

# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

186,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index  
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?  
Contact [book.department@intechopen.com](mailto:book.department@intechopen.com)

Numbers displayed above are based on latest data collected.  
For more information visit [www.intechopen.com](http://www.intechopen.com)



---

# Compton Effect in Graphene and in the Graphene-Like Dielectric Medium

Miroslav Pardy

Additional information is available at the end of the chapter

---

## Abstract

In the introduction and the second part of the chapter, we discuss the Compton effect in general, and the modern viewpoint on the 2-dimensional carbon crystals called graphene, where graphene unique properties arise from the collective behavior of electrons governed by the Dirac equation. The Dirac equation in graphene physics is used for so called pseudospin of pseudoelectron formed by the hexagonal lattice composed of the systems of two equilateral triangles with the corresponding particular wave functions. The total wave function of an electron moving in the hexagonal system is superposition of the particular wave functions. The crucial step in the graphene physics is the definition of the new spinor function where spinor function is solution of the Pauli equation in the nonrelativistic situation and Dirac equation of the generalized case. The corresponding mass of such effective electron is proved to be approximately zero.

In the third part of the chapter, we deal with the Dirac equation and its Volkov solution and in the fourth part of the chapter, we discuss the Volkov solution in a dielectric medium.

The fifth part of the chapter, deals with the Compton effect derived from Volkov solution of the Dirac equation while the sixth part of the chapter, deals with the calculation of the Compton effect with ultrashort laser pulse, where the pulse is of the Dirac delta-function form.

The seven part of the chapter, deals with Compton effect initiated by two orthogonal plane waves. We solve the Dirac equation for two different four-potentials of the plane electromagnetic waves and we specify the solutions of the Dirac equation for two orthogonal plane waves. The modified Compton formula for the scattering of two photons on an electron is determined.

The conclusion eighth part involves possible perspectives of the Compton effect with regard to the scientific and technological meaning of the results derived in our contribution.

**Keywords:** Compton effect, Dirac equation, Volkov solution, dielectric, laser pulse, graphene, silicene.

## 1. Introduction

The Compton effect is the light-particle interaction where the wavelength of scattered photon is changed. The difference between the Compton effect and the Einstein photoeffect consists in the fact that during the photoeffect, the energy of photon is transmitted to electron totally.

Compton used in his original experiment [1] the energy of the X-ray photon ( $\sim 20$  keV) which was very much larger than the binding energy of the atomic electron, so the electrons could be treated as being free. Compton scattering usually refers to the interaction involving only the electrons of an atom. However, the nuclear Compton effect was confirmed too. The effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon. The Compton experiment proved that light is composed of particle-like objects with energy  $E = \hbar\omega$ . The interaction between electrons and high-energy photons is such that the electron takes part of the photon initial energy, and a photon containing the remaining energy is emitted in a different direction from the original. If the scattered photon still has enough energy left, the process may be repeated.

Compton, in his paper [1], derived a simple formula relating the shift of wavelength to the scattering angle of the X-rays by postulating that each scattered X-ray photon interacted with only one electron. His paper involves the information on experiments for verification of the equation:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos \theta), \quad (1)$$

where  $\lambda'$  is wavelength of the scattered X-ray and  $\theta$  is the angle between the incident and scattered X-ray. The scattering was considered in the laboratory frame where electron was at rest. Let us remark immediately that eq. (1) has also a limit for  $m \rightarrow 0$ , if and only if  $\theta \rightarrow 0$ . It corresponds to the situation of graphene sheet, where the mass of the so-called pseudoelectron can be considered zero. This limit can be verified immediately by the visible light and not by the X-rays. In case of using the X-rays in graphene, we get still the original Compton process with the real electrons (ionization process in graphene) and not the process with the so-called pseudoelectrons. The considered process was the so-called one-photon process with the symbolic equation

$$\gamma + e \rightarrow \gamma + e. \quad (2)$$

The differential cross section corresponding to eq. (2) was derived by Klein and Nishina in the form

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \frac{e^4}{m^2 c^4} \left( \frac{\omega'}{\omega} \right)^2 \left( \frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2 \theta \right). \quad (3)$$

It is the ratio of the number of scattered photons into the unit solid angle  $\Omega$  over the number of incident photons. At the present time with the high-power lasers, there is a possibility

to realize so-called multiphoton scattering according to equation  $N\gamma + e \rightarrow M\gamma + e$ , where  $N$  and  $M$  are numbers of photons participating in the scattering.  $N$  photons are absorbed at a single point and, after some time,  $M$  photons are emitted at the distant point. Let us remark that in case of the Raman effect, the equation describing the Raman process is  $\gamma + A \rightarrow \gamma + A^*$  where  $A$  denotes atom, or molecule, and  $A^*$  denotes excited atom, or excited molecule. This Raman process involves also the interaction of the fullerene  $C_{60}$  with photons.

Equation (3) has also limit for  $m \rightarrow 0$ , if and only if the angle  $\theta$  is solution of eq.  $(\omega'/\omega + \omega/\omega' - \sin^2 \theta) = 0$ . Such limit corresponds to the Compton effect in graphene with zero mass of pseudoelectron.

We calculate the Compton process as a result of the Volkov solution [2] of the 3D and 2D Dirac equation. The Volkov solution involves not only the one-photon scattering but also the multiphoton scattering of photons on electron. In the time of the Compton experiment in 1922, the Volkov solution was not known, because the Dirac equation was published in 1928 and the Volkov solution in 1935 [2].

In the next part of the chapter, we discuss the modern viewpoint on the two-dimensional carbon crystals – graphene. Then we discuss the Dirac equation with plane wave and its Volkov solution including dielectric medium. Then we deal with the calculation of the Compton effect with ultrashort laser pulse in graphene and graphene-like sheet as silicene. We consider the case that a charged particle moves in the parallel direction to the silicene sheet. We include also the classical situation of a charged particle accelerated by the  $\delta$ -function form of impulsive force. Then, we discuss the corresponding quantum theory based on the Volkov solution of the Dirac equation. The modified Compton formula for frequency of photons generated by the laser pulse is derived. The discussion is extended to the Dirac equation for two different waves, and the Volkov solution is then determined for the orthogonal two plane waves. The last part of the chapter is the conclusion, where we consider the scientific and technological perspectives of the results derived in our contribution.

## 2. Graphene

Graphene is a two-dimensional carbon sheet which is very consistent (100 times stronger than steel). It is a very good conductor of heat and electricity. Graphene was investigated by theorists for decades; however, it was first generated in the laboratory in 2003. Being two-dimensional, it interacts in a special way with light and with other materials. Researchers have discovered the bipolar transistor effect, the charge ballistic transport, and the large quantum oscillations.

Technically, graphene is a crystalline allotrope of carbon with two-dimensional geometrical properties. More than 70 years ago, Peierls [3] and Landau [4] proved that the two-dimensional crystal is not stable from the viewpoint of thermodynamics and cannot exist. The thermodynamic displacements of atoms in such a crystal are of the same size as the distances between atoms at any finite temperature. Mermin [5] accepted the theoretical arguments in 1968, and it seemed that many experimental observations were in accord with the Landau-Peierls-Mermin theory.

However, in 2004, Geim and Novoselov [6], [7] with co-workers at the University of Manchester in England produced a crystalline sheet of carbon just one-atom thick. Then,

the Geim group was able to isolate graphene and was able to visualize the new crystal medium using a simple optical microscope. The Landau-Peierls-Mermin proof remained as the historical document.

After some time, the new sophisticated methods generating graphene sheets were invented. The graphene sheets were, e.g., synthesized by passing liquid ethanol droplets into an argon plasma. The authors of this method are Dato et al. [8].

Graphene is composed of the benzene rings ( $C_6H_6$ ) without their H-atoms. Graphene is only one of the crystalline forms of carbon which crystallize as diamond, graphite, fullerene ( $C_{60}$ ), carbon nanotube, and glassy carbon.

Unique physical properties of graphene are caused by the collective behavior of the quasiparticles called pseudoelectrons having pseudospins, which move according to the Dirac equation in the hexagonal lattice.

The Dirac fermions in graphene carry unit electric charge. Strong interactions between the electrons and the carbon atoms result in linear dispersion relation  $E = v_F|\mathbf{p}|$ , where  $v_F$  is the so-called Fermi-Dirac velocity,  $\mathbf{p}$  being the momentum of a pseudoelectron. The Fermi velocity is approximately only about 300 times less than the speed of light.

The pseudospin of the pseudoelectron follows from the hexagonal form of graphene. Every hexagonal cell system is composed of the systems of two equilateral triangles. The fermions in the triangle sub-lattice systems can be described by the wave functions  $\varphi_1$  and  $\varphi_2$ . Then the adequate wave function of the fermion moving in the hexagonal structure is their superposition, or  $\psi = c_1\varphi_1 + c_2\varphi_2$ , where  $c_1$  and  $c_2$  are functions of coordinate  $\mathbf{x}$  and functions  $\varphi_1, \varphi_2$  are functions of the wave vector  $\mathbf{k}$  and coordinate  $\mathbf{x}$ . The crucial step in graphene theory is the definition of the bispinor function with components  $\varphi_1, \varphi_2$  [9].

The relativistic generalization of nonrelativistic equation  $E = v_F|\mathbf{p}|$  is evidently the Dirac-Weyl equation for the description of neutrino which can be transcribed in four-component spinor form as

$$p_\mu \gamma^\mu = 0 \quad (4)$$

and it is possible to prove that this spinor function is solution of the Pauli equation in the nonrelativistic situation. The corresponding mass of such effective electron is proved approximately to be zero. So, it follows from this formalism that to describe the Compton effect on graphene is to solve the Compton effect with quasielectron with zero mass.

The introduction of the Dirac relativistic Hamiltonian in graphene physics has the physical meaning that we describe the graphene physics by means of electron-hole medium. It is the analogue of the Dirac theory of the electron-positron vacuum in quantum electrodynamics. However, the pseudoelectron and pseudospin in graphene physics are not an electron and the spin of quantum electrodynamics (QED), because QED is the relativistic quantum theory of the interaction of real electrons and photons where mass of an electron is defined by classical mechanics and not by collective behavior in the hexagonal sheet called graphene.

The graphene sheet can be considered as the special form of the more general 2D-graphene-like sheets, where, for instance, silicene has the similar structure as graphene [10].

On the other hand, amorphous solids – glasses – lack long-range translational periodicity in the atomic structure. However, due to chemical bonds, glasses do possess a high degree of short-range order with respect to local atomic polyhedra. It means that such structures can be considered as the graphene-like structures with the appropriate index of refraction, being necessary for the existence of the Čerenkov effect and the Compton effect in dielectric medium.

The last but not least graphene-like structure can be represented by graphene-based polaritonic crystal sheet [11], which can be used for the Čerenkov effect and the Compton effect in the graphene-like dielectric medium with the index of refraction  $n$ .

### 3. Volkov solution of the Dirac equation in vacuum

Volkov solution of the Dirac equation is the mathematical solution of the Dirac equation with the plane-wave potential. The derivation of the Volkov solution of the Dirac equation in vacuum is described in the textbook of Berestetskii et al. [12]. The four-potential in the Dirac equation

$$(\gamma(p - eA) - m)\psi = 0, \quad (5)$$

is

$$A^\mu = A^\mu(\varphi); \quad \varphi = kx. \quad (6)$$

We suppose that the four-potential satisfies the Lorentz gauge condition

$$\partial_\mu A^\mu = k_\mu (A^\mu)' = (k_\mu A^\mu)' = 0, \quad (7)$$

where the prime denotes derivation with regard to  $\varphi$ . From the last equation follows

$$kA = \text{const} = 0, \quad (8)$$

because we can put the constant to zero. The tensor of electromagnetic field is

$$F_{\mu\nu} = k_\mu A'_\nu - k_\nu A'_\mu. \quad (9)$$

Instead of the linear Dirac equation (5), we consider the quadratic equation, which we get by multiplying the linear equation by operator  $(\gamma(p - eA) + m)$ , [12]. We get

$$\left[ (p - eA)^2 - m^2 - \frac{i}{2} e F_{\mu\nu} \sigma^{\mu\nu} \right] \psi = 0. \quad (10)$$

Using  $\partial_\mu(A^\mu\psi) = A^\mu\partial_\mu\psi$ , which follows from eq. (7), and  $\partial_\mu\partial^\mu = \partial^2 = -p^2$ , with  $p_\mu = i(\partial/\partial x^\mu) = i\partial_\mu$ , we get the quadratic Dirac equation for the four-potential of the plane wave:

$$[-\partial^2 - 2i(A\partial) + e^2A^2 - m^2 - ie(\gamma k)(\gamma A')]\psi = 0. \quad (11)$$

We are looking for the solution of the last equation in the form

$$\psi = e^{-ipx}F(\varphi). \quad (12)$$

After insertion of this relation into (11), we get with ( $k^2 = 0$ )

$$\partial^\mu F = k^\mu F', \quad \partial_\mu\partial^\mu F = k^2 F'' = 0, \quad (13)$$

the following equation for  $F(\varphi)$ :

$$2i(kp)F' + [-2e(pA) + e^2A^2 - ie(\gamma k)(\gamma A')]F = 0. \quad (14)$$

The integral of the last equation is of the form

$$F = \exp \left\{ -i \int_0^{kx} \left[ \frac{e(pA)}{(kp)} - \frac{e^2}{2(kp)} A^2 \right] d\varphi + \frac{e(\gamma k)(\gamma A)}{2(kp)} \right\} \frac{u}{\sqrt{2p_0}}, \quad (15)$$

where  $u/\sqrt{2p_0}$  is the arbitrary constant bispinor.

All powers of  $(\gamma k)(\gamma A)$  above the first are equal to zero, since

$$(\gamma k)(\gamma A)(\gamma k)(\gamma A) = -(\gamma k)(\gamma k)(\gamma A)(\gamma A) + 2(kA)(\gamma k)(\gamma A) = -k^2 A^2 = 0. \quad (16)$$

Then we can write

$$\exp \left\{ e \frac{(\gamma k)(\gamma A)}{2(kp)} \right\} = 1 + \frac{e(\gamma k)(\gamma A)}{2(kp)}. \quad (17)$$

So, the solution is of the form

$$\psi_p = R \frac{u}{\sqrt{2p_0}} e^{iS} = \left[ 1 + \frac{e}{2kp} (\gamma k)(\gamma A) \right] \frac{u}{\sqrt{2p_0}} e^{iS}, \quad (18)$$

where  $u$  is an electron bispinor of the corresponding Dirac equation

$$(\gamma p - m)u = 0, \quad (19)$$

with the normalization condition  $\bar{u}u = 2m$ .

The mathematical object  $S$  is the classical Hamilton-Jacobi function, which was determined in the form

$$S = -px - \int_0^{kx} \frac{e}{(kp)} \left[ (pA) - \frac{e}{2}(A)^2 \right] d\varphi. \quad (20)$$

The current density is

$$j^\mu = \bar{\psi}_p \gamma^\mu \psi_p, \quad (21)$$

where  $\bar{\psi}_p$  is defined as the transposition of (18), or

$$\bar{\psi}_p = \frac{\bar{u}}{\sqrt{2p_0}} \left[ 1 + \frac{e}{2kp} (\gamma A)(\gamma k) \right] e^{-iS}. \quad (22)$$

After insertion of  $\psi_p$  and  $\bar{\psi}_p$  into the current density, we have

$$j^\mu = \frac{1}{p_0} \left\{ p^\mu - eA^\mu + k^\mu \left( \frac{e(pA)}{(kp)} - \frac{e^2 A^2}{2(kp)} \right) \right\}, \quad (23)$$

which is in agreement with the formula in the Meyer article [13].

If  $A^\mu(\varphi)$  are periodic functions, and their time-average value is zero, then the mean value of the current density is

$$\bar{j}^\mu = \frac{1}{p_0} \left( p^\mu - \frac{e^2}{2(kp)} \overline{A^2 k^\mu} \right) = \frac{q^\mu}{p_0}. \quad (24)$$

#### 4. Volkov solution in a dielectric medium

The mathematical approach to the situation where we consider plane wave solution in a medium is the same, only with the difference that the Lorentz condition must be replaced according to Schwinger et al. by the following one [14]:

$$\partial_\mu A^\mu = kA' = (\mu\varepsilon - 1)(\eta\partial)(\eta A) = (\mu\varepsilon - 1)(\eta k)(\eta A') \quad (25)$$

with the specification  $\eta^\mu = (1, \mathbf{0})$  as the unit time-like vector in the rest frame of the medium [14].

For periodic potential  $A^\mu$ , we then get from eq. (25) instead of  $kA = 0$  the following equation:

$$kA = (\mu\varepsilon - 1)(\eta k)(\eta A). \quad (26)$$

Then, we get instead of eq. (14) the following equation for function  $F(\varphi)$ :

$$2i(kp)F' + [-2e(pA) + e^2A^2 - ie(\gamma k)(\gamma A') - ie(\mu\varepsilon - 1)(\eta k)(\eta A')]F = 0. \quad (27)$$

The solution of the last equation is the solution of the linear equation of the form  $y' + Py = 0$ , and it means it is of the form  $y = C \exp(-\int P dx)$ , where  $C$  is some constant. So, we can write the solution as follows:

$$F = \exp \left\{ -i \int_0^{kx} \left[ \frac{e}{(kp)}(pA) - \frac{e^2}{2(kp)}A^2 \right] d\varphi + \frac{e(\gamma k)(\gamma A)}{2(kp)} + \frac{e}{2(kp)}\alpha \right\} \frac{u}{\sqrt{2p_0}}, \quad (28)$$

where

$$\alpha = (\mu\varepsilon - 1)(\eta k)(\eta A) \quad (29)$$

The wave function  $\psi$  is then the modified wave function (18), which we can write in the form

$$\psi_p = \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{e}{2(kp)} \right)^n (2\alpha)^{n-1} (\gamma k)(\gamma A) \right] \frac{u}{\sqrt{2p_0}} e^{iS} e^T, \quad (30)$$

where

$$T = \frac{e}{2(kp)} (\mu\varepsilon - 1)(\eta k)(\eta A), \quad (31)$$

and where we used in the last formula the following relation:

$$[(\gamma k)(\gamma A)]^n = (2\alpha)^{n-1} (\gamma k)(\gamma A). \quad (32)$$

So, we see that the influence of the medium on the Volkov solution is involved in  $\exp(T)$ , where  $T$  is given by eq. (31) and in the new term which involves the sum of the infinite number of coefficients.

## 5. Compton effect from Volkov solution

We determine the Compton process in vacuum as a result of the Volkov solution of the Dirac equation. The Volkov solution involves not only the one-photon scattering but also the multiphoton scattering of photons on electron. In the time of the Compton experiment in 1922, the Volkov solution was not known, because the Dirac equation was published in 1928 and the Volkov solution in 1935.

Now, let us consider electromagnetic monochromatic plane wave which is polarized in circle. We write the four-potential in the form

$$A = a_1 \cos \varphi + a_2 \sin \varphi, \quad (33)$$

where the amplitudes  $a_i$  are the same and mutually perpendicular, or

$$a_1^2 = a_2^2 = a^2, \quad a_1 a_2 = 0. \quad (34)$$

The Volkov solution for the standard vacuum situation is of the form

$$\begin{aligned} \psi_p = & \left\{ 1 + \left( \frac{e}{2(kp)} \right) [(\gamma k)(\gamma a_1) \cos \varphi + (\gamma k)(\gamma a_2) \sin \varphi] \right\} \frac{u(p)}{\sqrt{2q_0}} \times \\ & \exp \left\{ -ie \frac{(a_1 p)}{(kp)} \sin \varphi + ie \frac{(a_2 p)}{(kp)} \cos \varphi - iqx \right\}, \end{aligned} \quad (35)$$

where

$$q^\mu = p^\mu - e^2 \frac{a^2}{2(kp)} k^\mu. \quad (36)$$

because it follows from eq. (24).

We know that the matrix element  $M$  corresponding to the emission of photon by electron in the electromagnetic field is as follows [12]:

$$S_{fi} = -ie^2 \int d^4x \bar{\psi}_{p'} (\gamma e'^*) \psi_p \frac{e^{ik'x}}{\sqrt{2\omega'}}, \quad (37)$$

where  $\psi_p$  is the electron wave function before the interaction of electron with the laser pulse and  $\psi_{p'}$  is the electron wave function after emission of photon with components  $k'^\mu = (\omega', \mathbf{k}')$ . Symbol  $e'^*$  is the four polarization vector of emitted photon.

Then, we get the following linear combination in the matrix element:

$$e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \quad (38)$$

$$e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \cos \varphi \quad (39)$$

$$e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \sin \varphi. \quad (40)$$

where

$$\alpha_1 = e \left( \frac{a_1 p}{k p} - \frac{a_1 p'}{k p'} \right), \quad (41)$$

and

$$\alpha_2 = e \left( \frac{a_2 p}{k p} - \frac{a_2 p'}{k p'} \right). \quad (42)$$

Now, we can expand the exponential function into the Fourier transformation where the coefficients of the expansion will be  $B_s, B_{1s}, B_{2s}$ . So we write

$$e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} = e^{-i\sqrt{\alpha_1^2 + \alpha_2^2} \sin(\varphi - \varphi_0)} = \sum_{s=-\infty}^{\infty} B_s e^{-is\varphi} \quad (43)$$

$$e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \cos \varphi = e^{-i\sqrt{\alpha_1^2 + \alpha_2^2} \sin(\varphi - \varphi_0)} \cos \varphi = \sum_{s=-\infty}^{\infty} B_{1s} e^{-is\varphi} \quad (44)$$

$$e^{-i\alpha_1 \sin \varphi + i\alpha_2 \cos \varphi} \sin \varphi = e^{-i\sqrt{\alpha_1^2 + \alpha_2^2} \sin(\varphi - \varphi_0)} \sin \varphi = \sum_{s=-\infty}^{\infty} B_{2s} e^{-is\varphi}. \quad (45)$$

The coefficients  $B_s, B_{1s}, B_{2s}$  can be expressed by means of the Bessel function as follows [12]:

$$B_s = J_s(z) e^{is\varphi_0} \quad (46)$$

$$B_{1s} = \frac{1}{2} \left[ J_{s+1}(z) e^{i(s+1)\varphi_0} + J_{s-1}(z) e^{i(s-1)\varphi_0} \right] \quad (47)$$

$$B_{2s} = \frac{1}{2i} \left[ J_{s+1}(z)e^{i(s+1)\varphi_0} - J_{s-1}(z)e^{i(s-1)\varphi_0} \right], \quad (48)$$

where the quantity  $z$  is defined in [12] through  $\alpha$  components as follows:

$$z = \sqrt{\alpha_1^2 + \alpha_2^2} \quad (49)$$

and

$$\cos \varphi_0 = \frac{\alpha_1}{z}; \quad \sin \varphi_0 = \frac{\alpha_2}{z} \quad (50)$$

Functions  $B_s, B_{1s}, B_{2s}$  are related to one another as follows:

$$\alpha_1 B_{1s} + \alpha_2 B_{2s} = s B_s, \quad (51)$$

which follows from the well-known relation for Bessel functions:

$$J_{s-1}(z) + J_{s+1}(z) = \frac{2s}{z} J_s(z) \quad (52)$$

The matrix element (37) can be written in the form [12]

$$S_{fi} = \frac{1}{\sqrt{2\omega'2q_02q'_0}} \sum_s M_{fi}^{(s)} (2\pi)^4 i \delta^{(4)}(sk + q - q' - k'), \quad (53)$$

where the  $\delta$ -function involves the law of conservation in the form

$$sk + q = q' + k', \quad (54)$$

where, respecting eq. (24),

$$q^\mu = p^\mu - \frac{e^2}{2(kp)} \overline{A^2} k^\mu. \quad (55)$$

Using the last equation, we can introduce the so-called effective mass of electron immersed in the periodic wave potential as follows:

$$q^2 = m_*^2; \quad m_* = m \sqrt{\left(1 - \frac{e^2}{m^2} \overline{A^2}\right)} \quad (56)$$

Formula (56) represents the mass renormalization of an electron mass in the field  $A$ . In other words, the mass renormalization is defined by the equation

$$m_{\text{phys}} = m_{\text{bare}} + \delta m \quad (57)$$

where  $\delta m$  follows from eq. (56). The quantity  $m_{\text{phys}}$  is the physical mass that an experimenter would measure if the particle were subject to Newton's law  $\mathbf{F} = m_{\text{phys}}\mathbf{a}$ . In case of the periodic field of laser, the quantity  $\delta m$  has the finite value. The renormalization is not introduced here "by hands," but it follows immediately from the formulation of the problem of electron in the wave field.

We can write

$$q^2 = q'^2 = m^2(1 + \xi^2) \equiv m_*^2, \quad (58)$$

where for plane wave (35) with relations (36)

$$\xi = \frac{e}{m} \sqrt{-a^2}. \quad (59)$$

It may be easy to see that eq. (58) has very simple limit for  $m = 0$  which is the graphenic case. Or, in other words, with the help of eq. (59)  $m_*^2(m = 0) = -e^2 a^2$ .

According to [12], the matrix element in (53) is of the form

$$M_{fi}^{(s)} = -e\sqrt{4\pi\bar{u}}(p') \left\{ \left( (\gamma e') - e^2 a^2 \frac{(ke')}{2(kp)} \frac{(\gamma k)}{(kp')} \right) B_s + \right. \\ \left. e \left( \frac{(\gamma a_1)(\gamma k)(\gamma e')}{2(kp')} + \frac{(\gamma e')(\gamma k)(\gamma a_1)}{2(kp)} \right) B_{1s} + \right. \\ \left. e \left( \frac{(\gamma a_2)(\gamma k)(\gamma e')}{2(kp')} + \frac{(\gamma e')(\gamma k)(\gamma a_2)}{2(kp)} \right) B_{2s} \right\} u(p) \quad (60)$$

It is possible to show that the total probability of the emission of photons from unit volume in unit time is [12]

$$W = \frac{e^2 m^2}{4q_0} \sum_{s=1}^{\infty} \int_0^{u_s} \frac{du}{(1+u)^2} \times \\ \left\{ -4J_s^2(z) + \xi^2 \left( 2 + \frac{u^2}{1+u} \right) \left( J_{s+1}^2(z) + J_{s-1}^2(z) - 2J_s^2(z) \right) \right\}, \quad (61)$$

where

$$u = \frac{(kk')}{(kp_0)}, \quad u_s = \frac{2s(kp)}{m_*^2}, \quad z = 2sm^2 \frac{\xi}{\sqrt{1+\xi^2}} \sqrt{\frac{u}{u_s} \left(1 - \frac{u}{u_s}\right)}. \quad (62)$$

Variables  $u_{1,2}$  are to be expressed in terms of variables  $u$  and  $u_s$  from eq. (62).

When  $\xi \ll 1$  (the condition for perturbation theory to be valid), the integrand (61) can be expanded in powers of  $\xi$ . For the first term of the sum  $W_1$ , we get

$$W_1 \approx \frac{e^2 m^2}{4p_0} \xi^2 \int_0^{u_1} \left[ 2 + \frac{u^2}{1+u} - 4 \frac{u}{u_1} \left(1 - \frac{u}{u_1}\right) \right] \frac{1}{(1+u)^2} du =$$

$$\frac{e^2 m^2}{4p_0} \xi^2 \left[ \left(1 - \frac{4}{u_1} - \frac{8}{u_1^2}\right) \ln(1+u_1) + \frac{1}{2} + \frac{8}{u_1} - \frac{1}{2(1+u_1)^2} \right] \quad (63)$$

with  $u_1 \approx 2(kp)/m^2$ . It is possible to determine the second and the next harmonics as an analogy with the Berestetzkiĭ approach; however, the aim of this article was only to illustrate the influence of the dielectric medium on the Compton effect.

Let us consider eq. (54) in the form

$$sk + q - k' = q'. \quad (64)$$

Equation (64) has physical meaning for  $s = 1, 2, \dots, N$ ,  $s, N$  being positive integers.  $s = 1$  means the conservation of energy momentum of the one-photon Compton process and  $s = 2$  of the two-photon Compton process and  $s = N$  means the multiphoton interaction with  $N$  photons of laser beam with an electron. The multiphoton interaction is nonlinear and differs from the situation where electron scatters twice or more as it traverses the laser focus.

By analogy, the original Einstein photoelectric equation must be replaced by the more general multiphoton photoelectric equation in the form

$$s\hbar\omega = \frac{1}{2}mv^2 + E_i, \quad (65)$$

where  $E_i$  is the binding energy of the outermost electron in the atomic system. It means that the ionization effect occurs also in the case that  $\hbar\omega < E_i$  in case that the number of participating photons is  $s > E_i/\hbar\omega$ . We will not solve furthermore this specific problem.

We introduce the scattering angle  $\theta$  between  $\mathbf{k}$  and  $\mathbf{k}'$ . In other words, The scattering angle  $\theta$  is measured with respect to the incident photon direction. Then, with  $|\mathbf{k}| = n\omega$  and  $|\mathbf{k}'| = n\omega'$ , where  $n$  is index of refraction of the dielectric, we get from the squared eq. (64) in the rest system of electron, where  $q = (m_*, 0)$ , the following equation:

$$s \frac{1}{\omega'} - \frac{1}{\omega} = \frac{s}{m_*} (1 - n^2 \cos \theta), \quad (66)$$

which is a modification of the original equation for the Compton process

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m} (1 - \cos \theta). \quad (67)$$

Using relation  $m_*^2(m=0) = -e^2 a^2$ , we get eq. (66) for the situation of the Compton effect in the graphene sheet:

$$s \frac{1}{\omega'} - \frac{1}{\omega} = \frac{s}{m_*(m=0)} (1 - n^2 \cos \theta) = \frac{s}{e\sqrt{-a^2}} (1 - n^2 \cos \theta), \quad (68)$$

So, we see that the last Compton formula differs from the original one only by the existence of the renormalized mass and the occurrence of index of refraction.

We know that the original Compton formula can be written in the form suitable for the experimental verification, namely:

$$\Delta\lambda = 4\pi \frac{\hbar}{mc} \sin^2 \frac{\theta}{2}, \quad (69)$$

which was used by Compton for the verification of the quantum nature of light [1]. The limiting case with  $m \rightarrow 0$  has the appropriate angle limit  $\theta \rightarrow 0$ .

If we consider the Compton process in dielectric, then the last formula goes to the following form:

$$\Delta\lambda = 2\pi \frac{\hbar}{mc} (1 - n^2 \cos \theta). \quad (70)$$

It is evident that relation  $\lambda' - \lambda \geq 0$  follows from eq. (1). However, if we put

$$1 - n^2 \cos \theta \leq 0, \quad (71)$$

or equivalently

$$\frac{1}{n^2} \leq \cos \theta \leq 1, \quad (72)$$

then we see that for some angles determined by eq. (72), the relation  $\lambda' - \lambda \leq 0$  follows. This surprising result is the anomalous Compton effect which is caused by the index of refraction of the medium. To our knowledge, it was not published in the optical or particle journals.

The limiting case with  $m \rightarrow 0$  has the appropriate angle limit  $n^2 \cos \theta \rightarrow 1$ . It means that there is of the angle shift in this case in comparison with the Compton effect in vacuum.

The equation  $sk + q = q' + k'$  is the symbolic expression of the nonlinear Compton effect in which several photons are absorbed at a single point, but only a single high-energy photon is emitted. The second process where electron scatters twice or more as it traverses the laser focus is not considered here. The nonlinear Compton process was experimentally confirmed, for instance, by Bulla et al. [15].

The formula (66) can be also expressed in terms of  $\lambda$  as follows:

$$s\lambda' - \lambda = \frac{2\pi s}{m_*}(1 - n^2 \cos \theta) \quad (73)$$

where we have put  $\hbar = c = 1$ . In the case of the graphene two-dimensional carbon sheet with zero mass of the pseudoelectron, we replace the renormalized mass by  $m_*(m = 0) = e\sqrt{-a^2}$ , which is the new renormalized mass in graphene.

Formula (73) can be used for the verification of the Compton effect in a dielectric medium, and on the other hand, the index refraction follows from it in the following form:

$$n^2 = \frac{1}{\cos \theta} \left[ 1 - \frac{m_*}{2\pi s}(s\lambda' - \lambda) \right]. \quad (74)$$

It means, if we know the  $\theta, \lambda, \lambda', s, m_*$ , we are able to determine the index of refraction of some dielectric medium from the Compton effect. To our knowledge, this method was not published in the optical journals. In the graphene case, we write as usual  $m_*(m = 0) = e\sqrt{-a^2}$  in the last formula.

## 6. Compton effect initiated by a laser pulse

Let us start with the classical theory of interaction of particle with an impulsive force. We idealize the impulsive force by the dirac  $\delta$ -function. Newton's second law for the interaction of a massive particle with mass  $m$  with an impulsive force  $P\delta(t)$  is as follows:

$$m \frac{d^2 x}{dt^2} = P\delta(t), \quad (75)$$

where  $P$  is some constant.

Using the Laplace transform on the last equation, with

$$\int_0^{\infty} e^{-st} x(t) dt = X(s), \quad (76)$$

$$\int_0^{\infty} e^{-st} \ddot{x}(t) dt = s^2 X(s) - sx(0) - s\dot{x}(0), \quad (77)$$

$$\int_0^{\infty} e^{-st} \delta(t) dt = 1, \quad (78)$$

we obtain

$$ms^2 X(s) - msx(0) - m\dot{x}(0) = P. \quad (79)$$

For a particle starting from rest with  $\dot{x}(0) = 0, x(0) = 0$ , we get

$$X(s) = \frac{P}{ms^2}, \quad (80)$$

and using the inverse Laplace transform, we obtain

$$x(t) = \frac{P}{m} t \quad (81)$$

and

$$\dot{x}(t) = \frac{P}{m}. \quad (82)$$

Let us remark that if we express  $\delta$ -function by the relation  $\delta(t) = \dot{H}(t)$ ,  $H$  being defined as a step function, then from eq. (75) follows  $\dot{x}(t) = P/m$ , immediately. The physical meaning of the quantity  $P$  can be deduced from equation  $F = P\delta(t)$ . After  $t$ -integration, we have  $\int F dt = mv = P$ , where  $m$  is mass of a body and  $v$  its final velocity (with  $v(0) = 0$ ). It means that the value of  $P$  can be determined a posteriori and then this value can be used in more complex equations than eq. (75). Of course it is necessary to suppose that  $\delta$ -form of the impulsive force is adequate approximation of the experimental situation.

If we consider the  $\delta$ -form electromagnetic pulse, then we can write

$$F_{\mu\nu} = a_{\mu\nu} \delta(\varphi). \quad (83)$$

where  $\varphi = kx = \omega t - \mathbf{k}\mathbf{x}$ . In order to obtain the electromagnetic impulsive force in this form, it is necessary to define the four-potential in the following form:

$$A_\mu = a_\mu H(\varphi), \quad (84)$$

where function  $H$  is the Heaviside unit step function defined by the relation

$$H(\varphi) = \begin{cases} 0, & \varphi < 0 \\ 1, & \varphi \geq 0 \end{cases}. \quad (85)$$

If we define the four-potential by eq. (85), then the electromagnetic tensor with impulsive force is of the form

$$F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu = (k_\mu a_\nu - k_\nu a_\mu)\delta(\varphi) = a_{\mu\nu}\delta(\varphi). \quad (86)$$

To find motion of an electron in the  $\delta$ -form electromagnetic force, we must solve immediately the Lorentz equation, or, to solve Lorentz equation in general with four-potential  $A_\mu = a_\mu A(\varphi)$  and then replace the four-potential by the eta-function. Following Meyer [13], we apply his method and then replace  $A_\mu(\varphi)$  by  $a_\mu H(\varphi)$  in the final result.

The Lorentz equation reads:

$$\frac{dp_\mu}{d\tau} = \frac{e}{m} F_{\mu\nu} p^\nu = \frac{e}{m} (k_\mu a_\nu - k_\nu a_\mu) p^\nu A'(\varphi), \quad (87)$$

where the prime denotes derivation with regard to  $\varphi$ ,  $\tau$  is proper time, and  $p_\mu = m(dx_\mu/d\tau)$ . After multiplication of the last equation by  $k^\mu$ , we get with regard to the Lorentz condition  $0 = \partial_\mu A^\mu = a^\mu \partial_\mu A(\varphi) = k_\mu a^\mu A'$  or  $k \cdot a = 0$  and  $k^2 = 0$  the following equation:

$$\frac{d(k \cdot p)}{d\tau} = 0 \quad (88)$$

and it means that  $k \cdot p$  is a constant of the motion and it can be defined by the initial conditions, for instance, at time  $\tau = 0$ . If we put  $p_\mu(\tau = 0) = p_\mu^0$ , then we can write  $k \cdot p = k \cdot p^0$ . At this moment, we have

$$k \cdot p = \frac{mk \cdot dx}{d\tau} = m \frac{d\varphi}{d\tau}, \quad (89)$$

or

$$\frac{d\varphi}{d\tau} = \frac{k \cdot p^0}{m}. \quad (90)$$

So, using the last equation and relation  $d/d\tau = (d/\varphi)d\varphi/d\tau$ , we can write eq. (87) in the form

$$\frac{dp_\mu}{d\varphi} = \frac{e}{k \cdot p^0} (k_\mu a \cdot p - a_\mu k \cdot p^0) A'(\varphi) \quad (91)$$

giving (after multiplication by  $a^\mu$ )

$$\frac{d(a \cdot p)}{d\varphi} = -ea^2 A' \quad (92)$$

or

$$a \cdot p = a \cdot p^0 - ea^2 A. \quad (93)$$

Substituting the last formula into (91), we get

$$\frac{dp_\mu}{d\varphi} = e \left( a_\mu - \frac{k_\mu a \cdot p^0}{k \cdot p^0} \right) \frac{dA}{d\varphi} - \frac{e^2 a^2}{2k \cdot p^0} \frac{d(A^2)}{d\varphi} k_\mu. \quad (94)$$

This equation can be immediately integrated to give the resulting momentum in the form

$$p_\mu = p_\mu^0 - e \left( A_\mu - \frac{A^\nu p_\nu^0 k_\mu}{k \cdot p^0} \right) - \frac{e^2 A^\nu A_\nu k_\mu}{2k \cdot p^0}. \quad (95)$$

Now, if we put into this formula the four-potential (84) of the impulsive force, then for  $\varphi > 0$  when  $H > 1$ , we get

$$p_\mu = p_\mu^0 - e \left( a_\mu - \frac{a^\nu p_\nu^0 k_\mu}{k \cdot p^0} \right) - \frac{e^2 a^\nu a_\nu k_\mu}{2k \cdot p^0}. \quad (96)$$

The last equation can be used to determine the magnitude of  $a_\mu$ . It can be evidently expressed as the number of  $k$ -photons in electromagnetic momentum. For  $\varphi < 0$ , it is  $H = 0$  and, therefore,  $p_\mu = p_\mu^0$

It is still necessary to say what is the practical realization of the  $\delta$ -form potential. We know from the Fourier analysis that the Dirac  $\delta$ -function can be expressed by integral in the following form:

$$\delta(\varphi) = \frac{1}{\pi} \int_0^{\infty} \cos(s\varphi) ds. \quad (97)$$

So, the  $\delta$ -potential can be realized as the continual superposition of the harmonic waves. In case it will not be possible to realize it experimentally, we can approximate the integral formula by the summation formula as follows:

$$\delta(\varphi) \approx \frac{1}{\pi} \sum_0^{\infty} \cos(s\varphi). \quad (98)$$

Now, the quantum mechanical problem is to find solution of the Dirac equation with the  $\delta$ -form four-potential (84) and, from this solution, determine the quantum motion of the charged particle under this potential. After insertion of  $\Psi_p$  and  $\bar{\Psi}_p$  into the current density  $j^\mu = \Psi_p^* \gamma^\mu \Psi_p$ , we have

$$j^\mu = \frac{1}{p_0} \left\{ p^\mu - eA^\mu + k^\mu \left( \frac{e(pA)}{(kp)} - \frac{e^2 A^2}{2(kp)} \right) \right\}. \quad (99)$$

which is evidently related to eq. (23).

The so-called kinetic momentum corresponding to  $j^\mu$  is as follows:

$$J^\mu = \Psi_p^* (p^\mu - eA^\mu) \Psi_p = \bar{\Psi}_p \gamma^0 (p^\mu - eA^\mu) \Psi_p = \left\{ p^\mu - eA^\mu + k^\mu \left( \frac{e(pA)}{(kp)} - \frac{e^2 A^2}{2(kp)} \right) \right\} + k^\mu \frac{ie}{8(kp)p_0} F_{\alpha\beta} (u^* \sigma^{\alpha\beta} u), \quad (100)$$

where

$$\sigma^{\alpha\beta} = \frac{1}{2} (\gamma^\alpha \gamma^\beta - \gamma^\beta \gamma^\alpha). \quad (101)$$

Now, we express the four-potential by the step function. In this case, the kinetic momentum contains the tensor  $F_{\mu\nu}$  involving  $\delta$ -function. It means that there is a singularity at point  $\varphi = 0$ . This singularity plays no role in the situation for  $\varphi > 0$  because, in this case, the  $\delta$ -function is zero. Then, the kinetic momentum is the same as  $j^\mu$ . The emission of photons by electron in the delta-pulse force follows from the matrix element  $M$  corresponding to the emission of photon by electron in the electromagnetic field [12]:

$$M = -ie^2 \int d^4x \bar{\Psi}_{p'} (\gamma e'^*) \Psi_p \frac{e^{ik'x}}{\sqrt{2\omega'}}, \quad (102)$$

where  $\Psi_p$  is the wave function of an electron before interaction with a pulse and  $\Psi_{p'}$  is the wave function of an electron after interaction and emission of photon with components  $k'^{\mu} = (\omega', \mathbf{k}')$ . The symbol  $e'^*$  is the four-polarization vector of emitted photon.

If we write Volkov wave function  $\Psi_p$  in the form (18), then, for the impulsive vector potential (84), we have

$$S = -px - \left[ e \frac{ap}{kp} - \frac{e^2}{2kp} a^2 \right] \varphi, \quad R = \left[ 1 + \frac{e}{2kp} (\gamma k)(\gamma a) H(\varphi) \right]. \quad (103)$$

So, we get the matrix element in the form

$$M = g \int d^4x \bar{\Psi}_{p'} O \Psi_p \frac{e^{ik'x}}{\sqrt{2\omega'}}, \quad (104)$$

where  $O = \gamma e'^*$ ,  $g = -ie^2$  in case of the electromagnetic interaction and

$$\bar{\Psi}_{p'} = \frac{\bar{u}}{\sqrt{2p'_0}} \bar{R}(p') e^{-iS(p')}. \quad (105)$$

In such a way, using the above definitions, we write the matrix element in the form

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{2p'_0 2p_0} \int d^4x \bar{R}(p') O R(p) e^{-iS(p') + iS(p)} e^{ik'x}. \quad (106)$$

The quantity  $\bar{R}(p')$  follows immediately from eq. (103), namely:

$$\bar{R}' = \overline{\left[ 1 + \frac{e}{2kp'} (\gamma k)(\gamma a) H(\varphi) \right]} = \left[ 1 + \frac{e}{2kp'} (\gamma a)(\gamma k) H(\varphi) \right]. \quad (107)$$

Using

$$-iS(p') + iS(p) = i(p' - p) + i(\alpha' - \alpha)\varphi, \quad (108)$$

where

$$\alpha = \left( e \frac{ap}{kp} - \frac{e^2}{2} \frac{a^2}{kp} \right), \quad \alpha' = \left( e \frac{ap'}{kp'} - \frac{e^2}{2} \frac{a^2}{kp'} \right), \quad (109)$$

we get

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{2p'_0 2p_0} \int d^4x \bar{u} \bar{R}(p') OR(p) u e^{i(p'-p)x} e^{i(\alpha'-\alpha)\varphi} e^{ik'x}. \quad (110)$$

Putting

$$\bar{R}(p') OR(p) e^{i(\alpha'-\alpha)\varphi} = \frac{1}{2\pi} \int_{-\infty}^{\infty} D(s) e^{-is\varphi} ds \quad (111)$$

with the inverse transform

$$D(s) = \int_{-\infty}^{\infty} d\varphi e^{is\varphi} \bar{R}(p') OR(p) e^{i(\alpha'-\alpha)\varphi}, \quad (112)$$

we get after  $x$ -integration

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{\sqrt{2p'_0 2p_0}} \int ds \bar{u}(p') D(s) u(p) \delta^{(4)}(ks + p - k' - p'). \quad (113)$$

We see from the presence of the  $\delta$ -function in eq. (113) that during the process of the interaction of electron with the laser pulse, the energy-momentum conservation law holds good

$$sk + p = k' + p'. \quad (114)$$

The last equation has physical meaning for  $s = 1, 2, \dots, N$ ,  $N$  being a positive integer.  $s = 1$  means the conservation of energy momentum of the one-photon Compton process and  $s = 2$  of the two-photon Compton process and  $s = N$  means the multiphoton interaction with  $N$  photons of laser pulse with electron. The multiphoton interaction is nonlinear and differs from the situation where electron scatters twice or more as it traverses the laser focus. The analogical situation is valid for the photoelectric equation.

Now, let us determine  $D(s)$ . With regard to the mathematical relation  $H^2(\varphi) = H(\varphi)$ , we can put

$$\bar{R}(p') OR(p) = A + BH(\varphi) \quad (115)$$

where

$$A = \gamma e'^* \quad (116a)$$

and

$$B = \frac{e}{2(kp)} (\gamma e'^*) (\gamma k) (\gamma a) + \frac{e}{2(kp')} (\gamma a) (\gamma k) (\gamma e'^*) + \frac{e^2}{4(kp)(kp')} (\gamma a) (\gamma k) (\gamma e'^*) (\gamma k) (\gamma a). \quad (116b)$$

In such a way,

$$D(s) = A \int_{-\infty}^{\infty} e^{i(\alpha' - \alpha + s)\varphi} d\varphi + B \int_{-\infty}^{\infty} e^{i(\alpha' - \alpha + s)\varphi} H(\varphi) d\varphi = (2\pi)A\delta(\alpha' - \alpha + s) + (2\pi i)B \frac{1}{\alpha' - \alpha + s} \quad (117)$$

as a consequence of the integral

$$\int_0^{\infty} e^{-\varepsilon x} \sin mx dx = \frac{m}{\varepsilon^2 + m^2} \quad (118)$$

for  $m \rightarrow 0$ .

The total probability of the emission of photons during the interaction of the laser pulse with electron is as follows:

$$W = \int \frac{1}{2} \sum_{spin.polar.} \frac{|M|^2 d^3 p' d^3 k'}{VT (2\pi)^6}. \quad (119)$$

There are two substantial mathematical operations for the evaluation of  $W$ . The one step is to use, after performing  $|M|^2$ , the following mathematical identity:

$$(2\pi)^8 \delta^{(4)}(sk + p - p' - k') \delta^{(4)}(p + s'k - p' - k') = (2\pi)^4 VT \frac{\delta(s - s')}{\delta(0)} \delta^{(4)}(p + sk - p' - k'), \quad (120)$$

where  $V$  is the space volume and  $T$  is time interval, and the second step is the determination of Trace, because according to the quantum electrodynamics of a spin, it is possible to show that [12]

$$\frac{1}{2} \sum_{spin.polar.} |M|^2 = \frac{1}{2} \text{Tr} \left\{ (\gamma p' + m) D (\gamma p + m) \gamma^0 D^+ \gamma^0 \right\}, \quad (121)$$

where in our case, quantity  $D$  is given by eq. (117).

In order to determine  $Tr$  or spur of the combinations of  $\gamma$ -matrix, it is suitable to know some relations. For instance:

$$\text{Tr}(a\gamma)(b\gamma) = 4ab, \quad \text{Tr}(a\gamma)(b\gamma)(c\gamma) = 0, \quad (122)$$

$$\text{Tr}(a\gamma)(b\gamma)(c\gamma)(d\gamma) = 4[(ab)(cd) - (ac)(bd) + (ad)(bc)]. \quad (123)$$

Then,

$$\text{Tr} [(\gamma p' + m) D (\gamma p + m) \bar{D}] = S_1 + S_2 + S_3 + S_4; \quad \bar{D} = \gamma^0 D^+ \gamma^0 \quad (124)$$

with

$$S_1 = \text{Tr}[\gamma p' D \gamma p \bar{D}] \quad (125)$$

$$S_2 = \text{Tr}[m D \gamma p \bar{D}] \quad (126)$$

$$S_3 = \text{Tr}[m \gamma p' D \bar{D}] \quad (127)$$

$$S_4 = \text{Tr}[m^2 D \bar{D}], \quad (128)$$

where  $D$  is given by eq. (117) and  $A$  and  $B$  are given by eqs. (116b) and (116b).

Now, it is evident that the total calculation is complex and involves many algebraic operations with  $\gamma$ -matrices and  $\delta$ -functions. At this moment, we restrict the calculations to the most simple approximation where we replace the term in brackets in eq. (18) by unit, and so we write instead of eq. (18)

$$\Psi_p \sim \frac{u}{\sqrt{2p_0}} e^{iS} \quad (129)$$

which is usually used in similar form for the nonrelativistic calculations as it is discussed in [12]. Then,

$$M = \frac{g}{\sqrt{2\omega'}} \frac{1}{2p'_0 2p_0} \int dx^4 \bar{u}(\gamma e'^*) u e^{i(p'-p)x} e^{i(\alpha'-\alpha)\varphi} e^{ik'x} = \frac{g}{\sqrt{2\omega'}} \frac{1}{2p'_0 2p_0} \bar{u}(\gamma e'^*) u \delta^{(4)}(lk + p - p' - k'), \quad (130)$$

where

$$l = \alpha - \alpha'. \quad (131)$$

In this simplified situation,  $\bar{R}OR$  reduces to  $A = \gamma e'^*$ . Then, using relation  $\bar{\gamma}^\mu = \gamma^\mu$  with a consequence  $\bar{A} = A$  and relations (122) and (123), we get

$$S_1 = \text{Tr}[\gamma p' A \gamma p \bar{A}] = 4 [(p' e'^*)(p e'^*) - (pp')(e'^* e'^*) + (p' e'^*)(p e'^*)] \quad (132)$$

$$S_2 = \text{Tr}[m A \gamma p \bar{A}] = 0 \quad (133)$$

$$S_3 = \text{Tr}[m \gamma p' A \bar{A}] = 0 \quad (134)$$

$$S_4 = \text{Tr}[m^2 A \bar{A}] = 4m^2(e'^* e'^*). \quad (135)$$

At this moment, we can write probability of the process  $W$  in the form:

$$W = \int \frac{1}{2} \sum_{spin. polar.} \frac{|M|^2}{VT} \frac{d^3 p' d^3 k'}{(2\pi)^6} = \int \frac{d^3 p' d^3 k'}{(2\pi)^6} \frac{1}{2} (S_1 + S_2 + S_3 + S_4) \frac{1}{(2\pi)^2} \delta^{(4)}(lk + p - p' - k') = \int \frac{d^3 p' d^3 k'}{(2\pi)^6} \frac{1}{2} \delta^{(4)}(lk + p - p' - k') 4 \left\{ (p' e'^*)(p e'^*) + (e'^* e'^*)(m^2 - (pp')) \right\}. \quad (136)$$

The presence of the  $\delta$ -function in the last formula is expression of the conservation law  $lk + p = k' + p'$ , which we write in the form

$$lk + p - k' = p'. \quad (137)$$

If we introduce the angle  $\Theta$  between  $\mathbf{k}$  and  $\mathbf{k}'$ , then, with  $|\mathbf{k}| = \omega$  and  $|\mathbf{k}'| = \omega'$ , we get from the squared eq. (137) in the rest system of electron, where  $p = (m, 0)$ , the following equation:

$$l \frac{1}{\omega'} - \frac{1}{\omega} = \frac{l}{m} (1 - \cos \Theta); \quad l = \alpha - \alpha', \quad (138)$$

which is a modification of the original equation for the Compton process

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m} (1 - \cos \Theta). \quad (139)$$

We observe that the basic difference between single-photon interaction and  $\delta$ -pulse interaction is the factor  $l = \alpha - \alpha'$ .

We know that the last formula of the original Compton effect can be written in the form suitable for the experimental verification, namely:

$$\Delta\lambda = 4\pi \frac{\hbar}{mc} \sin^2 \frac{\Theta}{2}, \quad (140)$$

which was applied Compton for the verification of the quantum nature of light.

We can express eq. (138) in a new form. From equation  $lk + p = k' + p'$ , we get after multiplying it by  $k$  in the rest frame of electron

$$kp' = \omega m - \omega \omega' (1 - \cos \Theta) \quad (141)$$

Then,  $l$  in eq. (138) is given by the formula ( $a \equiv (v, \mathbf{w})$ ):

$$l = \frac{2evm - e^2 a^2}{2\omega m} - \frac{2ea p' - e^2 a^2}{2\omega [m - \omega' (1 - \cos \Theta)]}. \quad (142)$$

This equation (138) can be experimentally verified by the similar methods which were used by Compton for the verification of his formula. However, it seems that the interaction of the photonic pulse substantially differs from the interaction of a single photon with electron.

The equation  $lk + p = k' + p'$  is the symbolic expression of the nonlinear Compton effect in which several photons are absorbed at a single point, but only single high-energy photon is emitted. The second process where electron scatters twice or more as it traverses the laser

focus is not considered here. The nonlinear Compton process was experimentally confirmed, for instance, by Bulla et al. [15].

The present text is a continuation of the author discussion on laser acceleration [16, 17, 18], where the Compton model of laser acceleration was proposed.

The  $\delta$ -form laser pulses are here considered as an idealization of the experimental situation in laser physics. Nevertheless, it was demonstrated theoretically that at the present time, the zeptosecond and sub-zeptosecond laser pulses of duration  $10^{-21} - 10^{-22}$  s can be realized by the petawatt lasers [19].

## 7. Compton effect initiated by two orthogonal plane waves

The project with the two laser waves is the next goal and the future direction of the laser physics of elementary particles. The two laser beams can replace many laser beams in the thermonuclear reactor such as ITER in Cadarache near Aix-en-Provence in France. At the same time, the two-laser system can be considered in chemistry as a catalyzer which was not known before the laser physics.

We solve the Dirac equation for two different four-potentials of the plane electromagnetic waves. We specify the solutions of the Dirac equation for two orthogonal plane waves.

The modified Compton formula for the scattering of two photons on an electron is determined. The solution of the Dirac equation for two waves was found by Sen Gupta [20, 21] in the form of the Fourier series, however without immediate application. The solution of the Dirac equation for two waves with the perpendicular polarization was given by Lyulka [22-25] who described the decay of particles in two laser fields. The derivation of two-wave solution is not presented in his articles. So, we investigated the situation and presented new results.

The total four-potential  $V_\mu$  is a superposition of the potentials  $A_\mu$  and  $B_\mu$  as follows:

$$V_\mu = A_\mu(\varphi) + B_\mu(\chi), \quad (143)$$

where  $\varphi = kx$  and  $\chi = \kappa x$  and  $k \neq \kappa$ .

The Lorentz condition gives

$$\partial_\mu V^\mu = 0 = k_\mu \frac{\partial A^\mu}{\partial \varphi} + \kappa_\mu \frac{\partial B^\mu}{\partial \chi} = k_\mu A_\varphi^\mu + \kappa_\mu B_\chi^\mu, \quad (144)$$

where the subscripts  $\varphi, \chi$  denote partial derivatives. Equation (144) takes a more simple form if we notice that partial differentiation with respect to  $\varphi$  concerns only  $A_\mu$  and partial differentiation with respect to  $\chi$  concerns only  $B_\mu$ . So we write instead eq. (144).

$$\partial_\mu V^\mu = 0 = k_\mu (A^\mu)' + \kappa_\mu (B^\mu)' = kA' + \kappa B'. \quad (145)$$

Without loss of generality, we can write instead of eq. (145) the following one:

$$k_\mu(A^\mu)' = 0; \quad \kappa_\mu(B^\mu)' = 0, \quad (146)$$

or,

$$kA = \text{const} = 0; \quad \kappa B = \text{const} = 0, \quad (147)$$

putting integrating constant to zero.

The electromagnetic tensor  $F_{\mu\nu}$  is expressed in the new variables as in eq. (5)

$$F_{\mu\nu} = k_\mu A'_\nu - k_\nu A'_\mu + \kappa_\mu B'_\nu - \kappa_\nu B'_\mu. \quad (148)$$

Now, we write Dirac equation for the two potentials in the form

$$[-\partial^2 - 2ie(V\partial) + e^2V^2 - m^2 - \frac{i}{2}eF_{\mu\nu}\sigma^{\mu\nu}]\psi = 0, \quad (149)$$

where  $V = A + B$ ,  $F_{\mu\nu}$  is given by eq. (148) and the  $\sigma$ -term is defined as follows:

$$\frac{i}{2}eF_{\mu\nu}\sigma^{\mu\nu} = ie(\gamma k)(\gamma A') + ie(\gamma \kappa)(\gamma B') \quad (150)$$

We assume the solution of eq. (149) in the Volkov form, or

$$\psi = e^{-ipx}F(\varphi, \chi). \quad (151)$$

After performing all operations in eq. (149), we get the partial differential equation for function  $F(\varphi, \chi)$ :

$$\begin{aligned} & -2k\kappa F_{\varphi\chi} + (2ipk - 2ikB)F_\varphi + (2ip\kappa - 2ieA\kappa)F_\chi + \\ & (e^2(A+B)^2 - 2e(A+B)p - ie(\gamma k)(\gamma A_\varphi) - ie(\gamma \kappa)(\gamma B_\chi))F = 0. \end{aligned} \quad (152)$$

Equation (152) was simplified by the author [20], putting  $k\kappa = 0$ . However, to get the Compton effect-initiated two orthogonal waves, we ignore this simplification and write eq. (152) in the following form:

$$aF_{\varphi} + bF_{\chi} + cF = 2k\kappa F_{\varphi\chi}, \quad (153)$$

where

$$a = 2ipk - 2iekB; \quad b = 2ip\kappa - 2ie\kappa A \quad (154)$$

and

$$c = e^2(A + B)^2 - 2e(A + B)p - ie(\gamma k)(\gamma A') - ie(\gamma \kappa)(\gamma B') \quad (155)$$

and the term of the two partial derivations is not present because of  $k\kappa = 0$ .

For the field which we specify by the conditions

$$kB = 0; \quad \kappa A = 0; \quad AB = 0, \quad (156)$$

we have

$$2ipkF_{\varphi} + 2ip\kappa F_{\chi} + (e^2A^2 + e^2B^2 - 2epA - 2epB - ie(\gamma k)(\gamma A') - ie(\gamma \kappa)(\gamma B'))F = 2k\kappa F_{\varphi\chi}. \quad (157)$$

Now, let us put

$$F(\varphi, \chi) = X(\varphi)Y(\chi). \quad (158)$$

After insertion of eq. (158) into eq. (157) and division of the new equation by  $XY$ , we get the terms depending only on  $\varphi$  and on  $\chi$ . Or we get

$$\left( 2i(pk + ik\kappa) \frac{X'}{X} + e^2A^2 - 2epA - ie(\gamma k)(\gamma A') \right) + \left( 2i(p\kappa + ik\kappa) \frac{Y'}{Y} + e^2B^2 - 2epB - ie(\gamma \kappa)(\gamma B') \right) = 0 \quad (159)$$

So, there are terms dependent on  $\varphi$  and terms dependent on  $\chi$  only in eq. (159). The only possibility is that they are equal to some constant  $\lambda$  and  $-\lambda$ . Then,

$$2i(pk + ik\kappa)X' + (e^2A^2 - 2epA - ie(\gamma k)(\gamma A'))X = \lambda X \quad (160)$$

and

$$2i(pk + ik\kappa)Y' + (e^2B^2 - 2epB - ie(\gamma k)(\gamma B'))Y = -\lambda Y \quad (161)$$

We put  $\lambda = 0$  without loss of generality. In such a way, the solution of eq. (159) is the solution of two equations only. Because the form of every equation is similar to the form of eq. (14), we can write the solution of these equations as follows:

$$X = \left[ 1 + \frac{e}{2(kp + ik\kappa)} (\gamma k)(\gamma A) \right] \frac{u}{\sqrt{2p_0}} e^{iS_1}, \quad (162)$$

with

$$S_1 = \int_0^{\kappa x} \frac{e}{(kp + ik\kappa)} \left[ (pA) - \frac{e}{2}(A)^2 \right] d\varphi. \quad (163)$$

and

$$Y = \left[ 1 + \frac{e}{2(\kappa p + ik\kappa)} (\gamma \kappa)(\gamma B) \right] \frac{u}{\sqrt{2p_0}} e^{iS_2}, \quad (164)$$

with

$$S_2 = - \int_0^{\kappa x} \frac{e}{(\kappa p + ik\kappa)} \left[ (pB) - \frac{e}{2}(B)^2 \right] d\chi. \quad (165)$$

The total solution is then of the form

$$\psi_p = \left[ 1 + \frac{e}{2(kp + ik\kappa)} (\gamma k)(\gamma A) \right] \left[ 1 + \frac{e}{2(\kappa p + ik\kappa)} (\gamma \kappa)(\gamma B) \right] \frac{u}{\sqrt{2p_0}} e^{i(S_1(A) + S_2(B))}. \quad (166)$$

In the case of the two non-collinear laser beams, the problem was solved by Lyulka in 1974 for two linearly polarized waves [22]:

$$A = a_1 \cos \varphi; \quad B = a_2 \cos(\chi + \delta) \quad (167)$$

with the standard conditions for  $\varphi, \chi, k, \kappa$ , and  $\delta$  being the phase shift.

The two-wave Volkov solution is given by eq. (166), and the matrix elements with corresponding calculation ingredients are given by the standard method as it was shown by Lyulka [22].

It was shown in [22] that

$$q^\mu = p^\mu - e^2 \frac{a_1^2}{2(kp)} k^\mu - e^2 \frac{a_2^2}{2(\kappa p)} \kappa^\mu \quad (168)$$

and

$$m_*^2 = m^2 \left( 1 - \frac{e^2 a_1^2}{m^2} - \frac{e^2 a_2^2}{m^2} \right), \quad (169)$$

which is for the massless graphene limit, the following one  $m_*^2(m=0) = -e^2(a_1^2 + a_2^2)$ .

The matrix element involves the extended law of conservation. Namely:

$$sk + t\kappa + q = q' + k' + \kappa', \quad (170)$$

where the  $s$  and  $t$  are natural numbers. The last equation has natural interpretations. The photon object with momenta  $sk$  and  $t\kappa$  interacts with electron with momentum  $q$ . After interaction, the electron has a momentum  $q'$  and two photons are emitted with momenta  $k'$  and  $\kappa'$ .

We can write eq. (170) in the equivalent form:

$$sk + q - k' = q' + \kappa' - t\kappa. \quad (171)$$

From the squared form of the last equation and after some modification, we get the generalized equation of the double Compton process for  $s = t = 1$ :

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m_*} (1 - \cos \Theta) + \frac{\Omega' - \Omega}{\omega\omega'} - \frac{\Omega\Omega'}{\omega\omega'm_*} (1 - \cos \Xi), \quad (172)$$

where  $\Xi$  is the angle between the 3-momentum of the  $\kappa$ -photon and the 3-momentum of the  $\kappa'$ -photons with frequency  $\Omega$  and  $\Omega'$ , respectively. In the situation of graphene with the massless limit, we replace the renormalized mass in eq. (172) by  $m_*(m=0) = e\sqrt{-a_1^2 - a_2^2}$ .

Let us remark that if the frequencies of the photons of the first wave substantially differ from the frequencies of the photons of the second wave, then eq. (172) can be experimentally verified by the same method as the original Compton formula. To our knowledge, formula (172) is not involved in the standard textbooks on quantum electrodynamics.

## 8. Perspectives

We have considered the Compton effect in the framework of the Volkov solution of the Dirac equation assuming that the process occurred in vacuum and in medium with the index of refraction  $n$ . The determination of the index of refraction follows from the Compton effect. Mass renormalization of electron is involved in the Volkov solution.

The harmonic oscillator with frequency  $\omega_0$  and the dispersion theory leads to the known formula for the index of refraction of matter [26]. The index of refraction derived in the dispersion theory based on the damped oscillator is given by the formula [27]

$$n = 1 + 2\pi N \frac{e^2}{m} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}, \quad (173)$$

where  $N$  is number of electrons in the unit of volume. The modern aspects can be found in the Crenshaw article [28]. So, to consider the Compton effect in dielectric medium is the perspective problem.

The interesting result of our article is the derivation that for some scattering angles given by eq. (70), there exists the so-called anomalous Compton effect, where the wavelengths of scattered photons are shorter than the wavelengths of the original photons. To our knowledge, information of this effect was not published in the physical journals.

The Compton scattering is, at the present time, the elementary laboratory problem because for the monochromatic X-rays for  $\lambda = 1$ , the shift of wavelength is several percent. This is quantity which can be easily measured. On the other hand, the Compton wavelength shift for the visible light is only 0,01 percent. It means that the measurement of the Compton effect for the visible light in the dielectric medium involves the subtle approach.

We have also discussed the problem of the Dirac equation with the two-wave potentials of the electromagnetic fields. While the Volkov solution for one potential is well known for a long time, the Compton process with two beams was not investigated experimentally by any laboratory.

It is possible to consider the situation with the sum of  $N$  waves, or

$$V = \sum_{i=1}^N A_i(\varphi_i) \quad \varphi_i = k_i x. \quad (174)$$

The problem of the laser compression of target by many beams, involving the Compton effect, is one of the actual problems of the contemporary laser physics. The goal of the experiments is to generate the physical implosion in the spherical target. The light energy is absorbed by the target and generates a high-temperature plasma with high pressure of a few hundred megabars. For the process sufficiently spherically symmetric, the central area is heated up to 5–10 keV and fusion reaction starts [29]. The solution of that problem in the general form is a difficult one, and it can be solved only by the special laser institutions such as the Lebedev Institute of Physics, the Lawrence Livermore National Laboratory, and so on.

New experiments can be realized and new measurements performed by means of the laser pulses, giving new results and discoveries.

## Author details

Miroslav Pardy

\*Address all correspondence to: pamir@physics.muni.cz

Department of Physical Electronics, Masaryk University, Brno, Czech Republic

## References

- [1] Compton, A. A. (1923). A quantum theory of the scattering of X-rays by light elements, *Phys. Rev.*, 21(5), 483-502.
- [2] Volkov, D. M. (1935). Über eine Klasse von Lösungen der Diracschen Gleichung, *Z. Physik*, 94, 250-260.
- [3] Peierls, R. E. (1934). Bemerkungen über Umwandlungstemperaturen, *Helvetica Physica Acta* 7, 81; *ibid*: (1935). Quelques proprietes typiques des corps solides, *Ann. Inst. Henri Poincaré*, 5, 177.
- [4] Landau, L. D. (1937). Zur Theorie der Phasenumwandlungen II, *Phys. Z. Sowjetunion*, 11, 26.
- [5] Mermin, N. D. (1968). Crystalline order in two dimensions, *Phys. Rev.*, 176, 250.
- [6] Novoselov, K. S., Geim, A. K., Morozov, S. V., Jiang, D., Katsnelson, M. I., Grigorieva, I. V., Dubonos, S. V. and Firsov, A. A. (2005). Two-dimensional gas of massless Dirac fermions in graphene, *Nature*, 438/10.
- [7] Kane, Ch. L. (2005). Erasing electron mass, *Nature*, 438, 168.
- [8] Dato, A., Radmilovic, V., Lee, Z., Philips, J. and Frenklach, M. (2008). Substrate-free gas-phase synthesis of graphene sheets, *Nano Lett.*, 8(7), 2012.
- [9] Lozovi, Yu. E.; Merkulov, S. P. and Sokolik, A. A. (2008). Collective electron phenomena in graphene, *Uspekhi Fiz. Nauk*, 178(7), 758.
- [10] Ezawa, M. (2013). Photo-induced topological phase transition and single Dirac-cone state in silicene, *Phys. Rev. Lett.*, 110, 026603.
- [11] Bludov, Yu. V., Peres, N. M. R. and Vasilevskiy, M. I. (2012). Graphene-based polaritonic crystal, *Phys. Rev. B* 85, 245409.
- [12] Berestetskii, V. B., Lifshitz E. M. and Pitaevskii, L. P. (1989). Quantum Electrodynamics, (Nauka, Moscow).

- [13] Meyer, J. W. (1971). Covariant classical motion of electron in a laser beam, *Phys. Rev. D: Part. Fields*, 3(2), 621.
- [14] Schwinger, J., Tsai, W. Y. and Erber, T. (1976). Classical and quantum theory of synergic synchrotron-Čerenkov radiation, *Ann. Phys. (NY)*, 96(2), 303.
- [15] Bulla, C., *et al.*, (1996). Observation of nonlinear effects in Compton scattering, *Phys. Rev. Lett.*, 76, 3116.
- [16] Pardy, M. (1998). The quantum field theory of laser acceleration, *Phys. Lett. A*, 243, 223.
- [17] Pardy, M. (2001). The quantum electrodynamics of laser acceleration, *Radiat. Phys. Chem.*, 61, 321.
- [18] Pardy, M. (2003). Electron in the ultrashort laser pulse, *Int. J. Theor. Phys.*, 42(1), 99.
- [19] Pardy, M. (2004). Compton effect in dielectric medium, arXiv:hep-ph/0406008v1, 1 Jun.
- [20] Sen Gupta, N. D. (1967). On the solution of the Dirac equation in the field of two beams of electromagnetic radiation, *Zeitschrift für Physik*, 200, 13.
- [21] Pardy, M. (2006). Volkov solution for an electron in two wave fields, *Int. J. Theor. Phys.*, 45(3), 663.
- [22] Lyulka, V. A. (1974). Quantum effects in intense electromagnetic fields, *Sov. Phys. JETP*, 40, 815.
- [23] Lyulka, V. A. (1975). Decays of elementary particles in the field of of the intense electromagnetic wave, *Zh. Exp. Teor. Fiz.*, 69, 3(9), 800.
- [24] Lyulka, V. A. (1977). Quantum effects in the field of the non-chromatic electromagnetic wave, *Zh. Exp. Teor. Fiz.*, 72(3), 865.
- [25] Lyulka, V. A. (1985). Weak processes in the intense electron in the ultrashort laser pulse electromagnetic fields, *J. Nucl. Phys.*, 5(11), 1211.
- [26] Sokolov, A. A., Loskutov, Yu. M. and Ternov, I. M. (1962). *Quantum Mechanics*, (GIMP, RSFSR, Moscow). (in Russian).
- [27] Garbuny, M. (1965). *Optical Physics*, (Academic Press, New York, London).
- [28] Crenshaw, M. E. (2004). Quantum optics of dielectrics in momentum-space, *Opt. Commun.*, 235, 153.
- [29] Nakai, S. and Mima, K. (2004). Laser driven inertial fusion energy: present and perspective, *Rep. Prog. Phys.*, 67, 321.

