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Modelling of Bipedal Robots Using Coupled Nonlinear Oscillators

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1. Introduction

The first indications that the spinal marrow could contain the basic nervous system necessary to generate locomotion date back to the early 20th century. According to Mackay-Lyons (2002), the existence of nets of nervous cells that produce specific rhythmic movements for a great number of vertebrates is something unquestionable. Nervous nets in the spinal marrow are capable of producing rhythmic movements, such as swimming, jumping, and walking, when isolated from the brain and sensorial entrances. These specialised nervous systems are known as *nervous oscillators* or *central pattern generators* (CPGs).

Grillner (1985), Collins & Stewart (1993), Pearson (1993), and Collins & Richmond (1994) are some interesting works about the locomotion of vertebrates controlled by central pattern generators. According to Moraes (1999), the relation between spinal marrow and encephalus in the central nervous system of domestic animals is most significant than relation in human beings. This occurs because the most motor activities in the animals is performed by reflexes and not by the cerebral activity. In relation to the total activity of the central nervous system, it is estimate that exist approximately ten times more activity in the spinal marrow of dogs than in humans. However, the human locomotion is controlled, in part, by a central pattern generator, which is evidenced in the works as Calancie et al. (1994), Dimitrijevic et al. (1998) and Pinter & Dimitrijevic (1999).

Coupled nonlinear oscillators can be used in control systems of locomotion as pattern generators similar to the pattern of human gait, providing the approach trajectories of the legs. The central pattern generator is composed of a set of mutually coupled nonlinear oscillators, where each oscillator generates angular signals of reference for the movement of the legs. Each oscillator has its proper amplitude, frequency and parameters, and coupling terms makes the linking to the other oscillators. Some previous works on central pattern generators formed by nonlinear oscillators, applied in the locomotion of bipedal robots, can be seen in Bay & Hemami (1987), Dutra (1995), Zielinska (1996), Dutra et al. (2003) and Pina Filho et al. (2005).

The objective of this chapter is to present the modelling of a bipedal robot using a central pattern generator formed by a set of coupled nonlinear oscillators. We present some

concepts about the central nervous system of human being, including its main structures and characteristics, as well as its relation with the motor functions. Although people do not move in completely identical way, there are some characteristics of the gait that can be considered universal, and these similar points serve as base for the description of the patterns involved in the locomotion.

In the analyses, we consider a 2D model, with the three most important determinants of gait (the compass gait, the knee flexion of stance leg, and the plantar flexion of stance ankle), that performs only motions parallel to the sagittal plane. Using nonlinear oscillators with integer relation of frequency, we determine the transient motion and the stable limit cycles of the network formed by coupled oscillators, showing the behaviour of the hip angle and the knee angles. By changing a few parameters in the oscillators, modification of the step length and the frequency of the gait can be obtained.

The study of the utilisation of this system in the locomotion has great application in the project of autonomous robots and in the rehabilitation technology, not only in the project of prosthesis and orthosis, but also in the searching of procedures that help to recuperate motor functions of human beings.

2. The Central Nervous System

According to Cordeiro (1996), in neurological point of view, the human locomotion is started by impulses of cerebral cortex for the voluntary control and fine coordination; to these stimulations, are added the influences of the cerebellum (which becomes the coordinate gait), the vestibular system (which makes the maintenance of the balance), and the spinal marrow (which transmits the impulses to the motor organs through the peripheral nervous system, farther the maintenance of the posture).

The nervous system is constituted by the central and peripheral systems. The peripheral nervous system is constituted by nerves and ganglia. The central nervous system is divided in two main parts: encephalus (brain cavity) and spinal marrow (vertebral canal). The encephalus is subdivided in cerebrum (formed by diencephalus and telencephalus), cerebellum and cerebral trunk or brainstem (formed by bulbus, pons and mesencephalus). Figure 1 shows in detail the division of the central nervous system.

2.1 Cerebrum and Motor Cortex

The cerebrum is formed by two main parts: diencephalus and telencephalus. Telencephalus is the name given to the cerebral hemispheres, predominant in the superior vertebrates, separate by the superior longitudinal fissure. The diencephalus functions as transmitter of the sensorial information to the cerebral cortex, contained a great number of neural circuits related to the vital functions, such as: regulation of the body temperature, cardiac frequency and arterial pressure. In relation to the locomotion, it is involved in the regulation of the posture and movement.

The motor cortex is part of the cerebral cortex and its cells supply a direct canal between the cerebrum and the neurons of the spinal marrow, the so-called motoneurons. From studies with stimulation of the cortex performed by Penfield (1955), it was observed that the motor cortex is linked to all parts of the human body, including the inferior limbs, responsible for locomotion.

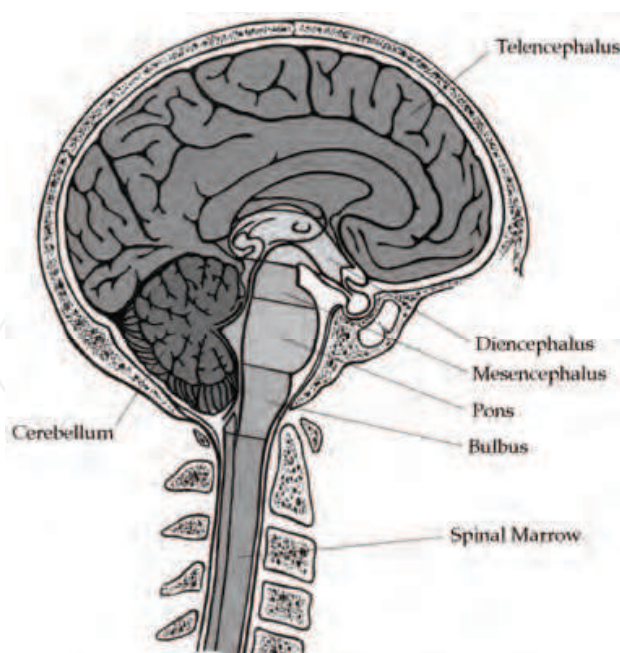


Fig. 1. Schematical vision of the division of central nervous system.

2.2 Cerebellum

The cerebellum plays an important part in the regulation of the fine and complex movements, as well as in the temporal and spatial determination of activation of the muscles in the course of movement or in the postural adjustment (Brandão, 2004).

In the locomotion movement, the cerebellum participates actively organising and updating, based in the analysis of sensorial information of position and speed of the limbs at every moment. This operation constitutes a planning at short term that overcoming the planning at long term performed by the cerebrum.

This functional characteristic of the cerebellum explains the reason for which in the learning of a movement, the same one is performed at first, slowly, with intense mental concentration. With training and consequent motor learning, the amount of pre-programmed movements increases, resulting in a greater easiness and quickness in the performing of the same ones.

2.3 Cerebral Trunk or Brainstem

The cerebral trunk is formed by three main parts: mesencephalus, pons and bulbus. Mesencephalus is the portion more cranial of the cerebral trunk, crossed for the cerebral aqueduct. The pons functions as station for the information proceeding from the cerebral hemispheres, and are going to the cerebellum. The bulbus contains nuclei (groupings of bodies of neurons) and tratus (bundle of nervous fibres), which lead the sensorial information for the superior centers of the cerebrum, as well as nuclei and canals which lead motor commands of the cerebrum for spinal marrow.

Parts of the central nervous system as pons, bulbus and spinal marrow, are endowed with programs of posture and movement, which are used by the organism when necessary, without the necessity of the involvement of regions located in a higher level in the central nervous system (Brandão, 2004). In fact, the main way of communication between the cerebrum and the rest of the organism is the spinal marrow, as it will be seen to follow.

2.4 Spinal Marrow

The spinal marrow is formed by nerves that if extend from the cerebral cortex or in some areas of the cerebral trunk and finish, in its great majority, in the somatic motoneurons (which make connection with the muscles that performing the conscientious movements). It is protected by the bones of the spinal column and its nervous fibres pass through small openings between each vertebra. In the superior vertebrates, the spinal marrow is more strong subordinated to the cerebrum, executing its orders.

The forefront of the spinal marrow contains the motor nerves (motoneurons), which transmit information to the muscles and stimulate the movement. The posterior part and the lateral parts contain the sensitive nerves, receiving information from the skin, joints, muscles and viscera.

The human gait requires a coordination of the muscular activity between the two legs, which is made by a flexible neural coupling to the level of the spinal marrow (Dietz, 2003). Thus, in the course of the locomotion, a disturbance in one of the legs leads to a pattern of proposital reply of the spinal marrow, characterising the existence of the so-called central pattern generator.

3. Locomotion Patterns

The choice of an appropriate pattern of locomotion depends on the combination of a central programming and sensorial data, as well as of the instruction for one determined motor condition. This information determines the way of organisation of the muscular synergy, which is planned for adequate multiple conditions of posture and gait (Horak & Nashner, 1986).

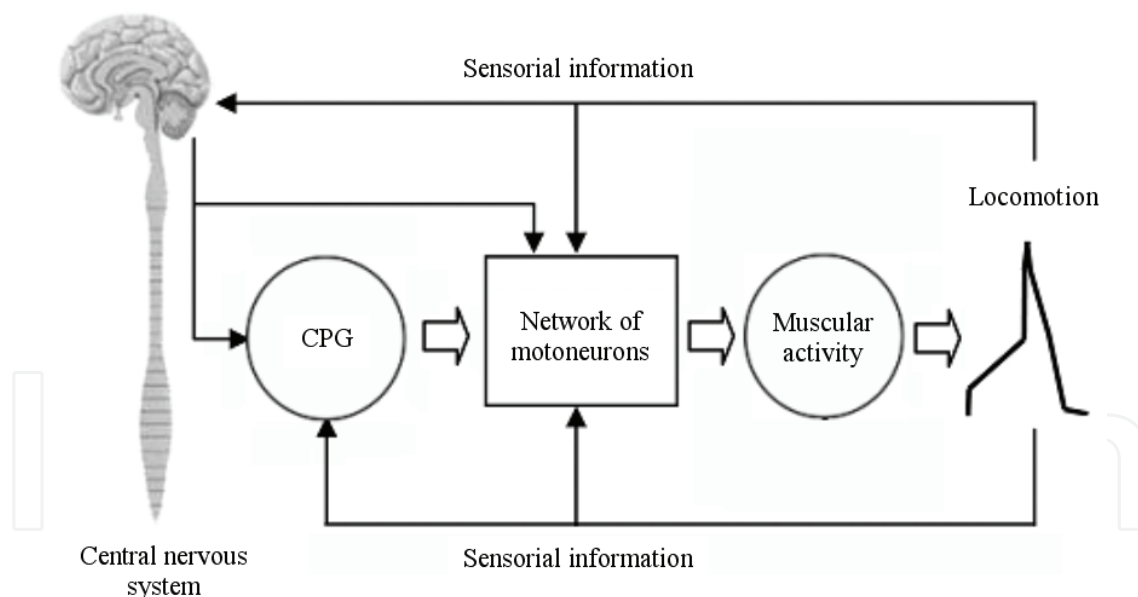


Fig. 2. Control system of the human locomotion.

Figure 2 presents a scheme of the control system of the human locomotion, controlled by the central nervous system, which the central pattern generator supplies a series of pattern curves for each part of the locomotor. This information is passed to the muscles by means of a network of motoneurons, and the conjoined muscular activity performs the locomotion. Sensorial information about the conditions of the environment or some disturbance are supplied as feedback of the system, providing a fast action proceeding from the central pattern generator, which adapts the gait to the new situation.

Despite the people not walk in completely identical way, some characteristics in the gait can be considered universal, and these similar points serve as base for description of patterns of the kinematics, dynamics and muscular activity in the locomotion.

In the study to be presented here, the greater interest is related to the patterns of the kinematics, in particular, of the hip and knee angles. From the knowledge of these patterns of behaviour, the simulation of the central pattern generator using the system of coupled oscillators becomes possible.

3.1 Anatomical Planes of Movement

In the study of the human movement, some definitions in relation to the planes of movement and regions of the body are adopted. The anatomical planes of movement are presented in Fig. 3, where: the sagittal plane divides the body in left side and right side; the frontal plane divides the body in a forepart region and posterior region; and the transverse plane divides the body in a superior part and another inferior.

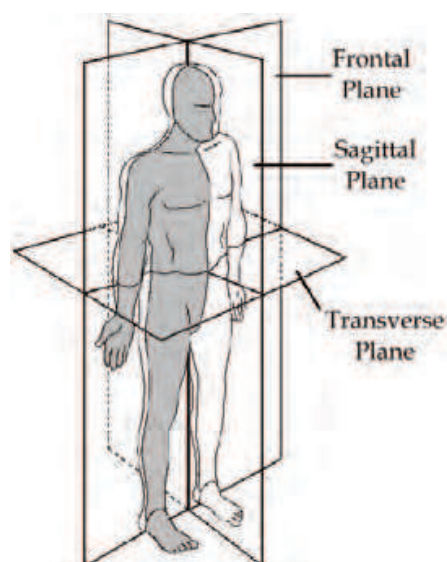


Fig. 3. Anatomical planes of movement.

3.2 The Articulation of the Hip

The movements of the hip are performed in an only articulation. The hip is the articulation of the inferior limb close to the pelvis, being the joint that links this limb to the remaining portion of the body. This articulation performs the function of a spherical joint and is formed by the head of femur and acetabulum, located in the pelvis.

The characteristics related to the amplitude of movement and stability are conditional for the functions of support of the weight and the locomotion, assumed for the inferior limb. These movements can be divided in movements of flexion or extension, adduction or abduction and external or internal rotation (Fig. 4).

The flexion of the hip is the movement that leads the forepart of the thigh to the meeting of the trunk, while in the extension of the hip the contrary movement occurs. These movements are performed around a perpendicular axle to the sagittal plane of the joint. The configuration of the knee has direct influence on the amplitude of flexion of the hip. With the extension of the knee, the maximum angle of flexion of the hip is 90 degrees, while for the flexion of the knee, the amplitude reaches or exceeds 120 degrees.

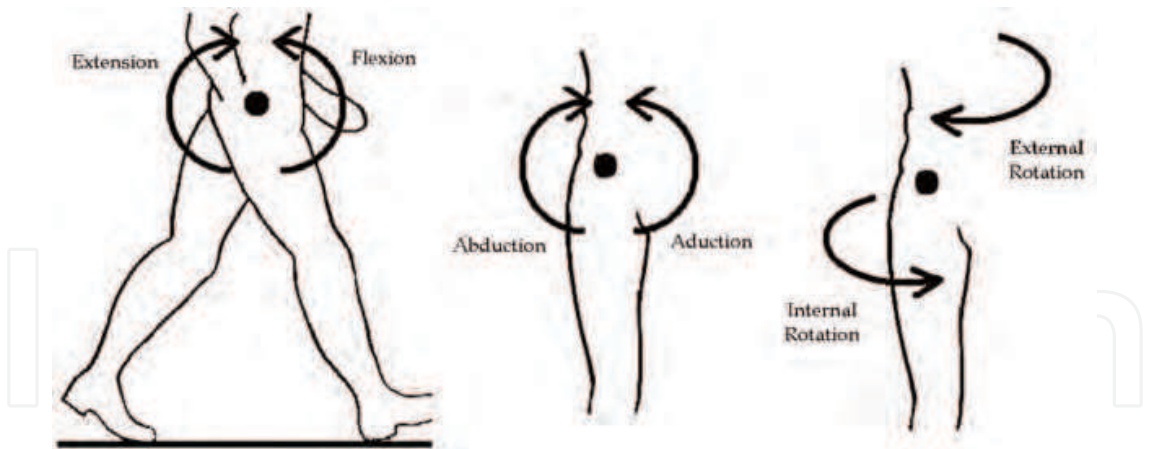


Fig. 4. Movements of the hip.

In the movement of abduction of the hip, the inferior limb is rotated directly for outside, moving away itself from the symmetry plane of the body, while the movement of adduction makes exactly the opposite.

The movements of external and internal rotation of the hip are performed around of the mechanical axle of the inferior limb. In the normal position of alignment, this axle if confuses with the vertical axle of the joint, and the movement of external rotation rotate the foot for outside, while the movement of internal rotation rotate the foot for inside (in the direction of the symmetry plane of the body).

3.3 Behaviour of the Hip in the course of Locomotion

Considering the three anatomical planes of movement, from the use of an optic-electronic system of three-dimensional kinematical analysis (Raptopoulos, 2003), it is possible to define the angular behaviour of the hip in the course of the locomotion cycle.

Figures 5 and 6 present the graphs of angular displacement and phase space of the hip in the sagittal plane, related to the movements of flexion and extension. Figures 7 and 8 are related to the movements of adduction and abduction in the frontal plane, while the Figures 9 and 10 show the behaviour of the hip to the movements of external and internal rotation in the transverse plane.

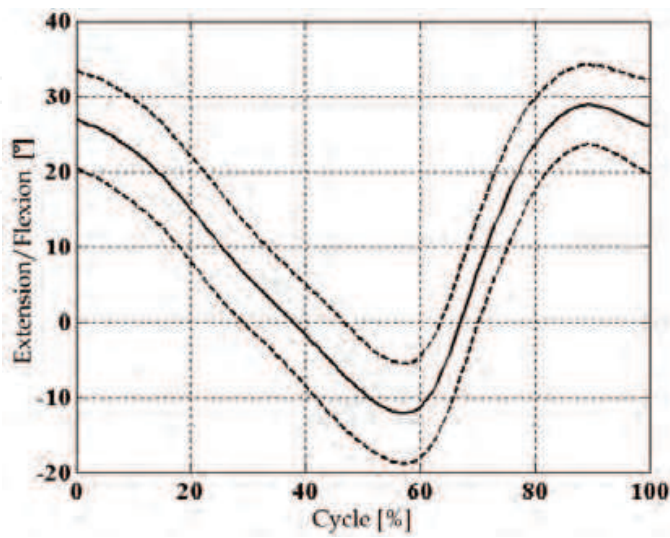


Fig. 5. Angular displacement of the hip in the sagittal plane (mean ± deviation).

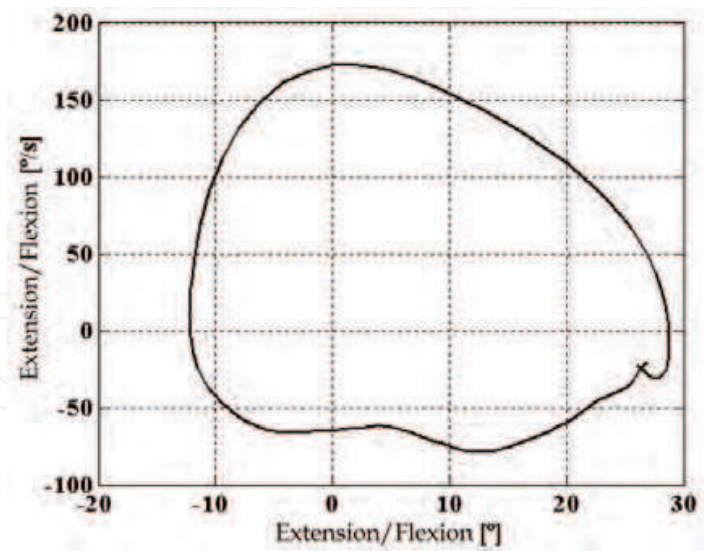


Fig. 6. Phase space of the hip in the sagittal plane.

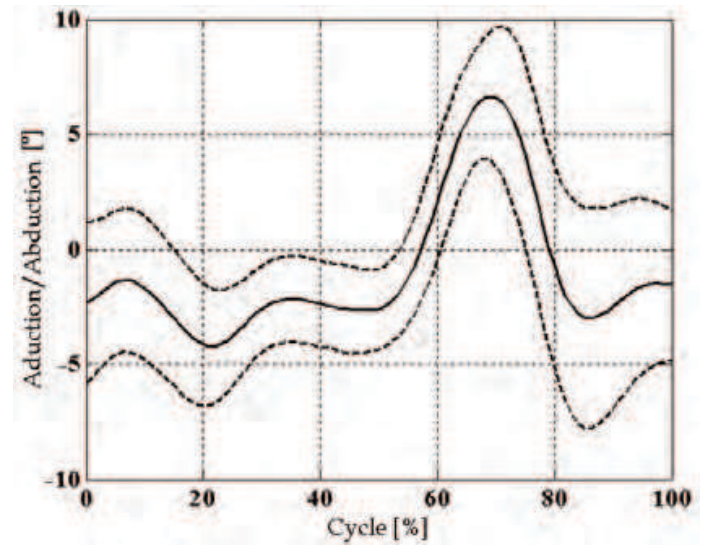


Fig. 7. Angular displacement of the hip in the frontal plane (mean \pm deviation).

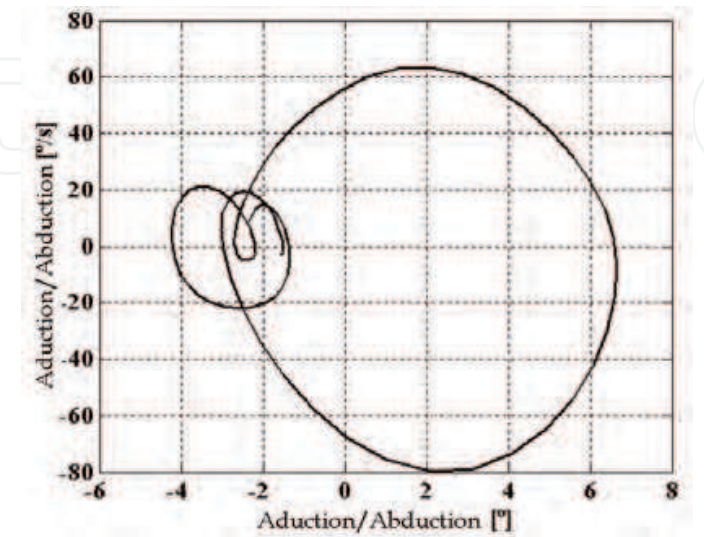


Fig. 8. Phase space of the hip in the frontal plane.

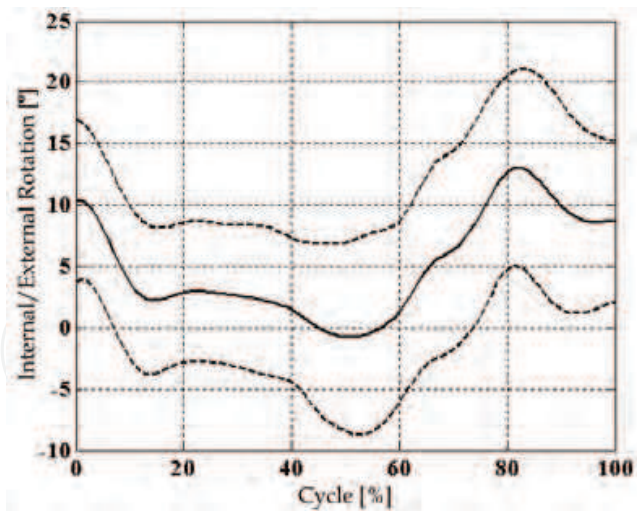


Fig. 9. Angular displacement of the hip in the transverse plane (mean \pm deviation).

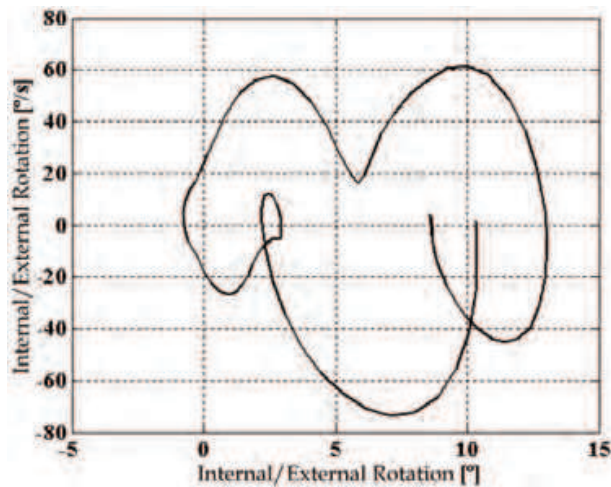


Fig. 10. Phase space of the hip in the transverse plane.

3.4 The Articulation of the Knee

The knee is the intermediate articulation of the inferior limb, being the joint that links femur to the tibia. This articulation have two degrees of freedom, being the flexion-extension the main movement, and the rotation around of the longitudinal axle of the leg is a auxiliary movement, which only appears when the knee is bent. This articulation is classified as a condilar joint, where a main movement occurs in the sagittal plane (movement of flexion and extension) and another one, secondary, occurs as composition of the rotation and translation of the segment of the tibia. Due to the restrictions imposed for the ligaments of the knee and the proper condilar surface, this does not present great amplitude of movement in the frontal and transverse planes in the course of the support phase. Figure 11 shows the movements of the knee in each one of the anatomical planes.

The flexion of the knee is a movement that approaches the posterior region of the leg to the posterior side of the thigh. The amplitude can come at 140 degrees with the hip in flexion, but it does not exceed 120 degrees with the hip in extension. The extension of the knee is defined as the movement that moves away the posterior side of the leg to the posterior side of the thigh. When the extension is extreme, this receives the name of *genu recurvatum* and has pathological causes.

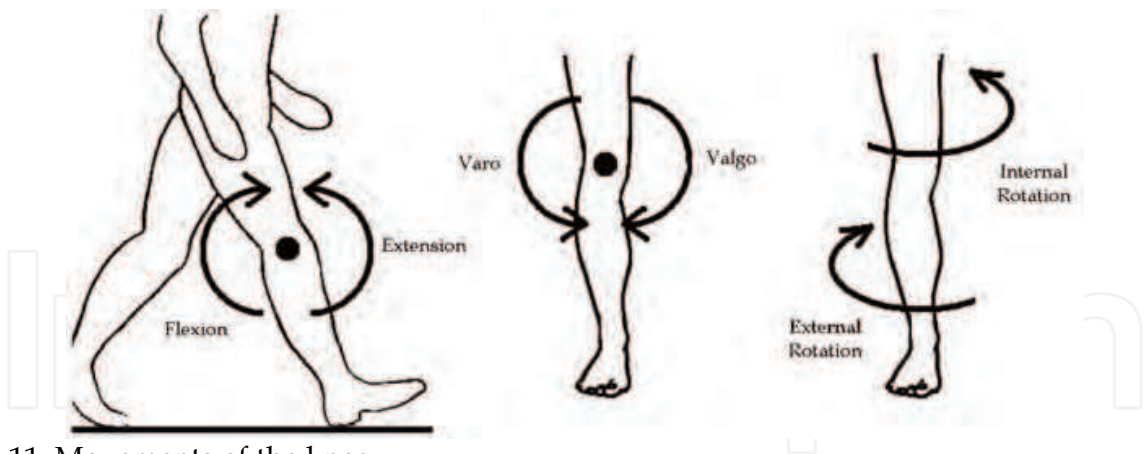


Fig. 11. Movements of the knee.

The knee presents lateral displacements that depend on the sex and pathological variations. When the knee is dislocated laterally, says that the articulation is in *genu varo*. When the knee is dislocated medially, says that the articulation is in *genu valgo*. The movement of axial rotation of the knee occurs around of its longitudinal axle and only it can be performed in flexion, therefore in extension the knee is braked. This braking mechanism is related to the articulate surfaces and has the function to increase the resistance of the knee to the torque that occurs in the course of the phase of contact of the limb with the ground. More details about the physiology of the articulations of the knee and other parts of the body can be seen in Kapandji (1980).

3.5 Behaviour of the Knee in the course of Locomotion

In similar way that performed for the hip, considering the three anatomical planes of movement, from the use of an optic-electronic system of three-dimensional kinematical analysis (Raptopoulos, 2003), it was possible to define the angular behaviour of the knee in the course of the locomotion cycle. Figures 12 and 13 present the graphs of angular displacement and phase space of the knee in the sagittal plane, related to the movements of flexion and extension. Figures 14 and 15 are related to the movements of genu varo and genu valgo in the frontal plane, while Figures 16 and 17 show the behaviour of the knee to the movements of external and internal rotation in the transverse plane.

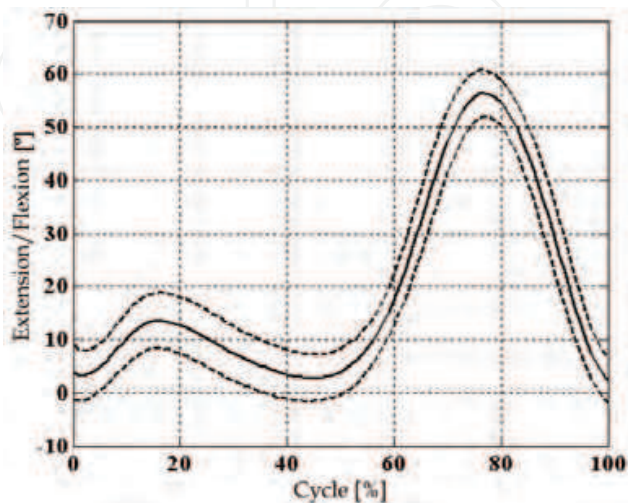


Fig. 12. Angular displacement of the knee in the sagittal plane (mean ± deviation).

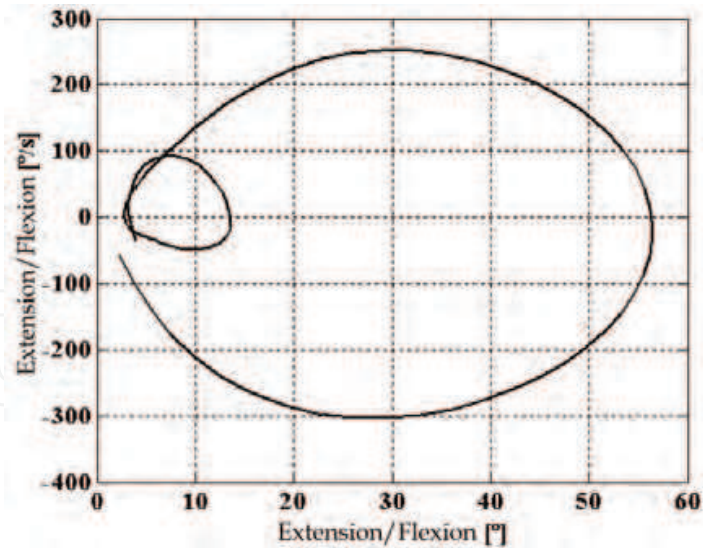


Fig. 13. Phase space of the knee in the sagittal plane.

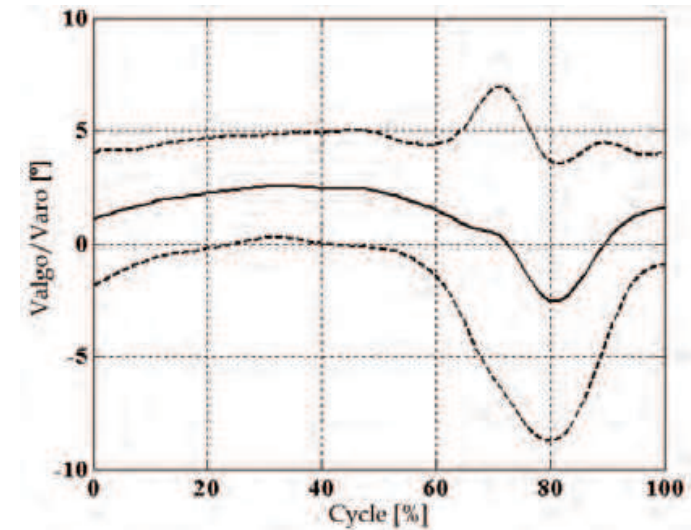


Fig. 14. Angular displacement of the knee in the frontal plane (mean \pm deviation).

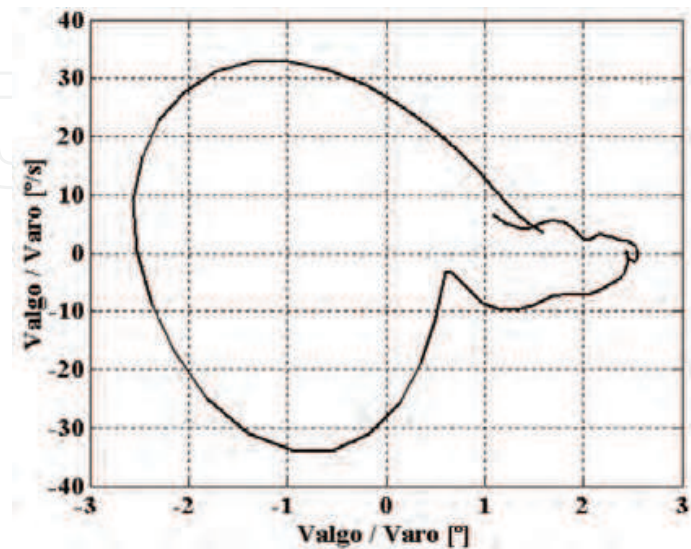


Fig. 15. Phase space of the knee in the frontal plane.

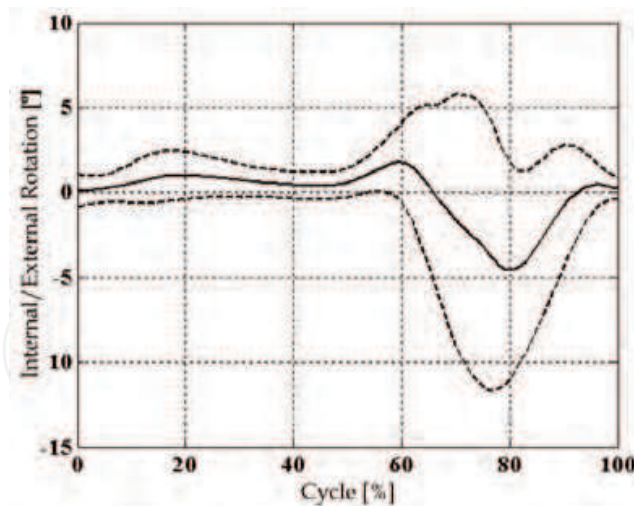


Fig. 16. Angular displacement of the knee in the transverse plane (mean \pm deviation).

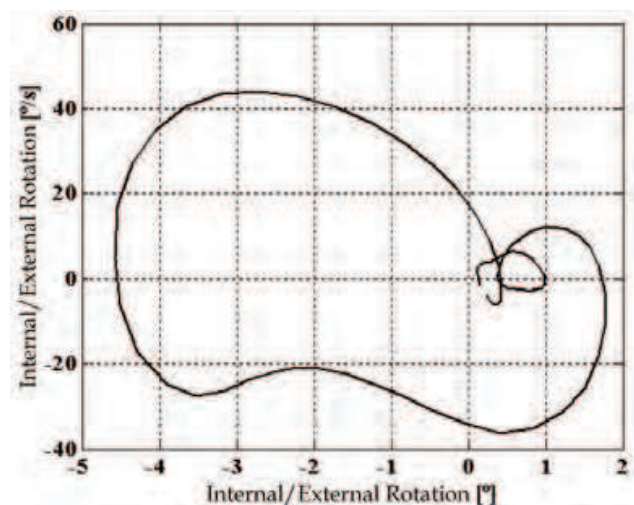


Fig. 17. Phase space of the knee in the transverse plane.

4. Mechanical Model

Before a model of central pattern generator can be applied to a physical system, the desired characteristics of the system must be completely determined, such as the movement of the leg or another rhythmic behaviour of the locomotor. Some works with description of the rhythmic movement of animals include Eberhart (1976), Winter (1983) and McMahon (1984), this last one presenting an ample study about the particularities of the human locomotion. To specify the model to be studied is important to know some concepts related to the bipedal gait, such as the determinants of gait.

The modelling of natural biped locomotion is made more feasible by reducing the number of degrees of freedom, since there are more than 200 degrees of freedom involved in legged locomotion. According to Saunders et al. (1953) the most important determinants of gait are: 1) the compass gait that is performed with stiff legs like an inverted pendulum. The pathway of the center of gravity is a series of arcs; 2) pelvic rotation about a vertical axis. The influence of this determinant flattens the arcs of the pathway of the center of gravity; 3) pelvic tilt, the effects on the non-weight-bearing side further flatten the arc of translation of the center of gravity; 4) knee flexion of the stance leg. The effects of this determinant combined with pelvic rotation and pelvic

tilt achieve minimal vertical displacement of the center of gravity; 5) plantar flexion of the stance ankle. The effects of the arcs of foot and knee rotation smooth out the abrupt inflexions at the intersection of the arcs of translation of the center of gravity; 6) lateral displacement of the pelvis.

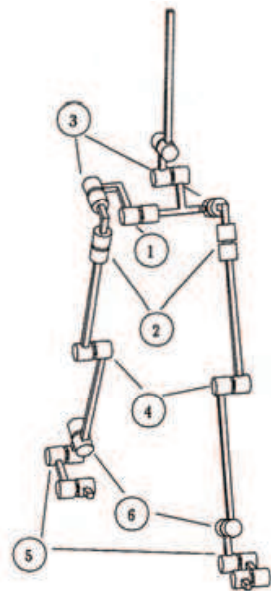


Fig. 18. Three-dimensional model with the six most important determinants of gait.

In order to perform these determinants of gait, a 3D model with 15 degrees of freedom (Fig. 18) is needed. The kinematical analysis, using the characteristic pair of joints method is presented in Saunders et al. (1953).

In order to simplify the investigation, a 2D model that performs motions parallel only to the sagittal plane will be considered. This model, depicted in Fig. 19, characterises the three most important determinants of gait, determinants 1 (the compass gait), 4 (knee flexion of the stance leg), and 5 (plantar flexion of the stance ankle). The model does not take into account the motion of the joints necessary for the lateral displacement of the pelvis, for the pelvic rotation, and for the pelvic tilt.

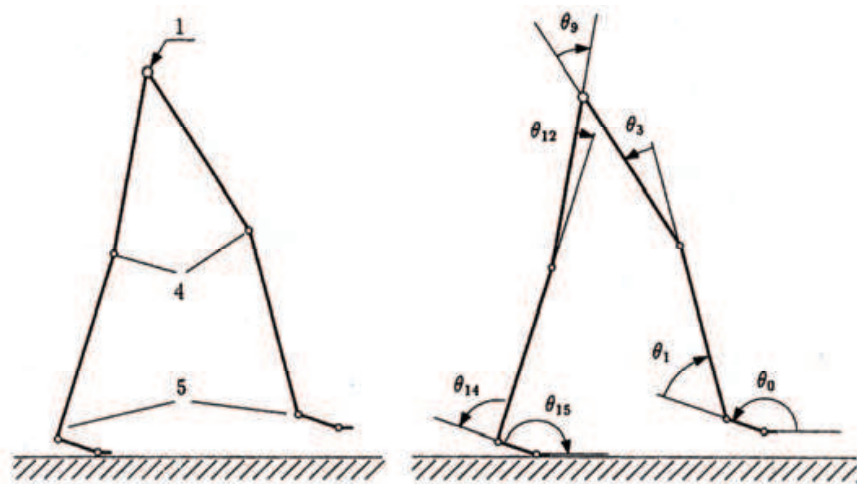


Fig. 19. 2D model with the most important determinants of gait and relative angles.

This model must be capable to show clearly the phenomena occurred in the course of the motion. The adopted model works with the hypothesis of the rigid body, where the natural structural movements of the skin and muscles, as well as bone deformities, are disregarded. The locomotion

cycle can be divided in two intervals: single support phase and double support phase. In single support phase, one of the legs performs the swinging movement, while the another one it is responsible for the support. The extremity of the support leg is assumed as not sliding. The double support phase is the phase where the transition of the legs occurs, the swinging leg becomes supporting leg and to another one it gets ready to initiate the swinging movement. This phase is initiated at the moment where the swinging leg touches the ground.

Considering the 2D model adopted in the Fig. 19, the knee angles θ_3 and θ_{12} , and the hip angle θ_9 , will be determined by a system of coupled nonlinear oscillators. Such oscillators must present an asymptotic behaviour, typical of the dissipative systems, and characterised for the existence of attractors, more specifically limit cycles, as it will be seen to follow.

5. Nonlinear Oscillators

Oscillators are used to describe mechanisms that repeat its action periodically, such as: some neurons, electric circuits, waves, cells, etc. The behaviour of an oscillator can be described by a differential equation, whose solution presents cyclical behaviour. Thus, nonlinear oscillators can be represented by nonlinear differential equations.

A question of particular interest in nonlinear systems is the existence of closed trajectories, which imply in periodic movement. Closed trajectories occur not only in conservative systems, but also in non-conservative systems, however in such systems, in the end of a cycle, the change of energy must be zero. This implies that during parts of the cycle the energy is wasted, and during other parts the system receives energy, what it keeps the balance of energy in zero. The closed trajectories in this in case receive the name of limit cycles. Limit cycles can be considered as movements of balance in which the system performs periodic movement (Meirovitch, 1975).

An important characteristic of the oscillators with limit cycles is that, while the amplitude of a closed trajectory for a conservative system depends initially on the energy supplied to the system, the amplitude of a limit cycle not only depends on the energy, but of the system parameters. One of the oscillators with limit cycle more known and used in diverse works about locomotion is the van der Pol oscillator. Another similar oscillator to the van der Pol, the Rayleigh oscillator, is less known and it was little explored, as well as its application in the locomotion. The system of coupled oscillators proposed in this work uses these Rayleigh oscillators.

5.1 Van der Pol Oscillator

Balthazar van der Pol (1889-1959) was a Dutch engineer who made dynamic experimental studies in the beginning of 20th century. He investigated electric circuits using vacuum tubes and verified that the circuits presented stable oscillations (limit cycles). Working with van der Mark, van der Pol was one of the first ones to present an article with experimental studies on chaos. He also performed diverse studies about the human heart, building models with the objective to study the dynamic stability of same one.

The equation of van der Pol is wellknown, originated from a simple circuit RLC in which was inserted an active nonlinear element. The form most common of the equation of van der Pol is given by the equation:

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0 \quad (1)$$

where μ is the damping term. More details and discussions about the equation of van der Pol, also considering forcing terms, can be seen in Jackson (1990) and Strogatz (1994).

5.2 Rayleigh Oscillator

In the course of 19th century, some works related with nonlinear oscillators had been performed, particularly in conjunction with models of musical instruments. At this time, the British mathematical physicist Lord Rayleigh (John William Strutt, 1842-1919) introduced an equation of the form:

$$mx'' + kx = ax' - b(x')^3 \quad (2)$$

(with nonlinear damping speed) to simulate the oscillations of a clarinet. The equation of Rayleigh used in the analyses will be:

$$\ddot{x} - \delta(1 - q\dot{x}^2)\dot{x} + \Omega^2(x - x_0) = 0 \quad \delta, q \geq 0 \quad (3)$$

where δ , q and Ω correspond to the parameters of the oscillator.

Despite the apparent similarity, the oscillators of van der Pol and Rayleigh present distinct behaviours. In electric point of view, the oscillators answer to an increase of voltage of different form. In the case of the van der Pol oscillator, an increase of the voltage implies in increase of the frequency, while in the Rayleigh oscillator it implies in an increase of the amplitude.

Considering $y = \dot{x}$, we have the following autonomous system:

$$\begin{cases} \dot{x} = y \\ \dot{y} = \delta(1 - qy^2)y - \Omega^2(x - x_0) \end{cases} \quad (4)$$

Choosing values for δ , q , Ω and x_0 , by means of a program that integrates the ordinary differential equations (ODE), is possible to plot the graphs of x and \dot{x} as function of the time and the trajectory in the phase space.

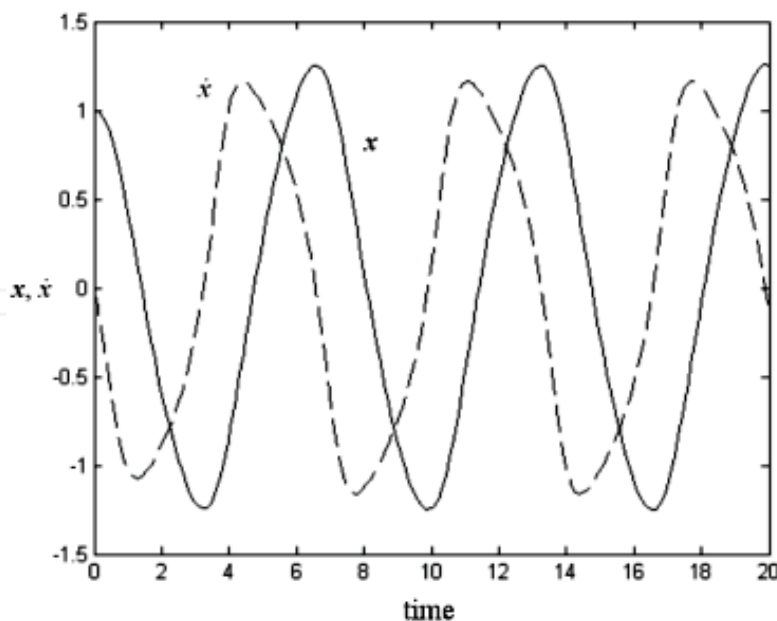


Fig. 20. Graph of x and \dot{x} as function of the time.

Making $\delta = q = \Omega = 1$ and $x_0 = 0$, and using the MATLAB®, the graphs for the Rayleigh oscillator had been generated, presented in Figures 20, 21 and 22. It observes the formation of the limit cycle (Fig. 21), that it implies in periodic movement that can be seen in Fig. 20.

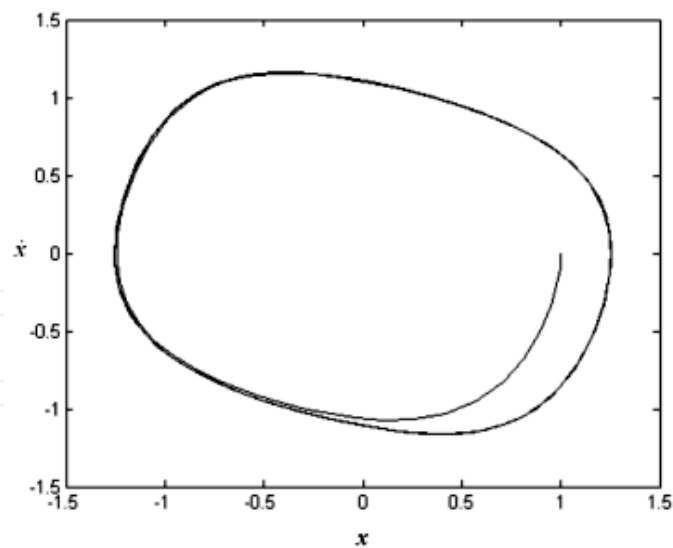


Fig. 21. Trajectory in the phase space (limit cycle).

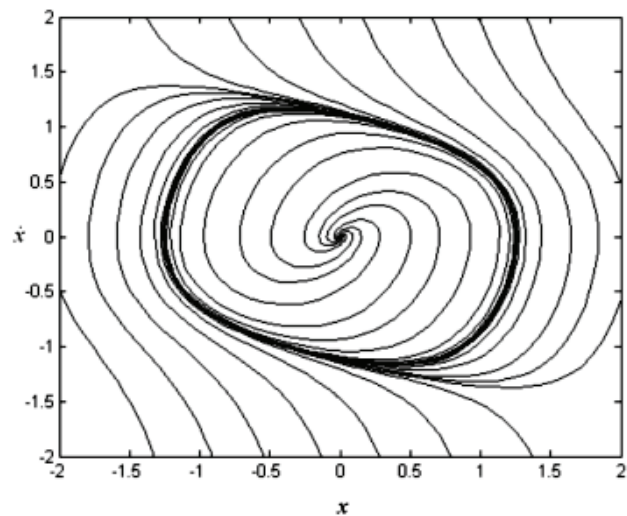


Fig. 22. Phase portrait.

5.3 Coupled Oscillators System

Oscillators are said coupled if they allow themselves to interact, in some way, one with the other, as for example, a neuron that can send a signal for another one in regular intervals. Mathematically speaking, the differential equations of the oscillators have coupling terms that represent as each oscillator interacts with the others.

According to Kozłowski et al. (1995), since the types of oscillators, the type and topology of coupling, and the external disturbances can be different, exist a great variety of couplings. In relation to the type of coupling, considering a set of n oscillators, exists three possible basic schemes (Low & Reinhall, 2001): 1) coupling of each oscillator to the closest neighbours, forming a ring (with the n -th oscillator coupled to the first one):

$$i = 1..n, \quad j = \begin{cases} i+1, n & i = 1 \\ i+1, i-1 & i = 2..n-1 \\ 1, i-1 & i = n \end{cases} ; \tag{5}$$

2) coupling of each oscillator to the closest neighbours, forming a chain (with the n -th oscillator not coupled to the first one):

$$i = 1..n, \quad j = \begin{cases} i+1 & i=1 \\ i+1, i-1 & i=2..n-1 \\ i-1 & i=n \end{cases} ; \quad (6)$$

3) coupling of each one of the oscillators to all others (from there the term "mutually coupled"):

$$i = 1..n, \quad j = 1..n, \quad j \neq i \quad (7)$$

This last configuration of coupling will be used in the analyses, since it desires that each one of the oscillators have influence on the others. Figure 23 presents the three basic schemes of coupling.

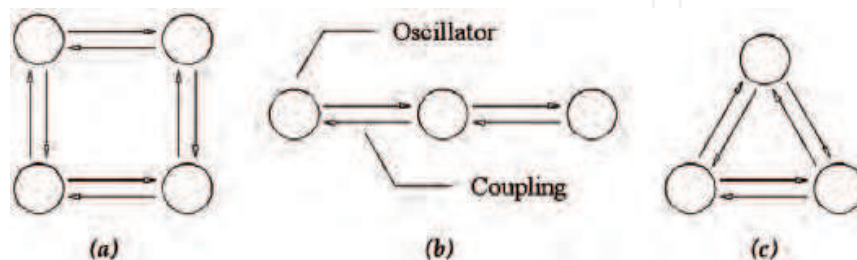


Fig. 23. Basic schemes of coupling: in ring (a), in chain (b) and mutually coupled (c).

5.4 Coupled Oscillators with the Same Frequency

From the equation (3), considering a net of n -coupled Rayleigh oscillators, and adding a coupling term that relates the velocities of the oscillators, we have:

$$\ddot{\theta}_i - \delta_i(1 - q_i \dot{\theta}_i^2) \dot{\theta}_i + \Omega_i^2(\theta_i - \theta_{i0}) - \sum_{j=1}^n c_{i,j}(\dot{\theta}_i - \dot{\theta}_j) = 0 \quad i = 1, 2, \dots, n \quad (8)$$

where δ_i , q_i , Ω_i and $c_{i,j}$ are the parameters of this system.

For small values of parameters determining the model nonlinearity, we will assume that the response is approximated by low frequency components from full range of harmonic response. Therefore periodic solutions can be expected, which can be approximated by:

$$\theta_i = \theta_{i0} + A_i \cos(\omega t + \alpha_i) \quad (9)$$

In this case, all oscillators have the same frequency ω . Deriving the equation (9) and inserting the solutions in (8), by the method of harmonic balance (Nayfeh and Mook, 1979), the following system of nonlinear algebraic equations are obtained:

$$\begin{cases} A_i(\Omega_i^2 - \omega^2) \cos \alpha_i + A_i \delta_i \omega \left(1 - \frac{3\omega^2 A_i^2 q_i}{4}\right) \sin \alpha_i + \omega \sum_{j=1}^n c_{i,j} (A_i \sin \alpha_i - A_j \sin \alpha_j) = 0 \\ A_i(\omega^2 - \Omega_i^2) \sin \alpha_i + A_i \delta_i \omega \left(1 - \frac{3\omega^2 A_i^2 q_i}{4}\right) \cos \alpha_i + \omega \sum_{j=1}^n c_{i,j} (A_i \cos \alpha_i - A_j \cos \alpha_j) = 0 \end{cases} \quad (10)$$

With this system of equations, the parameters q_i and Ω_i can be calculated:

$$q_i = \frac{4}{3\omega^2 A_i^2} + \frac{4}{3\omega^2 A_i^3 \delta_i} \sum_{j=1}^n c_{i,j} [A_i - A_j \cos(\alpha_i - \alpha_j)] \quad i = 1, 2, \dots, n \quad (11)$$

$$\Omega_i = \sqrt{\omega^2 - \frac{\omega}{A_i} \sum_{j=1}^n A_j c_{i,j} \sin(\alpha_i - \alpha_j)} \quad i=1, 2, \dots, n \quad (12)$$

Given the amplitude A_i and A_j , phase α_i and α_j , the frequency ω , and the chosen values of δ_i and $c_{i,j}$, the value of the parameters q_i and Ω_i can be calculated.

5.5 Coupled Oscillators with Integer Relation of Frequency

Oscillators of a coupling system, with frequency ω , can be synchronised with other oscillators with frequency $n\omega$, where n is an integer. In the study of human locomotion, we can observe that some degrees of freedom have twice the frequency of the others ($n = 2$).

Therefore, a net of coupled Rayleigh oscillators can be described as:

$$\ddot{\theta}_h - \delta_h (1 - q_h \dot{\theta}_h^2) \dot{\theta}_h + \Omega_h^2 (\theta_h - \theta_{ho}) - \sum_{i=1}^m c_{h,i} [\dot{\theta}_i (\theta_i - \theta_{io})] - \sum_{k=1}^n c_{h,k} (\dot{\theta}_h - \dot{\theta}_k) = 0 \quad (13)$$

where the term $c_{h,i} [\dot{\theta}_i (\theta_i - \theta_{io})]$ is responsible for the coupling between two oscillators with different frequencies, while the other term $c_{h,k} (\dot{\theta}_h - \dot{\theta}_k)$ makes the coupling between two oscillators with the same frequencies.

If the model nonlinearity is determined for small values of parameters, periodic solutions can be expected which can be approximated by the harmonic functions:

$$\theta_h = \theta_{ho} + A_h \cos(2\omega t + \alpha_h) \quad (14)$$

$$\theta_i = \theta_{io} + A_i \cos(\omega t + \alpha_i) \quad (15)$$

$$\theta_k = \theta_{ko} + A_k \cos(2\omega t + \alpha_k) \quad (16)$$

Deriving the equation (14-16) and inserting the solutions in (13), by the method of harmonic balance (Nayfeh and Mook, 1979), the following system of nonlinear algebraic equations are obtained:

$$\left\{ \begin{array}{l} A_h (\Omega_h^2 - 4\omega^2) \cos \alpha_h + 2A_h \delta_h \omega (1 - 3\omega^2 A_h^2 q_h) \sin \alpha_h + \omega \sum_{i=1}^m \frac{A_i^2}{2} c_{h,i} \sin 2\alpha_i + \\ \quad 2\omega \sum_{k=1}^n c_{h,k} (A_h \sin \alpha_h - A_k \sin \alpha_k) = 0 \\ A_h (4\omega^2 - \Omega_h^2) \sin \alpha_h + 2A_h \delta_h \omega (1 - 3\omega^2 A_h^2 q_h) \cos \alpha_h + \omega \sum_{i=1}^m \frac{A_i^2}{2} c_{h,i} \cos 2\alpha_i + \\ \quad 2\omega \sum_{k=1}^n c_{h,k} (A_h \cos \alpha_h - A_k \cos \alpha_k) = 0 \end{array} \right. \quad (17)$$

With this system of equations, the parameters q_k and Ω_k can be obtained:

$$q_h = \frac{1}{3\omega^2 A_h^2} + \frac{1}{12\omega^2 A_h^3 \delta_h} \sum_{i=1}^m A_i^2 c_{h,i} \cos(\alpha_h - 2\alpha_i) + \frac{1}{3\omega^2 A_h^3 \delta_h} \sum_{k=1}^n c_{h,k} [A_h - A_k \cos(\alpha_h - \alpha_k)] \quad (18)$$

$$\Omega_h = \sqrt{4\omega^2 + \frac{\omega}{2A_h} \sum_{i=1}^m A_i^2 c_{h,i} \sin(\alpha_h - 2\alpha_i) - \frac{2\omega}{A_h} \sum_{k=1}^n A_k c_{h,k} \sin(\alpha_h - \alpha_k)} \quad (19)$$

Given the amplitude A_h , A_i and A_k , phase α_h , α_i and α_k , the frequency ω , and the chosen values of δ_h , $c_{h,i}$ and $c_{h,k}$, the value of the parameters q_h and Ω_h can be calculated.

6. Analysis and Results of the Coupling System

To generate the motion of knee angles θ_3 and θ_{12} , and the hip angle θ_9 , as a periodic attractor of a nonlinear network, a set of three coupled oscillators had been used. These oscillators are mutually coupled by terms that determine the influence of each oscillator on the others (Fig. 24). How much lesser the value of these coupling terms, more “weak” is the relation between the oscillators.

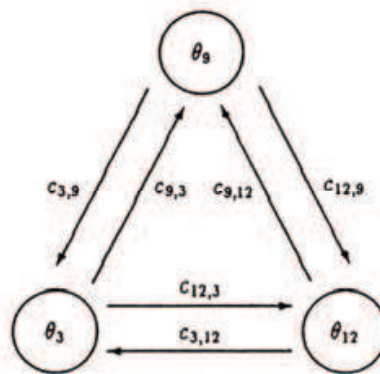


Fig. 24. Structure of coupling between the oscillators.

Considering Fig. 24, from the Equation (13) the coupling can be described for the equations:

$$\ddot{\theta}_3 - \delta_3 (1 - q_3 \dot{\theta}_3^2) \dot{\theta}_3 + \Omega_3^2 (\theta_3 - \theta_{3o}) - c_{3,9} [\dot{\theta}_9 (\theta_9 - \theta_{9o})] - c_{3,12} (\dot{\theta}_3 - \dot{\theta}_{12}) = 0 \quad (20)$$

$$\ddot{\theta}_9 - \delta_9 (1 - q_9 \dot{\theta}_9^2) \dot{\theta}_9 + \Omega_9^2 (\theta_9 - \theta_{9o}) - c_{9,3} [\dot{\theta}_3 (\theta_3 - \theta_{3o})] - c_{9,12} [\dot{\theta}_{12} (\theta_{12} - \theta_{12o})] = 0 \quad (21)$$

$$\ddot{\theta}_{12} - \delta_{12} (1 - q_{12} \dot{\theta}_{12}^2) \dot{\theta}_{12} + \Omega_{12}^2 (\theta_{12} - \theta_{12o}) - c_{12,9} [\dot{\theta}_9 (\theta_9 - \theta_{9o})] - c_{12,3} (\dot{\theta}_{12} - \dot{\theta}_3) = 0 \quad (22)$$

The synchronised harmonic functions, corresponding to the desired movements, can be writing as:

$$\theta_3 = \theta_{3o} + A_3 \cos(2\omega t + \alpha_3) \quad (23)$$

$$\theta_9 = A_9 \cos(\omega t + \alpha_9) \quad (24)$$

$$\theta_{12} = \theta_{12o} + A_{12} \cos(2\omega t + \alpha_{12}) \quad (25)$$

Considering $\alpha_3 = \alpha_9 = \alpha_{12} = 0$ and deriving the equation (23-25), inserting the solution into the differential equations (20-22), the necessary parameters of the oscillators (q_i and Ω_i , $i \in \{3, 9, 12\}$) can be determined. Then:

$$q_3 = \frac{4c_{3,12}(A_3 - A_{12}) + 4A_3\delta_3 + A_9^2 c_{3,9}}{12\omega^2 A_3^3 \delta_3} \quad (26)$$

$$\Omega_3 = 2\omega \quad (27)$$

$$q_9 = \frac{4}{3\omega^2 A_9^2} \quad (28)$$

$$\Omega_9 = \omega \tag{29}$$

$$q_{12} = \frac{4c_{12,3}(A_{12} - A_3) + 4A_{12}\delta_{12} + A_9^2c_{12,9}}{12\omega^2 A_{12}^3\delta_{12}} \tag{30}$$

$$\Omega_{12} = 2\omega \tag{31}$$

From equations (20-22) and (26-31), and using the MATLAB[®], the graphs shown in Fig. 25 and 26 were generated, and present, respectively, the behaviour of the angles as function of the time and the stable limit cycles of the oscillators. These results were obtained by using the parameters showed in Table 1, as well as the initial values provided by Table 2. All values were experimentally determined. In the Fig. 26, the great merit of this system can be observed, if an impact occurs and the angle of one joint is not the correct or desired, it returns in a small number of periods to the desired trajectory. Considering, for example, a frequency equal to 1 s⁻¹, with the locomotor leaving of the repose with arbitrary initial values: $\theta_3 = -3^\circ$, $\theta_9 = 40^\circ$ and $\theta_{12} = 3^\circ$, after some cycles we have: $\theta_3 = 3^\circ$, $\theta_9 = 50^\circ$ and $\theta_{12} = -3^\circ$.

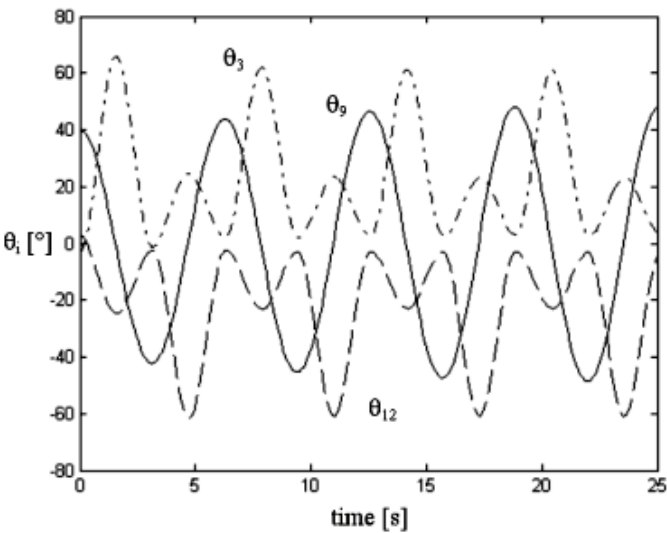


Fig. 25. Behaviour of θ_3 , θ_9 and θ_{12} as function of the time.

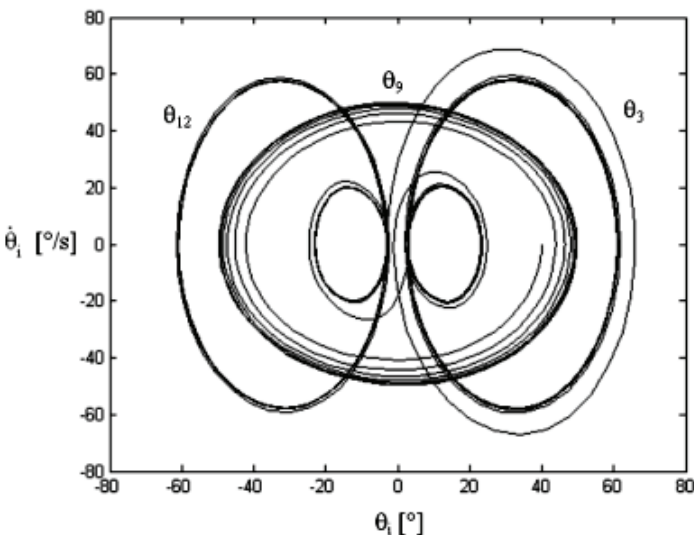


Fig. 26. Trajectories in the phase space (stable limit cycles).

$c_{3,9}$	$c_{9,3}$	$c_{3,12}$	$c_{12,3}$	$c_{9,12}$	$c_{12,9}$	ε_3	ε_9	ε_{12}
0.001	0.001	0.1	0.1	0.001	0.001	0.01	0.1	0.01

Table 1. Parameters of Rayleigh oscillators.

Cycle	A_3	A_9	A_{12}	θ_{3o}	θ_{9o}	θ_{12o}
$0 < \omega t \leq \pi$	-29	50	10	32	0	-13
$\pi < \omega t \leq 2\pi$	-10	50	29	13	0	-32

Table 2. Experimental initial values.

Comparing Fig. 25 and 26 with the experimental results presented in Section 3 (Fig. 5, 6, 12, 13), it is verified that the coupling system supplies similar results, what confirms the possibility of use of mutually coupled Rayleigh oscillators in the modelling of the CPG. Figure 27 shows, with a stick figure, the gait with a step length of 0.63 m. Figure 28 shows the gait with a step length of 0.38 m. Dimensions adopted for the model can be seen in Table 3. More details about the application of coupled nonlinear oscillators in the locomotion of a bipedal robot can be seen in Pina Filho (2005).

	Toes	Foot	Leg (below the knee)	Thigh
Length [m]	0.03	0.11	0.37	0.37

Table 3. Model dimensions.

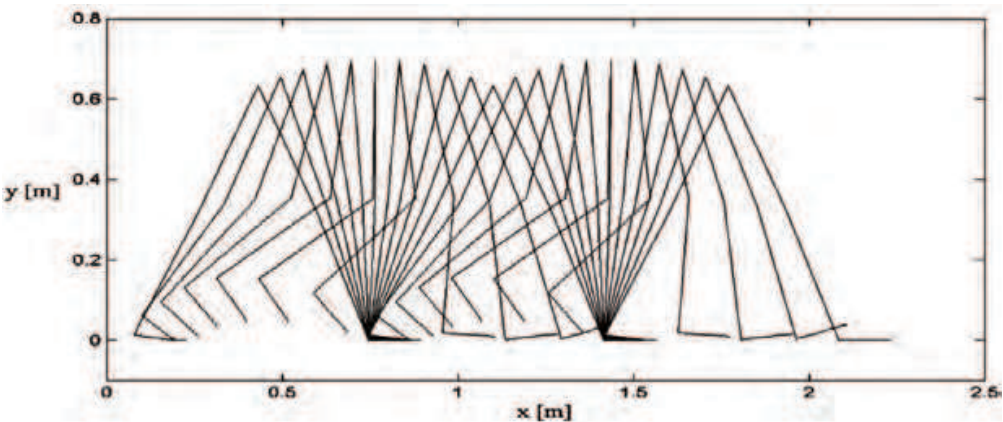


Fig. 27. Stick figure showing the gait with a step length of 0.63 m.

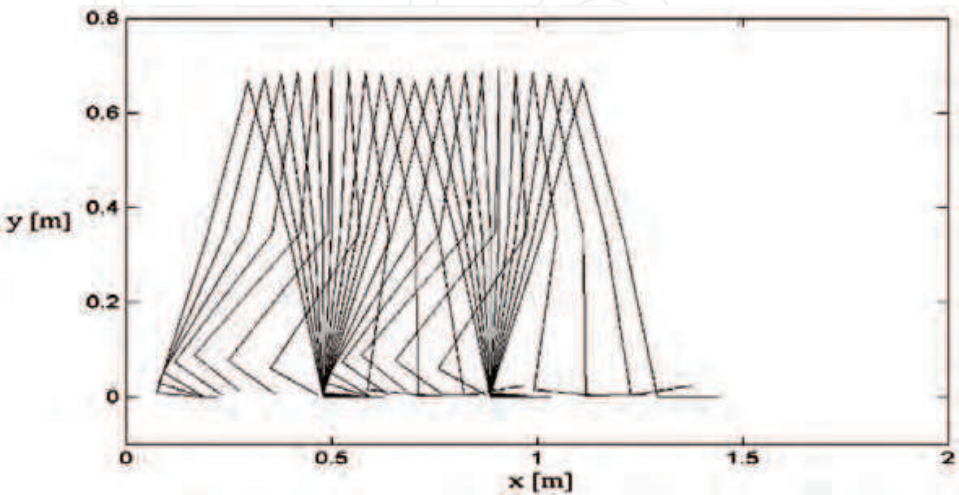


Fig. 28. Stick figure showing the gait with a step length of 0.38 m.

7. Conclusion

From presented results and their analysis and discussion, we come to the following conclusions about the modelling of a bipedal locomotor using mutually coupled oscillators: 1) The use of mutually coupled Rayleigh oscillators can represent an excellent way to signal generation, allowing their application for feedback control of a walking machine by synchronisation and coordination of the lower extremities. 2) The model is able to characterise three of the six most important determinants of human gait. 3) By changing a few parameters in the oscillators, modification of the step length and the frequency of the gait can be obtained. The gait frequency can be modified by means of the equations (23-25), by choosing a new value for ω . The step length can be modified by changing the angles θ_9 and θ_{12} , being the parameters q_i and Ω_i , $i \in \{3, 9, 12\}$, responsible for the gait transitions.

In future works, it is intended to study the behaviour of the ankles, as well as simulate the behaviour of the hip and knees in the other anatomical planes, thus increasing the network of coupled oscillators, looking for to characterise all determinants of gait, and consequently simulate with more details the central pattern generator of the human locomotion.

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