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Integral Sliding Modes with Block Control of Multimachine Electric Power Systems

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1. Introduction

Over last 15 years the problem of rotor angle stability of electric power systems (EPS) has received a great attention. A fundamental problem in the design of feedback controllers for EPS is that of robust stabilizing both rotor angle and voltage magnitude, and achieving a specified transient behavior. Robustness implies operation with adequate stability margins and admissible performance level in spite of plant parameters variations and in the presence of external disturbances.

The EPS have nonlinearities and are subject to variations as a result of a change in the systems loading and/or configuration. Then, the EPS are modeled as complex large-scale nonlinear systems and the generators may be interconnected over several kilometers in very large power systems. Thus, the controller design is a challenging problem. A complete centralized control scheme could be difficult to implement in EPS, due to the reliability and distortion in information transfer. On the other hand, accurate prediction of system responses and system robustness to disturbances under different operation conditions are guarantee by robust decentralized control schemes. The decentralized controllers are locally implemented, so do not need system information communication among subsystems. In each subsystem, the effects of the other subsystems are considered as a disturbance. To design decentralized control schemes for EPS, a controller is designed for each generator connected to the system.

The control schemes of power systems are commonly based on reduced order linearized model and classical control algorithms that ensure asymptotic stability of the equilibrium point under small perturbations (Anderson & Fouad, 1994, DeMello & Concordia, 1969). Improvements on linear techniques have been analyzed in (Wang et al., 1998, Djukanovic et at., 1998a, Djukanovic et al., 1998b). Nevertheless, these controllers have been designed by using linear models. To analyze the EPS entire operation region, nonlinear control design techniques are more appropriate. Various nonlinear techniques have been implemented, e.g., control based on direct Lyapunov method (Machowsky et al., 1999), feedback linearization (FL) technique (Akhkrif, et al, 1999, Wu & Malik, 2006,) including backstepping (Jung et al., 2005 King et al., 1994), intelligent neural networks (Venayagamoorthy et al., 2003, Mohagheghi et al., 2007), fuzzy logic (Yousef & Mohamed, 2004) and normal form analysis (Kshatriya, et al., 2005, Liu et al., 2006).

All of the mentioned controllers provide larger stability margins with respect to traditional ones. But these control schemes were designed for reduced order plant. The unmodelled electrical dynamics can affect the electromechanical dynamics in case of large perturbations. The detailed 7-th order model of synchronous machine (five equations for electrical dynamics and two for mechanical dynamics) has been considered and a nonlinear controller using this model and FL technique has been designed to enhance transient stability (Akhkrif, et al., 1999). The proposed nonlinear control law is a function of all plant parameters and disturbances. In practice some of these parameters are subjected to variations as a result of a change in the system loading and/or configuration. Since the detailed model is so involved, a direct use of the FL technique results in a computationally expensive control algorithm. Moreover, this control scheme does not take into account practical limitation on the magnitude of the excitation voltage, and an observer design problem was not solved.

On the other hand, sliding mode control (SMC), (Utkin, et al., 1999) is one of the most effective strategies to deal with robust nonlinear controllers. SMC enables high accuracy and robustness to disturbances and plant parameter variations. Moreover, the control variables of the basic sliding mode control law rapidly switch between extreme limits, which are ideal for the direct operation of the switched mode power converters of synchronous generators. Sliding mode controllers for power systems have been designed in (Dash et al., 1996, Bandal et al., 2005), however for reduced order plants only, the best of our knowledge. Application of these controllers to full order plant would cause undesirable chattering, since unmodelled dynamics can be excited.

In (Loukianov et al., 2004) it was designed a sliding mode controller to regulate the terminal voltage and power angle for a single machine infinite-bus system, based on the eighth order generator model (two equations for mechanical dynamics and six equations for electrical ones for thermo electrical power system). In this case, an information about the power angle reference, δ_{ref}, is required. To overcome this restriction, in (Loukianov et al., 2006) a decentralized robust sliding mode control scheme was proposed to regulate the voltages and stabilize the speed in a multi machine power system.

In this paper an eighth order model for each generator of the multimachine power systems is considered. Sliding mode controller is designed by using the combination of three techniques: block control (Loukianov, 1998), integral sliding mode control (Utkin *et al.*, 1999), and nested sliding mode control (Adhami-Mirhosseini and Yazdanpanah, 2005). The block control technique is used to design a nonlinear sliding surface in such a way that the sliding mode dynamics are represented by a linear system with desired eigenvalues. The integral sliding mode control combined with nested control technique are applied to reject perturbations. The controller designed in this way is computationally low demanding and takes into account structural constraints of the control input. The main feature of the proposed control scheme is robustness with respect to the both matched and unmatched perturbations and only local information is required. Moreover, a nonlinear observer for the unmeasureable estates of the systems such as the rotor fluxes of the generators is presented.

This chapter is organized as follows. Section 2 presents a general mathematical description of the EPS (nonlinear eight order electrical generator, electrical network and loads models). Section 3 deals with the problem of nonlinear robust controller for the class of the nonlinear systems represented in the nonlinear block controllable form, the Integral Sliding Modes with Block Control technique is analyzed. Section 4 shows the design of a nonlinear robust

control scheme for EPS, as well as a generator rotor fluxes observer. The results of the simulations in an equivalent of the WSCC, that illustrates the properties of the controller designed, can be found in section 5, followed by conclusions in section 6.

2. EPS Model

This section copes with the mathematical description of the EPS. The multimachine EPS model considers the generators model, the electrical network model and loads.

2.1 Generator model

The electrical dynamics comprised the field winding, rotor and stator windings, after the Park's transformation, can be expressed as follows (Anderson & Fouad, 1994):

$$\begin{bmatrix} d\boldsymbol{\lambda}_1/dt \\ d\mathbf{i}/dt \end{bmatrix} = \mathbf{A}(\omega) \cdot \begin{bmatrix} \boldsymbol{\lambda}_1 \\ \mathbf{i} \end{bmatrix} + \mathbf{T} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v} \end{bmatrix} \tag{1}$$

where $\boldsymbol{\lambda}_1 = (\lambda_f, \lambda_g, \lambda_{kd}, \lambda_{kq})^T$, $\mathbf{i} = (i_d, i_q)^T$, $\mathbf{v} = (v_d, v_q)^T$, $\mathbf{v}_1 = (v_f, \ 0, \ 0, \ 0)^T$, λ_f is the field flux, λ_{kd}, λ_{kq} and λ_g are the direct-axis and quadrature-axis damper windings fluxes respectively, i_d and i_q are the stator currents, ω is the angular speed, v_f is the excitation control input, v_d and v_q are the direct-axis and quadrature-axis terminal voltages, respectively. The matrices $\mathbf{A}(\omega) = -\mathbf{T} \left[\mathbf{R} \cdot \mathbf{L}^{-1} + \mathbf{W}(\omega) \right] \mathbf{T}^{-1}$, $\mathbf{T}$, $\mathbf{R}$, $\mathbf{L}$ and $\mathbf{W}(\omega)$ are defined in *Appendix*.

The complete mathematical description includes also the swing equation given by

$$\begin{aligned} d\delta/dt &= \omega - \omega_b \\ d\omega/dt &= (\omega_b/2H)(T_m - T_e) \end{aligned} \tag{2}$$

where δ is the power angle, ω_b is the rated synchronous speed, H is the inertia constant, T_m is the mechanical torque applied to the shaft, and T_e is the electromagnetic torque, expressed in terms of the linked fluxes and currents as follows:

$$T_e = a_1 \lambda_f i_q - a_2 \lambda_g i_d + a_3 \lambda_{kd} i_q - a_4 \lambda_{kq} i_d - a_5 i_d i_q \tag{3}$$

where $a_1, \ldots, a_5$ are constants defined in *Appendix*. The mechanical torque T_m it is assumed to be a slowly varying and bounded function of time. Thus:

$$\dot{T}_m = 0 . \tag{4}$$

Since the multimachine EPS has at least one more differential equation than is needed to solve the system, then, it is possible to define the angle relative to the generator 1 of the form:

$$\hat{\delta}_i = \delta_i - \delta_1, \quad i = 1, 2, \ldots, n$$

where n is the number of generators in the system. Thus

$$\frac{d\hat{\delta}_1}{dt} = 0, \qquad \frac{d\hat{\delta}_i}{dt} = \omega_i - \omega_1, \qquad i = 2,3,\ldots,n . \tag{5}$$

From (1)-(5), the nonlinear state-space presentation of the ith generator in the multimachine power system is derived of the form

$$\begin{bmatrix} \dot{\mathbf{x}}_{1i} \\ \dot{\mathbf{x}}_{2i} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{1i}(\mathbf{x}_i, \ \mathbf{i}_i) \\ \mathbf{f}_{2i}(\mathbf{x}_i, \ \mathbf{i}_i) \end{bmatrix} + \begin{bmatrix} \mathbf{b}_{1i} \\ 0 \end{bmatrix} v_{fi} + \begin{bmatrix} \mathbf{g}_i(\mathbf{x}_i, \ \mathbf{i}_i, T_{mi}) \\ 0 \end{bmatrix} \tag{6}$$

$$\mu \frac{d}{dt} \mathbf{i}_i = \mathbf{A}_{zi}(x_{2i})\mathbf{i}_i + \mathbf{f}_{zi}(\mathbf{x}_i) + \mathbf{b}_{zi} v_{fi} + \mathbf{H}_i \mathbf{v}_i \tag{7}$$

where $\mu = 1/\omega_b$ is a small parameter, $\mathbf{x}_i = (\mathbf{x}_{1i} \ \mathbf{x}_{2i})^T$, $\mathbf{x}_{1i} = (x_{1i}, x_{2i}, x_{3i})^T = (\hat{\delta}_i, \omega_i, \lambda_{fi})^T$,

$\mathbf{x}_{2i} = (x_{4i}, x_{5i}, x_{6i})^T = (\lambda_{gi}, \lambda_{kdi}, \lambda_{kqi})^T$, $\mathbf{f}_{1i} = \begin{bmatrix} x_{2i} - \omega_b \\ f_{\omega i}(\mathbf{x}_i, \mathbf{i}_i, T_{mi}) - q_i(\mathbf{x}_i, \mathbf{i}_i)x_{3i} \\ b_{i1}x_{3i} + b_{i2}x_{5i} + b_{i3}i_{di} \end{bmatrix}$, $\mathbf{f}_{2i}(\mathbf{x}_i, \mathbf{i}_i) = \mathbf{A}_{2i}\mathbf{x}_{2i} + \mathbf{d}_{i1}x_{3i} + \mathbf{D}_i\mathbf{i}_i$,

$f_{\omega i}(\cdot) = -d_{mi}(-a_{i2}x_{4i}i_{di} + a_{i3}x_{5i}i_{qi} - a_{i4}x_{6i}i_{di} + a_{i5}i_{di}i_{qi})$, $q_i(\cdot) = a_{i1}d_{m_i}i_{qi}$, $d_m = \dfrac{\omega_b}{2H}$,

$\mathbf{d}_{i1} = \begin{bmatrix} 0 \\ 0 \\ d_{i1} \end{bmatrix}$, $\mathbf{D}_i = \begin{bmatrix} 0 & c_{i3} \\ d_{i3} & 0 \\ 0 & e_{i3} \end{bmatrix}$, $\mathbf{A}_{2i} = \begin{bmatrix} c_{i1} & 0 & c_{i2} \\ 0 & d_{i2} & 0 \\ r_{i1} & 0 & r_{i2} \end{bmatrix}$, $\mathbf{b}_{1i} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{A}_{zi} = \begin{bmatrix} h_{i7} & k_{i6}x_{2i} \\ h_{i6}x_{2i} & k_{i7} \end{bmatrix}$, $\mathbf{H}_i = \begin{bmatrix} h_{i1} & 0 \\ 0 & k_{i1} \end{bmatrix}$,

$\mathbf{b}_{zi} = \begin{bmatrix} 0 \\ h_{i8} \end{bmatrix}$, $\mathbf{f}_{zi}(\mathbf{x}_i) = \begin{bmatrix} h_{i2}x_{3i} + h_{i3}x_{5i} + h_{i4}x_{2i}x_{4i} + h_{i5}x_{2i}x_{6i} \\ k_{i2}x_{4i} + k_{i3}x_{6i} + k_{i4}x_{2i}x_{3i} + k_{i5}x_{2i}x_{5i} \end{bmatrix}$, $\mathbf{v}_i = \begin{bmatrix} v_{di}, v_{qi} \end{bmatrix}^T$,

$\mathbf{g}_i(\mathbf{x}_i, \mathbf{i}_i, T_{mi}) = \begin{bmatrix} 0, & g_{2i}(\mathbf{x}_i, \mathbf{i}_i, T_{mi}), & 0 \end{bmatrix}^T$. The perturbation term $g_{2i}(\cdot)$ includes variations of the generator parameters in the function $f_{\omega i}(\cdot)$ and the mechanical torque T_{mi} (external disturbance), i. e.

$$g_{2i}(\cdot) = d_m T_{mi} - [\Delta a_{i2}x_{4i}i_{di} + \Delta a_{i3}x_{5i}i_{qi} + \Delta a_{i4}x_{6i}i_{di} + \Delta a_{i5}i_{di}i_{qi}], \ a_{ij} = a_{ij,n} + \Delta a_{ij}, \ j = 2,\ldots,5,$$

where $a_{ij,n}$ and Δa_{ij} are the nominal value and variation, respectively, of the parameter a_{ij}. Moreover $rank\{\mathbf{A}_{zi}\} = 2$ for all admissible values of x_{2i}.

To neglect the fast dynamics in the electric networks that in turn permits to simplify and simulate the complete power system by a differential algebraic equation (DAE) (Anderson & Fouad, 1994) we use the singular perturbation technique (Khalil, 1996). Thus, setting $\mu = 0$ in (7) results in

$$0 = \mathbf{A}_{zi}(x_{2i})\mathbf{i}_i + \mathbf{f}_{zi}(\mathbf{x}_i) + \mathbf{H}_i \mathbf{v}_i \tag{8}$$

The solution of (8) for $\mathbf{i}_i$ is calculated as

$$\mathbf{i}_i = \mathbf{g}_{zi}(\mathbf{x}_i, \mathbf{v}_i) \tag{9}$$

where $\mathbf{g}_{zi} = -\mathbf{A}_{zi}^{-1}(x_{2i})\left(\mathbf{f}_{zi}(\mathbf{x}_i) + \mathbf{H}_i\mathbf{v}_i\right)$. Finally, equations (6) and (9) give the following DAE system for the ith generator:

$$\dot{\mathbf{x}}_{1i} = \mathbf{f}_{1i}\left(\mathbf{x}_i, \mathbf{i}_i, T_{mi}\right) + \mathbf{b}_{1i}v_{fi} + \mathbf{g}_i\left(\mathbf{x}_i, \mathbf{i}_i, T_{mi}\right) \tag{10}$$

$$\dot{\mathbf{x}}_{2i} = \mathbf{f}_{2i}\left(\mathbf{x}_i, \mathbf{i}_i\right) \tag{11}$$

$$\mathbf{i}_i = \mathbf{g}_{zi}(\mathbf{x}_i, \mathbf{v}_i). \tag{12}$$

2.2 Electrical network model

Since the fast dynamics reduction for the generator was achieved in the last subsection, it is possible to neglect the dynamics of the loads and transmission lines. Then, considering the loads as constant impedances, the electrical network can be modeled using the phasorial nodal method. Moreover, all the nodes, except for the generator ones, can be reduced (Kron's reduction). Therefore the network algebraic equation can be expressed as (Anderson & Fouad, 1994)

$$\overline{\mathbf{I}} = \overline{\mathbf{Y}}\left(\delta_1, ..., \delta_n\right)\overline{\mathbf{V}} \tag{13}$$

where $\overline{\mathbf{V}} = \left[v_{d1} + jv_{q1}, \cdots, v_{dn} + jv_{qn}\right]^T$ and $\overline{\mathbf{I}} = \left[i_{d1} + ji_{q1}, \cdots, i_{dn} + ji_{qn}\right]^T$ are the complex terminal generators voltages and currents, respectively, $\overline{\mathbf{Y}}(\cdot)$ is the reduced transformed admittance matrix and its entry jk is given by:

$$\overline{\mathbf{Y}}_{jk} = \mathbf{Y}_{jk}e^{(\delta_j - \delta_k)}$$

with the elements $\mathbf{Y}_{jk}$ calculated by using the nodal method. It is more convenient to express the equation (13) of the form

$$\mathbf{I} = \tilde{\mathbf{Y}}\left(\delta_1, ..., \delta_n\right)\mathbf{V}, \quad \tilde{\mathbf{Y}}(\cdot) \in R^{2n \times 2n} \tag{14}$$

where $\mathbf{V} = \left[v_{d1}, v_{q1}, ..., v_{dn}, v_{qn}\right]^T$ and $\mathbf{I} = \left[\mathbf{i}_1^T ... \mathbf{i}_n^T\right]^T = \left[i_{d1}, i_{q1}, ..., i_{dn}, i_{qn}\right]^T$ are the phasors components of the voltages and currents, respectively. Thus, the multimachine EPS model is given by (10)-(12) and (14). It is important to note that the vector $\mathbf{I}$ coincides with the generator currents i_{di} and i_{qi}.

3. Integral Sliding Modes with Block Control

The Integral Sliding Modes with Block Control (ISM) technique (Huerta-Avila et al., 2007a, Huerta-Avila et al., 2007b) is shown in this section. The description of the ISM is presented in generic terms to show the generality of the approach. In the next section a robust controller for the electrical power system will be designed by using this methodology.

3.1 Problem statement

In this work, the class of nonlinear systems presented in the NBC (nonlinear block controllable) form is studied. The NBC form consist of r blocks (Loukianov, 1998):

$$\dot{\mathbf{x}}_i = \mathbf{f}_i\left(\overline{\mathbf{x}}_i\right) + \mathbf{B}_i\left(\overline{\mathbf{x}}_i\right)\mathbf{x}_{i+1} + \mathbf{g}_i\left(t,\mathbf{x}\right)$$
$$\dot{\mathbf{x}}_r = \mathbf{f}_r\left(\mathbf{x}\right) + \mathbf{B}_r\left(\mathbf{x}\right)\mathbf{u} + \mathbf{g}_r\left(t,\mathbf{x}\right), \quad i = 1,...,r-1, \quad (15)$$
$$\mathbf{y} = \mathbf{x}_1$$

where, $\mathbf{x} = \left[\mathbf{x}_1 ... \mathbf{x}_r\right]^T \in R^n$ is the state vector, $\mathbf{x}_i \in R^{n_i}$, $\overline{\mathbf{x}}_i = \left[\mathbf{x}_1 ... \mathbf{x}_i\right]^T$; $\mathbf{u} \in R^m$ is the control vector. Moreover, $\mathbf{f}(\cdot)$ and the columns of $\mathbf{B}(\cdot)$ are smooth vector fields, $\mathbf{g}_i(\cdot)$ is a bounded unknown perturbation term due to parameter variations and external disturbances, and

$$rank\left[\mathbf{B}_i\left(\mathbf{x}_1,...,\mathbf{x}_i\right)\right] = n_i, \ \forall \mathbf{x} .$$

The integers $n_1,...,n_r$ define the dimension of the ith block (system structure) and satisfy

$$n_1 \le n_2 \le \cdots \le n_r = m, \quad \sum_{i=1}^r n_i = n .$$

The control objective is to design a controller such that the output $\mathbf{y}$ in (15) tracks a desired reference $\mathbf{x}_{ref}(t)$ with bounded derivatives, in spite of unknown but bounded perturbations. To induce quasi sliding mode in the ith block of the system (15), the continuously differentiable sigmoid function $sigm(v/\varepsilon)$ defined as

$$sigm(v/\varepsilon) = \tanh(v/\varepsilon) , \quad \tanh(v/\varepsilon) = \frac{e^{v/\varepsilon} - e^{-v/\varepsilon}}{e^{v/\varepsilon} + e^{-v/\varepsilon}}$$

where $1/\varepsilon$ is the slope of the sigmoid function at $v = 0$, will be used since

$$\lim_{\varepsilon \to 0} sigm(v/\varepsilon) = sign(v) .$$

3.2 Control design

According to the block control technique (Loukianov, 1998), the state $\mathbf{x}_{i+1}, i = 1,...,r-1$ is considered as a virtual control vector in the ith block of the system (15). The design procedure is described in r steps.

Step 1. The control error in the first block of the system (15) is defined as

$$\mathbf{z}_1 = \mathbf{x}_1 - \mathbf{x}_{ref} := \boldsymbol{\psi}_1\left(\mathbf{x}_1\right)$$

then

$$\dot{\mathbf{z}}_1 = \mathbf{f}_1\left(\mathbf{x}_1\right) + \mathbf{B}_1\left(\mathbf{x}_1\right)\mathbf{x}_2 + \overline{\mathbf{g}}_1\left(t,\mathbf{x}\right) \quad (16)$$

with $\overline{\mathbf{g}}_1\left(t,\mathbf{x}\right) = \mathbf{g}_1\left(t,\mathbf{x}\right) - \dot{\mathbf{x}}_{ref}$.

And the virtual control $\mathbf{x}_2$ in (16) is redefined of the form

$$\mathbf{x}_2 = \mathbf{x}_{2,0} + \mathbf{x}_{2,1} \quad (17)$$

where the nominal part, $\mathbf{x}_{2,0}$ is selected to eliminate the old dynamics in (16) and introduce the new desired ones, $k_1\mathbf{z}_1$, $k_1 > 0$, i. e.

$$\mathbf{x}_{2,0} = -\mathbf{B}_1^+ (\mathbf{x}_1)\big(\mathbf{f}_1(\mathbf{x}_1) + k_1\mathbf{z}_1 - \mathbf{E}_1\mathbf{z}_2\big), \quad k_1 > 0 \tag{18}$$

where $\mathbf{z}_2 \in R^{n_2}$ is a new variables vector, $\mathbf{E}_1 = \begin{bmatrix} \mathbf{I}_{n_1} & 0 \end{bmatrix} \in R^{n_1 \times n_2}$ and $\mathbf{B}_1^+$ is the right pseudo-inverse of $\mathbf{B}_1$, defined as $\mathbf{B}_1^+ = \mathbf{B}_1^T(\mathbf{B}_1\mathbf{B}_1^T)^{-1}$.

In order to reject the perturbation term $\overline{\mathbf{g}}_1(t,\mathbf{x})$ in (16), the second part of the virtual control (17), $\mathbf{x}_{2,1}$ is designed by using the integral sliding mode technique (Utkin et al., 1999). The pseudo-sliding manifold $\mathbf{s}_1$ is chosen as

$$\mathbf{s}_1 = \mathbf{z}_1 + \boldsymbol{\sigma}_1 = 0 \ , \qquad \mathbf{s}_1, \ \boldsymbol{\sigma}_1 \in R^{n_1} \ . \tag{19}$$

Then, from (16)-(19) it follows

$$\dot{\mathbf{s}}_1 = -k_1\mathbf{z}_1 + \mathbf{E}_1\mathbf{z}_2 + \mathbf{B}_1(\mathbf{x}_1)\mathbf{x}_{2,1} + \overline{\mathbf{g}}_1(t,\mathbf{x}) + \dot{\boldsymbol{\sigma}}_1 \ . \tag{20}$$

Choosing the dynamics for the integral variable $\boldsymbol{\sigma}_1$ of the form

$$\dot{\boldsymbol{\sigma}}_1 = k_1\mathbf{z}_1 - \mathbf{E}_1\mathbf{z}_2, \quad \boldsymbol{\sigma}_1(0) = -\mathbf{z}_1(0) \tag{21}$$

the equation (20) becomes

$$\dot{\mathbf{s}}_1 = \mathbf{B}_1(\mathbf{x}_1)\mathbf{x}_{2,1} + \overline{\mathbf{g}}_1(t,\mathbf{x}) \ . \tag{22}$$

The control input $\mathbf{x}_{2,1}$ in (22) is selected as follows:

$$\mathbf{x}_{2,1} = -\rho_1(\mathbf{x}_1)\mathbf{B}_1^+ sigm(\mathbf{s}_1/\varepsilon_1) \tag{23}$$

where $sigm(\mathbf{s}_1/\varepsilon_1) = \big[sigm(s_{1,1}/\varepsilon_1),\ldots,sigm(s_{1,n_1}/\varepsilon_1)\big]^T$. Substituting (17), (18) and (23) in (16) results in

$$\dot{\mathbf{z}}_1 = -k_1\mathbf{z}_1 + \mathbf{E}_1\mathbf{z}_2 - \rho_1(\mathbf{x}_1)sigm(\mathbf{s}_1/\varepsilon_1) + \overline{\mathbf{g}}_1(t,\mathbf{x}) \ . \tag{24}$$

If the matrix $\mathbf{M}_1(\mathbf{x}_1) \in R^{(n_2-n_1) \times n_2}$ is chosen such that the square matrix $\tilde{\mathbf{B}}_2(\mathbf{x}_1) = \begin{bmatrix} \mathbf{B}_1(\mathbf{x}_1) & \mathbf{M}_1(\mathbf{x}_1)\end{bmatrix}^T$ has full rank, the new variables vector $\mathbf{z}_2$ can be obtained from equations (17), (18) and (23) as

$$\mathbf{z}_2 = \tilde{\mathbf{B}}_2\mathbf{x}_2 + \begin{bmatrix} \mathbf{f}_1(\mathbf{x}_1) - k_1\boldsymbol{\psi}_1(\mathbf{x}_1) - \rho_1(\mathbf{x}_1)sigm\left(\dfrac{\mathbf{s}_1}{\varepsilon_1}\right) \\[2mm] 0 \end{bmatrix} := \boldsymbol{\psi}_2(\overline{\mathbf{x}}_2) \tag{25}$$

where $\overline{\mathbf{x}}_2 = [\mathbf{x}_1 \ \mathbf{x}_2]^T$. The procedure describe above can be achieved in the ith block of (15) as follows.

Step i. At this step, the dynamics of the transformed i^{th} block of the system (15) are given by

$$\dot{\mathbf{z}}_i = \overline{\mathbf{f}}_i(\overline{\mathbf{x}}_i) + \overline{\mathbf{B}}_i(\overline{\mathbf{x}}_i)\mathbf{x}_{i+1} + \overline{\mathbf{g}}_i(t, \mathbf{x}) \tag{26}$$

where $\mathbf{z}_i \in R^{n_i}$ is a new variables vector, $\overline{\mathbf{g}}_i(t, \mathbf{x}) = \overline{\mathbf{g}}_{i-1}(t, \mathbf{x}) - d/dt\left[\rho_{i-1}(\overline{\mathbf{x}}_{i-1})sigm(\mathbf{s}_{i-1}/\varepsilon_{i-1})\right]$, $\mathbf{z}_i = \boldsymbol{\psi}_i(\overline{\mathbf{x}}_i)$ and $\overline{\mathbf{B}}_i = \tilde{\mathbf{B}}_i\mathbf{B}_i$. The virtual control $\mathbf{x}_{i+1}$ in (26) is redefined as

$$\mathbf{x}_{i+1} = \mathbf{x}_{i+1,0} + \mathbf{x}_{i+1,1}. \tag{27}$$

Taking into account the procedure achieved in *step 1*, $\mathbf{x}_{i+1,0}$ and $\mathbf{x}_{i+1,1}$ are selected, respectively, of the form

$$\mathbf{x}_{i+1,0} = -\overline{\mathbf{B}}_i^+(\overline{\mathbf{x}}_i)\left(\overline{\mathbf{f}}_i(\overline{\mathbf{x}}_i) + k_i\mathbf{z}_i - \mathbf{E}_i\mathbf{z}_{i+1}\right), \quad k_i > 0 \tag{28}$$

$$\mathbf{x}_{i+1,1} = -\rho_i(\overline{\mathbf{x}}_i)\mathbf{B}_i^+ sigm(\mathbf{s}_i/\varepsilon_i), \quad \rho_i > 0 \tag{29}$$

where $\mathbf{z}_{i+1} \in R^{n_{i+1}}$ is a new variables vector, $\mathbf{E}_i = [\mathbf{I}_{n_i} \ 0] \in R^{n_i \times n_{i+1}}$ and $\overline{\mathbf{B}}_i^+ = \overline{\mathbf{B}}_i^T(\overline{\mathbf{B}}_i\overline{\mathbf{B}}_i^T)^{-1}$. The proposed pseudo-sliding manifold and its derived dynamics, respectively, are:

$$\mathbf{s}_i = \mathbf{z}_i + \boldsymbol{\sigma}_i = 0 \ , \qquad \mathbf{s}_i, \ \boldsymbol{\sigma}_i \in R^{n_i} \ ,$$

$$\dot{\mathbf{s}}_i = -k_i\mathbf{z}_i + \mathbf{E}_i\mathbf{z}_{i+1} + \overline{\mathbf{B}}_i(\overline{\mathbf{x}}_i)\mathbf{x}_{i+1,1} + \overline{\mathbf{g}}_i(t, \mathbf{x}) + \dot{\boldsymbol{\sigma}}_i \ . \tag{30}$$

If $\boldsymbol{\sigma}_i$ satisfies

$$\dot{\boldsymbol{\sigma}}_i = k_i\mathbf{z}_i - \mathbf{E}_i\mathbf{z}_{i+1}, \quad \boldsymbol{\sigma}_i(0) = -\mathbf{z}_i(0) \tag{31}$$

the equation (30) can be rewritten as

$$\dot{\mathbf{s}}_i = -\rho_i(\overline{\mathbf{x}}_i)sigm(\mathbf{s}_i/\varepsilon_i) + \overline{\mathbf{g}}_i(t, \mathbf{x}), \quad \rho_i(\overline{\mathbf{x}}_i) > 0 \ .$$

The substitution of (28) and (29) in the block (26) yields

$$\dot{\mathbf{z}}_i = -k_i\mathbf{z}_i + \mathbf{E}_i\mathbf{z}_{i+1} - \rho_i(\overline{\mathbf{x}}_i)sigm(\mathbf{s}_i/\varepsilon_i) + \overline{\mathbf{g}}_i(t, \mathbf{x}) \ .$$

Again, choosing a $(n_{i+1} - n_i) \times n_{i+1}$ matrix $\mathbf{M}_i(\overline{\mathbf{z}}_i)$ such that the square matrix $\tilde{\mathbf{B}}_{i+1}(\overline{\mathbf{x}}_i) = [\overline{\mathbf{B}}_i(\overline{\mathbf{x}}_i) \ \mathbf{M}_i(\overline{\mathbf{x}}_i)]^T$ has full rank, the new variables vector $\mathbf{z}_{i+1}$ can be obtained from equations (26)-(29) as

$$\mathbf{z}_{i+1} = \tilde{\mathbf{B}}_{i+1}\mathbf{x}_{i+1} + \begin{bmatrix} \overline{\mathbf{f}}_i(\overline{\mathbf{x}}_i) - k_i\boldsymbol{\psi}_i(\overline{\mathbf{x}}_i) - \rho_i(\overline{\mathbf{x}}_i)sigm\left(\dfrac{\mathbf{s}_i}{\varepsilon_i}\right) \\ 0 \end{bmatrix}, i = 2, \ldots, r-1,$$

$$:= \boldsymbol{\psi}_{i+1}(\overline{\mathbf{x}}_{i+1}).$$

Step r. At the last step, the transformed complete system can be presented in the new variables $\mathbf{z}_1, \ldots, \mathbf{z}_r$ as

$$
\begin{aligned}
\dot{\mathbf{z}}_i &= -k_i \mathbf{z}_i + \mathbf{E}_i \mathbf{z}_{i+1} - \rho_i(\overline{\mathbf{x}}_i) sigm(\mathbf{s}_i / \varepsilon_i) + \overline{\mathbf{g}}_i(t, \mathbf{x}) \\
\dot{\mathbf{s}}_i &= -\rho_i(\overline{\mathbf{x}}_i) sigm(\mathbf{s}_i / \varepsilon_i) + \overline{\mathbf{g}}_i(t, \mathbf{x}) \\
\dot{\mathbf{z}}_r &= \overline{\mathbf{f}}_r(\mathbf{x}) + \overline{\mathbf{B}}_r(\mathbf{x})\mathbf{u} + \overline{\mathbf{g}}_r(t, \mathbf{x}), \quad i = 1, \ldots, r-1
\end{aligned}
\tag{32}
$$

where $\overline{\mathbf{B}}_r(\cdot) = \tilde{\mathbf{B}}_{r-1}(\cdot)\mathbf{B}_r(\cdot)$ has full rank since $n_r = m$. Design the control input $\mathbf{u}$ in (32) as

$$
\mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1
\tag{33}
$$

and define a sliding variable $\mathbf{s}_r \in R^{n_r}$ of the form

$$
\mathbf{s}_r = \mathbf{z}_r + \boldsymbol{\sigma}_r, \quad \boldsymbol{\sigma}_r \in R^{n_r}.
\tag{34}
$$

Then

$$
\dot{\mathbf{s}}_r = \overline{\mathbf{f}}_r(\mathbf{x}) + \overline{\mathbf{B}}_r(\mathbf{x})\mathbf{u}_0 + \overline{\mathbf{B}}_r(\mathbf{x})\mathbf{u}_1 + \overline{\mathbf{g}}_r(t, \mathbf{x}) + \dot{\boldsymbol{\sigma}}_r.
\tag{35}
$$

Choosing

$$
\dot{\boldsymbol{\sigma}}_r = -\overline{\mathbf{f}}_r(\mathbf{x}) - \overline{\mathbf{B}}_r(\mathbf{x})\mathbf{u}_0, \quad \boldsymbol{\sigma}_r(0) = -\mathbf{z}_r(0)
$$

simplifies the equation (35) to

$$
\dot{\mathbf{s}}_r = \overline{\mathbf{B}}_r(\mathbf{x})\mathbf{u}_1 + \overline{\mathbf{g}}_r(t, \mathbf{x}).
\tag{36}
$$

The second part of the control input (33) is selected as

$$
\mathbf{u}_1 = -\rho_r(\mathbf{x})\overline{\mathbf{B}}_r^{-1} sign(\mathbf{s}_r), \quad \rho_r(\mathbf{x}) > 0.
\tag{37}
$$

Under the condition $\rho_r(\mathbf{x}) > \left\| \overline{\mathbf{B}}_r^{-1}(\mathbf{x})\overline{\mathbf{g}}_r(t, \mathbf{x}) \right\|$ sliding mode occurs on the manifold $\mathbf{s}_r = 0$ (34) in a finite time. Solving (36) for $\mathbf{u}_{r,1}$, formally setting $\dot{\mathbf{s}}_r = 0$, shows

$$
\mathbf{u}_{1eq} = \overline{\mathbf{B}}_r^{-1}(\mathbf{x})\overline{\mathbf{g}}_r(t, \mathbf{x})
$$

where $\mathbf{u}_{1eq}(t, \mathbf{x})$ is the equivalent control (Utkin et al., 1999). Therefore, the integral control (37) rejects the perturbation term $\overline{\mathbf{g}}_r(t, \mathbf{x})$ in the last block of (32):

$$
\dot{\mathbf{z}}_r = \overline{\mathbf{f}}_r(\mathbf{x}) + \overline{\mathbf{B}}_r(\mathbf{x})\mathbf{u}_0 + \overline{\mathbf{B}}_r(\mathbf{x})\mathbf{u}_{1eq} + \overline{\mathbf{g}}_r(t, \mathbf{x})
$$

and we have

$$
\dot{\mathbf{z}}_r = \overline{\mathbf{f}}_r(\mathbf{x}) + \overline{\mathbf{B}}_r(\mathbf{x})\mathbf{u}_0.
$$

Now, choosing

$$\mathbf{u}_0 = -\overline{\mathbf{B}}_r^{-1}(\mathbf{x})\left[\overline{\mathbf{f}}_r(\mathbf{x}) + k_r\mathbf{z}_r\right], \; k_r > 0$$

the sliding mode dynamics are described by

$$\begin{aligned}
\dot{\mathbf{z}}_i &= -k_i\mathbf{z}_i + \mathbf{E}_i\mathbf{z}_{i+1} - \rho_i(\overline{\mathbf{x}}_i)sigm(\mathbf{s}_i/\varepsilon_i) + \overline{\mathbf{g}}_i(t,\mathbf{x}) \\
\dot{\mathbf{s}}_i &= -\rho_i(\overline{\mathbf{x}}_i)sigm(\mathbf{s}_i/\varepsilon_i) + \overline{\mathbf{g}}_i(t,\mathbf{x}) \\
\dot{\mathbf{z}}_r &= -k_r\mathbf{z}_r, \quad i = 1,\ldots,r-1.
\end{aligned} \tag{38}$$

Now, it is possible to establish the following result:

Theorem 1. If

H1) the unmatched $\overline{\mathbf{g}}_1(\cdot),\ldots,\overline{\mathbf{g}}_{r-1}(\cdot)$ and matched $\overline{\mathbf{g}}_r(\cdot)$ perturbations are bounded, i.e., there exist a known scalar function $\beta_i(\mathbf{x})$ such that

$$\left\|\overline{\mathbf{g}}_i(t,\mathbf{x})\right\| \le \beta_i(\mathbf{x}), \quad i = 1,\ldots,r$$

then, there exist constants $h_1,\ldots,h_{r-1}$ such that the states of the system (38), are uniformly bounded, i. e.

$$\left\|\mathbf{z}_i(t)\right\| \le h_i, \; i = 1,\ldots r-1.$$

Moreover the perturbed system (38) reaches to a neighborhood of the output $\mathbf{y} = \mathbf{x}_1$ in finite time and remains in this neighborhood.

Proof. The proof is constructive and consists of r steps, begin with the step r.

Step r. First, the sliding variable $\mathbf{s}_r$ stability is analyzed. Considering the Lyapunov function $\mathbf{V}_r = \mathbf{s}_r^T\mathbf{s}_r$, it follows:

$$\dot{\mathbf{V}}_r = \mathbf{s}_r^T\left[-\rho_r(\mathbf{x})sign(\mathbf{s}_r) + \overline{\mathbf{g}}_r(t,\mathbf{x})\right]. \tag{39}$$

Under the assumption H1, the equation (39) can be written as

$$\begin{aligned}
\dot{\mathbf{V}}_r &= \mathbf{s}_r^T\left[-\rho_r(\mathbf{x})sign(\mathbf{s}_r) + \beta_r(\mathbf{x})\right] \\
&\le \|\mathbf{s}_r\|\left[-\rho_r(\mathbf{x}) + \|\beta_r(\mathbf{x})\|\right].
\end{aligned} \tag{40}$$

From (40) it is easy to see that under the condition

$$\rho_r(\mathbf{x}) > \|\beta_r(\mathbf{x})\|$$

the derivative $\dot{\mathbf{V}}_r$ is definite negative and the equivalent control $\mathbf{u}_{r,1eq}(t,\mathbf{x})$ satisfies

$$\mathbf{u}_{r,1eq} = -\overline{\mathbf{g}}_r(t,\mathbf{x})$$

rejecting the perturbation term $\overline{\mathbf{g}}_r(t,\mathbf{x})$ in the last block of (38). Now, it is necessary to analyze the stability of the last block. Using the Lyapunov function $\mathbf{V}_r = \frac{1}{2}\mathbf{z}_r^T\mathbf{z}_r$, leads to

$$\dot{\mathbf{V}}_r \le -k_r\|\mathbf{z}_r\|^2, \; k_r > 0.$$

Thus, the trajectories of the last variables vector $\mathbf{z}_r$ are asymptotically stable.

Step r-1. Proceeding in similar way as in previous step, the Lyapunov function $\mathbf{V}_{r-1} = \mathbf{s}_{r-1}{}^T\mathbf{s}_{r-1}$ is proposed, then

$$\dot{\mathbf{V}}_{r-1} = \mathbf{s}_{r-1}{}^T \left[-\rho_{r-1}(\overline{\mathbf{x}}_{r-1})sigm(\mathbf{s}_{r-1}) + \overline{\mathbf{g}}_{r-1}(t,\mathbf{x}) \right]. \tag{41}$$

In the region $\|\mathbf{s}_{r-1}\| > \varepsilon_{r-1}$ the equation (41) becomes

$$\begin{aligned} \dot{\mathbf{V}}_{r-1} &= \mathbf{s}_{r-1}{}^T \left[-\rho_{r-1}(\overline{\mathbf{x}}_{r-1})sign(\mathbf{s}_{r-1}) + \overline{\mathbf{g}}_{r-1}(t,\mathbf{x}) \right] \\ &\leq \|\mathbf{s}_{r-1}\| \left[-\rho_{r-1}(\overline{\mathbf{x}}_{r-1}) + \|\overline{\mathbf{g}}_{r-1}(t,\mathbf{x})\| \right] \end{aligned}. \tag{42}$$

Moreover, under the condition $\rho_{r-1}(\overline{\mathbf{x}}_{r-1}) > \|\overline{\mathbf{g}}_{r-1}(t,\mathbf{z})\|$, $\|\mathbf{s}_{r-1}\|$ will be decreasing until it reaches the set $\{\|\mathbf{s}_{r-1}\| \leq \varepsilon_{r-1}\}$ in a finite time and it remains inside. The upper bound of this reaching time can be calculated by using the comparison lemma (Khalil, 1996) as follows:

$$t_{r-1} \leq \|\mathbf{s}_{r-1}(0)\| - \varepsilon_{r-1}.$$

Furthermore the equivalent control $\mathbf{x}_{r-1,1eq}$ fulfills

$$\dot{\mathbf{s}}_{r-1} = \mathbf{x}_{r-1,1eq} + \overline{\mathbf{g}}_{r-1}(t,\mathbf{z}) = \varepsilon_{r-1}\boldsymbol{\gamma}_{r-1} \tag{43}$$

where $\varepsilon_{r-1}\boldsymbol{\gamma}_{r-1}$ is the error introduced by using the control law (29). To analyze the stability of the *r-1* block of the system (38), the Lyapunov function $\mathbf{V}_{r-1} = \frac{1}{2}\mathbf{z}_{r-1}{}^T\mathbf{z}_{r-1}$ is considered and its time derivative is given by

$$\begin{aligned} \dot{\mathbf{V}}_{r-1} &= \mathbf{z}_{r-1}{}^T \left[-k_{r-1}\mathbf{z}_{r-1} + \mathbf{E}_{r-1}\mathbf{z}_r - \rho_{r-1}(\overline{\mathbf{x}}_{r-1})sigm\left(\mathbf{s}_{r-1}/\varepsilon_{r-1}\right) + \overline{\mathbf{g}}_{r-1}(t,\mathbf{z}) \right] \\ &\leq -k_{r-1}\|\mathbf{z}_{r-1}\|^2 + \|\mathbf{z}_{r-1}\| \left[\|\mathbf{z}_r\| - \rho_{r-1}(\overline{\mathbf{x}}_{r-1})sigm\left(\mathbf{s}_{r-1}/\varepsilon_{r-1}\right) + \overline{\mathbf{g}}_{r-1}(t,\mathbf{z}) \right]. \end{aligned}$$

In the region $\|\mathbf{s}_{r-1}\| > \varepsilon_{r-1}$, the derivative $\dot{\mathbf{V}}_{r-1}$ becomes

$$\begin{aligned} \dot{\mathbf{V}}_{r-1} &\leq -k_{r-1}\|\mathbf{z}_{r-1}\|^2 + \|\mathbf{z}_{r-1}\| \left[\|\mathbf{z}_r\| - \rho_{r-1}sign\left(\mathbf{s}_{r-1}/\varepsilon_{r-1}\right) + \overline{\mathbf{g}}_{r-1}(t,\mathbf{z}) \right] \\ &\leq -k_{r-1}\|\mathbf{z}_{r-1}\|^2 + \|\mathbf{z}_{r-1}\| \left[\|\mathbf{z}_r\| + \dot{\mathbf{s}}_{r-1} \right] \end{aligned}$$

and considering (43), it can be rewritten as

$$\dot{\mathbf{V}}_{r-1} \leq -k_{r-1}\|\mathbf{z}_{r-1}\|^2 + \|\mathbf{z}_{r-1}\| \left[\|\mathbf{z}_r\| + \varepsilon_{r-1}\boldsymbol{\gamma}_{r-1} \right]. \tag{44}$$

Suppose that $\varepsilon_{r-1}\boldsymbol{\gamma}_{r-1}$ satisfies the following bound:

$$\varepsilon_{r-1}\boldsymbol{\gamma}_{r-1} \leq \alpha_{r-1}\|\mathbf{z}_{r-1}\| + \beta_{r-1},\ \alpha_{r-1},\ \beta_{r-1} \in R .$$

Then it is possible to present the equation (44) of the form

$$\dot{\mathbf{V}}_{r-1} \le -k_{r-1}\|\mathbf{z}_{r-1}\|^2 + \|\mathbf{z}_{r-1}\|\big[\|\mathbf{z}_r\| + \alpha_{r-1}\|\mathbf{z}_{r-1}\| + \beta_{r-1}\big]$$
$$\le -\|\mathbf{z}_{r-1}\|\big[(k_{r-1} - \alpha_{r-1})\|\mathbf{z}_{r-1}\| - \|\mathbf{z}_r\| - \beta_{r-1}\big]$$

which is negative in the region

$$\|\mathbf{z}_{r-1}\| > \delta_{r-1}\|\mathbf{z}_r\| + \lambda_{r-1} \tag{45}$$

where $\delta_{r-1} = \dfrac{1}{k_{r-1} - \alpha_{r-1}}$ and $\lambda_{r-1} = \dfrac{\beta_{r-1}}{k_{r-1} - \alpha_{r-1}}$. Moreover δ_{r-1} and λ_{r-1} are positive for $k_{r-1} > \alpha_{r-1}$. Thus the trajectories of the vector state enter ultimately in the region defined by

$$\|\mathbf{z}_{r-1}\| \le \delta_{r-1}\|\mathbf{z}_r\| + \lambda_{r-1}.$$

Step i. The step *r-1* can be generalized for the block *i*, with *i=r-1, r-2, ..., 1*.
In the region $\|\mathbf{s}_i\| > \varepsilon_i$ the derivative of the Lyapunov function $\mathbf{V}_i = \mathbf{s}_i^T\mathbf{s}_i$, is calculated as

$$\dot{\mathbf{V}}_i = \mathbf{s}_i^T\big[-\rho_i(\overline{\mathbf{x}}_i)sign(\mathbf{s}_i) + \overline{\mathbf{g}}_i(t,\mathbf{x})\big]$$
$$\le \|\mathbf{s}_i\|\big[-\rho_i(\overline{\mathbf{x}}_i) + \|\overline{\mathbf{g}}_i(t,\mathbf{x})\|\big]. \tag{46}$$

Again, under the condition $\rho_i(\overline{\mathbf{x}}_i) > \|\overline{\mathbf{g}}_i(t,\mathbf{z})\|$, $\mathbf{s}_i$ enter in the region $\{\|\mathbf{s}_i\| \le \varepsilon_i\}$ in a finite time given by

$$t_i \le \|\mathbf{s}_i(0)\| - \varepsilon_i.$$

The equivalent control $\mathbf{x}_{i,1eq}$ satisfies

$$\dot{\mathbf{s}}_i = \mathbf{x}_{i,1eq} + \overline{\mathbf{g}}_i(t,\mathbf{z}) = \varepsilon_i\gamma_i. \tag{47}$$

Considering the function $\mathbf{V}_i = \frac{1}{2}\mathbf{z}_i^T\mathbf{z}_i$ inside the subspace $\|\mathbf{s}_i\| > \varepsilon_i$, it follows

$$\dot{\mathbf{V}}_i \le -k_i\|\mathbf{z}_i\|^2 + \|\mathbf{z}_i\|\bigg[\|\mathbf{z}_{i+1}\| - \rho_i(\overline{\mathbf{x}}_i)sign\bigg(\frac{\mathbf{s}_i}{\varepsilon_i}\bigg) + \overline{\mathbf{g}}_i(t,\mathbf{z})\bigg]$$
$$\le -k_i\|\mathbf{z}_i\|^2 + \|\mathbf{z}_i\|\big[\|\mathbf{z}_{i+1}\| + \dot{\mathbf{s}}_i\big]$$

and with (47), $\dot{\mathbf{V}}_i$ becomes

$$\dot{\mathbf{V}}_i \le -k_i\|\mathbf{z}_i\|^2 + \|\mathbf{z}_i\|\big[\|\mathbf{z}_{i+1}\| + \varepsilon_i\gamma_i\big]$$

Supposing that $\varepsilon_i\gamma_i$ fulfills

$$\varepsilon_i\gamma_i \le \alpha_i\|\mathbf{z}_i\| + \beta_i, \ \alpha_i, \beta_i \in R$$

then

$$\dot{\mathbf{V}}_i \le -\|\mathbf{z}_i\|\big[(k_i - \alpha_i)\|\mathbf{z}_i\| - \|\mathbf{z}_{i+1}\| - \beta_i\big]$$

which is negative in the region

$$\|\mathbf{z}_i\| > \delta_i \|\mathbf{z}_{i+1}\| + \lambda_i$$

where $\delta_i = \dfrac{1}{k_i - \alpha_i}$ and $\lambda_i = \dfrac{\beta_i}{k_i - \alpha_i}$, which are positive for $k_i > \alpha_i$. Therefore a solution for $\mathbf{z}_i$ is ultimately bounded by

$$\|\mathbf{z}_i\| \le \delta_i \|\mathbf{z}_{i+1}\| + \lambda_i \, .$$

Then with the bound

$$\varepsilon_i \gamma_i \le \alpha_i \|\mathbf{z}_i\| + \beta_i, \ i = 1, 2, \ldots, r-1$$

the convergence region is defined by:

$$\|\mathbf{z}_{r-1}\| > \delta_{r-1} \|\mathbf{z}_r\| + \lambda_{r-1} := h_{r-1}$$
$$\|\mathbf{z}_{r-2}\| > \delta_{r-2} \|\mathbf{z}_{r-1}\| + \lambda_{r-2} := h_{r-2}$$
$$\vdots$$
$$\|\mathbf{z}_1\| > \delta_1 \|\mathbf{z}_2\| + \lambda_1 := h_1.$$

4. PES control design

Since the subsystem (10) has the NBC form, the ISM technique will be applied to design a robust controller for EPS. First, the rotor speed stability will be achieved. Secondly, the terminal voltage generator controller is outlined. Then, a switching logic is proposed to coordinate the operation of both controllers. Finally, an EPS observer is introduced.

4.1 Integral Sliding Mode Speed Stabilizer (ISMSS)

To achieve the first control objective, that is, the rotor speed stability enhancement, define the control error as (Huerta-Avila et al., 2007a, Huerta-Avila et al., 2007b)

$$z_{2i} = x_{2i} - \omega_b. \tag{48}$$

Taking the time derivative of (48) along the trajectories of (10) yields

$$\dot{z}_{2i} = f_{\omega i}(\mathbf{x}_i, \mathbf{v}_i) - q_i(\mathbf{x}_i, \mathbf{v}_i)x_{3i} + g_{2i}(\mathbf{x}_i, \mathbf{v}_i, T_{mi}) \tag{49}$$

where $\mathbf{x}_i = (\mathbf{x}_{1i} \ \mathbf{x}_{2i})^T$, $q_i(t) > 0$, $\forall t > 0$.
Redefine the virtual control, x_{3i} in (49) as

$$x_{3i} = x_{3i,0} + x_{3i,1} \, . \tag{50}$$

The desired dynamics for z_{2i} is chosen of the form

$$\dot{z}_{2i} = -k_i z_{2i} + z_{3i} + q_i(\mathbf{x}_i, \mathbf{v}_i)x_{3i,1} + g_{2i}(\mathbf{x}_i, \mathbf{v}_i, T_{mi}), \ k_i > 0 \tag{51}$$

These dynamics can be obtained by choosing $x_{3i,0}$ as

$$x_{3i,0} = -\left[q_i(\mathbf{x}_i, \mathbf{v}_i)\right]^{-1}\left[f_{\omega i}(\mathbf{x}_i, \mathbf{v}_i) + k_i z_{2i} - z_{3i}\right] \tag{52}$$

where z_{3i} is a new variable. To design the second part of (50), $x_{3i,1}$, define a pseudo-sliding variable s_{2i} as

$$s_{2i} = z_{2i} + \sigma_{2i}$$

with the integral variable σ_{2i}. Using (49)-(51), it follows

$$\dot{s}_{2i} = -k_{0i} z_{2i} + z_{3i} + q_i(\mathbf{x}_i, \mathbf{v}_i)x_{3i,1} + g_{2i}(\mathbf{x}_i, \mathbf{v}_i, T_{mi}) + \dot{\sigma}_{2i} \tag{53}$$

Choosing

$$\dot{\sigma}_{2i} = k_{2i} z_{2i} - z_{3i}, \qquad \sigma_{2i}(0) = -z_{2i}(0)$$

the equation (53) becomes

$$\dot{s}_{2i} = g_{2i}(\mathbf{x}_i, \mathbf{v}_i, T_{mi}) + q_i(\mathbf{x}_i, \mathbf{v}_i)x_{3i,1}.$$

Select $x_{3i,1}$ of the form

$$x_{3i,1} = -\rho_{2i} sigm(s_{2i}/\varepsilon_i), \quad \rho_{2i} > 0. \tag{54}$$

Then, the sliding variable $s_{\omega i} = z_{3i}$ is defined from (50), (52) and (54) of the form

$$s_{\omega i} = f_{\omega i}(\mathbf{x}_i, \mathbf{v}_i) + q_i(\mathbf{x}_i, \mathbf{v}_i)x_{3i} + k_{0i} z_{2i} + \rho_{2i} sigm(s_{2i}/\varepsilon_i). \tag{55}$$

Thus, straightforward algebra reveals

$$\dot{s}_{\omega i} = f_{si}(\mathbf{x}_i, \mathbf{v}_i) + b_{si}(\mathbf{x}_i, \mathbf{v}_i)v_{fi} \tag{56}$$

where $f_{si}(\cdot)$ is a continuous function and $b_{si}(\cdot) = q_i(\cdot)b_{i4}$.

Considering (56), under the condition

$$k_{gi} > \left|b_{si}^{-1}(\mathbf{x}_i, \mathbf{v}_i)f_{si}(\mathbf{x}_i, \mathbf{v}_i)\right|$$

the proposed discontinuous control law

$$v_{fi} = -k_{gi} sign(s_{\omega i}), \qquad k_{gi} > 0 \tag{57}$$

ensures the convergence of the state to the manifold $s_{\omega i} = z_{3i} = 0$ (55) in a finite time (Utkin et al., 1999). The sliding mode motion on this manifold is governed by the reduced order system

$$\begin{aligned}
\dot{x}_{1i} &= z_{2i}, \\
\dot{z}_{2i} &= -k_{0i} z_{2i} - \rho_{2i} sigm(s_{2i}/\varepsilon_i) + g_{2i}(\mathbf{x}_i, \mathbf{v}_i, T_{mi}) \\
\dot{s}_{2i} &= -\rho_{2i} sigm(s_{2i}/\varepsilon_i) + g_{2i}(\mathbf{x}_i, \mathbf{v}_i, T_{mi})
\end{aligned} \tag{58}$$

$$\dot{\mathbf{x}}_{2i} = \mathbf{A}_{2i}\mathbf{x}_{2i} + \tilde{\mathbf{f}}_{2i}(\mathbf{x}_i, \mathbf{v}_i) \tag{59}$$

Now, choosing ε_i be sufficiently small and under the condition

$$\rho_{2i} > \left| g_{2i}(\mathbf{x}_i, \mathbf{v}_i, T_{mi}) \right|$$

a quasi sliding mode motion is enforced in a small ε_i-vicinity of $s_{2i} = 0$. Thus, if $\varepsilon_i \to 0$ then the perturbation term $g_{2i}(\mathbf{x}_i, \mathbf{v}_i, T_{mi})$ in (59) is rejected, and the linearized mechanical dynamics can be represented as

$$\begin{aligned}\dot{x}_{1i} &= z_{2i} \\ \dot{z}_{2i} &= -k_{0i} z_{2i}\end{aligned} \tag{60}$$

with the desired eigenvalue $-k_{0i}$.

The equation (59) represents the rotor flux internal dynamics. The matrix $\mathbf{A}_{2i}$ is Hurwitz and the nonvanishing perturbation $\tilde{\mathbf{f}}_{2i}(\mathbf{x}_i, \mathbf{V}_i)$ is a continuous function. Therefore there exists an admissible region where a solution $\mathbf{x}_{2i}(t)$ of (60) is ultimately bounded (Khalil, 1996). Moreover, the control error z_{2i} (48) tends exponentially to zero, and the angle x_{1i} tends to a constant steady state, δ_{ssi}.

Remark: Since the initial conditions of the EPS are availabe, it is possible to apply the integral sliding modes technique.

4.2 Sliding Mode Voltage Regulator

In this subsection, the voltage regulation problem is studied. The terminal voltage, v_{gi}, is defined as

$$v_{gi}^{\,2} = v_{di}^{\,2} + v_{qi}^{\,2} . \tag{61}$$

Using (8), v_{di} and v_{qi} are calculated of the form

$$\mathbf{v}_i = \begin{bmatrix} v_{di} \\ v_{qi} \end{bmatrix} = -\mathbf{H}_i^{-1}[\mathbf{A}_{zi}\mathbf{i}_i + \mathbf{f}_{zi}(\mathbf{x}_i)] . \tag{62}$$

Then, the dynamics for terminal voltage, v_{gi} can be obtained from (61), (62), (6), and (7) as (Loukianov, et al., 2006)

$$\dot{v}_{gi} = f_{vi}(\mathbf{x}_i, \mathbf{i}_i) + b_{vi}v_{fi} + g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi}) \tag{63}$$

where $f_{vi}(\mathbf{x}_i, \mathbf{i}_i)$ is the nominal part of the voltage dynamics and the perturbation term $g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi})$ contains parameter variations and external disturbances, $b_{vi} = h_{i2}b_{i4}$, $b_{vi}(t), \ \forall t \geq 0$. For the details see *Appendix*.

Defining the voltage control error

$$e_{vi} = v_{gi} - v_{refi}$$

and the control input v_{fi}

$$v_{fi} = v_{fi,0} + v_{fi,1} \tag{64}$$

we have

$$\dot{e}_{vi} = f_{vi}(\mathbf{x}_i, \mathbf{i}_i) + b_{vi}v_{fi,0} + b_{vi}v_{fi,1} + g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi}) \tag{65}$$

where v_{refi} is the constant reference voltage. To design a robust controller we use the integral sliding mode approach (Utkin et al., 1999). In order to reject the perturbation term $g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi})$ in (65) a sliding variable $s_{vi} \in R$ is formulated as

$$s_{vi} = e_{vi} + \sigma_{vi} \tag{66}$$

with the integral variable $\sigma_{vi} \in R$. Then from (65) and (66) it follows

$$\dot{s}_{vi} = f_{vi}(\mathbf{x}_i, \mathbf{i}_i) + b_{vi}v_{fi,0} + b_{vi}v_{fi,1} + g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi}) + \dot{\sigma}_{vi} \tag{67}$$

Choosing

$$\dot{\sigma}_{vi} = -f_{vi}(\mathbf{x}_i, \mathbf{i}_i) - b_{vi}v_{fi,0}, \quad \sigma_{vi}(0) = -e_{vi}(0)$$

results in

$$\dot{s}_{vi} = b_{vi}v_{fi,1} + g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi}) \tag{68}$$

Select $v_{fi,1}$ in (68) as

$$v_{fi,1} = -\rho_{2i}sign(s_{vi}), \quad \rho_{2i} > 0. \tag{69}$$

From (68), under the condition $\rho_{2i} > \left| b_{vi}^{-1}g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi}) \right|$ a sliding mode is enforced on the manifold $s_{vi} = 0$ (66) from the initial time instant $t = 0$. The equivalent control

$$v_{fi,1eq} = -b_{vi}^{-1}g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi})$$

calculated as a solution of $\dot{s}_{vi} = 0$ (67), compensates exactly the perturbation term $g_{vi}(\mathbf{x}_i, \mathbf{i}_i, T_{mi})$ in (63) (Utkin et al., 1999), and the sliding mode motion is described by the unperturbed system

$$\dot{e}_{vi} = f_{vi}(\mathbf{x}_i, \mathbf{i}_i) + b_{vi}v_{fi,0}. \tag{70}$$

Now, it is necessary to achieve the terminal voltage regulation, i. e. the control input $v_{fi,0}$ in (70) is selected of the form

$$v_{fi,0} = -k_g sign(e_{vi}) \tag{71}$$

From (70) and (71), we have

$$\dot{e}_{vi} = f_{vi}(\mathbf{x}_i, \mathbf{i}_i) - k_g b_{vi} sign(e_{vi}) . \tag{72}$$

Then, under the condition

$$k_g > \left| b_{vi}^{-1} f_{vi}(\mathbf{x}_i, \mathbf{i}_i) \right| \tag{73}$$

the terminal voltage control error e_{vi} tends to zero in a finite time (Utkin et al., 1999).

4.3 Control logic

There are two control objectives: the rotor speed stabilization and the terminal voltage regulation for each generator in the EPS. However, only one control input is available, the excitation voltage v_{fi}. Then, the following control logic is proposed:

$$v_{fi} = \begin{cases} -k_{gi} sign(s_{\omega i}), & if \ \ |s_{\omega i}| > \beta_i \\ -k_g sign(e_{vi}) - \rho_{2i} sign(s_{vi}), & if \ \ |s_{\omega i}| \le \beta_i \end{cases}, \quad \beta_i = \begin{cases} \beta_{1i} \ if \ \ |\beta_{vi}| > \beta_{3i} \\ \beta_{2i} \ if \ \ |\beta_{vi}| \le \beta_{3i} \end{cases} \tag{74}$$

with $\beta_{2i} < \beta_{1i}$. Basically, a hierarchical control action through the proposed logic (74) is presented. First, the mechanical dynamics is stabilized by means of the ISMSS, yielding the stabilization of the speed switching manifold $s_{\omega i}$. When $s_{\omega i}$ reaches to a region defined by β_{1i}, the control resources are dedicated to stabilize the terminal voltage error β_{vi}. After the convergence of β_{vi} such that $|\beta_{vi}| \le \beta_{3i}$, the control logic reduces the $s_{\omega i}$ boundary layer width from β_{1i} to β_{2i}. Thus, the controller maintains the value of $s_{\omega i}$ within desired accuracy $|s_{\omega i}| \le \beta_{2i}$ and $|e_{vi}| \le \beta_{3i}$. Figure 1 shows the schematic diagram of the proposed controller.
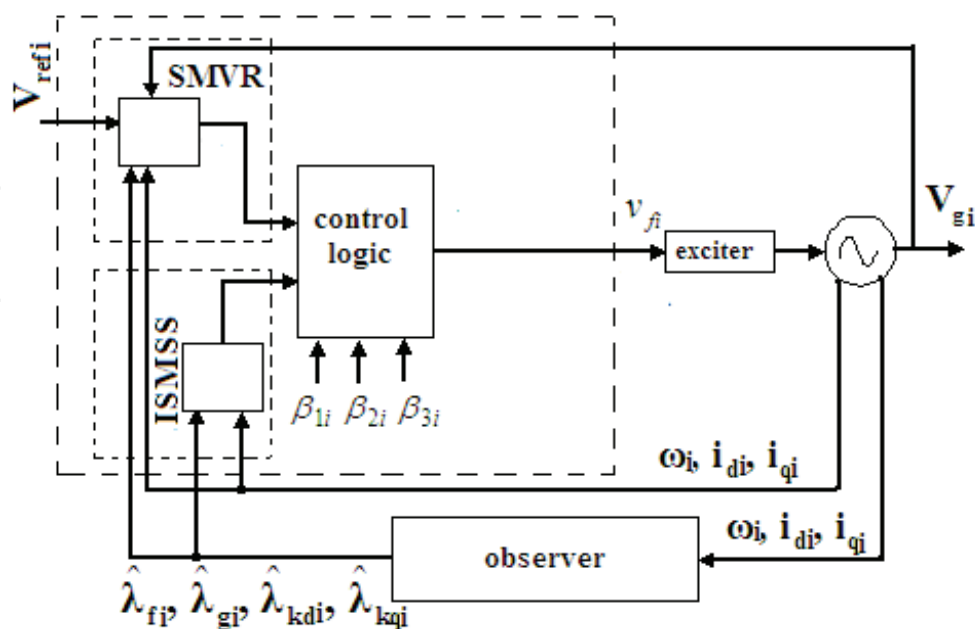

Figure 1. Proposed controller schematic diagram

4.4 EPS observer

Since the control scheme (74) needs the values of the rotor fluxes, it is neccesary to design a observer for the EPS. Assume that the power angle, x_{1i}, rotor speed, x_{2i} and stator currents i_{di} and i_{qi} can be measured.

The rotor fluxes x_{3i}, x_{4i}, x_{5i} and x_{6i} can be estimated by means of the following observer:

$$
\begin{bmatrix} \dot{\hat{x}}_{3i} \\ \dot{\hat{x}}_{4i} \\ \dot{\hat{x}}_{5i} \\ \dot{\hat{x}}_{6i} \end{bmatrix} = \begin{bmatrix} b_{i1}\hat{x}_{3i} + b_{i2}\hat{x}_{5i} + b_{i3}i_{di} \\ c_{i1}\hat{x}_{4i} + c_{i2}\hat{x}_{6i} + c_{i3}i_{qi} \\ d_{i1}\hat{x}_{3i} + d_{i2}\hat{x}_{5i} + d_{i3}i_{di} \\ r_{i1}\hat{x}_{4i} + r_{i2}\hat{x}_{6i} + r_{i3}i_{qi} \end{bmatrix} + \begin{bmatrix} b_{i4} \\ 0 \\ 0 \\ 0 \end{bmatrix} v_{fi} \tag{75}
$$

where $\hat{\mathbf{x}}_i = \begin{bmatrix} \hat{x}_{3i}, & \hat{x}_{4i}, & \hat{x}_{5i}, & \hat{x}_{6i} \end{bmatrix}^T$ are the estimate of the rotor fluxes. The convergence of the observer (75) can be analyzed by the error dynamics obtained from (75) and (6), given by the linear system:

$$
\dot{\mathbf{e}}_i = \mathbf{A}_{0i} \tag{76}
$$

with $\mathbf{e}_i = [e_{3i},\ldots,e_{6i}]$, $e_{ji} = x_{ji} - \hat{x}_{ji}$, $j = 3,\ldots,6$, $\mathbf{A}_{0i} = \begin{bmatrix} b_{i1} & 0 & b_{i2} & 0 \\ 0 & c_{i1} & 0 & c_{i2} \\ d_{i1} & 0 & d_{i2} & 0 \\ 0 & r_{4i} & 0 & r_{i2} \end{bmatrix}$.

The eigenvalues of the matrix $\mathbf{A}_{0i}$ calculated as

$$
p_{1,2i} = \frac{1}{2}(c_{i1} + r_{i2}) \pm \frac{1}{2}\sqrt{c_{i1}{}^2 + r_{i2}{}^2 - 2c_{i1}r_{i2} + 4c_{i2}r_{i1}},
$$

$$
p_{3,4i} = \frac{1}{2}(b_{i1} + d_{i2}) \pm \frac{1}{2}\sqrt{b_{i1}{}^2 + d_{i2}{}^2 - 2b_{i1}d_{i2} + 4b_{i2}d_{i1}}
$$

are real and negative. Therefore, the solution of the subsystem (76) is exponentially stable. The resulting estimates rotor fluxes are employed in the control logic (74) instead of the real variables.

5. Simulations results

The proposed control algorithm was tested on the equivalent model of the WSCC, (Western System Coordinating Council, Nine buses, three generators, three loads), fig. 2, (Anderson & Fouad, 1994). The parameters of the generators and network used in the simulation were taken from (Anderson & Fouad, 1994) (see *Appendix*).

Figures 3-8 depict results under four different events:

a.   at $t = 1$ s, experienced a pulse 0.5 p.u. for 1 s in the generator 2,

b.   at $t = 4$ s until $t = 4.15$ s, a three-phase short circuit is simulated in the terminals of generator 1,

c.   at $t = 10$ s, a three-phase short circuit during 150 ms is applied in the line 5-7 (see fig. 2); the fault is cleared by opening the line, and

d. at *t=15 s*, it was introduced a parametric variations, by incrementing up to 25% the parameters L_{mi} in the generators.

Figures 3 and 5 show the relative angles and speed response of the close-loop system, respectively with a type I excitation system with PSS (Anderson & Fouad, 1994, EPRI, 1977). Figure 8 show the proposed observer converge in spite of perturbations.

Figures 4-7 reveal some important aspects:

1. The state variables fastly reach a steady state condition after small and large disturbances, showing the robust stability of the closed-loop system.
2. The controller is able to improve both, the power system stabilization and the post-fault terminal voltage regulation.

Comparing the transient speed response of the generators in case of ISMSS /SMVR and AVR/ PSS controllers shown in Figures 6 and 5 respectively, we have some important observations:

1. The traditional AVR/PSS stabilizes the system. However, the transient response of the classical controller is more oscillatory than the response given by the proposed nonlinear ISMSS /SMVR one since the latter adds significantly better damping in the power oscillations. It is possible to observe that the overshoot and settling time are reduced as well.
2. The performance of the ISMSS /SMVR is robust under different operating conditions. Figures 4 and 6 show clearly that the robustness of the controller under generators parameters variations and changes on the network configuration, such as disconnection of lines and incrementing and /or decrementing of loads. Thus the performance of the proposed ISMSS /SMVR controller tends to be unaffected.
3. Since the ISMSS /SMVR adds additional damping, the transient response controller is better compared to other ones (see for instance (Ahmed at al., 1996)). With the ISMSS /SMVR, the settling time is lesser and the overshot is shorter than the shown by the suboptimal robust controller presented in (Ahmed at al., 1996).
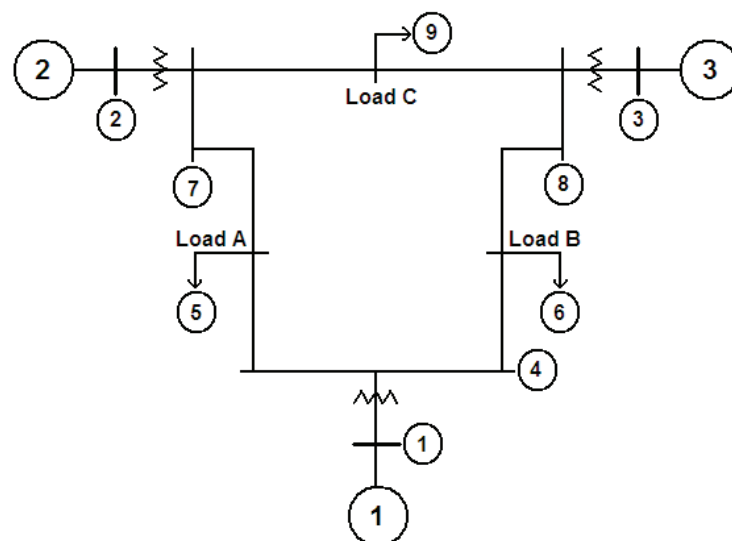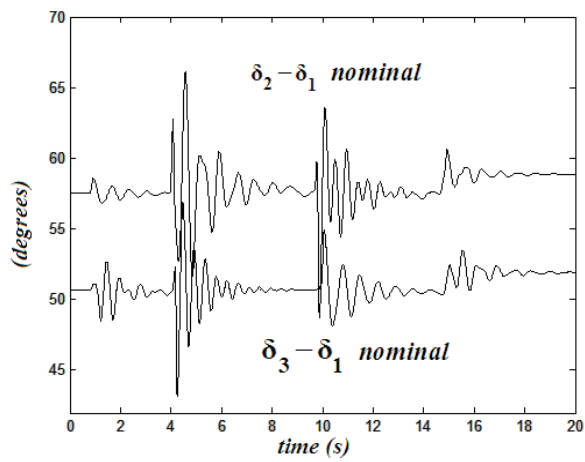

Figure 2. WSCC diagram

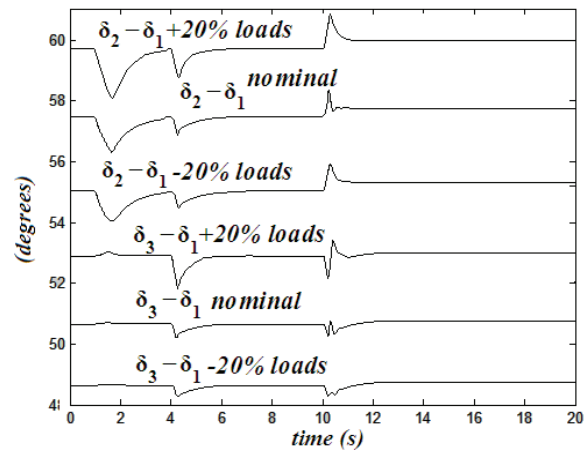Figure 3. Relative angles response with classical control

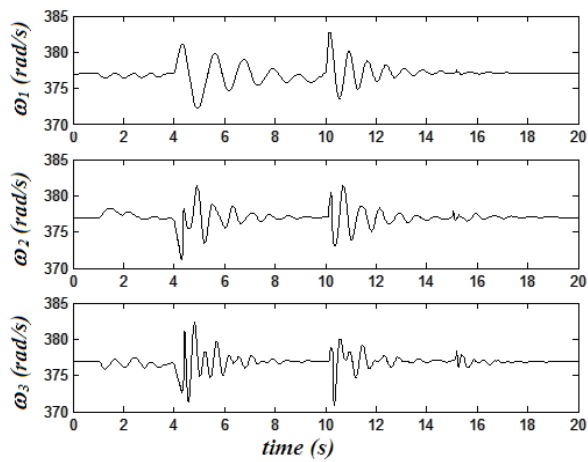Figure 4. Relative angles response with the proposed controller

Figure 5. Speed of the three generators response with classical control
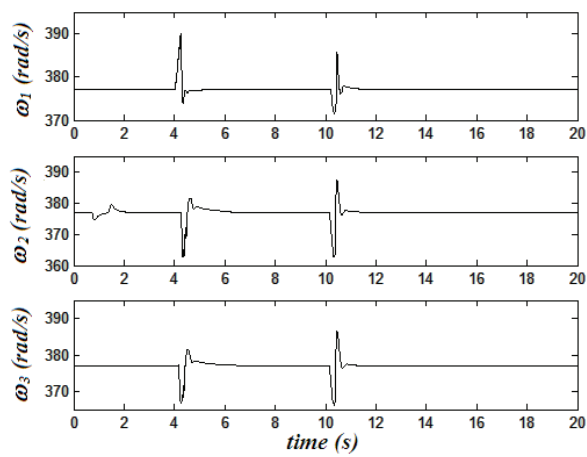
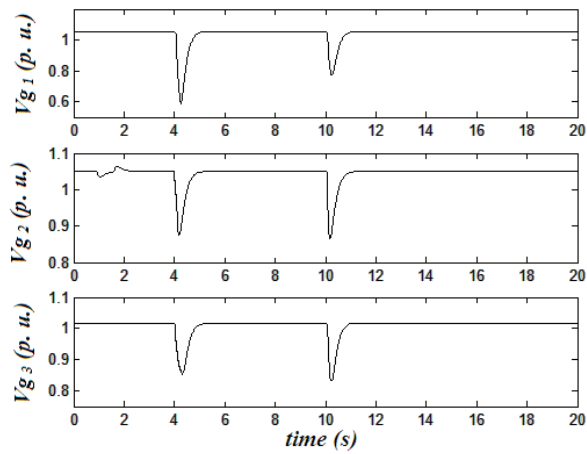Figure 6. Speed of the three generators response with the proposed controller

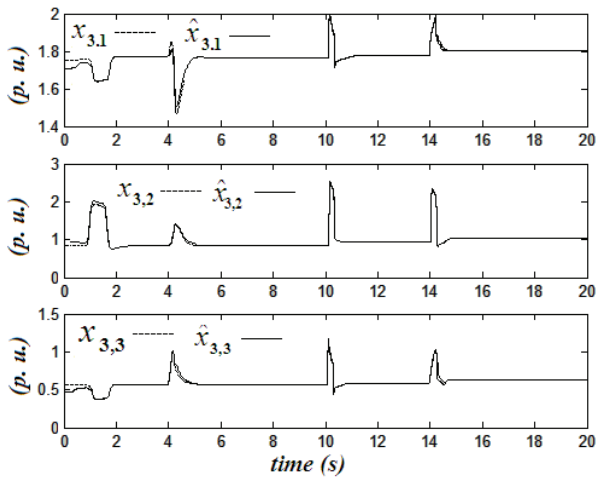Figure 7. Terminal voltage of the three generators response with the proposed controller

Figure 8. Field flux of the three generators response

6. Conclusions

The ISM with block control technique as a novel nonlinear control technique for the class of nonlinear systems presented in the NBC form was presented. The control methodology was explained step-by-step, and the stability conditions were found for each step. The ISM technique is robust under unknown but bounded matched and/or unmatched perturbations.

Then, in order to test the effectiveness of the ISM technique, a controller for EPS was designed. A plant model used for control is fully detailed nonlinear, and this model takes into account all interactions in power system between the electrical and mechanical dynamics and load constraints. With the proposed control scheme, the only local information is required. The stability analysis of the closed-loop EPS controller, including an observer was carried out. The designed ISMSS/SMVR was tested through simulation under the most important perturbations in the EPS:

1. Variation of the mechanical torque.
2. Large fault (a 150 ms short circuit).
3. Loads variations.
4. Generator parameter variations.

The simulation results show that the sliding mode controller with the proposed logic is able to achieve the mechanical dynamics and the generator terminal voltages robust stability under small and large disturbances.

The proposed performance of the nonlinear ISMSS/ SMVR control system (74) is independent from the operating point of the system. It is important to note that the proposed nonlinear control scheme ensures cancellation of the interactions between the subsystems provided an additional damping with respect to classical controllers.

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8. Appendix

8.1 Matrices used in generator model (1)

$$\mathbf{R} = diag\begin{bmatrix} R_f & R_g & R_{kd} & R_{kq} & -R_s & -R_s \end{bmatrix}, \mathbf{L} = \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{bmatrix}, \mathbf{W}(\omega) = \begin{bmatrix} 0 & 0 \\ 0 & I(\omega) \end{bmatrix} \in R^{6 \times 6},$$

$$I(\omega) = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}, \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} L_f & 0 & L_{md} & 0 \\ 0 & L_g & 0 & L_{mq} \\ L_{md} & 0 & L_{kd} & 0 \\ 0 & L_{mq} & 0 & L_{kq} \end{pmatrix} & \begin{pmatrix} -L_{md} & 0 \\ 0 & -L_{mq} \\ -L_{md} & 0 \\ 0 & -L_{mq} \end{pmatrix} \\ \begin{pmatrix} L_{md} & 0 & L_{md} & 0 \\ 0 & L_{mq} & 0 & L_{mq} \end{pmatrix} & \begin{pmatrix} -L_d & 0 \\ 0 & -L_q \end{pmatrix} \end{bmatrix}.$$

L_d and L_q are the direct-axis and quadrature-axis self-inductances, L_f is the field self-inductance, L_g, L_{kd} and L_{kq} are the damper windings self-inductances, L_{md} and L_{mq} are the direct-axis and quadrature-axis magnetizing inductances

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_4 & 0 \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{bmatrix}, \quad \begin{array}{l} \mathbf{T}_{21} = -[\mathbf{I}_2 - \mathbf{L}_{22}^{-1}\mathbf{L}_{21}\mathbf{L}_{11}^{-1}\mathbf{L}_{12}]^{-1}\mathbf{L}_{22}^{-1}\mathbf{L}_{21}\mathbf{L}_{11}^{-1}, \\ \mathbf{T}_{22} = -[\mathbf{I}_2 - \mathbf{L}_{22}^{-1}\mathbf{L}_{21}\mathbf{L}_{11}^{-1}\mathbf{L}_{12}]^{-1}\mathbf{L}_{22}^{-1}, \end{array} \quad \mathbf{I}_2 \text{ and } \mathbf{I}_4 \text{ are identity matrices of}$$

dimension 2 and 4, respectively.

8.2 Generators parameters

$$a_{i1} = \frac{L''_{di} - l_{ai}}{l_{fi}}, a_{i2} = -\frac{L''_{qi} - l_{ai}}{l_{gi}}, a_{i3} = \frac{L''_{di} - l_{ai}}{l_{kdi}}, a_{i4} = -\frac{L''_{qi} - l_{ai}}{l_{kqi}}, a_{i5} = L''_{qi} - L''_{di},$$

$$b_{i1} = -\frac{1}{\tau'_{d0i}}\left(1 + L_{mdi}\frac{L''_{di} - l_{ai}}{l_{kdi}l_{fi}}\right), b_{i2} = \frac{L_{mdi}}{\tau'_{d0i}}\frac{L''_{di} - l_{ai}}{(L'_{di} - l_{ai})l_{kdi}}, b_{i3} = -\frac{L_{mdi}}{\tau'_{d0i}}\frac{L''_{di} - l_{ai}}{L'_{di} - l_{ai}}, b_{i4} = \omega_s,$$

$$c_{i1} = -\frac{1}{\tau'_{q0i}}\left(1 + \frac{L''_{qi} - l_{ai}}{l_{kqi}l_{gi}}L_{mqi}\right), c_{i2} = \frac{L''_q - l_a}{(L'_q - l_a)l_{kq}}\frac{L_{mq}}{\tau'_{q0}}, c_{i3} = -\frac{L_{mqi}}{\tau'_{q0i}}\left(\frac{L''_{qi} - l_{ai}}{L'_{qi} - l_{ai}}\right), d_{i1} = \frac{L'_{di} - l_{ai}}{\tau''_{d0i}l_{fi}},$$

$$d_{i2} = -\frac{1}{\tau''_{d0i}}, d_{i3} = -\frac{L'_{di} - l_{ai}}{\tau''_{d0i}}, r_{i1} = \frac{L'_{qi} - l_{ai}}{\tau''_{q0i}l_{gi}}, r_{i2} = -\frac{1}{\tau''_{q0i}}, r_{i3} = -\frac{L'_{qi} - l_{ai}}{\tau''_{q0i}}, h_{i1} = -\frac{\omega_s}{L''_{di}},$$

$$h_{i2} = -\frac{1}{L''_{di}\tau'_{d0i}}\left(1 + L_{mdi}\frac{L'_{di} - l_{ai}}{l_{fi}l_{kdi}}\right)\frac{L'_{di} - l_{ai}}{l_{fi}} + \frac{1}{L''_{di}\tau''_{d0i}}\frac{L''_{di} - l_{ai}}{L''_{di}l_{kdi}}\frac{L'_{di} - l_{ai}}{l_{fi}}, h_{i4} = -\frac{1}{L''_{di}}\frac{L''_{qi} - l_{ai}}{l_{gi}},$$

$$h_{i5} = -\frac{L''_{qi} - l_{ai}}{L''_{di}l_{kqi}}, h_{i3} = \frac{L_{mdi}}{L''_{di}\tau'_{d0i}}\frac{L''_{di} - l_{ai}}{(L'_{di} - l_{ai})l_{kdi}}\frac{L''_{di} - l_{ai}}{l_{fi}} - \frac{1}{L''_{di}\tau''_{d0i}}\frac{L''_{di} - l_{ai}}{L''_{di}l_{kdi}}, h_{i6} = \frac{L''_{qi}}{L''_{di}}, h_{i8} = \frac{\omega_s(L''_{di} - l_{ai})}{L''_{di}l_{fi}},$$

$$k_{i1} = -\frac{\omega_s}{L''_{di}}, h_{i7} = -\frac{\omega_s r_{ai}}{L''_{di}} - \frac{L_{mdi}}{L''_{di}\tau'_{d0i}}\frac{L''_{di} - l_{ai}}{l_{fi}}\frac{L'_{di} - l_{ai}}{l_{fi}}\frac{L''_{di} - l_{ai}}{L'_{di} - l_{ai}} - \frac{(L'_{di} - l_{ai})}{L''_{di}\tau'_{d0i}}\frac{L''_{di} - l_{ai}}{L''_{di}l_{kdi}},$$

$$k_{i4} = -\frac{1}{L''_{qi}}\frac{L''_{qi} - l_{ai}}{l_{gi}}\frac{L'_{qi} - l_{ai}}{l_{fi}}, k_{i2} = \frac{L_{mqi}}{L''_{qi}\tau'_{q0i}}\frac{L''_{qi} - l_{ai}}{(L'_{qi} - l_{ai})l_{kqi}}\frac{L''_{qi} - l_{ai}}{l_{gi}} - \frac{1}{L''_{qi}\tau''_{q0i}}\frac{L''_{qi} - l_{ai}}{L''_{qi}l_{kdi}},$$

$$k_{i5} = -\frac{1}{L''_{qi}}\frac{L''_{qi} - l_{ai}}{L''_{qi}l_{kdi}}, k_{i6} = \frac{L''_{di}}{L''_{qi}},$$

$$k_{i3} = -\frac{\omega_s r_{ai}}{L''_{qi}} - \frac{L_{mqi}}{L''_{qi}\tau'_{q0i}}\frac{L''_{qi} - l_{ai}}{l_{gi}}\frac{L'_{qi} - l_{ai}}{l_{gi}}\frac{L''_{qi} - l_{ai}}{L'_{qi} - l_{ai}} - \frac{1}{L''_{qi}\tau'_{q0i}}(L'_{qi} - l_{ai})\frac{L''_{qi} - l_{ai}}{L''_{qi}l_{kdi}},$$

$$k_{i7} = -\frac{1}{L''_{qi}\tau'_{q0i}}\left(1 + L_{mqi}\frac{L'_{qi} - l_{ai}}{l_{gi}l_{kqi}}\right)\frac{L'_{qi} - l_{ai}}{l_{gi}} + \frac{1}{L''_{qi}\tau''_{q0i}}\frac{L''_{qi} - l_{ai}}{L''_{qi}l_{kdi}}\frac{L'_{qi} - l_{ai}}{l_{gi}}.$$

Generator	1	2	3
MVA	247.5	192.0	128.0
kV	16.5	18.0	13.8
P.F.	1.0	0.85	0.85
Type	Hydro	Steam	Steam
Speed	180 r/min	3600 r/min	3600 r/min
X_d	0.1460	0.8958	1.3125
X_q	0.0969	0.8645	1.2587
X_d'	0.0608	0.1198	0.1813
X_q'	0.0969	0.1969	0.2500
τ_{d0}'	8.9600	6.0000	5.8900
τ_{q0}'	0.0000	0.5350	0.6000
X_d''	0.0400	0.0600	0.0800
X_q''	0.0400	0.0600	0.0800
τ_{d0}''	0.2000	0.3000	0.4000
τ_{q0}''	0.2000	0.3000	0.4000
X_l	0.0336	0.0521	0.0742
r_a	0.0000	0.0000	0.0000
H	23.6400	6.4000	3.0100

Table 1. Parameters of generator model (6)-(7)

	Gen. 1	Gen. 2	Gen. 3		Gen. 1	Gen. 2	Gen. 3
a_1	0.1003	0.1644	0.0945	e_3	-5.000	-4.0	-2.5
a_2	1.13	1.1787	0.9458	h_1	-1256	-9424	-4712
a_3	0.0403	0.0119	0.0203	h_2	273.4	863.6	141.3
a_4	1.2552	1.0145	1.0239	h_3	0.5	-6.6	3.2
a_5	0.020	0.01	0.010	h_4	-31	-50.2	-16.4
b_1	-0.017	-0.0251	-0.0114	h_5	18.8	97.1	29.7
b_2	0.522	2.4483	1.8567	h_6	-0.1	-0.3	-0.3
b_3	-0.5075	-2.4185	-1.8659	h_7	-4.2	-25.4	-12.8
b_4	376.991	376.991	376.991	h_8	0.1	1.3	0.9
c_1	-0.07	-0.022	-0.0472	k_1	-1885	-7539	-5385
c_2	0.6453	10.6390	11.4979	k_2	1.7	11.8	6.4
c_3	-0.5348	-10.611	-11.4581	k_3	5.1	6.9	34.5
d_1	0.1360	-0.2257	0.2267	k_4	31.5	8.37	39.9
d_2	-3.79	-3.0659	-2.2838	k_5	0.5	3.3	0.7
d_3	-3.33	-3.333	-2.5	k_6	-5.7	-23.6	-13.5
e_1	0.2665	0.5792	0.4395	k_7	-0.1	-0.8	-1.1
e_2	-0.7899	-3.2871	-2.1290				

Table 2. Generators parameters

	Gen. 1	Gen. 2	Gen. 3		Gen. 1	Gen. 2	Gen. 3
a_1	0.2175	0.0916	0.03	e_3	-5.000	-4.0	-2.5
a_2	1.1324	1.1787	0.9458	h_1	-1256	-9424	-4712
a_3	0.0403	0.0119	0.0203	h_2	126.1	1549	233.1
a_4	1.2552	1.0145	1.0239	h_3	5	-6.8	3.2
a_5	0.020	0.010	0.010	h_4	-14.5	-100.3	-25.8
b_1	-0.003	-0.005	-0.0023	h_5	19	108.3	28.4
b_2	0.1044	0.4897	0.3713	h_6	-0.1	-0.3	-0.3
b_3	-0.0601	-0.0844	-0.0358	h_7	-3.2	-25.4	-12.8
b_4	376.991	376.991	376.991	h_8	1	1.3	0.9
c_1	-0.07	-0.022	-0.0472	k_1	-1885	-7539	-5385
c_2	0.6453	10.6390	11.4979	k_2	1.7	11.8	6.4
c_3	-0.5348	-10.611	-11.4581	k_3	5.1	69.2	34.5
d_1	0.1360	-0.2257	0.2267	k_4	31.5	83.7	39.9
d_2	-8.2182	-1.7090	-1.3842	k_5	1.1	1.8	0.4
d_3	-5.0	-3.333	-2.5	k_6	-5.7	-23.6	-13.5
e_1	0.2665	0.5792	0.4395	k_7	-0.1	-0.8	-1.1
e_2	-0.7899	-3.2871	-2.1290				

Table 3. Perturbed generators parameters

	Generator 1	Generator 2	Generator 3
k_{gi}	0.02	0.02	0.03
k_{0i}	7.5	5	6
ρ_{2i}	8	10	9
e_1	0.9	0.8	1.2
e_2	0.01	0.03	0.02
e_3	0.001	0.002	0.001

Table 4. Controllers parameters

8.3 Functions used in controllers design

$$f_{vi}\left(\mathbf{x}_i,\mathbf{i}_i\right) = v_{di}\left(\mathbf{x}_i,\mathbf{i}_i\right)\varphi_{1i}\left(\mathbf{x}_i,\mathbf{i}_i\right) + v_{qi}\left(\mathbf{x}_i,\mathbf{i}_i\right)\varphi_{2i}\left(\mathbf{x}_i,\mathbf{i}_i\right),\ b_{vi} = \left(h_{i2}b_{i4} + \frac{1}{k_{i1}}k_{i4}b_{i4}x_{2i}\right),$$

$$\varphi_{1i}\left(\mathbf{x}_i,\mathbf{i}_i\right) = -\frac{1}{h_{i1}}\left[\begin{array}{l} h_{i2}\left(b_{i1}x_{3i} + b_{i2}x_{5i} + b_{i3}i_{di}\right) + h_{i3}\left(d_{i1}x_{3i} + d_{i2}x_{5i} + d_{i3}i_{1i}\right) + \\ h_{i4}\left(\begin{array}{l} x_{2i}\left(c_{i1}x_{4i} + c_{i2}x_{6i} + c_{i3}i_{2i}\right) + \\ \left(f_{\omega i}(\mathbf{x}_i,\mathbf{i}_i,T_{mi}) - q_i(\mathbf{x}_i,\mathbf{i}_i)x_{3i}\right)x_{4i} \end{array}\right) + h_{i5}\left(\begin{array}{l} x_{2i}\left(r_{i1}x_{4i} + r_{i2}x_{6i} + r_{i3}i_{2i}\right) + \\ \left(f_{\omega i}(\mathbf{x}_i,\mathbf{i}_i,T_{mi}) - q_i(\mathbf{x}_i,\mathbf{i}_i)x_{3i}\right)x_{6i} \end{array}\right) \end{array}\right],$$

$$\varphi_{2i}\left(\mathbf{x}_i,\mathbf{i}_i\right) = -\frac{1}{k_{i1}}\left[\begin{array}{l} k_{i2}\dot{x}_{4i} + k_{i3}\left(r_{i1}x_{4i} + r_{i2}x_{6i} + r_{i3}i_{2i}\right) + \\ k_{i4}\left(x_{2i}\left(b_{i1}x_{3i} + b_{i2}x_{5i} + b_{i3}i_{1i}\right) + \left(f_{\omega i}(\mathbf{x}_i,\mathbf{i}_i,T_{mi}) - q_i(\mathbf{x}_i,\mathbf{i}_i)x_{3i}\right)x_{3i}\right) \\ + k_{i5}\left(x_{2i}\left(d_{i1}x_{3i} + d_{i2}x_{5i} + d_{i3}i_{1i}\right) + \left(f_{\omega i}(\mathbf{x}_i,\mathbf{i}_i,T_{mi}) - q_i(\mathbf{x}_i,\mathbf{i}_i)x_{3i}\right)x_{5i}\right) \end{array}\right],$$

$$g_{vi}\left(\mathbf{x}_i,\mathbf{i}_i\right) = h_{i6}\frac{d}{dt}i_{di} + h_{i7}\left(x_{2i}\frac{d}{dt}i_{qi} + \dot{x}_{2i}i_{qi}\right) + k_{i6}\left(x_{2i}\frac{d}{dt}i_{di} + \dot{x}_{2i}\right) + k_{i7}\frac{d}{dt}i_{qi} + h_{i8}\frac{d}{dt}v_{fi} + \Delta f_{vi}.$$

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