{1,2}(A_p) = 0.05 \pm 7.7458j. \quad (67)$$

Given controller structure and order ($A_c = A_{cm}$, $C_c = C_{cm}$), we are interested in finding the matrices B_c and D_c such that conditions (56), (57) hold with $\gamma = \gamma_m$.

At the first stage of tuning process described above we have obtained the following numerical solutions to dual LMI (56) with $\gamma = \gamma_m = 1.02$:

$$X = \begin{bmatrix} 0.0754 & -0.0003 & -0.0304 \\ -0.0003 & 0.0005 & -0.0001 \\ -0.0304 & -0.0001 & 1.0646 \end{bmatrix}, \quad Y = \begin{bmatrix} 13.4456 & 6.9922 & 0.3846 \\ 6.9922 & 1836.7693 & 0.3456 \\ 0.3846 & 0.3456 & 0.9503 \end{bmatrix}$$

for plant (1) with realization (65) and

$$X = \begin{bmatrix} 0.0687 & -0.0001 & 0.0000 \\ -0.0001 & 0.0011 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}, \quad Y = \begin{bmatrix} 14.5580 & 1.4755 & 0.0000 \\ 1.4755 & 873.4150 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

for plant (1) with realization (66).

At the second stage, solving LMI (60), (63), (64) subject to equality constraint (58) we have obtained the controller

$$\left[\begin{array}{c|c} A_{cm} & B_c^* \\ \hline C_{cm} & D_c^* \end{array} \right] = \left[\begin{array}{c|c} -23.33604 & -104.30004 \\ \hline -9.09 \cdot 10^{-7} & 52.71044 \end{array} \right] \quad (68)$$

for realization (66) and

$$\left[\begin{array}{c|c} A_{cm} & B_c^* \\ \hline C_{cm} & D_c^* \end{array} \right] = \left[\begin{array}{c|c} -23.33604 & -1.44589 \cdot 10^{-7} \\ \hline -9.09 \cdot 10^{-7} & 17.71328 \end{array} \right] \quad (69)$$

for realization (67). Denote that controller (68) results in $\|\Phi(s)\|_\infty = 1.0125 < \gamma = 1.02$, and controller (69) results in $\|\Phi(s)\|_\infty = 1.0069 < \gamma = 1.02$.

Simulation results for reference system (65), as well as for actual plants (66), (67) with controllers (68), (69), respectively, are presented in Fig. 1. The left red-coloured diagrams correspond to plant (66) and controller (68), whereas the right blue-coloured diagrams show transients and control for plant (67) and controller (69). The diagrams for the reference system are shown in black colour. At the top diagrams, the step responses of reference and actual plants are presented. The middle plots show the step responses of closed-loop reference and actual systems. The control signals generated by reference and adjusted controllers are given at the bottom diagrams. One can denote good visual proximity of step responses of the reference and adjusted closed-loop systems at the middle diagrams.

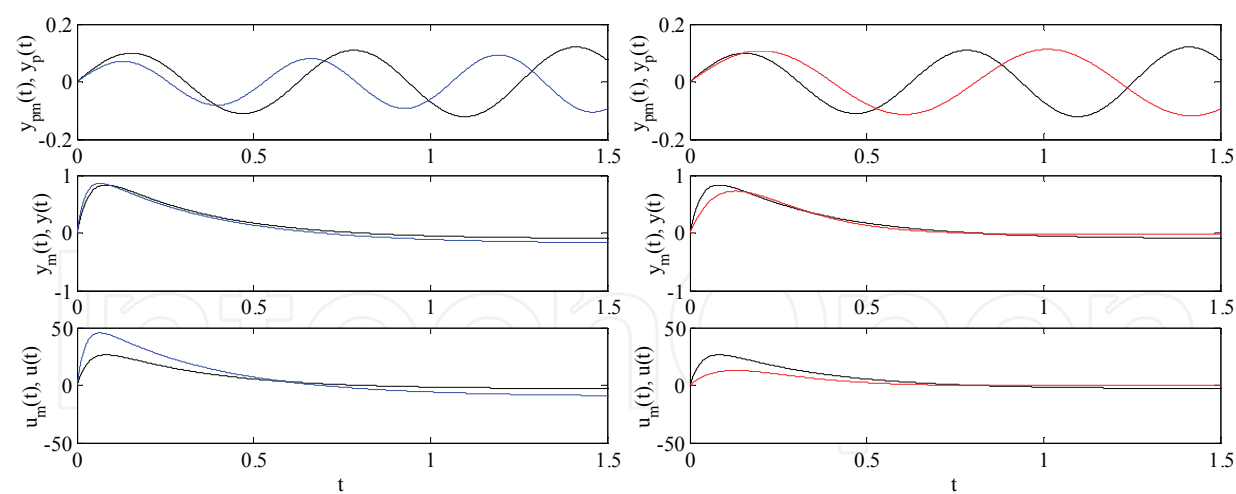


Figure 1. Step responses and control for reference and actual systems

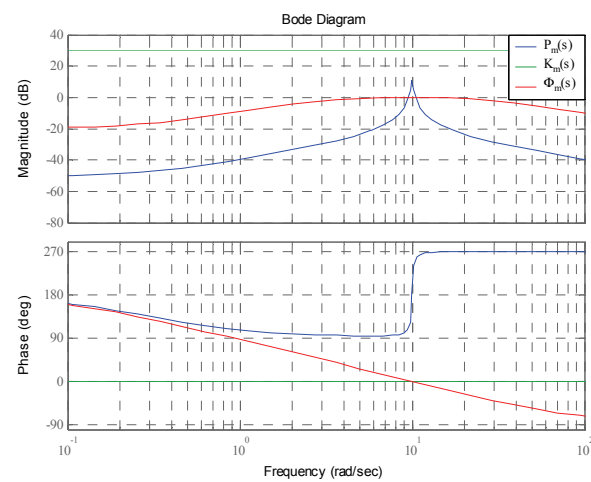


Figure 2. Bode diagram for reference system

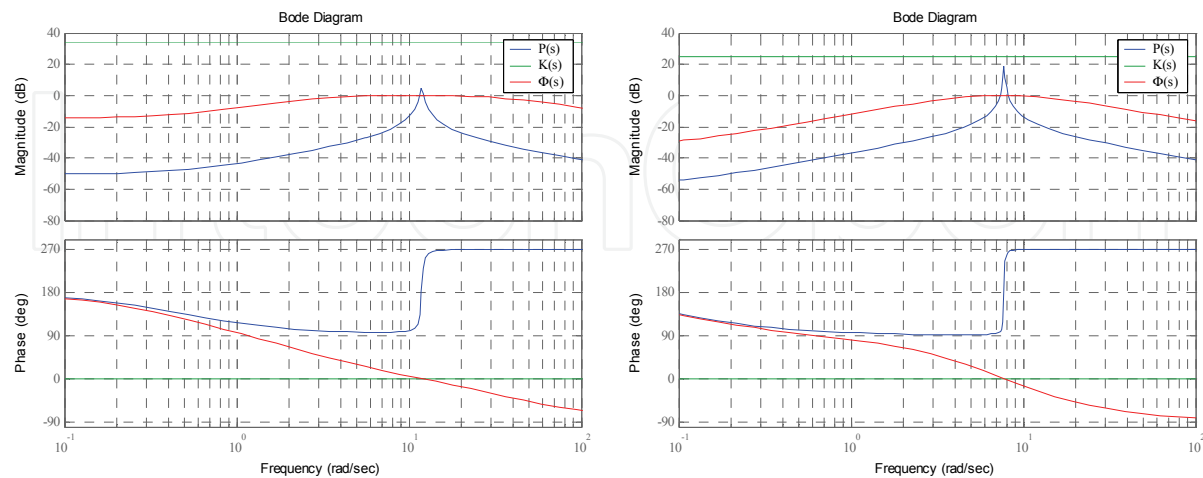


Figure 3. Bode diagrams for actual systems

The Bode diagrams for the reference and actual systems are shown in Fig. 2 and Fig. 3, correspondingly, including diagrams for plants (blue lines), controllers (green lines), and

closed-loop systems (red lines). At Fig. 3, the left plots correspond to plant (66) and controller (68), the right plots represent plant (67) and controller (69).

6. Conclusion

One of the main results of this Chapter consists in that the necessary minimum conditions for the functional given by \mathcal{H}_2 norm of the difference between the transfer functions of the closed-loop adjusted and reference systems coincide with the necessary minimum conditions for Frobenius norm of the controller tuning polynomial generated by these transfer functions that have been obtained earlier.

Theorem 2 shows that in spite of complexity of analytic expressions for the “direct” tuning functionals J_1 and J_2 , optimal values of the adjusted parameters can be found via comparatively simple pseudosolution of linear matrix algebraic equation. This approach ensures proximity of transient responses of the adjusted and reference systems and, consequently, the best (in sense of \mathcal{H}_2 norm) stability of performance indices of the adjusted system.

The properties of complete, partial, and weak adaptability of a system with respect to its output belongs to the system invariants. The adaptability criteria, just as Kalman’s criteria of controllability and observability, are formulated in terms of rank properties of the adaptability matrices. One of the main properties of the adaptability matrices is Toeplitz property.

Although \mathcal{H}_2 norm in functional J_2 is defined for the closed-loop systems, the elements of the adaptability matrices depend only on the coefficients of the characteristic polynomials, matrices and matrix coefficients of the resolvent series expansions of the plant, controller, and their reference models. An advantage of finding optimal controller parameters via the mentioned pseudosolution consists in that individual plant poles can be unstable on condition that all poles of adjusted closed-loop system are stable.

The main drawback of LQ and \mathcal{H}_2 optimal tuning algorithms consists in that the direct control of stability margin of the adjusted system is impossible. This drawback can be partially weakened by evaluating the characteristic polynomial of the closed-loop system or its roots. This drawback can be eliminated by use of \mathcal{H}_2 optimal tuning algorithm together with \mathcal{H}_∞ constraint.

Another one important result of this Chapter consists in the presented \mathcal{H}_2 optimal fixed-structure controller tuning algorithm with \mathcal{H}_∞ constraint for SISO systems represented by minimal state-space realization that can be easily extended onto MIMO systems. This approach is based on minimization of \mathcal{H}_2 criterion of proximity of transient responses of the closed-loop system and its implicit reference model subject to constraint onto \mathcal{H}_∞ norm of the transfer function of the closed-loop system formulated in terms of LMIs.

The obtained algorithms of optimal tuning of multiloop PID controller for bilinear MIMO plant have the same structure as the similar algorithms for linear MIMO plant (Morozov & Yadykin, 2004; Yadykin & Tchaikovsky, 2007). However, the optimal tuning procedures for the bilinear plant are more complex than similar procedures for the linear plant:

- Identification procedures for bilinear plants depend on operating point of the process, increment of piecewise-constant control, and its sign in various combinations. This gives rise to need in considering many modes of identification and tuning.

- The models of bilinear plant and reference system, as well as tuning criteria and algorithms have to be matched.
- Dynamics of transients in the adjusted system depends on the sign and magnitude of the test control increment. For positive increments, the transients, in general, accelerate and their decrement decrease, whereas for negative increments the transient decrement increase and it decelerate.

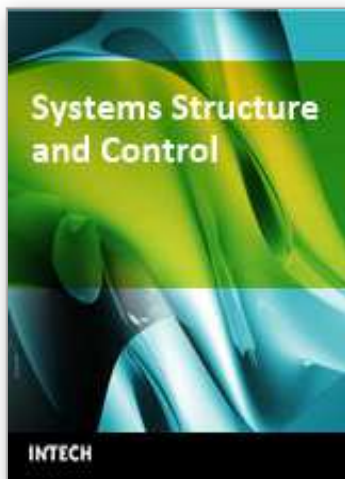
The obtained results can be also considered as a solution to the controller design problem for linear time invariant SISO and MIMO systems on the base of the constrained minimization of \mathcal{H}_2 norm of the difference between the transfer functions of the closed-loop designed and reference systems subject to constraint onto \mathcal{H}_∞ norm of the transfer function of the designed system established in terms of LMIs.

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