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# Optimal Allocation of Reliability in Series Parallel Production System

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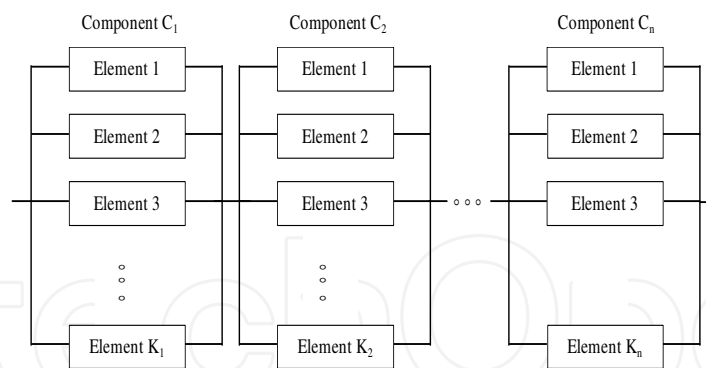
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## 1. Introduction

One of the most important problems in many industrial applications is the redundancy optimization problem. This latter is well known combinatorial optimization problem where the design goal is achieved by discrete choices made from elements available on the market. The natural objective function is to find the minimal cost configuration of a series-parallel system under availability constraints. The system is considered to have a range of performance levels from perfect working to total failure. In this case the system is called a *multi-state* system (MSS). Let consider a multi-state system containing  $n$  components  $C_i$  ( $i = 1, 2, \dots, n$ ) in series arrangement. For each component  $C_i$  there are various versions, which are proposed by the suppliers on the market. Elements are characterized by their cost, performance and availability according to their version. For example, these elements can represent machines in a manufacturing system to accomplish a task on product in our case they represent the whole of electrical power system (generating units, transformers and electric carrying lines devices). Each component  $C_i$  contains a number of elements connected in parallel. Different versions of elements may be chosen for any given system component. Each component can contain elements of *different* versions as sketched in figure 1.

A limitation can be undesirable or even unacceptable, where only identical elements are used in parallel (i.e. homogeneous system) for two reasons. First, by allowing different versions of the devices to be allocated in the same system, one can obtain a solution that provides the desired availability or reliability level with a lower cost than in the solution with identical parallel devices. Second, in practice the designer often has to include additional devices in the existing system. It may be necessary, for example, to modernize a production line system according to a new demand levels from customers or according to new reliability requirements.



**Figure 1.** Series Parallel Production System

### 1.1. Literature review

The vast majority of classical reliability or availability analysis and optimization assume that components and system are in either of *two* states (i.e., complete working state and total failure state). However, in many real life situations we are actually able to distinguish among various levels of performance for both system and components. For such situation, the existing dichotomous model is a gross oversimplification and so models assuming multi-state (degradable) systems and components are preferable since they are closer to reality. Recently much works treat the more sophisticated and more realistic models in which systems and components may assume many states ranging from perfect functioning to complete failure. In this case, it is important to develop MSS reliability theory. In this paper, an MSS reliability theory will be used, where the binary state system theory is extending to the multi-state case. As is addressed in recent review of the literature for example in (Ushakov, Levitin and Lisnianski, 2002) or (Levitin and Lisnianski, 2001). Generally, the methods of MSS reliability assessment are based on four different approaches:

- i. The structure function approach.
- ii. The stochastic process (mainly Markov) approach.
- iii. The Monte-Carlo simulation technique.
- iv. The universal moment generating function (UMGF) approach.

In (Ushakov, Levitin and Lisnianski, 2002), a comparison between these four approaches highlights that the UGF approach is fast enough to be used in the optimization problems where the search space is sizeable.

The problem of total investment-cost minimization, subject to reliability or availability constraints, is well known as the redundancy optimization problem (ROP). The ROP is studied in many different forms as summarized in (Tillman, Hwang and Kuo, 1977), and more recently in (Kuo and Prasad, 2000). The ROP for the multi-state reliability was introduced in (Ushakov, 1987). In (Lisnianski, Levitin, Ben-Haim and Elmakis, 1996) and (Levitin, Lisnianski, Ben-Haim and Elmakis, 1997), genetic algorithms were used to find the optimal or nearly optimal power system structure.

This work uses an *ant colony* optimization approach to solve the ROP for multi-state system. The idea of employing a colony of cooperating agents to solve combinatorial optimization problems was recently proposed in (Dorigo, Maniezzo and Colorni, 1996). The ant colony approach has been successfully applied to the classical traveling salesman problem (Dorigo and Gambardella, 1997), and to the quadratic assignment problem (Maniezzo and Colorni, 1999). Ant colony shows very good results in each applied area. It has been recently adapted for the reliability design of binary state systems (Liang and Smith, 2001). The ant colony has also been adapted with success to other combinatorial optimization problems such as the vehicle routing problem (Bullnheimer, Hartl and Strauss, 1997). The ant colony method has been used to solving the redundancy allocation problem (Nahas N., Nourelfath M., Aït-Kadi Daoud, 2006).

In this paper, we extend the work of other researchers by proposing ant colony system algorithm to solve the ROP characterised in the problem of optimization of the structure of power system where redundant elements are included in order to provide a desired level of reliability through optimal allocation of elements with different parameters (optimal structure with series-parallel elements) in continuous production system.

The use of this algorithm is within a general framework for the comparative and structural study of metaheuristics. In a first step the application of ant colonies in its primal form is necessary and thereafter in perspective the study will be completed.

## 1.2. Approach and outlines

The problem formulated in this chapter lead to a complicated combinatorial optimization problem. The total number of different solution to be examined is very large, even for rather small problems. An exhaustive examination of all possible solutions is not feasible given reasonable time limitations. Because of this, the ant colony optimization (or simply ACO) approach is adapted to find optimal or nearly optimal solutions to be obtained in a short time. The newer developed meta-heuristic method has the advantage to solve the ROP for MSS *without* the limitation on the diversity of versions of elements in parallel. Ant colony optimization is inspired by the behavior of real ant colonies that exhibit the highly structured behavior. Ants lay down in some quantity an aromatic substance, known as *pheromone*, in their way to food. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone is laid down by others ants, therefore the best path has more intensive pheromone and higher probability to be chosen.

During the optimization process, artificial ants will have to evaluate the availability of a given selected structure of the series-parallel system (electrical network). To do this, a fast procedure of availability estimation is developed. This procedure is based on a modern mathematical technique: the z-transform or UMGF which was introduced in (Ushakov, 1986). It was proven to be very effective for high dimension combinatorial problems: see e.g. (Ushakov, 2002), (Levitin, 2001). The universal moment generating function is an extension of the ordinary moment generating function (UGF) (Ross, 1993). The method developed in this chapter allows the availability function of repairable series-parallel MSS to be obtained using a straightforward numerical procedure.

## 2. Formulation of redundancy optimization problem

### 2.1. Series-parallel system with different redundant elements

Let consider a series-parallel system containing  $n$  subcomponents  $C_i$  ( $i = 1, 2, \dots, n$ ) in series as represented in figure 1. Every component  $C_i$  contains a number of different elements connected in parallel. For each component  $i$ , there are a number of element versions available in the market. For any given system component, different versions and number of elements may be chosen. For each subcomponent  $i$ , elements are characterized according to their version  $v$  by their cost ( $C_{iv}$ ), availability ( $A_{iv}$ ) and performance ( $\Sigma_{iv}$ ). The structure of system component  $i$  can be defined by the numbers of parallel elements (of each version)  $k_{iv}$  for  $1 \leq v \leq V_i$ , where  $V_i$  is a number of versions available for element of type  $i$ . Figure 2 illustrates these notations for a given component  $i$ . The entire system structure is defined by the vectors  $\mathbf{k}_i = \{k_{iv}\}$  ( $1 \leq i \leq n$ ,  $1 \leq v \leq V_i$ ). For a given set of vectors  $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$  the total cost of the system can be calculated as:

$$C = \sum_{i=1}^n \sum_{v=1}^{V_i} k_{iv} C_{iv} \quad (1)$$

### 2.2. Availability of reparable multi-state systems

The series-parallel system is composed of a number of failure prone elements, such that the failure of some elements leads only to a degradation of the system performance. This system is considered to have a range of performance levels from perfect working to complete failure. In fact, the system failure can lead to decreased capability to accomplish a given task, but not to complete failure. An important MSS measure is related to the ability of the system to satisfy a given demand.

In electric power systems, reliability is considered as a measure of the ability of the system to meet the load demand ( $D$ ), i.e., to provide an adequate supply of electrical energy ( $\Sigma$ ). This definition of the reliability index is widely used in power systems: see e.g., (Ross, 1993), (Murchland, 1975), (Levitin, Lisnianski, Ben-Haim and Elmakis, 1998), (Lisnianski, Levitin, Ben-Haim and Elmakis, 1996), (Levitin, Lisnianski, and Elmakis, 1997). The Loss of Load Probability index (LOLP) is usually used to estimate the reliability index (Billinton and Allan, 1990). This index is the overall probability that the load demand will not be met. Thus, we can write  $R = \text{Probab}(\Sigma \geq D)$  or  $R = 1 - \text{LOLP}$  with  $\text{LOLP} = \text{Probab}(\Sigma < D)$ . This reliability index depends on consumer demand  $D$ .

For reparable MSS, a multi-state steady-state availability  $E$  is used as  $\text{Probab}(\Sigma \geq D)$  after enough time has passed for this probability to become constant (Levitin, Lisnianski, Ben-Haim and Elmakis, 1998). In the steady-state the distribution of states probabilities is given by equation (2), while the multi-state stationary availability is formulated by equation (3):

$$P_j = \lim_{t \rightarrow \infty} [\text{Pr obab}(\Sigma(t) = \Sigma_j)] \quad (2)$$

$$E = \sum_{\Sigma_j \geq D} P_j \quad (3)$$

If the operation period  $T$  is divided into  $M$  intervals (with durations  $T_1, T_2, \dots, T_M$ ) and each interval has a required demand level ( $D_1, D_2, \dots, D_M$ , respectively), then the generalized MSS availability index  $A$  is:

$$A = \frac{1}{\sum_{j=1}^M T_j} \sum_{j=1}^M \text{Pr obab}(\Sigma \geq D_j) T_j \quad (4)$$

We denote by  $D$  and  $T$  the vectors  $\{D_j\}$  and  $\{T_j\}$  ( $1 \leq j \leq M$ ), respectively. As the availability  $A$  is a function of  $k_1, k_2, \dots, k_n, D$  and  $T$ , it will be written  $A(k_1, k_2, \dots, k_n, D, T)$ . In the case of a power system, the vectors  $D$  and  $T$  define the cumulative load curve (consumer demand). In reality the load curves varies randomly; an approximation is used from random curve to discrete curve see (Wood and Ringlee, 1970). In general, this curve is known for every power system.

### 2.3. Optimal design problem formulation

The multi-state system redundancy optimization problem of electrical power system can be formulated as follows: find the minimal cost system configuration  $k_1, k_2, \dots, k_n$ , such that the corresponding availability exceeds or equal the specified availability  $A_0$ . That is,

$$\text{Minimize } C = \sum_{i=1}^n \sum_{v=1}^{V_i} k_{iv} C_{iv} \quad (5)$$

$$\text{subject to } A(k_1, k_2, \dots, k_n, D, T) \geq A_0 \quad (6)$$

The input of this problem is the specified availability and the outputs are the minimal investment-cost and the corresponding configuration determined. To solve this combinatorial optimization problem, it is important to have an effective and fast procedure to evaluate the availability index for a series-parallel system of elements. Thus, a method is developed in the next section to estimate the value of  $A(k_1, k_2, \dots, k_n, D, T)$ .



### 3. Multi-state system availability estimation

The procedure used in this chapter is based on the universal  $z$ -transform, which is a modern mathematical technique introduced in (Ushakov, 1986). This method, convenient for numerical implementation, is proved to be very effective for high dimension combinatorial problems. In the literature, the universal  $z$ -transform is also called universal moment generating function (UMGF) or simply  $u$ -function or  $u$ -transform. In this chapter, we mainly use the acronym UMGF. The UMGF extends the widely known ordinary moment generating function (Ross, 1993).

#### 3.1. Definition and properties

The UMGF of a discrete random variable  $\Sigma$  is defined as a polynomial:

$$u(z) = \sum_{j=1}^J P_j z^{\Sigma_j} \quad (7)$$

where the variable  $\Sigma$  has  $J$  possible values and  $P_j$  is the probability that  $\Sigma$  is equal to  $\Sigma_j$ .

The probabilistic characteristics of the random variable  $\Sigma$  can be found using the function  $u(z)$ . In particular, if the discrete random variable  $\Sigma$  is the MSS stationary output performance, the availability  $E$  is given by the probability  $Probab(\Sigma \geq D)$  which can be defined as follows:

$$Probab(\Sigma \geq D) = \Psi(u(z)z^{-D}) \quad (8)$$

where  $\Psi$  is a distributive operator defined by expressions (9) and (10):

$$\Psi(Pz^{\sigma-D}) = \begin{cases} P, & \text{if } \sigma \geq D \\ 0, & \text{if } \sigma < D \end{cases} \quad (9)$$

$$\Psi\left(\sum_{j=1}^J P_j z^{\Sigma_j-D}\right) = \sum_{j=1}^J \Psi\left(P_j z^{\Sigma_j-D}\right) \quad (10)$$

It can be easily shown that equations (7) – (10) meet condition  $Probab(\Sigma \geq D) = \sum_{\Sigma_j \geq D} P_j$ . By using the operator  $\Psi$ , the coefficients of polynomial  $u(z)$  are summed for every term with  $\Sigma_j \geq D$ , and the probability that  $\Sigma$  is not less than some arbitrary value  $D$  is systematically obtained.

Consider single elements with total failures and each element  $i$  has nominal performance  $\Sigma_i$  and availability  $A_i$ . Then,  $Probab(\Sigma = \Sigma_i) = A_i$  and  $Probab(\Sigma = 0) = 1 - A_i$ . The UMGF of such an element has only two terms and can be defined as:

$$u_i = (1 - A_i)z^0 + A_i z^{\Sigma_i} = (1 - A_i) + A_i z^{\Sigma_i} \quad (11)$$

To evaluate the MSS availability of a series-parallel system, two basic composition operators are introduced. These operators determine the polynomial  $u(z)$  for a group of elements.

### 3.2. Composition operators

#### 3.2.1. Properties of the operators

The essential property of the UMGF is that it allows the total UMGF for a system of elements connected in parallel or in series to be obtained using simple algebraic operations on the individual UMGF of elements. These operations may be defined according to the physical nature of the elements and their interactions. The only limitation on such an arbitrary operation is that its operator  $\phi$  should satisfy the following Ushakov's conditions (Ushakov, 1986):

$$\begin{aligned} \phi(p_1 z^{g_1}, p_2 z^{g_2}) &= p_1 p_2 z^{\phi(g_1, g_2)}, \\ \phi(g) &= g, \\ \phi(g_1, \dots, g_n) &= \phi(\phi(g_1, \dots, g_k), \phi(g_{k+1}, \dots, g_n)), \\ \phi(g_1, \dots, g_k, g_{k+1}, \dots, g_n) &= \phi(g_1, \dots, g_k, g_{k+1}, \dots, g_n) \text{ for any } k. \end{aligned}$$

#### 3.2.2. Parallel elements

Let consider a system component  $m$  containing  $J_m$  elements connected in parallel. As the performance measure is related to the system productivity, the total performance of the parallel system is the *sum* of performances of all its elements. In power systems engineering, the term capacity is usually used to indicate the quantitative performance measure of an element (Lisnianski, Levitin, Ben-Haim and Elmakis, 1996). It may have different physical nature. Examples of elements capacities are: generating capacity for a generator, pipe capacity for a water circulator, carrying capacity for an electric transmission line, etc. The capacity of an element can be measured as a percentage of nominal total system capacity. In a manufacturing system, elements are machines. Therefore, the total performance of the parallel machine is the sum of performances (Dallery and Gershwin, 1992).

The  $u$ -function of MSS component  $m$  containing  $J_m$  parallel elements can be calculated by using the  $\Gamma$  operator:

$$u_p(z) = \Gamma(u_1(z), u_2(z), \dots, u_n(z)), \text{ where } \Gamma(g_1, g_2, \dots, g_n) = \sum_{i=1}^n g_i.$$

Therefore for a pair of elements connected in parallel:



$$\Gamma(u_1(z), u_2(z)) = \Gamma\left(\sum_{i=1}^n P_i z^{a_i}, \sum_{j=1}^m Q_j z^{b_j}\right) = \sum_{i=1}^n \sum_{j=1}^m P_i Q_j z^{a_i + b_j}.$$

Parameters  $a_i$  and  $b_j$  are physically interpreted as the respective performances of the two elements.  $n$  and  $m$  are numbers of possible performance levels for these elements.  $P_i$  and  $Q_j$  are steady-state probabilities of possible performance levels for elements.

One can see that the  $\Gamma$  operator is simply a product of the individual  $u$ -functions. Thus, the component UMGF is:

$$u_p(z) = \prod_{j=1}^{J_m} u_j(z).$$

Given the individual UMGF of elements defined in equation (11), we have:

$$u_p(z) = \prod_{j=1}^{J_m} (1 - A_j + A_j z^{\Sigma_i}).$$

### 3.2.3. Series elements

When the elements are connected in series, the element with the least performance becomes the bottleneck of the system. This element therefore defines the total system productivity. To calculate the  $u$ -function for system containing  $n$  elements connected in series, the operator  $\eta$  should be used:  $u_s(z) = \eta(u_1(z), u_2(z), \dots, u_m(z))$ , where  $\eta(g_1, g_2, \dots, g_m) = \min\{g_1, g_2, \dots, g_m\}$

so that

$$\eta(u_1(z), u_2(z)) = \eta\left(\sum_{i=1}^n P_i z^{a_i}, \sum_{j=1}^m Q_j z^{b_j}\right) = \sum_{i=1}^n \sum_{j=1}^m P_i Q_j z^{\min\{a_i, b_j\}}$$

Applying composition operators  $\Gamma$  and  $\eta$  consecutively, one can obtain the UMGF of the entire series-parallel system.

## 4. The ant colony optimization approach

The problem formulated in this chapter is a complicated combinatorial optimization problem. The total number of different solutions to be examined is very large, even for rather small problems. An exhaustive examination of the enormous number of possible solutions is not feasible given reasonable time limitations. Thus, because of the search space size of the ROP for MSS, a new meta-heuristic is developed in this section. This meta-heuristic consists in an adaptation of the ant colony optimization method.

### 4.1. The ACO principle

Recently, (Dorigo, Maniezzo and Coloni, 1996) introduced a new approach to optimization problems derived from the study of any colonies, called "Ant System". Their system inspired

by the work of real ant colonies that exhibit the highly structured behavior. Ants lay down in some quantity an aromatic substance, known as pheromone, in their way to food. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone is laid down by others ants, therefore the best paths have more intensive pheromone and higher probability to be chosen. This simple behavior explains why ants are able to adjust to changes in the environment, such as new obstacles interrupting the currently shortest path.

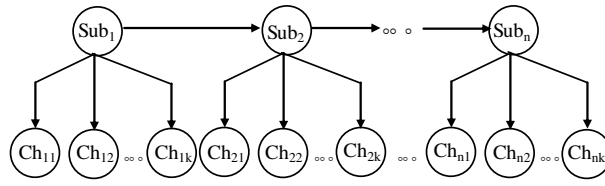
Artificial ants used in ant system are agents with very simple basic capabilities mimic the behavior of real ants to some extent. This approach provides algorithms called ant algorithms. The Ant System approach associates pheromone trails to features of the solutions of a combinatorial problem, which can be seen as a kind of adaptive memory of the previous solutions. Solutions are iteratively constructed in a randomized heuristic fashion biased by the pheromone trails, left by the previous ants. The pheromone trails,  $\tau_{ij}$ , are updated after the construction of a solution, enforcing that the best features will have a more intensive pheromone. An Ant algorithm presents the following characteristics. It is a natural algorithm since it is based on the behavior of ants in establishing paths from their colony to feeding sources and back. It is parallel and distributed since it concerns a population of agents moving simultaneously, independently and without supervisor. It is cooperative since each agent chooses a path on the basis of the information, pheromone trails, laid by the other agents with have previously selected the same path. It is versatile that can be applied to similar versions the same problem. It is robust that it can be applied with minimal changes to other combinatorial optimization problems. The solution of the travelling salesman problem (TSP) was one of the first applications of ACO.

Various extensions to the basic TSP algorithm were proposed, notably by (Dorigo and Gambardella, 1997a). The improvements include three main aspects: the state transition rule provides a direct way to balance between exploration of new edges and exploitation of a priori and accumulated knowledge about the problem, the global updating rule is applied only to edges which belong to the best ant tour and while ants construct solution, a local pheromone updating rule is applied. These extensions have been included in the algorithm proposed in this paper.

#### 4.2. ACO-based solution approach

In our reliability optimization problem, we have to select the best combination of parts to minimize the total cost given a reliability constraint. The parts can be chosen in any combination from the available components. Components are characterized by their reliability, capacity and cost. This problem can be represented by a graph (figure 2) in which the set of nodes comprises the set of subsystems and the set of available components (i.e.  $\max(M_j), j = 1..n$ ) with a set of connections partially connect the graph (i.e. each subsystem is connected only to its available components). An additional node (blank node) is connected to each subsystem.

In figure 2, a series-parallel system is illustrated where the first and the second subsystem are connected respectively to their 3 and 2 available components. The nodes  $cp_{i3}$  and  $cp_{i4}$ , represent



**Figure 2.** Definition of a series-parallel system with tree subsystems into a graph

the blank components of the two subsystems. At each step of the construction process, an ant uses problem-specific heuristic information, denoted by  $\eta_{ij}$  to choose the optimal number of components in each subsystem. An imaginary heuristic information is associated to each blank node. These new factors allow us to limit the search surfaces (i.e. tuning factors). An ant positioned on subsystem  $i$  chooses a component  $j$  by applying the rule given by:

$$j = \begin{cases} \arg \max_{m \in AC_i} ([\tau_{im}]^\alpha [\eta_{im}]^\beta) & \text{if } q \leq q_o \\ J & \text{if } q > q_o \end{cases} \quad (12)$$

and  $J$  is chosen according to the probability:

$$p_{ij} = \begin{cases} \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{m \in AC_i} [\tau_{im}]^\alpha [\eta_{im}]^\beta} & \text{if } j \in AC_i \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

$\alpha$  : The relative importance of the trail.

$\beta$  : The relative importance of the heuristic information  $\eta_{ij}$ .

$AC_i$ : The set of available components choices for subsystem  $i$ .

$q$ : Random number uniformly generated between 0 and 1.

The heuristic information used is :  $\eta_{ij} = 1/(1+c_{ij})$  where  $c_{ij}$  represents the associated cost of component  $j$  for subsystem  $i$ . A “tuning” factor  $t_i = \eta_{ij} = 1/(1+c_{i(M_i+1)})$  is associated to blank component  $(M_i+1)$  of subsystem  $i$ . The parameter  $q_o$  determines the relative importance of exploitation versus exploration: every time an ant in subsystem  $i$  have to choose a component  $j$ , it samples a random number  $0 \leq q \leq 1$ . If  $q \leq q_o$  then the best edge, is chosen (exploitation), otherwise an edge is chosen according to (12) (biased exploration).

The pheromone update consists of two phases: local and global updating. While building a solution of the problem, ants choose components and change the pheromone level on subsystem-component edges. This local trail update is introduced to avoid premature convergence

and effects a temporary reduction in the quantity of pheromone for a given subsystem-component edge so as to discourage the next ant from choosing the same component during the same cycle. The local updating is given by:

$$\tau_{ij}^{new} = (1 - \rho)\tau_{ij}^{old} + \rho\tau_o \quad (14)$$

where  $\rho$  is a coefficient such that  $(1-\rho)$  represents the evaporation of trail and  $\tau_o$  is an initial value of trail intensity. It is initialized to the value  $(n.TC_m)^{-1}$  with  $n$  is the size of the problem (i.e. number of subsystem and total number of available components) and  $TC_m$  is the result of a solution obtained through some simple heuristic.

After all ants have constructed a complete system, the pheromone trail is then updated at the end of a cycle (i.e. global updating), but only for the best solution found. This choice, together with the use of the pseudo-random-proportional rule, is intended to make the search more directed: ants search in a neighbourhood of the best solution found up to the current iteration of the algorithm. The pheromone level is updated by applying the following global updating rule:

$$\tau_{ij}^{new} = (1 - \rho)\tau_{ij}^{old} + \rho\Delta\tau_{ij}$$

$$\Delta\tau_{ij} = \begin{cases} \frac{1}{TC_{best}} & \text{if } (i, j) \in \text{best tour} \\ 0 & \text{otherwise} \end{cases}$$

### 4.3. The algorithm

An ant-cycle algorithm is stated as follows. At time zero an initialization phase takes place during which  $NbAnt$  ants select components in each subsystem according to the Pseudo-random-proportional transition rule. When an ant selects a component, a local update is made to the trail for that subsystem-component edge according to equation (13). In this equation,  $\rho$  is a parameter that determines the rate of reduction of the pheromone level. The pheromone reduction is small but sufficient to lower the attractiveness of precedent subsystem-component edge. At the end of a cycle, for each ant  $k$ , the value of the system's reliability  $R_k$  and the total cost  $TC_k$  are computed. The best feasible solution found by ants (i.e. total cost and assignments) is saved. The pheromone trail is then updated for the best solution obtained according to (13). This process is iterated until the tour counter reaches the maximum number of cycles  $NC_{max}$  or all ants make the same tour (stagnation behavior).

The followings are formal description of the algorithm.

1. Set  $NC:=0$  ( $NC$ : cycle counter)  
 For every edge  $(i,j)$  set an initial value  $\tau_{ij}(0)=\tau_o$
2. For  $k=1$  to  $NbAnt$  do

For i=1 to NbSubSystem do

For j=1 to MaxComponents do

Choose a component, including blanks, according to (1) and (2).

Local update of pheromone trail for chosen subsystem- component edge (i,j) :

$$\tau_{ij}^{new} = (1 - \rho)\tau_{ij}^{old} + \rho\tau_o$$

End For

End For

3. Calculate  $R_k$  (system reliability for each ant)

Calculate the total cost for each ant  $TC_k$

Update the best found feasible solution

4. Global update of pheromone trail:

For each edge  $(i,j) \in$  best feasible solution, update the pheromone trail according to:

$$\tau_{ij}^{new} = (1 - \rho)\tau_{ij}^{old} + \rho\Delta\tau_{ij}$$

$$\Delta\tau_{ij} = \begin{cases} \frac{1}{TC_{best}} & \text{if } (i, j) \in \text{best tour} \\ 0 & \text{otherwise} \end{cases}$$

End For

5. cycle=cycle +1

6. if  $(NC < NC_{max})$  and ( not stagnation behavior)

Then

Goto step 2

Else

Print the best feasible solution and components selection.

Stop.

## 5. Illustrative example

### Description of the system to be optimized

The power station coal transportation system which supplies the boilers is designed with five basic components as depicted in figure.3.

The process of coal transportation is: The coal is loaded from the bin to the primary conveyor (Conveyor 1) by the primary feeder (Feeder 1). Then the coal is transported through the conveyor 1 to the Stacker-reclaimer, when it is left up to the burner level. The secondary feeder (Feeder 2) loads the secondary conveyor (Conveyor 2) which supplies the burner feeding system of the boiler. Each element of the system is considered as unit with total failures.



Figure 3. Synoptic of the detailed power station coal transportation

Comp#	Vers#	Availability A	Cost C	Capacity $\Xi$
1	1	0.980	0.590	120
	2	0.977	0.535	100
	3	0.982	0.470	85
	4	0.978	0.420	85
	5	0.983	0.400	48
	6	0.920	0.180	31
	7	0.984	0.220	26
2	1	0.995	0.205	100
	2	0.996	0.189	92
	3	0.997	0.091	53
	4	0.997	0.056	28
	5	0.998	0.042	21
3	1	0.971	7.525	100
	2	0.973	4.720	60
	3	0.971	3.590	40
	4	0.976	2.420	20
4	1	0.977	0.180	115
	2	0.978	0.160	100
	3	0.978	0.150	91
	4	0.983	0.121	72
	5	0.981	0.102	72
	6	0.971	0.096	72
	7	0.983	0.071	55
	8	0.982	0.049	25
	9	0.977	0.044	25
5	1	0.984	0.986	100
	2	0.983	0.825	60
	3	0.987	0.490	40
	4	0.981	0.475	20

Acronyms:

**Comp #:** System component number.

**Vers #:** System version number.

Table 1. Characteristics of available system components on the market



<b>Demand level (%)</b>	100	80	50	20
<b>Duration (h)</b>	4203	788	1228	2536
<b>Probability</b>	0.479	0.089	0.140	0.289

Table 2. Parameters of the cumulative demand curve

<b>A0</b>	<b>A<sub>01</sub></b>	<b>Optimal Structure</b>	<b>Computed Availability A</b>	<b>Computed Cost C</b>
0.975	0.9760	1: Components 3 - 6 - 5 - 7 2: Components 2 - 3 - 4 - 4 3: Components 1- 4 4: Components 2 - 5 - 7 - 8 5: Components 3 - 3 - 4	0.9773	13.4440
0.980	0.9826	1: Components 2 - 2 2: Components 3 - 3 - 5 3: Components 2 - 3 - 3 4: Components 5 - 6 - 7 5: Components 3 - 3 - 4	0.9812	14.9180
0.990	0.9931	1: Components 2 - 1 2: Components 3 - 3 3: Components 2 - 2 - 3 4: Components 5 - 5 - 6 5: Components 2 - 2	0.9936	<b>16.2870</b>

Table 3. Optimal Solution Obtained By Ant Algorithm

Optimal availabilities obtained by Ant Algorithm were compared to availabilities given by genetic algorithm (presented by symbol  $A_0$  in table 3) in the reference (Levitin et al., 1997), and to those obtained by harmony search (presented by symbol  $A_{01}$  in table 3) given in (Rami et al., 2009).

For this type of problem, we define the minimal cost system configuration which provides the desired reliability level  $A \geq A_0$  (where  $A_0$  is given in (Levitin et al, 1997) taken as reference).

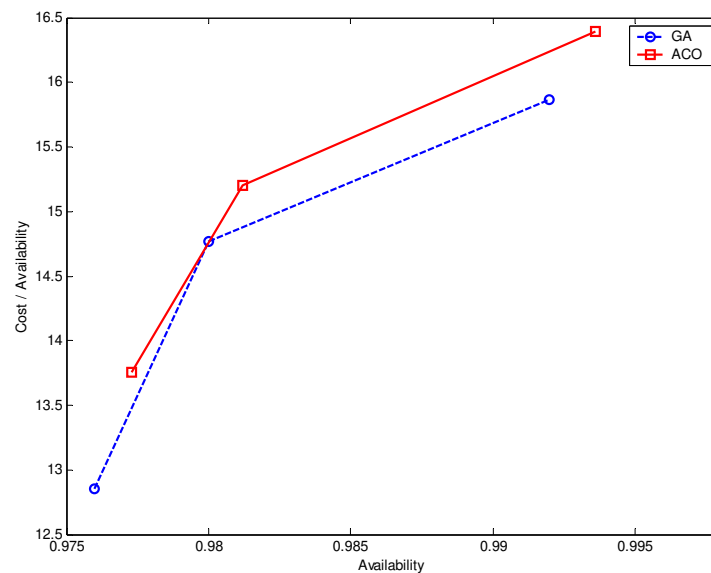
We will clearly remark the improvement of the reliability of the system at price equal compared to the two other methods.

We gave more importance to the reliability of the system compared to its cost what justifies the increase in the cost compared to the reference.

The compromise of the cost/reliability was treated successfully in this work.

The objective is to select the optimal combination of elements used in series-parallel structure of power system. This has to correspond to the minimal total cost with regard to the selected

level of the system availability. The ACO allows each subsystem to contain elements with different technologies. The ACO algorithm proved very efficient in solving the ROP and better quality results in terms of structure costs and reliability levels have been achieved compared to GA (Levitin et al., 1997).



**Figure 4.** Cost-availability rate of GA and ACO algorithm versus availability

A0	% of $A_{GA}$	% of $A_{ACO}$	% of C/A
0.975	0.1	0.23	58.5
0.980	0.0	0.12	13.3
0.990	0.2	0.36	39.4

**Table 4.** Comparison of Optimal Solutions Obtained by ACO and Genetic Algorithms For Different Availability Requirements

From figure 4 and the table, one can observe:

ACO achieved better quality results in terms of structure cost and reliability in different reliability levels (figure 4). We remark in all case, GA performed better by achieving a less expensive configuration, however ACO algorithm achieved a near optimal configuration with a slightly higher reliability level (table 4).

We take, for example, for reference reliability level ( $A_0 = 0.975$ , table 4), GA prove an augmentation of 0.1 percent compared to 0.23 percent given by ACO this for a difference in rate Cost-reliability of 58.3%. It is noticed, according to figure 4, that ACO tends, at equal price, to increase the reliability of the system.

## 6. Conclusion

A new algorithm for choosing an optimal series-parallel power structure configuration is proposed which minimizes total investment cost subject to availability constraints. This algorithm seeks and selects devices among a list of available products according to their availability, nominal capacity (performance) and cost. Also defines the number and the kind of parallel machines in each sub-system. The proposed method allows a practical way to solve wide instances of structure optimization problem of multi-state power systems without limitation on the diversity of versions of machines put in parallel. A combination is used in this algorithm is based on the universal moment generating function and an ant colony optimization algorithm.

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