Chapter 6

Unsteady Axial Viscoelastic Pipe Flows of an Oldroyd B Fluid

A. Abu-El Hassan and E. M. El-Maghawry

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/53638

1. Introduction

The unsteady flow of a fluid in cylindrical pipes of uniform circular cross-section has applications in medicine, chemical and petroleum industries [3,4,5]. For viscoelastic fluids, the unsteady axial decay problem for UCM fluid is considered by Rahman et al. [6]; and for Newtonian fluids as a special case. Rajagopal [7] has studied exact solutions for a class of unsteady unidirectional flows of a second-order fluid under four different flow situations. Atalik et al. [8] furnished a strong numerical evidence that non-linear Poiseuille flow is unstable for UCM, Oldroyd-B and Giesekus models. This fact is supported experimentally by Yesilata, [9]. The unsteady flow of a blood, considered as Oldroyd-B fluid, in tubes of rigid walls under specific APGs is concerned by Pontrelli, [10, 11].

Flow of a polymer solution in a circular tube under a pulsatile APG was investigated by Barnes et al. [12, 13].The same problem for a White-Metzner fluid is performed by Davies et al. [14] and Phan-Thien [15]. Recently, periodic APG for a second-order fluid has been studied by Hayat et al. [16]. Numerical simulation based on the role of the pulsatile wall shear stress in blood flow, is investigated by Grigioni et al. [1].

The present paper is concerned with the unsteady flow of a viscoelastic Oldroyd-B fluid along the axis of an infinite tube of circular cross-section. The driving force is assumed to be a time-dependent APG in the following three cases:

i. APG varies exponentially with time,

ii. Pulsating APG,

iii. A starting flow under a constant APG.

2. Formulation of the problem

The momentum and continuity equations for an incompressible and homogenous fluid are given by

$$\rho \frac{d\underline{q}}{dt} = -\nabla P + \nabla \cdot \underline{\underline{S}}, \tag{1}$$

and

$$\nabla \cdot \underline{q} = 0, \tag{2}$$

where ϱ is the material density, $\underline{q}$ is the velocity field, p is the isotropic pressure and $\underline{S}$ is the Cauchy or extra-stress tensor. The constitutive equation of Oldroyd-B fluid is written as

$$\underline{\underline{T}} = -p\underline{\underline{I}} + S; \underline{\underline{S}} + \lambda_1 \overset{\nabla}{\underline{\underline{S}}} = \mu\{\underline{\underline{A_1}} + \lambda_2 \overset{\nabla}{\underline{\underline{A_1}}}\} \tag{3}$$

where $\underline{T}$ is the total stress, $\underline{I}$ is the unit tensor, μ is a constant viscosity, λ_1 and λ_2, $(0 \le \lambda_2 \le \lambda_1)$ are the material time constants, termed as relaxation and retardation times; respectively. The deformation tensor $\underline{A}_1$ is defined by

$$\underline{\underline{A}}_1 = \underline{\underline{L}} + \underline{\underline{L}}^T; \underline{\underline{L}} = \nabla \underline{q}. \tag{4}$$

and "∇" denotes the upper convected derivative ; i.e. for a symmetric tensor $\underline{G}$ we get,

$$\overset{\nabla}{\underline{\underline{G}}} = \frac{\partial \underline{\underline{G}}}{\partial t} + \underline{q} \cdot \nabla \underline{\underline{G}} - \underline{\underline{G}} \cdot \underline{\underline{L}} - \underline{\underline{L}}^T \cdot \underline{\underline{G}}. \tag{5}$$

The symmetry of the problem implies that $\underline{S}$ and $\underline{q}$ depend only on the radial coordinate r in the cylindrical polar coordinates (r,θ,z) where the z-axis is chosen to coincide with the axis of the cylinder. Moreover, the velocity field is assumed to have only a z-component, i.e.

$$\underline{q} = (0, 0, \underline{w}), \tag{6}$$

which satisfies the continuity equation (2) identically. The substitution of Eq. (6), into Eqs. (1) and (3) yields the set of equations

$$S_{rz} + \lambda_1 \frac{\partial S_{rz}}{\partial t} = \mu(\frac{\partial w}{\partial r} + \lambda_2 \frac{\partial^2 w}{\partial r \partial t}), \tag{7}$$

$$\frac{\partial p}{\partial z} = \frac{\partial S_{rz}}{\partial r} + \frac{1}{r} S_{rz} - \rho \frac{\partial w}{\partial t}, \tag{8}$$

$$\frac{\partial p}{\partial r} = \frac{\partial p}{\partial \theta} = 0. \tag{9}$$

Equations (8) and (9) imply that the pressure function takes the form; $p = z \ f(t) + c$, so that

$$\frac{\partial p}{\partial z} = f(t). \tag{10}$$

The elimination of S_{rz} from (7) and (8) shows that velocity field $w(r, \ t)$ is governed by:

$$\rho(\lambda_1 \frac{\partial^2 w}{\partial t^2} + \frac{\partial w}{\partial t}) - \mu(1 + \lambda_2 \frac{\partial}{\partial t})(\frac{\partial^2 w}{\partial t^2} + \frac{1}{r}\frac{\partial w}{\partial r}) = -(1 + \lambda_1 \frac{\partial}{\partial t})\frac{\partial p}{\partial z}. \tag{11}$$

The non-slip condition on the wall and the finiteness of w on the axis give

$$w(r,t)|_{r=R} = 0 \ \text{ and } \ \frac{\partial w}{\partial r}|_{r=0} = 0. \tag{12}$$

Introducing the dimensionless quantities

$$\eta = \frac{r}{R}, \tau = \frac{\mu t}{\rho R^2}, \varphi = \frac{\mu L}{\Delta P R^2} w, \lambda = \frac{\lambda_2}{\lambda_1} \ \text{ and } \ H = \frac{\lambda_1 \mu}{\rho R^2} = \frac{We}{\mathrm{Re}}, \tag{13}$$

where R is the radius of the pipe, ΔP a characteristic pressure difference, L is a characteristic length, We and Re are the Weissenberg and Reynolds numbers; respectively, into Eqs. (10), (11) and (12) we get

$$H \frac{\partial^2 \varphi}{\partial \tau^2} + \frac{\partial \varphi}{\partial \tau} - [1 + \lambda H \frac{\partial}{\partial \tau}][\frac{\partial^2 \varphi}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial \varphi}{\partial \eta}] = [1 + H \frac{\partial}{\partial \tau}]\Psi(\tau), \tag{14}$$

with the BCs.

$$\varphi(1,\tau) = 0 \text{ and } \frac{\partial \varphi(0,\tau)}{\partial \eta} = 0, \tag{15}$$

and

$$\Psi(\tau) = -\frac{L}{\Delta p}\frac{\partial p}{\partial z} = -\frac{L}{\Delta p} f(t). \tag{16}$$

Equation (14) subject to BCs. (15) is to be solved for different types of APGs; i.e. different forms of the function $\Psi(\tau)$.

3. Pressure gradient varying exponentially with time

We consider the two cases of exponentially increasing and decreasing with time APGs separately.

3.1. Pressure gradient increasing exponentially with time

Let,

$$\Psi(\tau) = -\frac{L}{\Delta p}\frac{\partial p}{\partial z} = Ke^{\alpha^2 \tau}, \tag{17}$$

and assume that

$$\varphi(\eta,\tau) = g(\eta)e^{\alpha^2 \tau}, \tag{18}$$

where K and α are constants. The substitution of Eqs. (17) and (18) into Eq. (14) leads to

$$g'' + \frac{1}{\eta}g' - \frac{\alpha^2(H\alpha^2+1)}{\lambda H\alpha^2+1}g = -K\frac{H\alpha^2+1}{\lambda H\alpha^2+1}, \tag{19}$$

while the BCs. (15) reduce to

$$g(1) = 0, g'(0) = 0 \tag{20}$$

A solution of Eq. (19) subject to the BCs. (20) is

$$g(\eta) = \frac{K}{\alpha^2}[1 - \frac{I_0(\beta\eta)}{I_0(\beta)}], \tag{21}$$

where I_0 (x) is the modified Bessel-functions of zero-order, and

$$\beta^2 = \frac{\alpha^2(1 + H\alpha^2)}{1 + \lambda H\alpha^2} . \tag{22}$$

Therefore, the velocity field is given by

$$\varphi(\eta,\tau) = \frac{K}{\alpha^2}[1 - \frac{I_0(\beta\eta)}{I_0(\beta)}]e^{\alpha^2\tau}. \tag{23}$$

The solution given by Eq. (23) processes the following properties:

i. The time dependence is exponentially increasing such that for $\eta \neq 1$ $\lim_{\tau\to\infty}\phi(\eta, \tau) \to \infty$. It may be recommendable to choose another APG which increases up to a certain finite limit in order to keep $\phi(\eta, \tau)$ finite.

ii. The present solution depends on the parameter β in the same form as the solution for the UCM [6]. For any value of β the Oldroyd-B fluid exhibits the same form as the UCM- fluid. However, in the present case β depends on λ in addition to H and α^2. A close inspection show that $\lim_{\lambda\to 0}\beta^2 = \beta^2$ for the UCM-fluid while the $\lim_{\lambda\to 1}\beta^2 = \alpha^2$ which coincides with the case of the Newtonian fluid, [8].

iii. The parameter β is inversely proportional to λ where the decay rate increases by increasing the value of H. However, as mentioned above, as λ approaches the value $\lambda = 1$ all the curves matches together approaching the value $\beta^2 = \alpha^2$ asymptotically. The behavior of β as a function of λ, where H is taken as a parameter is shown in Fig. (1).

For small values of $|\beta|$ and by using the asymptotic expansion of I_0 (x),

it can be shown that the velocity profiles approaches the parabolic distribution;

$$\lim_{\beta\to 0}\varphi(\eta,\tau) = \frac{K(H\alpha^2 + 1)}{4(\lambda H\alpha^2 + 1)}(1 - \eta^2)e^{\alpha^2\tau} \tag{24}$$

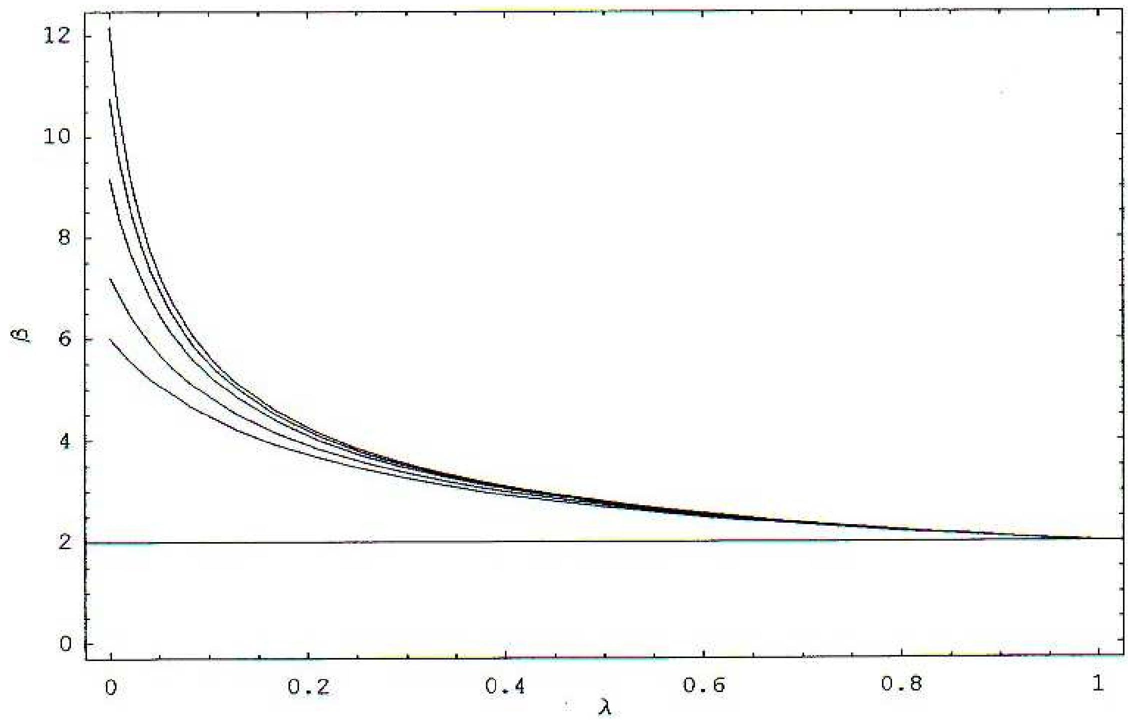

Figure 1. The $\lambda\beta$ - relation H= 2, 3, 5, 7, 9, (Bottom to top)

For the case of large $|\beta|$ the velocity distribution is given as;

$$\lim_{\beta \to \infty} \varphi(\eta,\tau) = \frac{K}{\alpha^2}[1 - \frac{1}{\sqrt{\eta}} e^{-\beta(1-\eta)}] e^{\alpha^2 \tau} \tag{25}$$

This solution is completely different from the parabolic distribution and it depends on η only in the neighborhood of the wall. Therefore, such a fluid exhibits boundary effects.

The rising-APG velocity field $\phi(\eta, \tau)$ is plotted in Figs. (2a) and (2b) as a function of η at different values of β for $\alpha = 2$ and $\alpha = 5$.

3.2. Pressure gradient decreasing exponentially with time

The solution at present is obtained from the previous case by changing α^2 by $-\alpha^{-2}$. Therefore,

$$\varphi(\eta,\tau) = -\frac{K}{\alpha^2}[1 - \frac{j_0(\beta_1 \eta)}{j_0(\beta_1)}] e^{-\alpha^2 \tau}. \tag{26}$$

where

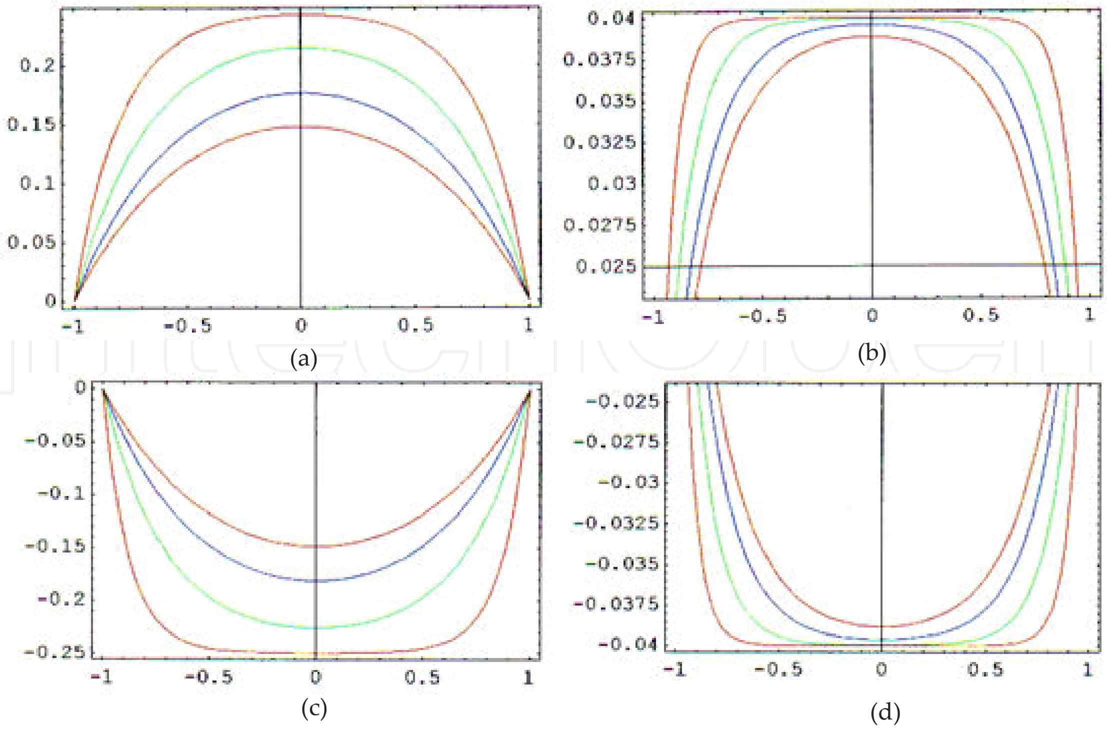

Figure 2. Rising – (a,b) APG velocity filed ;β=5.2, 3.5, 2.5, 2.1 (Bottom to top) Fig. (c) : Decreasing – APG velocity filed ;β=8.7, 3.9, 2.6, 2.1 (Top to Bottom) Fig. (d) : Decreasing – APG velocity filed ;β=16.4, 9.2, 6.5, 5.3 (Top to Bottom)

$$\beta_1{}^2 = \frac{\alpha^2(1-H\alpha^2)}{1-\lambda H\alpha^2} \tag{27}$$

The discussion of this solution is similar to the case of increasing APG except that the velocity decays exponentially with time and the value $\alpha^2 = 1/\lambda H$ is not permissible as it leads to infinite β_1^2 ; i.e.

$$\lim_{\alpha_1^2 \to 1/\ \lambda H} \beta_1^2 \to \infty \tag{28}$$

The two cases of small and large $|\beta_1|$ produce similar results as the previous solution. Thus

$$\underset{\beta_1 \to 0}{Lim}\, \phi(\eta,\tau) = -\frac{K}{4\alpha^2}\beta_1{}^2(1-\eta^2)e^{-\alpha\tau}, \tag{29}$$

and

$$\underset{\beta_1\to\infty}{Lim}\,\varphi(\eta,\tau)=-\frac{K}{\alpha^2}[1-\frac{1}{\sqrt{\eta}}\frac{\cos(\beta_1\eta-\frac{\pi}{4})}{\cos(\beta_1-\frac{\pi}{4})}]e^{-\alpha^2\tau}. \tag{30}$$

4. Pulsating pressure gradient

The present case requires the solution of Eq. (14) subject to BCs. (15) in the form

$$\Psi(\tau)=-\frac{L}{\Delta P}\frac{\partial p}{\partial z}=K\,e^{in\tau}; \qquad i=\sqrt{-1}, \tag{31}$$

K and n are constants. Assuming the velocity function has the form

$$\varphi(\eta,\tau)=re\left[f(\eta)e^{in\tau}\right], \tag{32}$$

$$\therefore \mathrm{f}''+\frac{1}{\eta}\mathrm{f}'-in\frac{(1+inH)}{(1+in\lambda H)}\mathrm{f}=-K\frac{(1+inH)}{(1+in\lambda H)}. \tag{33}$$

The solution of this equation satisfying the BCs. (15) is :

$$\mathrm{f}(\eta)=\frac{k}{in}[1-\frac{I_0(\beta\eta)}{I_0(\beta)}], \qquad \beta^2=in\frac{(1+inH)}{(1+in\lambda H)}. \tag{34}$$

Hence, the velocity distribution is given by:

$$\phi(\eta,\tau)=re\left\{\frac{k}{in}e^{in\tau}[1-\frac{I_0(\beta\eta)}{I_0(\beta)}]\right\}. \tag{35}$$

Obviously; for small $|\beta|$,

$$\underset{\beta\to 0}{Lim}\,f(\eta)=\frac{K}{in}\left(\frac{\beta^2(1-\eta^2)}{4}\right). \tag{36}$$

and for large $|\beta|$

http://dx.doi.org/10.5772/53638

$$\lim_{\beta\to\infty} \frac{I_0(\beta\eta)}{I_0(\beta)} = \frac{1}{\sqrt{\eta}} e^{-\beta(1-\eta)}, \tag{37}$$

So that,

$$\varphi(\eta,\tau) = re\left\{\frac{K}{in}[1 - \frac{1}{\sqrt{\eta}} e^{-\beta(1-\eta)} e^{in\tau}]\right\}, \tag{38}$$

where,

$$\beta^2 = \frac{in(1+inH)}{(1+in\lambda H)} = \frac{1}{1+n^2\lambda^2H^2}[n^2H(\lambda-1\} + in(1+n^2\lambda H^2)], \tag{39}$$

or simply,

$$\beta = \sqrt{\Re}\, e^{i\theta/2}, \tag{40}$$

$$\Re = \frac{n}{1+n^2\lambda^2H^2}\sqrt{n^2H^2(1-\lambda)^2 + (1+n^2\lambda H^2)^2}, \tag{41}$$

$$\frac{\theta}{2} = -\frac{1}{2}\mathrm{Tan}^{-1}\left[\frac{1+n^2\lambda H}{nH(1-\lambda)}\right]. \tag{42}$$

Substituting from Eqs.(40,41,42) into Eq. (37), we get:

$$\lim_{\beta\to\infty} \varphi(\eta,\tau) = \frac{k}{n}\left\{\sin n\tau - \frac{1}{\sqrt{\eta}} e^{-(1-\eta)\sqrt{\Re}\cos(\theta/2)} \sin[n\tau - (1-\eta\sqrt{\Re}\sin\frac{\theta}{2})]\right\}. \tag{43}$$

As $\lambda \to 0$, [6], $\Re \to r_1 = n\sqrt{1+n^2H^2}$ and $\frac{\theta}{2} \to \frac{\theta_1}{2} = -\frac{1}{2}Tan^{-1}(\frac{1}{nH})$.

Then

$$\varphi(\eta,\tau) = \frac{k}{n}\left\{\sin n\tau - \frac{1}{\sqrt{\eta}} e^{-(1-\eta)\sqrt{r_1}(\cos\theta_1/2)} \sin[n\tau - (1-\eta)\sqrt{r_1}(\sin\frac{\theta_1}{2})]\right\} \tag{44}$$

The velocity field $\varphi(\eta,\tau)$ is plotted in Figs. (3a) and (3b); respectively, against η for different values of β. The two limiting cases for small and large $|\beta|$ are represented in three-dimensional Figs. (4a) and (4b) in order to emphasize the oscillating properties of the solution.

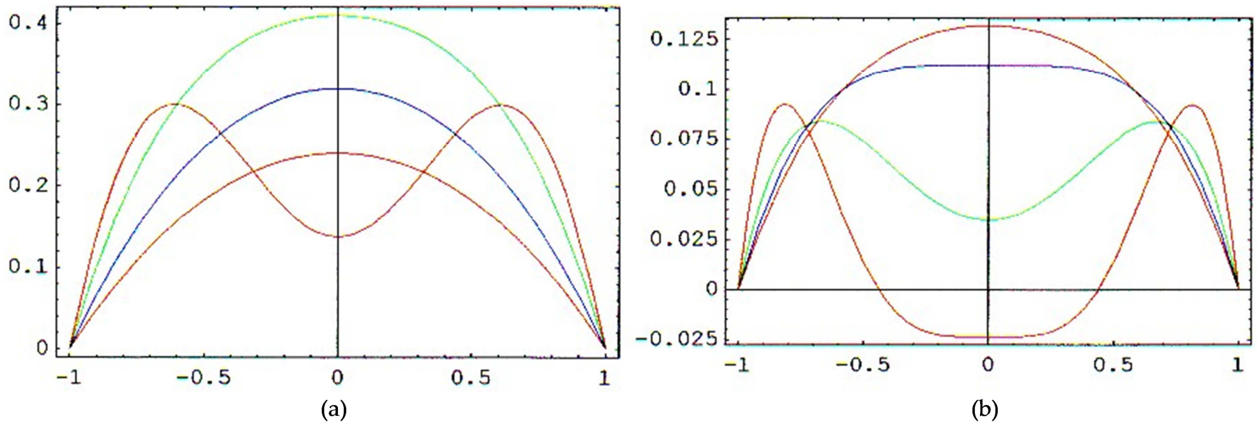

Figure 3. a) : Pulsating – APG ; n=2, H=5, $\beta=3.7$, 2.5, 1.8, 1.5 (b) : Pulsating – APG ; n=5, H=5, $\beta=6.8$, 4.1, 2.9, 2.4 [Top to Bottom for all]

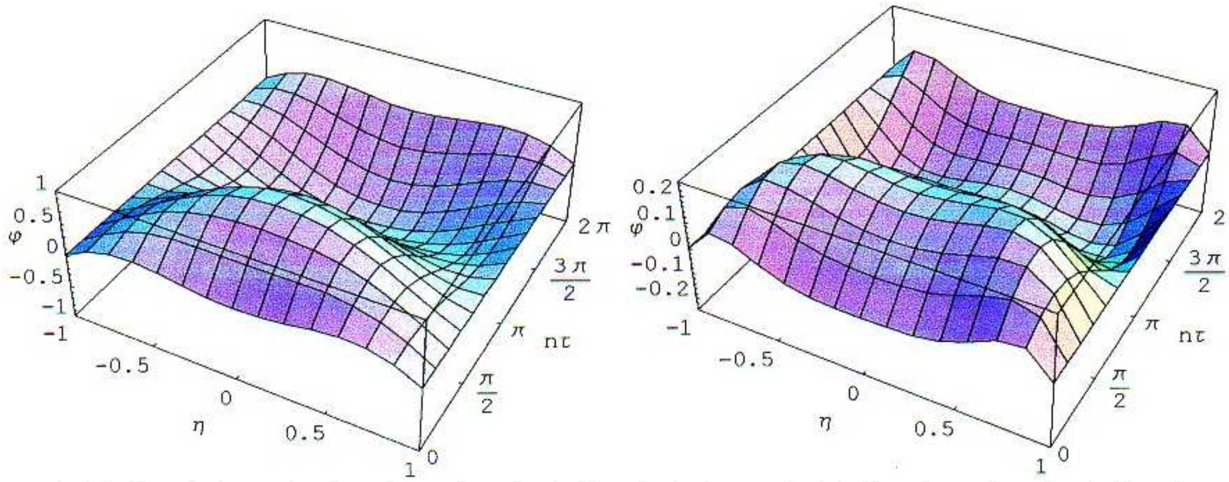

Figure 4. a): Pulsating-APG, n = 2, H = 5, at small $|\beta|$; $\beta=3.7$ (b): Pulsating-APG, n = 3, H = 5, at large $|\beta|$; $\beta=6.8$

5. Constant pressure gradient

Here we consider the flow to be initially at rest and then set in motion by a constant ABG "-K". Hence, $\Psi(\tau)$; Eq.(14), subject to BCs. (15) reduces to

$$\frac{L}{\Delta P}\frac{\partial P}{\partial z}=-K. \tag{45}$$

Therefore, we need to solve the equation

$$H\frac{\partial^2\Phi}{\partial\tau^2}+\frac{\partial\Phi}{\partial\tau}-[1+\lambda H\frac{\partial}{\partial\tau}][\frac{1}{\eta}\frac{\partial\Phi}{\partial\eta}+\frac{\partial^2\Phi}{\partial\eta^2}]=K, \tag{46}$$

subject to the boundary and initial conditions

$\phi(1,\tau)=0$, for $\tau\geq 0$,

$$\phi(\eta,0)=0, \text{for } 0\leq\eta\leq 1 \tag{47}$$

Equation (46) can be transformed to a homogenous equation by the assumption

$$\Phi(\eta,\tau)=\frac{K}{4}(1-\eta^2)-\psi(\eta,\tau), \tag{48}$$

where $\Psi(\eta,\tau)$ represents the deviation from the steady state solution. Hence,

$$[\frac{\partial}{\partial\tau}(1+H\frac{\partial}{\partial\tau})-(1+\lambda\, H\frac{\partial}{\partial\tau})(\frac{\partial^2}{\partial\eta^2}+\frac{1}{\eta}\frac{\partial}{\partial\eta})]\psi=0, \tag{49}$$

subject to the boundary and initial conditions

$$\Psi(1,\tau)=0 \text{ for } \tau\geq 0. \tag{50}$$

$$\psi(\eta,0)=\frac{K}{4}(1-\eta^2) \text{ for } 0\leq\eta\leq 1. \tag{51}$$

Assuming that $\psi(\eta,\ \tau)=F(\eta)\cdot G(\tau)$, Eq.(49) separates to

$$HG''+(1+\lambda H\alpha^2)G'+\alpha^2 G=0, \tag{52}$$

$$F''+\eta^{-1}(1+\lambda H\alpha^2)F'+\alpha^2 F=0. \tag{53}$$

Equation (52) has the solution,

$$G(\tau) = Ae^{\gamma_1 \tau} + Be^{\gamma_2 \tau} \tag{54}$$

where γ_1 and γ_2 are the roots of the Eq.(52). On the other hand, Eq. (53) has the solution

$$F(\eta) = J_0(\alpha_m \eta). \tag{55}$$

Therefore,

$$\gamma_{1,2} = \frac{-(1+\lambda H\alpha_m^2) \pm \sqrt{(1+\lambda H\alpha_m^2)^2 - 4\alpha_m^2 H}}{2H}. \tag{56}$$

The BCs. (50,51) implies that the constant α_m takes all zeros of the Bessel-function J_0 (α_1, α_2, ...). Hence,

$$\Psi(\eta,\tau) = \sum_{m=1}^{\infty} J_0(\alpha_m \eta) G(\tau), \tag{57}$$

$$\therefore \Psi(\eta,\tau) = \sum_{m=1}^{\infty} J_0(\alpha_m \eta)(A_m e^{\gamma_{1m}\tau} + B_m e^{\gamma_{2m}\tau}). \tag{58}$$

The initial condition (50) and BCs. (51) will not be sufficient to evaluate the constants A_m and B_m. Hence, it is required to employ another condition. We assume that G(τ) is smooth about the value τ = 0 and can be expanded in a power series about τ = 0. Assuming G(τ) to be linear function of τ in the domain about τ = 0, then $G''=0$ in Eq. (52). Hence

$$(1+\lambda H\alpha_m^2)G_m^{`}(0) + \alpha_m^2 G_m(0) = 0, \tag{59}$$

$$G_m(\tau) = A_m e^{\gamma_{1m}\tau} + B_m e^{\gamma_{2m}\tau}, \tag{60}$$

From which we obtain

$$A_m[(1+\lambda H\alpha_m^2)\gamma_{1m} + \alpha_m^2] + B_m[(1+\lambda H\alpha_m^2)\gamma_{2m} + \alpha_m^2] = 0. \tag{61}$$

To determine the constants A_m and B_m we firstly satisfy the remaining condition (51). Owing to Eq. (58) and the initial condition, Eq. (51), we notice that,

$$\Psi(\eta,0)=\sum_{m=1}^{\infty}(A_m+B_m)J_0(\alpha_m\eta)=\frac{K}{4}(1-\eta^2). \tag{62}$$

Via the Fourier–Bessel series, Eq. (62) leads to,

$$A_m+B_m=\frac{K}{2J_1^2(\alpha_m)}\int_0^1\eta(1-\eta^2)\,J_0(\alpha_m\eta)d\eta. \tag{63}$$

Performing this integration we get

$$A_m+B_m=\frac{2K}{\alpha_m^3J_1(\alpha_m)}-\frac{K}{\alpha_m^2}\frac{J_0(\alpha_m)}{J_1^2(\alpha_m)}. \tag{64}$$

From Eqs. (61) and (64) we obtain :

$$A_m=\frac{[(1+\lambda H\alpha_m^2)\gamma_{2m}+\alpha_m^2]}{(1+\lambda H\alpha_m^2)(\gamma_{2m}-\gamma_{1m})}\left[\frac{2K}{\alpha_m^3J_1(\alpha_m)}-\frac{K}{\alpha_m^2}\frac{J_0(\alpha_m)}{J_1^2(\alpha_m)}\right], \tag{65}$$

$$B_m=\frac{[(1+\lambda H\alpha_m^2)\gamma_{1m}+\alpha_m^2]}{(1+\lambda H\alpha_m^2)(\gamma_{1m}-\gamma_{2m})}\left[\frac{2K}{\alpha_m^3J_1(\alpha_m)}-\frac{K}{\alpha_m^2}\frac{J_0(\alpha_m)}{J_1^2(\alpha_m)}\right]. \tag{66}$$

Finally, the velocity field has the series representation

$$\phi(\eta,\ \tau)=\frac{K}{4}(1-\eta^2)-\sum_{m=1}^{\infty}\frac{J_0(\alpha_m\eta)}{(1+\lambda H\alpha_m^2)(\gamma_{2m}-\gamma_{1m})}\Big\{[(1+\lambda H\alpha_m^2)\gamma_{2m}+\alpha_m^2]e^{\gamma_{1m}\tau}$$

$$-[(1+\lambda H\alpha_m^2)\gamma_{1m}+\alpha_m^2]e^{\gamma_{2m}\tau}\Big\}\Big[\frac{2K}{\alpha_m^3J_1(\alpha_m)}-\frac{K}{\alpha_m^2}\frac{J_0(\alpha_m)}{J_1^2(\alpha_m)}\Big]. \tag{67}$$

The constant-APG velocity field $\varphi(\eta,\tau)$ as a function of η shown in Fig. (5).

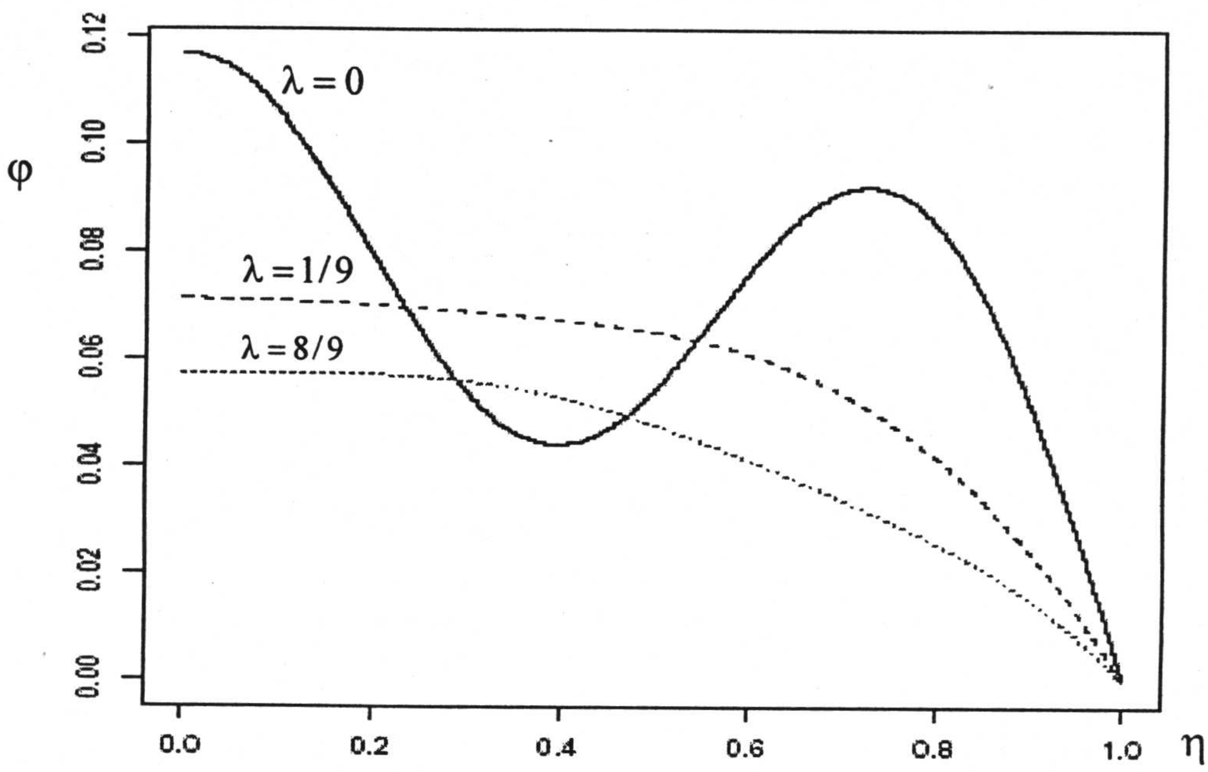

Figure 5. The velocity distribution for constant – APG taking H=0.2, τ=0.1 where the summation is taken for α_1=2.4, α_2=5.8, α_3=8.4

6. Results and discussion

The behavior of |β| as a function of λ where H is taken as a parameter is shown in Fig. (1). The behavior of β is inversely proportional to λ while it is fast-decreasing for higher H-values. For any β-value, the Oldroyd-B fluid exhibits the same form as the UCM-fluid. A close inspection of $\beta^2 = \alpha^2(1+\alpha^2 H)/(1+\lambda\alpha^2 H)$ shows that UCM-fluid is obtained by $\lim_{\lambda\to 0}\beta^2 = \beta^2$ while $\lim_{\lambda\to 0}\beta^2 = \alpha^2$ leads to the case of Newtonian fluid. For small values of |β| as well as |βη| and by using the asymptotic expansion of $I_0(x)$, it can be shown that the velocity profiles approaches the parabolic distribution.

For decay-APGs, Figs. (2a) and (2b) show that the velocity profiles of Oldroyd-B and UCM fluids are parabolic for small values of |βη| while for large |βη| they are completely different from this situation. The solutions depend on η only in the neighboring of the wall. Therefore, such fluids exhibit boundary layer effects [17]].

For pulsating-APG, the velocity distribution is represented in Figs. (3a) and (3b). The smallest value of β in both curves is almost parabolic as shown by Eq. (36) while the largest value exhibits boundary effect as reviled by Eq.(43). To emphasize the oscillating nature of the solution a three-dimensional diagrams (4a) and (4b) for the smallest and largest values of $|\beta|$ are respectively sketched.

Grigioni, et al [1], studided the behavior of blood as a viscoelastic fluid using the Oldroyd-B model. The results obtained for the velocity distribution stands in agreement with the obtained results in the present work.

Author details

A. Abu-El Hassan and E. M. El-Maghawry*

*Address all correspondence to: abd_galil@hotmail.com

Physics Department, Faculty of Science, Benha University, Egypt

References

[1] Grigioni, M., Daniele, C., and D' Avenio, G. The Role of Wall Shear Stress in Unsteady Vascular Dynamics, Vol. 7, No.3 Sep. (2002).

[2] Thurston, G. B., The viscoelasticity of human blood, Biophysical Journal, 12, 1205-1217(1972).

[3] Perspectives in fluid dynamics,A collective introduction to current research, Edited by: Batchelor, G.K., Moffatt, H.K. and Worster, M.G., Cambridge Univ. Press,(2000).

[4] Tucker, C. L., Fundamentals of computer modeling for polymer processing, III, HANSER Publishers, Munich (1989).

[5] Bird, R.B., Curtiss.C.F., Armstrong R.C. and Hassager, O., Dynamics of polymeric liquids, Vol.2, Wiley, New York (1987).

[6] Rahaman, K. D. and Ramkissoon, H., Unsteady axial viscoelastic pipe flows, J.Non-Newtonian Fluid Mech., 57 (1995) 27.

[7] Rajagopal, K.R., Int. J. Non-linear Mech. 17 (1982) 369.

[8] Atalik, K. and R. Keunings,R., J. Non-Newtonian Fluid mech. 102, (2002) 299.

[9] Yesilata, B, Fluid Dyn. Res. 31, 41 (2002).

[10] Pontrelli, G., Pulsatile blood flow in a pipe. Computers & Fluids, 27 (1998) 367.

[11] Pontrelli, G, Blood flow through a circular pipe with an impulsive pressure gradient. Math Models Methods in Appi Sci, 10 (2000) 187

[12] Bames, H.A., Townsend, P. and Walters, K., Rheol. Acta.10 (1971) 517.

[13] Bames, H.A., Townsend, P. and Walters, K., 244, Nature (1969) 585.

[14] Davies, J.M., Bhumiratana, S. and Bird, R.B. J. Non-Newtonian Fluid Mech, 3 (1977/1978) 237.

[15] Phan-Thien, N. and DudekJ, J.Non-Newtonian Fluid Mech., 11 (1982) 147.

[16] Hayat, T, Asghar, S. and Siddiqui, A.M., Some unsteady unidirectional flows of a Non -Newtonian fluid, Int. J. Eng. Science, 38 (2000) 337.

[17] Schlichting, H., Boundary- layer theory, McGraw-Hill, New York, 1968

IntechOpen