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Linear Approximation of Efficiency for Similar Non-Endoreversible Cycles to the Carnot Cycle

Delfino Ladino-Luna, Ricardo T. Páez-Hernández and Pedro Portillo-Díaz

Abstract

In the present paper the non-endoreversible Curzon-Ahlborn, Stirling and Ericsson cycles as models of thermal engines are discussed from the viewpoint of finite time thermodynamics. That is, it is propose the existence of a finite time of heat transfer for isothermal processes, but the cycles are analyzed assuming they are not endoreversible cycles, through a factor that represents the internal ireversibilities of them, so that the proposed heat engine models have efficiency closer to real engines. Some results of previous papers are used, and from the get expressions for the power output function and ecological function a methodology to obtain a linear approximation of efficiency including adequate parameters are shown, similar to those obtained in that previous paper used. Variable changes are made right, like those used previously.

Keywords: finite time thermodynamics, power output, ecological function, efficiency

1. Introduction

A valuable tool for validating and improving knowledge of nature is using models. A scientific model is an abstract, conceptual, graphic, or visual representation of phenomena, systems, or processes to analyze, describe, explain, simulate, explore, control, and predict these phenomena or processes. A model allows determining a final result from appropriate data. The creation of models is essential for all scientific activity. Moreover, a given physics theory is a model for studying the behavior of a complete system. The model is applied in all areas of physics, reducing the observed behavior to more basic fundamental facts, and helps to explain and predict the behavior of physical systems under different circumstances.

In classical equilibrium thermodynamics, the simplest model of an engine that converts heat into work is the Carnot cycle. The behavior of a heat engine working between two heat

reservoirs, modeled as this cycle, is expressed by the relation between the efficiency η and the ratio of temperatures of the heat reservoirs, T_C/T_H, with $0 < T_C/T_H < 1$, the Carnot efficiency $\eta_C = 1 - T_C/T_H$. The temperatures of reservoirs, cold and hot, respectively, are T_C and T_H (equal to those of heat engine), and η_C is a physical limit for any heat engine.

A more realistic cycle than the Carnot cycle is a modified cycle taking into account the processes time of heat transfer between the system and its surroundings, in which the working temperatures are different of those its reservoirs [1], obtaining the efficiency $\eta_{CAN} = 1 - \sqrt{T_C/T_H}$, first found in references [2] and [3], and known as Curzon–Ahlborn–Novikov–Chambadal efficiency. At present, the duration of heat transfer processes is important. Based on this model, at the end of the last century, a theory was developed as an extension of classical equilibrium thermodynamics, the finite time thermodynamics, in which the duration of the exchange processes heat becomes important.

Two operating regimens of a heat engine with the same type of parameters have been established: maximum power output regimen as in [1] and maximum effective power regimen, taking into account the entropy production through a function called ecological function, which represents the relationship between power output P and entropy production σ advanced in [4]. It is worth noting that there are other operating regimens such as maximum cycle efficiency or minimum entropy production. Thus, power output has been maximized in [1,5-7] among others, entropy production has been minimized in [8-10] among others, and the so-called ecological function has been maximized in [4, 11-13] among others. Also, cycles including internal irreversibilities in various aspects of operation of thermal engines have been analyzed in [14-19] and others, and the regions of existence of the objective functions listed above have been analyzed by a limited number of publications, in [9,20,21] and others. Notice that in almost all the above references, the time of the adiabatic processes in the so-called Curzon–Ahlborn cycle is assumed irrelevant because this time is considered very small compared with the total time of cycle. Nevertheless, a meticulous examination on the behavior of real engines leads to take into account the time of these adiabatic processes because these processes are only an approach to real processes in which there is no heat transfer.

An alternative to analyze the Curzon–Ahlborn cycle, taking into account some effects that are nonideal to the adiabatic processes through the time of these processes, is the model proposed in [5] and in [7]. It allows to find the efficiency of a cycle as a function of the compression ratio, $r_C = V_{max}/V_{min}$. When $r_C \to \infty$, $V_{max} >> V_{min}$, the Curzon–Ahlborn–Novikov–Chambadal efficiency is recovered. The non-endoreversible Curzon and Ahlborn cycle can be analyzed by means of the so-called non-endoreversibility parameter I_S, defined first in [14] and later in [15] and in [16], which can be used to analyze diverse particularities of cycles. Furthermore, this parameter leads to equality instead of Clausius inequality [14].

In the present paper, the performance of a non-endoreversible heat engine modeled as a Curzon–Ahlborn cycle is analyzed. The procedure in [5] is combined with the procedure in [16], arriving to linear approaches of the efficiency as a function of a parameter that contains the compression ratio in both regimens maximum power output and maximum ecological function. From the limit values of the non-endoreversibility parameter and the compression

ratio, the known expressions of the efficiency found in the literature of finite-time thermodynamics are recovered. Also, an analysis of the Stirling and Ericsson cycles is made, when the existence of a finite time for the heat transfer for isothermal processes is assumed, and assuming they are not endoreversible cycles, through the non-endoreversibility parameter that represents internal irreversibilities of them. Some results in [22] are used, and from the expressions obtained for the power output function and ecological function, the methodology to obtain a linear approximation of efficiency including an adequate parameter is shown, similar to those used in case of the Curzon–Ahlborn cycle. Variable changes are made right, like those used in [5] and in [23,24]. In order to make the present paper self-contained, a review of results for instantaneous adiabatic case is presented. All quantities have been taken in the International System of Measurement.

2. Linear approximation of efficiency: endoreversible Curzon–Ahlborn cycle

In a previous published chapter by InTech [25], we devoted to analyze the Curzon and Ahlborn cycle under the following conditions: without internal irreversibilities and non instantaneous adiabats. We have shown some results in case of the Newton heat transfer law (Newton cooling law) and the Dulong and Petit heat transfer law, namely, heat transfer law like $dQ/dt \propto (\Delta T)^k$, $k = 5/4$. Hence, we begin with a summary of the cited chapter.

2.1. Known results and basic assumptions

Since the pioneer paper [1], the so-called finite time thermodynamics has been development. They proposed a model of thermal engine shown in Figure 1, which has the mentioned Curzon–Ahlborn–Novikov–Chambadal efficiency, as a function of the cold reservoir temperature T_C and the hot reservoir temperature T_H, as follows:

$$\eta_{CAN} = 1 - \sqrt{T_C / T_H}, \tag{1}$$

In this cycle, $Q_H/T_{HW} = Q_C/T_{CW}$ is fulfilled. The entropy production during the exchange of heat between the system and its reservoirs is only taken into account. The working temperatures of substance are T_{HW} and T_{CW}, being $T_C < T_{CW} < T_{HW} < T_H$. In contrast, the Carnot efficiency is obtained when the temperatures of reservoirs are the same as the temperatures of the engine, which means $T_{HW} = T_H$ and $T_C = T_{CW}$ in Figure 1, namely,

$$\eta_C = 1 - \frac{T_C}{T_H} = 1 - \frac{T_{CW}}{T_{CH}} \tag{2}$$
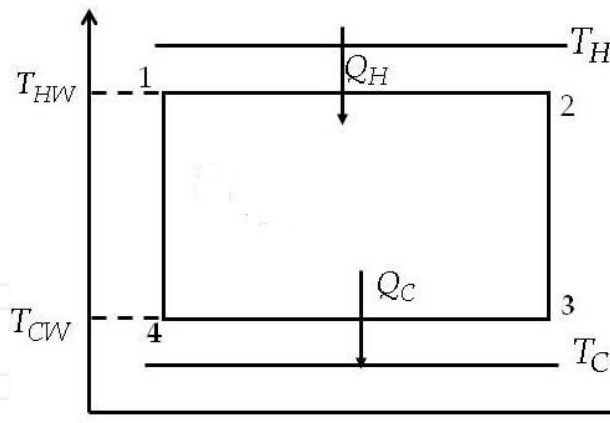
Figure 1. Curzon and Ahlborn cycle in the entropy S vs temperature T plane.

Equation (1) has been obtained at maximum power output regimen and recovered later by some procedures [5,10,26,27] among others. Moreover, in [4] was advanced an optimization criterion of merit for the Curzon and Ahlborn cycle, taking into account the entropy production, the ecological criterion, by maximization of the ecological function,

$$E = P - T_C \sigma, \tag{3}$$

where P is the power output, T_C is the temperature of cold reservoir, and σ is the total entropy production. The efficiency of Curzon and Ahlborn cycle now can be written as

$$\eta_E = 1 - \sqrt{\left(\varepsilon^2 + \varepsilon\right)/2}. \tag{4}$$

By contrast, following the procedure in [5], the form of the ecological function and its efficiency was found using the Newton heat transfer law and ideal gas as working substance in [12] and using the Dulong–Petit heat transfer law for ideal gas as working substance in [28]. Hence, as the upper limit of the efficiency of any heat engine is the Carnot efficiency, the temperatures of the reservoir equal those of the heat engine. Thus, the definition of efficiency of an engine working in cycles leads to the Carnot efficiency, fulfilling

$$\frac{Q_H - Q_C}{Q_H} \leq 1 - \frac{T_{CW}}{T_{HW}}. \tag{5}$$

With $\varepsilon \equiv T_C / T_H$, the following equations can be written: Carnot efficiency, $\eta_C \equiv 1 - \varepsilon$; Curzon–Ahlborn–Novikov–Chambadal efficiency: $\eta_{CAN} = 1 - \sqrt{\varepsilon}$; and ecological efficiency: $\eta_E = 1 - \sqrt{(\varepsilon^2 + \varepsilon)/2}$. Any efficiency can be written as

$$\eta = 1 - z(\varepsilon). \tag{6}$$

Thus, the problem of finding the efficiency of a heat engine modeled as a Curzon–Ahlborn cycle, maximizing power output or maximizing ecological function, becomes the problem of finding a function $z = z(\varepsilon)$. Substituting $z = z(\varepsilon)$ in Equation (6), one has

$$\eta = \eta(\varepsilon) \tag{7}$$

Similar results are obtained with a nonlinear heat transfer, like the Dulong and Petit heat transfer. Assuming the same thermal conductance α in two isothermal processes of the Curzon–Ahlborn cycle, the heat exchanged between the engine and its reservoirs could be in general as

$$\frac{dQ_H}{dt} = \alpha \left(T_H - T_{HM}\right)^k \quad \text{and} \quad \frac{dQ_H}{dt} = \alpha \left(T_C - T_{CW}\right)^k \qquad , k \geq 1. \tag{8}$$

By contrast, assuming the heat flows Q_H and Q_C, given by Newton's heat transfer law, the case $k = 1$ in Equation (8), the power output becomes

$$P = \frac{\alpha T_H \left(1 - z\right)\left[1 + \lambda \ln z\right]}{\frac{1}{1-u} + \frac{z}{uz - \varepsilon}}, \tag{9}$$

where R is the general constant of gases. The parameter $\gamma \equiv C_P / C_V$ has been used, and also the variables $u = T_{HW} / T_H$ and $z = T_{CW} / T_{HW}$ from which we obtain $P = P(u, z)$. The adiabatic processes are noninstantaneous. In fact, the total time of cycle is

$$t_{TOT} = t_1 + t_2 + t_3 + t_4, \tag{10}$$

being the times for the isothermal processes,

$$t_1 = \frac{RT_{HW}}{\alpha \left(T_H - T_{HW}\right)} \ln \frac{V_2}{V_1} \text{ and } t_3 = \frac{RT_{CW}}{\alpha \left(T_{CW} - T_C\right)} \ln \frac{V_4}{V_3}, \tag{11}$$

and the times for the adiabatic processes have been assumed to be

$$t_2 = f_1 \ln \frac{V_3}{V_2} \text{ and } t_4 = f_2 \ln \frac{V_4}{V_1}, \tag{12}$$

with

$$f_1 \equiv \frac{RT_{HW}}{\alpha\left(T_H - T_{HW}\right)} \text{ and } f_2 \equiv \frac{RT_{CW}}{\alpha\left(T_{CWw} - T_C\right)}. \tag{13}$$

The maximization conditions $\partial P / \partial u = 0$ and $\partial P / \partial z = 0$ lead to obtain (or, permit obtain)

$$u = \frac{z+\varepsilon}{2z} \text{ and } \left(z^2 - \varepsilon\right)\left(1 + \lambda \ln z\right) = \lambda\left(z - \varepsilon\right)\left(1 - z\right), \tag{14}$$

where λ represents the external parameter $\lambda = [(\gamma - 1)\ln(V_3/V_1)]^{-1}$, meaning that

$$P_{\max} = P_{\max}\left(u(z), z\right), \tag{15}$$

that is $P_{\max}$ is a projection on the (z, P) plane. It is also found that at the maximum power condition, z is given by a power series in λ, namely,

$$z_P = \sqrt{\varepsilon} + \frac{1}{2}\left(1 - \sqrt{\varepsilon}\right)^2 \lambda + \frac{1}{4}\left(1 - \sqrt{\varepsilon}\right)^2\left[\left(1 - \sqrt{\varepsilon}\right)^2 / 2\sqrt{\varepsilon} - \ln \varepsilon\right]\lambda^2 + O(\lambda^3) \tag{16}$$

Upon susbtituting Equation (16) in Equation (6) and because the terms in Equation (16) are positive, an upper bound for the efficiency is obtained when $\lambda = 0$, i.e., when the compression ratio $r_C = V_3/V_1$ goes to infinity, it results in the following:

$$\eta_{\max} = 1 - z_P\left(\lambda = 0\right) = \eta_{CAN}. \tag{17}$$

The equivalent of Equation (16) for the ecological function with this procedure was obtained in [12] by substituting Equation (9) in Equation (3), and the entropy production, $\sigma = \Delta S / t_{TOT}$. Using Equation (8) in the case $k = 1$, and the total time t_{TOT} given by Equations (10)–(13), the ecological function becomes

$$E = \alpha T_1 \frac{\left(1 + \varepsilon - 2z\right)\left(1 + \lambda \ln z\right)}{\frac{1}{1-u} + \frac{z}{zu - \varepsilon}}. \tag{18}$$

Upon maximizing the function $E = E(u, z)$ (ε is defined positive and λ is defined semi-positive, being external parameters), $\partial E / \partial u = 0$ and $\partial E / \partial z = 0$, for the first one $u = u(z)$ is as in case of

maximizing power output, and for the second one, the following relation between the variables z and u is obtained:

$$\left[2(1+\lambda \ln z)z - \lambda(1+\varepsilon - 2z)\right](z-\varepsilon)(zu-\varepsilon) = (1+\varepsilon - 2z)(1+\lambda \ln z)(1-u)\varepsilon z. \tag{19}$$

The equation that z obeys at the maximum of the ecological function is obtained as follows:

$$\left[2(1+\lambda \ln z)z - \lambda(1+\varepsilon - 2z)\right](z-\varepsilon) = (1+\varepsilon - 2z)(1+\lambda \ln z)\varepsilon. \tag{20}$$

We find, upon taking the implicit successive derivatives of Equation (20) with respect to λ, the following one-power series in λ :

$$z_E = \sqrt{\tfrac{1}{2}(\varepsilon+\varepsilon^2)}\left\{1+\left[\tfrac{1}{4}(1+3\varepsilon)\sqrt{\tfrac{2}{\varepsilon+\varepsilon^2}}-1\right]\lambda + \left(\frac{\tfrac{1}{16}(1+3\varepsilon)}{\varepsilon+\varepsilon^2} - \frac{1}{2}\sqrt{\frac{2}{\varepsilon+\varepsilon^2}}\ln\sqrt{\tfrac{1}{2}(\varepsilon+\varepsilon^2)}\right)\times \right.$$
$$\left.\left(1+3\varepsilon - 4\sqrt{\tfrac{1}{2}(\varepsilon+\varepsilon^2)}\right)\lambda^2 + O(\lambda^3)\right\} \tag{21}$$

Furthermore, using Equation (21), we can write the efficiency as a power series in λ,

$$\eta_E \equiv 1 - z_E(\varepsilon,\lambda). \tag{22}$$

When $\lambda = 0$, the corresponding ecological efficiency with instantaneous adiabats is,

$$\eta_{EO} = 1 - z_{EO}(\varepsilon,\lambda=0) = 1 - \sqrt{\tfrac{1}{2}(\varepsilon+\varepsilon^2)}, \tag{23}$$

which is the maximum possible one for this operating regimen. From Equations (16) and (21) a linear approximation for the efficiency η in terms of compression ratio can be derived, $r_C = V_3/V_1$, and of the ratio T_C/T_H. It can be verified that $r_C \to \infty$ and $\lambda \to 0$ lead to the Curzon–Ahlborn–Novikov–Chambadal efficiency, now written as $\eta_{CAN} \equiv \eta_P(\lambda=0) = \eta_{PO}$. From Equation (16), the linear approximation can be obtained:

$$\eta_{PL}(\lambda) = 1 - \sqrt{\varepsilon} - \tfrac{1}{2}(1-\sqrt{\varepsilon})^2\lambda, \tag{24}$$

and the corresponding linear approximation of ecological efficiency is as follows:

$$\eta_{EL}(\lambda) = 1 - \sqrt{\tfrac{1}{2}\left(\varepsilon^2 + \varepsilon\right)} - \left[\tfrac{1}{4}(1 + 3\varepsilon) - \sqrt{\tfrac{1}{2}\left(\varepsilon^2 + \varepsilon\right)}\right]\lambda. \tag{25}$$

As it is known in real compressors, the percent of volume in the total displacement of a piston into a cylinder is called the dead space ratio, and it is defined as $c = $(volume of dead space)/(volume of displacement) [29]. In the Curzon–Ahlborn cycle, r_C appears as the reciprocal of c. It is found that $3\% \leq c \leq 10\%$; hence, $100/3 \geq r_C \geq 100/10$ or $33 > r_C \geq 10$. Supposing power plants working as a Curzon–Ahlborn cycle, a linear approximation of efficiency, Equations (24) and (25), values of efficiency appear around the experimental values. As an example, Table 1 shows a comparison between real values and linear approximation values, $\gamma = 1.67$, and the closeness of the linear approximation, in case of some modern power plants.

Nuclear power plant	T_C (K)	T_H (K)	η_{obs}	η_{EL} , $10 \leq r_C < 33$
Doel 4 (Belgium),	283	566	0.35000	0.37944–0.38224
Almaraz II (Spain)	290	600	0.34500	0.39234–0.39539
Sizewell B (UK)	288	581	0.36300	0.38277–0.38563
Cofrentes (Spain)	289	562	0.34000	0.36844–0.37103
Heysham (UK)	288	727	0.40000	0.46036–0.46506

Table 1. Comparison of experimental efficiencies with linear ecological approximation

2.2. Nonlinear heat transfer law

The ecological efficiency has been calculated using Dulong and Petit's heat transfer law in [30], maximizing ecological function for instantaneous adiabats. When the time for all the processes of the Curzon and Ahlborn cycle is taken into account, efficiencies in both regimens, maximum power output and maximum ecological function, can be obtained, following the procedure employed. Suppose an ideal gas as a working substance in a cylinder with a piston that exchanges heat with the reservoirs, and using a heat transfer law of the form

$$\frac{dQ}{dt} = \alpha\left(T_f - T\right)_i^k, \tag{26}$$

where $k > 1$, α is the thermal conductance assumed the same for both reservoirs, dQ/dt is the rate of heat Q exchange, and T_i and T_f are the temperatures for the heat exchange process. From the first law of thermodynamics and under mechanical equilibrium condition, i.e., $p = p_{ext}$, because the working substance is an ideal gas, $U = U(T)$, one obtains

$$\frac{dQ}{dt} = p\frac{dV}{dt} \text{ or } \alpha\left(T_f - T_i\right)^k = \frac{RT_i}{V}\frac{dV}{dt}. \tag{27}$$

Equation (27) implies that the times along the isothermal processes in Figure 1 are, respectively,

$$t_1 = \frac{RT_{HW}}{\alpha\left(T_H - T_{HW}\right)^k}\ln\frac{V_2}{V_1} \text{ and } t_3 = \frac{RT_{CW}}{\alpha\left(T_{CW} - T_C\right)^k}\ln\frac{V_3}{V_4} \tag{28}$$

The corresponding heat exchanged Q_H and Q_C become, respectively,

$$Q_H = RT_{HW}\ln\frac{V_2}{V_1} \text{ and } Q_C = RT_{CW}\ln\frac{V_4}{V_3}, \tag{29}$$

where R is the universal gas constant and V_1, V_2, V_3, V_4, are the corresponding volumes for the states 1, 2, 3, and 4 in Figure 1 also. The times of the adiabatic processes are assumed as

$$t_2 = \frac{RT_{HW}}{\alpha\left(T_H - T_{HW}\right)^k\left(\gamma - 1\right)}\ln\frac{T_{HW}}{T_{CW}}, \text{ and } t_4 = \frac{-RT_{CW}}{\alpha\left(T_{CW} - T_C\right)^k\left(\gamma - 1\right)}\ln\frac{T_{CW}}{T_{HW}} \tag{30}$$

where $\gamma \equiv C_P/C_V$ has been used. With these results, the form for the power output is

$$P = \frac{T_1^k\alpha\left(1 - z\right)\left(1 + \lambda\ln z\right)}{\frac{1}{\left(1 - u\right)^k} + \frac{z}{\left(zu - \varepsilon\right)^k}}, \tag{31}$$

with the same used parameters. By means of $\partial P/\partial u = 0$ and $\partial P/\partial z = 0$, one obtains

$$u = \frac{z^{\frac{2}{k+1}} + \varepsilon}{z + z^{\frac{2}{k+1}}}, \tag{32}$$

and the resulting expression for the implicit function $z = z(\lambda, \varepsilon)$, for a given k,

$$\left[z^{\frac{2}{k+1}}\left(z - \varepsilon\right)\left(\lambda\left(1 - z\right) - z\left(1 + \lambda\ln z\right)\right) + zk\left(z^{\frac{2}{k+1}} + \varepsilon\right)\left(1 - z\right)\left(1 + \lambda\ln z\right)\right]\left(z^{\frac{2k}{k+1}} + z\right)$$
$$- z\left(1 - z\right)\left(1 + \lambda\ln z\right)\left[z^2 + \varepsilon z^{\frac{2k}{k+1}} + z^{\frac{2}{k+1}}\left(z - \varepsilon\right)\right] = 0. \tag{33}$$

With reasonable approximations, only for the exponents in Equation (33), the following can be obtained:

$$(1+\lambda)\big((k\varepsilon+zk)(1-z)-z(z-\varepsilon)\big)+\lambda(1-z)(1-\varepsilon)-(1+\lambda\ln z)(1-z)z=0. \tag{34}$$

Equation (34) allows to the explicit expression for the function $z=z(\varepsilon, k)$ when $\lambda=0$,

$$z_{OP}(\varepsilon,k)=\frac{(k-1)(1-\varepsilon)\pm\sqrt{(\varepsilon-1)^2(1-k)^2+4k^2\varepsilon}}{2k}. \tag{35}$$

Taking now $k=5/4$ in Equation (35), one obtains the following value for the physically acceptable and approximated solution of Equation (33), namely,

$$z_{OPDP}=\frac{1-\varepsilon+\sqrt{\varepsilon^2+98\varepsilon+1}}{10}. \tag{36}$$

The numerical results for $\eta_{OPDP}=1-z_{OPDP}$ are compared with η_{CAN} and the observed efficiency, η_{obs}, which are in good agreement with the reported values. Now, assuming that z obtained from Equation (34) can be expressed as a power series in the parameter λ, the expression for the efficiency at maximum power output regimen is as follows:

$$\eta_{PDP}=1-z_{PDP}(\lambda,\varepsilon)=1-z_{OPDP}\Big[1+B_1(\varepsilon)\lambda+B_2(\varepsilon)\lambda^2+O(\lambda^3)\Big]. \tag{37}$$

One can find B_j, $j=1, 2, ...etc.$, through successive derivatives respect to λ. The first one is

$$B_1(\varepsilon)=\frac{16(1-z_{OPDP})(\varepsilon-z_{OPDP})}{z_{OPDP}(5-4\varepsilon-40z_{OPDP})} \tag{38}$$

Now, the ecological function for Curzon and Ahlborn engine takes the form

$$E(u,z)=\frac{T_H^k\alpha(1+\lambda\ln z)(1+\varepsilon-2z)}{\frac{1}{(1-u)^k}+\frac{z}{(zu-\varepsilon)^k}}. \tag{39}$$

We find the function $z(\varepsilon)$ from the maximization of function $E(u, z)$ and the efficiency for $k=5/4$. Upon setting $\partial E/\partial u=0$ and $\partial E/\partial z=0$, one obtains from the first condition that

$$u = \frac{z^{\frac{2}{k+1}} + \varepsilon}{z + z^{\frac{2}{k+1}}}, \tag{40}$$

and from the second one,

$$\frac{\left(\left(1+\varepsilon-2z\right)\lambda - 2z\left(1+\lambda \ln z\right)\right)\left(zu-\varepsilon\right)}{\left(1+\lambda \ln z\right)\left(1+\varepsilon-2z\right)\left(zu-\varepsilon-kuz\right)z} - \frac{\left(1-u\right)^{k}}{\left(zu-\varepsilon\right)^{k} + z\left(1-u\right)^{k}} = 0. \tag{41}$$

Substituting now Equation (40) for u in Equation (41), one obtains the following expression:

$$\left(z^{2} + z^{\frac{k+3}{k+1}}\right)\left(z-\varepsilon\right)\left(-2\left(1+\lambda \ln z\right)z + \left(1+\varepsilon-2z\right)\lambda\right) =$$
$$= \left(z^{\frac{k+3}{k+1}} - \varepsilon z^{\frac{2}{k+1}} - \left(z^{\frac{k+3}{k+1}} + z\varepsilon\right)k\right)z\left(1+\lambda \ln z\right)\left(1+\varepsilon-2z\right). \tag{42}$$

The analytical solution of Equation (42) is not feasible when the exponents of z are not integers, which is the present case, $k = 5/4$. The numerical solution of Equation (42) shows that anyone solution falls into the region bounded by solutions for $\lambda = 0$ and $\lambda = 1$ [28]. It can be appreciated that within the values of $0 \leq \varepsilon \leq 1$, which are the only physically relevant, the curve represented by Equation (42) can be fitted with a parabolic curve. The simplest approximation that allows for a parabolic fit for $0 \leq \lambda \leq 1$ modifying the exponents leads to the approximate analytical expression for $z(\varepsilon, \lambda)$ as

$$2\left(-2\left(1+\lambda \ln z\right)z + \left(1+\varepsilon-2z\right)\lambda\right)\left(z-\varepsilon\right) - \left(1+\lambda \ln z\right)\left(1+\varepsilon-2z\right)\left(\left(z-\varepsilon\right) - \left(z+\varepsilon\right)k\right) = 0. \tag{43}$$

For the case $\lambda = 0$, that is instantaneous adiabats, and with $k = 5/4$, Equation (43) becomes

$$z_{OEDP} = \frac{1-\varepsilon + \sqrt{649\varepsilon^{2} + 646\varepsilon + 1}}{36}. \tag{44}$$

Any other root has no physical meaning because efficiencies must always be positive. Adequate comparison between fitted numerical values of η_{MEDP} in [30] and $\eta_{OEDP} = 1 - z_{OEDP}$ is in [28] and later in [25]. Assuming z given by Equation (43) as a power series in the parameter λ, efficiency can be found as follows:

$$\eta_{EDP} = 1 - z_{EDP}\left(\lambda, \varepsilon\right) = 1 - z_{OEDP}\left[1 + b_{1}\left(\varepsilon\right)\lambda + b_{2}\left(\varepsilon\right)\lambda^{2} + O\left(\lambda^{3}\right)\right]. \tag{45}$$

At last taking, $z_0 = z_{EDP}(\varepsilon, \lambda = 0) = z_{OEDP}(\varepsilon)$, and from Equation (43), coefficients are found by successively taking the derivative respect to λ and evaluating at $\lambda = 0$. The first one leads to the linear approximation for ecological efficiency since Equation (45), as follows:

$$b_1(\varepsilon) = \frac{-2z_0 + 2\varepsilon - 6z_0\varepsilon + 2\varepsilon^2 + 4z_0^2}{z_0\left(-9z_0 - \frac{1}{4}\varepsilon + \frac{1}{4}\right)}, \tag{46}$$

3. The non-endoreversible Curzon and Ahlborn cycle

By contrast, in finite time, thermodynamics is usually considered an endoreversible Curzon–Ahlborn cycle, but in nature, there is no endoreversible engine. Thus, some authors have analyzed the non-endoreversible Curzon and Ahlborn cycle. Particularly in [16] has been analyzed the effect of thermal resistances, heat leakage, and internal irreversibility by a non-endoreversibility parameter, advanced in [14],

$$I_S \equiv \frac{\Delta S_C}{\Delta S_H}, \tag{47}$$

where ΔS_C is the change of entropy during the exchange of heat from the engine to cold reservoir, and ΔS_H is the change of entropy during the exchange of heat from the hot reservoir to engine. The non-endoreversible Curzon–Ahlborn cycle is shown in Figure 2. The efficiency at maximum power output for instantaneous adiabats is

$$\eta_m = 1 - \sqrt{I_S \varepsilon}, \quad I_S > 1. \tag{48}$$
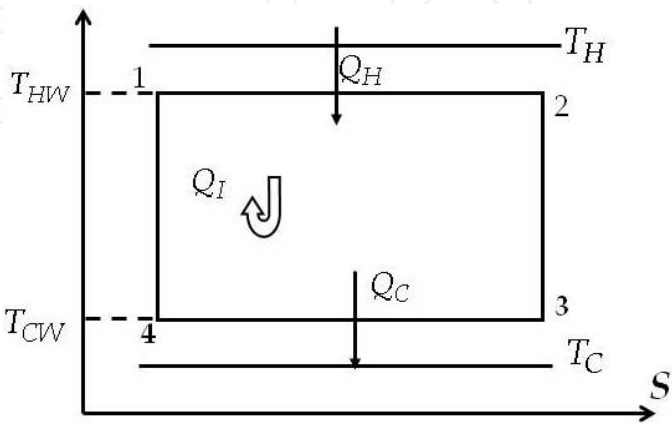
Figure 2. Curzon and Ahlborn cycle in the S-T plane. Q_I is a generated internally heat.

Following the procedure in [16], have been found expressions to measure possible reductions of undesired effects in heat engines operation [17], and has been pointed out that I_S is not dependent of ε and rewrote Equation (48) as

$$\eta_m = 1 - \sqrt{\frac{\varepsilon}{I}}, \quad I \equiv \frac{1}{I_S}, \quad 0 < I < 1. \tag{49}$$

Moreover, in [31] has been applied variational calculus showing that the saving function in [17] and modified ecological criteria are equivalent. In this section, internal irreversibilities are taken into account to obtain Equation (4), replacing $(\varepsilon^2 + \varepsilon)/2I$ instead $(\varepsilon^2 + \varepsilon)/2$ in case of a non-endoreversible Curzon and Ahlborn cycle. The procedure in [5] is combined with the cyclic model in [16] to obtain the form of power output function and of ecological function.

3.1. Curzon and Ahlborn cycle with instantaneous adiabats

Suppose a thermal engine working like a Curzon and Ahlborn cycle, in which an internal heat by internal processes of working fluid appears, assuming ideal gas as working fluid. The Clausius inequality with the parameter of non-endoreversibility becomes

$$I_S \frac{Q_H}{T_{HW}} - \frac{Q_C}{T_{CW}} = 0. \tag{50}$$

The changes of entropy are ΔS_C and ΔS_H during the heat exchange between the engine and its reservoirs. From Equation (50), $Q_C = (T_{CW}/T_{HW})I_S Q_H$, and clearly $I_S \geq 1$. Thus, the heat exchanges between the thermal engine and its reservoirs are

$$Q_H = R T_{HW} \ln \frac{V_2}{V_1} \text{ and } Q_C = \frac{T_{CW}}{T_{HW}} I_S R T_{HW} \ln \frac{V_2}{V_1}. \tag{51}$$

The volumes in the states of change of process in the cycle are V_1, V_2, V_3, V_4, and the total made work by the engine can be written as

$$W_I = R T_{HW} \left(1 - I_S \frac{T_{CW}}{T_{HW}}\right) \ln \frac{V_2}{V_1}, \tag{52}$$

Assume the exchange of heat as Equation (8) with $k = 1$ and an internal generated heat Q_I. For reversible adiabatic processes, $T V^{\gamma-1} = constant$, with $\gamma = C_P/C_V$, so that $V_2/V_1 = V_3/V_4$ is obtained. For instantaneous adiabatic processes, the total time is

$$t_{TOT} = \frac{RT_{HW}}{\alpha}\left[\frac{1}{T_H - T_{HW}} + \frac{I_S}{T_{CW} - T_C}\cdot\frac{T_{CW}}{T_{HW}}\right]\ln\frac{V_2}{V_1}, \tag{53}$$

with the changes $Z_I = I_S T_{CW}/T_{HW}$ and $u = T_{HW}/T_H$, the power output is

$$P_I = \alpha T_1(1 - Z_I)\left(\frac{1}{1-u} + \frac{I_S Z_I}{Z_I u - \varepsilon I_S}\right)^{-1}. \tag{54}$$

Also, the variation of entropy in the cycle can be written as

$$\Delta S_I = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = -R\frac{T_H}{T_C}(\varepsilon - Z_I)\ln\frac{V_3}{V_1}, \tag{55}$$

and Equation (18) is modified as

$$E_I = \alpha T_1(1 - 2Z_I + \varepsilon)\left(\frac{1}{1-u} + \frac{I_S Z_I}{Z_I u - \varepsilon I_S}\right)^{-1}. \tag{56}$$

Now, the following is obtained from the conditions $\partial P/\partial u = 0$ and $\partial P/\partial Z_I = 0$:

$$u = \left(Z_I + \varepsilon\sqrt{I_S}\right)\sqrt{I_S}\left[Z_I(1 + \sqrt{I_S})\right]^{-1}, \tag{57}$$

and a physically possible solution for Z_I is found, which leads to the efficiency

$$\eta_{CANI} = 1 - \sqrt{\frac{\varepsilon}{I}}, \tag{58}$$

when the change $I_S = 1/I$ proposed in [17] is used. Similar results can be obtained for the ecological function. Thus, from Equation (56) for the same variables u and Z_I, function $u = u(z)$ is obtained as Equation (57), and the physically possible solution of Z_I leads to ecological efficiency as in [23],

$$\eta_{EI} = 1 - \sqrt{\frac{\varepsilon^2 + \varepsilon}{2I}}, \tag{59}$$

For the suitable values of parameter $I = 1/I_S$, found in [17] as $0.8 \leq I \leq 0.9\,0$, Table 2 shows the values of the ecological efficiency, Equation (59), compared with the experimental values of

the efficiency, η_{obs}. The intervals of values of the efficiency are improved in the sense that they are nearer to the reported experimental values in literature.

Power Plant	$T_2(K)$	$T_1(K)$	η_{obs}	η_{EI}
Doel 4 (Belgium), 1985	283	566	0.35000	0.31535 to 0.3545
Almaraz II, Spain	290	600	0.34500	0.3306 to 0.36889
Sizewell B, UK	288	581	0.36300	0.3198 to 0.35821
Cofrentes, Spain	289	562	0.34000	0.30238 to 0.34228
Heysham, UK	288	727	0.40000	0.41206 to 0.44568

Table 2. Comparison of experimental efficiencies with efficiencies from Equation (25)

3.2. Curzon–Ahlborn cycle with noninstantaneous adiabats

In order to include the compression ratio in the analysis of Curzon and Ahlborn cycle, it is necessary to suppose finite time for the adiabatic processes. Hence, as it is known, with ideal gas as working fluid and using the Newton heat transfer law, the following can be written:

$$\frac{dQ}{dt} = p\frac{dV}{dt}, \tag{60}$$

and because $p = RT/V$, Equation (60) is now

$$\frac{dQ}{dt} = \frac{RT}{V}\frac{dV}{dt} = RT\frac{d}{dt}(\ln V). \tag{61}$$

Then again, internal energy U depends only on the initial and final states, so the adiabatic expansion in the cycle can be written as

$$\frac{1}{V}\frac{dV}{dt} = \alpha\frac{T_H - T_{HW}}{RT_{HW}}. \tag{62}$$

The integration of Equation (62) leads to the time of the adiabatic expansion in the cycle,

$$t_2 = \left(RT_{HW}\ln\frac{V_3}{V_2}\right)\left[\alpha(T_H - T_{HW})\right]^{-1}, \tag{63}$$

and taking into account the form that acquires the yielded heat Q_C based on the absorbed heat Q_H, Equation (51), the time of the adiabatic processes can be assumed as

$$t_{ad} = \left(RT_{HW} \ln \frac{V_3}{V_4} \right) \left[\alpha \left(T_H - T_{HW} \right) \right]^{-1} + \left[RT_{HW} (I_S \frac{T_{CW}}{T_{HW}}) \ln \frac{V_1}{V_4} \right] \cdot \left[\alpha \left(T_{CW} - T_C \right) \right]^{-1}, \tag{64}$$

and the total time of the non-endoreversible cycle is as follows:

$$t_{TOT}(ad) = \frac{RT_{HW}}{\alpha} \left[\frac{1}{T_H - T_{HW}} + \frac{I_s}{T_{CW} - T_C} \cdot \frac{T_{CW}}{T_{HW}} \right] \ln \frac{V_3}{V_1}, \tag{65}$$

So that a new expression for power output in the cycle using the changes of variables in Equation (54) and Equation (56) is found, namely,

$$P_{I\lambda} = \alpha T_H \left(1 - Z_I \right) \left(1 + \lambda \ln Z_I - \lambda \ln I_S \right) \cdot \left[\frac{1}{1-u} + \frac{Z_I I_S}{Z_I u - \varepsilon I_S} \right]^{-1}, \tag{66}$$

with $\lambda \equiv 1 / ((\gamma - 1) \ln r_C)$, and the compression ratio is $r_C \equiv V_3 / V_1$. The entropy production with the same changes of variables is found, and the new expression for ecological function is

$$E_{I\lambda} = \alpha T_H \left(1 - 2Z_I + \varepsilon \right) \left(1 + \lambda \ln Z_I - \ln I_S \right) \cdot \left[\frac{1}{1-u} + \frac{I_S Z_I}{Z_I u - \varepsilon I_S} \right]^{-1}. \tag{67}$$

In order to maximize power output, Equation (66), the conditions $\partial P_{I\lambda} / \partial u = 0$ and $\partial P_{I\lambda} / \partial Z_I = 0$ are necessary. Also, in order to maximize ecological function, Equation (67), the conditions $\partial E_{I\lambda} / \partial u = 0$ and $\partial E_{I\lambda} / \partial Z_I = 0$ are necessary. Hence, one can find the form of Z_I for each maximized features function. In case of maximizing power output, the following is obtained:

$$Z_I^2 + \lambda Z_I^2 \ln Z_I - \lambda Z_I^2 \ln I_S + \lambda \varepsilon I_S - \lambda Z_I - \lambda Z_I \varepsilon I_S + \lambda Z_I^2 - \varepsilon I_S - \lambda \varepsilon I_S \ln Z_I + \lambda \varepsilon I_S \ln I_S = 0, \tag{68}$$

and in case of maximizing ecological function, the following is obtained:

$$2Z_I^2 \left(1 - \lambda \ln I_S + \lambda \right) - \lambda Z_I \left(\varepsilon + 1 + 2\varepsilon I_S \right) + 2\lambda Z_I^2 \ln Z_I = \left(\varepsilon + 1 \right) \left(1 - \lambda - \lambda \ln I_S + \lambda \ln Z_I \right) \varepsilon I_S. \tag{69}$$

When $\lambda = 0$ and $I_S = 1$ the corresponding expressions shown in [5] and in [12] for maximum power output and maximum ecological function are recovered. Moreover, expressions of the Curzon–Ahlborn–Novikov–Chambadal efficiency and the ecological efficiency found in [1] and [4] are recovered. Non instantaneous adiabats imply $\lambda \neq 0$, and Z_I can be expanded in a power series of λ. The simplest expansion is the linear approach, so if Z_I is written for each case of objective function, one can obtain, respectively,

$$Z_{PI} = a_0 + a_1 \lambda + \cdots \text{ and } Z_{EI} = b_0 + b_1 \lambda + \cdots. \tag{70}$$

Parameters a_i and b_i can be calculated as in [5] and in [12]. They are small and greater than zero. They go to zero as $i \to \infty$; thus, it is possible to ensure the convergence of series. Linear approximation only requires finding a_0 and a_1 (or b_0 and b_1), which are, in case of maximum power output, as follows:

$$a_0 = \sqrt{\varepsilon I_S} \text{ and } a_1 = \tfrac{1}{2}\left(1 - \sqrt{\varepsilon I_S}\right), \tag{71}$$

and in case of maximum ecological function,

$$b_0 = \sqrt{\tfrac{1}{2}I_S\left(\varepsilon^2 + \varepsilon\right)} \text{ and } b_1 = \tfrac{1}{4}\left(1 + \varepsilon + 2\varepsilon I_S\right) - \sqrt{\tfrac{1}{2}I_S\left(\varepsilon^2 + \varepsilon\right)}. \tag{72}$$

The linear approximation of efficiency, at maximum power output and at maximum ecological function, can now be derived from Equation (71) or Equation (72), respectively, as

$$\eta_{CANL} = 1 - Z_{PI} \text{ or } \eta_{EL} = 1 - Z_{EI}. \tag{73}$$

It is important to note that compression ratio has no arbitrary values, as discussed in Section 2.1. Thus, for $r_C = 10$ and the extreme values of the range of values for I, $I = 1/I_S$, found in [17] for real engines with a gas as working fluid, namely, $I = 0.8$ and $I = 0.9$, the efficiency is obtained as a function of the parameter ε.

4. Stirling and Ericsson cycles

As it is known, the thermal engines can be endothermic or exothermic. Among the first engines, the best known are Otto and Diesel, and among the second two engines, very interesting and similar to the theoretical Carnot engine are Stirling and Ericsson engines [32,33]. In particular, a Stirling engine is a closed-cycle regenerative engine initially used for various applications, and until the middle of last century, they were manufactured on a large scale. However, the development of internal combustion engines from the mid-nineteenth century and the improvement in the refining of fossil fuels influenced the abandonment of the Stirling and Ericsson engines in the race for industrialization, gradually since the early twentieth century. Reference [34] is an interesting paper devoted to Stirling engine.

In the classical equilibrium thermodynamics, Stirling and Ericsson cycles have an efficiency that goes to the Carnot efficiency, as it is shown in some textbooks. These three cycles have the common characteristics, including two isothermal processes. The objection to the classical point of view is that reservoirs coupled to the engine modeled by any of these cycles do not have the same temperature as the working fluid because this working fluid is not in direct thermal contact with the reservoir. Thus, an alternative study of these cycles is using finite

time thermodynamics. Thus, since the end of the previous century, and on recent times, the characteristics of Stirling and Ericsson engines have resulted in renewed interest in the study and design of such engines, and in the analysis of its theoretical idealized cycle, as it is shown in many papers, [22,35-37] among others. Nevertheless, the discussion on these engines and its theoretical model has not been exhausted.

In this section, an analysis of the Stirling and Ericsson cycles from the viewpoint of finite time thermodynamics is made. The existence of finite time for heat transfer in isothermal processes is proposed, but the cycles are analyzed assuming they are not endoreversible cycles, through the factor that represents their internal irreversibilities [14], so that the proposed heat engine model is closer to a real engine. Some results in reference [22] are used, and a methodology to obtain a linear approximation of efficiency, including adequate parameters, is shown. Variable changes are made right, like those used in [5] and in [23,25]. This section is a summary of obtained results in [38].

4.1. Stirling cycle

Now, as it is known, Stirling cycle consists of two isochoric processes and two isothermal processes. At finite time, the difference between the temperatures of reservoirs and the corresponding operating temperatures is considered, as shown in Figure 3. To construct expressions for power output and ecological function for this cycle, some initial assumptions are necessary. First, the heat transfer is supposed as Newton's cooling law for two bodies in thermal contact with temperatures T_i and T_f, $T_i > T_f$, with a rapidity of heat change dQ/dt, and a constant thermal conductance α, which for convenience is assumed to be equal in all cases of heat transfer as follows:
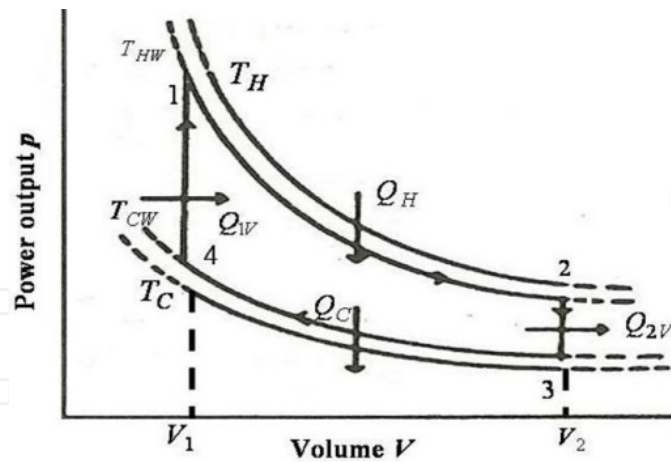
$$\frac{dQ}{dt} = \alpha\left(T_i - T_f\right). \tag{74}$$

On the other hand, it is assumed that the internal processes of the system cause irreversibilities that can be represented by the factor I_S previously presented, so from the second law of thermodynamics, the following can be written:

$$Q_C = \frac{T_{CW}}{T_{HW}} I_S Q_H. \tag{75}$$

Power output is defined as

$$P = \frac{Q_H - Q_C}{t_{TOT}}. \tag{76}$$

Figure 3. Idealized Stirling cycle at the V–p (volume vs pressure) plane.

With ideal gas as working substance for an isothermal process, the equation of state leads to

$$\frac{RT_i}{V}\frac{dV}{dt} = \alpha\left(T_i - T_f\right). \tag{77}$$

An assumption for the cycle is that heating and cooling at constant volume is performed as

$$\left|\frac{dT}{dt}\right| = r_V = constant, \tag{78}$$

where it is not difficult to show that it meets

$$\left|Q_{1V}\right| = \left|Q_{2V}\right|. \tag{79}$$

By contrast, from the equilibrium conditions, it can be assumed

$$\frac{dU}{dt} = \frac{dQ}{dt} = C_V\frac{dT}{dt} = r_V C_V, \tag{80}$$

and the heating and cooling, respectively, from the first law of thermodynamics are

$$Q_{1V} = C_V\left(T_{HW} - T_{CW}\right) = \Delta U_{41} \text{ and } Q_{2V} = C_V\left(T_{CW} - T_{HW}\right) = \Delta U_{23}, \tag{81}$$

and the time for each isochoric processes is given as

$$t_V = \frac{1}{r_V}\left(T_{HW} - T_{CW}\right).$$

(82)

The time for the isothermal processes can be found from Equation (77) as

$$t_1 = \frac{RT_{HW}}{\alpha\left(T_H - T_{HW}\right)}\ln\frac{V_2}{V_1} \text{ and } t_2 = -\frac{RT_{CW}}{\alpha\left(T_{CW} - T_C\right)}\ln\frac{V_1}{V_2}.$$

(83)

The negative sign in t_2 is just because there is no negative time, the total time of cycle is

$$t_{TOT} = \frac{RT_{HW}}{\alpha\left(T_H - T_{HW}\right)}\ln\frac{V_2}{V_1} - \frac{RT_{CW}}{\alpha\left(T_{CW} - T_C\right)}\ln\frac{V_1}{V_2} + \frac{2}{r_V}\left(T_{HW} - T_{CW}\right).$$

(84)

Since its definition and taking into account Equation (76), the power output of cycle is written as

$$P_{SI} = \frac{Q_H - I_S\frac{T_{CW}}{T_{HW}}Q_H}{t_1 + t_2 + 2t_V}.$$

(85)

Now, with the change of variables used in the previous section in Equations (54) and (56), and taking into account the ratio of temperatures of the heat reservoirs, used in Equations (1) and (2), with the parameter $\lambda = [(\gamma - 1)\ln(V_2/V_1)]^{-1}$ that includes the compression ratio of cycle, in this case $V_2/V_1 = r_C$, the power output of Stirling cycle takes the form

$$P_{SI} = \frac{\alpha T_H\left(1 - Z_I\right)}{\frac{1}{1-u} + \frac{I_S Z_I}{Z_I u - I_S \varepsilon} + \frac{2\alpha T_H}{C_V I_S r_V}\left(I_S - Z_I\right)\lambda}.$$

(86)

The optimization conditions $(\partial P_{SI}/\partial u)_{Z_I=const}=0$ and $(\partial P_{SI}/\partial Z_I)_{u=const}=0$ permit find the function $Z_I = Z_I(\varepsilon, I_S, \lambda)$. From the first one, $u = u(Z_I, I_S)$ is obtained as

$$u = \frac{\left(Z_I + \varepsilon\sqrt{I_S}\right)\sqrt{I_S}}{Z_I\left(1 + \sqrt{I_S}\right)},$$

(87)

and from the second one, a solution physically adequate Z_{IP} can be obtained by

$$\frac{\left(1+\sqrt{I_S}\right)^2 r_V C_V I_S + 2\alpha T_H \lambda \left(I_S - 2Z_I + \varepsilon I_S\right)}{-\left(Z_I - \varepsilon I_S\right) + \left(1 - Z_I\right)} =$$
$$= \frac{Z_I \left(1+\sqrt{I_S}\right)^2 r_V C_V I_S + 2\alpha T_H \lambda \left(Z_I - \varepsilon I_S\right)\left(I_S - Z_I\right)}{\left(1 - Z_I\right)\left(Z_I - \varepsilon I_S\right)}. \tag{88}$$

Thus, that the efficiency at maximum power output can be written as

$$\eta_{SIP} = 1 - Z_{IP}\left(\varepsilon, I_S, \lambda\right). \tag{89}$$

For known values of parameters C_V, α, and T_H, in the limit $\lambda \to 0$, namely, $V_2/V_1 \to \infty$, the efficiency of non-endoreversible Stirling cycle, η_{SIP}, goes to the efficiency for the non-endoreversible Curzon and Ahlborn cycle, as can be seen from Equation (86),

$$\eta_{SIP} \to \eta_m = 1 - \sqrt{\tfrac{\varepsilon}{I}}. \tag{90}$$

The analysis for ecological function is similar to power output, and also leads to similar results. The shape of function $u = u(Z_I, I_S, \varepsilon)$ is the same as in Equation (87), but the form of $Z_I = Z_I(\varepsilon, I_S, \lambda)$ changes. Because heating and cooling in both isochoric and isobaric processes are considered constant, and taking into account Equations (75) and (78), the change of entropy can be taken only for isothermal processes. Then, the change of entropy for the non-endoreversible cycle considered is

$$\Delta S = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = -\frac{Q_H}{T_H} + I_S \frac{T_{CW}}{T_{HW}}\frac{Q_H}{T_C}, \tag{91}$$

which leads to the ecological function as

$$E_{SI} = \frac{Q_H}{t_{TOT}}\left(1 - 2I_S \frac{T_{CW}}{T_{HW}} + \frac{T_C}{T_H}\right) = \frac{R T_{HW}}{t_{tot}}\left(1 - 2I_S \frac{T_{CW}}{T_{HW}} + \frac{T_C}{T_H}\right)\ln\frac{V_2}{V_1}, \tag{92}$$

Where t_{TOT} is as Equation (84). With the same parameters definite in the previous section, ecological function can be written now as

$$E_{SI} = \frac{\alpha T_H \left(1 - 2Z_I + \varepsilon\right)}{\frac{1}{1-u} + \frac{I_S Z_I}{Z_I u - I_S \varepsilon} + \frac{2\alpha T_H}{C_V I_S r_V}\left(I_S - Z_I\right)\lambda}. \tag{93}$$

As in the case of power output, in order to find the efficiency at maximum ecological function, there are two conditions, namely, $(\partial E_{SI} / \partial u)_{Z_I = const} = 0$ and $(\partial E_{SI} / \partial Z_I)_{u = const} = 0$. These conditions lead to obtaining the parameter u as in Equation (87) and also $Z_{IE} = Z_{IE}(\varepsilon, I_S, \lambda)$ as an adequate solution for the second condition by the relation,

$$\frac{Z_I \left(1 + \sqrt{I_S}\right)^2 r_V C_V I_S + 2 T_H \alpha \lambda \left(Z_I - \varepsilon I_S\right)\left(I_S - Z_I\right)}{\left(1 - 2 Z_I + \varepsilon\right)\left(Z_I - \varepsilon I_S\right)} =$$
$$= \frac{\left(1 + \sqrt{I_S}\right)^2 r_V C_V I_S + 2 T_H \alpha \lambda \left(I_S - 2 Z_I + \varepsilon I_S\right)}{\left(1 - 2 Z_I + \varepsilon\right) - 2\left(Z_I - \varepsilon I_S\right)} \tag{94}$$

The efficiency for the Stirling cycle at maximum ecological function can be written now as

$$\eta_{SIE} = 1 - Z_{IE}\left(\varepsilon, I_S, \lambda\right), \tag{95}$$

and $\lambda \to 0$ implies η_{SIE} goes to the efficiency for the non-endoreversible Curzon–Ahlborn cycle,

$$\eta_{SIE} \to \eta_{EI} = 1 - \sqrt{\tfrac{\varepsilon + \varepsilon^2}{2I}}. \tag{96}$$

The existence of a finite heat transfer in the isothermal processes is affected with the assumption of a non-endoreversible cycle with ideal gas as working substance. Power output and ecological function have also an issue that shows direct dependence on the temperature of the working substance. Expressions obtained with the changes of variables have the virtue of leading directly to the shape of the efficiency through Z_I function. Thus, in classical equilibrium thermodynamics, the Stirling cycle has its efficiency like the Carnot cycle efficiency; in finite time thermodynamics, this cycle has an efficiency in their limit cases as the Curzon–Ahlborn cycle efficiency.

4.2. Ericsson cycle

The Ericsson cycle consisting of two isobaric processes and two isothermal processes is shown in Figure 4. Now, it follows a similar procedure as in the Stirling cycle case. Thus, the hypothesis on constant heating and cooling, now at constant pressure, is expressed as

$$\left|\frac{dT}{dt}\right| = r_p = \text{constant}. \tag{97}$$

It is true that

$$\left|Q_{1p}\right|=\left|Q_{2p}\right|. \tag{98}$$

The equilibrium condition now is

$$\frac{dU}{dt}=C_p\frac{dT}{dt}-p\frac{dV}{dt}=r_pC_p-p\frac{dV}{dt}, \tag{99}$$

and the time for a constant pressure process is given as

$$t_p=\frac{T_{HW}}{r_p}\left(1-\frac{T_{CW}}{T_{HW}}\right). \tag{100}$$

The time for the isothermal processes can also be obtained from Equation (77) and can be written as

$$t_1=\frac{RT_{HW}}{\alpha\left(T_H-T_{HW}\right)}\ln\frac{V_2}{V_1}, t_2=-\frac{RT_{CW}}{\alpha\left(T_{CW}-T_C\right)}\ln\frac{V_4}{V_3}, \tag{101}$$

and the total time of cycle is now

$$t_{TOT}=\frac{RT_{HW}}{\alpha(T_H-T_{HW})}\ln\frac{V_2}{V_1}-\frac{RT_{CW}}{\alpha(T_{CW}-T_C)}\ln\frac{V_4}{V_3}+\frac{2}{r_p}(T_{HW}-T_{CW}), \tag{102}$$

so the power output of cycle from its definition and taking into account Equation (76) remains

$$P=\frac{Q_H-I_S\frac{T_{CW}}{T_{HW}}Q_H}{t_1+t_2+2t_p}. \tag{103}$$

With the change of variables used in the previous section, now the expression for the power output of the non-endoreversible Ericsson cycle is

$$P_{EI}=\frac{\alpha T_H\left(1-Z_I\right)}{\frac{1}{1-u}+\frac{Z_I}{Z_Iu-I_S\varepsilon}+\frac{2\alpha T_H}{C_VI_Sr_p}\left(I_S-Z_I\right)\lambda}, \tag{104}$$

which is essentially found for the Stirling cycle, with factor r_p instead of r_v. For extreme conditions, $(\partial P_{EI}/\partial u)_{u=const}=0$ and $(\partial P_{EI}/\partial u)_{Z_I=const}=0$ are obtained again using Equation (87), allowing us to find a physically acceptable solution $Z_{IP}=Z_{IP}(\varepsilon, I_S, \lambda)$ by
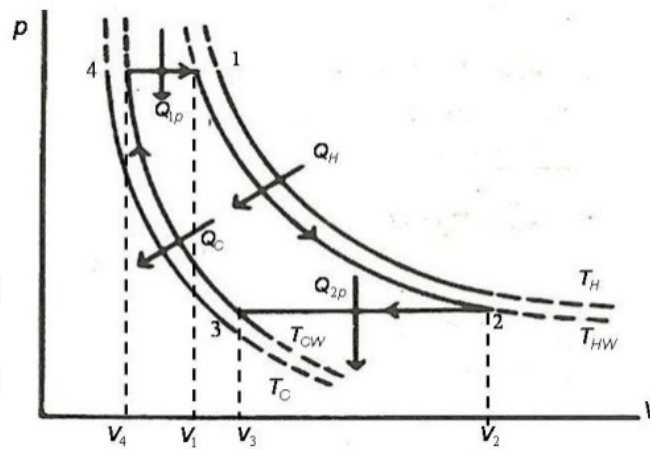
Figure 4. Idealized Ericsson cycle at the $V - p$ (volume vs pressure) plane.

$$
\frac{\left(1+\sqrt{I_S}\right)^2 r_p C_V I_S + 2\alpha T_H \lambda \left(I_S - 2Z_I + \varepsilon I_S\right)}{-\left(Z_I - \varepsilon I_S\right) + \left(1 - Z_I\right)} =
$$
$$
= \frac{Z_I \left(1+\sqrt{I_S}\right)^2 r_p C_V I_S + 2\alpha T_H \lambda \left(Z_I - \varepsilon I_S\right)\left(I_S - Z_I\right)}{\left(1 - Z_I\right)\left(Z_I - \varepsilon I_S\right)}
$$

(105)

Thus, at maximum power output regimen, the efficiency of non-endoreversible Ericsson cycle is

$$
\eta_{EIP} = 1 - Z_{IP}\left(\varepsilon, I_S, \lambda\right).
$$

(106)

The analysis for the case of ecological function is similar to the case of power output and also leads to similar results. The shape of the function $u = u(Z_I, I_S, \varepsilon)$ is the same as in Equation (87), but the form of $Z_I = Z_I(\varepsilon, I_S, \lambda)$ changes. Thus, because heating and cooling in isobaric processes are considered constant, the change of entropy can be taken only for the isothermal processes. Hence, for the non-endoreversible Ericsson cycle considered, we have

$$
\Delta S = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = -\frac{Q_H}{T_H} + I_S \frac{T_{CW}}{T_{HW}} \frac{Q_H}{T_C},
$$

(107)

from which the ecological function for the Ericsson cycle can be written as

$$
E_{EI} = \frac{\alpha T_H \left(1 - 2Z_I + \varepsilon\right)}{\frac{1}{1-u} + \frac{Z_I}{Z_I u - I_S \varepsilon} + \frac{2\alpha T_H}{C_V I_S r_p}\left(I_S - Z_I\right)\lambda},
$$

(108)

where the parameter r_p takes the adequate value depending on the cycle analyzed. As in the case of power output, there are two conditions for maximum ecological function, namely,

$(\partial E_{EI}/\partial u)_{z_I=const}=0$ and $(\partial E_{EI}/\partial Z_I)_{u=const}=0$. These conditions lead to obtain parameter u as in Equation (87) and also $Z_{EI}=Z_{EI}(\varepsilon, I_S, \lambda)$ by

$$\frac{Z_I\left(1+\sqrt{I_S}\right)^2 r_p C_V I_S + 2T_H \alpha\lambda\left(Z_I - \varepsilon I_S\right)\left(I_S - Z_I\right)}{\left(1-2Z_I+\varepsilon\right)\left(Z_I - \varepsilon I_S\right)} =$$
$$= \frac{\left(1+\sqrt{I_S}\right)^2 r_p C_V I_S + 2T_H \alpha\lambda\left(I_S - 2Z_I + \varepsilon I_S\right)}{\left(1-2Z_I+\varepsilon\right) - 2\left(Z_I - \varepsilon I_S\right)} \tag{109}$$

The efficiency for Ericsson cycle at maximum ecological function can be written now as

$$\eta_{EIE} = 1 - Z_{IE}\left(\varepsilon, I_S, \lambda\right). \tag{110}$$

5. Concluding remarks

The developed methodology leads directly to appropriate expressions of the objective functions simplifying the optimization process. This methodology shows the consequences of assuming non-endoreversible cyle in the process of isothermal heat transfer through the factor $I_S = 1/I$, which represents the internal irreversibilities of cycle, so that the proposed heat engine model is closer to a real engine. By contrast, as the known Carnot theorem provided a level of operation of heat engines, the Curzon and Ahlborn cycle provides levels of operation of such engines closer to reality. In this sense, the same manner within the context of classical equilibrium thermodynamics shows that in any cycle formed by two isothermal processes and any other pair of the same processes (isobaric, isochoric, and adiabatic), efficiency tends to Carnot cycle efficiency. In the context of finite time thermodynamics, any cycle as previously mentioned has an efficiency, which tends to Curzon and Ahlborn cycle efficiency. The above statements are independent if the cycle is considered endoreversible or non-endoreversible.

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