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Chapter 5

–5/3 Kolmogorov Turbulent Behaviour and Intermittent Sustainable Energies

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Additional information is available at the end of the chapter

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Abstract

The massive integration of sustainable energies into electrical grids (non-interconnected or connected) is a major problem due to their stochastic character revealed by strong fluctuations at all scales. In this paper, the scaling behaviour or power law correlations and the nature of scaling behaviour of sustainable resource data such as flow velocity, atmospheric wind speed, solar global solar radiation and sustainable energy such as, wind power output, are highlighted. For the first time, Fourier power spectral densities are estimated for each dataset. We show that the power spectrum densities obtained are close to the 5/3 Kolmogorov spectrum. Furthermore, the multifractal and intermittent properties of sustainable resource and energy data have been revealed by the concavity of the scaling exponent function. The proposed analysis frame allows a full description of fluctuations of processes considered. A good knowledge of the dynamic of fluctuations is crucial to management of the integration of sustainable energies into a grid.

Keywords: turbulence, kolmogorov spectrum, intermittency, multifractality

1. Introduction

The installed capacity for energy from solar farms, wind farms and marine energy systems is constantly increasing in response to worldwide interest in low-emissions power sources and a desire to decrease the dependence on petroleum. The variability and unpredictability of this kind of resources over short time scales remains a major problem, as its penetration of this energy into the electric grid is limited. Hence, a good knowledge of renewable resource

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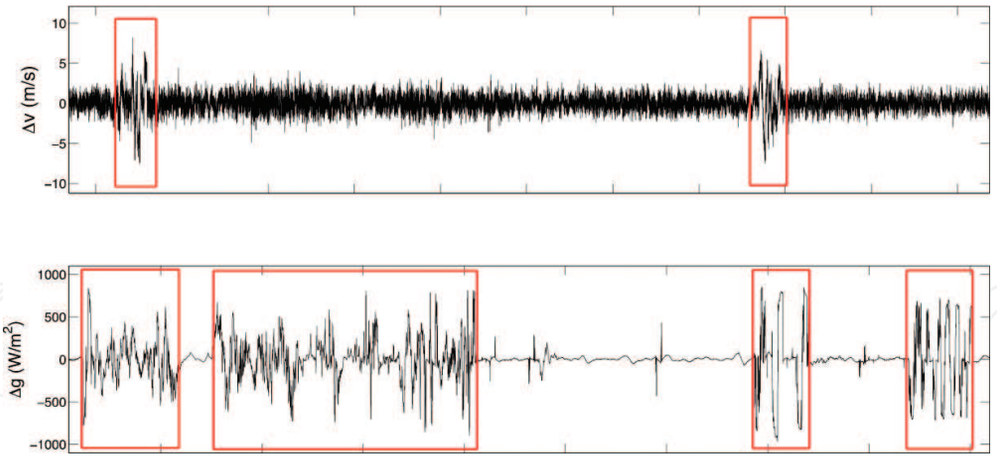


Figure 1. Examples of temporal increments of atmospheric wind speed Δv and the global solar radiation Δg. These sequences show intermittent bursts.

variations and intermittency is of real practical importance in managing the electrical network integrating this kind of energy.

Figure 1 illustrates examples of temporal increments of atmospheric wind speed and global solar radiation for a time scales $\Delta v = 5$ min. We can observe the existence of intermittent bursts. Following, the disciplinary field, the concept of intermittency can be defined differently [1, 2]. In the wind and solar energies fields, the concept of intermittency is often defined as the variability [2]. In turbulence field, Batchelor and Townsend have observed the intermittency experimentally for the first time in 1949 [3] and formalized in the multifractal framework after the seminal works of Kolmogorov [4]. The meaning of intermittency can change according to the authors. Frisch defines an intermittent signal if "it displays activity during only a fraction time, which decreases with the scale under consideration". According to Pope, a motion "sometimes turbulent and sometimes non-turbulent" characterizes an intermittent flow. In the engineering field, the intermittency is considered as a transition between a laminar and turbulent flows [1].

Here, the concept of intermittency in the fully developed turbulence framework is used, with with the help of multifractal analysis. This allows a better description of a stochastic signal at all scales and all intensities.

Multifractal analysis techniques have encountered an amount success through several disciplinary fields, such as, for instance, turbulence [5–8], finance [9–11], physiology [12], rainfall [13, 14] and geophysics [15, 16].

In this chapter, the intermittent properties of renewable resources data (wind speed, solar radiation and flow velocity data) and sustainable energy data (power output data from WECS and marine energy systems) are investigated using a classical multifractal analysis method, structure functions analysis.

The structure of this chapter is as follows. Section 2 describes briefly the fully developed turbulence framework. Section 3 presents the results analysis.

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2. Fully developed turbulence framework

2.1. Richardson's cascade and Kolmogorov theory

The intuitive scheme of Richardson has largely inspired numerous authors in the turbulence field. Richardson provided a poetic form of energetic cascade [17] this is represented by a schematic illustration of Kolmogorov-Obhukov given in (**Figure 2**):

"Big whirls have little whirls that feed on their velocity,

And little whirls have lesser whirls

And so on to viscosity in the molecular sense"

The mathematical formalization of this scheme is given in 1940s by Kolmogorov who postulated the local-similarity hypothesis, i.e. small-scale turbulence is homogeneous and statistically isotropic in the inertial sub-range and hypothesized that velocity fluctuations Δv between two points separated by a distance r depend only on the average dissipation rate ε. This translates into the following expression for the squared fluctuations $S_2(r) = \left(\Delta v_r\right)^2 = \left(v_{x+r} - v_x\right)^2$ [4]:

$$S_2(r) \approx \varepsilon^{2/3} r^{2/3} \tag{1}$$

This has been generalized, considering the structure functions for moments of order $q > 0$ of the absolute spatial velocity increments as follows [18]:

$$S_q(r) \approx \varepsilon^{q/3} r^{q/3} \tag{2}$$

This leads to the famous K41 linear law (when there is no intermittency):

$$\zeta(q) = \frac{q}{3} \tag{3}$$

where $\zeta(q)$ is the scaling exponent of the structure functions:

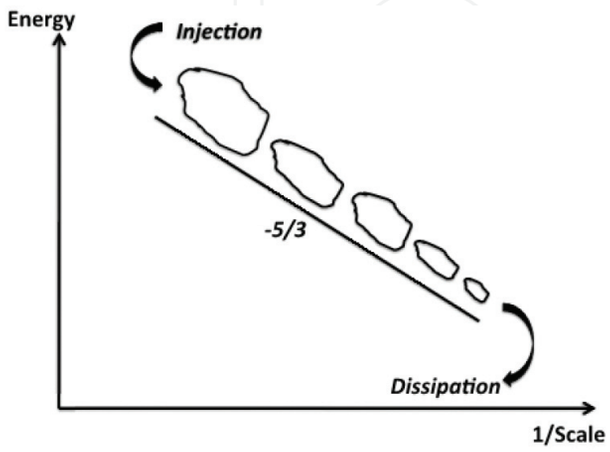


Figure 2. A schematic illustration of Kolmogorov-Obhukov spectrum that a −5/3 slope, based on Richardson's cascade concepts.

$$S_q(r) \approx r^{\zeta(q)} \tag{4}$$

This leads to the following expression for the power spectrum of velocity fluctuations in the Fourier space:

$$E(k) \approx k^{-5/3} \tag{5}$$

where k is the wave number.

In 1949, the experimental works of Batchelor and Townsend [3] highlighted the nonlinearity of the scaling exponent $\zeta(q)$ contrary to the K41 prediction. This nonlinearity indicates the intermittent character of the dissipation energy, caused by the inhomogeneity and anisotropy of the turbulent flow. To take intermittency into account, many theoretical formulations have been provided for a quantitative description of cascade processes and fitting the scaling exponent function $\zeta(q)$. The log-normal model was the first prediction describing the intermittency of the fully turbulence [18]:

$$\zeta(q) = \frac{q}{3} + \frac{\mu}{18}(3q - q^2) \tag{6}$$

where μ is the intermittency parameter. Thereafter, others models have been proposed. The most used are given in Section 2.3.

2.2. A description of scale invariance and multifractal framework

2.2.1. Self-similarity and scale invariance

The idea of describing natural phenomena by the study of statistical scaling laws is not recent [19]. Self-similarity has been widely observed in nature: self-similarity concept being the simplest form of scale invariance. A process $x(t)$ is self-similar if these statistical properties remains unchanged with the process $a^H x(t/a)$ obtained by simultaneously dilating the time axis by a factor $a > 0$, and the amplitude axis by a factor a^{-H}. H is called the self-similarity or Hurst parameter. This parameter provides information on the variability degree of process. A primitive model of self-similar signals is the fractional Brownian motion (fBm) $B_H(t)$ [20] for illustration, (**Figure 3**) shows a protion of flow velocity u dilated in the box, exibithing the statistical self-similarity features of flow velocity signal considered in this study.

The Fourier spectral density $E(f)$ of scale invariance or self-similar processes follows a power law obtained over a range of frequency f:

$$E(f) \sim f^{-\beta} \tag{7}$$

where β is the spectral exponent. According to some authors [19, 21, 22], it defines the degree of stationary of the signal:

- $\beta < 1$, the process is stationary
- $\beta > 1$, the process is no stationary
- $1 < \beta < 3$, the process is no stationary with increments stationary.

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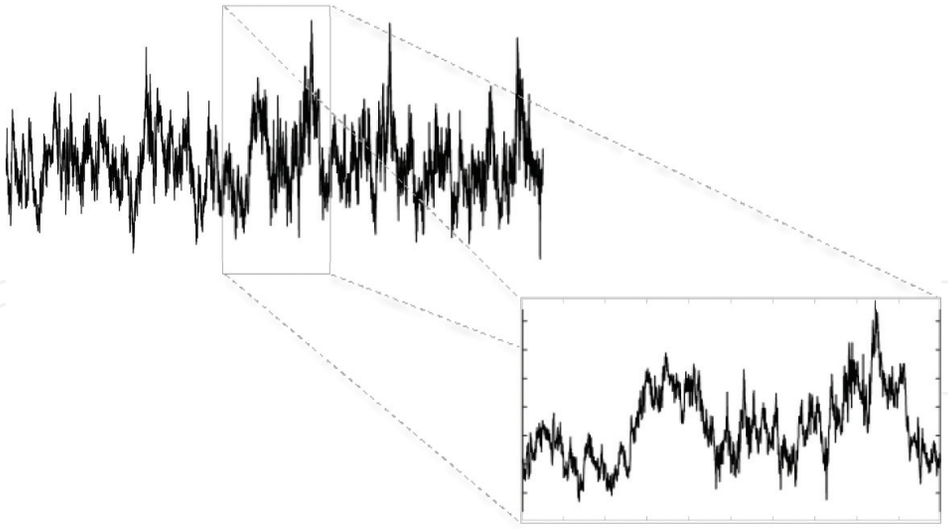


Figure 3. A portion of flow velocity u dilated in the box. This shows the statistical self-similarity features of flow velocity u.

2.2.2. Multifractal framework

The mathematical multifractal framework was appeared with the cascade multiplicative emergence in order to consider the intermittency of the energy dissipation in Turbulence.

Multiscaling concept allows the statistical description of stochastic signals for the modelling of physical systems, using multifractal technique analyses.

If $x(t)$ is a stochastic signal function of time, his scaling behaviour is highlighted when the time absolute time increments $|\Delta x| = |x(t+\tau) - x(t)|$, more precisely, the structure functions of order q respect the following relationship [5]:

$$S_q(\tau) = (|\Delta x|) \approx \tau^{\zeta(q)} \tag{8}$$

where τ is a time lag and ζ is the scaling exponent function. The full $(q, \zeta(q))$ curve for integer and non-integer q moments provides a full characterization of signal considered at all scales and at all intensities. The parameter $\zeta(2) = \beta - 1$ relates the second order moment to the β Fourier power spectrum scaling exponent. The parameter $H = \zeta(1)$ is the Hurst exponent with $0 < H < 1$. This parameter defines the degree of roughness or smoothness of a measured signal: more H is, the more the signal is smooth. The values of the $\zeta(q)$ function are estimated from the slope of the $S_q(\tau)$ versus τ in a log-log representation for all moments q. Concerning the scaling behaviour, the scaling exponent function is useful to characterize the statistics of a stochastic process. For a linear scaling function of the form qH, the signal is said to be monofractal; Brownian motion is described by $H = 1/2$, fractional Brownian motion is described by $0 < H < 1$, and homogeneous non-intermittent turbulence is described by $H = 1/3$. While for a nonlinear scaling exponent function, the signal is said to be multifractal. **Figure 4** illustrates the scaling behaviour of the $\zeta(q)$ function for instance a monofractal and multifractal processes. Furthermore, the concavity of $\zeta(q)$ function gives an indication on the intermittency degree of process considered: the more concave the curve is, the more intermittent the process [5, 22].

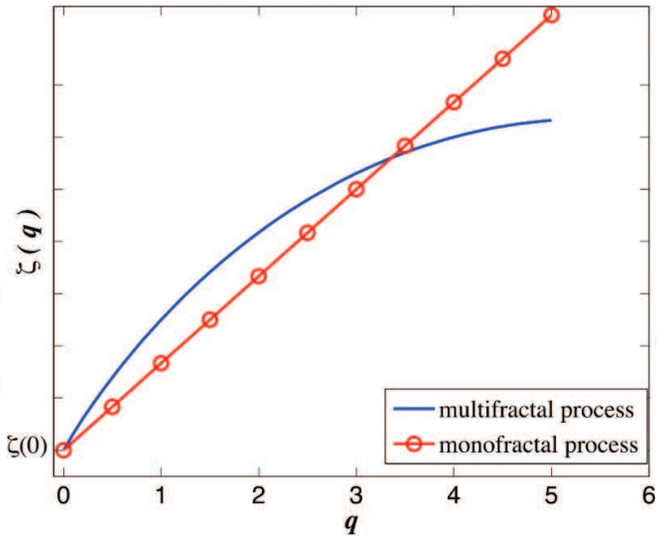


Figure 4. Examples of scaling exponent functions $\zeta(q)$ for a monofractal and a multifractal processes. The scaling exponent functions represented are linear and nonlinear (concave), respectively, for monofractal and multifractal processes.

2.3. Some multifractal models

Several models have been proposed to fit the scaling exponent function $\zeta(q)$ since in the literature, for instance, the "black and white" model [23], the log-normal model [18] and the log-stable model [22].

The "black and white" model proposed by Frisch et al. in 1978 is the simplest model [23]:

$$\zeta(q) = qH - \mu(q-1) \tag{9}$$

where H is the Hurst exponent and μ the intermittency parameter.

The classical lognormal model of the form:

$$\zeta(q) = qH - \frac{\mu}{2}(q^2 - q) \tag{10}$$

The log-stable or log-Lévy model proposed by Schertzer and Lovejoy in 1987 [22]:

$$\zeta(q) = qH - \frac{C_1}{(\alpha-1)}(q^\alpha - q) \tag{11}$$

where H is the Hurst exponent. The parameter C_1 is the fractal co-dimension measuring the mean intermittency: the larger C_1, the more the signal is intermittent. Furthermore, $0 < C_1 < d$ with d the dimension space (here $d = 1$). The multifractal Lévy parameter $0 < \alpha < 2$ inquires on the degree of multifractality i.e., how fast the inhomogeneity increases with the order of the

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moments. Furthermore, $\alpha = 0$ corresponds to the monofractal case and $\alpha = 2$ corresponds to the multifractal log-normal case.

In this chapter, we consider the log-normal model that provides a reasonable fit for the scaling exponent of data considered. In [24], the log-stable is considered for the global solar radiation data.

3. Results

In this chapter, we present analysis results from multiple time series sampled at different sampling rates and at different places. The atmospheric wind speed u was measured with a sampling frequency of 20 Hz during 40 h, on the wind energy site production of Petit-Canal in Guadeloupe an island located at 16°15′N latitude and 60°30′W longitude. The wind power output P was measured at the same place, with a sampling frequency of 1 Hz over a one-year period. A 10 MW wind farm delivers this wind power output. The global solar radiation measurements G was collected with a sampling frequency of 1 Hz over a one-year period, at the University site of Pointe-à-Pitre in Guadeloupe. The flow velocity measurements were generated from the facilities of the wave and current flume tank of IFREMER (French Research Institute for Exploitation of the Sea) in Boulogne-sur-mer (North of France). The data are collected with a sampling frequency of 100 Hz. **Figure 5** illustrates extract of signals considered. All the signals fluctuate over a large range scales showing the intermittent nature of sustainable resources and energy considered in this study.

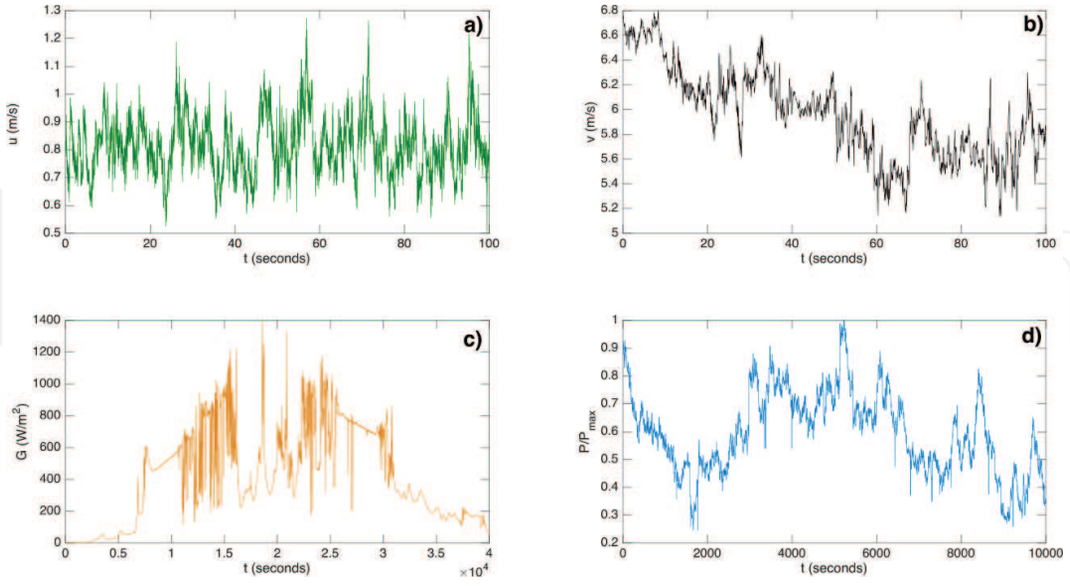


Figure 5. Examples of extract of signal considered: (a) flow velocity u, (b) atmospheric wind speed, (c) global solar radiation G, (d) normalized wind power output delivered by a wind farm. All the signals display strong fluctuations at all scales.

3.1. Fourier analysis of sustainable energy data and −5/3 Kolmogorov spectrum

The Fourier power spectral density separates and measures the amount of variability occurring in different frequency bands. In this section, the Fourier power spectral densities are estimated for our database in order to detect scale invariance. For a scale invariant signal, the following scaling power law is obtained over a range of frequency f:

$$E(f) \approx f^{-\beta} \tag{12}$$

where β is the exponent spectral.

Figure 6 shows the Fourier power spectral densities of databases described above, compared to the −5/3 Kolmogorov spectrum (red straight line), log-log representation. The spectra computed follow a power law of the $f^{-\beta}$ with β close to 5/3. As expected, the atmospheric wind and the flow velocity spectra demonstrate a scaling behaviour for the respective frequencies from about $f = 0.1$–10 Hz and $f = 0.1$–50 Hz with $\beta = 1.67$ close to the 5/3 Kolmogorov value [4, 25]. This is consistent with the values obtained for the inertial range in previous studies [26–28]. The wind power output spectrum displays a power law with $\beta = 1.68$ close to the 5/3 Kolmogorov value, for frequencies from about $f = 10^{-4}$ to 0.5 Hz. In 2007, Apt has shown that the wind power output from a wind turbine, follows a Kolmogorov spectrum over more than four orders of magnitude in frequency [29]. In [30], we show the wind power output spectrum with an exponent spectral close to the 5/3 value, which is observed for particular conditions.

The global solar radiation spectrum shows also a power law behaviour with $\beta = 1.66$ close to the 5/3 Kolmogorov value for frequencies from about $f = 0.7 \times 10^{-4}$ to 0.07 Hz. This scale invariance is indirectly linked to scale invariance of cloud field transported by atmospheric turbulence. In [31], a power law is also observed for the spectrum of cloud radiances obtained

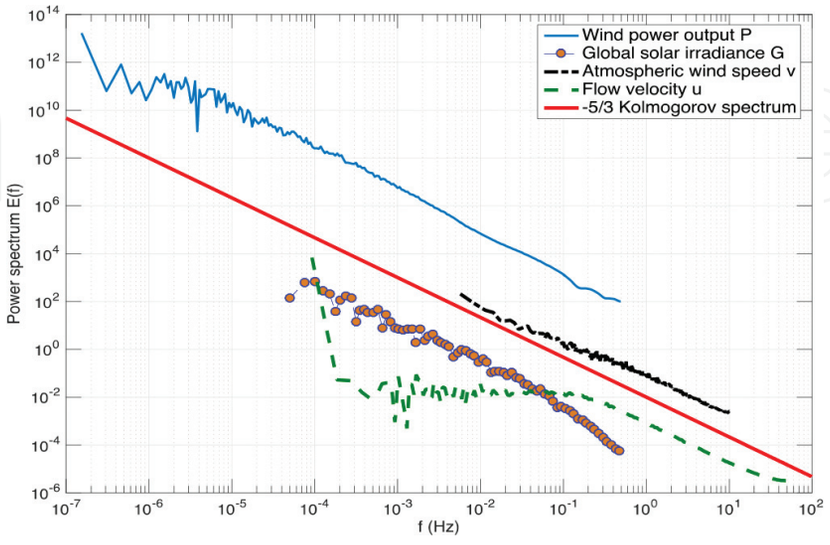


Figure 6. The Fourier power spectral densities for each dataset, compared with the −5/3 Kolmogorov spectrum.

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from ground-based photography: the exponent spectral $\beta = 1.67$ is observed for clouds over ocean.

In summary, the spectra of sustainable data considered in this study, display power law behaviour with an exponent spectral close to the 5/3 Kolmogorov value. The slight difference with the exact 5/3 value is usually caused by intermittency effects [5, 22].

Furthermore, the Fourier power spectrum is a second order statistic providing information on medium level fluctuations, and consequently, its slope is not sufficient to fully describe a scaling process. Multifractal analysis is a natural generalization to fully study the scaling behaviour of a nonlinear phenomenon using, for example, the qth order structure functions.

3.2. Multifractal analysis of sustainable energy data

In order to qualify the nature of scaling behaviour (monofractal or multifractal), a multifractal analysis using qth order structure functions is applied to sustainable energy data to determine the scaling exponents $\zeta(q)$. For each dataset, the structure functions are computed on the temporal increments Δx as defined above. The details concerning the scale range of τ and q are given in the following references [24, 32–34]. As shown in [24, 32–34], the straight lines of structure functions indicate that the scaling of the relationship is well respected. Consequently, the scaling exponents $\zeta(q)$ are extracted from the slopes of the straight lines using a linear regression. **Figure 7** represents the scaling exponents $\zeta(q)$ corresponding to each dataset compared with a model proposed by Kolmogorov, the linear model K41, $\zeta(q) = q/3$. We can see that the scaling exponents $\zeta(q)$ obtained are nonlinear and concave. This highlights the

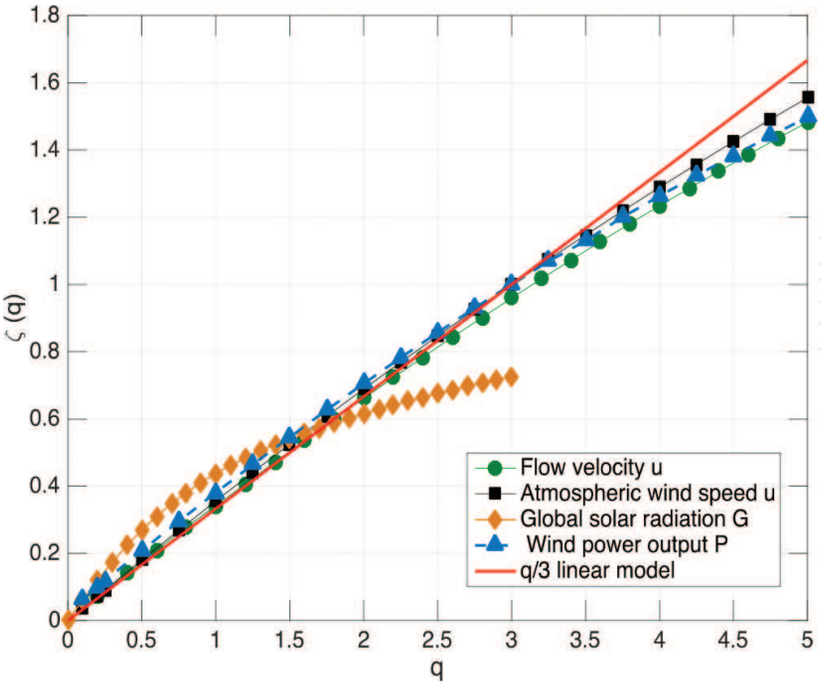


Figure 7. The scaling exponent functions $\zeta(q)$ for each dataset compared with the linear non-intermittent K41 model.

	H	$\zeta(2)$	μ
Flow velocity u	0.34	0.66	0.02
Atmospheric wind speed v	0.35	0.68	0.02
Wind power output P	0.38	0.70	0.06
Global solar radiation G	0.43	0.61	0.25

Table 1. Hurst exponent $H = \zeta(1)$, $\zeta(2)$ linked to the exponent spectral by $\zeta(2) = \beta - 1$ and the intermittency parameter μ estimated for each dataset.

multifractal and intermittent character of considered sustainable data here. Furthermore, the degree of concavity gives an indication on the degree of intermittency: the more concave the scaling exponent curve is, the more intermittent the process. We recall that the intermittency parameter can be estimated by $\mu = 2\zeta(1) - \zeta(2)$ with $0 < \mu < 1$. **Table 1** draws up some parameters for each dataset: H the Hurst exponent, $\zeta(2)$, and the intermittency parameter μ.

As shown in **Figure 6** and indicated in **Table 1**, the global solar radiation G is the most intermittent.

4. Conclusion

This work highlights the intermittency and the scale invariance properties of flow velocity u, atmospheric wind speed v, wind power output P and global solar radiation G data, at all intensities and at all scales, in the fully developed turbulence framework.

We have shown for all datasets over the period encountered:

- The presence of a scaling regime or power law correlation of the form $f^{-\beta}$ over a broad range of time scales, in the Fourier space. The exponent spectral β is close to the exact 5/3 Kolmogorov value for all the datasets.
- The nature of the scaling behaviour for each dataset is determined using qth order structure functions analysis. The nonlinearity and the concavity of the scaling exponent functions $\zeta(q)$ obtained reveal the intermittent and the multifractal properties of datasets considered in this manuscript. This could result from the complex interaction of the turbulent atmospheric and the energy converter systems such as, for example, wind turbine.

With the increase in sustainable energies, a good knowledge of their nonstationary and intermittent properties is crucial. The fully developed turbulence framework is a relevant frame to analysis stochastic processes such as those considered in this manuscript. It allows providing a sharp description of fluctuations of processes at all scales and at intensities. The Hurst and the intermittency parameters can be used in stochastic simulations based on multifractal cascade model, as performed in [33]. Here, with a dynamical modelling of fluctuations sustainable energy considered, the interest could be, for instance, to test the stability evaluation of electricity grid.

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