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# Comparison of Spatial Interpolation Techniques Using Visualization and Quantitative Assessment

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Additional information is available at the end of the chapter

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## Abstract

Spatial interpolation has been widely and commonly used in many studies to create surface data based on a set of sampled points, such as soil properties, temperature, and precipitation. Currently, there are many commercial Geographic Information System (GIS) or statistics software offering spatial interpolation functions, such as inverse distance weighted (IDW), kriging, spline, and others. To date, there is no “rule of thumb” on the most appropriate spatial interpolation techniques for certain situations, though general suggestions have been published. Many studies rely on quantitative assessment to determine the performance of spatial interpolation techniques. Most quantitative assessment methods provide a numeric index for the overall performance of an interpolated surface. Although it is objective and convenient, there are many facts or trends not captured by quantitative assessments. This study used 2D visualization and 3D visualization to identify trends not evident in quantitative assessment. This study also presented a special case, a closed system in which all interpolated surfaces should sum up to 100%, to demonstrate the interaction between interpolated surfaces that were created separately and independently.

**Keywords:** spatial interpolation, quantitative assessment, 2D visualization, 3D visualization, performance

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## 1. Introduction

Spatial interpolation is the process of using a set of point data to create surface data [1, 2]. A point data set has data values only for certain locations, such as field work locations, within the study area. Surface data divides the study area into cells, with a data value for each cell. With surface data, there is often a data value for every location inside the study area, whether

it was sampled or not. Though a set of point data is more manageable in terms of labor, budget, and time; surface data are more useful and practical in many disciplines, such as precision agriculture, particularly with variable rate applications [3–9].

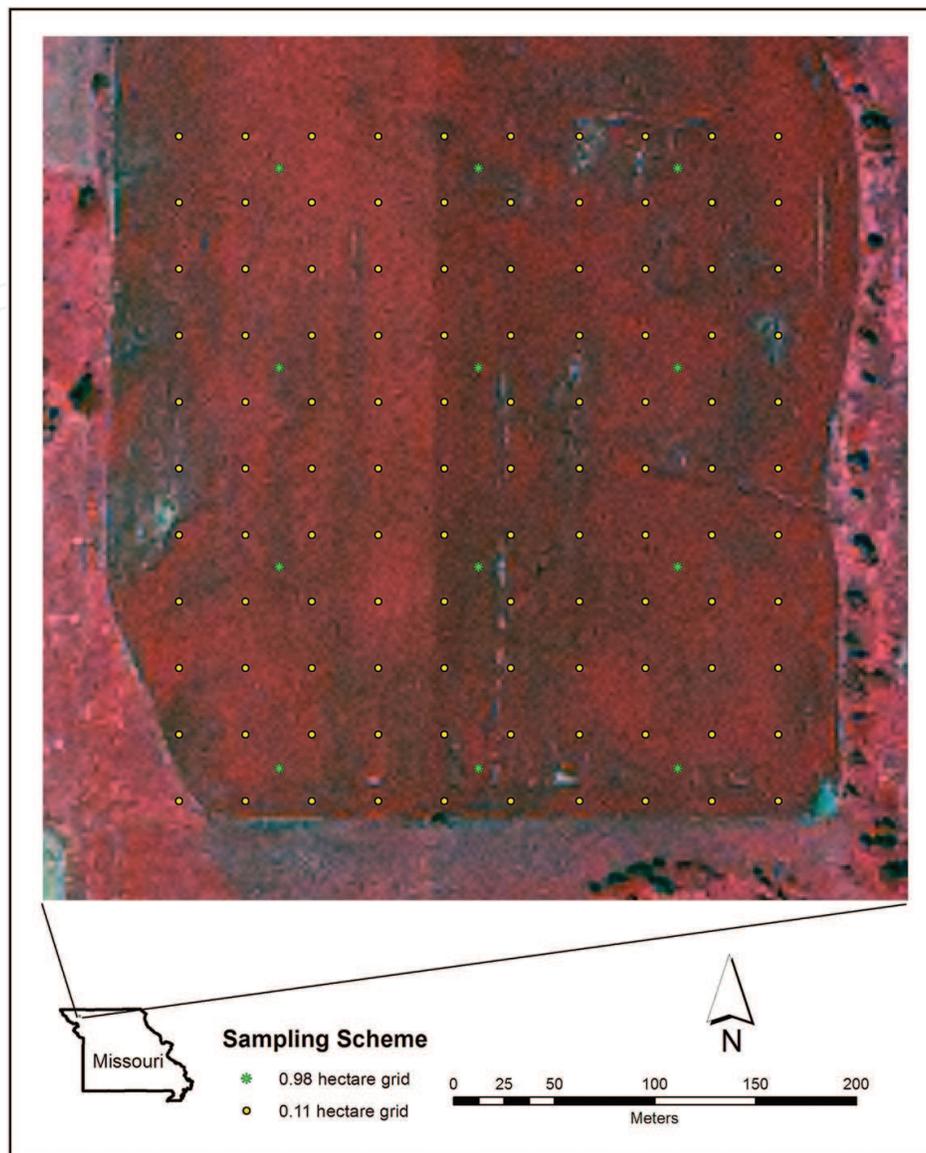
There are many spatial interpolation algorithms available in the literature, as well as in commercial GIS or statistics software [1, 10]. Each algorithm typically requires different parameters. Even with the same algorithm and same input data points, these different parameters can create different surfaces.

Evaluation of interpolated surfaces is difficult and often times overlooked. In most spatial interpolation studies, quantitative assessment was the only method used to evaluate the resultant surfaces. Most quantitative methods provide a numeric index for overall performance. Such a numeric index is easy to understand and convenient [10–13]. However, interpolated surfaces cannot be described by one numeric index, as many characteristics cannot be observed or evaluated by quantitative assessments. To date, there is no “rule of thumb” on which spatial interpolation techniques are most appropriate for certain situations [14].

The purpose of this chapter is to demonstrate a comprehensive approach to evaluate spatial interpolation, including: common quantitative assessment, 2D visualization, and 3D visualization. This chapter also presents a special case, a closed system consisting of three variables. Spatial interpolation techniques were applied to the three variables separately and independently to create surfaces. 2D visualization and 3D visualization then were used to evaluate whether the interpolated surfaces met the requirements for a closed system. This chapter is organized as follows: Section 2—study area and data, Section 3—spatial interpolation methods, Section 4—quantitative assessments, Section 5—2D and 3D visualization, Section 6—special case of a closed system, and Section 7—conclusions.

## 2. Study area and data

The study area is a 12.15 ha field located at the northwest Missouri State University R.T. Wright Farm near Maryville, MO, USA (**Figure 1**). This field was managed under a corn-soybean rotation [14]. Soils within the field were mapped as mollisols. Soil samples were collected in January 2006 using five soils sampling schemes outlined in a previous study [15]: 0.11 ha grid with 110 samples, 0.98 ha grid with 12 samples, 3.04 ha grid with four samples, topography-based composite with three samples, and whole-field composite with only one sample. The soil pH value of the 110 sample points from the 0.11 ha grid was the input data for spatial interpolation in this study. Point sampling was used to collect grid-based samples; five 1.27 cm diameter soil cores to a 15.24 cm depth were randomly collected from a 1.0 m<sup>2</sup> area around the predetermined cell sample point. The five soil cores were composited to form the sample for each respective grid sample location. Soil pH was as determined using the standard laboratory method of the United States Department of Agriculture [16].



**Figure 1.** Study area: R. T. Wright University Farm in northwest Missouri, with NAIP (National Agricultural Imagery Program) 2006 CIR (color infrared) display.

### 3. Spatial interpolation methods

Spatial interpolation, or spatial prediction, is a process to estimate values of locations that were not surveyed based on a network of points with known values [1, 2, 10, 11]. In most cases, the input data is a network of points, while the output is a surface that divides the study area into small cells with a data value for each cell. There are two basic assumptions for spatial interpolation. First is spatial autocorrelation, which is best explained by Tobler's first law of geography "everything is related to everything else, but near things are more related than distant things" [17]. The second assumption is that values are smooth and continuous over space. Many spatial interpolation techniques were developed based on these

two assumptions. Commercial GIS or statistical software provides several spatial interpolation functions, such as inverse distance weighted (IDW), kriging, spline, and others.

Although there are many options for spatial interpolation, to date, there is no “rule of thumb” on which technique is best under what certain circumstances. Even with the same technique and same input point data, different parameters may result in different surfaces. Potentially, a given set of points and a given spatial interpolation technique can generate many different surfaces [10, 14]. Therefore, it is important to evaluate and understand the accuracy and reliability of surface data generated from spatial interpolation. In this study, IDW, kriging, and spline will be used to demonstrate the process to evaluate and visualize spatial interpolation surfaces.

### 3.1. Inverse distance weighted

Inverse distance weighted is a deterministic estimation method where values at unmeasured points are determined by a linear combination of values at nearby measured points. Among available parameters, the power parameter can significantly affect the results. As the power parameter increases, IDW acts similarly to the nearest neighbor interpolation method in which the interpolated value is close to the value of the nearest measured value. The advantages of IDW are that it is simple, easy to understand, and efficient. Disadvantages are that it is sensitive to outliers and there is no indication of error [1].

Schloeder et al. [18] compared IDW, kriging, and spline spatial interpolation methods. They concluded that IDW and kriging performed similarly and that both are more accurate than the spline interpolation method. Mueller et al. [19] compared IDW and kriging on soil properties. Though individual performance differed greatly depending on the existence of spatial structure and sampling density, they concluded little difference between the overall performances between IDW and kriging. Kravchenko [20] conducted another study to compare IDW and kriging on soil properties. He reported that spatial structure significantly affected the accuracy of interpolation performance. He also reported that known variograms can greatly improve kriging performance, which may result in a better performance than IDW. Lu and Wong [21] developed a new form of IDW, which estimated data values at an unsampled location based on spatial pattern found in its neighborhood. As already reported in Refs. [19, 20], Lu and Wong [21] also found that variograms may greatly affect the performance of kriging. Their new form of IDW may perform better than kriging without variograms.

### 3.2. Kriging

Kriging is a stochastic method similar to IDW in that it also uses a linear combination of weights at known locations to estimate the data value of an unknown location. Variogram is an important input in kriging interpolation. It is a measure of spatial correlation between two points. With known variograms, weights can change according to the spatial arrangement of the samples. A major advantage of kriging is that, in addition to the estimated surface, kriging also provides a measure of error or uncertainty of the estimated surface. A disadvantage is that it requires substantially more computing time and more input from users, compared to IDW and spline [1].

Bekele et al. [22] compared several spatial interpolation methods, including kriging and IDW. They found that kriging generally performed better than IDW. However, they concluded that a regression-based autocorrelated error model was overall a more flexible method for interpolation. Laslett et al. [23] compared kriging and spline spatial interpolation methods and found that kriging produced better and more accurate surface than spline. Gotway et al. [24] compared kriging and IDW, and reported that kriging performed better than IDW and was relatively more stable because it was less dependent on spatial structure or soil sampling. Bishop and McBratney [25] conducted a study to explore the effect of having secondary data (such as color aerial photos) in the interpolation process. They reported an improved kriging performance.

### 3.3. Spline

Spline is a deterministic method to represent two-dimensional curves on three-dimensional surfaces. It can be imagined as fitting a flexible surface through a set of known points using a mathematical function. A major advantage of spline is that it can create fairly accurate and visually appealing surfaces based on only a few sample points. Disadvantages of spline are that the resultant surface may have different minimum and maximum values from the input data set, it is sensitive to outliers, and there is no indication of errors [1].

Laslett et al. [26] conducted an early study to evaluate and compare the performance of different spatial interpolation methods, including kriging, IDW, spline, and others. They reported though each method may perform better than others under certain situations, overall spline and kriging performed relatively better than IDW. Voltz and Webster [27] compared kriging and spline on soil properties, and concluded that kriging performed overall better than spline. Robinson and Metternicht [28] compared spline, kriging, and IDW interpolations methods on soil properties. They reported that no single method was suitable for all situations. Simpson and Wu [29] compared IDW, kriging, and spline on interpolating lake depth, and reported that spline produced the most accurate results with less than the ideal amount of sampled points.

## 4. Quantitative assessment

Based on a previous study [14], six interpolated surfaces were chosen for demonstration purposes. They are IDW (parameters: power 2, 10 neighbors), spline (parameters: tension, 10 neighbors), kriging (parameters: circular, 10 neighbors), IDW (parameters: power 4, 20 neighbors), spline (parameters: thin plate, 20 neighbors), and kriging (parameters: exponential, 20 neighbors). Each surface was evaluated by cross validation (Jackknifing) by the 110 points from the 0.11 ha grid [10]. This validation process will go through iterations till all points were processed and validated. In each iteration, one sample point with known data value was discarded, and the remaining sample points were used to predict the value at the location of the discarded point. The known data values were compared to their counterpart predicted values and a measure of prediction accuracy was calculated.

Four error measures were used as accuracy index [14]. They are (1) mean absolute error (MAE), see Eq. (1) [12, 30]; (2) root mean square errors (RMSE), see Eq. (2) [12]; (3) systematic

root mean square errors (RMSEs), see Eq. (3) [31]; and (4) unsystematic root mean square errors (RMSEu), see Eq. (4) [31]. Readings from the accuracy index, the lower values mean less errors, and therefore, higher accuracies and better performances.

$$\frac{\sum_{i=1}^n |P_i - S_i|}{n} \quad (1)$$

where  $n$  is the sample size,  $P_i$  is the predicted value at point  $i$ , and  $S_i$  is the sampled value at point  $i$ .

$$\sqrt{\frac{\sum_{i=1}^n (P_i - S_i)^2}{n}} \quad (2)$$

$$\sqrt{\frac{\sum_{i=1}^n (\hat{P}_i - S_i)^2}{n}} \quad (3)$$

where  $\hat{P}_i$  is the estimated value at point  $i$ , by the best-fit regression function specific to each interpolation surface.

$$\sqrt{\frac{\sum_{i=1}^n (P_i - \hat{P}_i)^2}{n}} \quad (4)$$

**Table 1** summarizes these four error measures for these six interpolated surfaces. At first glance, they are quite compatible with each, meaning a similar performance. With closer examinations, one may notice that spline (parameter: thin plate, 20 neighbors) seems to have higher error measures, meaning more errors, and therefore worse performance. This particular interpolation has 0.3481 in MAE measure, while other surfaces are between 0.2925

	MAE	RMSE	RMSEs	RMSEu
IDW, power 2, N 10	0.2930	0.3671	0.3164	0.1712
Spline, tension, N 10	0.2957	0.3702	0.3279	0.1813
Kriging, circular, N 10	0.2926	0.3669	0.3255	0.1669
IDW, power 4, N 20	0.2965	0.3702	0.3310	0.1815
Spline, thin plate, N 20	0.3481	0.4408	0.3508	0.3167
Kriging, exponential, N 20	0.2925	0.3661	0.3357	0.1540

N: neighbor parameter.

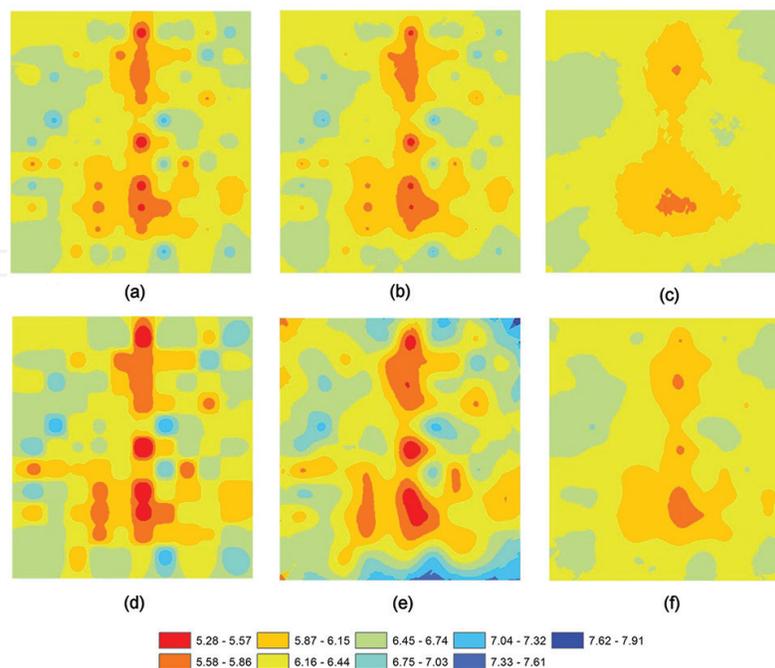
**Table 1.** Cross validation (Jackknifing) by 110 sample points from 0.11 ha grid.

and 0.2965; 0.4408 in RMSE measure while others between 0.3661 and 0.3702; and 0.3167 in RMSEu measure while others between 0.1540 and 0.1815. Among these four error measures, spline (parameter: thin plate, 20 neighbors) interpolation has considerably higher values than the other surfaces in three measures. On the other hand, IDW and kriging seem to perform similarly with compatible error measures.

## 5. Visualization of spatial interpolation

### 5.1. 2D visualization

**Figure 2** shows these six interpolated surfaces in a flat 2D visualization environment. With visual inspection, one may notice that among these three surfaces with 10 neighbors, kriging (parameter: circular, 10 neighbors) appears differently. One may describe it as smoother with less extreme values (because of less red colors and blue colors). On the other hand, IDW (parameter: power 2, 10 neighbors) and spline (parameter: tension, 10 neighbors) seem to appear similarly. The same observation can be made in the group of three surfaces with 20 neighbors. Kriging (parameter: exponential, 20 neighbors) appears smoother than other two surfaces. IDW (parameter: power 4, 20 neighbors) and spline (parameter: thin plate, 20 neighbors) seem to appear similarly. Comparison between the group of 10 neighbors and the group of 20 neighbors, one may observe another interesting trend that the group of 20 neighbors generally appears to have more extreme values, with more red colors and blue colors, than the group of 10 neighbors.



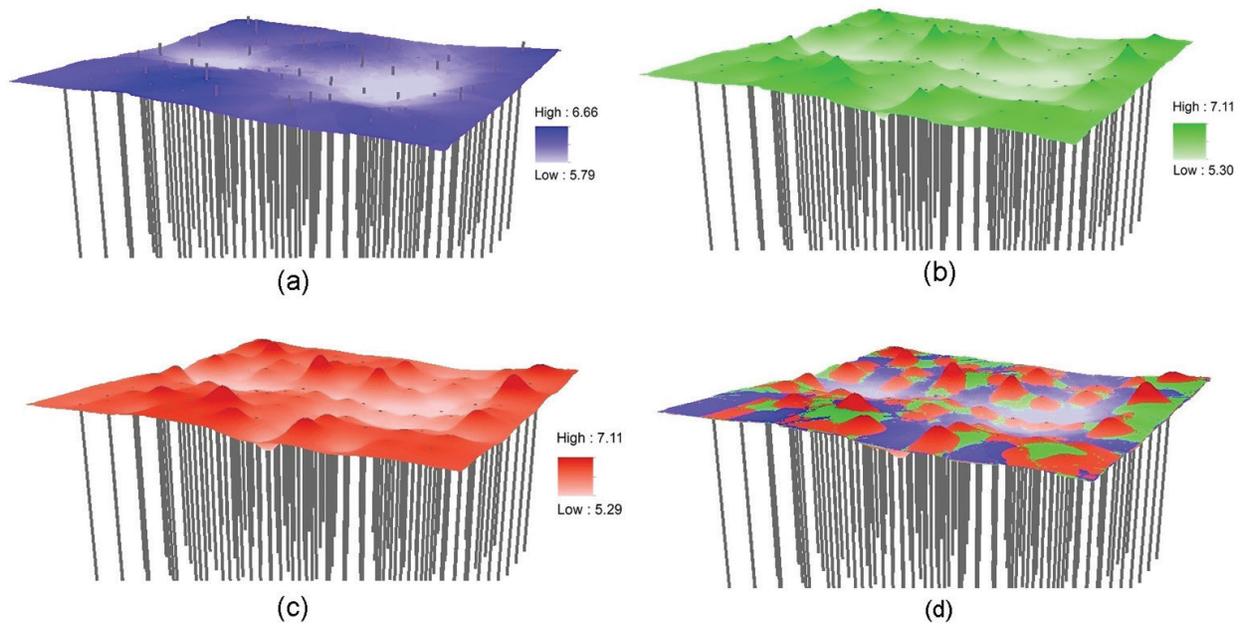
**Figure 2.** Six interpolated surfaces with their parameters. N: neighbor. (a) IDW, power 2, N 10; (b) spline, tension, N 10; (c) kriging, circular, N 10; (d) IDW, power 4, N 20; (e) spline, thin plate, N 20; (f) kriging, exponential, N 20.

Appearing smoother with less extreme values is not necessarily an indication of good performance or bad performance. It is just a characteristic of the overall trend of the interpolated surface, which was not revealed by quantitative assessment, such as four error measures shown earlier. An initial visual inspection of the interpolated surfaces already revealed a different observation from quantitative assessment. In quantitative assessment, it was observed that IDW and kriging performed similarly, and both are better than spline. With initial visual inspection, it was observed that IDW and spline performed similarly, while kriging performed differently, not necessarily in a better or worse way. Such difference warrants a further examination with visualization tools.

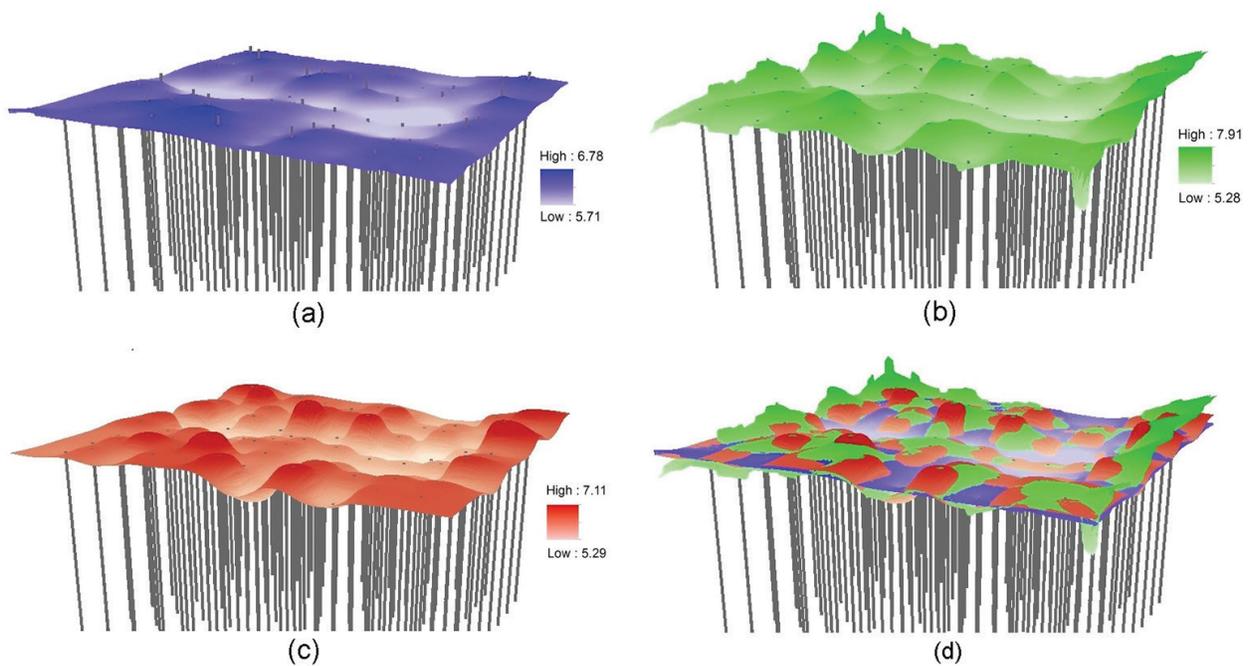
## 5.2. 3D visualization

**Figure 3** shows the group of three surfaces with 10 neighbors in 3D visualization. One can confirm the trend observed in the 2D visualization that kriging (parameters: circular, 10 neighbors) appears smoother than IDW (parameters: power 2, 10 neighbors) and spline (parameters: tension, 10 neighbors). This particular 3D visualization reveals even more trends that cannot be observed in quantitative assessment. In **Figure 3**, gray bars indicate locations of sample points, with bar height equaling data values. One may notice that kriging (parameters: circular, 10 neighbors) does not quite match the sampled data. Bars poke out (or appear above) the interpolated surface, indicating that the interpolated surface has data values less than actual sampled data. This is an indication of inexact interpolation [10], meaning the predicted data value at the sampled location is different from actual data value sampled at this same location. It implies that kriging (parameters: circular, 10 neighbors) underestimated data values, compared to actual data values. This phenomena (bar poking out of the surface) is less evident for spline (parameters: tension, 10 neighbors), and almost nonexistent for IDW (parameters: power 2, 10 neighbors). This implies that, in this study, kriging and spline are inexact interpolations, while IDW is an exact interpolation. There are parameters that can control exact or inexact interpolation in kriging or spline. Unfortunately, for most food producers, novice GIS users, or the general public, they are not familiar with exact or inexact interpolation. Chances are they do not know how to control the exact or inexact interpolation, and will end up like this study with some inexact interpolations, which is not revealed in quantitative assessment.

**Figure 4** shows the group of three surfaces with 20 neighbors in 3D visualization. One can observe the same trend that kriging (parameters: exponential, 20 neighbors) appears smoother than other two surfaces, with evident bars poking out of the surface. Comparing the group of 10 neighbors and the group of 20 neighbors, one may notice a difference in overall surface appearance. Taking IDW for example, IDW (parameters: power 2, 10 neighbors) has some pointy peaks, while IDW (parameters: power 4, 20 neighbors) appears duller. Same can be observed between spline (parameters: tension, 10 neighbors, pointy) and spline (parameters: thin plate, 20 neighbors, duller). One may also notice another abnormality on the south and north edges of spline (parameters: thin plate, 20 neighbors). There are some extreme peaks or villages among these two edges. This is also visible in the 2D visualization in **Figure 2(e)**, where some clusters of blue colors appear along the south edge and the north edge of the study area. Such clusters of blue colors are only visible in this particular interpolation.



**Figure 3.** 3D visualization of three interpolations with 10 neighbor points. Each interpolation is displayed with a continuous tone, lighter colors for lower values, and stronger colors for higher values. View at the image from southwest. Soil sample data are displayed as gray bars, height of bars indicates data values. (a) Kriging, circular; (b) spline, tension; (c) IDW, power 2; (d) kriging, circular, spline, tension, and IDW, power 2 three interpolations.



**Figure 4.** 3D visualization of three interpolations with 20 neighbor points. Each interpolation is displayed with a continuous tone, lighter colors for lower values, and stronger colors for higher values. View at the image from southwest. Soil sample data are displayed as gray bars, height of bars indicates data values. (a) Kriging, exponential; (b) spline, thin plate; (c) IDW, power 4; (d) kriging, exponential, spline, thin plate, and IDW, power 4 three interpolations.

	Min.	Max.	Mean	S.D.
IDW, power 2, N 10	5.29	7.11	6.30	0.24
Spline, tension, N 10	5.30	7.11	6.34	0.23
Kriging, circular, N 10	5.80	6.65	6.30	0.17
IDW, power 4, N 20	5.29	7.11	6.31	0.31
Spline, thin plate, N 20	5.29	7.91	6.34	0.35
Kriging, exponential, N 20	5.71	6.78	6.30	0.18
110 samples from 0.11 ha	5.29	7.11	6.27	0.38

N: neighbor points.

**Table 2.** Descriptive statistics for six interpolation results and the original sample set

**Table 2** shows the descriptive statistics for these six interpolated surfaces, as well as the original sample data set (110 points from 0.11-ha grid). One may notice that only IDW surfaces have the exact minimum and maximum values as the original sample data. Overall, kriging has a smaller range (difference between minimum and maximum) than spline. Spline (parameters: thin plate, 20 neighbors) has the largest range, as observed in **Figures 2(e)** and **4(b)**.

In summary, different assessment methods reveal different characteristics of these interpolations. The quantitative assessment indicated that IDW and kriging performed similarly, and both better than spline. 2D visualization indicated that IDW and spline performed similarly, while kriging performed differently, not necessarily in a better or worse way. 3D visualization indicated that IDW is an exact interpolation, while kriging and spline are inexact interpolations. It was also revealed that kriging has the tendency to underestimate data values, compared to actual data values. Spline had the tendency to generate extreme data values along edges of the study area. Quantitative assessment is widely and commonly used in most spatial interpolation studies. Although 2D and 3D visualization tools do not provide quantitative indication of good or bad performance, they both revealed something quantitative assessment failed to report.

## 6. Interactions between spatial interpolations

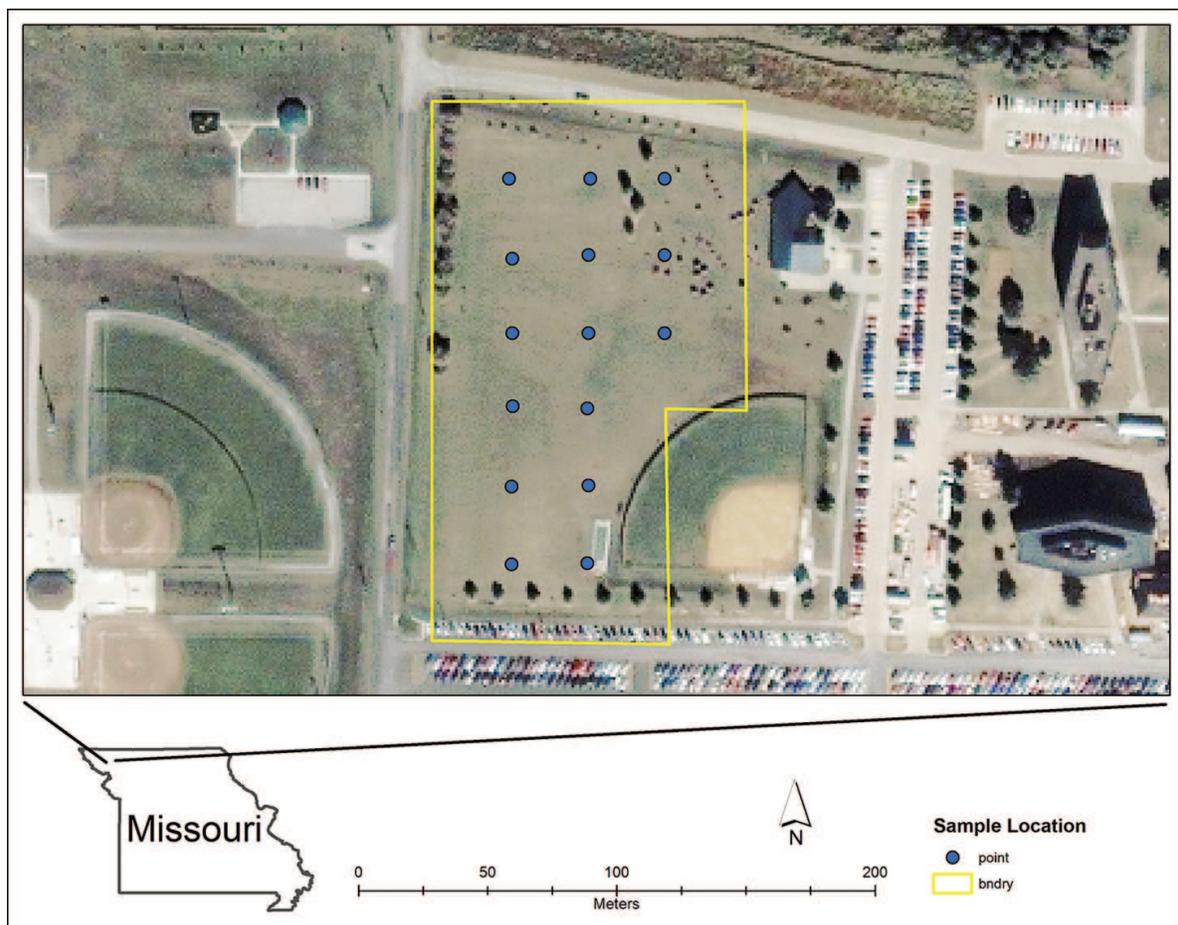
So far, we have examined spatial interpolations on the individual surface level. As discussed earlier, it is difficult to determine which one performed better than others, based on one assessment method. Different assessment methods reveal different characteristics of interpolations. It is essential to understand these interpolated surfaces from all available assessment methods.

There are occasions where spatial interpolations were used to estimate a single variable in a larger project where multiple variables consist of a closed system. The V-I-S (vegetation-

impervious surface-soil) model commonly used in modeling physical urban areas [32–34] is an example of such a closed system. In the V-I-S model, urban areas are represented by composition of vegetation, impervious surface, and soil. For example, industrial areas may be made of 50% impervious surface, 20% vegetation, and 30% soil, while low density residential areas may be made of 30% impervious surface, 60% vegetation, and 10% soil. The sum of V, I, and S percentage should be 100%, i.e., a closed system. When surveying V, I, and S percentage with field work, image processing, or photo interpretation, one can assure that surveyed data values sum up to 100%, meeting the closed system requirements. When doing the spatial interpolation to generate surfaces of V, I, and S percentages, special attention should be paid to the interactions between variables or surfaces.

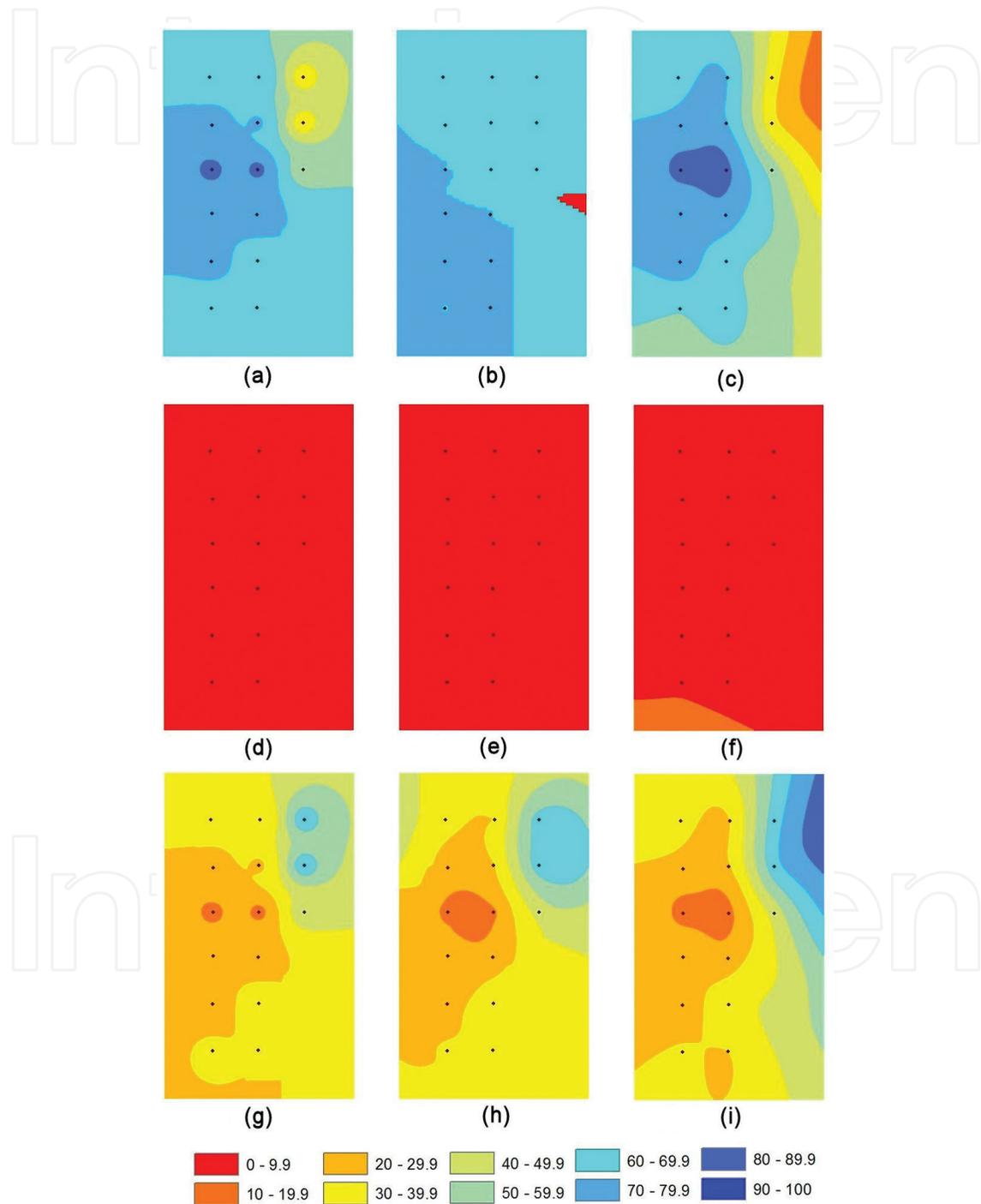
### 6.1. Data and spatial interpolation in a closed system

A small experiment was conducted to demonstrate how individual spatial interpolation interacts with each other on a closed system. Fifteen points were visited and V, I, and S percentages were sampled in a grass field in Northwest Missouri State University in Maryville, MO, USA (see **Figure 5**). This field is grassy, with scattered trees, bushes, and pitches of



**Figure 5.** Study area: a grass field in Northwest Missouri State University, with 2003 IKONOS image (R, G, B/3, 2, 1) true color display.

soil. Impervious surface can only be found on the edges (roads and parking lots). Each point is 30 m away from its immediate four neighbors. At each point location, 100 samples were taken, with each sample verified as either vegetation, impervious surface, or soil. All 100 samples were then summed and converted to V, I, and S percentage for that point location. Most points have various amounts of vegetation and soil, with no impervious surface,



**Figure 6.** Nine interpolated surfaces for percentage vegetation, impervious surface, and soil, created by IDW, kriging, and spline spatial interpolation methods, respectively. (a) Veg: idw; (b) Veg: kriging; (c) Veg: spline; (d) Imp: idw; (e) Imp: kriging; (f) Imp: spline; (g) Soil: idw; (h) Soil: kriging; (i) Soil: spline.

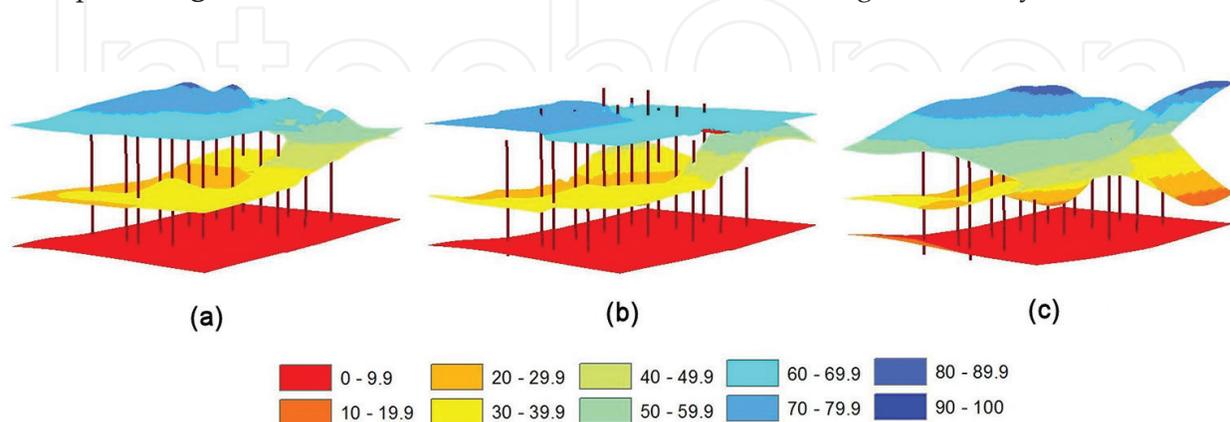
except two points near the south edge of the study area, which is close to parking lots where impervious surface exists.

Three spatial interpolations were chosen for demonstration purposes. They are: IDW (parameters: power 2, 10 neighbors), spline (parameters: tension, 10 neighbors), and kriging (parameters: circular, 10 neighbors). Each interpolation was applied to create V, I, and S surfaces. In total, there were nine surfaces generated. **Figure 6** shows these nine interpolated surfaces. One may quickly observe how differently these surfaces appear, especially among these vegetation percent surfaces. One may also notice that among three impervious surfaces, only spline surface shows data values greater than 10, which is along the south edge. Among three vegetation surfaces, only spline shows data values in orange or red colors (very low) near the northeast corner. Among three soil surfaces, only spline shows data values in blue colors (very high) near the northeast corner. These are extreme values near edges of interpolated surfaces, a trend associated with spline interpolation, as observed in the earlier examples, also shown in **Figures 2(e)** and **4(b)**, as well as discussed in Ref. [14].

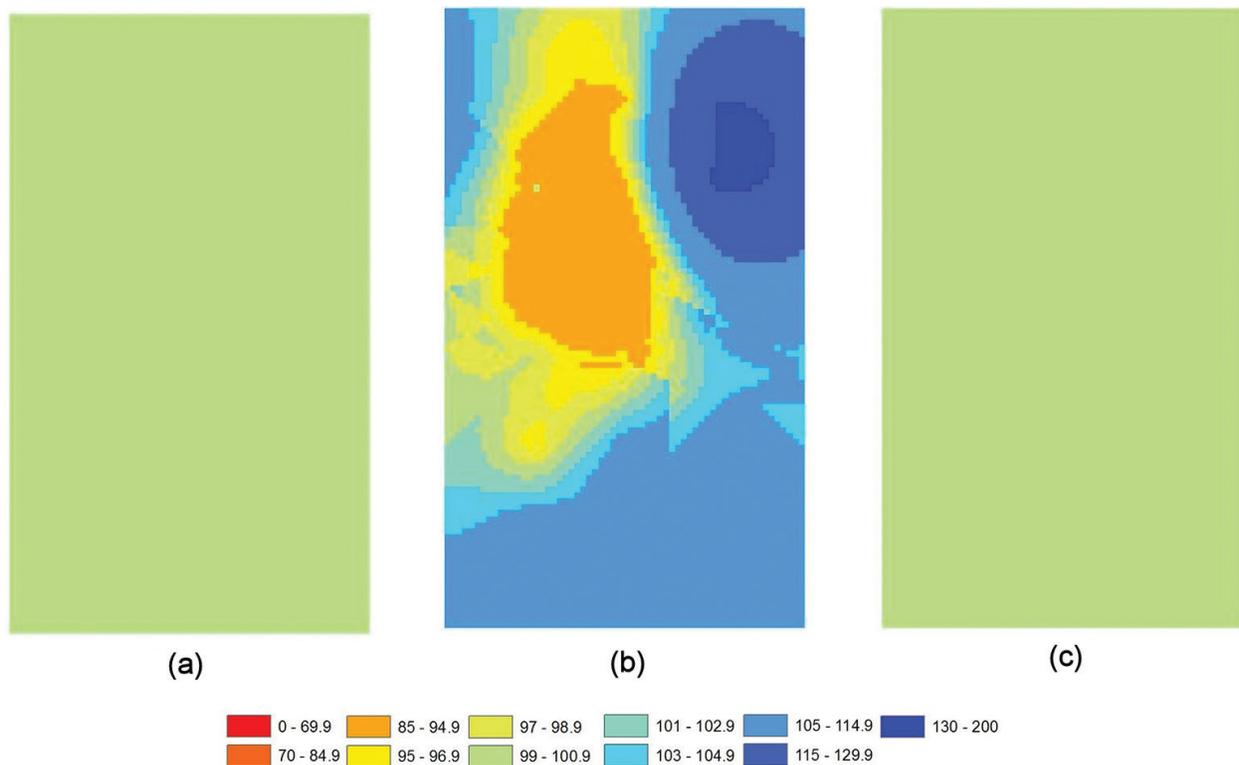
## 6.2. Evaluation and visualization of spatial interpolation in a closed system

**Figure 7** shows these surfaces in 3D visualization, looking from the southeast. **Figure 7(a)** shows the three percentage surfaces generated by IDW, top surface for vegetation, middle surface for soil, and bottom surface for impervious surface. **Figure 7(b)** shows the three percentage surfaces generated by kriging, and **Figure 7(c)** for spline. Bars indicate locations of sampled points. Height of bars equals the percent of vegetation. One may observe that bars poking out of kriging vegetation surface, means an inexact interpolation. One may also observe the extreme data values on spline surfaces. In this 3D visualization, it is evident that three interpolation methods performed very differently.

When adding three surfaces generated by IDW together, because it is a closed system, all cells supposedly should have a data value close to 100%. So do three surfaces generated by kriging and spline. **Figure 8** shows the sum of three surfaces generated by IDW, kriging, and spline. **Figure 8(a)** shows the sum of V, I, and S surfaces generated by IDW. One can



**Figure 7.** 3D visualization of V, I, and S percentage surface. Top surface is for vegetation, middle for soil, and bottom for impervious surface. Bars indicate locations of sampled points. Height of bars equals vegetation percent. (a) IDW; (b) kriging; (c) spline.



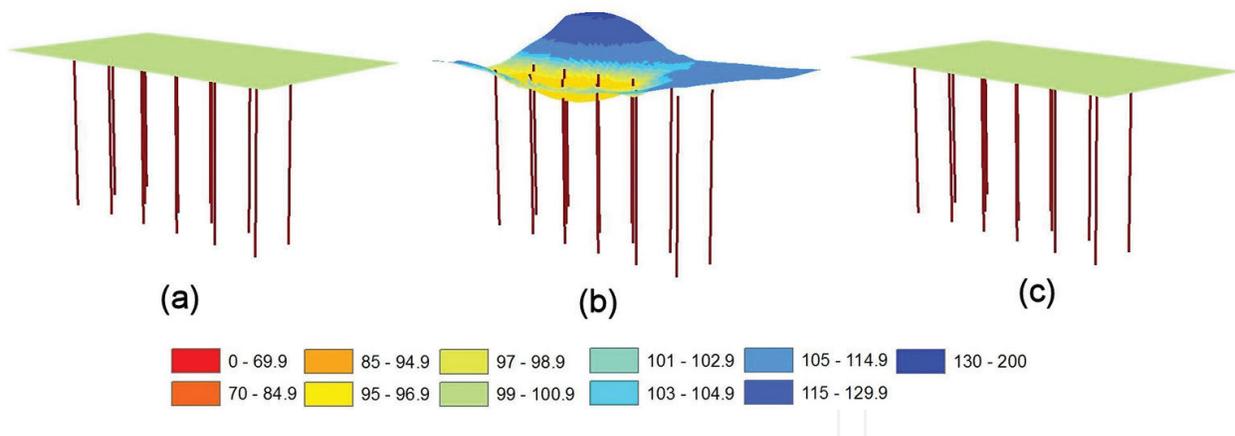
**Figure 8.** 2D visualization of sum surfaces estimated by (a) IDW, (b) kriging, and (c) spline.

	Min.	Max.	Mean	S.D.
Sum surface estimated by IDW	100	100	100	0
Sum surface estimated by kriging	85.12	132.00	104.68	9.69
Sum surface estimated by spline	100	100	100	0

**Table 3.** Descriptive statistics for three sum surfaces estimated by IDW, kriging, and spline

observe that there is no major variation from 100% in sum percentage as all cells fall into the category of 99–100.9 range. One may also observe the same trend for spline as shown in **Figure 8(c)**. However, kriging as displayed in **Figure 8(b)** shows a lot of variations from 100% in the sum of V, I, and S percentage. This is another evidence of inexact interpolation, as the interpolated data are not true to the sampled data even at the exact location where it is sampled. **Table 3** shows descriptive statistics for these sum surfaces. One may clearly see that kriging is the only interpolation method that failed to meet the closed system requirement (sum of all variables equals to 100%) when individual variable is interpolated separately and independently.

**Figure 9** shows these three sum surfaces in a 3D visualization environment. Bars indicate the locations of sampled data. Height of bars is set at 100, the requirement for a closed system.



**Figure 9.** 3D visualization of sum surfaces estimated by (a) IDW, (b) kriging, and (c) spline. Bars indicate location of sampled data, with height set to 100.

One may again observe the bars overreach or underreach the kriging sum surface, an indication of inexact interpolation. On the other hand, IDW and spline seem to quite meet the 100% requirement.

It has to be noted that in this experiment, there are only 15 sample points. It is a very small number of samples. The results in this experiment can be biased due to small sample. Nevertheless, some interesting trends were observed by 2D and/or 3D visualization, which was not evident in quantitative assessment. When examining the interactions between interpolated surfaces in a closed system, both IDW and spline met the requirement, i.e., summing variables to 100%, even though each surface was generated from one variable separately and independently. On the other hand, kriging failed to meet this requirement. It was observed again that kriging is an inexact interpolation. Furthermore, it was also observed that spline, as reported earlier in this study and in Ref. [14], had the tendency to generate extreme values along edges of the study area.

## 7. Conclusion

In this study, three spatial interpolation algorithms (IDW, kriging, and spline) were applied to a set of soil pH value data to demonstrate the complexity of the process to validate the results of spatial interpolation. Three methods of validation were used: quantitative assessment, 2D visualization, and 3D visualization. Each validation method revealed different characteristics of each spatial interpolation. With quantitative assessment, it was observed that IDW and kriging performed similarly, and both are better than spline. With 2D visualization, it was observed that IDW and spline performed similarly, while kriging performed differently, not necessarily in a good or bad way. With 3D visualization, it was observed that kriging is an inexact interpolation. It was also observed that spline had a tendency to create extreme values along edges of the study area.

Another experiment was conducted to demonstrate the interactions between interpolated surfaces, especially in a closed system. There were three variables in this closed system, each represented a percentage of a specific land cover in an urban area. In a closed system, these three variables should sum up to 100%. Three spatial interpolation algorithms (IDW, kriging, and spline) were applied to each variable separately and independently. These interpolated surfaces were then added up to form a sum surface. It was observed that both IDW and spline successfully met the requirement, making the sum surface 100% for all cells, while kriging failed to meet this requirement.

In conclusion, each spatial interpolation algorithm performed differently. One has to be careful on evaluation of the results. Though quantitative assessment is commonly and widely used in most spatial interpolation studies, it is essential to understand that evaluation of a spatial interpolation should not rely on quantitative assessment alone. 2D visualization and 3D visualization can reveal some facts that cannot be observed in quantitative assessment.

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