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Event-Triggered Static Output Feedback Simultaneous H_∞ Control for a Collection of Networked Control Systems

Sheng-Hsiung Yang and Jenq-Lang Wu

Abstract

This chapter considers the design of event-triggered static output feedback simultaneous H_∞ controllers for a collection of networked control systems (NCSs). It is shown that conventional point-to-point wiring delayed static output feedback simultaneous H_∞ controllers can be obtained by solving linear matrix inequalities (LMIs) with a linear matrix equality (LME) constraint. Based on an obtained simultaneous H_∞ controller, an L_2-gain event-triggered transmission policy is proposed for reducing the network usage. An illustrative example is presented to verify the obtained theoretical results.

Keywords: networked control systems, simultaneous stabilization, event-triggered, static output feedback, H_∞ control.

1. Introduction

A networked control system (NCS) is a feedback control system with feedback loop closed through a communication network. As the signal in an NCS is exchanged via a network, the network-induced delay, packet dropout, and limited network bandwidth can degrade the control performance. Many results have been proposed for dealing with these issues [1–5]. In the early stages, the studies on NCSs were mainly based on periodic task models [4–6]. The number of data packets to be transmitted will be large as the sampling period is small. This leads to a conservative usage of network resources and possibly leads to a congested network traffic. Therefore, how to design networked feedback controllers to achieve desired performance with low network usage is an important issue in NCSs.

Recently, some sporadic task models have been presented in NCSs without degrading system performance. An important approach is the event-triggered scheme [7–26]. In [7], the state transmitting and the control signal updating events were triggered only if the error between the current measured state and the last transmitted state is larger than a threshold condition. In [8], event-triggered distributed NCSs with transmission delay were studied. Based on the designed event-triggered policy, an allowable upper bound of the transmission delay was derived. In [9], for distributed control systems, an implementation of event-triggering control policy in sensor-actuator network was introduced. In [10], the authors concerned with the design of event-triggered state feedback controllers for distributed NCSs with transmission delay and possible packet dropout. Under the proposed triggering policy, the tolerable packet delay and packet dropout were derived. In [11], an event-triggered control policy was developed for discrete-time control systems. In [12], under stochastic packet dropouts, an event-triggered control law for NCSs was calculated by the proposed algorithms. In [13], an event-triggered scheme was developed for uncertain NCSs under packet dropout. In [14], an event-based controller and a scheduler scheme were proposed for NCSs under limited bandwidth. The NCSs were modeled as discrete-time switched control systems. A sufficient condition for the existences of event-based controllers and schedulers was derived by the LMI optimization approach. Recently, the event-triggered scheme has been extended to H_∞ control of NCSs for achieving the disturbance attenuation performance [15–21]. In [15] and [18], with considering transmission delays, event-triggered H_∞ state feedback controllers for NCSs were proposed. Criterion for stability and criterion for co-designing both the controller gains and the trigger parameters were derived. In [16], an event-triggered state feedback control scheme was proposed for guaranteeing finite L_2-gain stability of a linear control system. In [17], an event-triggered state feedback H_∞ controller for sampled-data control system was proposed. In [19], the design of event-triggered networked feedback controllers for discrete-time NCS was considered. In [20], based on Lyapunov-Krasovskii function, an event-triggered state feedback H_∞ controller was derived for NCSs under time-varying delay and quantization.

All the results in [7–20] are derived in the assumption that the system states are available for measurement. For practical control systems, system states are often unavailable for direct measurement. In the literature, only few results have been proposed for output-based event-triggered NCSs [22–26]. In [22], a dynamic output feedback event-triggered controller for NCSs was proposed for guaranteeing the asymptotic stability. In [23] and [24], by the passivity theory approach, output-based event-triggered policies were derived for guaranteeing the satisfaction of L_2-gain requirements of dynamic output feedback NCSs in the presence of time-varying delays. The synthesis of controllers has not been discussed. In [25] and [26], under nonuniform sampling, new output-based event-triggered H_∞ transmission policies were proposed of NCSs under time-varying transmission delays. Furthermore, the design of static output feedback H_∞ controllers for NCSs was discussed. Conditions for the existence of H_∞ controllers were presented in terms of bilinear matrix inequalities. A non-convex minimization problem must be solved to get a static output feedback H_∞ controller.

On the other hand, few results have been proposed in the literature for simultaneous stabilization of NCSs. The consideration of simultaneous stabilization is important since it allows us to design highly reliable controllers that are able to accommodate possible element failures in control systems. As the signal transmitted through network, the solvability of simultaneous stabilization problem of NCSs is quite different to that of point-to-point wiring control systems. Only few results have been proposed for relevant issues [21, 27]. In [27], based on the average dwell time approach, the simultaneous stabilization for a collection of NCSs was considered. A sufficient condition for guaranteeing simultaneous stabilization was proposed. In [21], under the assumption that the network communication channel is ideal (no delay, no packet dropout, and no quantization error), we considered the design of state feedback event-triggered simultaneous H_∞ transmission policies for a collection of NCSs. Under the proposed event-triggered transmission policies, the L_2-gain stability of all the closed-loop NCSs can be guaranteeing under low network usages.

It is known that static output feedback controllers are preferred in practical applications since their implementations are much easier than dynamic output feedback controllers. However, the design of static output feedback controllers is much more difficult than dynamic ones. In this chapter, we extend our previous work [21] to static output feedback case. Furthermore, we consider the network-induced time-varying delay that has not been considered in [21]. We develop an event-triggered static output feedback simultaneous H_∞ transmission policy for a collection of continuous-time linear NCSs under time-varying delay. It is shown that, under mild assumptions, conventional point-to-point wiring delayed static output feedback simultaneous H_∞ controllers can be obtained by solving LMIs with a LME constraint. Based on the obtained static output feedback simultaneous H_∞ controllers, an event-triggered transmission policy was derived for reducing network usage. Different to the results presented in [25] and [26] that only considering the design of an event-triggered H_∞ controller for a single system, this chapter considers the design of a fixed event-triggered H_∞ controller that is able to L_2-stabilize a collection of NCSs simultaneously. By the proposed method, highly reliable NCSs that are able to accommodate possible element failures with low network usage can be designed. Even simplifying our results to the single system case, our method for designing static output feedback H_∞ controllers is quite different from those in [25] and [26]. In [25] and [26], a non-convex minimization problem must be solved for getting a static output feedback H_∞ controller. Moreover, the obtained controller can only guarantee uniform ultimate boundedness but not internal stability. In our approach, (simultaneous) static output feedback H_∞ controllers are obtained by solving LMIs with a LME constraint. Moreover, internal stabilities of the closed-loop NCSs can be guaranteed.

2. Problem formulation and preliminaries

In this section, the problem to be solved is formulated and some preliminaries are given. For simplifying the expressions, we use the same notations x, u, w, and z to denote the states, control inputs, exogenous inputs, and the controlled outputs of all considered systems.

2.1. Problem formulation

Consider a collect of continuous-time control systems:

$$
\begin{aligned}
\dot{x}(t) &= A_j x(t) + B_{1j} w(t) + B_{2j} u(t), \quad j = 1, 2, \ldots, N \\
z(t) &= C_{1j} x(t) + D_{11j} w(t) + D_{12j} u(t) \\
y(t) &= C_{2j} x(t)
\end{aligned}
\tag{1}
$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, $z(t) \in R^s$ is the controlled output, $y(t) \in R^l$ is the measured output, $w(t) \in R^r$ is the exogenous input, and A_j, B_{1j}, B_{2j}, C_{1j}, D_{11j}, D_{12j}, and C_{2j} are constant matrices with appropriate dimensions. Here, for convenience, we assume $C_{2j} = C_2$, $j = 1, 2, \ldots, N$. Suppose that (A_j, B_{2j}) are stabilizable and (C_2, A_j) are detectable for each $j \in \{1, 2\ldots, N\}$. Furthermore, assume that $\gamma^2 I - D_{11j}^T D_{11j} > 0$ for all $j \in \{1, 2\ldots, N\}$.

In this chapter, we consider the case that the feedback loop of system (1) is closed through a real-time network, but not by the conventional point-to-point wiring. Suppose that the sensor node keeps measuring the output signal y, but not all the sampled data need to be sent to the controller node. The data transmission at the sensor node is not periodic. Let t_i ($i = 1,2,\ldots$) be the time that the i-th transmission occurs at the sensor nodes. In this case, the controller node receives the networked feedback data $y(t_i)$ and updates the control signal at time $t_i + \tau_i$, $i = 1,2, \ldots$, where $\tau_i \in [\tau_{d\min}, \tau_{d\max}]$ is the transmission delay. That is,

$$
u(t) = F y(t_i), t_i + \tau_i \leq t < t_{i+1} + \tau_{i+1}, i = 1, 2, \ldots
\tag{2}
$$

where F is the feedback gain to be designed later. With the same controller (2), the closed-loop systems are:

$$
\begin{aligned}
\dot{x}(t) &= A_j x(t) + B_{1j} w(t) + B_{2j} F C_2 x(t_i), \quad t_i + \tau_i \leq t < t_{i+1} + \tau_{i+1}, j = 1, 2, \ldots, N \\
z(t) &= C_{1j} x(t) + D_{11j} w(t) + D_{12j} F C_2 x(t_i)
\end{aligned}
\tag{3}
$$

If the measured data is not critical for L_2-gain stability, it will not be sent for saving the network usage. In this case, the controller node does not update the control signal. If the measured data is critical, it will be sent through the network to the controller node, and the controller will update the control signal.

Our main goal is to design an event-triggered transmission rule to determine whether the currently measured data should be sent to the controller node, such that, under the transmis-

sion delay, all possible closed-loop systems in (3) are internally stable and satisfy, for a given constant $\gamma > 0$ and for any $T > 0$ and $w \in L_2[0, T]$,

$$\int_0^T z^T(t)z(t)\mathrm{d}t \le \gamma_0^2 \int_0^T w^T(t)w(t)\mathrm{d}t, \text{ for some } \gamma_0 < \gamma \tag{4}$$

Note that, a practical control system may have several different dynamic modes since it may have several different operating points (please see e.g., the ship steering control problem considered in [28]). On the other hand, for achieving higher reliability of a practical control system, we may want to design a controller to accommodate possible element failures. With considering possible element failures, a control system can have several different dynamic modes (see e.g., the reliable control problem for active suspension systems considered in [29]). The problem we considered has a practical importance owing to its high applicability in designing robust and/or reliable controllers.

2.2. Preliminaries

The following Lemmas will be used later.

Lemma 1 [30]: For any vectors $X, Y \in R^n$ and any positive definite matrix $G \in R^{n \times n}$, the following inequality holds:

$$2X^T Y \le X^T G X + Y^T G^{-1} Y \quad \blacksquare$$

Lemma 2 [31]: For any given matrices $\Pi < 0$ and $\Phi = \Phi^T$, and any scalar λ, the following inequality holds:

$$\Phi \Pi \Phi \le -2\lambda\Phi - \lambda^2 \Pi^{-1} \quad \blacksquare$$

For convenience, define $x_t(s) = x(t+s), \ \forall s \in [-\tau_{\max}, 0]$.

Lemma 3 (*Lyapunov–Krasovskii Theorem*) [32]: Consider a time-delay system:

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)), \ \forall t \ge 0 \tag{5}$$

with $\tau(t) \in [0, \tau_{\max}], \ \forall t \ge 0$. Suppose that $x(t) = \psi(t), \ \forall t \in [-\tau_{\max}, 0]$. If there exists a function

$$V : C([-\tau_{\max}, 0], R^n) \to R$$

and a scalar $\varepsilon > 0$, such that, for all $\varphi \in C([-\tau_{\max}, 0], R^n)$, $V(\varphi) \geq \varepsilon \|\varphi(0)\|^2$, and, along the solutions of (5),

$$\frac{\mathrm{d}V(x_t)}{\mathrm{d}t}\bigg|_{x_t = \varphi} \leq -\varepsilon \|\varphi(0)\|^2,$$

then the system (5) is asymptotically stable. ∎

3. Main results

We first consider the design of the event-triggered transmission policy under the assumption that we have a delayed simultaneous H_∞ controller, and then show how to derive simultaneous H_∞ controller under transmission delay.

3.1. Event-triggered transmission policy for NCSs under time-varying delay

Define the equivalent time-varying delay

$$\tau(t) = t - t_i, \ t_i + \tau_i \leq t < t_{i+1} + \tau_{i+1}, \ i = 1, 2, \dots.$$

It is clear that

$$\tau(t) \in \left[\tau_{\min}, \ \tau_{\max} \right] \forall t \geq 0, \text{and } \dot{\tau} = 1 \text{ almost everywhere} \tag{6}$$

where $\tau_{\min} \equiv \min_{i \in N} \{\tau_i\} = \tau_{d\min}$ and $\tau_{\max} \equiv \max_{i \in N} \{t_{i+1} - t_i + \tau_{i+1}\} = \max_{i \in N} \{t_{i+1} - t_i\} + \tau_{d\max}$. Then, the systems in (3) can be equivalently described as

$$\begin{aligned}
\dot{x}(t) &= A_j x(t) + B_{1j} w(t) + B_{2j} F C_2 x(t - \tau(t)), \quad j = 1, 2, \dots, N \\
z(t) &= C_{1j} x(t) + D_{11j} w(t) + D_{12j} F C_2 x(t - \tau(t))
\end{aligned} \tag{7}$$

To derive an event-triggered transmission policy in the presence of transmission delay, assume that, for the systems in (1), we have a conventional delayed static output feedback simultaneous H_∞ controller:

$$u(t) = F y(t - \tau(t)) \tag{8}$$

which is such that all of the possible closed-loop systems in (7) are internally stable and satisfy the condition (4) for $\tau(t) \in [\tau_{min}, \tau_{max}]$. How to get such a delayed static output feedback simultaneous H_∞ controller will be discussed later.

Define the error signal:

$$e(t) = y(t) - y(t_i), t_i \le t < t_{i+1} \tag{9}$$

We have the following results.

Theorem 1: Consider the systems in (1). Suppose that the controller (8) is such that all the closed-loop systems in (7) are internally stable and satisfy the condition (4). If there exist matrices $P_j > 0$, $Q_j > 0$, G_{1j}, G_{2j}, G_{3j}, and G_{4j}, $j=1,2,\ldots,N$, of appropriate dimensions, and scalars $\varepsilon_j > 0$, $j=1,2,\ldots,N$, satisfying the following LMIs:

$$\begin{bmatrix} \Phi_j & \Xi_j & G_{3j}^T & P_j B_{1j} + G_{4j}^T + C_{1j}^T D_{11j} & \tau_{max} A_j^T Q_j & \tau_{max} G_{1j} \\ * & \Sigma_j & -G_{3j}^T & -G_{4j}^T + C_2^T F^T D_{12j}^T D_{11j} & \tau_{max} C_2^T F^T B_{2j}^T Q_j & \tau_{max} G_{2j} \\ * & * & -\varepsilon_j I & 0 & 0 & \tau_{max} G_{3j} \\ * & * & * & D_{11j}^T D_{11j} - r^2 I & \tau_{max} B_{1j}^T Q_j & \tau_{max} G_{4j} \\ * & * & * & * & -\tau_{max} Q_j & 0 \\ * & * & * & * & * & -\tau_{max} Q_j \end{bmatrix} < 0, \tag{10}$$

where

$$\Phi_j = A_j^T P_j + P_j A_j + C_{1j}^T C_{1j} + C_2^T C_2 + G_{1j} + G_{1j}^T$$

$$\Xi_j = P_j B_{2j} F C_2 + C_{1j}^T D_{12j} F C_2 - G_{1j} + G_{2j}^T$$

$$\Sigma_j = C_2^T F^T D_{12j}^T D_{12j} F C_2 - G_{2j} - G_{2j}^T$$

then all the networked closed-loop systems in (7) are internally stable and satisfy the condition (4) if the following condition holds:

$$\|e(t)\| < \min_{j \in \{1,2,\ldots,N\}} \frac{1}{\sqrt{\varepsilon_j}} \cdot \|y(t)\|, t_i \le t < t_{i+1} \tag{11}$$

Proof: For the systems in (7), choose the candidate storage functions:

$$V_j(x(t)) = x^T(t)P_jx(t) + \int_{-\tau_{max}}^{0} \int_{t+\sigma}^{t} \dot{x}^T(\theta)Q_j\dot{x}(\theta)d\theta d\sigma, \ j = 1,2,\ldots,N.$$

Define

$$\hat{H}_{dj}(x(t),x(t_i),e(t),w(t)) \equiv \dot{V}_j(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)$$
$$w(t) + y^T(t)y(t) - \varepsilon_j e^T(t)e(t), \ j = 1,2,\ldots,N$$

Along the solutions of the *j*-th system, we have

$$\hat{H}_{dj} = 2x^T(t)P_j\dot{x}(t) - \int_{t-\tau_{max}}^{t} \dot{x}^T(\theta)Q_j\dot{x}(\theta)d\theta + \tau_{max}\dot{x}^T(t)Q_j\dot{x}(t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t)$$
$$+ y^T(t)y(t) - \varepsilon_j e^T(t)e(t) + 2\eta^T(t)G_j\left(x(t) - x(t_i) - \int_{t_i}^{t} \dot{x}(\theta)d\theta\right)$$

where $\eta(t) = [x^T(t) \ \ x^T(t_i) \ \ e^T(t) \ \ w^T(t)]^T$ and $G_j = [G_{1j}^T \ \ G_{2j}^T \ \ G_{3j}^T \ \ G_{4j}^T]^T$. Then,

$$\hat{H}_{dj} = 2x^T(t)P_j\left(A_jx(t) + B_{1j}w(t) + B_{2j}FC_2x(t-\tau(t))\right) + x^T(t)C_{1j}^TC_{1j}x(t)$$

$$+2x^T(t)C_{1j}^TD_{12j}FC_2x(t-\tau(t)) + x^T(t-\tau(t))C_2^TF^TD_{12j}^TD_{12j}FC_2x(t-\tau(t))$$

$$+w^T(t)D_{11j}^TD_{11j}w(t) + 2x^T(t)C_{1j}^TD_{11j}w(t) + 2x^T(t-\tau(t))C_2^TF^TD_{12j}^TD_{11j}w(t)$$

$$-r^2w^T(t)w(t) - \int_{t-\tau_{max}}^{t} \dot{x}^T(\theta)Q_j\dot{x}(\theta)d\theta$$

$$+\tau_{max}x^T(t)A_j^TQ_jA_jx(t) + \tau_{max}w^T(t)B_{1j}^TQ_jB_{1j}w(t)$$

$$+\tau_{max}x^T(t-\tau(t))C_2^TF^TB_{2j}^TQB_{2j}FC_2x(t-\tau(t))$$

$$+2\tau_{max}x^T(t)A_j^TQ_jB_{1j}w(t) + 2\tau_{max}x^T(t)A_j^TQ_jB_{2j}FC_2x(t-\tau(t))$$

$$+2\tau_{max}x^T(t-\tau(t))C_2^TF^TB_{2j}^TQ_jB_{1j}w(t)$$

$$+x^T(t)C_2^T C_2 x(t) - \varepsilon_j e^T(t)e(t) + 2\eta^T(t)G_j\left(x(t) - x(t_i) - \int_{t_i}^{t} \dot{x}(\theta)\mathrm{d}\theta\right) \tag{12}$$

From the definition of $\tau_{\max}$, it is clear that $\tau_{\max} \geq t - t_i$ as $t \in [t_i + \tau_i, \ t_{i+1} + \tau_{i+1})$. As a result,

$$-\int_{t-\tau_{\max}}^{t} \dot{x}^T(\theta)Q_j\dot{x}(\theta)\mathrm{d}\theta \leq -\int_{t_i}^{t} \dot{x}^T(\theta)Q_j\dot{x}(\theta)\mathrm{d}\theta. \tag{13}$$

By (12), (13), and the Jensen integral inequality [33], we can show that

$$\hat{H}_{dj} \leq \eta^T(t)\begin{bmatrix} \Phi_j & P_jB_{2j}FC_2 + C_{1j}^T D_{12j}FC_2 - G_{1j} + G_{2j}^T & G_{3j}^T & P_jB_{1j} + G_{4j}^T + C_{1j}^T D_{11j} \\ * & C_2^T F^T D_{12j}^T D_{12j}FC_2 - G_{2j} - G_{2j}^T & -G_{3j}^T & -G_{4j}^T + C_2^T F^T D_{12j}^T D_{11j} \\ * & * & -\varepsilon_j I & 0 \\ * & * & * & D_{11j}^T D_{11j} - r^2 I \end{bmatrix}\eta(t)$$

$$+\tau_{\max}x^T(t)A_j^T Q_j A_j x(t) + \tau_{\max}w^T(t)B_{1j}^T Q_j B_{1j}w(t)$$

$$+\tau_{\max}x^T(t-\tau(t))C_2^T F^T B_{2j}^T Q_j B_{2j}FC_2 x(t-\tau(t))$$

$$+2\tau_{\max}x^T(t)A_j^T Q_j B_{1j}w(t) + 2\tau_{\max}x^T(t)A_j^T Q_j B_{2j}FC_2 x(t-\tau(t))$$

$$+2\tau_{\max}x^T(t-\tau(t))C_2^T F^T B_{2j}^T Q_j B_{1j}w(t) + \tau_{\max}\eta^T(t)G_j Q_j^{-1}G_j^T\eta(t) \tag{14}$$

Then, by Schur complement and after some manipulations, it can be proved that if (10) holds, we have

$$\hat{H}_{dj}(x(t), x(t_i), e(t), w(t)) < \text{ for all } \eta(t) \neq 0$$

.

That is, under (11),

$$\dot{V}_j(x(t)) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) < 0, \ \eta(t) \neq 0 \tag{15}$$

This shows that the j-th closed-loop system in (7) satisfies condition (4). To prove the internal stability, by letting $w(t)=0$ in (15) yields (note that j can be any number belonging to $\{1,2,…,N\}$)

$$\dot{V}_j(x(t)) < -z^T(t)z(t) \leq 0, \forall x(t) \neq 0.$$

That is, the j-th closed-loop system is internally stable. Note that j can be any number belonging to $\{1,2,\ldots,N\}$. The above proof shows that all the closed-loop systems are internally stable and satisfy condition (4). ∎

Remark 1: Note that condition (11) is checked at the sensor node but not the controller node. In practice, the transmission event is triggered by the condition

$$\|e(t)\| \geq \eta \cdot \min_{j \in \{1,2,\ldots,N\}} \frac{1}{\sqrt{\varepsilon_j}} \cdot \|y(t)\|$$

for some constant $0 < \eta < 1$. In general we set η near to 1. ∎

3.2. Synthesis of static output feedback delayed simultaneous H_∞ controllers

In this subsection, we introduce how to derive a conventional delayed simultaneous static output feedback H_∞ controller (8) such that all of the closed-loop systems (7) are internally stable and satisfy the condition (4). We have the following results.

Lemma 4: Consider the systems in (1). For given positive scalars λ and $\tau_{\max}$, if there exist matrices $S > 0$, $Q > 0$, T_{1j}, T_{2j}, T_{3j}, $j=1,2,\ldots,N$, and matrices M and L of appropriate dimensions, satisfying the following LMIs and LME :

$$\begin{bmatrix} \Lambda_j & \zeta_j & B_{1j} + T_{3j}^T + SC_{1j}^T D_{11j} & \tau_{\max} SA_j^T & SC_{1j}^T & \tau_{\max} T_{1j} \\ * & -T_{2j} - T_{2j}^T & -T_{3j}^T + C_2^T L^T D_{12j}^T D_{11j} & \tau_{\max} C_2^T L^T B_{2j}^T & C_2^T L^T D_{12j}^T & \tau_{\max} T_{2j} \\ * & * & D_{11j}^T D_{11j} - r^2 I & \tau_{\max} B_{1j}^T & 0 & \tau_{\max} T_{3j} \\ * & * & * & -\tau_{\max} Q^{-1} & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & \tau_{\max}\left(-2\lambda S + \lambda^2 Q^{-1}\right) \end{bmatrix} < 0 \quad (16)$$

$$MC_2 = C_2 S \quad (17)$$

where $\Lambda_j = SA_j^T + A_j S + T_{1j} + T_{1j}^T$ and $\zeta_j = B_{2j} L C_2 - T_{1j} + T_{2j}^T$, then the feedback law (8) with $F = L M^{-1}$ is a simultaneous H_∞ controller for the systems in (1).

Proof: Let $P = S^{-1}$. Choose a candidate storage function

$$V(x(t)) = x^T(t)Px(t) + \int_{-\tau_{max}}^{0} \int_{t+\sigma}^{t} \dot{x}^T(\theta)Q\dot{x}(\theta)d\theta d\sigma$$

and define

$$H_{dj}(x(t),w(t)) \equiv \dot{V}(x(t)) + (C_{1j}x(t) + D_{11j}w(t) + D_{12j}u(t))^T (C_{1j}x(t) + D_{11j}w(t) + D_{12j}u(t))$$
$$- \gamma^2 w^T(t)w(t), \quad j = 1,2,...,N.$$

Define

$$\mu(t) = \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ w(t) \end{bmatrix}, T_j = \begin{bmatrix} PT_{1j}P \\ PT_{2j}P \\ T_{3j}P \end{bmatrix}$$

Then, along the trajectories of the j-th system,

$$H_{dj} = 2x^T(t)P\dot{x}(t) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) - \int_{t-\tau_{max}}^{t} \dot{x}^T(\theta)Q\dot{x}(\theta)d\theta + \tau_{max}\dot{x}^T(t)Q\dot{x}(t)$$
$$+ 2\mu^T(t)T_j \left(x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(\theta)d\theta \right)$$

$$= 2x^T(t)P\left(A_j x(t) + B_{1j}w(t) + B_{2j}FC_2 x(t-\tau(t)) \right) + x^T(t)C_{1j}^T C_{1j}x(t)$$

$$+ 2x^T(t)C_{1j}^T D_{12j}FC_2 x(t-\tau(t)) + x^T(t-\tau(t))C_2^T F^T D_{12j}^T D_{12j}FC_2 x(t-\tau(t))$$

$$+ w^T(t)D_{11j}^T D_{11j}w(t) + 2x^T(t)C_{1j}^T D_{11j}w(t) + 2x^T(t-\tau(t))C_{2j}^T F^T D_{12j}^T D_{11j}w(t)$$

$$- r^2 w^T(t)w(t) - \int_{t-\tau_{max}}^{t} \dot{x}^T(\theta)Q\dot{x}(\theta)d\theta + \tau_{max}x^T(t)A_j^T QA_j x(t)$$

$$+ \tau_{max}w^T(t)B_{1j}^T QB_{1j}w(t) + \tau_{max}x^T(t-\tau(t))C_2^T F^T B_{2j}^T QB_{2j}FC_2 x(t-\tau(t))$$

$$+ 2\tau_{max}x^T(t)A_j^T QB_{1j}w(t) + 2\tau_{max}x^T(t)A_j^T QB_{2j}FC_2 x(t-\tau(t))$$

$$+2\tau_{\max}x^T(t-\tau(t))C_2^TF^TB_{2j}^TQB_{1j}w(t)+2\mu^T(t)T_j\left(x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^t \dot{x}(\theta)d\theta\right) \tag{18}$$

By Lemma 1 and the Jensen integral inequality [33], we can show that

$$-2\mu^T(t)T_j\int_{t-\tau(t)}^t \dot{x}(\theta)\mathrm{d}\theta \le \tau_{\max}\mu^T(t)T_jQ^{-1}T_j^T\mu(t)+\int_{t-\tau(t)}^t \dot{x}^T(\theta)Q\dot{x}(\theta)\mathrm{d}\theta \tag{19}$$

As a result,

$$H_{dj} \le 2x^T(t)P\left(A_jx(t)+B_{1j}w(t)+B_{2j}FC_2x(t-\tau(t))\right)+x^T(t)C_{1j}^TC_{1j}x(t)$$

$$+2x^T(t)C_{1j}^TD_{12j}FC_2x(t-\tau(t))+x^T(t-\tau(t))C_2^TF^TD_{12j}^TD_{12j}FC_2x(t-\tau(t))$$

$$+w^T(t)D_{11j}^TD_{11j}w(t)+2x^T(t)C_{1j}^TD_{11j}w(t)+2x^T(t-\tau(t))C_2^TF^TD_{12j}^TD_{11j}w(t)$$

$$-r^2w^T(t)w(t)-\int_{t-\tau(t)}^t \dot{x}^T(\theta)Q\dot{x}(\theta)\mathrm{d}\theta$$

$$+\tau_{\max}x^T(t)A_j^TQA_jx(t)+\tau_{\max}w^T(t)B_{1j}^TQB_{1j}w(t)$$

$$+\tau_{\max}x^T(t-\tau(t))C_2^TF^TB_{2j}^TQB_{2j}FC_2x(t-\tau(t))+2\tau_{\max}x^T(t)A_j^TQB_{1j}w(t)$$

$$+2\tau_{\max}x^T(t)A_j^TQB_{2j}FC_2x(t-\tau(t))+2\tau_{\max}x^T(t-\tau(t))C_2^TF^TB_{2j}^TQB_{1j}w(t)$$

$$+2\mu^T(t)T_jx(t)-2\mu^T(t)T_jx(t-\tau(t))+\tau_{\max}\mu^T(t)T_jQ^{-1}T_j^T\mu(t)+\int_{t-\tau(t)}^t \dot{x}^T(\theta)Q\dot{x}(\theta)\mathrm{d}\theta$$

$$=\mu^T(t)\begin{bmatrix} \Theta_j & PB_{2j}FC_2+C_{1j}^TD_{12j}FC_2-PT_{1j}P+PT_{2j}^TP & PB_{1j}+PT_{3j}^T+C_{1j}^TD_{11j} \\ * & C_2^TF^TD_{12j}^TD_{12j}FC_2-PT_{2j}P-PT_{2j}^TP & -PT_{3j}^T+C_2^TF^TD_{12j}^TD_{11j} \\ * & * & D_{11j}^TD_{11j}-r^2I \end{bmatrix}\mu(t)$$

$$+\tau_{\max}x^T(t)A_j^TQA_jx(t)+\tau_{\max}w^T(t)B_{1j}^TQB_{1j}w(t)$$

$$+\tau_{max}x^T(t-\tau(t))C_2^TF^TB_{2j}^TQB_{2j}FC_2x(t-\tau(t))+2\tau_{max}x^T(t)A_j^TQB_{1j}w(t)$$

$$+2\tau_{max}x^T(t)A_j^TQB_{2j}FC_2x(t-\tau(t))+2\tau_{max}x^T(t-\tau(t))C_2^TF^TB_{2j}^TQB_{1j}w(t)$$

$$+\tau_{max}\mu^T(t)T_jQ^{-1}T_j^T\mu(t)$$

$$\equiv \mu^T(t)\Omega_j\mu(t)$$

where

$$\Theta_j = PA_j + A_j^TP + C_{1j}^TC_{1j} + PT_{1j}P + PT_{1j}^TP$$

and

$$\Omega_j = \begin{bmatrix} \Theta_j & PB_{2j}FC_2 + C_{1j}^TD_{12j}FC_2 - PT_{1j}P + PT_{2j}^TP & PB_{1j} + PT_{3j}^T + C_{1j}^TD_{11j} \\ * & C_2^TF^TD_{12j}^TD_{12j}FC_2 - PT_{2j}P - PT_{2j}^TP & -PT_{3j}^T + C_2^TF^TD_{12j}^TD_{11j} \\ * & * & D_{11j}^TD_{11j} - r^2I \end{bmatrix}$$
$$+ \begin{bmatrix} \tau_{max}A_j^TQA_j & \tau_{max}A_j^TQB_{2j}FC_2 & \tau_{max}A_j^TQB_{1j} \\ * & \tau_{max}C_2^TF^TB_{2j}^TQB_{2j}FC_2 & \tau_{max}C_2^TF^TB_{2j}^TQB_{1j} \\ * & * & \tau_{max}B_{1j}^TQB_{1j} \end{bmatrix} + \tau_{max}T_jQ^{-1}T_j^T$$

By noting (17) and the Schur complement, we know that $\Omega_j < 0$ if $\hat{\Omega}_j < 0$, where

$$\hat{\Omega}_j = \begin{bmatrix} \Psi_j & \delta_j & PB_{1j} + PT_{3j}^T + C_{1j}^TD_{11j} & \tau_{max}A_j^T & C_{1j}^T & \tau_{max}PT_{1j}P \\ * & -PT_{2j}P - PT_{2j}^TP & -PT_{3j}^T + C_2^TF^TD_{12j}^TD_{11j} & \tau_{max}C_2^TF^TB_{2j}^T & C_2^TF^TD_{12j}^T & \tau_{max}PT_{2j}P \\ * & * & D_{11j}^TD_{11j} - r^2I & \tau_{max}B_{1j}^T & 0 & \tau_{max}T_{3j}P \\ * & * & * & -\tau_{max}Q^{-1} & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\tau_{max}Q \end{bmatrix}$$

with

$$\Psi_j = PA_j + A_j^T P + PT_{1j} P + PT_{1j}^T P$$
$$\delta_j = PB_{2j} FC_2 - PT_{1j} P + PT_{2j}^T P$$

Moreover, $\hat{\Omega}_j < 0$ if and only if $\widetilde{\Omega}_j < 0$, where $\widetilde{\Omega}_j$ is the matrix obtained by pre- and post-multiplying $\hat{\Omega}_j$ by $diag\{S \quad S \quad I \quad I \quad I \quad S\}$:

$$\widetilde{\Omega}_j = \begin{bmatrix} S\Psi_j S & S\delta_j S & B_{1j} + T_{3j}^T + SC_{1j}^T D_{11j} & \tau_{max} SA_j^T & SC_{1j}^T & \tau_{max} T_{1j} \\ * & -T_{2j} - T_{2j}^T & -T_{3j}^T + SC_2^T F^T D_{12j}^T D_{11j} & \tau_{max} SC_2^T F^T B_{2j}^T & SC_2^T F^T D_{12j}^T & \tau_{max} T_{2j} \\ * & * & D_{11j}^T D_{11j} - r^2 I & \tau_{max} B_{1j}^T & 0 & \tau_{max} T_{3j} \\ * & * & * & -\tau_{max} Q^{-1} & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\tau_{max} SQS \end{bmatrix}$$

By Lemma 2, it follows that $\widetilde{\Omega}_j < 0$ (and then $\Omega_j < 0$) if (16) and (17) hold. This proves that the feedback law (8) with $F = L M^{-1}$ is a simultaneous static output feedback H_∞ controller for all the systems in (1). ∎

4. An illustrative example

Suppose that a control system operates at three different operating points. The dynamics at these operating points are different. Suppose that it behaves in the following three possible modes:

$$\dot{x}(t) = A_j x(t) + B_{1j} w(t) + B_{2j} u(t)$$
$$z(t) = C_{1j} x(t) + D_{11j} w(t) + D_{12j} u(t), \, j = 1, 2, 3 \tag{20}$$
$$y(t) = C_2 x(t)$$

where

$$A_1 = \begin{bmatrix} 0.211 & -1.471 & -0.361 \\ -0.585 & -1.683 & 0.729 \\ -1.811 & 0.64 & -2.287 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.696 \\ 0.385 \\ 0.176 \end{bmatrix}, B_{21} = \begin{bmatrix} -1.824 \\ -1.182 \\ 2.564 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 0.686 & -0.421 & -2.211 \end{bmatrix}, D_{111} = 1.164, D_{121} = 0.665$$

$$C_2 = \begin{bmatrix} 0.657 & 0.265 & -1.288 \\ -0.439 & 0.336 & -0.246 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.406 & -1.525 & 0.321 \\ 0.625 & -1.145 & 1.239 \\ -4.185 & 1.212 & -1.431 \end{bmatrix}, B_{12} = \begin{bmatrix} -0.559 \\ 0.47 \\ -0.679 \end{bmatrix}, B_{22} = \begin{bmatrix} -0.591 \\ 1.521 \\ 2.351 \end{bmatrix},$$

$$C_{12} = \begin{bmatrix} -0.829 & 0.451 & 2.395 \end{bmatrix}, D_{112} = -1.523, D_{122} = -0.414,$$

$$C_2 = \begin{bmatrix} 0.657 & 0.265 & -1.288 \\ -0.439 & 0.336 & -0.246 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.121 & -1.235 & 0.261 \\ 0.625 & -0.733 & 0.639 \\ -2.106 & 0.55 & 1.147 \end{bmatrix}, B_{13} = \begin{bmatrix} 0.509 \\ 0.585 \\ -0.776 \end{bmatrix}, B_{23} = \begin{bmatrix} -0.824 \\ 1.202 \\ 3.514 \end{bmatrix},$$

$$C_{13} = \begin{bmatrix} 0.686 & -0.421 & -2.211 \end{bmatrix}, D_{113} = 1.164, D_{123} = 0.569,$$

$$C_2 = \begin{bmatrix} 0.657 & 0.265 & -1.288 \\ -0.439 & 0.336 & -0.246 \end{bmatrix}.$$

We want to design a static output feedback event-triggered H_∞ controller that is able to L_2-stabilize the system at all the three possible operating points with $\gamma = 7$. Suppose that the minimal and maximal transmission delays are $\tau_{d\min} = 0.1$ms and $\tau_{d\max} = 0.45$ms, respectively. We first need to derive a conventional simultaneous static output feedback H_∞ controller for all the modes in (20) and then, based on the obtained controller, we can obtain an event-triggered transmission policy.

Given $\lambda = 0.6$ and $\tau_{\max} = 0.1\ s$, by solving (16) and (17) we can get a simultaneous H_∞ controller

$$u(t) = Fy(t - \tau(t)) = \begin{bmatrix} 0.885 & -1.559 \end{bmatrix} y(t - \tau(t))$$

With this controller, by solving (10) we can get solutions:

$$P_1 = \begin{bmatrix} 112.141 & -30.286 & -9.24 \\ -30.286 & 113.675 & 14.086 \\ -9.24 & 14.086 & 47.207 \end{bmatrix} > 0 \quad P_2 = \begin{bmatrix} 60.909 & -1.957 & 8.043 \\ -1.957 & 42.793 & -1.25 \\ 8.043 & -1.25 & 71.935 \end{bmatrix} > 0$$

$$P_3 = \begin{bmatrix} 129.678 & -14.921 & -18.771 \\ -14.921 & 63.544 & -18.135 \\ -18.771 & -18.135 & 40.175 \end{bmatrix} > 0 \quad Q_1 = \begin{bmatrix} 297.0174 & -97.1611 & 42.8020 \\ -97.1611 & 345.4888 & 58.5580 \\ 42.8020 & 58.5580 & 134.3278 \end{bmatrix} > 0$$

$$Q_2 = \begin{bmatrix} 157.5111 & 14.4987 & 6.6046 \\ 14.4987 & 120.8614 & 11.6833 \\ 6.6046 & 11.6833 & 117.3282 \end{bmatrix} > 0 \quad Q_3 = \begin{bmatrix} 282.727 & -17.226 & -12.768 \\ -17.226 & 61.7053 & -10.87 \\ -12.768 & -10.87 & 103.349 \end{bmatrix} > 0$$

$$G_{11} = \begin{bmatrix} -217.2794 & 119.9043 & 11.8195 \\ 39.8052 & 12.3064 & -28.9204 \\ 46.2486 & -53.3922 & -72.2522 \end{bmatrix} \quad G_{21} = \begin{bmatrix} 217.253 & -42.004 & 10.029 \\ 19.619 & 73.242 & 0.249 \\ -88.031 & 31.144 & 156.995 \end{bmatrix}$$

$$G_{31} = \begin{bmatrix} -9.24 & 14.086 & 47.207 \\ 60.91 & -1.957 & 42.793 \end{bmatrix} \quad G_{41} = \begin{bmatrix} -40.504 & 9.752 & 16.069 \end{bmatrix}$$

$$G_{12} = \begin{bmatrix} -134.794 & 35.386 & 65.797 \\ 73.976 & -46.104 & -60.232 \\ 118.174 & -34.092 & -46.002 \end{bmatrix} \quad G_{22} = \begin{bmatrix} 170.595 & -39.806 & -49.551 \\ -7.746 & 56.83 & 17.2 \\ -21.124 & 16.814 & 140.097 \end{bmatrix}$$

$$G_{32} = \begin{bmatrix} -9.2402 & 14.0864 & 47.2070 \\ 60.9096 & -1.9572 & 42.7932 \end{bmatrix} \quad G_{42} = \begin{bmatrix} -7.001 & -8.089 & 21.297 \end{bmatrix}$$

$$G_{13} = \begin{bmatrix} -150.599 & 38.683 & -6.694 \\ 69.354 & -43.986 & -33.385 \\ 126.071 & -47.435 & -130.298 \end{bmatrix} \quad G_{23} = \begin{bmatrix} 258.559 & -32.327 & -49.367 \\ -40.332 & 58.951 & 15.119 \\ -142.399 & 35.247 & 141.214 \end{bmatrix}$$

$$G_{33} = \begin{bmatrix} -9.24 & 14.086 & 47.207 \\ 60.91 & -1.957 & 42.793 \end{bmatrix} \quad G_{43} = \begin{bmatrix} -78.396 & 1.269 & 45.971 \end{bmatrix}$$

$$\varepsilon_1 = 38.2561, \ \varepsilon_2 = 72.7127, \text{ and } \varepsilon_3 = 72.8613$$

According to Theorem 1 and Remark 1, the event-triggered policy is (let $\eta = 0.99$):

$$\|e(t)\| \geq \eta \cdot \min_{j \in \{1,2,3\}} \frac{1}{\sqrt{\varepsilon_j}} \cdot \|y(t)\| = 0.1116 \|y(t)\| \tag{21}$$

With the triggering condition (21), the sensor node can determine whether the currently measured data must be transmitted. If the currently measured data is such that condition (21) is violated, it will be discarded for reducing network usage. If the measured data is such that condition (21) holds, it will be sent to the controller node for updating the control signal.

By simulation, for guaranteeing the simultaneous L_2–gain stability, the number of transmission events at the sensor node of the first system is 64 in the first 10 s. The average inter-transmitting time is 0.1563 s. The number of transmission events at the sensor node of the second system is 585. The average inter-transmitting time is 0.0171 s. The number of transmission events at the sensor node of the third system is 595. The average inter-transmitting time is 0.0168 s. **Figures 1–3** are the responses of the event-triggered and non-event-triggered closed-loop systems under the same initial condition $x(0)=[1 \ \ -1 \ \ 1]^T$ and the same exogenous disturbance $w(t)=(3\sin(8t)+2\cos(5t))\times e^{-0.5t}$ (shown in **Figure 4**). It is clear that the proposed event-triggered policy guarantees simultaneous L_2–gain stability under low network usages. Moreover, it can be seen that the responses of closed-loop systems controlled by the event-triggered controller and the non-event-triggered controller are almost the same. That is, the obtained event-triggered controller, in a very low network usage rate, can perform almost the same control performance as the conventional non-event-triggered controller. A low network usage rate will in general lead to a good quality of network service.
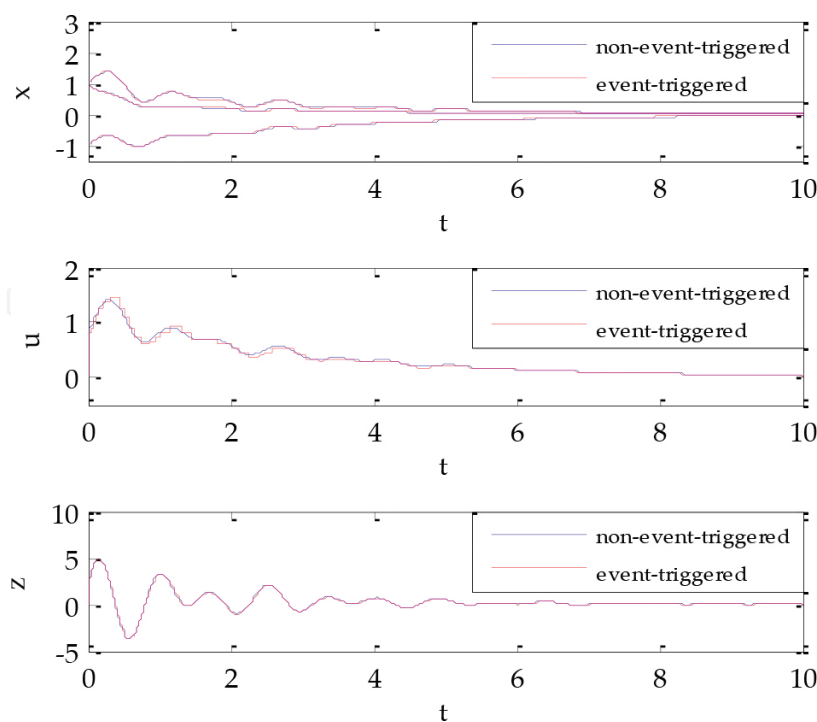

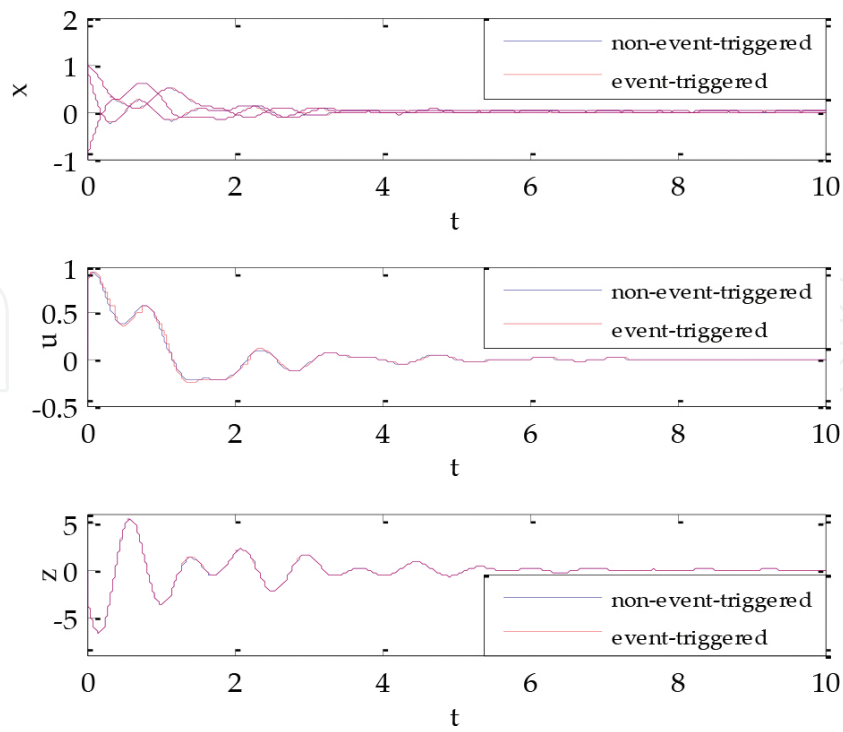**Figure 1.** Responses of the first closed-loop NCS.
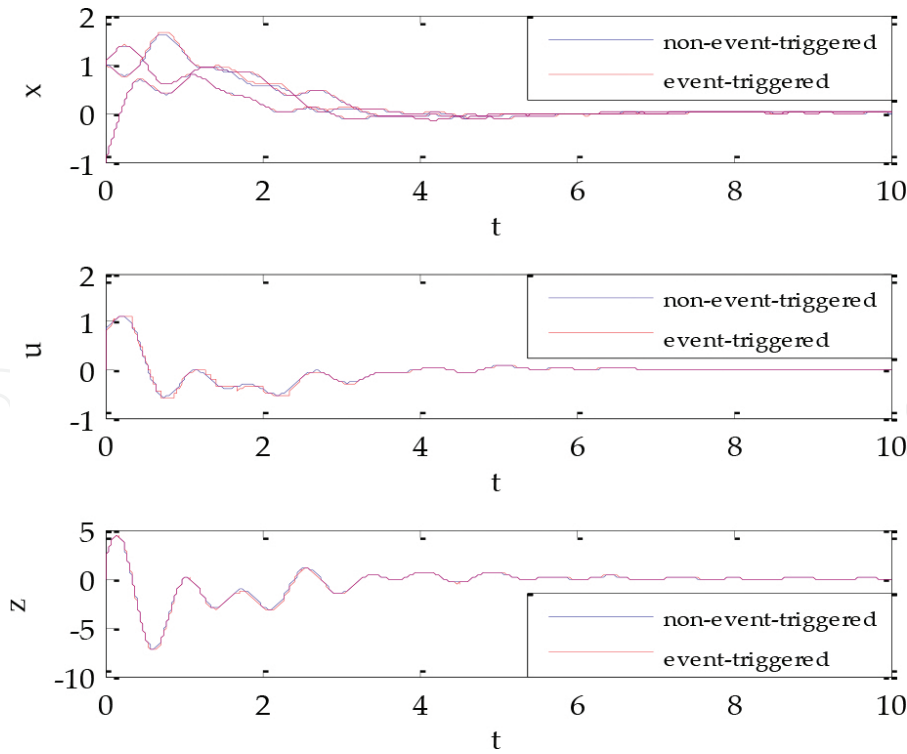
Figure 2. Responses of the second closed-loop NCS.
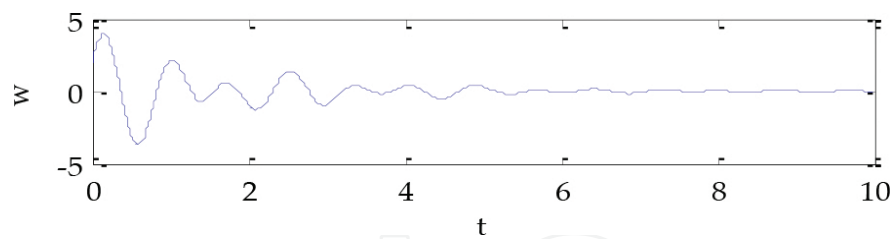

Figure 3. Responses of the third closed-loop NCS.

Figure 4. Disturbance input.

5. Conclusions

In this chapter, we develop an event-triggered static output feedback simultaneous H_∞ transmission policy for NCSs under time-varying transmission delay. With the proposed method, we do not need to switch controllers or event-triggered policies for an NCS with several different operating points. Moreover, the reliability of NCSs can be improved as possible element failures can be accommodated. The implementation of the obtained event-triggered simultaneous H_∞ controller is easy as it is in the static output feedback framework. One weakness of our result is that the conditions for the existence of static output feedback simultaneous H_∞ controllers are represented in terms of LMIs with a LME constraint. Standard LMI tools cannot be directly applied to find the solutions. Possible issues for further study include finding less conservative event-triggered transmission policies, considering the possibility of packet dropouts, and relaxing the continuous monitoring requirement at the sensor node by replacing the event-triggered scheme with a self-triggered one.

Acknowledgements

Nomenclatures

R^n real vector of dimension n.

$R^{n \times m}$ real $n \times m$ matrix.

$\|\cdot\|$ the Euclidean vector norm.

M^T (resp., M^{-1}) the transpose (resp., inverse) of matrix M.

$M > 0$ (resp., $M \geq 0$) the matrix M is positive definite (resp., positive semidefinite).

$M = \begin{bmatrix} A & B \\ * & C \end{bmatrix}$ the symbol * denotes the symmetric terms in a symmetric matrix

I the identity matrix of appropriate dimension.

$\text{diag}\{\cdots\}$ the block diagonal matrix.

$\min z(\cdot)$ the minimum value of $z(\cdot)$.

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