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Multi-objective Uniform-diversity Genetic Algorithm (MUGA)

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1. Introduction

Optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. Basically, the optimization process is defined as finding a set of values for a vector of design variables so that it leads to an optimum value of an objective or cost function. In such single-objective optimization problems, there may or may not exist some constraint functions on the design variables and they are respectively referred to as constrained or unconstrained optimization problems. There are many calculus-based methods including gradient approaches to search for mostly local optimum solutions and these are well documented in (Arora, 1989; Rao, 1996). However, some basic difficulties in the gradient methods such as their strong dependence on the initial guess can cause them to find a local optimum rather than a global one. This has led to other heuristic optimization methods, particularly Genetic Algorithms (GAs) being used extensively during the last decade. Such nature-inspired evolutionary algorithms (Goldberg, 1989; Back et al., 1997) differ from other traditional calculus based techniques. The main difference is that GAs work with a population of candidate solutions, not a single point in search space. This helps significantly to avoid being trapped in local optima (Renner & Ekart, 2003) as long as the diversity of the population is well preserved.

In multi-objective optimization problems, there are several objective or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that as one objective function improves, another deteriorates. Therefore, there is no single optimal solution that is best with respect to all the objective functions. Instead, there is a set of optimal solutions, well known as Pareto optimal solutions (Srinivas & Deb, 1994; Fonseca & Fleming, 1993; Coello Coello & Christiansen, 2000; Coello Coello & Van Veldhuizen, 2002), which distinguishes significantly the inherent natures between single-objective and multi-objective optimization problems. V. Pareto (1848-1923) was the French-Italian economist who first developed the concept of multi-objective optimization in economics (Pareto, 1896). The concept of a Pareto front in the space of objective functions in multi-objective optimization problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. Evidently, changing the vector of design variables in such a Pareto optimal

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solutions consisting of these non-dominated solutions would not lead to the improvement of all objectives simultaneously. Consequently, such change leads to a deterioration of at least one objective to an inferior one. Thus, each solution of the Pareto set includes at least one objective inferior to that of another solution in that Pareto set, although both are superior to others in the rest of search space.

The inherent parallelism in evolutionary algorithms makes them suitably eligible for solving MOPs. The early use of evolutionary search is first reported in 1960s by Rosenberg (Rosenberg, 1967). Since then, there has been a growing interest in devising different evolutionary algorithms for MOPs. Basically, most of them are Pareto-based approaches and use the well-known non-dominated sorting procedure. In such Pareto-based approaches, the values of objective functions are used to distinguish the non-dominated solutions in the current population. Among these methods, the Vector Evaluated Genetic Algorithm (VEGA) proposed by Schaffer (Schaffer, 1985), Fonseca and Fleming's Genetic Algorithm (MOGA) (Fonseca & Fleming, 1993), Non-dominated Sorting Genetic Algorithm (NSGA) by Srinivas and Deb (Srinivas & Deb, 1994), and Strength Pareto Evolutionary Algorithm (SPEA) by Zitzler and Thiele (Zitzler & Thiele, 1998), and the Pareto Archived Evolution Strategy (PAES) by Knowles and Corne (Knowles & Corne, 1999) are the most important ones. A very good and comprehensive survey of these methods has been presented in (Coello Coello, 1999; Deb, 2001; Khare et al., 2003). Coello (Coello Coello, home page) has also presented an internet based collection of many papers as a very good and easily accessible literature resource. Basically, both NSGA and MOGA as Pareto-based approaches use the revolutionary non-dominated sorting procedure originally proposed by Goldberg (Goldberg, 1989).

There are two important issues that have to be considered in such evolutionary multi-objective optimization methods: driving the search towards the true Pareto-optimal set or front and preventing premature convergence or maintaining the genetic diversity within the population (Toffolo & Benini, 2003). The lack of elitism was also a motivation for modification of that algorithm to NSGA-II (Deb et al., 2002) in which a direct elitist mechanism, instead of a sharing mechanism, has been introduced to enhance the population diversity. This modified algorithm represents the state-of-the-art in evolutionary MOPs (Coello Coello & Becerra, 2003). A comparison study among SPEA and other evolutionary algorithms on several problems and test functions showed that SPEA clearly outperforms the other multi-objective EAs (Zitzler et al., 2000). Some further investigations reported in reference (Toffolo & Benini, 2003) demonstrated, however, that the elitist variant of NSGA (NSGA-II) equals the performance of SPEA. Despite its popularity and effectiveness, NSGA-II is modified in this work to enhance its diversity preserving mechanism especially for problems with more than two objective functions.

In this chapter, a new simple algorithm in conjunction with the original Pareto ranking of non-dominated optimal solutions is proposed and tested for MOPs including some test functions and engineering problems in power and energy conversion. In the Multi-objective Uniform-diversity Genetic Algorithm (MUGA), a ϵ-elimination diversity approach is used such that all the clones and/or ϵ-similar individuals based on normalized Euclidean norm of two vectors are recognized and simply eliminated from the current population. The superiority of MUGA is shown in comparison with NSGA-II in terms of diversity of population and Pareto fronts both for bi-objective and multi-objective optimization problems.

2. Multi-objective optimization

Multi-objective optimization which is also called multicriteria optimization or vector optimization has been defined as finding a vector of decision variables satisfying constraints to give optimal values to all objective functions (Coello Coello & Christiansen, 2000; Homaifar et al., 1994) . In general, it can be mathematically defined as:

find the vector $X^* = \left[x_1^*, x_2^*, ..., x_n^*\right]^T$ to optimize:

$$F(X) = \left[f_1(X), f_2(X), ..., f_k(X)\right]^T \tag{1}$$

subject to *m* inequality constraints:

$$g_i(X) \le 0 \quad , \quad \text{i} = 1 \text{ to } \text{m} \tag{2}$$

and *p* equality constraints:

$$h_j(X) = 0 \quad , \quad \text{j} = 1 \text{ to } \text{p} \tag{3}$$

where $X^* \in \Re^n$ is the vector of decision or design variables, and $F(X) \in \Re^k$ is the vector of objective functions. Without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on the Pareto approach can be conducted using some definitions:

Definition of Pareto dominance

A vector $U = \left[u_1, u_2, ..., u_k\right] \in \Re^k$ dominates to vector $V = \left[v_1, v_2, ..., v_k\right] \in \Re^k$ (denoted by $U \prec V$) if and only if $\forall i \in \{1,2,...,k\},\ u_i \le v_i \ \wedge\ \exists j \in \{1,2,...,k\} : u_j < v_j$. It means that there is at least one u_j which is smaller than v_j whilst the rest u's are either smaller or equal to corresponding v's.

Definition of Pareto optimality

A point $X^* \in \Omega$ (Ω is a feasible region in $\Re^n$ satisfying equations (2) and (3)) is said to be Pareto optimal (minimal) with respect to all $X \in \Omega$ if and only if $F(X^*) \prec F(X)$. Alternatively, it can be readily restated as $\forall i \in \{1,2,...,k\}$, $\forall X \in \Omega - \{X^*\}\ \ f_i(X^*) \le f_i(X) \ \wedge$ $\exists j \in \{1,2,...,k\} : f_j(X^*) < f_j(X)$. It means that the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

Definition of Pareto Set

For a given MOP, a Pareto set $\mathcal{P}^*$ is a set in the decision variable space consisting of all the Pareto optimal vectors, $\mathcal{P}^* = \{X \in \Omega \mid \nexists\, X' \in \Omega : F(X') \prec F(X)\}$. In other words, there is no other X' in Ω that dominates any $X \in \mathcal{P}^*$.

Definition of Pareto front

For a given MOP, the Pareto front $\mathcal{PF}^*$ is a set of vectors of objective functions which are obtained using the vectors of decision variables in the Pareto set $\mathcal{P}^*$, that is,

$\mathcal{PF}^* = \{F(X) = (f_1(X), f_2(X), ..., f_k(X)) : X \in \mathcal{P}^*\}$. Therefore, the Pareto front $\mathcal{PF}^*$ is a set of the vectors of objective functions mapped from $\mathcal{P}^*$.

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. Therefore, most difficulties and deficiencies within the classical methods in solving multi-objective optimization problems are eliminated. For example, there is no need for either several runs to find the Pareto front or quantification of the importance of each objective using numerical weights. It is very important in evolutionary algorithms that the genetic diversity within the population be preserved sufficiently. This main issue in MOPs has been addressed by much related research work (Toffolo & Benini, 2003). Consequently, the premature convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front if such genetic diversity is well provided. The Pareto-based approach of NSGA-II (Deb et al., 2002) has been recently used in a wide range of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different levels of Pareto frontiers. However, the crowding approach in such a state-of-the-art MOEA (Coello Coello & Becerra, 2003) works efficiently for two-objective optimization problems as a diversity-preserving operator which is not the case for problems with more than two objective functions. The reason is that the sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper-boxes. Thus, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property. In order to show this issue more clearly, some basics of NSGA-II are now represented. The entire population R_t is simply the current parent population P_t plus its offspring population Q_t which is created from the parent population P_t by using usual genetic operators. The selection is based on non-dominated sorting procedure which is used to classify the entire population R_t according to increasing order of dominance (Deb et al., 2002). Thereafter, the best Pareto fronts from the top of the sorted list is transferred to create the new parent population P_{t+1} which is half the size of the entire population R_t. Therefore, it should be noted that all the individuals of a certain front cannot be accommodated in the new parent population because of space. In order to choose exact number of individuals of that particular front, a crowded comparison operator is used in NSGA-II to find the best solutions to fill the rest of the new parent population slots. The crowded comparison procedure is based on density estimation of solutions surrounding a particular solution in a population or front. In this way, the solutions of a Pareto front are first sorted in each objective direction in the ascending order of that objective value. The crowding distance is then assigned equal to the half of the perimeter of the enclosing hyper-box (a rectangular in bi-objective optimization problems). The sorting procedure is then repeated for other objectives and the overall crowding distance is calculated as the sum of the crowding distances from all objectives. The less crowded non-dominated individuals of that particular Pareto front are then selected to fill the new parent population. It must be noted that, in a two-objective Pareto optimization, if the solutions of a Pareto front are sorted in a decreasing order of importance to one objective, these solutions are then automatically ordered in an increasing order of importance to the second objective. Thus, the hyper-boxes surrounding an individual solution remain unchanged in the objective-wise sorting procedure of the crowding distance of NSGA-II in the two-objective Pareto optimization problem. However, in multi-objective Pareto optimization problems with more

than two objectives, such sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper-boxes. Thus, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property for the multi-objective Pareto optimization problems with more than two objectives.
In our work, a new method is presented to modify NSGA-II so that it can be safely used for any number of objective functions (particularly for more than two objectives) in MOPs. Such a modified MOEA is then used for four-objective thermodynamic optimization of subsonic turbojet engines.

3. Multi-objective Uniform-diversity Genetic Algorithm (MUGA)

The multi-objective uniform-diversity genetic algorithm (MUGA) uses non-dominated sorting mechanism together with a ε-elimination diversity preserving algorithm to get Pareto optimal solutions of MOPs more precisely and uniformly.

3.1 The non-dominated sorting method

The basic idea of sorting of non-dominated solutions originally proposed by Goldberg (Goldberg, 1989) used in different evolutionary multi-objective optimization algorithms such as in NSGA-II by Deb (Deb et al., 2002) has been adopted here. The algorithm simply compares each individual in the population with others to determine its non-dominancy. Once the first front has been found, all its non-dominated individuals are removed from the main population and the procedure is repeated for the subsequent fronts until the entire population is sorted and non-dominately divided into different fronts.
A sorting procedure to constitute a front could be simply accomplished by comparing all the individuals of the population and including the non-dominated individuals in the front. Such procedure can be simply represented as following steps:

1-Get the population (pop)
2-Include the first individual {ind(1)} in the front P as P*(1), let P*_size=1;*
3-Compare other individuals {ind (j), j=2, Pop_size)} of the pop with { P(K), K=1, P*_size} of the P*;*
If ind(j)<P(K) replace the P*(K) with ind(j)*
If P(K)<ind(K), j=j+1, continue comparison;*
Else include ind(j) in P, P*_size= P*_size+1, j=j+1, continue comparison;*
4-End of front P;*

It can be easily seen that the number of non-dominated solutions in P^* grows until no further one is found. At this stage, all the non-dominated individuals so far found in P^* are removed from the main population and the whole procedure of finding another front may be accomplished again. This procedure is repeated until the whole population is divided into different ranked fronts. It should be noted that the first rank front of the final generation constitute the final Pareto optimal solution of the multi-objective optimization problem.

3.2 The ε-elimination diversity preserving approach

In the ε-elimination diversity approach that is used to replaced the crowding distance assignment approach in NSGA-II (Deb et al., 2002), all the clones and ε-similar individuals

are recognized and simply eliminated from the current population. Therefore, based on a value of ε as the elimination threshold, all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ε-similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ε-similarity in the space of objectives will not be eliminated from the population. The pseudo-code of the ε-elimination approach is depicted in Fig. 1. Evidently, the clones and ε-similar individuals are replaced from the population by the same number of new randomly generated individuals. Meanwhile, this will additionally help to explore the search space of the given MOP more effectively. It is clear that such replacement does not appear when a front rather than the entire population is truncated for ε-similar individual.

```
ε-elim= ε-elimination(pop)                    // pop includes design variables and
                                              objective function
i=1; j=1;
get K (K=1 for the first front);
While i,j <pop_size

 e(i,j)=  ‖ X(i,:),X(j,:) ‖ / ‖ X(i,:) ‖ ; X(i),X(j) ϵ P*_k Ụ PF*_k  //finding mean value of ε

                                                          within pop.
end
ε=mean(e);
i=1;
until i+1<pop_size;
j=i+1
        until j<pop_size
        if e(i,j)<ε
        then {pop}={pop}/ {pop(j)}          //remove the ε-similar individual
        j=j+1
        end
i=i+1
end
```

Fig. 1. The ε-elimination diversity preserving pseudo-code

3.3 The main algorithm of MUGA

It is now possible to present the main algorithm of MUGA which uses both non-dominated sorting procedure and ε-elimination diversity preserving approach which is given in Fig.2. It first initiates a population randomly. Using genetic operators, another same size population is then created. Based on the ε-elimination algorithm, the whole population is then reduced by removing ε-similar individuals. At this stage, the population is re-filled by randomly generated individuals which helps to explore the search space more effectively. The whole population is then sorted using non-dominated sorting procedure. The obtained fronts are

then used to constitute the main population. It must be noted that the front which must be truncated to match the size of the population is also evaluated by ε-elimination procedure to identify the ε-similar individuals. Such procedure is only performed to match the size of population within ±10 percent deviation to prevent excessive computational effort to population size adjustment. Finally, unless the number of individuals in the first rank front is changing in certain number of generations, randomly created individuals are inserted in the main population occasionally (e.g. every 20 generations of having non-varying first rank front).

```
Get N                          //population size
t=1 ;                          //set generation number
Random_N(Pt);                  //generate the first population (P1) randomly
Qt=Recomb(Pt)                  //generate population Qt from Pt by genetic operators
Rt=Pt Ụ Qt                     //union of both parent and offspring population
Rt′=ε-elimination (Rt)         //remove ε-similar individuals in Rt
Rt″= Rt′ Ụ Random_(Rt_size-R′t_size) (Pt′)    //add random individuals to fill Rt to 2N

Do non-dominate sorting procedure (Rt″)    //Rt″=P*1Ụ P*2Ụ…ỤP*k  where k is total
                                                                number of fronts
i=1
Pt+1=Θ
While not Pt+1_size>N                  //includes fronts into new population
                  Pt+1= Pt+1Ụ P*i
                  i=i+1
end
N′=N- Pt+1_size
While not (0.9 N′< Pt+1_size<1.1 N′)          //remove the ε-similar individuals within
                                                the tolerance of ±10 percent
        F′=ε-elimination (P*i-1)
      If F′_size< N′
        e=1.1*e
        else
        e=0.9 * e          //adjust the value of threshold to get the right population
                           size of the last front
        end
end
t=t+1                                  //Start next generation
```

Fig. 2. The pseudo-code of the main algorithm of MUGA

4. Numerical results of MUGA using test functions

In this section four test functions which have been widely used in literature (Deb et al., 2002) are adopted here to test and compare the effectiveness of MUGA with that of NSGA-II. These test functions are all bi-objective and have no constraint. A generation number of 250 with a population size of 100 have been used in all experiments. The probabilities of crossover and mutation have been chosen as 0.9 and 0.1, respectively. Each test function has

been run for 5 times to compute the mean and variance of the metric of non-uniformity of the solutions obtained in the final Pareto front.

In order to evaluate the diversity of the obtained Pareto front, a metric, Δ, has been adopted here to measure the spread and uniformity of the achieved non-dominated solutions along a Pareto front (Deb et al., 2002). Such metric basically calculates the relative Euclidean distance of consecutive solutions from their average value. Hence, a lower value of Δ (zero in ideal case) indicates a better uniformly spread non-dominated solutions. It is therefore possible to simply compare the performance of MUGA with that of NSGA-II in term of uniformity using the same metric.

Four different functions which have been used to test and compare the results of MUGA with those of NSGA-II are as follows:

$$1\text{-}\begin{cases} f_1(x) = x^2 \\ f_2(x) = (x-2)^2 \end{cases} \qquad x \in [-1000, 1000]$$

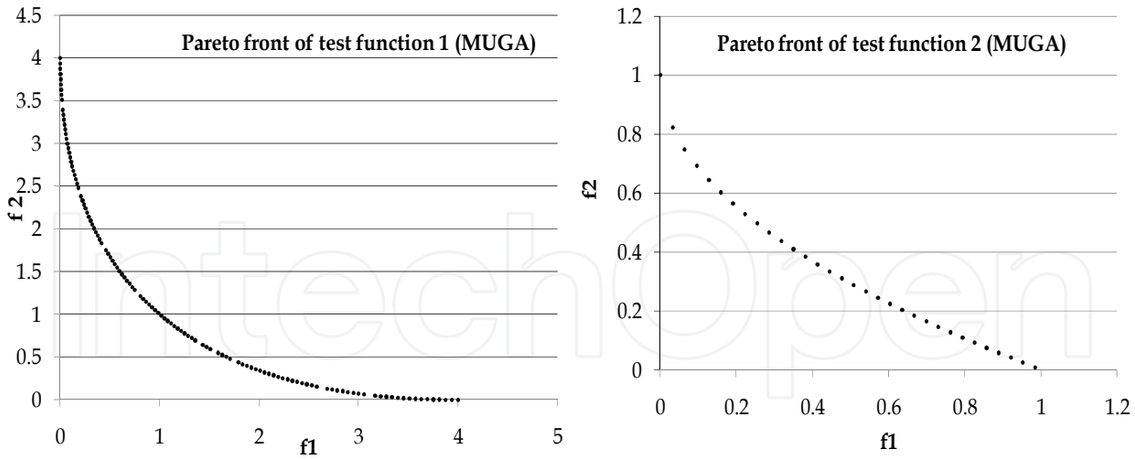
$$2\text{-}\begin{cases} f_1(x) = x_1 \\ f_2(x) = g(x)\left[1 - \sqrt{x_i / g(x)}\right] \\ g(x) = 1 + 9\left(\sum_{i=2}^{n} x_i\right) \Big/ (n-1) \end{cases} \qquad x \in [0,1], n = 30$$

$$3\text{-}\begin{cases} f_1(x) = x_1 \\ f_2(x) = g(x)\left[1 - \left(x_i / g(x)\right)^2\right] \\ g(x) = 1 + 9\left(\sum_{i=2}^{n} x_i\right) \Big/ (n-1) \end{cases} \qquad x \in [0,1], n = 30$$

$$4\text{-}\begin{cases} f_1(x) = \sum_{i=1}^{n-1}\left(-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right) \\ f_2(x) = \sum_{i=1}^{n}\left(|x_i|^{0.8} + 5\sin x_i^3\right) \end{cases} \qquad x \in [-5,5], n = 3$$

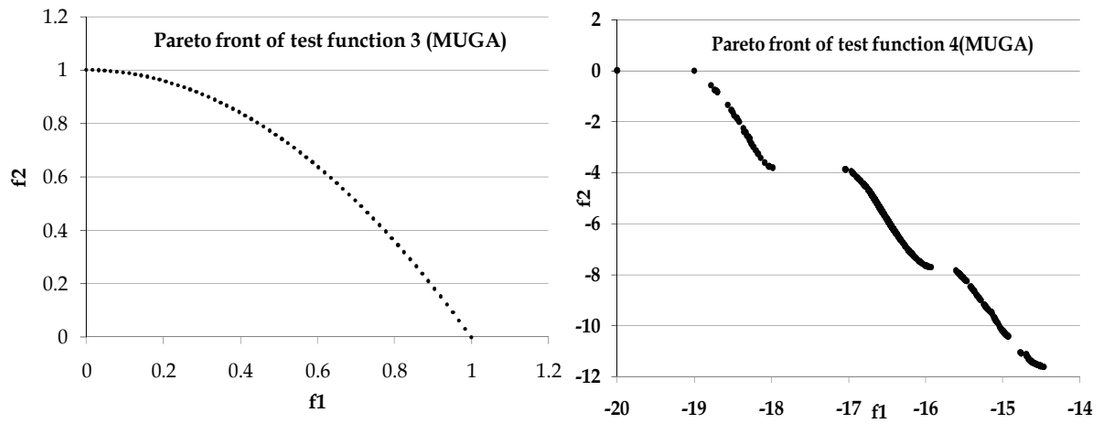
Figure 3 depicts the Pareto fronts obtained for test functions 1 and 2 using MUGA. Figure 4 depicts the same for test functions 3 and 4. The uniformity of the well spread-out of the non-dominated solutions is evident from these figures.

In order to compare the uniformity of the results of this work (MUGA) with those of NSGA-II, Table 1 shows the means and variances of metric Δ of both methods for multiple runs (Deb et al., 2002).



(a) (b)

Fig. 3. Pareto fronts obtained by MUGA: (a) Test function 1 (b):Test function 2



(a) (b)

Fig. 4. Pareto fronts obtained by MUGA: (a) Test function 3 (b):Test function 4

Methods	Test function 1	Test function 2	Test function 3	Test function 4
NSGA-II	0.449265	0.463292	0.435112	0.442195
	0.002062	0.041622	0.024607	0.001498
SPEA	1.021110	0.784525	0.755148	0.852490
	0.004372	0.004440	0.004521	0.002619
PAES	1.063288	1.229794	1.165942	1.079838
	0.002868	0.004839	0.007682	0.013772
MUGA (this work)	0.162595	0.273347	0.225211	0.402798
	2.9E-06	0.000261	2.1E-07	0.0

Table 1. Comparison of mean and variance of metric Δ of different methods (Deb et al., 2002) with those of MUGA (shaded rows are mean values and un-shaded rows are variances)

It is very evident from Table 1 that the performance of MUGA is better than that of other methods in achieving lower Δ in obtaining more uniform non-dominated solutions for these test functions. Further, the very small value of variances of that metric obtained in multiple runs simply demonstrates the robustness of finding uniform Pareto fronts in MOPs using MUGA.

5. Multi-objective thermodynamic optimization of turbojet engines with two design variables

Turbojet engines use air as the working fluid and produce thrust based on the variation of kinetic energy of burnt gases after combustion. The study of the thermodynamic cycle of a turbojet engine involves different thermo-mechanical aspects such as specific thrust, thermal and propulsive efficiencies, and thrust-specific fuel consumption (Atashkari, et al., 2005). A detailed description of the thermodynamic analysis and equations (Mattingly, 1996) of ideal turbojet engines is given in Appendix A (Atashkari, et al., 2005). This elementary thermodynamic model is sufficient to capture the principles of behaviour and interactions among different input and output parameters in a multi-objective optimal sense. Furthermore, this provides a suitable real-world engineering benchmark for comparing purpose between MOEA using the new diversity preserving mechanism of this work.

The input parameters involved in such thermodynamic analysis in an ideal turbojet engine given in Appendix A are flight Mach number (M_0), input air temperature (T_0, K), specific heat ratio (γ), heating value of fuel (h_{pr}, kJ/kg), exit burner total temperature (T_{t4}, K), and pressure ratio, π_c. The output parameters involved in the thermodynamic analysis in the ideal turbojet engine given in Appendix A are, specific thrust, (ST, N/kg/s), fuel-to-air ratio (f), thrust-specific fuel consumption ($TSFC$, kg/s/N), thermal efficiency (η_t), and propulsive efficiency (η_p). However, in the multi-objective optimization study, some input parameters are already known or assumed as, T_0 = 216.6 K, γ =1.4, h_{pr} =48000 kJ/kg, and T_{t4} = 1666 K. The input flight Mach number $0 < M_0 \leq 1$ and the compressor pressure ratio $1 \leq \pi_c \leq 40$ are considered as design variables to be optimally found based on multi-objective optimization of 4 output parameters, namely, ST, $TSFC$, η_t, and η_p.

5.2 Two-objective thermodynamic optimization of turbojet engines

In order to investigate the optimal thermodynamic behaviour of subsonic turbojet engines, 5 different sets, each including two objectives of the output parameters, are considered individually. Such pairs of objectives to be optimized separately have been chosen as (η_p, $TSFCx10^5$), (η_p, ST), (η_t, $TSFCx10^5$), (η_t, ST), and (η_p, η_t). Evidently, it can be observed that η_p, η_t, and ST are maximized whilst $TSFC$ is minimized in those sets of objective functions.

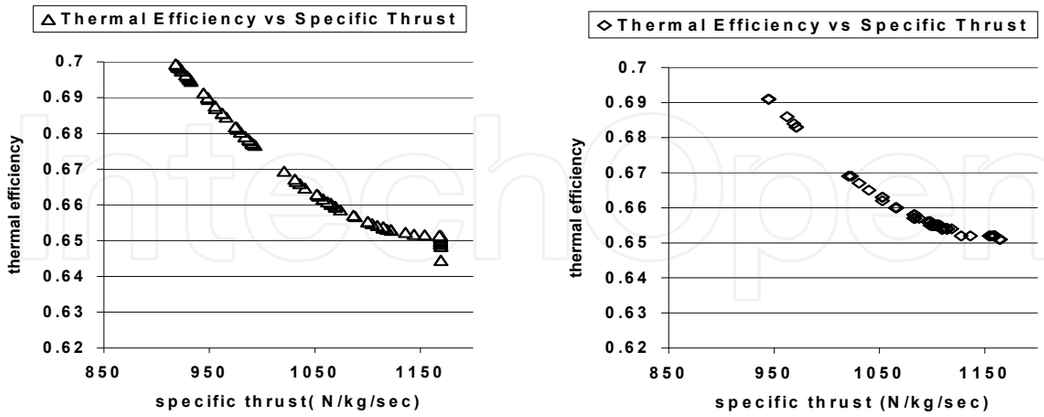
A population size of 100 has been chosen in all runs with crossover probability P_c and mutation probability P_m as 0.8 and 0.1, respectively.

The results of the two-objective optimizations considering those 5 different combinations of objectives are summarized in Table 2. Some Pareto fronts of each pair of two objectives have been shown through figures (5-6) using both the approach of this work and that of NSGA-II.

	Pairs of objectives in two-objective optimizations						
	(ηp, *TSFC*)		**(ηt, *TSFC*)**	**(ηp, *ST*)**		**(ηt, *ST*)**	**(η_p, η_t)**
	$0 < \eta p \leq 0.39$	$0.4 < \eta p \leq 0.55$	$.65 \leq \eta t \leq 0.7$	$0.41 < \eta p \leq 0.5$	$0 < \eta p\ 0.39$	$0.64 \leq \eta t\ 0.7$	$0.4 \leq \eta p \leq 0.56$
	$2.1 \leq TSFC \leq 2.43$	$3.16 \leq TSFC \leq 6.8$	$2.1 \leq TSFC \leq 2.43$	$515 \leq ST \leq 817$	$906 \leq ST \leq 1169$	$890 \leq ST \leq 1169$	$0.16 \leq \eta t \leq 0.55$
M_0	$0 < M_o \leq 1$	1	$0 < M_o \leq 1$	$0.85 \leq M_o \leq 1$	$0 < M_o \leq 1$	$0 < M_o \leq 1$	1
π_c	$\pi c = 40$	$1.0 \leq \pi c \leq 8.25$	$\pi c = 40$	$1.2 \leq \pi c \leq 4.28$	$13.5 \leq \pi c \leq 39.3$	$37.3 \leq \pi c \leq 40$	$1 \leq \pi c \leq 8.78$

Table 2. Values of decision variables and objective functions in various two-objective optimizations (Atashkari, et al., 2005)

It is clear from these figures that choosing appropriate values for the decision variables, namely flight Mach number (M_0) and pressure ratio (π_c), to obtain a better value of one objective would normally cause a worse value of another objective. However, if the set of decision variables is selected based on each of a Pareto front, it will lead to the best possible combination of that pair of objectives. In other words, if any other pair of decision variables M_0 and π_c is chosen, the corresponding values of the particular pair of objectives will locate a point inferior to that Pareto front. The inferior area in the space of objective functions (plane in these cases) for figures (5-6) are in fact bottom/left sides. A better diversity of results obtained using the approach of this work than those of NSGA-II can also observed in these figures. Evidently, figures (5-6) reveal some important and interesting optimal relationships among the thermodynamic parameters in the ideal thermodynamic cycle of turbojet engines that may not have been found without a multi-objective optimization approach. Such relationships have been called a worthwhile task for a designer by Deb in (Deb, 2003). These figures and the associated values of the decision variables and the objective functions given in Table 1 simply covers all the 4 objectives studied in the two-objective Pareto optimization.

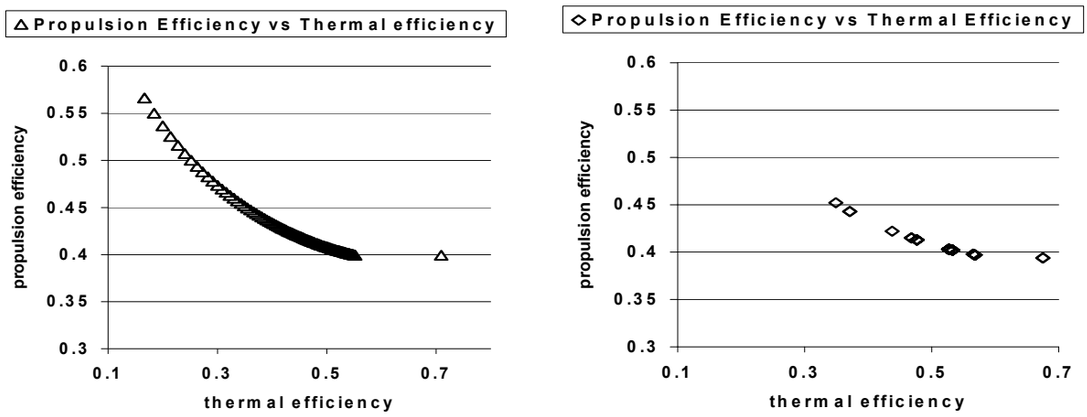


(a) (b)

Fig. 5. Pareto front of thermal efficiency and specific thrust in 2-objective optimization: (a) MUGA (b) NSGA-II

However, other pairs of objective functions in the two-objective Pareto optimization together with their associated values of the decision variables have been shown in Table 1. A

careful investigation of these Pareto optimization results reveals some interesting and informative design aspects. It can be observed that a small value of pressure ratio (π_c <8.7) is required in large value of Mach number (0.85<M_0 <1) when high value of η_p is important to the designer (0.4 <η_p <0.55). In this case both *ST* and *TSFC* get their worse values (*ST* becomes smaller and *TSFC* becomes larger), whilst η_t varies between small and medium values (0.16<η_t<0.55) depending on the value of flight Mach number. However, with high value of pressure ratio (37<π_c<40) in a wide range of flight Mach number (0<M_0 <1), *TSFC*, *ST*, and η_t improve whilst η_p cannot be better than 0.4. The specific values of these objectives depend on the exact value of flight Mach number which have been given in Table 1. However, such important and worthwhile information can be simply discovered using a four-objective Pareto optimization, which will be presented in the next section.



(a) (b)

Fig. 6. Pareto front of propulsive efficiency and thermal efficiency in 2-objective optimization: (a) MUGA (b) NSGA-II

Moreover, figures (5-6) also reveal some important and interesting optimal relationships of such objective functions in ideal thermodynamic cycle of turbojet engines that may have not been known without a multi-objective optimization approach. For example, figure (3) demonstrates that the optimal behaviours of η_t with respect to *ST* can be readily represented by

$$\eta_t \propto (ST)^2 \tag{4}$$

Figure (4) represents a non-linear optimal relationship of η_t and η_p in the form of

$$\eta_t \propto (\eta_p)^2 \tag{5}$$

It should be noted that these relationships, which have been obtained from the two-objective Pareto optimization results, are valid when the corresponding two-objective optimization of such functions is of importance to the designer and, in fact, demonstrates the optimal compromise of such pairs of objectives.

5.3 Four-objective thermodynamic optimization of turbojet engines

A multi-objective thermodynamic optimization including all four objectives simultaneously can offer more choices for a designer. Moreover, such 4-objective optimization can subsume

all the 2-objective optimization results presented in the previous section. Therefore, in this section, four objectives, namely, *TSFC*, *ST*, η_p, and η_t, are chosen for multi-objective optimization in which *ST*, η_p, and η_t are maximized whilst *TSFC* is minimized simultaneously. A population size of 200 has been chosen with crossover probability P_c and mutation probability P_m as 0.8 and 0.02, respectively.

Figure (7) depicts the non-dominated individuals in both 4-objective and previously obtained 2-objective optimization in the plane of (η_t-*ST*). Such non-dominated individuals in both 4 and 2-objective optimization have alternatively been shown in the plane of (η_p-η_t) in figure (8). It should be noted that there is a single set of individuals as a result of 4-objective optimization of *TSFC*, *ST*, η_p, and η_t that are shown in different planes together with the corresponding 2-objective optimization results. Therefore, there are some points in each plane that may dominate others in the same plane in the case of 4-objective optimization. However, these individuals are all non-dominated when considering all four objectives simultaneously. By careful investigation of the results of 4-objective optimization in each plane, the Pareto fronts of the corresponding two-objective optimization can now be observed in these figures. It can be readily observed that the results of such 4-objective optimization include the Pareto fronts of each 2-objective optimization and provide, therefore, more optimal choices for the designer.

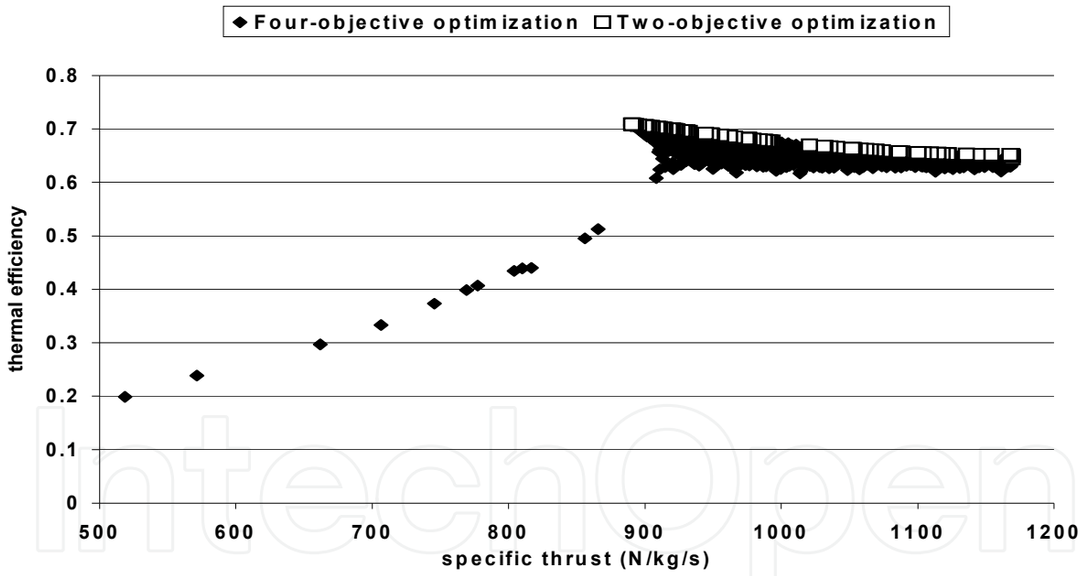


Fig. 7. Thermal efficiency variation with specific thrust in both 4-objective & 2-objective optimisation

The non-dominated individuals obtained in 4-objective optimization demonstrate some interesting behaviours in terms of design variables. Two different parts can be easily observed in figures (7-8). One of these parts which is less populated corresponds to high value of pressure ratio ($0.4<\eta_p<0.55$), unlike the rest of objective functions which all together degrades in their values simultaneously, that is, $3<TSFC\text{x}10^5<6.3$, $515<ST<890$, $0.2<\eta_t<0.52$. The corresponding values of objectives for the second part can be given as, $0<\eta_p<0.4$, $2<TSFC\text{x}10^5<3$, $900<ST<1169$, $0.6<\eta_t<0.71$ which can be appropriately chosen by the

designer. Such facts would be very important to the designer to switch from one optimal solution to another for achieving different trade-off requirements of the objectives (Deb, 2003).

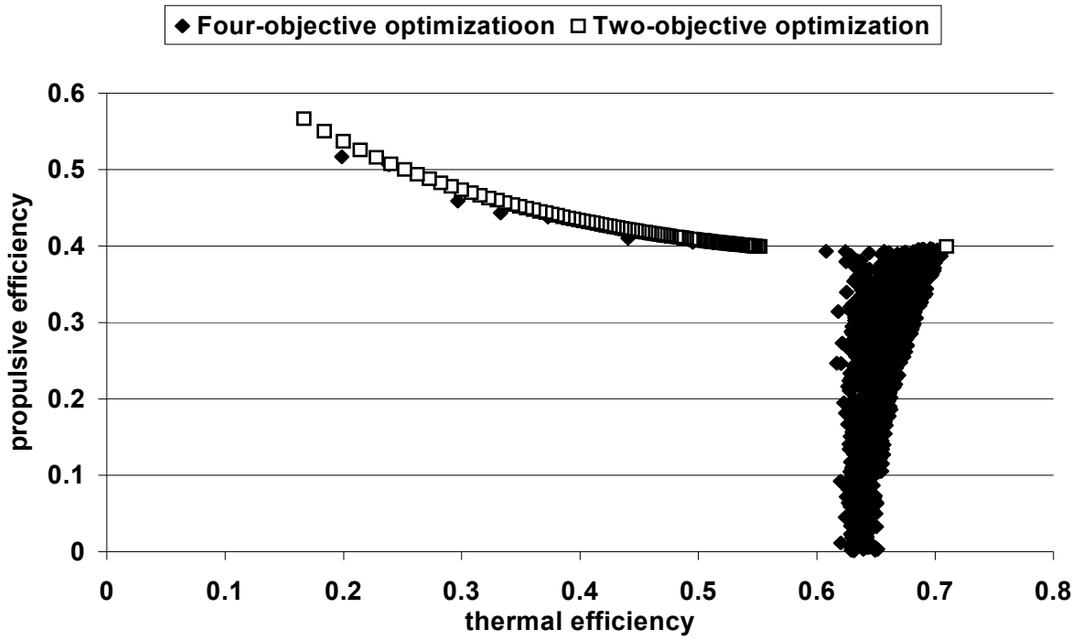


Fig. 8. Propulsive efficiency variation with thermal efficiency in both 4-objective & 2-objective optimization

Additionally, there are some more profound optimal design relationships among the objective functions and the decision variables which have been discovered by the four-objective thermodynamic Pareto optimization of ideal turbojet engines. Such important optimal design facts could not have been found without the multi-objective Pareto optimization. Firstly, figure (9) shows the variation of 4 optimized objective functions *ST*, *TSFC*, η_p, and η_t with the pressure ratio. It can be seen that for pressure ratio less than 14, three objectives *ST*, *TSFC*, and η_t become worse, unlike η_p which gradually starts getting better. The slope of such degradation for *ST*, *TSFC*, and η_t becomes faster especially in *TSFC* and η_t when the pressure ratio becomes smaller than 6. However, for high pressure ratios, the variation of optimal values of *TSFC* and η_t are small whilst there are a wide range of selections for $\eta_p \approx 0.4$. Secondly, figure (10) demonstrates the behaviours of *ST* and η_p with respect to flight Mach number in high pressure ratios. It can be readily seen that the optimal values of *ST* changes linearly with M_0, that is

$$ST = -264.75\, M_0 + 1164.5 \tag{7}$$

with a R-squared value of 0.999. The optimal relationship of η_p with M_0 is non-linear and is represented as

$$\eta_p = -0.0977\, (M_0)^2 + 0.491\, M_0 + 0.0013 \tag{8}$$

with a R-squared value of 0.998.

Therefore, such multi-objective optimization of *ST*, *TSFC*, η_p, and η_t provide optimal choices of design variables based on Pareto non-dominated points.

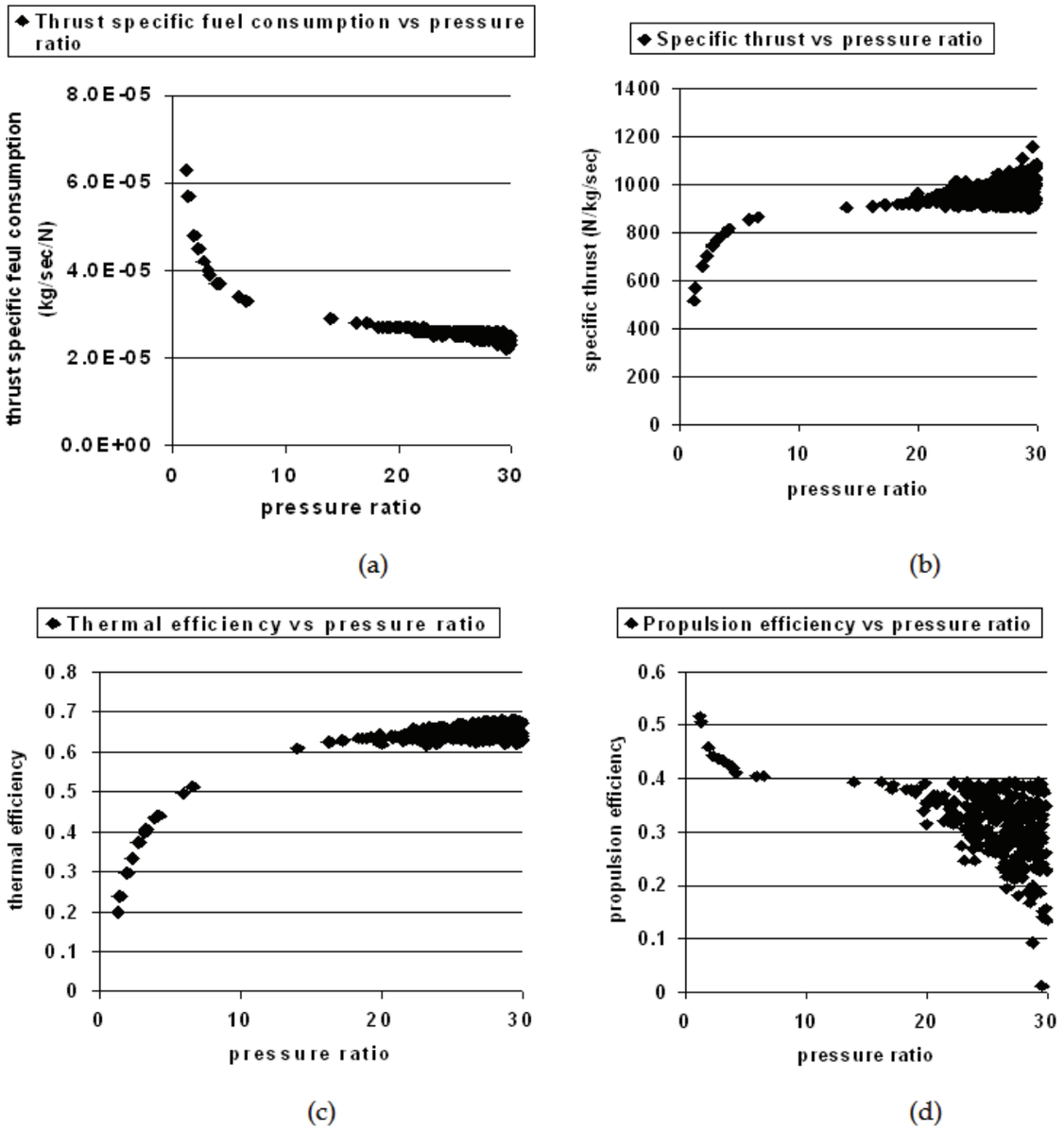


Fig. 9. Variation of four objective functions with pressure ratio in 4-objective optimization

6. Conclusion

A new multi-objective uniform-diversity genetic algorithm (MUGA) has been proposed and successfully used for some test functions and for thermodynamic cycle optimization of ideal turbojet engines. It has been shown that the performance of this algorithm is superior to that

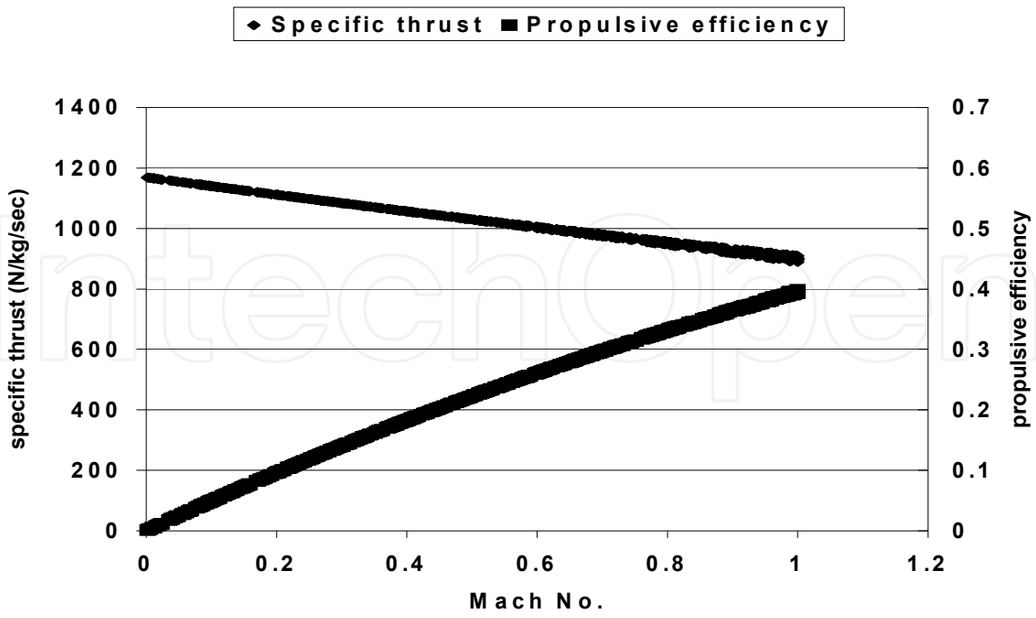


Fig. 10. Relationships of specific thrust & propulsive efficiency with flight Mach No. in 4-objective optimization (Atashkari, et al., 2005)

of NSGA-II in terms of diversity and the uniformity of Pareto front obtained for both 2-objective and 4-objective optimization processes. The robustness of uniform Pareto fronts obtained using MUGA has been shown by the very small values of variance of the metric Δ in multiple runs in comparisons with that of other methods. Further, such multi-objective optimization led to the discovering of important relationships and useful optimal design principles in thermodynamic optimization of ideal turbojet engines both in the space of objective functions and decision variables. The evolutionary multi-objective optimization process has helped to discover important relationships with relatively few efforts of modeling preparation that would otherwise have required at least a very through mathematical analysis. If the underlying objective modeling becomes more complex (like deviating from the ideality of components behaviour) evolutionary multi-objective optimization process may even be expected to become the sole present-time means of attaining respective solutions.

Appendix A

Thermodynamic model of ideal turbojet engine

Assumptions: Inlet diffuser, compressor, turbine and exit nozzle, all operate isentropically. No pressure loss in the burner.

f =(fuel/air)<<1, P_e (turbojet exit pressure)= P_0 (ambient pressure), C_P =1.004 (kJ/kg.K)

$T_0 = 216.6$ K, $\gamma = 1.4$, $h_{PR} = 48000 \frac{kJ}{kg}$, $T_{t4} = 1666$ K (in 2 design variables), g_c=1 (kg-m/(N-s^2))

Input parameters:

$$M_0, T_0(K), \gamma, c_P(\frac{kJ}{kg.K}), h_{PR}(\frac{kJ}{kg}), T_{t4}(K), \pi_c$$

Output parameters:

$$ST = \frac{F}{\dot{m}_0}(\frac{N}{kg/s}), SFC(\frac{kg/s}{N}), \eta_t, \eta_P$$

Equations:

$$R = \frac{\gamma - 1}{\gamma}.c_p \quad \text{(A-1)}$$

$$a_0 = \sqrt{\gamma R g_c T_0} \quad \text{(A-2)}$$

$$\tau_r = 1 + \frac{\gamma - 1}{2} M_0^2 \quad \text{(A-3)}$$

$$\tau_\lambda = \frac{T_{t4}}{T_0} \quad \text{(A-4)}$$

$$\tau_c = (\pi_c)^{(\gamma - 1)/\gamma} \quad \text{(A-5)}$$

$$\tau_t = 1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1) \quad \text{(A-6)}$$

$$\frac{V_9}{a_0} = \sqrt{\frac{2}{\gamma - 1}\frac{\tau_\lambda}{\tau_r \tau_c}(\tau_r \tau_c \tau_t - 1)} \quad \text{(A-7)}$$

$$ST = \frac{F}{\dot{m}_0} = \frac{a_0}{g_c}\left(\frac{V_9}{a_0} - M_0\right) \quad \text{(A-8)}$$

$$f = \frac{c_P T_0}{h_{PR}}(\tau_\lambda - \tau_r \tau_c) \quad \text{(A-9)}$$

$$SFC = \frac{f}{ST} \quad \text{(A-10)}$$

$$\eta_t = 1 - \frac{1}{\tau_r \tau_c} \quad \text{(A-11)}$$

$$\eta_p = \frac{2M_0}{V_9/a_0 + M_0} \quad \text{(A-12)}$$

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