Chapter 7

Radiation and Propagation of Waves in Magnetic Materials with Helicoidal Magnetic Structure

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Additional information is available at the end of the chapter

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Abstract

In this chapter, we are shortly reviewing some problems of electromagnetic and acoustic wave propagation and radiation in the magnets with helicoidal spin structure. We show the band structure of the coupled wave spectrum in the materials. The band gap width depends on the spiral angle (or, equivalently, on external magnetic field value). Interaction of spin and electromagnetic waves leads to opening the gap in spin-electromagnetic dispersion. This gap leads to opacity window in reflection spectrum of spiral magnet plate. The opacity window closes at phase transition into collinear ferromagnetic state and reaches a maximum at simple spiral state. At the frequencies near band gap boundaries, the rotation of polarization plane of propagating electromagnetic wave is observed. Account of interaction of spin and electromagnetic waves with acoustic subsystem leads to opening the gap in spin-acoustic spectrum. This gap leads to some features in electromagnetic reflectance spectrum and to rotation of acoustic wave polarization plane, i.e. to acoustic Faraday effect. We also show the possibility of acoustic and electromagnetic wave radiation by helicoidal magnets at phase transition into collinear ferromagnetic state. Some features of electromagnetic waves generation by spiral magnets placed in homogeneous magnetic field with harmonical time-dependence are also discussed.

Keywords: spiral magnets, helicoidal magnetic structure, electromagnetic waves, spin waves, acoustic waves, Faraday effect, acoustic Faraday effect

1. Introduction

The magnets with spiral (helicoidal, or modulated) magnetic structure have been under investigation for a long time [1, 2]. These materials include the rare-earth metals, compounds based on them, and some of the compounds based on transition metals. Such materials may be semiconductors, as well as dielectrics. The reasons of such ordering may be the following: competing exchange interaction of atoms in the first and the second coordination spheres, asymmetric exchange, relativistic interaction (Dzyaloshinskii-Moriya interaction). The main characteristic of modulated structures is a modulation wave number $q = 2\pi/L$, where L is the modulation period. The modulation period of such structures is much greater than the lattice constant of the crystal. Modulation period of helicoidal magnets usually changes continuously with the change of temperature. It may be incommensurable with the crystal's lattice constants. This is the reason why modulated, or long-periodic, magnetic structures are frequently called "incommensurable" ones.

Modulated magnetic structure leads to sufficient features of dynamical properties of helicoidal magnets comparing with the properties of collinear ferro- and anti-ferromagnets. Thus, the spectrum of spin waves has a band structure. It has no activation frequency when wave number of propagating wave is equal to the modulation wave number [2], i.e. $k = q$, while in collinear ferro-and anti-ferromagnets this situation is observed at $k = 0$. Magnetoelastic waves in ferromagnets with spiral structure have been studied in details, for example, in works [3–8]. Works [4–6] are devoted to investigation of spin and acoustic oscillations interaction in spiral magnets without taking into account the spontaneous deformations in the ground state. These deformations were consistently considered in work [8]. Interaction of spin and electromagnetic waves in magnetic dielectrics with collinear magnetic structure has been investigated in [9]. Peculiarities of spin waves and helicons hybridization have been considered in [10]. The result of this work has been generalized for the case of arbitrary propagation direction in [11]. Later, coupled spin-Alfven waves in ferromagnetic metals [12], and coupled plasmon-spin-electromagnetic waves in ferromagnetic semiconductors and metals with different anisotropy types [13] have been studied. These investigations show that interaction of the spin subsystem with the electromagnetic waves usually leads to the shift of activation frequency of quasi-spin branch: an additional term of electromagnetic nature, namely magnetostatic frequency, appears. This interaction may also lead to reduce the phase speed of electromagnetic wave.

Modulated magnetic structure may manifest itself in the modulation of electromagnetic characteristics of the magnet. Phenomenological approach has been successfully applied to investigate the magneto optical effects in the medium with the simple spiral structure [14]. Some features of light propagation have been predicted in works [15, 16]. Authors of these works have not taken into account the dynamics of magnetic subsystem. Such approach is valid for optical frequencies, when magnetic subsystem does not keep pace with changes of field's magnetic component. As for investigation of electrodynamical processes at lower frequencies, taking into account dynamics of magnetic subsystem may lead to qualitatively new effects.

Coupled spin-electromagnetic and spin-acoustic-electromagnetic waves have been investigated in detail for magnets with the simple spiral structure in [17, 18]. Wave hybridization in such magnets leads to opening the gaps in spectrums of electromagnetic waves, as well as in acoustic ones. Due to the band structure of the spectrum, some features in electromagnetic and acoustic reflectance may be observed. All these features should have a place in magnets with conical spiral ordering as well.

A special kind of dynamic effects in materials with a modulated magnetic structure can occur during phase transitions, accompanied by appearance/disappearance of the modulated structure. Similar effects are well known for periodically distributed sources of different nature and they usually consist of different wave radiation. Thus, nanostructured metallic films may be sources of radiation when they are irradiated by laser pulses [19, 20]. Theoretical model for describing this effect has been proposed in [21]. Terahertz wave radiation may occur when magnons are excited by femtosecond laser pulses in anti-ferromagnets [22], or by infrared pulses in diluted magnetic semiconductors [23]. In helicoidal magnets, the spin structure is reorganized during phase transition. This may also lead to radiation of electromagnetic waves and sound. Recently [24], we have observed experimentally electromagnetic waves radiation by Heusler alloy during magneto-structural phase transition, possible mechanisms and theory of this effect have been discussed as well. We have supposed that the main contribution is caused by moving dislocations with dipole moment. Here, we show that in spiral magnets during orientational magnetic phase transition, electromagnetic radiation may be comparable to and even be more than in Heusler alloys.

Nowadays, scientific interest on the investigation of helicoidal magnets is caused by the fact that in such magnets, spontaneous polarization may appear frequently due to specific magnetic ordering. Such materials, combining both electric and magnetic properties, are called multiferroics. First, in 2003 such effect was observed in $TbMnO_3$ [25], then in 2004 in $TbMn_2O_5$ [26], and many other materials. Experiments clearly show the coupling between spontaneous polarization and modulated magnetic ordering. Phenomenologically, coupling of polarization and non-uniform magnetic ordering may be described by energy term of third order $PM\partial M$, which is allowed by symmetry. This coupling is linear according to polarization. Thus, even weak interaction of such type will lead to electric polarization at corresponding magnetic arrangement. It this chapter we will not consider multiferroic materials, but qualitatively all results are valid even in case of multiferroics. Details of peculiarities of wave interaction in helicoidal multiferroics may be found, for example, in our previous works [27–29].

This chapter consists of introduction, two sections and conclusions. In the first section, we review some peculiarities of electromagnetic and acoustic wave propagation in helical magnets. The results cover the coupled wave spectrum, electromagnetic reflection from half-space and plate of magnet with the spiral magnetic structure, Faraday rotation, and acoustic Faraday rotation (i.e. rotation of polarization plane of acoustic wave). In the second section, we consider the possibility of electromagnetic and acoustic wave radiation in spiral magnets during the phase transition. An infinitely fast phase transition and the linear change of the transverse magnetization component are considered as model processes.

2. Influence of electromagnetic-spin-acoustic interaction on wave propagation

This section is mainly based on results of our previous investigations [30–33].

Let us consider the magnet with spiral magnetisation distribution at the ground state. Such magnetic order may be realized if free energy of the crystal has the following form:

$$F = \frac{\alpha}{2}\left(\frac{\partial M_i}{\partial x_i}\right)^2 + \frac{\beta_1}{2}M_z^2 + \frac{\beta_2}{2}M_z^4 - M_z H + b_{ijlm}M_i M_j u_{lm} + c_{ijlm}u_{ij}u_{lm} + F_{in}, \tag{1}$$

where $\mathbf{M} = (M_x, M_y, M_z)$ is the magnetization of the crystal, α is the exchange constant, β_1 and β_2 are anisotropy constants, b_{ijlm} are magnetostriction constants, c_{ijlk} are elasticity constants, i, j, l, $k = x$, y, z. The sum is assumed where indexes are repeated. The term F_{in} corresponds to inhomogeneous magnetization distribution at the ground state. It may have an exchange nature (such structures we will call "exchange spiral"): $F_{in} = \gamma\left(\partial^2 M_i/\partial x_i^2\right)/2$. Another reason may be relativistic interaction (such situation we will call "relativistic spiral"): $F_{in} = \alpha_1 \mathbf{M} rot \mathbf{M}$. In (1) we suppose that external magnetic field is directed along z-axis, which corresponds to anisotropy axis.

For concreteness, we will assume that crystal with exchange spiral structure is from hexagonal crystal system (D_{3d}, C_{3v},D_6, D_{6h}). Magnets with relativistic spiral structure will be assumed as isotropic ones. Their elastic properties may be defined by two characteristics (Lamé constants λ and μ), and magnetostriction will be characterized by one "effective" constant b.

Spiral magnetization distribution may be described by the following coordinate dependence of magnetization components:

$$M_x = M_0 \sin\theta \cos qz,\ M_y = M_0 \sin\theta \sin qz,\ M_z = M_0 \cos\theta. \tag{2}$$

In (2), $q = 2\pi/L$ is the spiral wave number, L is the spiral period, θ is the spiral angle, M_0 is the magnetization amplitude. In case of $\theta = 0$, one will have a ferromagnetic ground state, while $\theta = \pi/2$ corresponds to simple spiral structure. When $0 < \theta < \pi/2$, the "ferromagnetic spiral" phase is realized.

Minimizing the free energy (1) with appropriate expression for F_{in}, taking into account ground state magnetization distribution of spiral type (2), we will get the conditions of the phase stability. For crystals with exchange spiral, these conditions are the following: $\gamma > 0$, $\alpha < 0$, $q = (-\alpha/2\gamma)^{1/2}$. For relativistic spiral, one will have $\alpha_1 \neq 0$, $\alpha > 0$, $q = \alpha_1/\alpha$. In both cases an external magnetic field is coupled with the spiral angle:

$$M_0 \cos\Theta\left[\tilde{\beta}_1 + h_{me} + \left(\tilde{\beta}_2 - h_{me}/M_0^2\right)M_0^2\cos^2\Theta + \alpha q^2 + \tilde{\Delta}\right] + H = 0,$$

where β_1 and β_2 are anisotropy constants renormalized by magnetostriction [28], and $h_{me}=(b_{11}-b_{12})^2 M_0^2/(c_{11}-c_{12})$, $\tilde{\Delta}=\gamma q^4$ for magnets with exchange spiral structure, while $h_{me}=b^2 M_0^2/2\mu$, $\tilde{\Delta}=-2\alpha_1 q$ for relativistic ones.

There is a critical magnetic field value H_{cr}, when the phase transition into collinear ferromagnetic state occurs. Diagonal components of the equilibrium stress tensor are homogeneous and depend on the spiral angle, while the components u_{xz} and u_{yz} depend on both spiral angle and z-coordinate, $u_{xy}=0$.

To obtain the spectrum of coupled spin, acoustic and electromagnetic waves, one should use the Landau-Lifshitz and Maxwell equations with equation of motion of the elastic medium. Simplifying such system of equations by small oscillations method, linearizing and taking the circular components, one will obtain the following:

$$\begin{aligned}
&\pm\omega m_\pm(k) = \cos\theta\left[\omega_{2k}^\pm + \frac{1}{2}\omega_{me4}\sin^2\theta\right]m_\pm(k) + \frac{1}{2}\omega_{me4}\sin^2\theta\cos\theta m_\pm(k\mp 2q) - \\
&-\omega_{1k\pm q}\sin\theta m_z(k\mp q) + igb_{44}M_0^2 k\left[\frac{1}{2} - \frac{3}{2}\sin^2\theta\right]u_\pm(k) - \\
&-\frac{i}{2}gM_0{}^2 b_{44}\sin^2\theta(k\pm 2q)u_\pm(k\mp 2q) - ig(b_{33}-b_{31})M_0^2\sin 2\theta(k\pm q)u_z(k\mp q) + \\
&+gM_0\sin\theta h_z(k\mp q) - gM_0\cos\theta h_\pm(k),
\end{aligned}$$

$$\begin{aligned}
&\omega m_z(k) = \frac{1}{2}\sin\theta\left[\omega_{2k-q}^- m_-(k-q) - \omega_{2k+q}^+ m_+(k+q)\right] - \\
&-\frac{i}{4}gb_{44}M_0^2\sin 2\theta\left[(k-q)u_-(k-q) - (k+q)u_+(k+q)\right] + \\
&+\frac{1}{2}gM_0\sin\theta\left[h_+(k+q) - h_-(k-q)\right],
\end{aligned}$$

$$\left[\omega^2 - s_t^2 k^2\right]u_\pm(k) = \frac{i}{\rho}kb_{44}M_0\left[\sin\theta m_z(k\mp q) + \cos\theta m_\pm(k)\right],$$

$$\left[\omega^2 - s_l^2 k^2\right]u_z(k) = -2i(b_{33}-b_{31})kM_0\cos\theta m_z(k)/\rho,$$

$$\left[\omega^2 - k^2 v^2\right]h_\pm(k) = -\omega^2 4\pi m_\pm(k), \quad h_z(k) = -4\pi m_z(k).$$

This system of equations should be complemented by the condition of magnetization vector permanence $|\mathbf{M}|=const$, which for Fourier components has a form:

$$\sin\theta\left[m_-\left(k-q\right)+m_+\left(k+q\right)\right]+2\cos\theta m_z\left(k\right)=0.$$

In all equations above we have used the following notations: $\mathbf{h}(k)$, $\mathbf{m}(k)$, $\mathbf{u}(k)$ are Fourier components of corresponding vector $\mathbf{h}$, $\mathbf{m}$, $\mathbf{u}$; $v=c/\sqrt{\varepsilon}$ is electromagnetic wave speed in the medium; ε is permittivity of the crystal; $\omega_{2k}^{\pm}=\omega_{20}+gM_0L_{\perp}^{\pm}(k)$, $\omega_{1k}=\omega_{10}+gM_0\sin^2\theta L_{\parallel}(k)=0$, $L_{\perp}^{\pm}(k)=-\alpha(q^2-k^2)-\gamma(q^4-k^4)+2\alpha_1(q\mp k)$, $L_{||}(k)=-\alpha(q^2-k^2)-\gamma(q^4-k^4)+2\alpha_1 q$, $s_t^2=c_{44}/\rho$, $s_l^2=c_{33}/\rho$, $\omega_{me4}=gM_0h_{me4}=gb_{44}^2M_0^3/c_{44}$, $\omega_{10}=gM_0[h_{me4}-\sin^2\theta(\beta_1+(\beta_2+2\beta_2)M_0^2\cos^2\theta+h_{me}\sin^2\theta)]$.

Dispersion equation may be obtained from this system of equations. Solution of dispersion equation gives the propagation constant k for the waves with opposite circular polarization. We should note that due to the symmetry of the problem, the propagation of the wave with fixed circular polarization along z-axis is similar to the propagation of the wave with opposite circular polarization in inverse direction. Detailed investigation shows that waves with opposite circular polarizations have different propagation constants. This situation is valid for electromagnetic waves, as well as for acoustic ones. The difference in propagation constants will lead to different speed of waves with opposite circular polarization, hence, the rotation of polarization plane of linearly polarized wave. Rotation angle may be calculated as $\Delta\varphi=(k_+-k_-)l/2$, where k_+ and k_- are propagation constants of the waves with corresponding circular polarization, and l is the distance, which the wave goes in the medium.

For investigation of electromagnetic wave reflection from the plate of magnet with spiral magnetic structure, one should add the boundary conditions: $H_{x,y}^{(out)}=H_{x,y}^{(in)}$, $E_{x,y}^{(out)}=E_{x,y}^{(in)}$, $D_z^{(out)}=D_z^{(in)}$, $\partial m_i/\partial z = 0$, and $\sigma_{jz} = 0$. Indexes (in) and (out) denote the fields inside and outside the magnet. We suppose that the outer medium is a vacuum. When one investigates the reflection of electromagnetic waves from semi-infinite structure, these boundary conditions should be applied to the single boundary.

For numerical estimations we will use following constant values [2]: $b_{ij}\sim 20$ erg/(Oe × cm^4), $\varrho\sim 10$ g/cm^3, $v_t\sim 3\times 10^5$ cm/s, $v_l\sim 5\times 10^5$ cm/s, $M_0\sim 500$ Oe. The period of the structure for relativistic interaction-caused spiral magnet is usually much larger that in case of exchange-caused spiral structures. For example, $Fe_xCo_{1-x}Si$ alloys, which symmetry allows Dzyaloshinskii-Moriya interaction, for $x = 0.3$ in spiral state has a modulation period $L = 230$ nm ($q\sim 3\times 10^5$ cm^{-1}) [34]. Other examples of the magnets with Dzyaloshinskii-Moriya interaction-caused spiral structures are FeGe ($L = 70$ nm, $q\sim 8\times 10^5$ cm^{-1}) [35], and MnSi ($L = 18$ nm, $q\sim 3\times 10^6$ cm^{-1}) [36]. Different modulated states exist in erbium single crystal due to the competing exchange interaction. In conical state, wave number of the structure is $5c^*/21$ ($c^* = 2\pi/c$ is inverse lattice constant, $c = 0.56$ nm is lattice constant) [37], i.e. $q\sim 3\times 10^7$ cm^{-1}. We will use $q\sim 10^5$ cm^{-1} and $q\sim 10^8$ cm^{-1}, consequently.

First of all, let us discuss the influence of spin-electromagnetic interaction on wave propagation. For such purposes, the problem may be simplified. We may neglect the elasticity and magnetostriction terms in free energy (1). In such case, dispersion equation may be calculated in form:

$$\left[2\omega^2\left(\omega^2-v^2k^2\right)+\left(\left(\omega^2-v^2k^2\right)\Omega_k^+-\omega^2\omega_M\right)\left(\Omega_{1M}\sin^2\theta-2\omega\cos\theta\right)\right]\times$$
$$\left[\begin{array}{l}2\omega^2\left(\omega^2-v^2\left(k-2q\right)^2\right)+\\+\left(\left(\omega^2-v^2\left(k-2q\right)^2\right)\Omega_{k-2q}^--\omega^2\omega_M\right)\left(\Omega_{1M}\sin^2\theta+2\omega\cos\theta\right)\end{array}\right]-$$
$$-\Omega_{1M}^2\sin^4\theta\left(\left(\omega^2-v^2k^2\right)\Omega_k^+-\omega^2\omega_M\right)\left(\left(\omega^2-v^2\left(k-2q\right)^2\right)\Omega_{k-2q}^--\omega^2\omega_M\right)=0.$$

Here, $\omega_M=4\pi gM_0$ is magnetostatic frequency, $\Omega_{1M}=\Omega_{1k-q}+\omega_M$, $\Omega_{k-2q}^-=gM_0\left(\alpha\left(q^2-(k-2q)^2\right)+\gamma\left(q^4-(k-2q)^4\right)-2\alpha_1(k-q)\right)$, $\Omega_k^+=gM_0\left(\alpha\left(q^2-k^2\right)+\gamma\left(q^4-k^4\right)+2\alpha_1(k-q)\right)$, $\Omega_{1k-q}=gM_0\left(\beta_1+3\beta_2M_0^2\cos^2\theta-\alpha\left(q^2-(k-q)^2\right)+\gamma\left(q^4-(k-q)^4\right)+2\alpha_1q\right)$. For relativistic spiral, one should put $\alpha_1=0$, while in exchange spiral case $\gamma=0$.

Results of calculations are shown in **Figure 1**.

Figure 1a shows the dispersion of coupled spin-electromagnetic waves in spiral magnets with different spiral angles (or external magnetic field values). One can see that the spectrum has a band structure. The band gap is observed near the point where non-interacting dispersion curves are crossed. This band gap appears due to the resonant interaction of spin and electromagnetic waves in a magnet. From **Figure 1a** we can see that with decrease of angle, the gap shifts toward lower frequencies and its width decreases. Calculations show that in case of exchange spiral, a zone of opacity is much narrower than in case of relativistic spiral, and lies at higher frequencies. It should be also noted that the magnitude of the interaction of spin and electromagnetic waves depends on the spiral angle.

Figure 1b shows the frequency dependence of the reflectance of electromagnetic wave from the plate of spiral magnet with width $l=100$ μm for different spiral angles. One can see that reflectance is near to unit at frequencies corresponding to the band gap of the spectrum of coupled waves, i.e. opacity window is observed. This window exists only for spiral states with $\theta\neq 0$, while it disappears in the collinear ferromagnetic state.

The results of calculating polarization plane rotation are shown in **Figure 1c** and **d**. One can see that rotation angle depends on external magnetic field almost linearly at frequencies upper and lower the band gap. The maximum values of rotation correspond to the band gap boundaries.

Let us now investigate the effect of acoustic subsystem on wave propagation in the magnet. For such purposes we should solve all equations for exact free energy expression (1). The details of this investigation may be found in work [33].

Calculations of the spectrum of coupled spin-electromagnetic-acoustic waves in spiral magnets show that despite the gap corresponding to spin-electromagnetic interaction, an additional gap corresponding to spin-acoustic interaction is opened. This gap reaches a maximal width at ferromagnetic spiral state with $\theta=\pi/4$. The spectrum of coupled waves near $k=0$ is shown in **Figure 2a**.

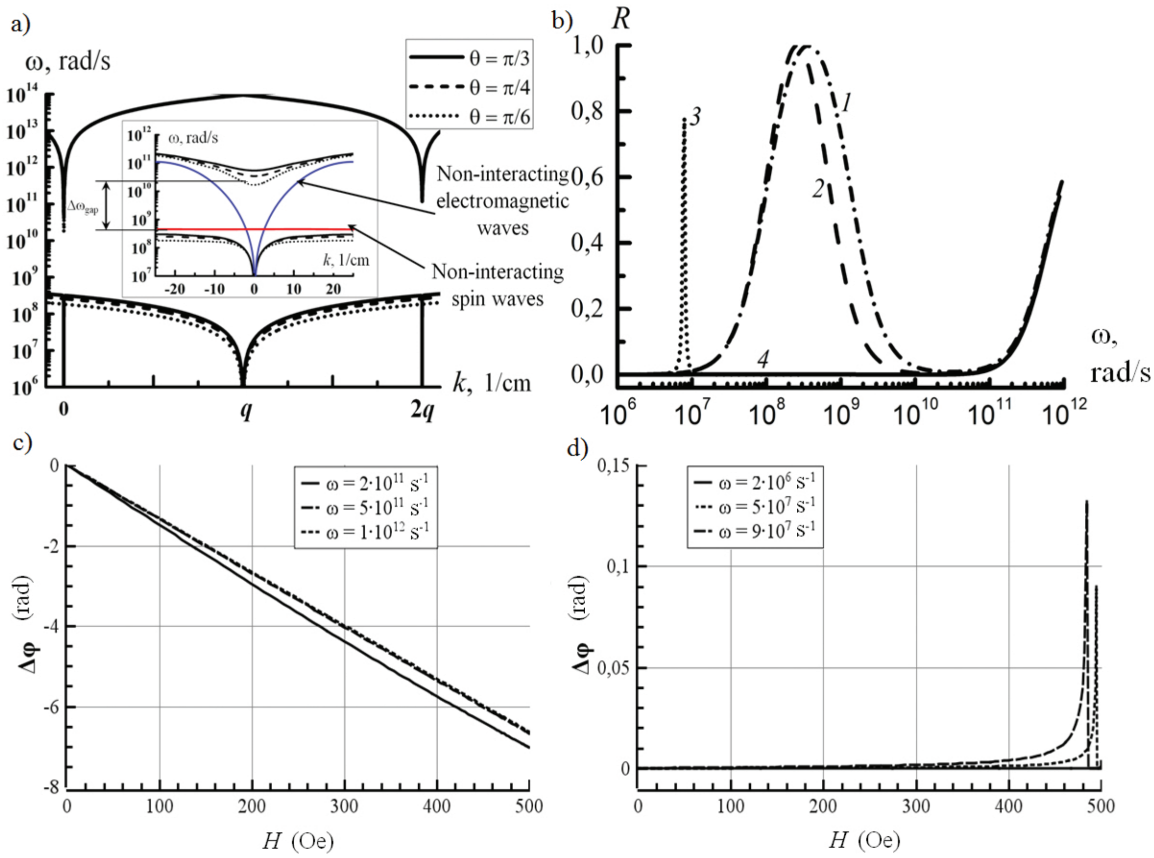

Figure 1. Dispersion of coupled spin-electromagnetic waves in spiral magnets with different spiral angles (a). On the inset the small k region is shown. Frequency dependence of the reflectance of electromagnetic wave from the plate of spiral magnet with width l = 100 μm for different spiral angles (b). Electromagnetic waves polarization plane rotation: near upper frequency of band gap (c), and near lower frequency of band gap (d).

Figure 2b shows results of investigation of electromagnetic waves reflection from semi-infinite spiral magnet. One can see that besides the opacity window corresponding to spin-electromagnetic interaction, there are some features at the frequencies of spin-acoustic interaction. This effect corresponds to resonant interaction of electromagnetic, spin and acoustic waves. The width of such acoustically caused electromagnetic opacity window depends on the spiral angle (or, equivalently, on external magnetic field value), as well.

The gap opening in spin-acoustic spectrum leads to the possibility of polarization plane rotation for acoustic waves, i.e. acoustic Faraday effect. The results of calculation of rotation angle at the sample distance of 1 cm for different frequencies are shown in **Figure 2c**. The breaks on the curves correspond to band gap in the spectrum, which moves with the change of the external magnetic field. One can see that maximal rotation of the polarization plane is observed near the band gap.

The results we have discussed in this section show that spiral magnets may be perspective materials for electromagnetic and acoustic wave manipulation (such as filtering, polarizing, and phase shifting).

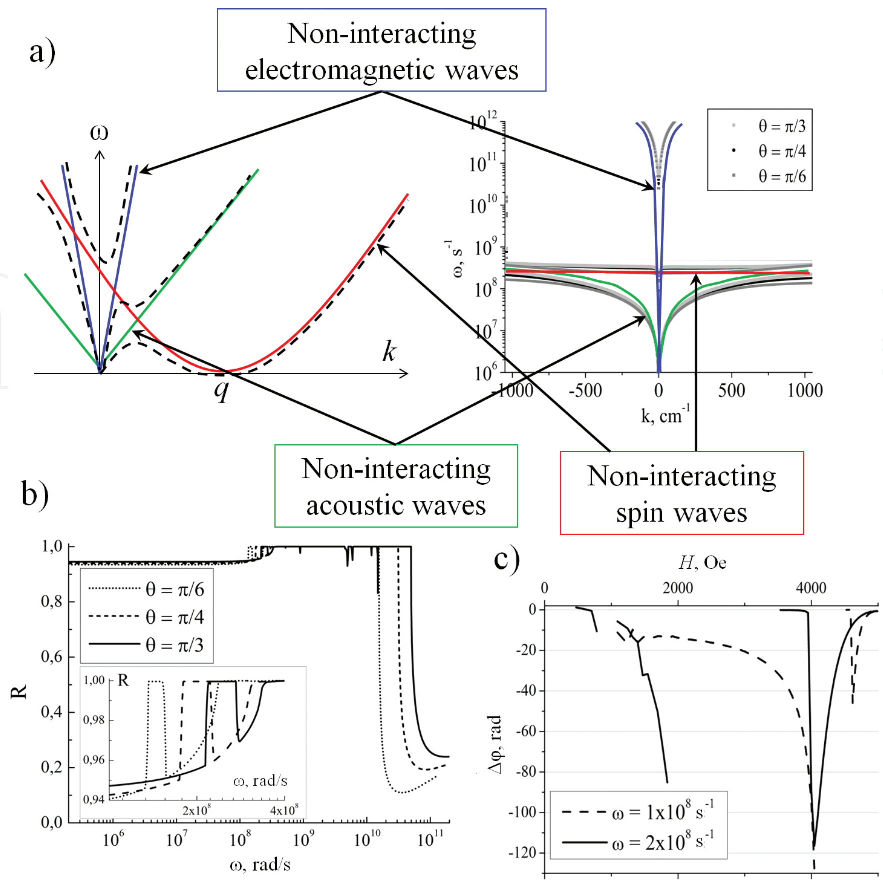

Figure 2. Spectrum of coupled waves (a) (left – schematic spectrum, right – calculated spectrum near $k = 0$). Electromagnetic waves reflection from semi-infinite spiral magnet (b). On the inset the feature associated with the acoustic gap is shown. Acoustic Faraday effect (c).

3. Radiation of electromagnetic and acoustic waves by helicoidal magnets at phase transitions

In this section we partly follow our previous works [38, 39].

In context of macroscopic electrodynamics, electromagnetic wave radiation may be described by the following equation:

$$\Delta\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{4\pi}{c^2}\left[\frac{\partial}{\partial t}\left(\mathbf{j} + \frac{\partial\mathbf{P}}{\partial t} + c\,\mathrm{rot}\,\mathbf{M}\right) + c^2\,\mathrm{grad}\,\mathrm{div}\left(\mathbf{P}\right)\right] \quad (3)$$

In (3) **E** is electric field strength, **P** and **M** are polarization and magnetization of the medium, consequently, **j** is the current density, c is speed of light in the vacuum. Assuming magnetic material to be non-conductive with dielectric constant ε, Eq. (3) may be simplified:

$$\Delta \mathbf{E} - \frac{\varepsilon}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \mathbf{j_M}}{\partial t} \tag{4}$$

In (4) magnetic current density has been introduced: $\mathbf{j_M} = c\cdot\mathrm{rot}(\mathbf{M})$. For spiral magnets with magnetization distribution given by (2), one may calculate magnetic current density $\mathbf{j_M} = (j_0\cos(qz), j_0\sin(qz), 0)$, $j_0 = -cqM_0\sin(\theta)$. So, we may conclude that any process with changing the spiral wave number q, the amplitude of magnetization vector M_0, or the spiral angle θ will be accompanied by electromagnetic radiation.

The form of radiation Eq. (4) allows one to see that time-varying non-uniform magnetization distribution is equivalent to time-varying distributed conductive currents. Generally, this equation should be solved together with the magnetization motion equation (usually, Landau-Lifshitz equation with additional relaxation terms). This system of equations can be solved only numerically. However, it is known that the frequency of precession of the magnetization vector in ferromagnetic materials (ferromagnetic resonance frequency) is about 10–100 GHz, and the relaxation time of magnetization to equilibrium state is of the order of few microseconds at usual conditions. In this case, if we are interested in processes with characteristic frequencies far from resonance, the motion of magnetization can be neglected, and one can assume that magnetization has always ground state value (2). This makes reasonable the assumption that only spiral angle changes during magnetic field induced phase transition.

For investigation of sound waves generation, we should solve equation of motion for elastic medium:

$$\rho\, \partial^2 u_i \big/ \partial t^2 = \partial\sigma_{ij} \big/ \partial x_j \,,\; \sigma_{ij} = \partial F \big/ \partial u_{ij} \,. \tag{5}$$

In (5), F denotes free energy density defined by Eq. (1) for spiral magnets. Eq. (5), similarly to the case of electromagnetic waves, may be transformed into wave equations with non-zero source functions, which in general may depend on both coordinate and time. It is convenient to introduce circular components $(u, M)_{\pm} = (u, M)_x \pm i(u, M)_y$, transversal and longitudinal sound velocities $v_t = (2c_{44}/)^{1/2}$ and $v_l = (c_{33}/)^{1/2}$, consequently. Calculations show that only transverse sound will be excited. We will have

$$\frac{\partial^2 u_{\pm}}{\partial t^2} - v_t^2\frac{\partial^2 u_{\pm}}{\partial z^2} = \pm\frac{iqb_{44}M_0^2}{\rho}\sin(2\theta)\exp(\pm iqz). \tag{6}$$

Let's consider an infinite crystal. At initial time, it will be assumed that there was no electromagnetic radiation, i.e., we should solve the Eq. (4) or (6) with zero initial conditions. In this case, solutions of Eq. (4) or (6) can be obtained from the Duhamel's principle [40].

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Here, we would not solve the problem of phase transition kinetics. We will consider only two simplest cases: an infinitely fast phase transition and linear variation of source functions. Comparing Eqs. (6) and (4) one may note that source function for radiation of electromagnetic waves is proportional to time derivative of sin[θ(t)], while in case of acoustic waves it is defined by time dependence of sin[2θ(t)]. Thus, maximal radiation of electromagnetic waves may be expected at phase transition from simple spiral state to collinear ferromagnetic one, while maximal acoustic radiation should be observed at phase transition from "ferromagnetic spiral" phase with θ = π/4 to collinear ferromagnetic state.

Let us assume that sin[θ(t)] = Θ(-t); Θ(t) is the step-like Heaviside function; ∂Θ(-t)/∂t = −δ(t); δ(t) is the Dirac's delta function. This situation corresponds to instantaneous actuation of magnetic field with the value H_{cr} at time t = 0, and collapse of the spin spiral. Such a process is quite unreal since magnetization requires finite relaxation time to get to the final collinear ferromagnetic state. However, despite the unreality of the case, its analysis allows us to estimate an upper limit of emitted wave amplitude. Calculations show that electromagnetic field has a form:

$$E_{\pm}(z,t) = E_x(z,t) \pm iE_y(z,t) = \mp 4\pi i M_0 \varepsilon^{-1/2} \sin(qvt) \exp[\mp iqz] \tag{7}$$

In case of linear time dependence of sin[θ(t)]: sin[θ(t)] = Pt, at $t \le P^{-1}$, $P = const$, we will have:

$$E_{\pm}(z,t) = \pm 4\pi i M_0 P (qc)^{-1} \left[1-\cos(qvt)\right] \exp[\mp iqz],\ t \le P^{-1} \tag{8}$$

It is seen that in contrast to the fast actuation of the magnetic field, in addition to the standing wave, a constant component of the electric field appears as well. The amplitude and the constant component are directly proportional to the speed of the state change P.

Solutions given by expressions (7) and (8) are superposition of waves travelling to the left and to the right, which, due to unbound periodic source, give a standing wave by adding each other. In the real material, multiple reflections of the excited waves from the boundaries and some part of electromagnetic energy will be emitted beyond the material. If refractive index of the environment is the same as for crystal (i.e. mediums are ideally conjugated) with thickness d, two electromagnetic wave pulses with wave number q and frequency qv will be emitted from each sample's side.

Maximum sound waves generation will take place at phase transition from θ = π/4 to collinear ferromagnetic state. Mathematically, it may be expressed as sin[2θ(t)] = Θ(t). In such case, solution may be easily calculated:

$$u_{\pm}(z,t) = \pm i\, b_{44} M_0^2 \exp[\pm iqz]\left[1-\cos(qv_t t)\right] / \rho q v_t^2 \tag{9}$$

Let us consider now the model process, when $\sin[2\theta(t)] = \eta t$, $t < \eta^{-1}$ (i.e. linear time-dependence of source function). Calculations show that the following sound oscillations will be excited:

$$u_{\pm}(z,t) = \pm\eta i b_{44} M_0^2 \rho^{-1} q^{-2} v_t^{-3} \left[q v_t t - \sin(q v_t t) \right] \exp[\pm iqz], \ t < \eta^{-1}. \tag{10}$$

The frequency of standing electromagnetic waves in formulas (7) and (8) is $\omega = qv \sim 10^{15} - 10^{16}$ s^{-1}. At "slow" change of the state, in mode determined by formula (8), the signal amplitude is proportional to P. For $P \sim 10^7$ s^{-1}, the amplitude of emitted waves is small: $E \sim 10^{12}$ CGSE (energy density is about 10^{-11} J/cm^3). For ultrafast magnetization reversal processes, the time of magnetization switch is usually about 10^{-12} s, or $P \sim 10^{12}$ s^{-1}, and we have $E \sim 10^2$ CGSE (energy density is about 10^{-3} J/cm^3). The maximal radiation corresponding to the situation of the instantaneous change of the state has amplitude of electric field $E \sim 10^3$ CGSE (energy density is about 0.1 J/cm^3). This radiation will be emitted like a pulse. Characteristic time length of this pulse is $d/v + P^{-1}$. For sample with $d \sim 1$ μm, we will have $d/v \sim 10^{-14}$ s. This time is much smaller than usual magnetization switching time, so in real experiments, characteristic time length of the pulse should be comparable with switching time for $d \ll vP^{-1}$. Despite low energy density of the pulse, it may have valuable power density. In case of $P = 10^{12}$ s^{-1} this value may reach about 1 GW/cm^3.

For acoustic wave generation in case of infinitely fast phase transition, we will have the oscillations frequency $\omega = qv_t$. For Dzyaloshinskii-Moriya interaction-caused spiral magnets the linear frequency $f = \omega/(2\pi) \sim 5$ GHz, and amplitude is $u_{\pm} \sim 10^{-5}$ cm. In exchange-caused case $f \sim 5$ THz, $u_{\pm} \sim 10^{-8}$ cm. Both cases correspond to hypersound oscillations. In case of "slow" phase transition sound oscillations with the same frequency are excited. Amplitude of excited oscillations linearly depends on the "phase transition speed" η. We may calculate that $u_{\pm} \sim 10^{-21} \times \eta$ cm and $u_{\pm} \sim 10^{-27} \times \eta$ cm for Dzyaloshinskii-Moriya and exchange interaction-caused spiral magnets, respectively. For example, ultrafast magnetization reversal processes have time of magnetization switch of about 10^{-12} s, or $\eta \sim 10^{12}$ s^{-1}. The amplitudes will have values $u_{\pm} \sim 10^{-9}$ cm and $u_{\pm} \sim 10^{-15}$ cm for Dzyaloshinskii-Moriya and exchange interaction-caused spiral magnets, consequently. One can see that in case of exchange spiral structure, the amplitude is negligibly small. In experiments it is possible to detect the pulse of sound with characteristic time length of about $\tau \sim d/v_t$, where d is the sample size. For sample size $d \sim 1$ mm we will have time length $\tau \sim 10^{-6}$ s.

The abovementioned results make one believe that a similar effect may be observed when the spiral magnet is placed in homogeneous periodic magnetic field, i.e. when the spiral angle periodically changes near its equilibrium value. For investigation of this problem, let us consider the spiral magnet in time-dependent external magnetic field $H_z(t) = H_0 + h_0(t)$, where H_0 is a constant magnetic field defining the ground state and equilibrium value of spiral angle, and $h_0(t) = h_0\exp[i\Omega t]$ is a periodic component. For simplicity, we will assume that the periodic component of the field has small amplitude. In such context, magnetization may be described by static component $\mathbf{M_0}$, which is described by Eq. (2) with the spiral angle defined by H_0, and by dynamical component $\boldsymbol{m}(z,t)$. Dynamical component may be obtained from Landau-Lifshitz equation. We will neglect relaxation processes for simplicity. Oscillated components may be assumed in form of Fourier series $\boldsymbol{m}(z,t) = \Sigma \boldsymbol{m}_n(t)\exp[-inqz]$. Calculations show that longitudinal periodic magnetic field will excite only harmonics of m_x and m_y with $n = \pm 1$. We will have the following:

$$\begin{aligned} &\frac{\partial^2 m_\pm}{\partial z \partial t} = \frac{-gq\Omega M_0 \sin\theta h_{0z} e^{i\Omega t}}{\Omega_{res}^2 - \Omega^2} \left\{ \pm 2g\alpha_1 q M_0 \cos\theta e^{\mp iqz} + \left(\Omega \pm g\alpha_1 q M_0 \cos\theta\right) e^{\pm iqz} \right\}; \\ &m_\pm = m_x \pm i m_y; \, \Omega_{res}^2 = 5\alpha_1^2 g^2 q^2 M_0^2 \cos^2\theta. \end{aligned} \tag{11}$$

One can see that the source function defined by Eq. (11) has a resonance at $\Omega = \Omega_{res}$. This is due to resonant behaviour of corresponding susceptibility component. In real materials, due to unavoidable damping processes, the source function will have a finite value even at resonant frequency. We will take into account these processes formally by putting $\Omega + i\delta$ instead of Ω, where δ is a formal damping parameter.

Solving Eq. (4) with use of Green's function formalism for magnet's size d, we will have the following stationary electric field amplitudes at the left boundary of the sample:

$$\begin{aligned} &E_\pm(0,t) = \frac{i2\pi g M_0 \sin\theta h_{0z} e^{i\Omega t} e^{-i\Omega d/v}}{\Omega_{res}^2 - (\Omega + i\gamma)^2} \times \\ &\times \left\{ \frac{\Omega + i\gamma \pm g\alpha_1 q M_0 \cos\theta}{\Omega \mp qv} \left[qv\left(e^{i\Omega d/v} - 1\right) \pm \Omega\left(1 - e^{\pm iqd}\right) \right] \mp \right. \\ &\left. \mp \frac{2g\alpha_1 q M_0 \cos\theta}{\Omega \pm qv} \left[qv\left(e^{i\Omega d/v} - 1\right) \mp \Omega\left(1 - e^{\mp iqd}\right) \right] \right\} \end{aligned} \tag{12}$$

One may calculate the energy of emitted waves $W_\pm = |E_\pm|^2/8\pi$. Dependencies of this energy from the frequency and from the equilibrium spiral angle are shown in **Figure 3**.

One can see that intensities of left- and right-polarized emitted waves differ from each other. When $0 < \theta < \pi/2$ (i.e. when the static component of magnetization is tilted along spiral axis) the energy of negatively polarized waves is greater than that of positively polarized ones. When $\theta = \pi/2$, energies of waves for both polarizations are equal. When $\pi/2 < \theta < \pi$, the energy of positively polarized waves prevails. The increase of damping parameter leads to decrease of emitted wave energy, especially near the resonance. When the damping parameter becomes much greater than the resonant frequency, the energy of emitted waves slowly varies with the frequency, equal for waves of both polarizations, and has a maximum at $\theta = \pi/2$.

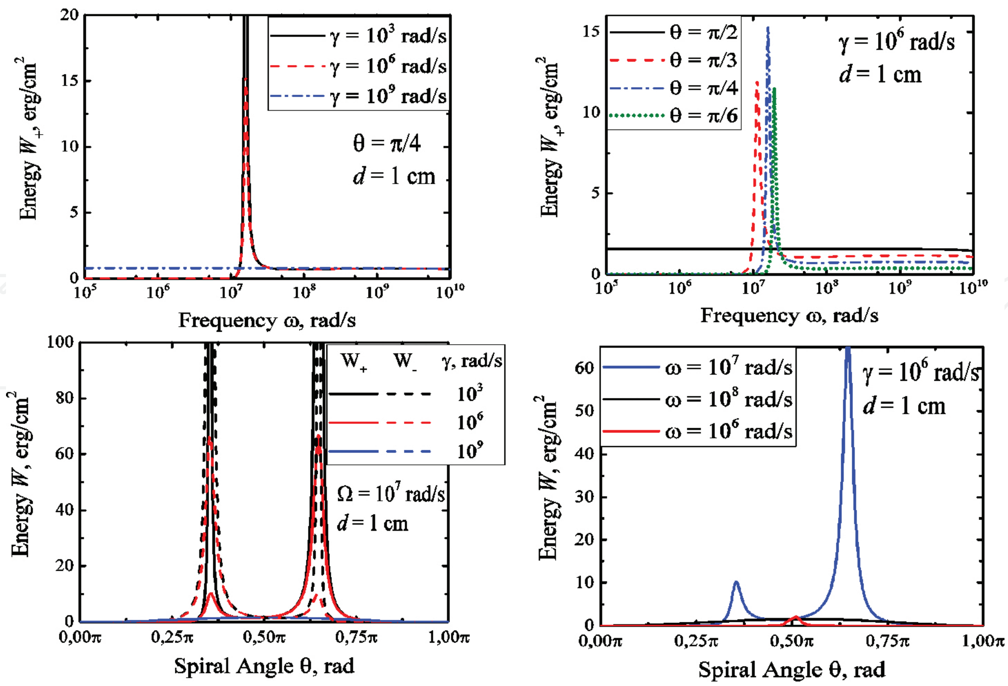

Figure 3. Energy of emitted electromagnetic waves versus frequency of exciting field and the static value of spiral angle.

Similarly, one may consider acoustic wave excitation in spiral magnets in homogeneous harmonical magnetic field. Wave equations may be written as follows:

$$\begin{aligned}
&\frac{\partial^2 u_\pm}{\partial z^2} - \frac{\ddot{u}_\pm}{v_t} = -\frac{b_{44}}{c_{44}}\left(M_{z0}\frac{\partial m_\pm}{\partial z} + m_z \frac{\partial M_{\pm 0}}{\partial z}\right);\\
&\frac{\partial^2 u_z}{\partial z^2} - \frac{\ddot{u}_z}{v_l} = -\frac{2b_{13}}{c_{33}}\left(M_{x0}\frac{\partial m_x}{\partial z} + m_x\frac{\partial M_{x0}}{\partial z} + M_{y0}\frac{\partial m_y}{\partial z} + m_y\frac{\partial M_{y0}}{\partial z}\right).
\end{aligned} \tag{13}$$

Both longitudinal and transversal acoustic waves may be excited. Transverse magnetization oscillations will excite both longitudinal and transverse sound, while longitudinal magnetization oscillations affect only transverse sound. Similar to calculations made in [41], for simplicity we will suppose that magnetization oscillations are uniform, i.e. $\partial m_i(z,t)/\partial z \approx m_{i0}[\delta(z)-\delta(z-d)]\exp(-i\omega t)$, here $\delta(z)$ is Dirac's delta-function, d is the sample size. Wave equations may be solved by Green's function formalism. Green's functions for longitudinal and transverse acoustic waves $G_{t,l}(z, z_0)$ consist of two terms. The first one $(i/2k_{t,l})\exp(ik_{t,l}|z-z_0|)$ corresponds to the sound excited in point z_0 and propagating in +z-direction. The second one $(i/2k_{t,l})\exp(ik_{t,l}|z+z_0|)$ corresponds to the wave excited in the same point, but propagating in opposite direction. In both cases, $k_{t,l} = \omega/v_{t,l}$. Complete Green's function will be as follows:

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$$G_{t,l}\left(z,z_0\right)=ik_{t,l}^{-1}\exp\left(-ik_{t,l}z\right)\cos\left(k_{t,l}z_0\right). \tag{14}$$

Calculations show that amplitudes of excited waves are:

$$\begin{aligned}
u_{\pm} &= -\frac{ib_{44}M_0}{k_t c_{44}}\left\{\begin{aligned}&m_{\pm 0}\cos\theta\left[1-\cos\left(k_t d\right)\right]\pm\\ &\pm iq\,\frac{m_{z0}\sin\theta}{q^2-k_t^2}\left[\begin{aligned}&k_t\exp\left(\pm iqd\right)\sin\left(k_t d\right)\mp\\ &\mp iq\left(1-\exp\left(\pm iqd\right)\cos\left(k_t d\right)\right)\end{aligned}\right]\end{aligned}\right\};\\
u_z &= -\frac{4ib_{13}M_0\sin\theta}{k_l c_{33}}\left\{\begin{aligned}&m_{x0}\frac{k_l}{q^2-k_l^2}\left[q\sin qd\sin k_l d-k_l\cos qd\cos k_l d\right]-\\ &-m_{y0}\sin qd\left[\cos k_l d-\frac{q\sin k_l d}{q+k_l}\right]\end{aligned}\right\}.
\end{aligned} \tag{15}$$

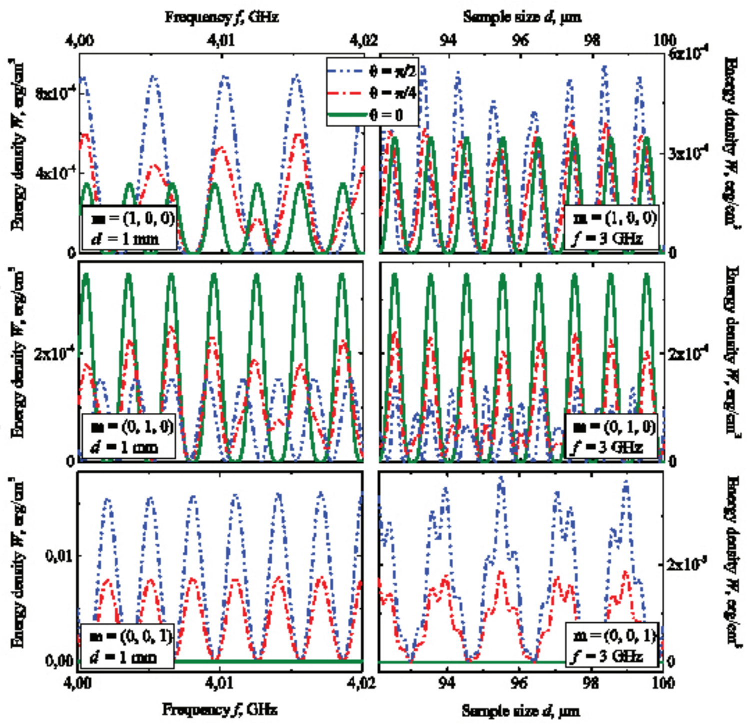

Figure 4. Fragments of acoustic waves energy density dependencies from frequency and sample size for different polarization of excited field. Wave number of spiral is $q = 10^5$ cm^{-1}.

Energy density of excited waves is $W = c_{33}(u_{zz})^2/2 + 2c_{44}[(u_{xz})^2+(u_{yz})^2]$. Fragments of its dependencies from frequency and sample size for different polarization of excited field are shown in **Figure 4**. One can see that these dependencies have a complex oscillating behaviour. Oscillations are caused by interference of sound waves propagating from opposite sample boundaries. Another reason of oscillations is periodicity of source function. Eq. (15) have resonances at $k_{t,l} = q$. At microwave frequencies these resonances may be observed in magnets with relativistic spiral structure ($q \sim 10^5$ cm^{-1}, $\omega_{res} \sim qv_{t,l} \sim 10^{10}$ rad/s), while in magnets with exchange spiral structure, resonance frequencies correspond to terahertz frequency range ($q \sim 10^8$ cm^{-1}, $\omega_{res} \sim qv_{t,l} \sim 10^{13}$ rad/s).

4. Conclusions

We have reviewed some features of electromagnetic and acoustic wave propagation in magnets with helicoidal spin order. There is an opacity window in reflectance spectrum of electromagnetic waves from the plate of spiral magnet. The width of this window has a maximal value at phase of simple spiral, and decreases with decrease of spiral angle (or, equivalently, with increase of external magnetic field). Field dependencies of polarization plane rotation angle have a resonant behaviour. Maximum rotation may be observed near boundaries of band gaps for both electromagnetic and acoustic waves. Both electromagnetic and acoustic waves may be radiated by spiral magnets during the phase transition. The amplitude of radiated waves strictly depends on the speed of phase transition.

In general, the results show the possibility of practical applications of spiral magnets for electromagnetic and acoustic wave manipulation and generation.

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