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The Measurement Problem in Quantum Mechanics Revisited

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1. Introduction

In a paper entitled "Against 'measurement'," J. Bell points out [1]: "Here are some words which, however legitimate and necessary in application, have no place in a *formulation* with any pretention to physical precision: *system, apparatus, environment, microscopic, macroscopic, reversible, irreversible, observable, information, measurement*… On this list of bad words… the worst of all is measurement."

To begin with, let us recall that none of the words of the previous list is included in classical theories. Classical Mechanics, Electromagnetism, Statistical Physics and other *classical theories speak about what happens, not about what is observed or measured*; they assume the behavior of apparatuses or measuring devices ruled by the same laws which govern processes where man-made objects are absent; they treat on the same footing microscopic and macroscopic objects; and they offer no room for notions such as environment or information.

The situation is radically different in quantum mechanics. Orthodox Quantum Mechanics, the theory formulated by J. von Neumann in the late 1920, consists of five axioms, and two of them refer to measurements. One of them, which is a generalization of Born's Postulate, refers to the possible results of a measurement, and their corresponding probabilities. The other one, the Projection Postulate, refers to the system's state once the measurement process is completed. In addition to these issues, present since the quantum mechanical formalism was established, in the following years other problems were unveiled and the Projection Postulate became the principal target of criticisms. In particular, it was pointed out that this postulate introduces a subjective element into the theory; it conflicts with the Schrödïnger equation; and it implies a kind of action-at-a-distance.

This chapter critically reviews quantum measurement, starting with contributions by E. Schrödinger and M. Born dating from 1926. Schrödinger proposed an electromagnetic interpretation and Born a probabilistic interpretation of the wave function; the latter implies that quantum mechanics has to be considered a probabilistic theory. In 1927 Einstein objected the idea that quantum mechanics is a complete theory of individual processes. Several ways to face the measurement problem are reported and discussed, among them: Dirac's notion of observation; Bohr's point of view; von Neumann's theory of measurement; Margenau's rejection of the Projection Postulate; the Many Worlds Interpretation; and Decoherence. Brief references are made to Schrödinger cat, EPR paradox, Bell's inequalities and quantum teleportation. A comparison between the characteristics of spontaneous processes and those of measurement processes highlights why so many scientists are disappointed with Orthodox Quantum Mechanics formalism, and in particular with its Projection Postulate.

In the last sections of the chapter we deal with the following items: (i) Conservation laws are strictly valid in spontaneous processes and have only a statistical sense in measurement processes; (ii) Ad-hoc use of the Projection Postulate; (iii) Introduction of the essential concepts involved in the Spontaneous Projection Approach; and (iv) Formal treatment of the ideal measurement scheme in the framework of this approach.

2. Born's probabilistic interpretation of the wave function

The mathematical formalism of quantum mechanics was completed in 1926, the theory already exhibiting a spectacular success in accounting for nearly every spectroscopic phenomena. E. Schrödinger largely contributed to the achievement of these goals by showing that his own formalism and Heisenberg´s matrix calculus are mathematically equivalent. The only major problem left seemed to be interpreting the function $\psi(x, y, z, t)$ which satisfies a wave equation (at present called the Schrödinger equation). Then, in view that *e.g.* the hydrogen atom emits electromagnetic waves whose frequency and polarization should be related to the initial and final atom states, Schrödinger thought it necessary to ascribe to the function ψ an electromagnetic character. A further elaboration of this idea led him to interpret quantum theory as a simple classical theory of waves. In his view, *physical reality consists of waves and waves only*. [2] This interpretation of quantum theory did not convince many physicists and soon several objections were raised, among them that ψ undergoes a discontinuous change during a process of measurement.

M. Born proposed also in 1926 a *probabilistic interpretation of the wave function* (sometimes called statistical interpretation of the wave function) and, as a consequence, that *quantum mechanics should be considered a probabilistic theory*. Summarizing his interpretation, one could say that $|\psi|^2 \, d\tau$ measures the probability of *finding* the particle within the volume dτ, the particle being as a mass point having at each instant both a definite position and a definite momentum. In 1954 Born was awarded the Nobel Price "for his fundamental work in quantum mechanics and especially for his statistical interpretation of the wave function." When explaining why he did not follow Schrödinger's interpretation of the function ψ, he pointed out that "every experiment by Franck [a physicist working close to Born's institute] and his assistants on electron collisions appeared to me as a new proof of the corpuscular nature of the electron." [3]

Born's interpretation was adopted by most leading physicists, included W. Heisenberg. In a letter sent to Born's wife on March 3, 1926, A. Einstein said: "The Heisenberg-Born concepts leave us all breathless, and have made a deep impression on all theoretically oriented people. Instead of dull resignation, there is now a singular tension in us sluggish people." [4] Einstein's conception of quantum mechanics, expressed in this letter, pleased Heisenberg and Born. A few months later, however, Einstein wrote to Born: "Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one'. I, at any rate, am convinced that He is not playing at dice." [4] Note that *Einstein was not rejecting the probabilistic interpretation of the wave function* ψ; he was expressing dissatisfaction with loss of determinism. In his view, a probabilistic theory cannot be complete since "the real thing" should be described by a deterministic theory. His position became more explicit in the Fifth Solvay Congress (October 24 to 29, 1927).

During this congress, Born and Heisenberg presented a paper on matrix mechanics and the probabilistic interpretation of the wave function ψ. At the end of their lecture they made this provocative statement: "We maintain that *quantum mechanics is a complete theory*; its basic physical and mathematical hypotheses are not further susceptible of modifications." [2, emphases added] During the discussion which followed, H. Lorentz objected the rejection of determinism in atomic physic, as proposed by the majority of speakers. Although admitting that Heisenberg's indeterminacy relations impose a limitation on observation, he objected the notion of probability as an axiom a priori, at the beginning of the interpretation, instead of putting it at the end, as a conclusion of theoretical considerations. Finally, he declared: "*Je pourrais toujours garder ma foi déterministe pour les phénomènes fondamentaux… Est-ce qu'un esprit plus profond ne pourrait pas se rendre compte des mouvements de ces électrons? Ne pourrait-on pas garder le déterminisme en faisant l'objet d'une croyance? Faut-il nécessairement exiger l'indéterminisme en principe*?" [2]

After the intervention of other speakers, A. Einstein intervened to point out that the theory of quanta may be considered from two different viewpoints. To illustrate his assertion he referred to the following experiment; see Figure: "A particle (photon or electron) impinges normally on a diaphragm with slit O so that the ψ-wave associated with the particle is diffracted in O. A scintillation-screen… in the shape of a hemisphere is placed behind O so as to show the arrival of a particle…" [2] Then Einstein asserted:
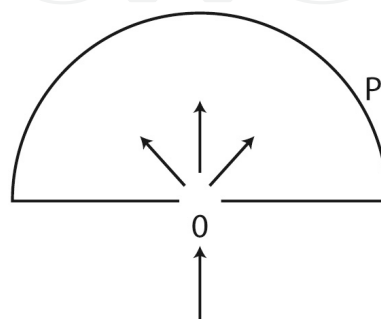

Figure 1. The experiment analyzed by Einstein in the Fifth Solvay Congress

According to viewpoint I, the… waves do not represent one individual particle but rather an ensemble of particles distributed in space. Accordingly, the theory provides information not on an individual process but rather on an ensemble of them. Thus $|\psi(r)|^2$ expresses the probability (probability density) that there exists at *r some* particle of the ensemble [*une certain particule du nuage*].

According to viewpoint II quantum mechanics is considered as a complete theory of individual processes… each particle moving toward the screen in the shape of a hemisphere is described as a wave packet which, after diffraction at O arrives at a certain point P on the screen, and $|\psi(r)|^2$ expresses the probability… that at a given moment one and the same particle shows its presence at *r*… If $|\psi|^2$ is interpreted according to II, then, as long as no localization has been effected, the particle must be considered as potentially present with almost constant probability over the whole area of the screen; however, as soon as it is localized, a peculiar action-at-a-distance must be assumed to take place which prevents the continuously distributed wave in space from producing an effect at *two* places on the screen.

"It seems to me"-Einstein concluded-"that this difficulty cannot be overcome unless the description of the process in terms of the Schrödinger wave is supplemented by some detailed specification of the localization of the particle during its propagation… If one works only with Schrödinger waves, the interpretation II of $|\psi|^2$, I think, contradicts the postulate of relativity." [2]

Many years after this memorable meeting Born and Einstein continued to discuss about this subject. Their divergence is illustrated in Einstein's letter to Born dated September 7, 1944: "We have become Antipodean in our scientific expectations. You believe in the God who plays dice, and I in complete law and order in a world which objectively exists, and which I, in a wildly speculative way, am trying to capture. I firmly believe, but I hope that someone will discover a more realistic way, or rather a more tangible basis than it has been my lot to find. Even the great initial success of the quantum theory does not make me believe in the fundamental dice-game, although I am well aware that our younger colleagues interpret this as a consequence of senility. No doubt the day will come when we will see whose instinctive attitude was the correct one." [4]

Original Born's probabilistic interpretation of the wave function ψ enjoyed great success in the analysis of atomic scattering, but soon it was evident that it confronted serious difficulties to explain other phenomena. [2] In any case, a generalization of Born's probabilistic interpretation of the wave function ψ was included by J. von Neumann as the third postulate of his quantum mechanics formulation; see next section.

3. The formalism of orthodox quantum mechanics

Von Neumann formulated quantum mechanics as an operator calculus in Hilbert space; the German version of his "Mathematical Foundations of Quantum Mechanics" was published for the first time in 1932. [5] A couple of years earlier P. A. M. Dirac published his celebrated

treatise "The Principles of Quantum Mechanics." [6] The essential of the theory was presented there and, even though von Neumann admitted that Dirac's formalism could 'scarcely be surpassed in brevity and elegance,' he criticized it as deficient in mathematical rigor." [2] Many other versions of quantum mechanics followed these pioneer works, most of them motivated by the desire of solving "the measurement problem." But, in general, von Neumann's formulation continued to be preferred to other approaches and, at present, it is frequently the only one taught at the academy. We shall refer to it as Orthodox Quantum Mechanics (OQM).

The primitive (undefined) notions of OQM are *system, physical quantity* and *state*; and the formalism can be summarized in the following way [2]:

a. To every system corresponds a Hilbert space H whose vectors (state vectors, wave functions) completely describe the states of the system.

b. To every physical quantity A corresponds uniquely a self-adjoint operator A acting in H. It has associated the eigenvalue equations

$$\mathbf{A}\left|\mathbf{a}_j^v\right\rangle = \mathbf{a}_j\left|\mathbf{a}_j^v\right\rangle \tag{1}$$

(v is introduced in order to distinguish between the different eigenvectors that may correspond to one eigenvalue a_j), and the closure relation

$$\sum_{j,v}\left|\mathbf{a}_j^v\right\rangle\left\langle\mathbf{a}_j^v\right| = I \tag{2}$$

is fulfilled (here I is the identity operator). If j or v is continuous, the respective sum has to be replaced by an integral.

c. For a system in the state $|\Phi\rangle$ the probability that the result of a measurement of A lies between a' and a'' is given by $\|\Psi\|^2$, where $\|\Psi\|$ is the norm of $|\Psi\rangle = (I_{a''} - I_{a'})|\Phi\rangle$ and I_a is the resolution of identity belonging to A.

d. The evolution in time of the state vector $|\Phi\rangle$ is determined by the Schrödinger equation.

e. Projection Postulate: If a measurement of A yields a result between a' and a'', then the state of the system immediately after the measurement is an eigenfunction of $(I_{a''} - I_{a'})$.

Many prominent authors of quantum mechanics textbooks adopt the primitive notions *system, physical quantity* and *state*, either in an explicit or implicit way. They also take as valid *the first four postulates* of the previous formalism with little or none modification. The exact formulations of these axioms due to some conspicuous authors can be found in: [7-10].

4. The measurement problem and the statistical interpretation of quantum mechanics

The problem pointed out by Einstein at the Fifth Solvay Congress has been considered one of the most serious flaws that quantum mechanics confronts. Some years later, in 1935, he published with B. Podolsky and N. Rosen their celebrated paper "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" [11] This article prompted H. Margenau to consider the Projection Postulate as indicative of a defect in the formalism of quantum mechanics and to suggest that it should be abandoned [2]; one of the main reasons to do so being that this postulate contradicts the more fundamental Schrödinger equation of motion. As an example, Margenau considered the measurement of the coordinate of a particle which initially has a definite momentum and argued: as the value of the position (and then the state ψ after the measurement) cannot be predicted, the Hamiltonian of interaction between the particle and the measuring device cannot be a unique operator as usually encountered in the formalism.

In the following we reproduce his argument in case the operator A representing the physical quantity A to be measured has a discrete non-degenerate spectrum, the eigenfunctions of A being $\psi_j(j=1, 2, \cdots)$. Let t be the time the measurement process starts and $t + \Delta t$ the time such a process is over. We shall call H_0 the Hamiltonian of the particle before t, H_M the term due to its interaction with the measuring device and $H = H_0 + H_M$ the total Hamiltonian acting on the particle in the time interval $(t, t + \Delta t)$. If φ is the state of the particle at t, assuming that during the time interval $(t, t + \Delta t)$ the Schrödinger equation rules the process, the state of the particle at $t + \Delta t$ should be $\psi = \varphi + \Delta\varphi$, where

$$\Delta\varphi = \left(\Delta t\, H\, /\, i\hbar\right)\varphi \tag{3}$$

and

$$\psi = \left\{1 + \left[\Delta t\left(H_0 + H_M\right)\, /\, i\hbar\right]\right\}\varphi \tag{4}$$

($\hbar$ being Planck's constant). Now, on the one hand the several possible states of the particle immediately after the measurement has been completed should be one of the ψ_j; the uncontrollable character of the measurement process implies that it is not possible to predict which one of them will result. But, on the other hand, ψ given by (4) is just one function, whatever the specific form of H_M may be.

Margenau's suggestion to abandon the Projection Postulate and the arguments which support this idea were included in a manuscript he sent to Einstein on November 13, 1935. Einstein, however, replied: "the formalism of quantum mechanics requires inevitable the following postulate: 'If a measurement performed upon a system yields a value *m*, then the same measurement performed immediately afterwards yields again the value *m* with certainty.' He

illustrated this postulate by the example of a quantum of light which, if it has passed a polarizer P_1, is known to pass with certainty a second polarizer P_2 with orientation parallel to P_1." [2]

According to Einstein a particle should always be considered as possessing a definite though perhaps unknown position, even when no such definite position is described by the wave function ψ. [12] In his "Reply to Criticism," he asserts: "One arrives at very implausible theoretical conceptions if one attempts to maintain the thesis that the statistical quantum theory is in principle capable of producing a complete description of an individual physical system. On the other hand, those difficulties of theoretical interpretation disappear if one views the quantum-mechanical description as the description of ensembles of systems." [13] It is not surprising that Einstein, as Margenau, uphold the Statistical Interpretation of Quantum Mechanics (SIQM). According to this approach, "*a pure state* Φ (and hence also a general state) provides a description of certain statistical properties of an *ensemble* of similarly prepared systems, but need not provide a complete description of an individual system." [14] By contrast, Postulate A of OQM explicitly establishes that a pure state Φ completely describes the state of an individual system. *We have then two versions of quantum mechanics which do not deal with the same referent: OQM refers to individual systems and SIQM to ensembles of similarly prepared systems.* In addition: As already mentioned, in classical theories measurement processes are supposed to be ruled by the same laws which govern spontaneous processes. By contrast, in OQM spontaneous processes follow Schrödinger evolutions (given by Postulate D) and measurement processes are ruled by the Projection Postulate (Postulate E). *This is a very important difference between OQM and classical theories*. SIQM avoids this difference by adopting formalism where no mention to measurement is made. In particular, instead of Postulate C (see Section 3), SIQM states: "The only values which an observable [physical quantity or dynamical variable represented by a self-adjoint operator] *may take on* are its eigenvalues…" [14; emphasis added] *So, where OQM talks about the possible results of a measurement, SIQM talks about the values which an observable may take on. SIQM is a theory* which has a referent and postulates different from OQM; in particular, SIQM and OQM *include different generalizations of Born's probabilistic interpretation of the wave function*. By contrast, *Born's probabilistic interpretation of the wave function is not a theory*. This is a difference between Born's probabilistic interpretation of the wave function and SIQM worth to be stressed.

D. Bes points out: "rather than dwell on philosophical interpretations of equations, most physicists proceed to carry out many exciting applications of quantum mechanics." [10] On their side, M. Tegmar and J. A. Wheeler argue: "This approach proved stunningly successful. Quantum mechanics [we would say OQM or similar versions of quantum mechanics] was instrumental in predicting antimatter, understanding radioactivity (leading to nuclear power), accounting for the behavior of materials such as semiconductors, explaining superconductivity and describing interactions such as those between light and matter (leading to the invention of the laser) and of radio waves and nuclei (leading to magnetic resonance imaging)." [15] By contrast, SIQM has not been so successful. This is, we think, the main reason why *most physicists and chemists prefer versions of quantum mechanics which refer to individual systems and, in particular, OQM.*

5. Von Neumann's theory of measurement

Von Neumann's formalism is generally based on the so-called Copenhagen Interpretation whose founding father was mainly N. Bohr; even if E. Schrödinger, W. Heisenberg, M. Born and other physicists also made very important contributions to this interpretation. But Bohr did not care about the absence of quantum mechanics formalism, let alone a theory of measurement. In his view, any formalism would become meaningful only if it was possible to interpret it in terms of classical concepts. So he assigned a double nature to the measuring apparatus: on the one hand it should behave as a classical object; on the other hand, it should follow quantum mechanical laws. As a result, his position remained a somewhat questionable or, at least, obscure. [2]

Contrary to Bohr, von Neumann treated the measuring apparatus as a purely quantum system. His starting point was the assumption that "there are two kinds of changes of quantum mechanical states: (1) 'the discontinuous, non-causal and instantaneously acting experiments or measurements,' which he called 'arbitrary changes by measurements'; and (2) 'continuous and causal changes in the course of time,' which evolve in accordance with the equations of motion and which he called 'automatic changes.' The former, or briefly, 'processes of the first kind' are irreversible whereas the latter, the 'processes of the second kind,' are reversible." [2]

In the first place von Neumann showed how the formalism of quantum mechanics is capable of accounting consistently for the operation of the measuring apparatus. Then he continued his analysis by regarding the measurement process as consisting of two stages: (I) the interaction between the object and the apparatus, and (II) the act of observation. In the following we summarize his arguments.

Stage I does not present any conceptual difficulty. Let S be the operator representing the physical quantity S to be measured. We assume that it has the eigenvalues $s_j (j=1, 2, \cdots)$ and the corresponding eigenvectors are σ_j (for simplicity we refer to the discrete non-degenerate case). If before the interaction of the system (object) with the apparatus the pure state of the system is

$$\sigma = \sum_n c_n \sigma_n \tag{5}$$

during the interaction the object is coupled to the measuring apparatus designed to measure S and, once the interaction ceased, the system+apparatus is in the state

$$\psi = \sum_n c_n \sigma_n \alpha_n \tag{6}$$

where α_n would be the apparatus state if the system state before the interaction were σ_n. Let us stress that ψ, being causally determined, is a pure state as long as the combined system +apparatus remains isolated.

Stage II. The conceptual difficulties concerning the measurement problem become apparent at this stage. "Von Neumann was fully aware that the knowledge of the state of the combined system does not suffice to infer the state of the object or the value of S. If it could be ascertained that after the interaction the apparatus is in the state α_j, it would be known that the object is in the state α_j and S has the value s_j. But how can we find out whether the apparatus is in the state α_j? It may be suggested that one couple the apparatus to a second measuring device. This proposal, however, would lead to an infinite regress… But clearly, von Neumann reasoned, a measurement must be a finite operation; usually it is completed by an act of observing the pointer position of [the apparatus]. The process leading to this result, von Neumann conclud-ed, can therefore no longer be of the second kind but has to be a discontinuous, non-causal, and instantaneous act." [2]

Where and how does this act take place? In our view, von Neumann's answer to this question is not satisfactory. In his Mathematical Foundations of Quantum Mechanics he asserts [16]: "We must always divide the world into two parts, the one being the observed system, the other the observer. In the former, we can follow up all physical processes (in principle at least) arbitrarily precisely. In the latter, this is meaningless. The boundary between the two is arbitrary to a very large extent… That this boundary can be pushed arbitrarily deeply into the interior of the body of the actual observer is the content of the principle of the psycho-physical parallelism – but this does not change the fact that in each method of description the boundary must be put somewhere… Now quantum mechanics describes the events which occur in the observed portion of the world, so long as they do not interact with the observing portion, with the aid of the process [of the second kind], but as soon as such interaction occurs, *i.e.* a measurement, it requires the application of [a] process [of the first kind]."

M. Jammer points out that "this argument for the indispensability of processes of the first kind also seems to suggest that these processes do not occur in the observed portions of the world, however deeply in the observer's body the boundary is drawn. They can thus occur only in his consciousness. *A complete measurement, according to von Neumann's theory, involves therefore the consciousness of the observer.*" [2; emphases added]

6. Entangled states: The Schrödinger cat and the EPR paradox

The state ψ given by (6) is an *entangled* state where each term in the sum is the product of a possible state σ_n of a microsystem, the corresponding final state α_n of the apparatus, and the number c_n. So, as long as the total system+apparatus remains isolated, we have a linear superposition of different states of the apparatus, the coefficients being $c_n \sigma_n$.

Entangled states do not have a classical equivalent and are an unavoidable consequence of the superposition principle, considered by some physicists the fundamental principle of quantum mechanics. Schrödinger showed how strange some entangled states are with his well-known example of the cat: Imagine that the microsystem is a radioactive element with two possible states: σ_1 (atom non-decayed), and σ_2 (atom decayed). If the atom decays, a mechanism is activated and

kills the cat, a macrosystem with the possible states α_1 (cat alive, if the atom has not decayed) and α_2 (cat dead, if the atom has decayed). Then, if at a given instant the probability the radioactive element has of being decayed is ½, the coefficients take on the value $c_2 = c_1 = \sqrt{(½)}$ and

$$\psi = \sqrt{(½)}\sigma_1\alpha_1 + \sqrt{(½)}\sigma_2\alpha_2 \tag{7}$$

This entangled state is a superposition in which the two states "cat alive" and "cat dead" are mixed or smeared together by equal amounts. Following von Neumann, one should say that only through the act of observation, that is, looking at the cat, the system is thrown into a definite state. On his hand, Schrödinger asserts [2]: "*states of a macroscopic system which could be told apart by a macroscopic observation are distinct from each other whether observed or not.*" So, in his view, "it would be naïve to consider the ψ-function in (7) as depicting the reality." [2]

In [11] Einstein, Podolsky and Rosen demonstrate that the idea that "the wave function *does* contain a complete description of the physical reality of the system in the state to which it corresponds… together with the criterion of reality [see below] leads to a contradiction." Referring to this paper, frequently people speak of EPR paradox, but in fact one should talk about the EPR theorem. It states: "*if the predictions of quantum mechanics are correct (even for systems made of remote correlated particles) and if physical reality can be described in a local (or separable) way, then quantum mechanics is necessarily incomplete: some elements of reality exists in Nature that are ignored by this theory.*" [17]

At a first glance the Schrödinger cat and the EPR paradox look very different. Nevertheless, they share the conceptual problem implied in entangled states and, in this sense, it could be said that they are variations on the same theme. To pin point what we mean, let us suppose that instead of having a particle and a measuring apparatus (as in the previous section), or a radioactive element and a cat (as in the previous example) we have two spin ½ particles which propagate in opposite directions after leaving the source where they have been emitted in a singlet spin state

$$\left|\Psi\right\rangle = \sqrt{(½)}\left|1:+,\ 2:-\right\rangle - \sqrt{(½)}\left|1:-,\ 2:+\right\rangle \tag{8}$$

In this entangled state "spin up" and "spin down" of particle 1 are mixed or smeared together by equal amounts. The same is valid for particle 2.

Let us suppose that after leaving the source every interaction between both particles ceases. Then, if one of them is submitted to a measurement of spin in a direction orthogonal to that of propagation, quantum mechanics tells us that a measurement of spin in the same direction (orthogonal to the direction of propagation) upon the other particle will yield the opposite value to that obtained in the first measurement: for instance, if the first result is $+\hbar/2$, the other will be $-\hbar/2$ *with certainty*, and this must happen independently of the distance between both

particles. So, by measuring the spin of the particle going to one side, *e.g.* particle 1, it is possible to know the spin of the particle going to the other side, *i.e.* particle 2, without performing any measurement upon it or disturbing it in any way.

Now, on the one hand the EPR criterion of reality states [11]: "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." And, on the other hand, according to the condition of completeness formulated by EPR [11]: "every element of the physical reality must have a counterpart in the physical theory." So, applying the EPR criterion of reality we can conclude that the spin of the non-disturbed particle (particle 2), which has for instance the value $-\hbar/2$, is an element of reality. But this supposed element of reality is, however, absent from the state (8) where "spin up" and "spin down" of particle 2 are mixed or smeared together by equal amounts.

Einstein, Podolsky and Rosen end their article with the assertion [11]: "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

7. Controversies about the projection postulate and the theory of measurement

Most authors agree on the following point: neither the primitive notions nor the first four postulates of OQM are controversial. But this is the case neither of the Projection Postulate nor of the Theory of Measurement. The list of authors who have tried to solve the measurement problem in quantum mechanics is very long. In the following we shall sum up and comment the points of view of a few of them.

7.1. Dirac's notion of observation

Referring to an experiment with a single obliquely polarized photon incident on a crystal of tourmaline, Dirac says: "When we make the photon meet a tourmaline crystal, we are subjecting it to an observation. We are observing whether it is polarized parallel or perpendicular to the optic axis. *The effect of making this observation is to force the photon entirely into the state of parallel or entirely into the state of perpendicular polarization*. It has to make a sudden jump from being partly in each of these two states to being entirely in one or the other of them. Which of the two states it will jump into cannot be predicted, but is governed only by probability laws. If it jumps into the parallel state it gets absorbed and if it jumps into the perpendicular state it passes through the crystal and appears on the other side preserving this state of polarization." [6; emphases added]

Our comments: Dirac seems to suggest that these jumps (or projections), even if induced by observations, happen in the real, material world. We ask: what would happen if a photon polarized by nature (i.e. without the intervention of humans) meets a tourmaline crystal?

Would it jump from being partly in each of these two states to being entirely in one or the other? Or would it remain in an entangled state with the tourmaline crystal like that of the system+apparatus given by (6)?

7.2. Landau and Lifshitz' point of view

Following Bohr, L. Landau and E. Lifshitz deal with the measurement problem in the following terms [18]: "The possibility of a quantitative description of the motion of an electron requires the presence also of physical objects which obey classical mechanics to a sufficient degree of accuracy [for brevity the authors speak here of 'an electron,' meaning in general any object of a quantum nature, i.e. a particle or system of particles obeying quantum mechanics and not classical mechanics]. If an electron interacts with such a 'classical object', the state of the latter is, generally speaking, altered… In this connection the 'classical object' is usually called *apparatus*, and its interaction with the electron is spoken of as *measurement*. However, it must be emphasized that we are not discussing a process of measurement in which the physicist-observer takes part. *By measurement*, in quantum mechanics, we understand any process of interaction between classical and quantum objects, occurring apart from and independently of any observer."

They further add [18]: "[Let us] consider a system consisting of two parts: a classical apparatus and an electron (regarded as a quantum object). The process of measurement consists in these two parts coming in interaction with each other, as a result of which the apparatus passes from its initial state into some other; from this change of state we draw conclusions concerning the state of the electron. The states of the apparatus are distinguished by the values of some physical quantity (or quantities) characterizing it – the 'readings of the apparatus'. We conventionally denote this quantity by g, and its eigenvalues by $g_n \cdots$ [we shall] suppose the spectrum discrete. The states of the apparatus are described by means of quasi-classical wave functions which we shall denote by $\Phi_n(\xi)$, where the suffix n corresponds to the 'reading' g_n of the apparatus, and ξ denotes the set of its coordinates. The classical nature of the apparatus appears in the fact that, at any given instant, we can say with certainty that it is in one of the known states Φ_n with some definite value of the quantity g; for a quantum system such an assertion would, of course, be unjustified."

It follows the description of an analogous process to that mentioned in Stage I of von Neumann's theory of measurement: If $\Phi_0(\xi)$ is the wave function of the initial state of the apparatus (before the measurement), and $\Psi(q)$ some arbitrary normalized initial wave function of the electron (q denoting its coordinates), the initial wave function of the whole system is the product

$$\Psi\big(q\big)\,\Phi_0\big(\xi\big) \tag{9}$$

Then, applying the equations of quantum mechanics, we can in principle follow the change of the total system wave function with time. The measurement process finished, we can expand this wave function in terms of the Φ_n and obtain the sum

$$\sum_n A_n(q)\Phi_n(\xi) \tag{10}$$

where the $A_n(q)$ are some functions of q.

At this point Landau and Lifshitz assert [18]: "The classical nature of the apparatus, and the double role of classical mechanics as both the limiting case and the foundation of quantum mechanics, now make their appearance. As mentioned above, the classical nature of the apparatus means that, at any instant, the quantity g (the 'reading of the apparatus') has some definite value. This enables us to say that the state of the system apparatus+electron after the measurement will in actual fact be described, not by the entire sum (10), but by only the one term which corresponds to the 'reading' g_n of the apparatus,

$$A_n(q)\,\Phi_n(\xi) \tag{11}$$

It follows from this that $A_n(q)$ is proportional to the wave function of the electron after the measurement."

Our comments: We have already pointed out that Stage I of the measurement process does not involve any conceptual difficulty. In addition, there is no substantial difference between the analysis due to Landau and Lifshitz, which leads to sum (10), and that due to von Neumann, which leads to sum (6). The problem arises in Stage II, where the reduction to one and only one term of sum (6) or of sum (10) must be achieved. This problem is faced in different ways by different authors: von Neumann, for whom the apparatus is a purely quantum system, makes appeal to observer's consciousness; Landau and Lifshitz, for whom quantum measurements occur apart from and independently of any observer, make appeal to the classical character of the apparatus.

7.3. Bunge's epistemological realism

In [19] M. Bunge asserts: "The main epistemological problem about quantum theory is whether it is compatible with epistemological realism. (The latter is a family of epistemologies which assume that (a) the world exists independently of the knowing subject, and (b) the task of science is to produce maximally true conceptual models of reality…)"

On the one hand, in Bunge's view the question of reality has nothing to do with scientific problems such as whether all properties have sharp values, and whether all behavior is causal. On the other hand, he thinks the Schrödinger equation rules every quantum process. Then, when referring to projections, he says [19]: "we would like to see a rigorous *proof* that the projection, or something close to it, occurs partly as a consequence of the Schrödinger equation,

not as a result of an arbitrary decision of an omnipotent Observer placed above the laws of nature. More precisely, we should like to derive a projection (or semi-projection) *theorem* from physical (quantum and classical) first principles. And we should like to have a proof that the projection (or semi-projection) is *a swift but not instantaneous process caused by certain interactions*, in particular those between quanton and apparatus." And in [20]: "one should attempt to *deduce* the reduction of the state function instead of *postulating* it."

Our comments: In [21] and [22] we have asserted that quantum theory is compatible with realism. And we also think that the question of reality has nothing to do with scientific problems such as whether all properties have sharp values or not and whether all behavior is causal or not. We fully agree with Bunge on these points. Nevertheless, we would add to the list of scientific problems which have nothing to do with the question of reality: the issue of action-at-a-distance and the validity of conservation laws, in particular conservation of energy. Concerning this last point, H. Poincaré declares: *"[cette loi] ne peut avoir qu'une signification, c'est qu'il y a une propriété commune à tous les possibles; mais dans l'hypothèse déterministe il n'y a qu'un seul possible et alors la loi n'a plus de sens. Dans l'hypothèse indéterministe, au contraire, elle en prendrait un, même si on voulait l'entendre dans un sens absolue… "* [23] This remark seems to us pertinent for, if there are quantum processes not ruled by deterministic laws, one could suspect that conservation laws are not valid in these kinds of processes.

Now, concerning Bunge's suggestion: it would be grateful to see the Projection Postulate deduced from the Schrödinger equation; the problem is to know whether achieving this task is possible or not. In 1935 Margenau showed that the Projection Postulate contradicts the more fundamental Schrödinger equation of motion; see Section 4. And according to Bes, "because of the linearity of the Schrödinger evolution, there is no mechanism to stop the evolution and yield a single result for the measurement: *the state reduction is beyond the scope of the Schrödinger evolution."* [10; emphases added] So, as long as these assertions have not proven wrong, we do not see in which way somebody could be inspired to face the task Bunge proposes us.

7.4. The many worlds interpretation

In [24] H. Everett proposes an alternative to observation-triggered wave. He assumes that the equations of physics that model the time evolution of systems without observers are sufficient for modeling systems which do contain observers. As a result, the universe which includes the system, the measuring apparatus and the observer, always evolves in agreement with the Schrödinger equation, even when the observer performs a measurement. In this approach the system+apparatus+observer+environment splits into as many branches as results of the measurement are possible. All possibilities are realized at the same time and these branches coexist without interfering, so the component of the observer in one branch is unaware of the others, and he/she perceives what happens as if the system state has been projected. But this is a delusion of the mind of the observer for "there does not exist anything like a single state for one subsystem…" [24]

Everett originally called his approach the "Correlation Interpretation," where correlation refers to entanglement, as that obtained at the end of Stage I of von Neumann's theory of

measurement; see sum (6) in Section 5 and sum (10) in Section 7.2. The phrase "many-worlds" is due to B. DeWitt, who was responsible for the wider popularization of Everett's theory. [25]

Our comments: Since each component of the observer is condemned to remain in his/her branch there is no way he/she could know what is happening in the others. As a consequence, Bunge says, "this solution to the contradictions generated by the orthodox version of the projection hypothesis is unscientific because the splitting is unobservable, so the conjecture is untestable." [19] We fully share this assertion.

7.5. Decoherence

Decoherence is a process which prevents different elements in the quantum superposition of the total system's wave function from interfering with each other. So, it has been said, "it looks and smells as a collapse." [15]

W. Zurek, one of its conspicuous defenders, introduces the concept of decoherence in the following way [26-27]: Let $|\uparrow\rangle$ and $|\downarrow\rangle$ be the orthonormal states of a particle of spin ½ in interaction with a detector whose orthonormal states are $|d_\uparrow\rangle$ and $|d_\downarrow\rangle$. If the detector begins in the $|d_\downarrow\rangle$ state and "clicks," $|\uparrow\rangle|d_\downarrow\rangle \to |\uparrow\rangle|d_\uparrow\rangle$ when the spins are in the state $|\uparrow\rangle$ but remains unchanged otherwise.

If before the interaction the particle is in the pure state $|\psi_S\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$, the composite system SD (system+detector) starts as $|\Phi^i\rangle = |\psi_S\rangle|d_\downarrow\rangle$ and the interaction results in the evolution of $|\Phi^i\rangle$ into the correlated state $|\Phi^c\rangle$

$$\left|\Phi^i\right\rangle = \left(\alpha\left|\uparrow\right\rangle + \beta\left|\downarrow\right\rangle\right)\left|d_\downarrow\right\rangle \to \alpha\left|\uparrow\right\rangle\left|d_\uparrow\right\rangle + \beta\left|\downarrow\right\rangle\left|d_\uparrow\right\rangle = \left|\Phi^c\right\rangle \tag{12}$$

The corresponding pure state density matrix is

$$\begin{aligned}
\rho^c &= \left|\Phi^c\right\rangle\left\langle\Phi^c\right| \\
&= \left|\boldsymbol{\alpha}\right|^2\left|\uparrow\right\rangle\left\langle\uparrow\right|\left|\mathbf{d_\uparrow}\right\rangle\left\langle\mathbf{d_\uparrow}\right| + \alpha\beta^*\left|\uparrow\right\rangle\left\langle\downarrow\right|\left|\mathbf{d_\uparrow}\right\rangle\left\langle\mathbf{d_\downarrow}\right| \\
&+ \boldsymbol{\alpha}^*\boldsymbol{\beta}\left|\downarrow\right\rangle\left\langle\uparrow\right|\left|\mathbf{d_\downarrow}\right\rangle\left\langle\mathbf{d_\uparrow}\right| + \left|\boldsymbol{\beta}\right|^2\left|\downarrow\right\rangle\left\langle\downarrow\right|\left|\mathbf{d_\downarrow}\right\rangle\left\langle\mathbf{d_\downarrow}\right|
\end{aligned} \tag{13}$$

As happened in the analyses performed by von Neumann and by Landau and Lifshitz, this first stage of the detection process is a Schrödinger evolution which does not involve any conceptual difficulty.

Now, there are two branches of the detector state in this correlated state $|\Phi^c\rangle$, but we know the alternatives are distinct outcomes rather than a mere superposition of states. Nevertheless, cancelling the off-diagonal terms, which express quantum correlations, the reduced density matrix results:

$$\rho^r = |\alpha|^2 |\uparrow\rangle\langle\uparrow||d_\uparrow\rangle\langle d_\uparrow| + |\beta|^2 |\downarrow\rangle\langle\downarrow||d_\downarrow\rangle\langle d_\downarrow| \tag{14}$$

In Zurek's view, the key advantage of ρ^r over ρ^c is that its coefficients $|\alpha|^2$ and $|\beta|^2$ may be interpreted as classical probabilities. Unitary evolution condemns every closed quantum system to 'purity.' Yet if the outcomes of a measurement are to become independent, with consequences that can be explored separately, a way must be found to dispose of the excess of information (contained in the off-diagonal terms). This disposal can be caused by interaction with the degrees of freedom external to the system, which we shall summarily refer to as the 'environment'… [26]

Following the first step of the measurement process –establishment of the correlation as shown in (12)– the environment E initially in the state $|\varepsilon_0$, becomes correlated with SD (system +detector):

$$\begin{aligned} |\Phi^c\rangle|\varepsilon_0\rangle &= \left(\alpha|\uparrow\rangle|d_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle\right)|\varepsilon_0\rangle \\ &\rightarrow \alpha|\uparrow\rangle|d_\uparrow\rangle|\varepsilon_\uparrow\rangle + \beta|\downarrow\rangle|d_\downarrow\rangle|\varepsilon_\downarrow\rangle = |\Phi\rangle \end{aligned} \tag{15}$$

with obvious notation. "This final state extends the correlation beyond the system-detector pair. When the states of the environment corresponding to spin up and spin down states of the detector are orthogonal, we can take the trace over the uncontrolled degrees of freedom to get the same results as the reduced matrix." [26] The density matrix that describes the detector-system combination obtained by ignoring (tracing over) the uncontrolled (and unmeasured) degrees of freedom is

$$\rho_{SD} \equiv Tr_E |\Phi\rangle\langle\Phi| = \sum_i \langle\varepsilon_i|\Phi\rangle\langle\Phi|\varepsilon_i\rangle = \rho^r \tag{16}$$

which coincides with the reduced matrix given by (14).

It has been proven that for large classical objects decoherence would be virtually instantaneous because of the high probability of interaction of such systems with some environmental quantum. A quantitative model due to Zurek [26] illustrates the gradual cancellation of the off-diagonal elements with decoherence over time.

Our comments: If we want to describe processes ruled by the Schrödinger equation, disposing of terms which give an account for something that is happening is not a good idea. When SD is coupled to the environment E, the Schrödinger evolution leads the total system SDE to the pure state $|\Phi\rangle$, *it does not lead the SD system to the mixture* ρ^r. In addition, the *mixture* ϱ^r is *unique* and completely different from the SD *pure states* $|\psi_\uparrow\rangle = |\uparrow\rangle|d_\uparrow\rangle$ and $|\psi_\downarrow\rangle = |\downarrow\rangle|d_\downarrow\rangle$ which are, according to the Projection Postulate, the *only two* possible final states of SD. So, in our view decoherence does not provide a solution to the measurement problem. In [28] we have

advanced similar arguments to object the contributions of Griffith, Gell-mann, Hartle and Omnès.

Other authors have criticized the solution to the measurement problem which involves decoherence. In particular, in [29] it is asserted that to obtain ρ^r "… an appeal has been made that goes beyond the ordinary Schrödinger equation, to a prior split of [the total] physical system into microscopic system S, detector D and environment E. But no rules have ever been given for making such a split, and certainly a physical system does not come with a subsystem containing a little sign reading, 'I am the environment: Trace over me.' Without such rules one cannot, in the general case, apply the environment-trace prescription…" And in [17] F. Laloë points out: "Indeed, in common life as well as in laboratories, one never observes superposition of results; we observe that Nature seems to operate in such a way that a single result always emerges from a single experiment; this will never be explained by the Schrödinger equation, since all that it can do is to endlessly extend its ramifications into the environment, without ever selecting one of them only."

7.6. Complementing Schrödinger dynamics

In order to find a solution to the measurement problem keeping as valid the individual interpretation of the state vector, other theories close to, but different from, quantum mechanics have been proposed. In these theories, the Schrödinger equation is complemented in a way that leads to spontaneous collapses. This is the case of those developed by D. Bohm [30-31], G. Ghiradi, A. Rimini and T. Weber [32], L. Diosi [33], and E. Joos and H. D. Zeh [34]. Ballentine [35] has demonstrated that these theories violate energy conservation and are incompatible with the existence of stationary states. Let us summarize two of these contributions and reproduce some additional comments on them.

i.   In the theory of measurement proposed by Bohm [27-28] the state function $|\psi\rangle$ refers to an ensemble and every particle of the ensemble has a position x, which is a hidden variable. In addition to the usual potential V(x), a quantum potential U(x) is introduced. This allows Bohm to explain in an elegant way the double-slit experiment. In the EPR experiment disturbances from one particle to the other are transmitted instantaneously by the potential U(x).

ii.   In CSL (Continuous Spontaneous Localization) theory [32], particles can undergo spontaneous wave-function collapses. For individual particles, these collapses happen probabilistically and will occur at a given rate with high probability but not with certainty; groups of particles behave in a statistically regular way, however. Since experimental physics has not already detected an unexpected spontaneous collapse, it can be argued that CSL collapses happen extremely rarely. The authors suggest that the rate of spontaneous collapse for an individual particle is of the order of once every hundred million years.

In two interesting comments F. Laloë [17] emphasizes that in those theories which modify the Schrödinger equation (i) "new constants appear which may in a sense look like ad hoc constants, but actually have an important conceptual role: They define the limit between the

microscopic and macroscopic world (or between reversible and irreversible evolution); the corresponding border is no longer ill-defined, as opposed to the situation, for instance, in the Copenhagen Interpretation." And (ii) "in the initial Bohm-Bub theory, a complete collapse of the wave function is never obtained in any finite time. The same feature actually exists in CSL: *There is always what is called a 'tail' and even when most of the wave function goes to the component corresponding to one single outcome of an experiment, there always remains a tiny component on the others* (extremely small and continuously going down in size)." [17, emphases added]

8. Measurement processes versus spontaneous processes

In "Against 'measurement'," J. Bell complains about quantum mechanics formulations in the following terms [1]: "Surely, after 62 years, we should have an exact formulation of some serious part of quantum mechanics? By 'exact' I do not of course mean 'exactly true'. I mean only that the theory should be fully formulated in mathematical terms, with nothing left to the discretion of the theoretical physicist… until workable approximations are needed in applications. By 'serious' I mean that some substantial fragment of physics should be covered. Nonrelativistic 'particle' quantum mechanics, perhaps with the inclusion of the electromagnetic field and a cut-off interaction, is serious enough. For it covers 'a large part of physics and the whole of chemistry'; see [36]. I mean too, by 'serious', that 'apparatus' should not be separated off from the rest of the world into black boxes, as if it were not made of atoms and not ruled by quantum mechanics."

In the following table the most significant differences between measurement processes and spontaneous processes are reported.

Spontaneous processes	Measurement processes
The observer plays no role	The observer plays a paramount role
The state vector $\lvert \psi(t) \rangle$ is necessarily continuous	In general the state vector $\lvert \psi(t) \rangle$ is projected
The superposition principle is valid: there is interference	Superposition breaks down: interference is lost
The process is ruled by deterministic laws	The process is ruled by probability laws
Every action is localized	There is a kind of action-at-a-distance
Conservation laws are strictly valid	They have only a statistical sense

The mere comparison of the characteristics of both kinds of processes facilitates the understanding of why so many scientists are disappointed with quantum mechanics formalism.

At this stage it seems superfluous to comment on the first three lines of the previous table. Concerning determinism (fourth line of the table), let us recall that during the Fifth Solvay Congress, *i.e.* less than a century ago, H. Lorentz expressed his dissatisfaction with the rejection of determinism in atomic physic. Nowadays the notion of indeterminism is normally accepted,

despite many scientists' aspirations for a version of quantum theory based on deterministic laws, and the "Old One" not playing at dice.

Something similar happened with the idea of action-at-a-distance (fifth line of the table) pointed out by Einstein in the Fifth Solvay Congress. First this notion was rejected by the majority of scientists. Then, in 1964 J. Bell proved a theorem stating that a local hidden variable theory cannot reproduce all statistical predictions of quantum mechanics [37]: More precisely, he showed that in the framework of any deterministic and local theory the correlations between some properties of two particles should satisfy an inequality (Bell's inequality) and that this inequality could be violated if the two particles were in an entangled state like that given by (8). In the following years many experiments yielded results which are compatible with the predictions of quantum mechanics and violate Bell's inequality. [38-41]

Now the door was opened to explore an even more strange and fascinating phenomenon: quantum teleportation. [44] This is a process by which quantum information (*e.g.* the exact state of an atom or photon) can be transmitted from one location to another, with the help of classical communication and previously shared quantum entanglement between the sending and receiving location. Because it depends on classical communication, which cannot proceed faster than the speed of light, it cannot be used for superluminal transport or communication. The seminal paper first expounding the idea was published in 1993. Since then, quantum teleportation has been realized in various physical systems. At present the record distance for quantum teleportation is 143 km (89 mi) with photons, and 21 m with material systems. In August 2013, the achievement of "fully deterministic" quantum teleportation, using a hybrid technique, was reported. On 29 May 2014, scientists announced a reliable way of transferring data by quantum teleportation. Quantum teleportation of data had been done earlier but with highly unreliable methods. The important point in what concerns the measurement problem is that, thanks to these astonishing results, the idea that projections imply a peculiar action-at-a-distance is nowadays frequently accepted.

On the last line of the previous table one reads: Conservation laws are strictly valid in spontaneous processes and have only a statistical sense in measurement processes. We have dealt with this subject a few years ago, but surely our results are not known by everybody. So in the next section we shall reproduce the essential of the paper where this problem is discussed; see [45]

9. Validity of conservation laws in spontaneous processes and in measurement processes

In the framework of OQM, in general, physical quantities are not sharp. "A popular working rule of pragmatic quantum mechanics says that a physical quantity has no value before a measurement." [46] Now, if the operator A_S represents the physical quantity A_S referred to the individual system S, when the system state is $|\Phi_S\rangle$ the mean value of A_S can be defined as

$$\left\langle A_S \right\rangle = \left\langle \Phi_S \middle| A_S \middle| \Phi_S \right\rangle \tag{17}$$

see for instance [7]. Hence, even if the physical quantity A_S has in general no value, it has a mean value $\left\langle A_S \right\rangle$ which is perfectly sharp.

A necessary condition for the physical quantity A_S to be conserved is that $\left\langle A_S \right\rangle$ be a constant. If H_S is the Hamiltonian of S, the validity of conditions

$$\partial A_S / \partial t = 0 \tag{18}$$

and

$$\left[A_S, H_S \right] = 0 \tag{19}$$

ensure that in those processes that are governed by the Schrödinger equation $\left\langle A_S \right\rangle$ remains a constant in time for every state $| \Phi_S \rangle$. As a consequence, according to OQM there is no inconvenience in saying that if conditions (18) and (19) are fulfilled, A_S is conserved in spontaneous processes.

We shall now address the problem of the validity of conservation laws when a measurement of A_S is performed; for simplicity we shall deal with the discrete case. Let $a_k (k = 1, 2, \cdots)$ be an eigenvalue of the operator A_S, g_k its degree of degeneracy and $| a_k^v \rangle (v = 1, 2, \cdots g_k)$ an eigenvector corresponding to the eigenvalue a_k. We shall assume that $| m_0$ represents the initial state of a measuring device M of A_S, and $| \psi_k^v \rangle$ the orthonormal states of S+M when the measurement process is over. To ensure that measurements of A_S can be performed according to the ideal measurement scheme, we shall suppose that A_S commutes with every operator representing another conserved quantity referred to S+M. [47-51]

According to the ideal scheme the transition

$$\left| a_k^v \right\rangle \left| m_0 \right\rangle \rightarrow \left| \psi_k^v \right\rangle \tag{20}$$

has a probability of one, hence it can be assumed that *it is a result of the Schrödinger evolution.*

Let A be the operator representing a physical quantity A referred to S+M, and H be its Hamiltonian. We can then write

$$H = H_S + H_M + H_{int} \tag{21}$$

where H_M refers to M, and H_{int} is due to the interaction between S and M. We assume that the conditions

$$\partial A / \partial t = 0 \tag{22}$$

and

$$\left[A, H\right] = 0 \tag{23}$$

are fulfilled. If at t_0 (when the interaction between S and M starts) it is possible to write

$$A = A_S + A_M \tag{24}$$

(where A_M refers to M), we have

$$\left\langle A \right\rangle_k^v (t_0) = \left\langle a_k^v \middle| A_S \middle| a_k^v \right\rangle + \left\langle m_0 \middle| A_M \middle| m_0 \right\rangle = a_k + \left\langle m_0 \middle| A_M \middle| m_0 \right\rangle \tag{25}$$

And, since at t_f (when the interaction between S and M is over)

$$\left\langle A \right\rangle_k^v (t_f) = \left\langle \psi_k^v \middle| A \middle| \psi_k^v \right\rangle \tag{26}$$

the validity of (22) and (23) implies that $\langle A \rangle_k^v(t_f) = \langle A \rangle_k^v(t_0)$, and hence

$$\left\langle \psi_k^v \middle| A \middle| \psi_k^v \right\rangle = a_k + \left\langle m_0 \middle| A_M \middle| m_0 \right\rangle \tag{27}$$

for every v. As $\left\langle \psi_k^v \mid A \mid \psi_k^v \right\rangle$ does not depend on v, it can be written

$$\left\langle A \right\rangle_k (t_f) = \left\langle A \right\rangle_k^v (t_f) = a_k + \left\langle m_0 \middle| A_M \middle| m_0 \right\rangle \tag{28}$$

This relation must necessarily be fulfilled in the ideal measurement scheme. As a consequence, it can be said that *in those cases where the initial state of S is an eigenstate of the operator A_S representing the physical quantity A_S to be measured, the corresponding conservation law of A is valid.* This result can also be seen as a natural consequence of the hypothesis that the process described by (20) is governed by the Schrödinger equation.

$$\left| \Phi_S \left(t_0 \right) \right\rangle = \sum_{l,\mu} c_l^\mu \left| a_l^\mu \right\rangle \tag{29}$$

(where at least two coefficients c_l^μ and $c_{l'}^{\mu'}$ with $l \neq l'$ are non-null) and the Schrödinger equation rules the measurement process, then the Hamiltonian H, referred to S+M, induces the evolution

$$\sum_{l,\mu} c_l^\mu \left| a_l^\mu \right\rangle \left| m_0 \right\rangle \rightarrow \sum_{l,\mu} c_l^\mu \left| \psi_l^\mu \right\rangle \tag{30}$$

Making

$$\left\langle A \right\rangle \left(t_0 \right) = \left\langle \Phi_S \left(t_0 \right) \right| \left\langle m_0 \left| A \right| \Phi_S \left(t_0 \right) \right\rangle \left| m_0 \right\rangle = \sum_{l,\mu} \left| c_l^\mu \right|^2 a_l + \left\langle m_0 \left| A_M \right| m_0 \right\rangle \tag{31}$$

and

$$\left\langle A \right\rangle \left(t_f \right) = \left(\sum_{l,\mu} c_l^{\mu *} \left\langle \psi_l^\mu \right| \right) A \left(\sum_{l',\mu'} c_{l'}^{\mu'} \left| \psi_{l'}^{\mu'} \right\rangle \right) \tag{32}$$

the validity of (22) and (23) allow us to ensure that $\langle A \rangle (t_0) = \langle A \rangle (t_f)$: *As long as the Schrödinger equation rules the process, the mean value of the physical quantity A, referred to the total system S+M, remains a constant* and the state of S+M continues to be the superposition which appears in (30).

But in view of the Projection Postulate such a superposition is broken down. Hence, the change of S+M is not given by (30) and the transition

$$\sum_{l,\mu} c_l^\mu \left| a_l^\mu \right\rangle \left| m_0 \right\rangle \rightarrow \sum_\mu c_k^\mu \left| \psi_k^\mu \right\rangle \tag{33}$$

has probability $\sum_\mu | c_k^\mu |^2$ to happen. In this last case,

$$\left\langle A \right\rangle_k \left(t_f \right) = a_k + \left\langle m_0 \left| A_M \right| m_0 \right\rangle \tag{34}$$

as stated in (28). As a consequence, it results

$$\langle \mathbf{A} \rangle_k \left(\mathbf{t}_f \right) \neq \langle \mathbf{A} \rangle \left(\mathbf{t}_0 \right) \tag{35}$$

for every k, even though conditions (22) and (23) are fulfilled.

It is worth noticing that inequalities (35) are obtained under the assumptions that the individual interpretation of the state vector and the Projection Postulate are valid. In this case the condition that $\langle A \rangle$ be a constant, a necessary condition for A to be conserved, is not satisfied. We are thus forced to conclude that *if the initial state of S is not an eigenvector of A_S, the physical quantity A is not conserved in processes of measurement of* A_S. In [52-55] we give examples of processes of measurement of the type analyzed in this section; and in [45] we deal with the continuous case. The same result is obtained.

A similar conclusion resulting from a different analysis has been obtained by P. Pearle. [56] He says that "it should first be noted that quantum theory itself, with the reduction postulate indiscriminately applied, does not necessarily satisfy the conservation laws..." In his view, "this is a serious problem for quantum theory with a reduction postulate."

Our next step is to calculate the *average* of $\langle A \rangle_k(t_f)$ when the process of measurement of A_S is repeated many times. Let f_k be the frequency corresponding to the possible results $a_k (k = 1, 2, \cdots)$ and to the mean value $\langle A \rangle_k(t_f)$. If the process is repeated N times, the resulting average is

$$\overline{A} = \sum_k f_k \langle A \rangle_k \left(t_f \right) = \sum_k f_k a_k + \langle m_0 | A_M | m_0 \rangle \tag{36}$$

where (34) has been taken into account. Now, if N is big enough, we can assert that $f_k \approx \sum_\mu | c_k^\mu |^2$ and, in view of (31) we obtain

$$\overline{A} \approx \langle A \rangle \left(t_0 \right) \tag{37}$$

To sum up, in individual processes of measurement of A_S the conservation law of A is in general not valid; but this law still has a statistical sense.

10. Ad-hoc use of the projection postulate

We shall start this section with some remarks concerning the concept of probabilities. Following tradition, we are going to adopt the expression *subjective probabilities* for probabilities related to the lack of knowledge in processes governed by deterministic laws; and *objective probabilities* for probabilities where the process is not ruled by deterministic laws. Accordingly, there is no room for objective probabilities in classical mechanics, electromagnetism and relativity. Moreover, as in the framework of OQM every spontaneous process is ruled by the

Schrödinger equation, which is a deterministic equation, objective probabilities have nothing to do with these kinds of processes.

Then, we explicitly state that *a system cannot be in two different states at the same time*. Hence, if the system is in the state $|\psi(t_0)\rangle$ at time t_0 and the process is ruled by the Schrödinger equation, there is no more than one possibility: *at time t its state must certainly be*

$$|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle \qquad (38)$$

where $U(t, t_0)$ is the evolution operator. As a consequence, if at time t_0 the system is in the state $|\psi(t_0)\rangle$ and the process is spontaneous, the *objective probability* the system has of being in $|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle$ at time t is P=1 and the *objective probability* the system has of being in another, different state from $|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle$, is P=0.

There is no doubt that quantum mechanics has been extremely successful in explaining radioactivity, electron-phonon scattering, interactions between light and matter and many other phenomena which involve, supposedly, only *spontaneous processes*. So, in principle one could expect that the analysis of these processes does not involve projections (for they play a role just in cases measurements are performed). Nevertheless, reading quantum mechanics textbooks one is forced to conclude the opposite; see for instance any book of the following list: [6-10, 18, 57].

To deal with spontaneous processes involved in phenomena such as those previously mentioned, in most cases time-dependent perturbation theory is necessary. In Dirac's view, "[time-dependent perturbation theory] must be used for solving all problems involving a consideration of time." [6] And W. Heitler states: "for all problems of physical interest the application of [time-dependent] perturbation theory is beyond doubt." [57] So let us examine in which way this theory is needed to confront these kinds of problems. According to Dirac, "with [time-dependent perturbation theory] one takes a stationary state of the unperturbed system and sees how it varies with time under the influence of the perturbation." [6] And "the perturbation causes the state to change." [6] We are going to analyze this point in detail.

Consider a system with Hamiltonian H_0 which does not depend explicitly on time. It is assumed that the eigenvalues equations of H_0 have previously been solved. We shall denote by ε_k and $|\varphi_k\rangle$ ($k=1, 2, \cdots$) its eigenvalues and eigenvectors, respectively; for simplicity we shall deal with the discrete non-degenerate case. Then, if at t_0 a perturbation W(t) depending explicitly on time is added to H_0, the Hamiltonian of the system for $t > t_0$ becomes

$$H(t) = H_0 + W(t) \qquad (39)$$

and the system evolves according to the Schrödinger equation

$$i\hbar d\left|\psi\left(t\right)\right\rangle / dt = \left[H_0 + W\left(t\right)\right]\left|\psi\left(t\right)\right\rangle \tag{40}$$

"The solution $\left|\psi(t)\right\rangle$ of this first-order differential equation which corresponds to the initial condition $\left|\psi(t_0)\right\rangle = \left|\varphi_i\right\rangle$ is unique." [9] Then it is said that at time t_f the probability $P_{if}(t_f)$ of *finding* the system in another eigenstate $\left|\varphi_f\right\rangle$ of H_0 is

$$P_{if}\left(t_f\right) = \left|\left\langle \varphi_f \left|\psi\left(t_f\right)\right.\right\rangle\right|^2 \tag{41}$$

Taking into account what has been previously said, we shall write $\left|\psi(t_f)\right\rangle = U(t_f, t_0)\left|\psi(t_0)\right\rangle$. Now, to find *a system which at t_f certainly is in the state* $\left|\psi(t_f)\right\rangle = U(t_f, t_0)\left|\psi(t_0)\right\rangle$ in another, different state like $\left|\varphi_f\right\rangle$ is a task impossible to achieve. By contrast, to find such a system *immediately after* t_f in $\left|\varphi_f\right\rangle$ is a task possible to achieve but it requires a measurement to be performed at time t_f. And which one should be the physical quantity to be measured? The answer is not obvious for, in particular, if this physical quantity were the energy, the system should not be projected to $\left|\varphi_f\right\rangle$, an eigenstate of the operator H_0 which does not represent the energy of the system at time $t > t_0$. This last remark, however, does not apply in cases where W(t) is a perturbing interaction limited in time and it can be considered that W(t_f)=0. [8]

But let us come back to the declared aim of time-dependent perturbation theory. Conspicuous authors make statements such as "Our objective is to calculate transition amplitudes between the relevant unperturbed eigenstates, owing to the presence of the perturbation…" [8]; "we want to study the transitions which can be induced by the perturbation…" [9]; "the transition probability between the initial state $\left|\varphi_i\right\rangle$ and the final state $\left|\varphi_f\right\rangle$ [is] induced by the perturbation…" [10] As the perturbation W(t) modifies the Hamiltonian, it is evident that the state $\left|\psi(t_f)\right\rangle = U(t_f, t_0)\left|\psi(t_0)\right\rangle$ resulting when W(t) is applied will be different from the state $\left|\psi(t_f)\right\rangle$ resulting when W(t) is absent. *But perturbations do not induce transitions.* In this sense Messiah is very clear. Referring to the objective of time-dependent perturbation theory, he asserts: "*Supposons qu'à l'instant initial t_0, le système se trouve dans l'un des états propres de H_0, l'état a par exemple. Nous nous proposons de calculer la probabilité de le trouver à l'instant t dans un autre état propre de H_0, l'état b par exemple, dans l'éventualité d'une mesure à cet instant*" [7]; we emphasize: *dans l'éventualité d'une mesure à cet instant*. On the contrary, other authors seem to have forgotten that in the framework of OQM measurements are absolutely necessary in order to obtain the transition probability $P_{if}(t_f)$.

11. Who is afraid of the projection postulate?

C. M. Caves asserts [58]: "Mention collapse of the wave function and you are likely to encounter vague uneasiness or, in extreme cases, real discomfort. This uneasiness can usually be traced

to a feeling that a wave-function collapse lies 'outside' quantum mechanics. The real quantum mechanics is said to be the unitary Schrödinger evolution; wave-function collapse is regarded as an ugly duckling of questionable status, dragged in to interrupt the beautiful flow of Schrödinger evolution."

Projections are disliked for many reasons; one of them is that they imply discontinuities. But is there a way of give an account for processes of emission and absorption of light without invoking discontinuities? We think there is not, as shown in the following.

To start with, let us face this question in an intuitive and nearly classic way. Consider the absorption of one photon by one atom. We shall assume that (i) initially the atom and the photon exist as separated things; (ii) the photon can only travel with a speed c and has an energy $\hbar\omega$ at each instant; and (iii) the energy of the system atom+photon is conserved in the process of absorption. Note that the photon cannot be absorbed through a swift, not instantaneous change: *either it is, travels with speed c and carries the energy $\hbar\omega$ or it is not. This implies that the photon must be absorbed by the atom at once and that the energy of the atom must be increased in an instantaneous way.* [59] In more elaborated treatments of the subject, probabilities of projections and hence, indirectly, state function discontinuities are mentioned frequently. For instance, in the classical textbook of Heitler one reads: " $\mid c \cdots_{n_\lambda}(t) \mid {}^2$ is the probability for *finding* n_1 photons of type 1, n_λ photons of type λ, etc."; "The probability for *finding* the system at time t in the state n when it was in state 0 at t=0 is thus…"; "We now calculate the probabilities $\mid b_n(t) \mid {}^2$ for *finding* the system in a state n at the time t." [57, emphases added]

On his side, Jammer points out a serious problem which becomes apparent when the notion of projections is rejected: "As long as a quantum mechanical one-body or many-body system does not interact with macroscopic objects, as long as its motion is described by the deterministic Schrödinger time-dependent equation, no events could be considered to take place in the system. Even such elementary process as the scattering of a particle in a definite direction could not be assumed to occur (since this would require a 'reduction of the wave packet' without an interaction with a macroscopic body). In other words, if the whole physical universe were composed only of microphysical entities, as it should be according to the atomic theory, it would be a universe of evolving potentialities (time-dependent ψ-functions) but not of real events." [2]

A few authors have considered the possibility that projections may happen at the microscopic level. One of them is H. Primas, for whom "the reality of the breakdown of the superposition principle of traditional quantum mechanics on the molecular level is dramatically demonstrated by the terrible Contergan tragedy which caused many severe birth defects." [46] And Bell complains: "during 'measurement' the linear Schrödinger evolution is suspended and an ill-defined 'wave-function collapse takes over. There is nothing in the mathematics to tell what is 'system' and what is 'apparatus' nothing to tell which natural processes have the special status of 'measurements'. Discretion and good taste, born from experience, allow us to use quantum theory with marvelous success, despite the ambiguity of the concepts named above in quotation marks." [60] In [28] we have given an answer to the question "which natural

processes have the special status of measurements?" In the next section we shall summarize the most important points of our approach.

12. The spontaneous projections approach

In the Spontaneous Projection Approach (SPA) it is assumed that two kinds of processes, irreducible to one another, occur in nature: (i) the strictly continuous and causal ones, which are governed by the Schrödinger equation and (ii) those implying discontinuities, which are ruled by probability laws. A postulate ensuring the statistical sense of conservation laws is adopted. Taking into account this postulate the concept of preferential states is introduced. If the system does not have preferential states, the Schrödinger evolution follows. By contrast, if the system has preferential states projections may happen. Spontaneous and measurement processes are treated on the same footing.

SPA is compatible with epistemological realism: we assume that the world exists independently of the knowing subject and that it is possible to know it, at least in a partial way. So our discourse will be about what happens, not about what is measured or observed. (This does not mean, obviously, that it has to be right; it could happen that it be completely wrong.) We share Bunge's assertion [19]: "the question of reality has nothing to do with scientific problems such as whether all properties have sharp values and whether all behavior is causal." And, as we have already said, we would add to the list of scientific problems which have nothing to do with the question of reality the issue of action-at-a-distance and the validity of conservation laws in individual processes.

The primitive (undefined) notions of SPA are: *system, physical quantity, state system and probability*; the term probability will be used as a synonym of *objective probability*; see Section 10. Nevertheless, we have taken into account Bell's remark [1]: "The concepts 'system', 'apparatus' 'environment' immediately imply an artificial division of the world, and an intention to neglect, or take only schematic account of, the interaction across the split." We do not make such an artificial division for *apparatus and environment are absent of SPA postulates*. And systems mean either objects or collections of objects.

The two first postulates of SPA coincide with those of OQM. They state:

Postulate I: To every system corresponds a Hilbert space H whose vectors (state vectors, wave functions) completely describe the state of the system.

Postulate II: To every physical quantity A corresponds uniquely a self-adjoint operator A acting in H. It has associated the eigenvalue equations

$$A\left|a_j^v\right\rangle = a_j\left|a_j^v\right\rangle \tag{42}$$

(v is introduced in order to distinguish between the different eigenvectors that may correspond to one eigenvalue a_j), and the closure relation

$$\sum_{j,\nu} \left| a_j^\nu \right\rangle \left\langle a_j^\nu \right| = I \tag{43}$$

is fulfilled (here I is the identity operator). If j or ν is continuous, the respective sum has to be replaced by an integral.

Postulate III: If the conditions

$$\partial A / \partial t = 0 \tag{44}$$

and

$$\left[H,\, A \right] = 0 \tag{45}$$

are fulfilled (here H is the Hamiltonian of the system), and there is a *generic* orthonormal set $\{ | u_k \rangle \}(k = 1, 2, \cdots)$ such that the normalized state $| \Phi \rangle$ of the system can be written

$$\left| \varphi \right\rangle = \sum_k c_k \left| u_k \right\rangle \tag{46}$$

the validity of

$$\left\langle \Phi \right| A \left| \Phi \right\rangle = \sum_k \left| c_k \right|^2 \left\langle u_k \right| A \left| u_k \right\rangle \tag{47}$$

is a necessary condition for projections of the state $| \Phi \rangle$, given by (46), to the vectors of the set $\{ | u_k \rangle \}$ to happen, *i.e.* for jumps like $| \Phi \rangle \rightarrow | u_1 \rangle$, or $| \Phi \rangle \rightarrow | u_2 \rangle$ etc., to occur.

Comments: (i) By definition, A is a constant of the motion if it satisfies conditions (44) and (45). (ii) Postulate III ensures the statistical sense of the conservation of the physical quantity A. [28]

Hypothesis: A system in the state $| \Phi \rangle$ has tendency to jump to the eigenstates of its constants of the motion.

Comments: (iii) This tendency should not become actualized if the projections it induces results in a violation of Postulate III or lead the state vector outside the Hilbert space. (iv) Taking into account this Hypothesis and Postulate III, the concept of *preferential states* is introduced; see [28, 61]. For simplicity, instead of dealing with the general case, here we shall refer to the following one: Let H, A and B be three operators representing respectively the energy, the physical quantity A and the physical quantity B of the system. It will be assumed that they have discrete spectra and satisfy (44), and that $\{H, A, B\}$ is the *unique* complete set of compatible operators of the system. The vectors of its common basis will be denoted by $| E_p, a_q, b_r \rangle$, where E_p, a_q and b_r are respectively the eigenvalues of H, A and B. (v) On the one hand, taking into

account the previous hypothesis, we can say that the system's state $|\Phi$ has tendency to be projected to the eigenvectors of H, to the eigenvectors of A and to the eigenvectors of B. On the other hand, as the relations

$$\langle\Phi|H|\Phi\rangle = \sum_{p,q,r}|c_{p,q,r}|^2\langle E_p,a_q,b_r|H|E_p,a_q,b_r\rangle \tag{48}$$

$$\langle\Phi|A|\Phi\rangle = \sum_{p,q,r}|c_{p,q,r}|^2\langle E_p,a_q,b_r|A|E_p,a_q,b_r\rangle \tag{49}$$

and

$$\langle\Phi|B|\Phi\rangle = \sum_{p,q,r}|c_{p,q,r}|^2\langle E_p,a_q,b_r|B|E_p,a_q,b_r\rangle \tag{50}$$

are satisfied for the state $|\Phi\rangle = \sum_{p,q,r} c_{p,q,r}|E_p, a_q, b_r\rangle$, Postulate III does not prohibit projections like $|\Phi\rangle \rightarrow |E_p, a_q, b_r\rangle$. Then we state:

Definition: The preferential states of the system are the common eigenstates of H, A and B.

Comment: (vi) The previous definition is valid in cases restrictions established in Comment (iii) are fulfilled; in this particular case the preferential states do not depend on $|\Phi\rangle$. Cases where $[A, B] \neq 0$ or where the spectrum of H is partially continuous, have been analyzed in [28, 61]. In these last cases the preferential states depend on $|\Phi\rangle$. (vii) As we have assumed that $\{H, A, B\}$ is the unique complete set of compatible operators of the system, the set of preferential states $\{|E_p, a_q, b_r\rangle\}$ is necessarily unique. *The condition of uniqueness of the set of preferential states remains valid in the general case.* [28, 61]

Postulate IV: The system's state $|\Phi\rangle$ can be projected to the state $|u_j\rangle$ if and only if $|u_j\rangle$ is a preferential state. If the system in the state $|\Phi\rangle$ does not have preferential states, *the Schrödinger evolution must follow.*

Postulate V: Let $|u_k\rangle (k = 1, 2, \cdots)$ be the preferential states of the system in the state

$$|\Phi(t)\rangle = \sum_k c_k(t)|u_k(t)\rangle \tag{51}$$

where $c_k(t) = \langle u_k(t)|\Phi(t)\rangle$. In the small interval $(t, t + dt)$ the system's state can undergo the following changes:

$$\left|\Phi(t)\right\rangle \to \left|\Phi(t+dt)\right\rangle = \left|u_j(t)\right\rangle \tag{52}$$

with probability $dP_j(t) = |c_j(t)|^2(dt/\tau)$; or

$$\left|\Phi(t)\right\rangle \to \left|\Phi_{Sch}(t+dt)\right\rangle = U(t+dt,t)\left|\Phi(t)\right\rangle \tag{53}$$

with probability $dP_{Sch}(t) = 1 - dt/\tau$, where $U(t+dt, t)$ is the evolution operator,

$$\tau \Delta H = \hbar/2 \tag{54}$$

and

$$(\Delta H)^2 = \left\langle \Phi(t)\left|H^2\right|\Phi(t)\right\rangle - \left\langle \Phi(t)\left|H\right|\Phi(t)\right\rangle \tag{55}$$

Comments: (viii) The change given by (53) is a Schrödinger evolution and those given by (52) are projections to the preferential states of the system in the state $|\Phi(t)\rangle$ (ix) Since preferential states are members of an orthonormal set of vectors, a system's state projected to a preferential state remains there evolving in agreement with the Schrödinger equation. (x) The state $|\Phi_{Sch}(t)\rangle$ may be considered as un unstable state that can decay to one of the preferential states $|u_j(t)\rangle$, the relaxation time being τ. Calling $P_{Sch}(t)$ to the probability that the system's state has not been projected to any preferential state in the interval (0, t) the well-known exponential decay law is obtained; see [28, 61].

13. The ideal measurement scheme in the framework of SPA

Let us start this section with the question: "what can be observed?" In his answer Bell quotes Einstein saying "it is theory which decides what is 'observable'." He adds: "I think he was right – 'observation' is a complicated and theory-laden business." [1] We agree with these assertions.

Consider, for instance, the determination of the energy levels of the Hg atom in the Franck-Hertz experiment, where a curve of electrical current versus the applied voltage is obtained; this curve presents peaks of the current at regular intervals of voltage. [62] Relating the values of the voltage where the peaks are located to the first excited energy level of the atom requires a quite elaborate theory of what is happening inside the tube. But once the way the device works has been understood, the Franck-Hertz experiment provides a *direct* measurement of the energy difference between the quantum states of the atom: it appears on the dial of a voltmeter! It is worth stressing that no entanglement is invoked in the analysis of this experi-

ment and the same is true of many others related, *e.g.*, to blackbody radiation, photoelectric effect and Compton shift. By contrast, *in the ideal measurement scheme entanglements are unavoidable.* This is for instance the case of the system *photon* meeting the device tourmaline crystal mentioned by Dirac; see Section 7.1.

In the following we shall address the conceptual problem of the ideal measurement scheme in the framework of SPA. We are going to analyze the measurement of the physical quantity A_S pertaining to the system S; for simplicity we shall deal with the discrete non-degenerate case. Let $a_k (k = 1, 2, \cdots N)$ be an eigenvalue of the operator A_S representing A_S and $|a_k\rangle$ the corresponding eigenvector. The operator A_S acts in the Hilbert space H_S of S and its extension $A = A_S \otimes I_M$ (here I_M is the identity operator in the Hilbert space H_M of M) acts in the Hilbert space H_{S+M} of S+M. The Hamiltonian of the total system S+M will be denoted by H, the operator B will represent a physical quantity B referred to S+M, the initial state of the measuring device M of A_S will be denoted by $|m_0\rangle$, and the state of the total system S+M at time t by $|\Phi(t)\rangle$.

In a first step we shall suppose that at t_0, when the interaction between S and M starts, the state of S is $|a_k\rangle$ and that of S+M is

$$\left|\Phi_k\left(t_0\right)\right\rangle = \left|a_k\right\rangle\left|m_0\right\rangle \tag{56}$$

It is easily verified that the state $|\Phi_k(t_0)\rangle$ is an eigenstate of A corresponding to the non-degenerate eigenvalue a_k. In addition, if [A, B]=0 and H, A, and B are constants of the motion, the state $|\Phi_k(t_0)\rangle$ will be a common eigenstate of these three operators. Hence, if {H, A, B} is the unique complete set of compatible operators of the system, according to SPA the state $|\Phi_k(t_0)\rangle$ will be a preferential state of the system S+M. As we have already pointed out, it must remain evolving in agreement with the Schrödinger equation (see previous section). So at time t the state of S+M will be

$$\left|\Phi\left(t\right)\right\rangle = \left|\Phi_k\left(t\right)\right\rangle = U\left(t, t_0\right)\left|\Phi_k\left(t_0\right)\right\rangle \tag{57}$$

as it happens in the traditional treatment of the ideal measurement scheme.

Note that if H, A, and B are constants of the motion but [A, B]≠0, the operators A and B do not have a common basis. In this case it has been shown that collapses to the basis of the eigenvectors common to H and A violate the statistical sense of the conservation of B, hence Postulate III of SPA prevents these projections; in the same way it is concluded that collapses to the eigenvectors common to H and B are forbidden [28]: as stated in the traditional treatment, for the ideal measurement scheme to be valid, the measured physical quantity must be compatible with every conserved quantity referred to S+M.

Now we shall consider the case where the initial state of S is $\sum_{k=1}^{N} c_k \mid a_k \rangle$. The initial state of S+M will be

$$\left| \Phi\left(t_0\right) \right\rangle = \sum_{k=1}^{N} c_k \left| a_k \right\rangle \left| m_0 \right\rangle = \sum_{k=1}^{N} c_k \left| \Phi_k\left(t_0\right) \right\rangle \tag{58}$$

Postulate V of SPA tells us that at $t > t_0$ the state of S+M can be

one of its N preferential states $\mid \Phi_k(t) \rangle\rangle$, in case in the interval (t_0, t) the state of S+M has been projected; or

$\mid \Phi_{Sch}(t) \rangle = U(t, t_0) \mid \Phi(t_0) \rangle$, in case in the interval (t_0, t) the state of S+M has not been projected and hence its behavior has been ruled by the Schrödinger equation.

Which one of these (N+1) states will result at time t cannot be predicted, but each one of them has an associated probability given by Postulate V. In case $t \gg \tau$, the relaxation time given by (54), the probability the system has to remain in the state $\mid \Phi_{Sch}(t) \rangle$ goes to zero and all we can "observe" is the result corresponding to one of the preferential states onto which the system can decay.

14. Conclusions

OQM formalism includes two different laws: a strictly continuous and causal Schrödinger evolution which governs spontaneous processes and the Projection Postulate, a rule implying discontinuities and changes of the state vector in agreement with probability laws. On the one hand, the inclusion in the formalism of two laws irreducible to one another has been a source of dissatisfaction from quantum mechanics birth. On the other hand, OQM (which includes the Projection Postulate) has been extremely successful in the area of experimental predictions; and *even if the Projection Postulate should be applied only in cases where measurements are performed, in the present work we have shown that it is also used ad-hoc, when needed to explain processes which supposedly are spontaneous.*

Some authors have suggested that measurement processes could be a particular kind of natural processes. But, then, we confront the problem pointed out by Bell [60]: there is nothing in OQM formalism to tell which natural processes have the special status of measurements, *i.e.* to decide whether one or the other law rules the process.

Looking for a solution to this problem, we have proposed a Spontaneous Projection Approach (SPA) to quantum mechanics, a theory where *spontaneous and measurement processes are treated on the same footing* and the behavior of macroscopic and microscopic objects are ruled by the same laws. The first step to achieve this objective is to admit that *projections can occur spontaneously in nature, even in closed systems, without being acted by any external perturbation.* But, then, the theory must say in which situations and to which vectors the state vector can collapse, and

which are the corresponding probabilities. These goals have been achieved in the framework of SPA.

It is worth stressing that our approach does not modify OQM in a substantial way: it does not change the Schrödinger equation and it recovers a version of Born postulate where no reference to measurements is made. So, in general its predictions coincide with those of OQM.

Concerning the treatment of the ideal measurement scheme in the framework of SPA, we are aware of its limitations derived, among other reasons, from the hypotheses introduced "for simplicity." For instance, we have considered that there are only three relevant physical quantities referred to the total system (which includes the measuring apparatus), that the operators which represent them are constants of the motion, and that the physical quantity to be measured is represented by an operator having discrete non-degenerate spectrum. Our treatment, however, has the merit of predicting results which completely agree with those obtained in the framework of OQM, without having recourse to the observer consciousness, to the macroscopic character of the measurement device, or to interactions with the environment producing decoherence, something that in the long term looks and smells like a collapse. We should also stress that in other theories such us that due to Ghirardi, Rimini and Weber, even when most of the wave function goes to the component corresponding to one single outcome of an experiment, there always remains a 'tail', *i.e.* a tiny component of the system's state on the others. By contrast, in SPA the system's state either evolves according to the Schrödinger equation, or is at once entirely projected into one of its preferential states.

To end this chapter let us highlight the most important differences between SPA and OQM:

i.   SPA is compatible with epistemological realism.

ii.   In SPA projections occurring in spontaneous processes such as those involved in radioactivity, interactions between light and matter, etc., are not surreptitiously but explicitly included. In this sense it could be said that SPA enjoys of a coherence which is absent from OQM.

iii. Differing from OQM, SPA yields an expression for the probability of transitions to the continuum which is valid for every time and, except for some minimal restrictions, for every added potential. We have pointed out in [61] that these predictions could be experimentally tested.

Theories which include only deterministic laws in their formalism can give an account for nothing but "automatic changes." On the contrary, by including probabilistic laws in its formalism, SPA opens the door to novelty.

Acknowledgements

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