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Differential Neural Networks Observers: development, stability analysis and implementation

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1. Introduction

The control and possible optimization of a dynamic process usually requires the complete on-line availability of its state-vector and parameters. However, in the most of practical situations only the input and the output of a controlled system are accessible: all other variables cannot be obtained on-line due to technical difficulties, the absence of specific required sensors or cost (Radke & Gao, 2006). This situation restricts possibilities to design an effective automatic control strategy. To this matter many approaches have been proposed to obtain some numerical approximation of the entire set of variables, taking into account the current *available* information. Some of these algorithms assume a complete or partial knowledge of the system structure (mathematical model). It is worth mentioning that the influence of possible disturbances, uncertainties and nonlinearities are not always considered.

The aforementioned researching topic is called *state estimation, state observation* or, more recently, *software sensors design*. There are some classical approaches dealing with same problem. Among others there are a few based on the Lie-algebraic method (Knobloch et. al., 1993), Lyapunov-like observers (Zak & Walcott, 1990), the high-gain observation (Tornambe 1989), optimization-based observer (Krener & Isidori 1983), the reduced-order nonlinear observers (Nicosia et. al.,1988), recent structures based on sliding mode technique (Wang & Gao, 2003), numerical approaches as the set-membership observers (Alamo et. al., 2005) and etc. If the description of a process is incomplete or partially known, one can take the advantage of the function approximation capacity of the Artificial Neural Networks (ANN) (Haykin, 1994) involving it in the observer structure designing (Abdollahi et. al., 2006), (Haddad, et. al. 2007), (Pilutla & Keyhani, 1999).

There are known two types of ANN: *static* one, (Haykin, 1994) and *dynamic* neural networks (DNN). The first one deals with the class of global optimization problems trying to adjust the weights of such ANN to minimize an identification error. The second approach, exploiting the feedback properties of the applied Dynamic ANN, permits to avoid many problems related to global extremum searching. Last method transforms the learning process to an adequate feedback design (Poznyak et. al., 2001). Dynamic ANN's provide an effective instrument to attack a wide spectrum of problems, such as parameter identification, state estimation, trajectories tracking, and etc. Moreover, DNN demonstrates remarkable identification properties in the presence of uncertainties and external disturbances or, in other words, provides the *robustness* property.

In this chapter, we discuss the application of a special type of observers (based on the DNN) for the state estimation of a class of uncertain nonlinear system, which output and state are affected by bounded external perturbations. The chapter comprises four sections. In the first section the fundamentals concerning state estimation are included. The second section introduces the structure of the considered class of Differential Neural Network Observers (DNNO) and their main properties. In the third section the main result concerning the stability of estimation error, with its analysis based on the Lyapunov-Like method and Linear Matrix Inequalities (LMI) technique is presented. Moreover, the DNN dynamic weights boundedness is stated and treated as a second level of the learning process (the first one is the learning laws themselves). In the last section the implementation of the suggested technique to the chemical soil treatment by ozone is considered in details.

2. Fundamentals

2.1 Estimation problem

Consider the nonlinear continuous-time model given by the following ODE:

$$\begin{gathered}\frac{d}{dt}x(t) = f\left(x(t),u(t)\right) + \xi(t),\ x(0)\text{ is fixed}\\ y(t) = Cx(t) + \eta(t)\end{gathered} \tag{1}$$

where

$x(t) \in \Re^n$	-	state-vector at time $t \geq 0$,
$y(t) \in \Re^m$	-	corresponding measurable output,
$C \in \Re^{m \times n}$	-	the known matrix defining the state-output transformation,
$u(t) \in \Re^r$	-	the bounded control action $(r \leq n)$ belonging to the following admissible set

$$U^{adm} := \left\{u(t)\ :\ \left\|u(t)\right\| \leq \Upsilon_u < \infty\right\},$$

$\xi(t)$ and $\eta(t)$	-	noises in the state dynamics and in the output, respectively, $f : \Re^{n \times r} \to \Re^n$.

The software sensor design, also called *state estimation (observation) problem,* consists in designing a vector-function $\hat{x}(t) \in \Re^n$, called *"estimation vector"*, based only the available data information (measurable) $\left\{y(t), u(t)\right\}_{\tau \in [0,t]}$ in such a way that it would be "closed" to its real (but non-measurable) state-vector $x(t)$. The measure of that "closeness" depends on the accepted assumptions on the state dynamics as well as the noise effects. The most of observers usually have ODE-structure:

$$\frac{d}{dt}\hat{x}(t) = F\left(\hat{x}(t), u(t), y_{\tau \in [0,t]}, t\right),\ \hat{x}_0 \text{ is a fixed vector} \tag{2}$$

Here the mapping $F : \Re^n \times \Re^r \times L^m \times \Re^+ \to \Re^n$ defines the structure of the observer to be implemented.

2.2 Physical Constraints of the state vector

To realize the state observation objective, many authors have taken advantages of the physical state constraints. Some examples of these techniques employing *"a priori"* information on states are: interval observers (Dochain, 2003) and moving horizon state estimation (Valdes-González et. al., 2003). In the present study, some physical restrictions are considered and using previous results given in (García, et. al. 2007). The main property of an observer, which are looked for, is to keep the generated state estimates $\hat{x}(t)$ within the given compact set X (even in the presence of noise), that is:

$$\hat{x}(t) \in X \tag{3}$$

In different problems the *compact set* X has a concrete physical sense. For example, the dynamic behaviors of some reagents, participating in chemical reactions, always keep their nonnegative current values. Similar remark seems to be true for other physical variables such as temperature, pressure, light intensity and etc. To complete (3) the next *projectional observer* is proposed:

$$\hat{x}(t) = \pi_X \left\{ \hat{x}(t-h(t)) + \int_{\tau = t-h(t)}^{t} F\left(\hat{x}(\tau), u(\tau), y_{s \in [0,\tau]}, \tau\right) d\tau \right\},\ t > h(0) \tag{4}$$

Here $h(t) \in C^1$ fulfills $\dot{h}(t) \le 0$. The operator $\pi_X\{\cdot\}$ is the projector to the given convex compact set X possessing the property

$$\left\| \pi_X\{x\} - z \right\| \le \left\| x - z \right\| \tag{5}$$

for any $x \in \Re^n$ and any $z \in X$. The operator $\pi_X\{\cdot\}$ may be defined by different ways. Two examples of $\pi_X\{\cdot\}$ are given below.

Example 1 (Saturation function):

$$\pi_X\{x\} = \left[sat(x_1) \ \ \ldots \ \ sat(x_n) \right]^{\mathrm{T}} \tag{6}$$

where for any i=1..n

$$sat(x_i) := \begin{cases} (x_i)^- & x_i \le (x_i)^- \\ x_i & (x_i)^- < x_i < (x_i)^+ \\ (x_i)^+ & x_i \ge (x_i)^+ \end{cases} \tag{7}$$

with $(x_i)^- < (x_i)^+$ as an extreme point *a priori* known.

Example 2 (Simplex): If X is the n-simplex, i.e.,

$$X = \left\{ z \in R^n \ : \ z_i \geq 0 \ (i = 1,...,n), \ \sum_{i=1}^{n} z_i = 1 \right\} \tag{8}$$

then $\pi_X\{x\}$ can be found numerically by at least within n-steps. The case $n = 3$ is illustrated by Figure 1.

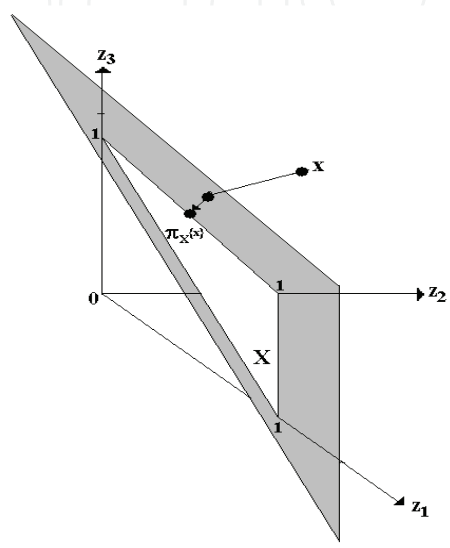


Figure 1. Projectional operator over a simplex (n=3)

An important point is that with the projectional operator implementations the trajectories $\{\hat{x}(t)\}$, generated by (4), are not differentiable for any $t \geq h(t) > 0$.

3 Structures of DNN Observers

3.1 State estimation under complete information

If the right-hand side $f(x(t))$ of the dynamics (1) is known then the structure F of the observer (4) is usually selected in the, so-called, Luenberger-type form:

$$F(\hat{x}(t), u(t), y(t), t) = f(\hat{x}(t), u(t)) + K(t)(y(t) - C\hat{x}(t)) \tag{9}$$

So, it repeats the dynamics of the plant and, additionally, contains the correction term, proportional to the output error (see, for example Yaz & Azemi, 1994; Poznyak, 2004). The adequate selection of the matrix-gain $K(t)$ provides a good-enough state estimation.

3.2 Differential Neural Network Observer, the "grey-box" case

In the case when the right-hand side $f(x,u)$ of the dynamics (1) is unknown, there is suggested to apply some guessing of it, say, $\bar{f}(x(t), u(t) \mid W(t))$ where $\bar{f} \in \Re^n$ defines the approximating map depending on the time-varying parameters $W(t)$, which should be adjusted by a "*adaptation law*" suggested by a designer or derived, using some stability analysis method. According to the DNN-approach (Poznyak et. al., 2001) we may decompose $\bar{f}(x(t),u(t)\,|\,W(t))$ in two parts: first one, approximates the linear dynamics part by a *Hurwitz* fixed matrix $A \in \Re^{n\times n}$ (selected by the designer) and the second one, uses the ANN reconstruction property for the nonlinear part by means of variable time parameters $W_{1,2}(t)$ with a set of basis functions, that is,

$$\begin{gathered}\bar{f}\Big(x(t),u(t)\,|\,W_{1,2}(t)\Big) := Ax(t) + W_1(t)\sigma(x(t)) + W_2(t)\varphi(x(t))u(t) \\ A \in \Re^{n\times n},\ W_1(t) \in \Re^{n\times p},\ \sigma(\cdot) \in \Re^{p\times 1} \\ W_2(t) \in \Re^{n\times q},\ \varphi(\cdot) \in \Re^{q\times r}\end{gathered} \tag{10}$$

The activation vector (the basis) function $\sigma(\cdot)$ and matrix-function $\varphi(\cdot)$ are usually selected as functions with *sigmoid-type components*, i.e.:

$$\sigma_j(x(t)) := a_j\left(1 + b_j\, exp\left(-\sum_{j=1}^{n} c_j x_j(t)\right)\right)^{-1},\ j=\overline{1,n} \tag{11}$$

and

$$\varphi_{i,j}(x(t)) := a_{i,j}\left(1 + b_{i,j}\, exp\left(-\sum_{s=1}^{n} c_{i,s} x_s(t)\right)\right)^{-1},\ i=\overline{1,q};\ j=\overline{1,r} \tag{12}$$

It is easy to see that the activation functions satisfy the following sector conditions

$$\left\|\sigma(x(t)) - \sigma(x'(t))\right\|^2_{\Lambda_\sigma} \le L_\sigma \left\|x(t) - x'(t)\right\|^2_{\Lambda_\sigma} \tag{13}$$

$$\left\|\varphi(x(t)) - \varphi(x'(t))\right\|^2_{\Lambda_\varphi} \le L_\varphi \left\|x(t) - x'(t)\right\|^2_{\Lambda_\varphi} \tag{14}$$

and stay bounded on $\Re^n$. In (10), the constant parameter A, as well as the time-varying parameters $W_{1,2}(t)$, should be properly adjusted to guarantee a good state approximation. Notice that for any fixed matrices $W_{1,2}(t) = \hat{W}_{1,2}$ the dynamics (1) always could be represented as

$$\begin{gathered}\frac{d}{dt}x(t) = Ax(t) + \hat{W}_1\sigma(x(t)) + \hat{W}_2\varphi(x(t))u(t) + \tilde{f}(t) + \xi(t) \\ \tilde{f}(t) := f(x(t)) - \bar{f}\Big(x(t)\,|\,\hat{W}_{1,2}\Big)\end{gathered} \tag{15}$$

where $\tilde{f}(t)$ is referred to as a modeling error vector-field called the "*unmodelled dynamics*". In view of the corresponding boundedness property, the following inequality for the unmodelled dynamics $\tilde{f}(t)$ takes place:

$$
\begin{gathered}
\left\|\tilde{f}(t)\right\|^2_{\Lambda_f} \leq \tilde{f}_0 + \tilde{f}_1 \left\|x(t)\right\|^2_{\Lambda^1_{\tilde{f}}} \\
\tilde{f}_0, \tilde{f}_1 > 0;\ \Lambda_f, \Lambda^1_{\tilde{f}} > 0,\ \Lambda_f = \Lambda_f^T,\ \Lambda^1_{\tilde{f}} = \left(\Lambda^1_{\tilde{f}}\right)^T
\end{gathered} \tag{16}
$$

3.3 Structure DNN observers considering state physical constraints

Introduce the following projectional DNNO:

$$
\begin{gathered}
\hat{x}(t) = \pi_X\left\{\hat{x}(t-h(t)) + \int\limits_{\tau=t-h(t)}^{t} \left[A\hat{x}(\tau) + W_1(\tau)\sigma(\hat{x}(\tau)) + W_2(\tau)(\varphi(x(\tau))u(\tau) + Ke(\tau)\right]d\tau\right\} \\
e(t) := y(t) - C\hat{x}(t)
\end{gathered} \tag{17}
$$

Here the weights matrices $W_1(t)$ and $W_2(t)$ supply the adaptive behavior to this class of observers if they are adjusted by an adequate manner. We derived (see Appendix) the following nonlinear weight *updating* laws based on the Lyapunov-like stability analysis:

$$
\left.
\begin{gathered}
\frac{d}{dt}W_1(t) = -\frac{k_1^{-1}(t)}{2}P\Omega(t)\sigma^T(\hat{x}(t)) - \frac{dk_1(t)}{dt}\tilde{W}_1(t) \\
\Omega(t) := \Pi\tilde{W}(t)\sigma(\hat{x}(t)) + 2N_{\varpi}C^T e(t-h(t));\qquad \tilde{W}_1(t) := W_1(t) - \hat{W}_1; \\
\Pi = \left(N_{\varpi}\left(\varpi\Lambda_3 + C^T\Lambda_2 C\right)N_{\varpi}P + I\right)
\end{gathered}
\right\} \tag{18}
$$

$$
\left.
\begin{gathered}
\frac{d}{dt}W_2(t) = -\frac{k_2^{-1}(t)}{2}P\Phi(t)u^T(\tau)\varphi^T(\hat{x}(\tau)) - \frac{dk_2(t)}{dt}\tilde{W}_2(t) \\
\Phi(t) := \Xi\tilde{W}_2(\tau)(\varphi(\hat{x}(\tau))u(\tau) + 2N_{\varpi}C^T e(t-h(t));\qquad \tilde{W}_2(t) := W_2(t) - \hat{W}_2; \\
\Xi = \left(N_{\varpi}\left(\varpi\Lambda_7 + C^T\Lambda_6 C\right)N_{\varpi}P + I\right)
\end{gathered}
\right\} \tag{19}
$$

where:

$$
N_{\varpi} = \left(C^T C + \varpi I\right)^{-1},\ \varpi > 0
$$

To improve the behavior of this adaptive laws, the matrix $\hat{W}_{1,2}$ can be "provided" by one of the, so-called, *training algorithms* (see, for example, Chairez et. al., 2006; Stepanyan & Hovakimyan, 2007). Both present least square solutions considering some identification structure for possible set of fictitious values or even an available set of directly measured data of the process.

4. DNN Observers Stability

4.1 Behavior of weights dynamics

Here we wish to show that under the adapting weights laws (18) and (19) the weights $W_1(t)$ and $W_2(t)$ are bounded.

Theorem 1 (bounded adaptive weights): If $k_i(t)$ $(\mathrm{i}=1,2)$ in (18) and (19) satisfy

$$\begin{gathered}
\frac{d}{dt}k_1(t) \le -\frac{2(k_1(t))^2 \left| tr\left\{\tilde{W}_1^T(t)P\Omega(t)\sigma^T(\hat{x}(t))\right\}\right|}{tr\left\{\tilde{W}_1^T(t)\tilde{W}_1(t)\right\} + ck_1(t)\left[k_1(t)-k_{1min}\right]} \\
\frac{d}{dt}k_2(t) \le -\frac{2(k_2(t))^2 \left| tr\left\{\tilde{W}_2(t)P\Phi(t)u^T(t)\varphi^T(\hat{x}(t))\right\}\right|}{tr\left\{\tilde{W}_2(t)^T\tilde{W}_2(t)\right\} + ck_2(t)\left(k_2(t)-k_{2,min}\right)}
\end{gathered} \tag{20}$$

then $tr\left\{\tilde{W}_1^T(t)\tilde{W}_1(t)\right\}$ is monotonically non-increasing function.

Proof: Considering the dynamics for the weight matrix $\tilde{W}_1(t)$ and the following candidate Lyapunov function $V_w(t)$.

$$V_w(t) := \frac{1}{2}tr\left\{\tilde{W}_1^T(t)\tilde{W}_1(t)\right\} + \frac{c}{4}\left[k_1(t)-k_{1min}\right]_+^2 \tag{21}$$

where

$$[z(t)]_+ := \begin{cases} z(t) & z(t)\ge 0 \\ 0 & z(t)<0 \end{cases} \tag{22}$$

Then, one has

$$\frac{d}{dt}V_w(t) := tr\left\{\tilde{W}_1^T(t)\left(\frac{d}{dt}W_1(t)\right)\right\} + 2^{-1}c\frac{d(k_1(t))}{dt}\left[k_1(t)-k_{1min}\right]_+^2 \tag{23}$$

By (18) it follows

$$\begin{gathered}
\frac{d}{dt}V_w(t) = tr\left\{\tilde{W}_1^T(t)\left(-\frac{k_1^{-1}(t)}{2}\left[P\Omega(t)\sigma^T(\hat{x}(t)) - \frac{d(k_1(t))}{dt}\tilde{W}_1(t)\right]\right)\right\} + \\
2^{-1}c\frac{d(k_1(t))}{dt}\left[k_1(t)-k_{1min}\right]_+ \le \frac{k^{-1}(t)}{2}\left| tr\left\{\tilde{W}_1^T(t)P\Omega(t)\sigma^T(\hat{x}(t))\right\}\right| + \\
2^{-1}\frac{d(k_1(t))}{dt}\left(k_1^{-1}(t)tr\left\{\tilde{W}_1^T(t)\tilde{W}_1(t)\right\} + 2^{-1}c\left[k_1(t)-k_{1min}\right]_+\right)
\end{gathered} \tag{24}$$

The property $\frac{d}{dt}V_w(t) \le 0$ results from (20).

Some examples of $k_i(t)$ $(i=1,2)$ are given below

a. Introduce the following auxiliary function

$$s\left(\tilde{W}_1^T(t), e(t-h(t))\right) := \frac{k_1^{-1}(t)\left| tr\left\{\tilde{W}_1^T(t) P\Omega(t)\sigma^T(\hat{x}(t))\right\}\right.}{c\left[k_1(t)-k_{1,min}\right]_+}$$

And select

$$k_1(t) := \frac{k(0)}{1+a\left(\tilde{W}_1^T(t), e(t-h(t))\right)exp(bt)} + k_{min,\,j},\quad k_{min,\,j} > 0$$

$$\frac{d(k_1(t))}{dt} := -k_1(0)\frac{a\left(\tilde{W}_1^T(t), e(t-h(t))\right)b_j\, exp(bt)}{1+a\left(\tilde{W}_1^T(t), e(t-h(t))\right)exp\left(b_j t\right)} < -s\left(\tilde{W}_1^T(t), e(t-h(t))\right)$$

Leading to

$$a\left(\tilde{W}_1^T(t), e(t-h(t))\right)exp(bt)\left(k(0)b - s\left(\tilde{W}_1^T(t), e(t-h(t))\right)\right) > s\left(\tilde{W}_1^T(t), e(t-h(t))\right)$$

The last inequality is fulfilled if the weight dependent parameter $a\left(\tilde{W}_1^T(t), e(t)\right)$ is selected as

$$a\left(\tilde{W}_1^T(t), e(t-h(t))\right) > s\left(\tilde{W}_1^T(t), e(t-h(t))\right)exp(-bt)\Psi^{-1}$$

$$\Psi := k(0)b - s\left(\tilde{W}_1^T(t), e(t-h(t))\right)$$

b. Analogously, for $\tilde{W}_2^T(t)$:

$$s\left(\tilde{W}_2^T(t), e(t-h(t))\right) := \frac{k_2^{-1}(t)\left| tr\left\{\tilde{W}_2^T(t) P\Phi(t)u^T(\tau)\varphi^T(\hat{x}(\tau))\right\}\right.}{c\left[k_2(t)-k_{2,min}\right]_+}$$

$$k_2(t) := \frac{k(0)}{1+a\left(\tilde{W}_2^T(t), e(t-h(t))\right)exp(bt)} + k_{min,\,j},\quad k_{min,\,j} > 0$$

$$\frac{d}{dt}k_2(t) := -k_2(0)\frac{a\left(\tilde{W}_2^T(t), e(t-h(t))\right)b_j\, exp(bt)}{1+a\left(\tilde{W}_2^T(t), e(t-h(t))\right)exp\left(b_j t\right)} < -s\left(\tilde{W}_2^T(t), e(t-h(t))\right)$$

It is worth to notice that the learning law (18) and (19) must be realized on-line in parallel with the gain-parameter adaptation procedure (20). By this reason, this structure can be considered as a second adaptation level.

4.2 Main theorem on an upper bound for the observation error

For the stability analysis of the proposed DNNO, the next assumptions are accepted:

A1) the function $f:\Re^n \to \Re^n$ is Lipschitz continuous in $x \in X$, that is, for all $x, x' \in X$ there exist constants $L_{1,2}$ such that

$$\begin{gathered}\|f(x,u,t)-f(y,v,t)\| \le L_1\|x-y\|+L_2\|u-v\| \\ \|f(0,0,t)\|^2 \le C_1;\ x,y \in \Re^n;\ u,v \in \Re^m;\ 0 \le L_1, L_2 < \infty\end{gathered} \tag{25}$$

A2) The pair (A,C) is observable, that is, there exists a gain matrix $K \in \Re^{n\times m}$ such that matrix

$$\tilde{A}(K) := A - KC \tag{26}$$

is stable (Hurwitz).

A3) The noises $\xi(t)$ and $\eta(t)$ in the system (1) are uniformly (on t) bounded such that

$$\|\xi(t)\|^2_{\Lambda_\xi} \le \Upsilon_\xi,\ \|\eta(t)\|^2_{\Lambda_\eta} \le \Upsilon_\eta \tag{27}$$

where Λ_ξ and Λ_η are known "normalizing" non-negative definite matrices, which permit to operate with vectors having components of different physical nature (for example, meters, voltage and etc.).

Theorem *(Upper error for DNNO)*. Under assumptions *A1-A3* and if there exist matrices $\Lambda_i = \Lambda_i^T > 0$, $\Lambda_i \in \Re^{n\times n}$, $i = 1 \ldots 10$, $Q_0 \in \Re^{n\times n}$, $K \in \Re^{n\times m}$ and positive parameters ϖ, μ_1, μ_2 and μ_3 such that the following LMI

$$\begin{bmatrix} \begin{matrix} -\Gamma(K,\varpi,\mu_1,\mu_2) & P \\ P & R \end{matrix} & 0 & 0 & 0 \\ 0 & \begin{matrix} \Theta_1 & \tilde{A}^T(K)P \\ P\tilde{A}(K) & \mu_1 P \end{matrix} & 0 & 0 \\ 0 & 0 & \begin{matrix} \Theta_2 & \hat{W}_1^T(K)P \\ P\hat{W}_1 & \mu_2 P \end{matrix} & 0 \\ 0 & 0 & 0 & \begin{matrix} \Theta_3 & \hat{W}_2^T(K)P \\ P\hat{W}_2 & \mu_3 P \end{matrix} \end{bmatrix} > 0 \tag{28}$$

with $tr\{\Theta_i\} < 1,\quad i = 1,2,3$ and

$$\Gamma(K,\delta,\mu_1,\mu_2) = \left[\tilde{A}^T(K)P + P\tilde{A}(K) + Q(\delta,\mu_1,\mu_2,\mu_3) \right]$$
$$R^{-1} = \Lambda_1^{-1} + \Lambda_9^{-1} + \Lambda_{10}^{-1} + \hat{W}_1 \Lambda_5^{-1} (\hat{W}_1)^T + \hat{W}_2 \Lambda_8^{-1} (\hat{W}_2)^T$$
$$Q(\delta,\mu_1,\mu_2,\mu_3) = \left[\|\Lambda_5\| L_\sigma + \|\Lambda_8\| L_\varphi \Upsilon_u^2 + \mu_1 + \mu_2 L_\sigma + \mu_3 \Upsilon_u^2 L_\varphi \right] I$$
$$+ \varpi\left(\Lambda_3^{-1} + \Lambda_7^{-1} \right) + Q_0$$

has positive definite solution **P**, then the projectional DNNO, with the weight's learning laws, given by (18), (19), (20) and with $h(t)$ satisfying

$$\lim_{t\to\infty} h(t) \to \varepsilon, 0 < \varepsilon << 1 \tag{29}$$

Provides the following upper bound for the "averaged estimation" error

$$\begin{aligned} &\overline{\lim_{T\to\infty}} \frac{1}{T} \int_{\tau=0}^{T} \left(\delta^T(\tau - h(\tau)) Q_0 \delta(\tau - h(\tau)) \right) d\tau \leq \\ &\|\Lambda_9\| \left(\left(\|K\| \left\| \Lambda_\eta^{-1} \right\|^{1/2} \Upsilon_\eta + \left\| \Lambda_\xi^{-1} \right\|^{1/2} \Upsilon_\xi \right) \right)^2 \\ &+ \|\Lambda_{10}\| \left\| \Lambda_{\tilde{f}}^{-1} \right\| \left[\tilde{f}_0 + \tilde{f}_1 \|x(t)\|^2_{\Lambda_{\tilde{f}}^1} \right] + \|K\|^2 \|P\| \left\| \Lambda_\eta^{-1} \right\|^{1/2} \Upsilon_\eta \\ &+ \|P\| \left\| \Lambda_{\tilde{f}}^{-1} \right\| \left[\tilde{f}_0 + \tilde{f}_1 \left\| \Lambda_{\tilde{f}}^1 \right\| \mathtt{Diam}(x)^2 \right] + \|P\| \left\| \Lambda_\xi^{-1} \right\| \Upsilon_\xi + 2\Upsilon_\eta \end{aligned} \tag{30}$$

where $Diam(x) = \sup_{x,z\in X} \|x - z\|$, and $\delta(t) := \hat{x}(t) - x(t)$ is the state estimation error. The proof of this theorem is presented in the appendix A.

Remark 1: It is easy to see that in the absence of noises ($\eta(t) = \xi(t) = 0$) and unmodelled dynamics ($\tilde{f} = 0$), we can prove that:

$$\overline{\lim_{T\to\infty}} \frac{1}{T} \int_{\tau=0}^{T} \left(\delta^T(\tau - h(\tau)) Q_0 \delta(\tau - h(\tau)) \right) d\tau \to 0 \tag{31}$$

5. Numerical Example Implementation

5.1 Algorithm of Implementation

As it follows from the presentation above, to realized the suggested approach one needs to fulfill the following steps:

- Define the projector.
- Select Matrices A and $\hat{W}$ (some hints are given in Chairez, et. al. 2006; Stepanyan & Hovakimyan, 2007).
- Select K such that $A - KC$ is stable, with C defined by the output of the system.

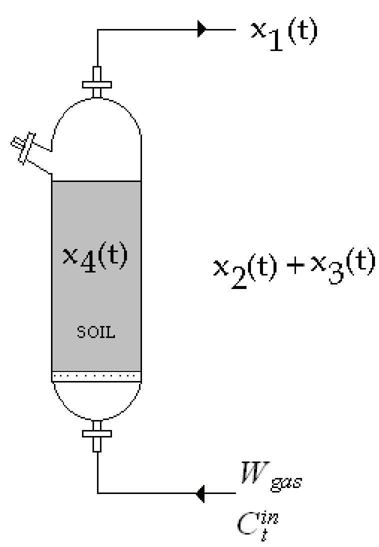
- Find P as the solution of the LMI problem (28).
- Introduce P into the adapting weight law (18), (19) and (20) and realized them on-line.

5.2 DNNO implementation (Contaminated Soil Treatment by Ozonation)

High oxidation process employing ozone is one of the most recent approaches in the treatment of the contaminated soil with chemical compounds such as polyaromatic hydrocarbons. The next simplified model (32) describes the ozonization of one contaminant in the solid and gas phases in a semi-continuous reactor (Poznyak T., et. al. 2007).

$$\begin{gathered}
\frac{d}{dt}x_1(t) = V_{gas}^{-1}\left[W_{gas}C^{in} - W_{gas}x_1(t) - k_1x_{4,t}x_3(t) - K_t^{abs}\left(Q_{max}^{free_abs} - x_2(t)\right)\right] \\
\frac{d}{dt}x_2 = K_t^{abs}\left(Q_{max}^{free_abs} - x_2(t)\right) \\
\frac{d}{dt}x_{3,t} = k_1x_4(t)x_3(t) \\
\frac{d}{dt}x_4(t) = -k_1G^{-1}x_4(t)x_3(t)
\end{gathered} \tag{32}$$

Here in (32) $y(t) = x_1(t) + \eta(t)$ (see Figures 2 and 3) is the ozone concentration (mole/L) at the output of the reactor assumed to be on-line measurable, $x_2(t)$ (mole) is the ozone amount absorbed by the soil, which is not reacting with the contaminant, $x_3(t)$ (mole) is the ozone amount absorbed by the soil and reacting with the contaminant, and $x_4(t)$ (mole/g) is the current contaminant concentration, C^{in} is the ozone concentration at the reactor input (mole/L), $Q_{max}^{free\_abs}$ is the maximum amount of ozone, which can be absorbed by the soil, $Wgas$ is the gas flow (L/s) (established as a constant value), $Vgas$ is the volume of the gas phase.



(L).

Figure 2. Contaminated soil ozonation procedure in a semi-continuous batch reactor

It is worth notice that the model is employed only as a data source; any structural information (mathematical model) has been used in the projectional DNNO design.
The convex compact set X according to the physical system constrictions is given as:

$$X:=\left\{\begin{array}{c} 0 \le x_1(t) \le x_1(t) \\ 0 \le x_2(t) \le Q_{max}^{free\_abs} \\ 0 \le x_3(t) \le V_{gas} C^{in} \\ 0 \le x_4(t) \le x_4(t) \end{array}\right\} \tag{33}$$

Projectional operator is defined as in (6), and the corresponding observer parameters are defined by:

$$A = \begin{bmatrix} -2.6 & 0 & 0 & 0 \\ 0 & -1.6 & 0 & 0 \\ 0 & 0 & -2.24 & 0 \\ 0 & 0 & 0 & -0.46 \end{bmatrix}, K = \begin{bmatrix} 0.01 \\ 0.01 \\ -0.0001 \\ -0.1 \end{bmatrix} \tag{34}$$

Figures 4-7 represent the results of x_3 and x_4 estimation from the measurable output. We have compared the projectional DNNO against a DNNO without projection operator, it means, with and without considering physical restrictions in the DNNO structure. Simulation have been realized in the presence of "quasi-white noise" $\eta(t)$ (amplitude $= 0.6 \times 10^{-5}$) and with the same initial conditions in both cases.

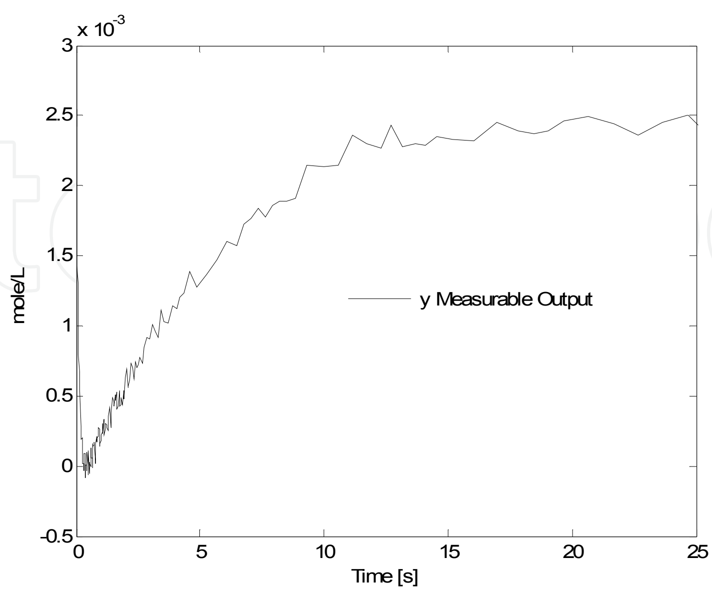


Figure 3. Measurable output (available information)

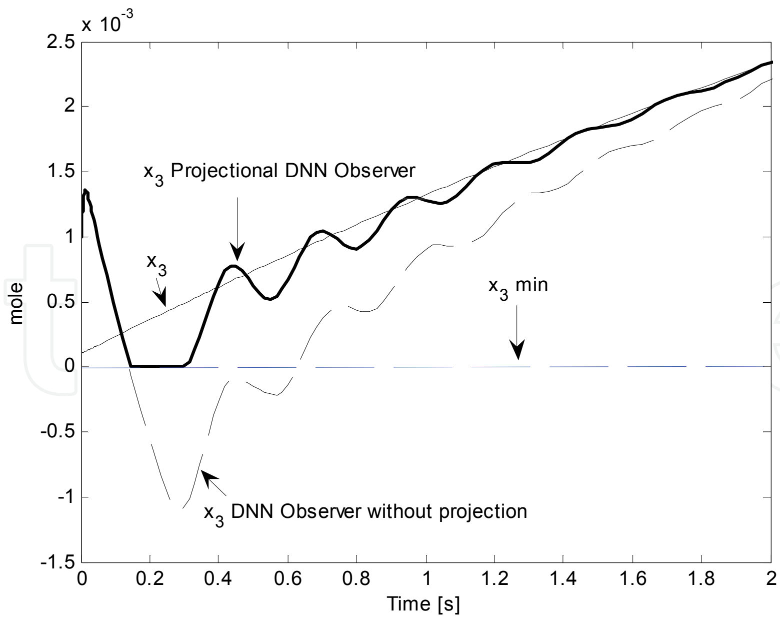


Figure 4. Estimation of $x_3(t)$ (2 s)

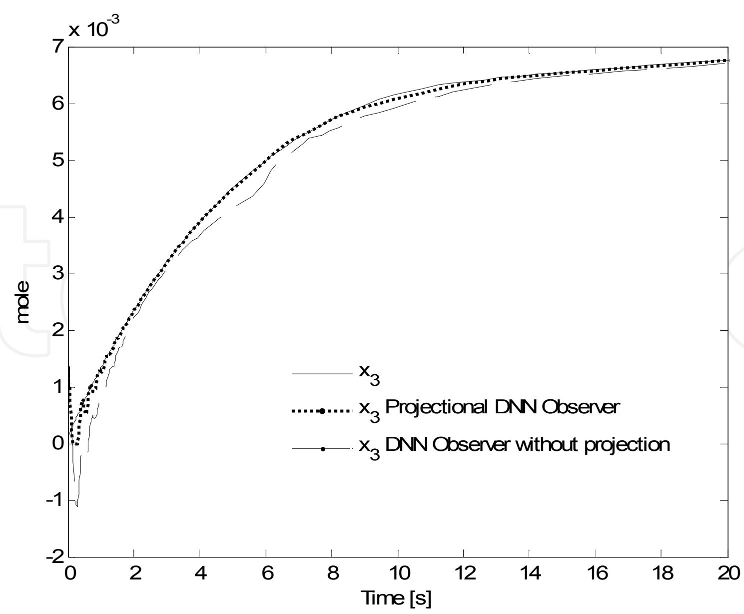


Figure 5. Estimation of $x_3(t)$ (20 s)

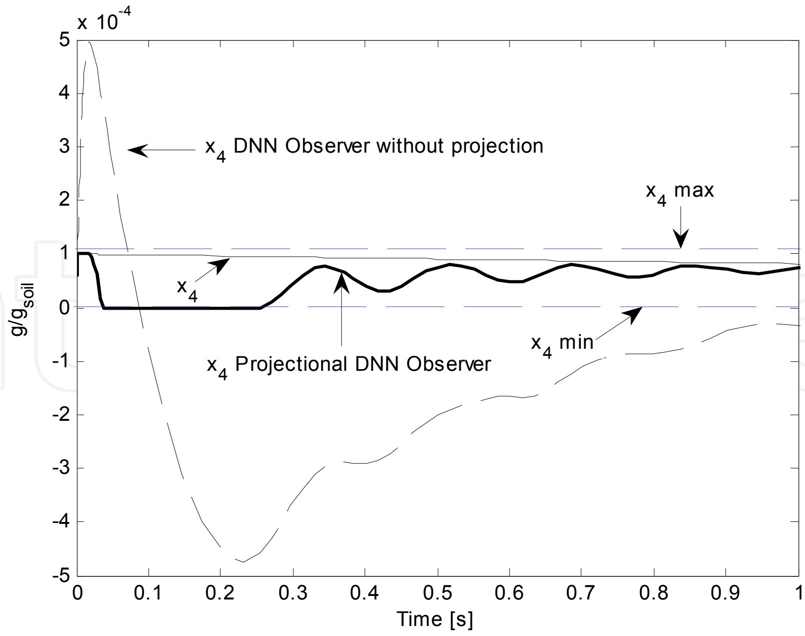


Figure 6. Estimation of $x_4(t)$ (1 s)

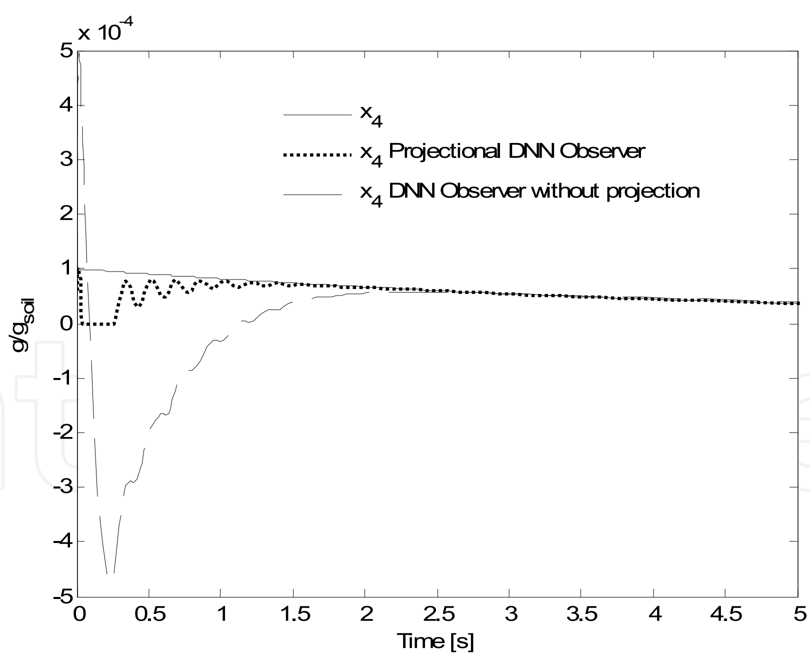


Figure 7. Estimation of $x_4(t)$ (5 s)

As it can be seen, the projectional DNNO has significantly better quality in state estimation, especially in the beginning of the process, when negative values and over-estimation have been obtained by a non-projectional DNNO.

6. Conclusion and future work

The complete convergence analysis for this class of adaptive observer is presented. Also the boundedness property of the adaptive weights in DNN was proven. Since the projection method leads to discontinuous trajectories in the estimated states, a nonstandard Lyapunov - Krasovski functional is applied to derive the upper bound for estimation error (in "average sense"), which depends on the noise power (output and dynamics disturbances) and on an unmodelled dynamic. It is shown that the asymptotic stability is attained when both of these uncertainties are absent. The illustrative example confirms the advantages, which the suggested observers have being compared with traditional ones.

Appendix (proof of Theorem 2)

Evidently that

$$\|\delta(t') - \delta(t-h)\| \le L_{\delta} \|t' - (t - h(t))\|$$

$$\|\eta(t)\| = \sqrt{\left(\Lambda_{\eta}^{1/2}\eta(t), \Lambda_{\eta}^{-1}\Lambda_{\eta}^{1/2}\eta(t)\right)} \le \sqrt{\left\|\Lambda_{\eta}^{-1}\right\| \|\eta(t)\|_{\Lambda_{\eta}}^{2}} \le \left\|\Lambda_{\eta}^{-1}\right\|^{1/2} \Upsilon_{\eta}$$

$$\|\xi(t)\| \le \left\|\Lambda_{\xi}^{-1}\right\|^{1/2} \Upsilon_{\xi}$$

$$\|\tilde{f}(t)\| \le \left\|\Lambda_{\tilde{f}}^{-1}\right\|^{1/2} \left[\tilde{f}_0 + \tilde{f}_1 \|x(t)\|_{\Lambda_{\tilde{f}}^{1}}^{2}\right]^{1/2}$$

where $\delta(t') := \hat{x}(t') - x(t')$ is the state estimation error at time t.
Consider the next "nonstandard" Lyapunov-Krasovskii ("energetic") function

$$V(t) = \int_{t-h(t)}^{t} \left[\|\delta(\tau)\|_{p}^{2} + k(\tau) tr\left\{\tilde{W}^{T}(\tau)\tilde{W}(\tau)\right\}\right] d\tau$$

where $\tilde{W}(\tau\tau) = W(\tau(-\hat{W}$. Since the problem under consideration contains uncertainties and external output disturbances we won't demonstrate that the time-derivative of this energetic function is strictly negative. Instead, we will use it to obtain an upper bound for the averaged state estimation error. Taking time derivative of Lyapunov-Krasovski function and considering the property (5), the assumption *A2*, and in view of (29) we have:

$$\frac{d}{dt}V(t) \le$$
$$\left\| \pi_X \left\{ \hat{x}(t-h(t)) + \int_{\tau=t-h(t)}^{t} \left[A\hat{x}(\tau) + W_1(\tau)\sigma(\hat{x}(\tau)) + W_2(\tau)(\varphi(\hat{x}(\tau))u(\tau) + K(Cx(\tau) + \eta(\tau) - C\hat{x}(t))) \right] d\tau \right\} \right.$$
$$\left. - x(t) \right\|_p^2 -$$
$$\left\| \delta(t-h(t)) \right\|_p^2 +$$
$$k_1(t)\mathbf{tr}\left\{\widetilde{W}_1^T(t)\widetilde{W}_1(t)\right\} - k_1(t-h(t))\mathbf{tr}\left\{\widetilde{W}_1^T(t-h(t))\widetilde{W}_1(t-h(t))\right\} +$$
$$k_2(t)\mathbf{tr}\left\{\widetilde{W}_2^T(t)\widetilde{W}_2(t)\right\} - k_2(t-h(t))\mathbf{tr}\left\{\widetilde{W}_2^T(t-h(t))\widetilde{W}_2(t-h(t))\right\} \le$$
$$\left\| \hat{x}(t-h(t)) + \int_{\tau=t-h(t)}^{t} \left[A\hat{x}(\tau) + W_1(\tau)\sigma(\hat{x}(\tau)) + W_2(\tau)(\varphi(\hat{x}(\tau))u(\tau) + K(\eta(\tau) - C\delta(\tau)) \right] d\tau \right.$$
$$\left. - x(t-h(t)) - \int_{\tau=t-h(t)}^{t} \left[Ax(t) + \hat{W}_1\sigma(x(t)) + \hat{W}_2(\varphi(x(\tau))u(\tau) + \tilde{f}(\tau) + \xi(\tau) \right] d\tau \right\|_p^2 - \left\| \delta(t-h(t)) \right\|_p^2$$
$$+ k_1(t)\mathbf{tr}\left\{\widetilde{W}_1^T(t)\widetilde{W}_1(t)\right\} - k_1(t-h(t))\mathbf{tr}\left\{\widetilde{W}_1^T(t-h(t))\widetilde{W}_1(t-h(t))\right\}$$
$$+ k_2(t)\mathbf{tr}\left\{\widetilde{W}_2^T(t)\widetilde{W}_2(t)\right\} - k_2(t-h(t))\mathbf{tr}\left\{\widetilde{W}_2^T(t-h(t))\widetilde{W}_2(t-h(t))\right\}$$

Taking into account that

$$\|a+b\|_P^2 = \|a\|_P^2 + \|b\|_P^2 + 2(Pa, b)$$

Defining:

$$\begin{gathered} \widetilde{A} \ := A - KC, \\ \widetilde{W}_i(t) := W_i(t) - \hat{W}_i \quad i = 1,2 \\ \widetilde{\sigma}(t) := \sigma(\hat{x}(t)) - \sigma(x(t)) \\ \widetilde{\varphi}(t) \ := \varphi(\hat{x}(t)) - \varphi(x(t)) \end{gathered}$$

we derive

$$\dot{V} \le a(t) + \beta(t) +$$
$$k_1(t)tr\left\{\widetilde{W}_1^T(t)\widetilde{W}_1(t)\right\} - k_1(t-h(t))tr\left\{\widetilde{W}_1^T(t-h(t))\widetilde{W}_1(t-h(t))\right\} +$$
$$k_2(t)tr\left\{\widetilde{W}_2^T(t)\widetilde{W}_2(t)\right\} - k_2(t-h(t))tr\left\{\widetilde{W}_2^T(t-h(t))\widetilde{W}_2(t-h(t))\right\}$$

where:

$$\alpha(t) := \left\| \int_{\tau=t-h(t)}^{t} \left[\widetilde{A}\delta(\tau) + \widetilde{W}_1(\tau)\sigma(\hat{x}(\tau)) + \hat{W}_1\widetilde{\sigma}(\tau) + \widetilde{W}_2(\tau)(\varphi(\hat{x}(\tau))u(\tau) + \right. \right.$$
$$\left. \left. \hat{W}_2\widetilde{\varphi}(\tau)u(\tau) + K\eta(\tau) - \xi(\tau) - \tilde{f}(\tau) \right] d\tau \right\|_2^P$$
$$\beta(t) := \left(2P\delta(t-h), \int_{\tau=t-h(t)}^{t} \left[\widetilde{A}\delta(\tau) + \widetilde{W}_1(\tau)\sigma(\hat{x}(\tau)) + \hat{W}_1\widetilde{\sigma}(\tau) + \widetilde{W}_2(\tau)(\varphi(\hat{x}(\tau))u(\tau) \right. \right.$$
$$\left. \left. + \hat{W}_2\widetilde{\varphi}(\tau)u(\tau) + K\eta(\tau) - \xi(\tau) - \tilde{f}(\tau) \right] d\tau \right)$$

The term $\beta(t)$ is expanded as

$$\beta(t)=2\left(P\delta(t-h(t)),\int_{\tau=t-h(t)}^{t}\tilde{A}\delta(\tau)d\tau\right)+$$
$$2\left(P\delta(t-h(t)),\int_{\tau=t-h(t)}^{t}\tilde{W}(\tau)\sigma\hat{x}(\tau)d\tau\right)+2\left(P\delta(t-h(t)),\int_{\tau=t-h(t)}^{t}\hat{W}_1\tilde{\sigma}(\tau)d\tau\right)$$
$$+2\left(P\delta(t-h(t)),\int_{\tau=t-h(t)}^{t}\tilde{W}_2(\tau)(\varphi(\hat{x}(\tau))u(\tau)d\tau\right)$$
$$+2\left(P\delta(t-h(t)),\int_{\tau=t-h(t)2}^{t}\hat{W}_2\tilde{\varphi}(\tau)u(\tau)d\tau\right)$$
$$+2\left(P\delta(t-h(t)),\int_{\tau=t-h(t)}^{t}(K\eta(\tau)-\xi(\tau))d\tau\right)-2\left(P\delta(t-h(t)),\int_{\tau=t-h(t)}^{t}\tilde{f}(\tau)d\tau\right)$$

Similarly, we can estimate α_t by the Jensen's inequality we get

$$\alpha(t):=\left\|\int_{\tau=t-h(t)}^{t}\left[\tilde{A}\delta(\tau)+\tilde{W}_1(\tau)\sigma(\hat{x}(\tau))+\hat{W}_1\tilde{\sigma}(\tau)+\tilde{W}_2(\tau)(\varphi(\hat{x}(\tau))u(\tau)\right.\right.$$
$$\left.\left.+\hat{W}_2\tilde{\varphi}(\tau)u(\tau)+K\eta(\tau)-\xi(\tau)-\tilde{f}(\tau)\right]d\tau\right\|_2^P\leq$$
$$8\left\{\int_{\tau=t-h(t)}^{t}\left(\left\|\tilde{A}\delta(\tau)\right\|_p^2+\left\|\tilde{W}_1(\tau)\sigma(\hat{x}(\tau))\right\|_p^2+\left\|\hat{W}_1\tilde{\sigma}(\tau)\right\|_p^2+\left\|\tilde{W}_2(\tau)(\varphi(\hat{x}(\tau))u(\tau)\right\|_p^2\right)d\tau\right.$$
$$\left.+\int_{\tau=t-h(t)}^{t}\left(\left\|\hat{W}_2\tilde{\varphi}(\tau)u(\tau)\right\|_p^2+\left\|K\eta(\tau)\right\|_p^2+\left\|\tilde{f}(\tau)\right\|_p^2+\left\|\xi(\tau)\right\|_p^2\right)d\tau\right\}$$

Each term of α_t and $\beta(t)$ is upper bounded, next facts are used. Norm inequality $\|AB\|\leq\|A\|\|B\|$ and the matrix inequality

$$XY^T+YX^T\leq X\Lambda\Lambda^T+Y\Lambda^{-1}Y^T$$

valid for any $X,Y\in R^{r\times s}$ and any $0<\Lambda=\Lambda^T\in R^{s\times s}$ (Poznyak, 2001).
It also necessary to represents the state estimation error δ_t as a function of the available output, the estimation error e_t :

$$-e(t)=\hat{y}(t)-y(t)=C\hat{x}(t)-Cx(t)-\eta(t)$$
$$-C^Te(t)=C^T(C\delta(t)-\eta(t))$$
$$-C^Te(t)+C^T\eta(t)=C^TC\delta(t)+\varpi\delta(t)-\varpi\delta(t)$$
$$-C^Te(t)+C^T\eta(t)+\varpi\delta(t)=\left(C^TC+\varpi I\right)\delta(t)$$

Giving

$$\delta(t)=N_{\varpi}\left(-C^Te(t)+C^T\eta(t)+\varpi\delta(t)\right)$$

where:

$$N_{\varpi} := \left(C^T C + \varpi I\right)^{-1}$$

and ϖ is a small positive scalar. Taking into account all these facts next estimation is obtained:

$$\frac{d}{dt}V(t) \le h(t)\delta_{t-h(t)}^T \Big[\tilde{A}^T P + P\tilde{A} +$$
$$P\left(\Lambda_1^{-1} + \hat{W}_1 \Lambda_5^{-1}(\hat{W}_1)^T + \hat{W}_2 \Lambda_8^{-1}(\hat{W}_2)^T + \Lambda_9^{-1} + \Lambda_{10}^{-1}\right)P +\Big)$$
$$\left(\|\Lambda_5\| L_\sigma + \|\Lambda_8\| L_\varphi \Upsilon_u^2 + \mu_1 + \mu_2 L_\sigma + \mu_3 \Upsilon_u^2 L_\varphi\right) I + \varpi\left(\Lambda_3^{-1} + \Lambda_7^{-1}\right) + Q_0 \Big]\delta_{t-h(t)} +$$

$$h(t)^3 \left[\|\Lambda_1\| \|\tilde{A}\|^2 \frac{L_\delta^2}{4} + \|\Lambda_5\| \frac{L_\sigma L_\delta^2}{3} + \mu_2 \frac{L_\sigma L_\delta^2}{3} + \mu_1 \frac{L_\delta^2}{3} + \mu_3 \frac{\Upsilon_u^2 L_\varphi L_\delta^2}{3} + \|\Lambda_8\| \frac{L_\varphi \Upsilon_u^2 L_\delta^2}{3} \right] +$$
$$h(t) \left[\|\Lambda_9\| \left(\left(\|K\| \left\|\Lambda_\eta^{-1}\right\|^{1/2} \Upsilon_\eta + \left\|\Lambda_\xi^{-1}\right\|^{1/2} \Upsilon_\xi \right) \right)^2 + \|\Lambda_{10}\| \left\|\Lambda_{\tilde{f}}^{-1}\right\| \left[\tilde{f}_0 + \tilde{f}_1 \|x(t)\|^2_{\Lambda_{\tilde{f}}^1} \right] + \right.$$
$$\|K\|^2 \|P\| \left\|\Lambda_\eta^{-1}\right\|^{1/2} \Upsilon_\eta + \|P\| \left\|\Lambda_{\tilde{f}}^{-1}\right\| \left[\tilde{f}_0 + \tilde{f}_1 \left\|\Lambda_{\tilde{f}}^1\right\| Diam(x)^2 \right] + \Big)$$
$$\left. \|P\| \left\|\Lambda_\xi^{-1}\right\| \Upsilon_\xi + 2\Upsilon_\eta - \delta_{t-h(t)}^T Q_0 \delta_{t-h(t)} \right] + \int_{\tau=t-h(t)}^{t} 2\left(e^T(t-h(t)) C N_{\varpi} P \tilde{W}_1(\tau) \sigma(\hat{x}(\tau))\right) d\tau$$
$$+ \int_{\tau=t-h(t)}^{t} \left[\sigma^T(\hat{x}(\tau)) \tilde{W}_1^T(\tau) P N_{\varpi} \left(C\Lambda_2 C + \varpi\Lambda_3\right) N_{\varpi} P \tilde{W}_1(\tau) \sigma(\hat{x}(\tau)) \right] d\tau$$

$$+ \int_{\tau=t-h(t)}^{t} \sigma^T\left(\hat{x}_\tau\right) \tilde{W}_1^T(\tau) P \tilde{W}_\tau \sigma\left(\hat{x}_\tau\right) d\tau + k_1(t) tr\left\{\tilde{W}_1^T(t) \tilde{W}_1(t)\right\} -$$
$$k_1(t-h(t)) tr\left\{\tilde{W}_1^T(t-h(t)) \tilde{W}_1(t-h(t))\right\} +$$
$$\int_{\tau=t-h(t)}^{t} 2\left(e^T(t-h(t)) C N_{\varpi} P \tilde{W}_2(\tau) (\varphi(\hat{x}(\tau)) u(\tau)\right) d\tau$$
$$+ \int_{\tau=t-h(t)}^{t} u^T(\tau) \varphi^T(\hat{x}(\tau)) \tilde{W}_2^T(\tau) P N_{\varpi} \left(C^T \Lambda_6 C + \varpi\Lambda_7\right) N_{\varpi} P \tilde{W}_2(\tau) (\varphi(\hat{x}(\tau)) u(\tau) d\tau$$
$$+ \int_{\tau=t-h(t)}^{t} u^T(\tau) (\varphi(\hat{x}(\tau))^T \tilde{W}_2^T(\tau) P \tilde{W}_2(\tau) (\varphi(\hat{x}(\tau)) u(\tau) d\tau +$$
$$k_2(t) tr\left\{\tilde{W}_2^T(t) \tilde{W}_2(t)\right\} - k_2(t-h(t)) tr\left\{\tilde{W}_2^T(t-h(t)) \tilde{W}_2(t-h(t))\right\}$$

Considering

$$
\begin{gathered}
\tilde{A}^T(K)P + P\tilde{A}(K) + PR^{-1}P + Q(\delta,\mu_1,\mu_2,\mu_3) \leq 0 \\
R^{-1} = \Lambda_1^{-1} + \hat{W}_1\Lambda_5^{-1}(\hat{W}_1)^T + \hat{W}_2\Lambda_8^{-1}(\hat{W}_2)^T + \Lambda_9^{-1} + \Lambda_{10}^{-1} \\
Q(\delta,\mu_1,\mu_2,\mu_3) = \left[\|\Lambda_5\|L_\sigma + \|\Lambda_8\|L_\varphi\Upsilon_u^2 + \mu_1 + \mu_2 L_\sigma + \mu_3\Upsilon_u^2 L_\varphi\right]I \\
+\varpi\left(\Lambda_3^{-1} + \Lambda_7^{-1}\right) + Q_0
\end{gathered}
$$

implies:

$$
\begin{gathered}
\int_{\tau=t-h(t)}^{t} tr\Big\{\tilde{W}_1^T(\tau)P\Big[2N_\varpi C^T e(t-h(t)) + N_\varpi\left(\varpi\Lambda_3 + C^T\Lambda_2 C\right)N_\varpi P\tilde{W}_1(\tau)\sigma(\hat{x}(\tau)) \\
+\tilde{W}_1(\tau)\sigma(\hat{x}(\tau))\Big]\sigma^T(\hat{x}(\tau))\Big\}d\tau \\
+k_1(t)tr\left\{\tilde{W}_1^T(t)\tilde{W}_1(t)\right\} - k_1(t-h(t))tr\left\{\tilde{W}_1^T(t-h(t))\tilde{W}_1(t-h(t))\right\} = 0
\end{gathered}
$$

that can be obtained selecting

$$
\begin{gathered}
\frac{d}{dt}W_1(t) = \\
-\frac{k_1(t)^{-1}}{2}\Big\{P\Big[2N_\varpi C^T e(t\text{-}h(t)) + N_\varpi\left(\varpi\Lambda_3 + C^T\Lambda_2 C\right)N_\varpi P\tilde{W}_1(\tau)\sigma(\hat{x}(\tau)) + \\
\tilde{W}_1(\tau)\sigma(\hat{x}(\tau))\Big]\sigma^T(\hat{x}(\tau)) - \\
\frac{dk_1(t)}{dt}\tilde{W}_1(t)\Big\}
\end{gathered}
$$

Analogously, for the second adaptive law

$$
\begin{gathered}
\int_{\tau=t-h(t)}^{t} \mathbf{tr}\Big\{\tilde{W}_2^T(\tau)P\Big[2N_\varpi C^T e(t-h(t)) + N_\varpi\left(C^T\Lambda_6 C + \varpi\Lambda_7\right)N_\varpi P\tilde{W}_1(\tau)(\varphi(\hat{x}(\tau))u(\tau) \\
+\tilde{W}_2(\tau)(\varphi(\hat{x}(\tau))u(\tau)\Big]u^T(\tau)\varphi^T(\hat{x}(\tau))\Big\}d\tau \\
+k_2(t)\mathbf{tr}\left\{\tilde{W}_2^T(t)\tilde{W}_2(t)\right\} - k_2(t-h(t))\mathbf{tr}\left\{\tilde{W}_2^T(t-h(t))\tilde{W}_2(t-h(t))\right\} = 0
\end{gathered}
$$

leading to

$$
\begin{gathered}
\frac{d}{dt}W_2(t) = \\
-\frac{k_2(t)^{-1}}{2}\Big\{P\Big[2N_\varpi C^T e(t\text{-}h(t)) + N_\varpi\left(C^T\Lambda_6 C + \varpi\Lambda_7\right)N_\varpi P\tilde{W}_1(\tau)(\varphi(\hat{x}(\tau))u(\tau) + \\
\tilde{W}_2(\tau)(\varphi(\hat{x}(\tau))u(\tau)\Big]u^T(\tau)\varphi^T(\hat{x}(\tau)) \\
-\frac{dk_2(t)}{dt}\tilde{W}_2(t)\Big\}
\end{gathered}
$$

Finally:

$$\frac{d}{dt}V(t)\le h(t)^3\left[\|\Lambda_1\|\|\tilde{A}\|^2\frac{L_\delta^2}{4}+\|\Lambda_5\|\frac{L_\sigma L_\delta^2}{3}+\mu_2\frac{L_\sigma L_\delta^2}{3}+\mu_1\frac{L_\delta^2}{3}+\mu_3\frac{\Upsilon_u^2 L_\varphi L_\delta^2}{3}+\|\Lambda_8\|\frac{L_\varphi\Upsilon_u^2 L_\delta^2}{3}\right]$$
$$+h(t)\left[\|\Lambda_9\|\left(\left(\|K\|\left\|\Lambda_\eta^{-1}\right\|^{1/2}\Upsilon_\eta+\left\|\Lambda_\xi^{-1}\right\|^{1/2}\Upsilon_\xi\right)\right)^2+\|\Lambda_{10}\|\left\|\Lambda_{\tilde{f}}^{-1}\right\|\left[\tilde{f}_0+\tilde{f}_1\|x(t)\|^2_{\Lambda^1_{\tilde{f}}}\right]\right.$$
$$+\|K\|^2\|P\|\left\|\Lambda_\eta^{-1}\right\|^{1/2}\Upsilon_\eta+\|P\|\left\|\Lambda_{\tilde{f}}^{-1}\right\|\left[\tilde{f}_0+\tilde{f}_1\left\|\Lambda^1_{\tilde{f}}\right\|Diam(x)^2\right]$$
$$\left.+\|P\|\left\|\Lambda_\xi^{-1}\right\|\Upsilon_\xi+2\Upsilon_\eta-\delta^T_{t-h(t)}Q_0\delta_{t-h(t)}\right]$$

or in the short form:

$$\frac{d}{dt}V(t)\le h(t)\left(h(t)^2 a+b-\delta^T(t-h(t))Q_0\delta(t-h(t))\right)$$

where

$$a:=\|\Lambda_1\|\|\tilde{A}\|^2\frac{L_\delta^2}{4}+\|\Lambda_5\|\frac{L_\sigma L_\delta^2}{3}+\mu_2\frac{L_\sigma L_\delta^2}{3}+\mu_1\frac{L_\delta^2}{3}+\mu_3\frac{\Upsilon_u^2 L_\varphi L_\delta^2}{3}+\|\Lambda_8\|\frac{L_\varphi\Upsilon_u^2 L_\delta^2}{3}$$
$$b:=\|\Lambda_9\|\left(\left(\|K\|\left\|\Lambda_\eta^{-1}\right\|^{1/2}\Upsilon_\eta+\left\|\Lambda_\xi^{-1}\right\|^{1/2}\Upsilon_\xi\right)\right)^2+\|\Lambda_{10}\|\left\|\Lambda_{\tilde{f}}^{-1}\right\|\left[\tilde{f}_0+\tilde{f}_1\|x(t)\|^2_{\Lambda^1_{\tilde{f}}}\right]$$
$$+\|K\|^2\|P\|\left\|\Lambda_\eta^{-1}\right\|^{1/2}\Upsilon_\eta+\|P\|\left\|\Lambda_{\tilde{f}}^{-1}\right\|\left[\tilde{f}_0+\tilde{f}_1\left\|\Lambda^1_{\tilde{f}}\right\|Diam(x)^2\right]+\|P\|\left\|\Lambda_\xi^{-1}\right\|\Upsilon_\xi+2\Upsilon_\eta$$

So,

$$\delta^T(t-h(t))Q_0\delta(t-h(t))\le\left(ah(t)^2+b\right)-\frac{dV(t)}{dt}\frac{1}{h(t)}$$

And integrating, we obtain

$$\int_{\tau=0}^{T}\delta^T(\tau-h(\tau))Q_0\delta(\tau-h(t)(\tau))d\tau\le\int_{\tau=0}^{T}\left[\left(ah(\tau)^2+b\right)-\frac{dV(t)}{dt}\frac{1}{h(\tau)}\right]d\tau$$

And hence,

$$-\int_{\tau=0}^{T}\frac{dV_\tau}{h(\tau)}=-\int_{\tau=0}^{T}d\left(\frac{V_\tau}{h(\tau)}\right)+\int_{\tau=0}^{T}\frac{V_\tau}{h(\tau)^2}h(\tau)d\tau\le$$
$$-\int_{\tau=0}^{T}d\left(\frac{V_\tau}{h(\tau)}\right)=-\frac{V_t}{h(t)}+\frac{V_0}{h(0)}\le\frac{V_0}{h(0)}$$

This implies

$$\int_{\tau=0}^{T}\delta^T(\tau-h(t)(\tau))Q_0\delta(\tau-h(t)(\tau))d\tau\le a\int_{\tau=0}^{T}h(t)^2d\tau+bT+\frac{V_0}{h(0)}$$

Dividing by T and taking the upper limit we finally get (30).

8. References

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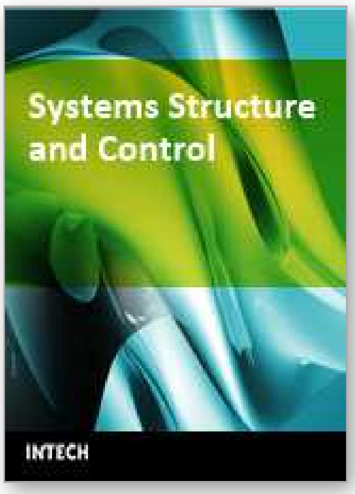
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