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Control Designs for Linear Systems Using State-Derivative Feedback

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1. Introduction

From classical control theory, it is well-known that state-derivative feedback can be very useful, and even in some cases essential to achieve a desired performance. Moreover, there exist some practical problems where the state-derivative signals are easier to obtain than the state signals. For instance, in the following applications: suppression of vibration in mechanical systems, control of car wheel suspension systems, vibration control of bridge cables and vibration control of landing gear components. The main sensors used in these problems are accelerometers. In this case, from the signals of the accelerometers it is possible to reconstruct the velocities with a good precision but not the displacements. Defining the velocities and displacement as the state variables, then one has available for feedback the state-derivative signals. Recent researches about state-derivative feedback design for linear systems have been presented. The procedures consider, for instance, the pole placement problem (Abdelaziz & Valášek, 2004; Abdelaziz & Valášek, 2005), and the design of a Linear Quadratic Regulator (Duan et al., 2005). Unfortunately these results are not applied to the control of uncertain systems or systems subject to structural failures. Another kind of control design is the use of state-derivative and state feedback. It has been used by many researches for applications in descriptor systems (Nichols et al., 1992; A. Bunse-Gerstner & Nichols, 1999; Duan et al., 1999; Duan & Zhang, 2003). However, usually these designs are more complex than the design procedures with only state or state-derivative feedback.

In this chapter two new control designs using state-derivative feedback for linear systems are presented. Firstly, considering linear descriptor plants, a simple method for designing a state-derivative feedback gain using methods for state feedback control design is proposed. It is assumed that the descriptor system is a linear, time-invariant, Single-Input (SI) or Multiple-Input (MI) system. The procedure allows that the designers use the well-known state feedback design methods to directly design state-derivative feedback control systems. This method extends the results described in (Cardim et al., 2007) and (Abdelaziz & Valášek, 2004) to a more general class of control systems, where the plant can be a descriptor system. As the first design can not be directly applied for uncertain systems, then a design considering LMI formulation is presented. This result can be used to solve systems with polytopic uncertainties in the plant parameters, or subject to structural failures. Furthermore, it can include as design specifications the decay rate and bounds on the output

peak, and on the state-derivative feedback matrix K . When feasible, LMI can be easily solved using softwares based on convex programming, for instance MATLAB. These new control designs allow new specifications, and also consider a broader class of plants than the related results available in the literature (Abdelaziz & Valášek, 2004; Duan et al., 2005; Assunção et al., 2007c). The proposed method extends the results presented in (Assunção et al., 2007c), because it can also be applied for the control of uncertain systems subject to structural failures. Examples illustrate the efficiency of these procedures.

2. Design of State-Derivative Feedback Controllers for Descriptor Systems Using a State Feedback Control Design

In this section, a simple method for designing a state-derivative feedback gain using methods for state feedback control design, where the plant can be a descriptor system, is proposed.

2.1 Statement of the Problem

Consider a controllable linear descriptor system described by

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \quad (1)$$

where $E \in \mathbb{R}^{n \times n}$, $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the control input vector. It is assumed that $1 \leq m \leq n$, and also, $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are time-invariant matrices. Now, consider the state-derivative feedback control

$$u(t) = -K_d \dot{x}(t). \quad (2)$$

Then, the problem is to obtain a state-derivative feedback gain K_d , using state feedback techniques, such that the poles of the controlled system (1), (2) are arbitrarily specified by a set $\{\lambda_1, \lambda_2, \dots, \lambda_n\}$, where $\lambda_i \in \mathbb{C}$ and $\lambda_i \neq 0$, $i = 1, 2, \dots, n$, such that this closed-loop systems presents a suitable performance. The motivation of this study was to investigate the possibility of designing state-derivative gains using state feedback design methods. This procedure allows the designers to use well-known methods for pole-placement using state feedback, available in the literature, for state-derivative feedback design (Chen, 1999; Valášek & Olgac, 1995a; Valášek & Olgac, 1995b). To establish the proposed results, consider the following assumptions:

- (A) $\text{rank}[E \mid B] = n$;
- (B) $\text{rank}[A] = n$;
- (C) $\text{rank}[B] = m$.

Remark 1. It is known (Bunse-Gerstner et al, 1992; Duan et al, 1999) that if Assumption (A) holds, then there exists K_d such that:

$$\text{rank}[E + BK_d] = n. \quad (3)$$

Assumption (B) was also considered in (Abdelaziz & Valášek, 2004) and, as will be described below, is important for the stability of the system (1), with the proposed method and the control law $u = -K_d \dot{x}$. Assumption (C) means that B is a full rank matrix. For K_d

such that (3) holds, then from (2) it follows that (1) can be rewrite such as a standard linear system, given by:

$$E\dot{x}(t) = Ax(t) - BK_d\dot{x}(t), \quad (4)$$

$$\dot{x}(t) = (E + BK_d)^{-1} Ax(t). \quad (5)$$

From (5) note that if $\text{rank}(A) < n$, then the controlled system (1), (2) given by (5) is unstable, because it presents at least one pole equal to zero. It is known that the stability problem for descriptor systems is much more complicated than for standard systems, because it is necessary to consider not only stability, but also regularity (Bunse-Gerstner et al., 1992; S. Xu & J. Lam, 2004). In this work, a descriptor system is regular if it has uniqueness in the solutions and avoid impulsive responses. In the next section, the proposed method is presented.

2.2 Design of State-Derivative Feedback Using a State Feedback Design

Lemma 1 below will be very useful in the analysis of the method that solves the proposed problem.

Lemma 1. Consider a matrix $Z \in \mathbb{R}^{n \times n}$, with $\text{rank}(Z) = n$ and eigenvalues equal to $\lambda_1, \lambda_2, \dots, \lambda_n$.

Then, the eigenvalues of Z^{-1} are the following: $\lambda_1^{-1}, \lambda_2^{-1}, \dots, \lambda_n^{-1}$.

Proof: For each eigenvalue $\lambda \in \{ \lambda_1, \lambda_2, \dots, \lambda_n \}$ of Z , there exists an eigenvector v such that

$$Zv = \lambda v. \quad (6)$$

Considering that $\text{rank}(Z) = n$, then $\lambda \neq 0$. Therefore, from (6),

$$v = Z^{-1}\lambda v \Rightarrow \lambda^{-1}v = Z^{-1}v, \quad (7)$$

and so λ^{-1} is an eigenvalue of Z^{-1} .

Remark 2. Consider that $\lambda = a + jb$ is an eigenvalue of Z . Then, from Lemma 1,

$\lambda^{-1} = (a + jb)^{-1} = \frac{a}{a^2 + b^2} - j \frac{b}{a^2 + b^2}$ is also an eigenvalue of Z^{-1} . Therefore, note that the real parts of the

λ and λ^{-1} present the same signal. So, if Z is Hurwitz (it has all eigenvalues with negative real parts), then Z^{-1} will be also Hurwitz.

Now, the main result of this section will be presented.

Theorem 1. Define the matrices:

$$A_n = A^{-1}E \quad \text{and} \quad B_n = -A^{-1}B \quad (8)$$

and suppose that (A_n, B_n) is controllable. Let K_d be a state feedback gain, such that $\{ \lambda_1^{-1}, \lambda_2^{-1}, \dots, \lambda_n^{-1} \}$ are the poles of the closed-loop system

$$\dot{x}_n(t) = A_n x_n(t) + B_n u_n(t), \quad (9)$$

$$u_n(t) = -K_d x_n(t), \quad (10)$$

where $\lambda_i \in \mathbb{C}$ and $\lambda_i \neq 0$, $i = 1, 2, \dots, n$, are arbitrarily specified. Then, for this gain K_d , $\{\lambda_1, \lambda_2, \dots, \lambda_n\}$ are the poles of the controlled system with state-derivative feedback (1), (2) and also, the condition (3) holds.

Proof: Considering that (A_n, B_n) is controllable, then one can find a state feedback gain K_d such that the controlled system with state feedback (9), (10), given by

$$\dot{x}_n(t) = (A_n - B_n K_d) x_n(t). \quad (11)$$

has poles equal to $\lambda_1^{-1}, \lambda_2^{-1}, \dots, \lambda_n^{-1}$ (Chen, 1999). Now, from $A_n = A^{-1}E, B_n = -A^{-1}B$ and $\lambda_i \neq 0$, $i = 1, 2, \dots, n$, note that

$$(A_n - B_n K_d)^{-1} = [A^{-1}(E + BK_d)]^{-1} \quad (12)$$

$$= (E + BK_d)^{-1} A \quad (13)$$

and from (11) and Lemma 1, $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of $(E + BK_d)^{-1} A$. Therefore (3) holds, the state-derivative feedback system (1) and (2) can be described by (5) and presents poles equal to $\lambda_1, \lambda_2, \dots, \lambda_n$.

This result is a generalization of the methods proposed in (Abdelaziz & Valášek, 2004) and (Cardim et al., 2007), because it can be applied in the control of descriptor systems (1), with $\det(E) = 0$.

2.3 Examples

The effectiveness of the proposed methods designs is demonstrated by simulation results.

First Example

A simple electrical circuit, can be represented by the linear descriptor system below (Nichols et al, 1992):

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad (14)$$

where x_1 is the current and the x_2 is the potential of a capacitor. In this system one has:

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (15)$$

Consider the pole placement as design technique, using the state derivative feedback (2) with the feedback gain matrix K_d . In this example, the suitable closed-loop poles for the controlled system (2) and (14) are the following:

$$\lambda_1 = -2 + li, \quad \lambda_2 = -2 - li$$

Note that, the system (14) with the control signal (2) satisfies the Assumptions A, B and C. From (8) one has:

$$A_n = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_n = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad (16)$$

and (A_n, B_n) is controllable.

From Theorem 1, the poles for the new closed-loop system with state feedback (9) and (10) with A_n and B_n given in (8) are the following:

$$\lambda_1^{-1} = -0.40 - 0.20i, \quad \lambda_2^{-1} = -0.40 + 0.20i.$$

So, one can obtain by using the command *acker* of MATLAB (Ogata, 2002), the feedback gain matrix K_d below:

$$K_d = [-0.20 \quad -0.80]. \quad (17)$$

Figures 1 and 2 show the simulation results of the controlled system (5) with the initial condition $x(0) = [1 \ 0]^T$. In this example the validity and simplicity of the proposed method can be observed.

Example 2

Consider a linear descriptor MI system described by the following equations:

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -0.800 & 0.020 \\ -0.020 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0.050 & 1 \\ 0.001 & 0 \end{bmatrix} u(t), \quad (18)$$

where $u(t) = [u_1(t) \ u_2(t)]^T$.

The wanted poles for closed-loop system with the control law $u(t) = -K_d \dot{x}(t)$ are given by:

$$\lambda_1 = -2 + 1i, \quad \lambda_2 = -2 - 1i.$$

Observe that, the system (18) with the control signal (2) satisfies the Assumptions A, B and C. From (8) one has:

$$A_n = \begin{bmatrix} -50 & 0 \\ -2000 & 0 \end{bmatrix}, \quad B_n = \begin{bmatrix} 0.050 & 0 \\ -0.500 & -50 \end{bmatrix}, \quad (19)$$

and (A_n, B_n) is controllable.

From Theorem 1, the poles for the new closed-loop system with state feedback (11), with A_n and B_n given in (19) are the following:

$$\lambda_1^{-1} = -0.40 - 0.20i, \quad \lambda_2^{-1} = -0.40 + 0.20i.$$

So, with these parameters, one can obtain with the command *place* of MATLAB, the feedback gain matrix K_d below:

$$K_d = \begin{bmatrix} -992.0000 & 4.0000 \\ 49.9240 & -0.0480 \end{bmatrix}. \quad (20)$$

Figures 3 and 4 show the simulation results of the controlled system with state-derivative feedback, given by (2), (18) and (20) that can be described by (5), with the initial condition $x_0 = [1\ 0]^T$.

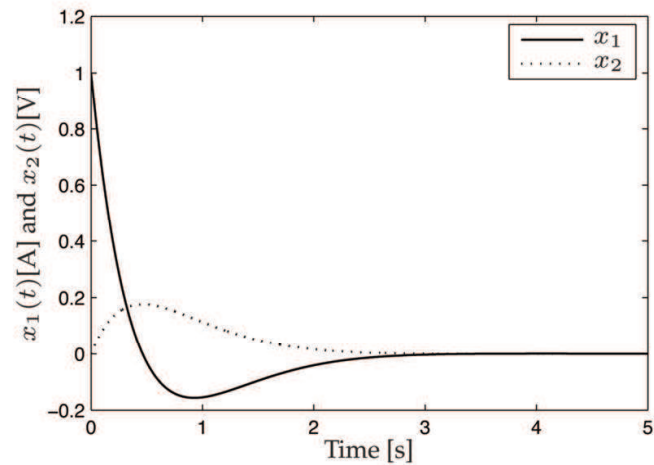


Figure 1. Transient response of the controlled system (Example 1), for $x_0 = [1\ 0]^T$

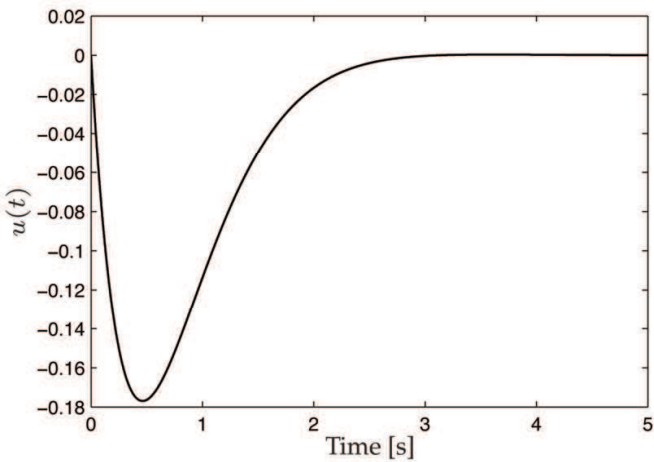


Figure 2. Control inputs of the controlled system (Example 1), for $x_0 = [1\ 0]^T$

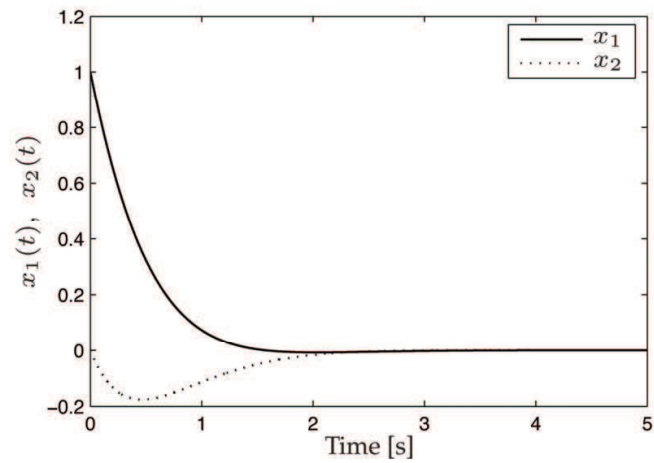


Figure 3. Transient response of the controlled system (Example 2), for $x_0 = [1\ 0]^T$

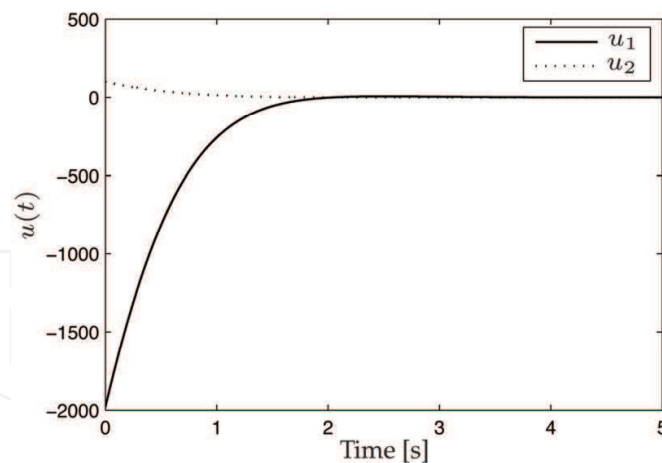


Figure 4. Control inputs of the controlled system (Example 2), for $x_0 = [1 \ 0]^T$

Example 3

In this example, is considered that the matrix $E = I$. So, the system (1) is in the standard space state form. The idea was to show that, for the case where $\det(E) \neq 0$, the proposed method is also valid.

Consider the mechanical system shown in Figure 5. It is a simple model of a controlled vibration absorber, in the sense of reducing the oscillations of the masses m_1 and m_2 . In this case, the model contains two control inputs, $u_1(t)$ and $u_2(t)$. This system is described by the following equations (Cardim et al., 2007):

$$\begin{cases} m_1 \ddot{y}_1(t) + b_1(\dot{y}_1(t) - \dot{y}_2(t)) + k_1 y_1(t) = u_1(t), \\ m_2 \ddot{y}_2(t) + b_1(\dot{y}_2(t) - \dot{y}_1(t)) + k_2 y_2(t) = u_2(t). \end{cases} \quad (21)$$

The state space form of the mechanical system in Figure 5 is represented in equation (1) considering as state variables $x(t) = [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T$, where $x_1(t) = y_1(t)$, $x_2(t) = \dot{y}_1(t)$, $x_3(t) = y_2(t)$, $x_4(t) = \dot{y}_2(t)$, $u(t) = [u_1(t) \ u_2(t)]^T$ and:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b_1}{m_2} & \frac{-k_2}{m_2} & \frac{-b_1}{m_2} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}. \quad (22)$$

For a digital simulation of the control system, assume for instance that $m_1 = 10\text{kg}$, $m_2 = 30\text{kg}$, $k_1 = 2.5\text{kN/m}$, $k_2 = 1.5\text{kN/m}$ and $b_1 = 30\text{Ns/m}$. Consider the pole placement as design technique, and the following closed-loop poles for the controlled system:

$$\lambda_1 = -10, \quad \lambda_2 = -15, \quad \lambda_{3,4} = -2 \pm 10i.$$

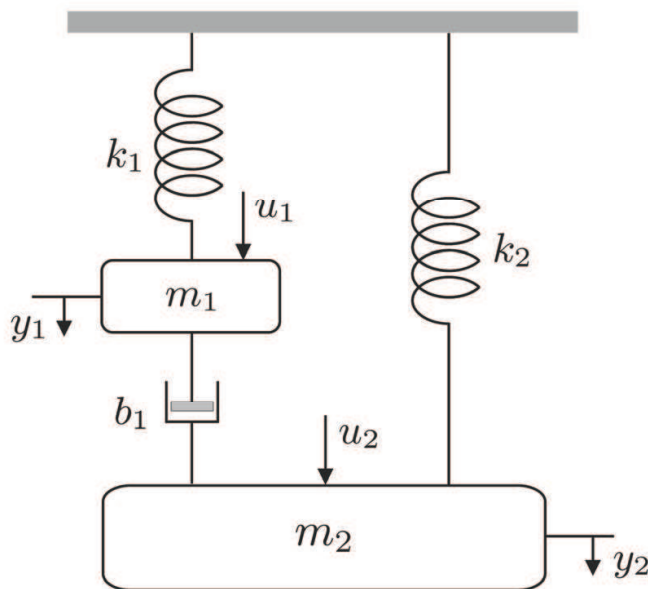


Figure 5. Multivariable (MI) mass-spring system with damping

With these parameters and from (8), one has:

$$A_n = \begin{bmatrix} -0.0120 & -0.0040 & 0.0120 & 0 \\ 1.0000 & 0 & 0 & 0 \\ 0.0200 & 0 & -0.0200 & -0.0200 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}, B_n = \begin{bmatrix} 0.4000 \times 10^{-3} & 0 \\ 0 & 0 \\ 0 & 0.6667 \times 10^{-3} \\ 0 & 0 \end{bmatrix}, \quad (23)$$

and (A_n, B_n) is controllable.

From Theorem 1, the poles for the new closed-loop system with state feedback (11), with A_n and B_n given in (23) are the following:

$$\lambda_1^{-1} = -0.1000, \quad \lambda_2^{-1} = -0.0667, \quad \lambda_{3,4}^{-1} = -0.0192 \pm 0.0962i.$$

So, with these parameters, one can obtain through the command *place* of MATLAB, the feedback gain matrix K_d below:

$$K_d = \begin{bmatrix} 178.9532 & -6.4647 & 323.3542 & 19.8478 \\ -79.6370 & -11.4321 & 152.3204 & -26.1863 \end{bmatrix}. \quad (24)$$

Figures 6 and 7 show the simulation results of the controlled system (1), (2), (22), (24), that can be given by (5), with the initial condition $x(0) = [0.1 \ 0 \ 0.1 \ 0]^T$.

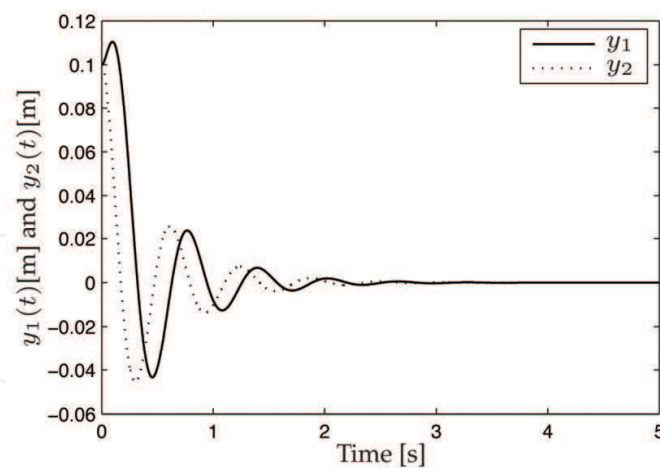


Figure 6. Transient response of the controlled system (Example 3), with $x(0) = [0.1 \ 0 \ 0.1 \ 0]^T$

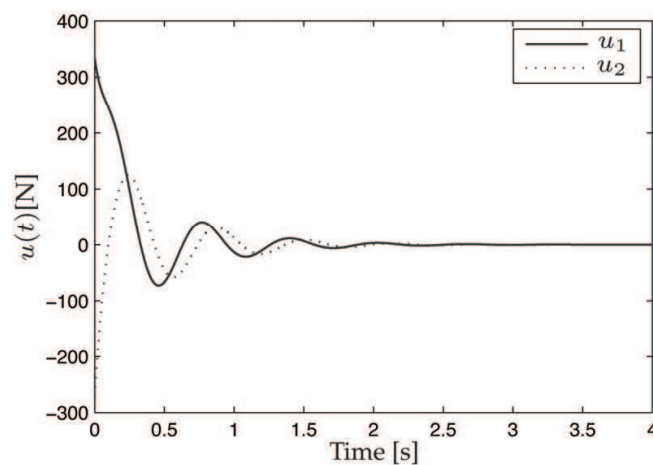


Figure 7. Control inputs of the controlled system (Example 3), with $x(0) = [0.1 \ 0 \ 0.1 \ 0]^T$

3. LMI-Based Control Design for State-Derivative Feedback

Consider the linear time-invariant uncertain polytopic system, described as convex combinations of the polytope vertices:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r_a} \alpha_i A_i x(t) + \sum_{j=1}^{r_b} \beta_j B_j u(t), \\ &= A(\alpha)x(t) + B(\beta)u(t),\end{aligned}\tag{25}$$

and

$$\left. \begin{aligned} \alpha_i &\geq 0, & i &= 1, \dots, r_a, & \sum_{i=1}^{r_a} \alpha_i &= 1, \\ \beta_j &\geq 0, & j &= 1, \dots, r_b, & \sum_{j=1}^{r_b} \beta_j &= 1, \end{aligned} \right\}\tag{26}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, r_a and r_b are the numbers of polytope vertices of the matrices $A(\alpha)$ and $B(\beta)$, respectively. For $i = 1, \dots, r_a$ and $j = 1, \dots, r_b$ one has: $A_i \in \mathbb{R}^{n \times n}$ and $B_j \in \mathbb{R}^{n \times m}$ are constant matrices and α_i and β_j are constant and unknown real numbers.

From (8) and (25), one has:

$$A_n = A(\alpha)^{-1}E \quad \text{and} \quad B_n = -A(\alpha)^{-1}B(\beta), \quad (27)$$

Then, for the control design of the system (25) with Theorem 1, is necessary to know the real numbers α_i and β_j . However, in the practical problems these parameters are unknown.

Therefore, Theorem 1 can not be directly applied in the control design of the system (25). For the solution of this problem, in this section sufficient Linear Matrix Inequalities (LMI) conditions for asymptotic stability of linear uncertain systems using state-derivative feedback are presented. The LMI formulation has emerged recently (Boyd et al., 1994) as an useful tool for solving a great number of practical control problems such as model reduction, design of linear, nonlinear, uncertain and delayed systems (Boyd et al., 1994; Assunção & Peres, 1999; Teixeira et al., 2001; Teixeira et al., 2002; Teixeira et al., 2003; Palhares et al., 2003; Teixeira et al., 2005; Assunção et al., 2007a; Assunção et al., 2007b; Teixeira et al., 2006). The main features of this formulation are that different kinds of design specifications and constraints that can be described by LMI, and once formulated in terms of LMI, the control problem, when it presents a solution, can be efficiently solved by convex optimization algorithms (Nesterov & Nemirovsky, 1994; Boyd et al., 1994; Gahinet et al., 1995; Sturm, 1999). The global optimum is found with polynomial convergence time (El Ghaoui & Niculescu, 2000). The state-derivative feedback has been examined with various approaches (Abdelaziz & Valášek, 2004; Kwak et al., 2002; Duan et al., 2005; Cardim et al., 2007), but neither them can be applied for uncertain systems or systems subject to structural failures (Isermann, 1997; Isermann & Ballé, 1997; Isermann, 2006). Robust state-derivative feedback LMI-based designs for linear time-invariant and time-varying systems were recently proposed in (Assunção et al., 2007c), but the results does not consider structural failures in the control design. Structural failures appear of natural form in the systems, for instance, in the following cases: physical wear of equipments, or short circuit of electronic components.

Recent researches for detection of the structural failures (or faults) in systems, have been presented in LMI framework (Zhong et al., 2003; Liu et al., 2005; D. Ye & G. H. Yang, 2006; S. S. Yang & J. Chen, 2006).

In this section, we will show that it is possible to extend the presented results in (Assunção et al., 2007c), for the case where there exist structural failures in the plant. A fault-tolerant design is proposed. The methods can include in the LMI-based control designs the specifications of bounds: on the decay rate, on the output peak, and on the state-derivative feedback matrix K . These design procedures allow new specifications and also, they consider a broader class of plants than the related results available in the literature.

3.1 Statement of the Problem

Consider a homogeneous linear time-invariant system given by

$$\dot{x}(t) = A_N x(t) \quad (28)$$

It is known from literature that the linear system (28) is asymptotically stable if there exist a symmetric matrix P satisfying the Lyapunov conditions (Boyd et al., 1994):

$$\left. \begin{array}{l} P > 0, \\ \text{and} \\ A_N' P + P A_N < 0. \end{array} \right\} \quad (29)$$

This result is useful for the design of the proposed controller.

In this work, structural failure is defined as a permanent interruption of the system's ability to perform a required function under specified operating conditions (Isermann & Ballé, 1997).

Systems subject to structural failures can be described by uncertain polytopic systems (25) (see Section 3.5 for details). Now, suppose that all poles of (25) are different from zero (the matrix $A(\alpha)$ must have a full rank). Then, the proposed problem is defined below.

Problem 1: Find a constant matrix $K \in \mathbb{R}^{m \times n}$ such that the following conditions hold:

1. $(I + B(\beta)K)$ has a full rank;
2. the closed-loop system (25) with the state-derivative feedback control

$$u(t) = -K\dot{x}(t), \quad (30)$$

is asymptotically stable.

Note that from (25) and (30) it follows that

$$\dot{x}(t) = A(\alpha)x(t) - B(\beta)K\dot{x}(t)$$

or

$$(I + B(\beta)K)\dot{x}(t) = A(\alpha)x(t).$$

When $(I + B(\beta)K)$ has a full rank, the closed-loop system is well-defined and given by

$$\dot{x}(t) = (I + B(\beta)K)^{-1} A(\alpha)x(t). \quad (31)$$

This condition was also assumed in other related researches (Kwak et al., 2002; Abdelaziz & Valášek, 2004; Assunção et al., 2007c; Cardim et al., 2007).

3.2 Robust Stability Condition for State-derivative Feedback

The main results of this section is presented in the next theorem, that solves Problem 1 (Assunção et al., 2007c). For the proof of this theorem, the following result will be useful.

Remark 3. Recall that for any nonsymmetric matrix M ($M \neq M'$), $M \in \mathbb{R}^{n \times n}$, if $M + M' < 0$, then M has a full rank.

Theorem 2. A sufficient condition for the solution of Problem 1 is the existence of matrices $Q = Q'$ and Y , where $Q \in \mathbb{R}^{n \times n}$ and $Y \in \mathbb{R}^{m \times n}$, such that:

$$Q > 0, \quad (32)$$

$$QA'_i + A_i Q + B_j YA'_i + A_i Y' B'_j < 0 \quad (33)$$

where $i = 1, \dots, r_a$ and $j = 1, \dots, r_b$. Furthermore, when (32) and (33) hold, a state-derivative feedback matrix that solves the Problem 1 is given by:

$$K = YQ^{-1} \quad (34)$$

Proof: Supposing that (32) and (33) hold, then multiplying both sides of (33) by $\alpha_i \beta_j$, for $i = 1, \dots, r_a$ and $j = 1, \dots, r_b$ and considering (26), it follows that

$$\begin{aligned} & \alpha_i \beta_j (QA'_i + A_i Q + B_j YA'_i + A_i Y' B'_j) < 0, \quad \forall i, j, \\ & \Leftrightarrow \sum_{i=1}^{r_a} \sum_{j=1}^{r_b} \alpha_i \beta_j (QA'_i + A_i Q + B_j YA'_i + A_i Y' B'_j) = \\ & Q \left(\sum_{i=1}^{r_a} \alpha_i A_i \right)' + \left(\sum_{i=1}^{r_a} \alpha_i A_i \right) Q + \left(\sum_{j=1}^{r_b} \beta_j B_j \right) Y \left(\sum_{i=1}^{r_a} \alpha_i A_i \right)' \\ & + \left(\sum_{i=1}^{r_a} \alpha_i A_i \right) Y' \left(\sum_{j=1}^{r_b} \beta_j B_j \right)' < 0 \end{aligned} \quad (35)$$

Then, from (25) one has

$$QA(\alpha)' + A(\alpha)Q + B(\beta)YA(\alpha)' + A(\alpha)Y'B(\beta)' < 0.$$

Replacing $Y = KQ$ and $Q = P^{-1}$ one obtains

$$\begin{aligned} & P^{-1}A(\alpha)' + A(\alpha)P^{-1} + B(\beta)KP^{-1}A(\alpha)' + A(\alpha)P^{-1}K'B(\beta)' = \\ & (I + B(\beta)K)P^{-1}A(\alpha)' + A(\alpha)P^{-1}(I + B(\beta)K)' < 0 \end{aligned} \quad (36)$$

From Remark 3, it follows that the matrix $((I + B(\beta)K)P^{-1}A(\alpha)')'$ has a full rank, and so the matrices $(I + B(\beta)K)$ and $A(\alpha)'$ have a full rank too. Now, premultiply by $P(I + B(\beta)K)^{-1}$, postmultiply by $[(I + B(\beta)K)']^{-1}P$ in both sides of (36) and replace $A_N(\alpha, \beta) = (I + B(\beta)K)^{-1}A(\alpha)$ to obtain

$$\begin{aligned} & A(\alpha)'[(I + B(\beta)K)']^{-1}P + P(I + B(\beta)K)^{-1}A(\alpha) = \\ & A_N(\alpha, \beta)'P + PA_N(\alpha, \beta) < 0 \end{aligned} \quad (37)$$

Observe that, when the LMI (32) and (33) hold, the system (31) satisfies the Lyapunov conditions (29), considering $A_N(\alpha, \beta) = (I + B(\beta)K)^{-1}A(\alpha)$. Therefore, when the LMI (32) and (33) hold the system (31) is asymptotically stable and a solution that solves Problem 1 is given by (34).

When (32) and (33) are feasible, they can be easily solved using available softwares, such as LMISol (de Oliveira et al, 1997), that is a free software, or MATLAB (Gahinet et al, 1995; Sturm, 1999). These algorithms have polynomial time convergence.

Remark 4. From the analysis presented in the proof of Theorem 2, after equation (36), note that when (32) and (33) are feasible, the matrix $A(\alpha)$, defined in (25), has a full rank. Therefore, $A(\alpha)$ with a full rank is a necessary condition for the application of Theorem 2. Moreover, from (25), observe that for $\alpha_i = 1$ and $\alpha_k = 0$, $i \neq k$, $i, k = 1, 2, \dots, r_a$, then $A(\alpha) = A_i$. So, if $A(\alpha)$ has a full rank, then A_i , $i = 1, 2, \dots, r_a$ has a full rank too.

Usually, only the stability of a control system is insufficient to obtain a suitable performance. In the design of control systems, the specification of the decay rate can also be very useful.

3.3 Decay Rate Conditions

Consider, for instance, the controlled system (31). According to (Boyd et al., 1994), the decay rate is defined as the largest real constant γ , $\gamma > 0$, such that

$$\lim_{t \rightarrow \infty} e^{\gamma t} \|x(t)\| = 0$$

holds, for all trajectories $x(t)$, $t \geq 0$.

One can use the Lyapunov conditions (29) to impose a lower bound on the decay rate, replacing (29) by

$$P > 0, \text{ and } A_N(\alpha, \beta)'P + PA_N(\alpha, \beta) < -2\gamma P. \quad (38)$$

where γ is a real constant (Boyd et al., 1994). Sufficient conditions for stability with decay rate for Problem 1 are presented in the next theorem (Assunção et al., 2007c).

Theorem 3. The closed-loop system (31), given in Problem 1, has a decay rate greater or equal to γ if there exist a symmetric matrix $Q \in \mathbb{R}^{n \times n}$ and a matrix $Y \in \mathbb{R}^{m \times n}$ such that

$$Q > 0 \quad (39)$$

$$\begin{bmatrix} QA_i' + A_iQ + B_jYA_i' + A_iY'B_j' & Q + B_jY \\ Q + Y'B_j' & -Q/(2\gamma) \end{bmatrix} < 0 \quad (40)$$

where $i = 1, \dots, r_a$ and $j = 1, \dots, r_b$. Furthermore, when (39) and (40) hold, then a robust state-derivative feedback matrix is given by:

$$K = YQ^{-1}. \quad (41)$$

Proof: Following the same ideas of the proof of Theorem 2, multiply both sides of (40) by $\alpha_i\beta_j$, for $i = 1, \dots, r_a$ and $j = 1, \dots, r_b$ and consider (26), to conclude that

$$\begin{bmatrix} QA(\alpha)' + A(\alpha)Q + B(\beta)YA(\alpha)' + A(\alpha)Y'B(\beta)' & Q + B(\beta)Y \\ Q + Y'B(\beta)' & -Q/(2\gamma) \end{bmatrix} < 0$$

Now, using the Schur complement (Boyd et al., 1994), the equation above is equivalent to:

$$\begin{aligned}
&QA(\alpha)' + A(\alpha)Q + B(\beta)YA(\alpha)' + A(\alpha)Y'B(\beta)' \\
&+ (Q + B(\beta)Y)2\gamma Q^{-1}(Q + B(\beta)Y)' < 0
\end{aligned} \tag{42}$$

Replacing $Y = KQ$ and $Q = P^{-1}$ one obtains

$$\begin{aligned}
&(I + B(\beta)K)P^{-1}A(\alpha)' + A(\alpha)P^{-1}(I + B(\beta)K)' \\
&+ (I + B(\beta)K)P^{-1}(2\gamma P)P^{-1}(I + B(\beta)K)' = \\
&(I + B(\beta)K)P^{-1}A(\alpha)' + A(\alpha)P^{-1}(I + B(\beta)K)' \\
&+ (I + B(\beta)K)(2\gamma P^{-1})(I + B(\beta)K)' < 0
\end{aligned} \tag{43}$$

Premultiplying by $P(I + B(\beta)K)^{-1}$, posmultiplying by $[(I + B(\beta)K)']^{-1}P$ in both sides of (43) and replacing $A_N(\alpha, \beta) = (I + B(\beta)K)^{-1}A(\alpha)$ one obtain

$$\begin{aligned}
&A(\alpha)'[(I + B(\beta)K)']^{-1}P + P(I + B(\beta)K)^{-1}A(\alpha) + 2\gamma P < 0 \\
&\Leftrightarrow A_N(\alpha, \beta)'P + PA_N(\alpha, \beta) < -2\gamma P,
\end{aligned} \tag{44}$$

that is equivalent to the Lyapunov condition (38). Then, when (39) and (40) hold, the system (31) satisfies the Lyapunov conditions (38), considering $A_N(\alpha, \beta) = (I + B(\beta)K)^{-1}A(\alpha)$. Therefore, the system (31) is asymptotically stable with a decay rate greater or equal to γ , and a solution for the problem can be given by (41).

Due to limitations imposed in the practical applications of control systems, many times it should be considered output constraints in the design.

3.4 Bounds on Output Peak

Consider that the output of the system (25) is given by:

$$y(t) = Cx(t), \tag{45}$$

where $y(t) \in \mathbb{R}^p$ and $C \in \mathbb{R}^{p \times n}$. Assume that the initial condition of (25) and (45) is $x(0)$. If the feedback system (31) and (45) is asymptotically stable, one can specify bounds on output peak as described below:

$$\max \|y(t)\|_2 = \max \sqrt{y'(t)y(t)} < \xi_0 \tag{46}$$

for $t \geq 0$, where ξ_0 is a known positive constant. From (Boyd et al., 1994), (46) is satisfied when the following LMI hold:

$$\begin{bmatrix} 1 & x(0)' \\ x(0) & Q \end{bmatrix} > 0, \tag{47}$$

$$\begin{bmatrix} Q & QC' \\ CQ & \xi_0^2 I \end{bmatrix} > 0, \tag{48}$$

and the LMI that guarantee stability (Theorem 2), given by (32) and (33), or stability and decay rate (Theorem 3), given by (39) and (40).

In some cases, the entries of the state-derivative feedback matrix K must be bounded. In (Assunção et al., 2007c) is presented an optimization procedure to obtain bounds on the state-derivative feedback matrix K , that can help the practical implementation of the controllers. The result is the following:

Theorem 4. *Given a constant $\mu_0 > 0$, then the specification of bounds on the state-derivative feedback matrix K can be described by finding the minimum value of $\beta, \beta > 0$, such that $KK' < \beta I / \mu_0^2$. The optimal value of β can be obtained by the solution of the following optimization problem:*

$$\begin{aligned} & \min \beta \\ & \text{s.t.} \\ & \begin{bmatrix} \beta I & Y \\ Y' & I \end{bmatrix} > 0, \end{aligned} \quad (49)$$

$$Q > \mu_0 I, \quad (50)$$

(Set of LMI),

where the Set of LMI can be equal to (33), or (40), with or without the LMI (47) and (48).

Proof: See (Assunção et al., 2007c) for more details.

In the next section, a numerical example illustrates the efficiency of the proposed methods for solution of Problem 1.

3.5 Example

The presented methods are applied in the design of controllers for an uncertain mechanical system subject to structural failures. For the designs and simulations, the software MATLAB was used.

Active Suspension Systems

Consider the active suspension of a car seat given in (E. Reithmeier and G. Leitmann, 2003; Assunção et al., 2007c) with other kind of control inputs, shown in Figure 8. The model consists of a car mass M_c and a driver-plus-seat mass m_s . Vertical vibrations caused by a street may be partially attenuated by shock absorbers (stiffness k_1 and damping b_1). Nonetheless, the driver may still be subjected to undesirable vibrations. These vibrations, again, can be reduced by appropriately mounted car seat suspension elements (stiffness k_2 and damping b_2). Damping of vibration of the masses M_c and m_s can be increased by changing the control inputs $u_1(t)$ and $u_2(t)$. The dynamical system can be described by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k_1 - k_2}{M_c} & \frac{k_2}{M_c} & \frac{-b_1 - b_2}{M_c} & \frac{b_2}{M_c} \\ \frac{k_2}{m_s} & \frac{-k_2}{m_s} & \frac{b_2}{m_s} & \frac{-b_2}{m_s} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{M_c} & \frac{-1}{M_c} \\ 0 & \frac{1}{m_s} \end{bmatrix} u(t), \quad (51)$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}. \quad (52)$$

The state vector is defined by $x(t) = [x_1(t) \ x_2(t) \ \dot{x}_1(t) \ \dot{x}_2(t)]^T$.

As in (E. Reithmeier and G. Leitmann, 2003), for feedback only the accelerations signals $\ddot{x}_1(t)$ and $\ddot{x}_2(t)$ are available (that are measured by accelerometer sensors). The velocities $\dot{x}_1(t)$ and $\dot{x}_2(t)$ are estimated from their measured time derivatives. Therefore the accelerations and velocities signals are available (derivative of states), and so one can use the proposed method to solve the problem.

Consider that the driver weight can assume values between 50kg and 100kg. Then the system in Figure 8 has an uncertain constant parameter m_s such that, $70\text{kg} \leq m_s \leq 120\text{kg}$. Additionally, suppose that can also happen a fail in the damper of the seat suspension (in other words, the damper can break after some time). The fault can be described by a polytopic uncertain system, where the system parameters without failure correspond to a vertice of the polytopic, and with failures, the parameters are in another vertice. Then, one can obtain the polytopic plant given in (25) and (26), composed by the polytopic sets due the failures and the uncertain plant parameters.

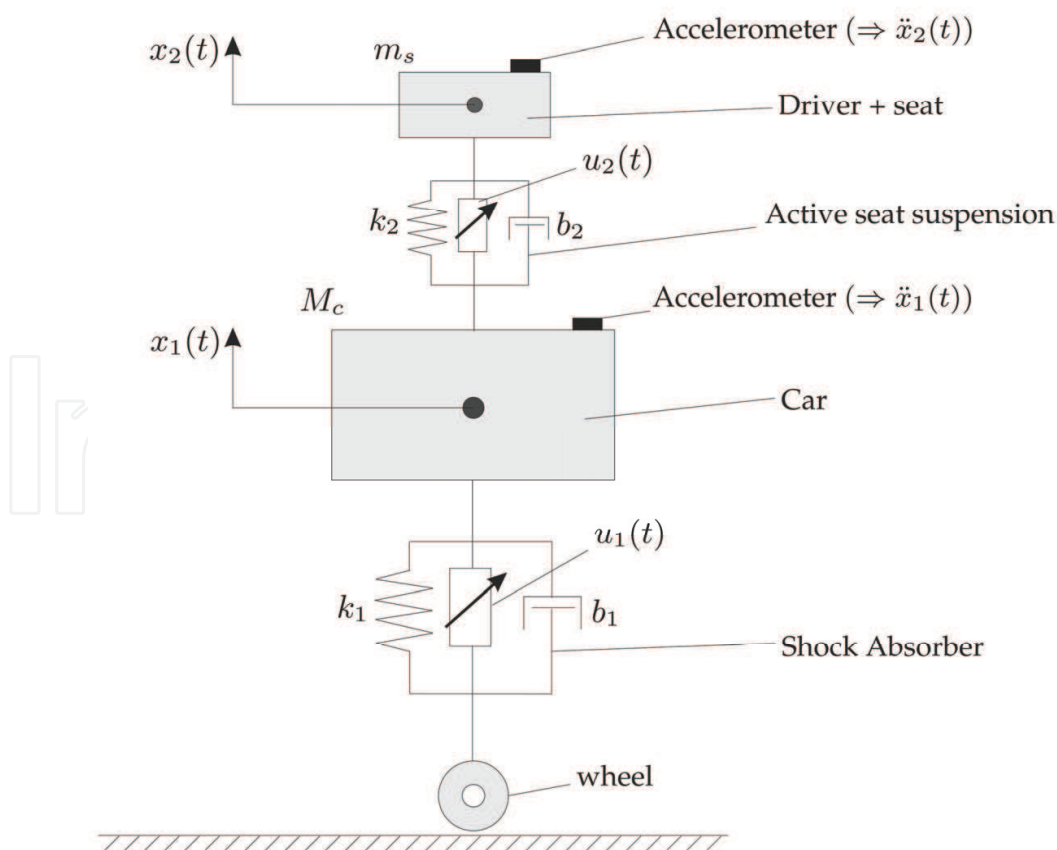


Figure 8. Active suspension of a car seat

The damper of the seat suspension b_2 can be considered as an uncertain parameter such that: $b_2 = 5 \times 10^2 \text{Ns/m}$ while the damper is working and $b_2 = 0$ when the damper is broken. Hence, and supposing $M_c = 1500 \text{kg}$ (mass of the car), $k_1 = 4 \times 10^4 \text{N/m}$ (stiffness), $k_2 = 5 \times 10^3 \text{N/m}$ (stiffness) and $b_1 = 4 \times 10^3 \text{Ns/m}$ (damping), the plant (51) and (52) can be described by equations (25), (26) and (45), and the matrices A_i and B_j , where $r_a = 4$, $r_b = 2$, are given by:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -30 & 3.33 & -3 & 0.33 \\ 71.43 & -71.43 & 7.143 & -7.143 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -30 & 3.33 & -3 & 0.33 \\ 41.67 & -41.67 & 4.167 & -4.167 \end{bmatrix},$$

while the damper is working (in this case $b_2 = 5 \times 10^2 \text{Ns/m}$, $m_s = 70 \text{kg}$ in A_1 and $m_s = 120 \text{kg}$ in A_2),

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -30 & 3.33 & -2.67 & 0 \\ 71.43 & -71.43 & 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -30 & 3.33 & -2.67 & 0 \\ 41.67 & -41.67 & 0 & 0 \end{bmatrix},$$

when the damper is broken (in this case $b_2 = 0$, $m_s = 70 \text{kg}$ in A_3 and $m_s = 120 \text{kg}$ in A_4) and

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 6.67 \times 10^{-4} & -6.67 \times 10^{-4} \\ 0 & 1.43 \times 10^{-2} \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 6.67 \times 10^{-4} & -6.67 \times 10^{-4} \\ 0 & 8.33 \times 10^{-3} \end{bmatrix},$$

because the input matrix $B(\beta)$ depends only on the uncertain parameter m_s (in this case $m_s = 70 \text{kg}$ in B_1 and $m_s = 120 \text{kg}$ in B_2). Specifying an output peak bound $\xi_0 = 300$, an initial condition $x(0) = [0.1 \ 0.3 \ 0 \ 0]^T$ and using the MATLAB (Gahinet et al, 1995) to solve the LMI (32) and (33) from Theorem 2, with (47) and (48), the feasible solution was:

$$Q = \begin{bmatrix} 2.4006 \times 10^4 & 2.2812 \times 10^4 & -4.1099 \times 10^4 & -2.6578 \times 10^4 \\ 2.2812 \times 10^4 & 2.3265 \times 10^4 & -2.1628 \times 10^4 & -2.9019 \times 10^4 \\ -4.1099 \times 10^4 & -2.1628 \times 10^4 & 5.29 \times 10^5 & 8.3897 \times 10^4 \\ -2.6578 \times 10^4 & -2.9019 \times 10^4 & 8.3897 \times 10^4 & 1.8199 \times 10^5 \end{bmatrix},$$

$$Y = \begin{bmatrix} -7.9749 \times 10^6 & -3.0334 \times 10^7 & -4.4436 \times 10^6 & 6.5815 \times 10^8 \\ 1.7401 \times 10^6 & 2.2947 \times 10^6 & -8.0344 \times 10^6 & -1.616 \times 10^7 \end{bmatrix}.$$

From (34), we obtain the state-derivative feedback matrix below:

$$K = \begin{bmatrix} 2.894 \times 10^3 & 923.6 & -442.06 & 4.3902 \times 10^3 \\ -498.14 & 471.29 & -22.567 & -75.996 \end{bmatrix}. \tag{53}$$

The locations in the s-plane of the eigenvalues λ_i , for the eight vertices (A_i, B_j) , $i = 1, 2, 3, 4$ and $j = 1, 2$, of the robust controlled system, are plotted in Figure 9. There exist four eigenvalues for each vertex.

Consider that driver weight is 70kg, and so $m_s = 90\text{kg}$. Using the designed controller (53) and the initial condition $x(0)$ defined above, the controlled system was simulated. The transient response and the control inputs (30), of the controlled system, while the damper is working are presented in Figures 10 and 11. Now suppose that happen a fail in the damper of the seat suspension b_2 after 1s (in other words, $b_2 = 5 \times 10^2\text{Ns/m}$ if $t \leq 1\text{s}$ and $b_2 = 0$ if $t > 1\text{s}$). Then, the transient response and the control inputs (30), of the controlled system, are displayed in Figures 12 and 13. The required condition $\max \sqrt{y'(t)y(t)} < \xi_0 = 300$ was satisfied.

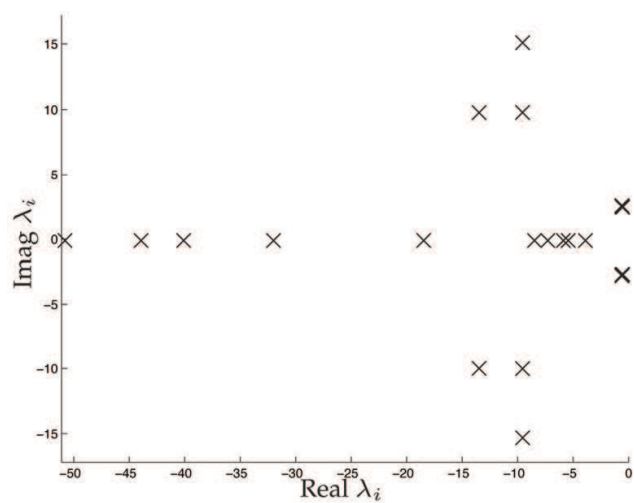


Figure 9. The eigenvalues in the eight vertices of the controlled uncertain system

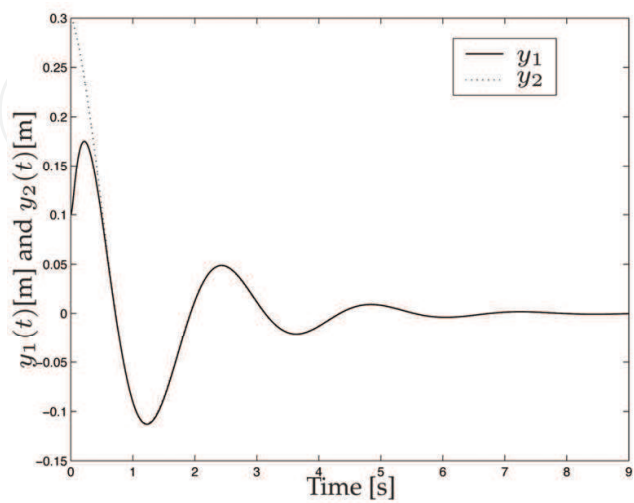


Figure 10. Transient response of the system with the damper working

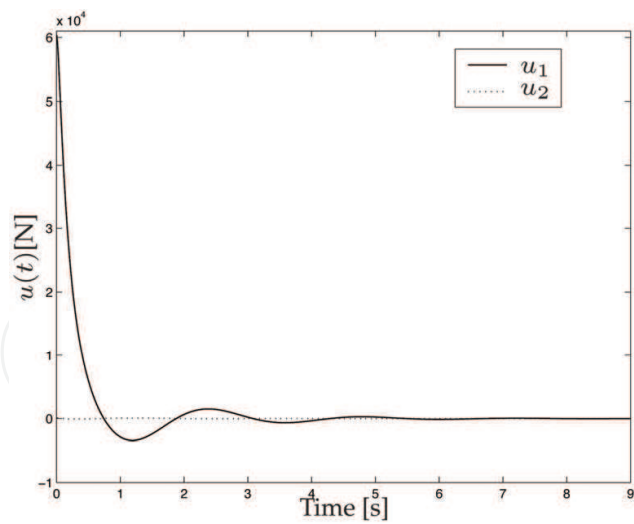


Figure 11. Control inputs of the controlled system with the damper working

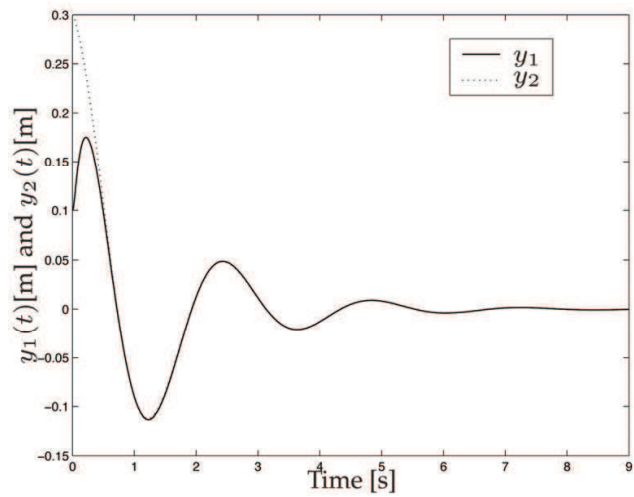


Figure 12. Transient response of the system with a fail in the damper b_2 after 1s

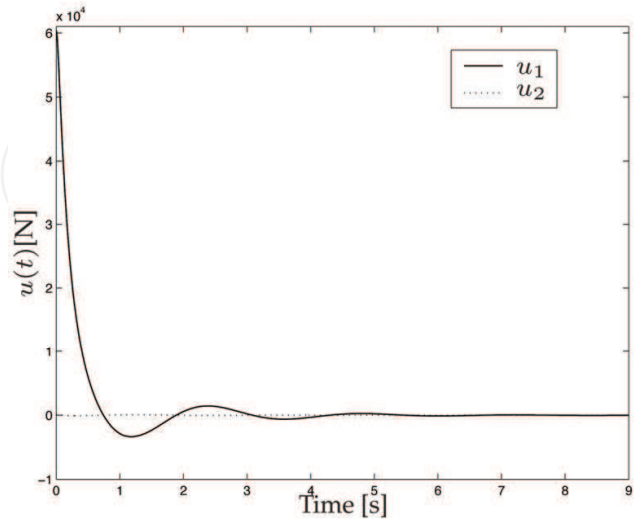


Figure 13. Control inputs of the controlled system with a fail in the damper b_2 after 1s

Observe in Figures 10 and 12, that the happening of a fail in the damper b_2 does not change the settling time of the controlled system, and had little influence in the control inputs. Furthermore, as discussed before, considering $m_s = 90\text{kg}$ and the controller (53), the matrix $(I + B(\beta)K)$ has a full rank ($\det(I + B(\beta)K) = 0.85868 \neq 0$).

There exist problems where only the stability of the controlled system is insufficient to obtain a suitable performance. Specifying a lower bound for the decay rate equal $\gamma = 3$, to obtain a fast transient response, Theorem 3 is solved with (47) and (48) ($\xi_0 = 300$). The solution obtained with the software MATLAB was:

$$Q = \begin{bmatrix} 3.9195 \times 10^3 & 3.1064 \times 10^3 & -2.6316 \times 10^4 & -1.6730 \times 10^4 \\ 3.1064 \times 10^3 & 3.6868 \times 10^3 & -1.3671 \times 10^4 & -1.8038 \times 10^4 \\ -2.6316 \times 10^4 & -1.3671 \times 10^4 & 5.3775 \times 10^5 & 1.0319 \times 10^5 \\ -1.6730 \times 10^4 & -1.8038 \times 10^4 & 1.0319 \times 10^5 & 1.9587 \times 10^5 \end{bmatrix},$$

$$Y = \begin{bmatrix} 4.3933 \times 10^7 & 2.8021 \times 10^7 & -7.9356 \times 10^8 & -1.6408 \times 10^8 \\ 1.3888 \times 10^6 & 1.8426 \times 10^6 & -9.1885 \times 10^6 & -1.69 \times 10^7 \end{bmatrix}.$$

From (41), we obtain the state-derivative feedback matrix below:

$$K = \begin{bmatrix} -621 & 3.8664 \times 10^3 & -1.452 \times 10^3 & 230.33 \\ -313.58 & 365.55 & -8.79 & -74.77 \end{bmatrix} \quad (54)$$

The locations in the s-plane of the eigenvalues λ_i , for the eight vertices (A_i, B_j) , $i = 1, 2, 3, 4$ and $j = 1, 2$, of the robust controlled system, are plotted in Figure 14. There exist four eigenvalues for each vertice.

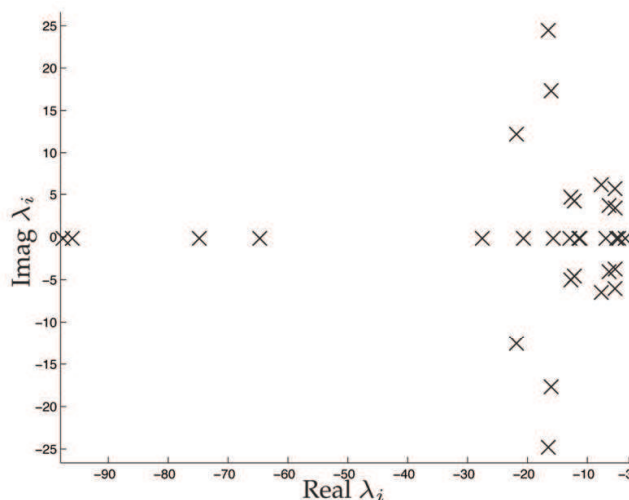


Figure 14. The eigenvalues in the eight vertices of the controlled uncertain system

From Figure 14, one has that all eigenvalues of the vertices have real part lower than $-\gamma = -3$. Therefore, the controlled uncertain system has a decay rate greater or equal to γ . Again, considering that $m_s = 90\text{kg}$ and using the designed controller (54) the matrix $(I + B(\beta)K)$ has a full rank ($\det(I + B(\beta)K) = 0.026272$). For the initial condition $x(0)$ defined above, the controlled system was simulated. The transient response and the control inputs (30) of the controlled system are presented in Figures 15, 16, 17 and 18, respectively.

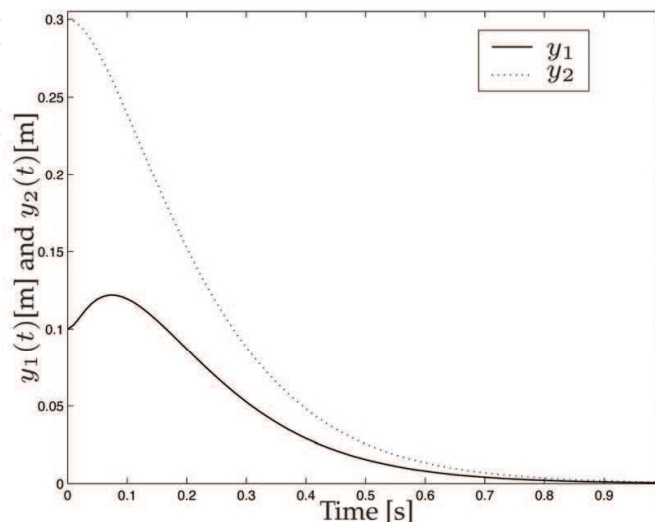


Figure 15. Transient response of the system with the damper working

Observe that, the settling time in Figures 15 and 17 are smaller than the settling time in Figures 10 and 12, where only stability was required and also, $\max \sqrt{y'(t)y(t)}$ is equal to $0.31623 < \xi_0 = 300$. Then, the specifications were satisfied by the designed controller (54). Moreover, the happening of a fail in the damper b_2 does not significantly change the settling time (Figures 15 and 17) of the controlled system. In spite of the change in the control inputs from Figures 16 and 18, the fail in the damper does not changed the maximum absolute value of the control signal ($u(t) = 1.1161 \times 10^5 \text{N}$).

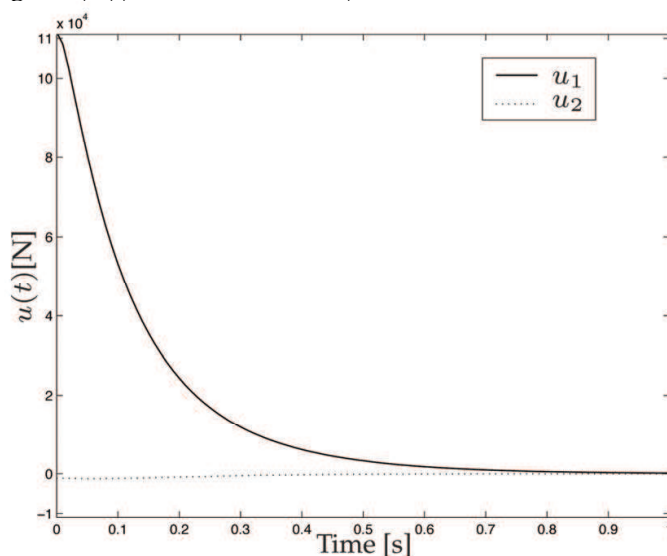


Figure 16. Control inputs of the controlled system with the damper working

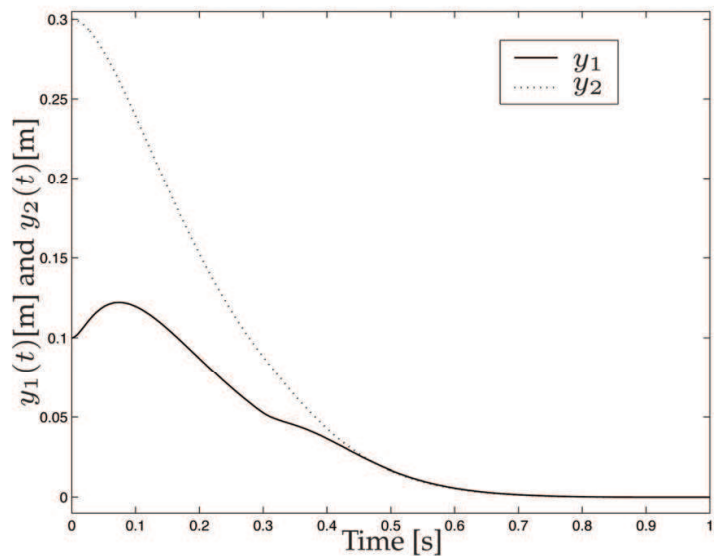


Figure 17. Transient response of the system with a fail in the damper b_2 after 0.3s

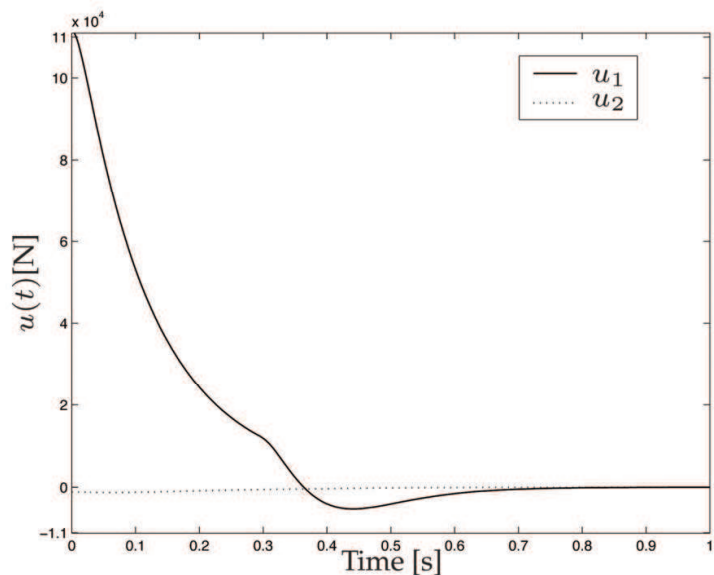


Figure 18. Control inputs of the controlled system with a fail in the damper b_2 after 0.3s

Note that some absolute values of the entries of (53) and (54) are great values and it could be a trouble for the practical implementation of the controller. For the reduction of this problem in the implementation of the controller, the specification of bounds on the state-derivative feedback matrix K can be done using the optimization procedure stated in Theorem 4, with $\mu_0 = 0.1$. The optimal values, obtained with the software MATLAB, for Theorem 4 considering: (33) for stability, or (40) for stability with bound on the decay rate ($\gamma = 3$), and (47) and (48) ($\xi_0 = 300$) are displayed in Table 1. Considering that $m_s = 90\text{kg}$ and the initial condition $x(0)$ defined above, the transient response and the control inputs obtained by Theorem 4 considering (33) or (40), are displayed in Figures 19, 20, 21 and 22 respectively.

Theorem 4 with (33)	Theorem 4 with (40)
$Q = \begin{bmatrix} 1.2265 & 1.5357 & -1.667 & -5.8859 \\ 1.5357 & 2.5422 & 0.6289 & -5.1654 \\ -1.667 & 0.6289 & 27.177 & 30.007 \\ -5.8859 & -5.1654 & 30.007 & 67.502 \end{bmatrix}$	$Q = \begin{bmatrix} 0.16831 & 0.088439 & -0.52166 & -0.25122 \\ 0.088439 & 0.56992 & -0.07813 & -2.3703 \\ -0.52166 & -0.07813 & 5.1595 & -2.9849 \\ 0.25122 & -2.3703 & -2.9849 & 43.238 \end{bmatrix}$
$Y = \begin{bmatrix} 17.423 & 19.928 & -13.793 & 12.407 \\ -25.896 & 20.088 & -2.8711 & 0.69624 \end{bmatrix}$	$Y = \begin{bmatrix} 918.06 & 749.73 & -3.3745 \times 10^3 & 204.86 \\ 30.057 & 468.97 & -102.46 & -3.5475 \times 10^3 \end{bmatrix}$
$K = \begin{bmatrix} 39.536 & -6.5518 & -2.7229 & 4.3402 \\ -276.41 & 173.56 & -17.953 & -2.829 \end{bmatrix}$	$K = \begin{bmatrix} 4.7321 \times 10^3 & 859.72 & -121.49 & 70.976 \\ -559.07 & 664.62 & -98.521 & -55.661 \end{bmatrix}$

Table 1. The solutions with Theorem 4

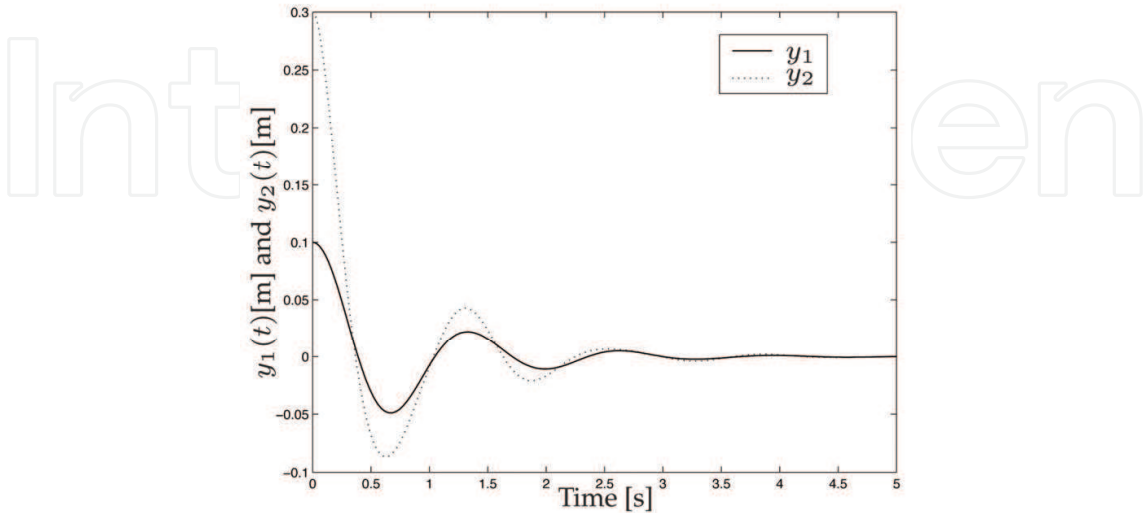


Figure 19. Transient response of the system with a fail in the damper b_2 after 1s, obtained with Theorem 4 and (33)

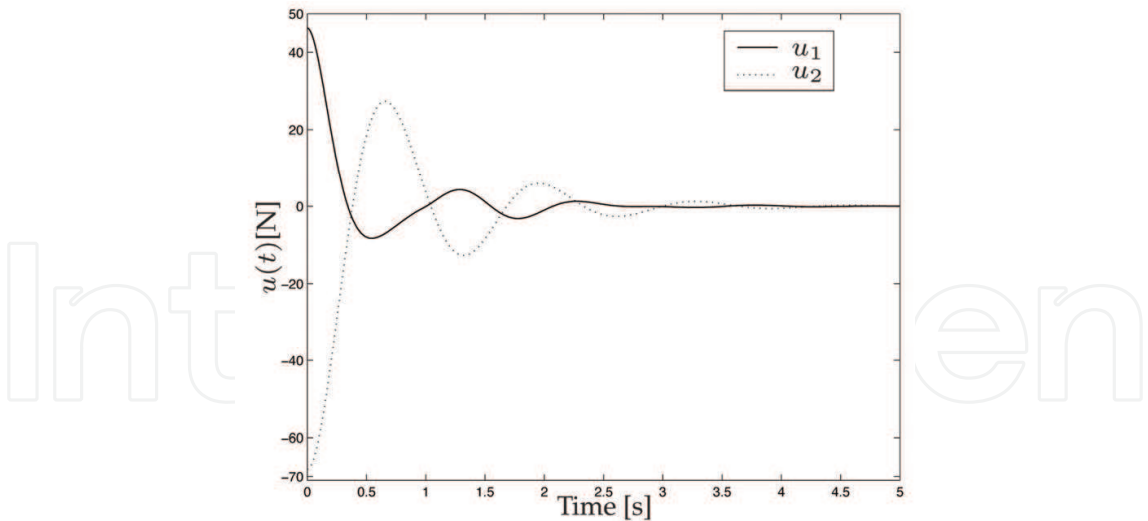


Figure 20. Control inputs of the controlled system with a fail in the damper b_2 after 1s

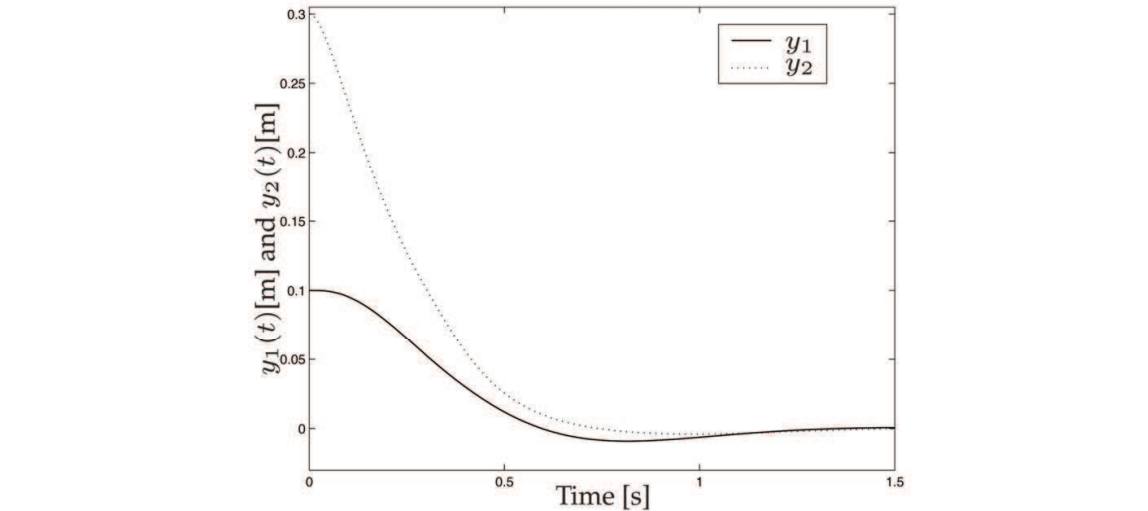


Figure 21. Transient response of the system with a fail in the damper b_2 after 0.3s, obtained with Theorem 4 and (40)

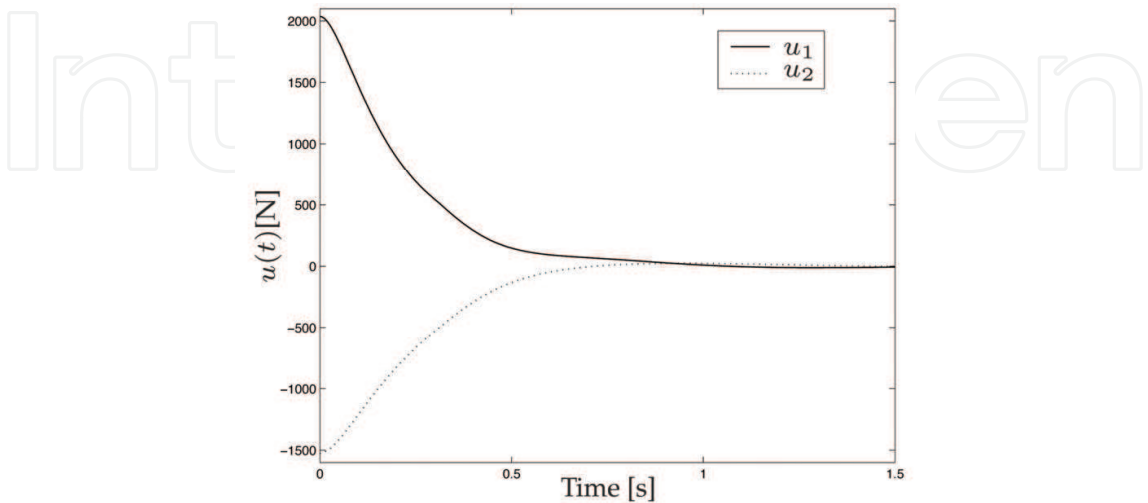


Figure 22. Control inputs of the controlled system with a fail in the damper b_2 after 0.3s

The matrix norm of the controller (53) obtained with Theorem 2 is equal to $\|K\| = 5.3628 \times 10^3$ and the maximum absolute value of the control signal is $u(t) = 6.0356 \times 10^4 \text{N}$, while that the matrix norm of the same controller obtained with Theorem 4 considering (33) is equal to $\|K\| = 328.96$ and the maximum absolute value of the control signal is $u(t) = 68.111 \text{N}$.

Then, Theorem 4 was able to stabilize the controlled system with a smaller state-derivative feedback matrix gain. The similar form, the maximum absolute value of the control signal $u(t)$ from (54), obtained with Theorem 3 is $u(t) = 1.1161 \times 10^5 \text{N}$, and of the same controller obtained with Theorem 4 considering (40) is $u(t) = 2.0362 \times 10^3 \text{N}$. This example shows that the proposed methods are simple to use and it is easy to specify the constraints in the design.

4. Conclusions

In this chapter two new control designs using state-derivative feedback for linear systems were presented. Firstly, considering linear descriptor plants, a simple method for designing a state-derivative feedback gain (K_d) using methods for state feedback control design was proposed. The descriptor linear systems must be time-invariant, Single-Input (SI) or Multiple-Input (MI) system. The procedure allows that the designers use the well-known state feedback design methods to directly design state-derivative feedback control systems. This method extends the results described in (Cardim et al, 2007) and (Abdelaziz & Valášek, 2004) to a more general class of control systems, where the plant can be a descriptor system. As the first design can not be directly applied for uncertain systems, then a design considering sufficient stability conditions based on LMI for state-derivative feedback, that provide an extension of the methods presented in (Assunção et al., 2007c) were presented. The designers can include in the LMI-based control design, the specification of the decay rate and bounds on output peak and on state-derivative feedback gains. The plant can be subject to structural failures. So, in this case, one has a fault-tolerant design. Furthermore, the new design methods allow a broader class of plants and performance specifications, than the related results available in the literature, for instance in (E. Reithmeier and G. Leitmann, 2003; Abdelaziz & Valášek, 2004; Duan et al., 2005; Assunção et al., 2007c; Cardim et al., 2007). The presented method offers LMI-based designs for state-derivative feedback that, when feasible, can be efficiently solved by convex programming techniques. In Sections 2.3 and 3.5, the validity and simplicity of the new control designs can be observed with some numerical examples.

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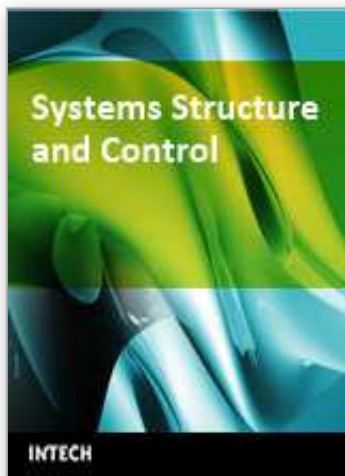
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