Chapter 5

Formation of Product Properties Determining Its Quality in a Multi-Operation Technological Process

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Additional information is available at the end of the chapter

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1. Introduction

Quality management of manufacture products requires knowledge of the values and interaction of all factors which form the quality. The mathematical description or the model of the process for obtaining the required product properties which correspond to the specified quality are needed for this purpose in the first place.

One of the most widespread processes in machine-building manufacture is the multi-operation technological process. As known, formation of product properties starts from receiving blank parts or raw materials to the enterprise warehouse for subsequent processing or reprocessing. After blanking operations, the main technological operations (TOs) are performed, which in most cases are concluded by final assembling. Sometimes final surface finishing and/or deposition of coating is performed after assembling.

During formation of product properties it is necessary to take into account the measurement errors which inevitably appear during quality control at each TO. In general, the technological process may be considered as a set of successive technologic states (TS) E[1)][1], in which the property index (PI) or a set of PIs obtained at the completed TO have passed quality control and keep their values unchanged. This allows representing the technological process in the form of a tuple

$$E_1 \prec E_2 \prec \ldots \prec E_r \prec \ldots \prec E_{s\text{-}1} \prec E_s,\ r = \overline{1,s} \tag{1}$$

where:

$\prec$ is the symbol of ordered preference in the sense of closeness to the final TS;

r and s are the subscripts of current TS and final TS, respectively.

The question now arises: what should be regarded as parallel transformation of the properties considered here? Undoubtedly, assembling TOs should. Here this tuple is expressed in another form:

$$(E_1, E_2, \ldots, E_r, \ldots, E_{s-1})^T \prec E_s, \tag{2}$$

where T is the sign of transposition of several E_r in vectorial form of recording.[1]

In case of such, so to say, 'existential' approach to formation of product properties, TS E_r must be considered as achieving of the prescribed value by property P_r at the completed TO or, in vectorial form, as achieving of the prescribed values by a set of properties (P_r), which is testified by the PIs obtained as the result of post-operation check.

For the development of mathematical model of formation of product properties (expressed by relevant PIs) during technological process, it is essential to represent each TO in the form of elementary oriented graph (fig.1), which nodes correspond to adjacent TSs (preceding TS E_{r-1} and subsequent TS E_r), respectively [1]. Graph edge r oriented at TS E_r is symbolizing a TO or, if it is principally significant, a technological step, during which the property P_r or properties (P_r) are transformed from TS E_{r-1} into TS E_r, as shown in fig. 1 a and 1b, respectively.

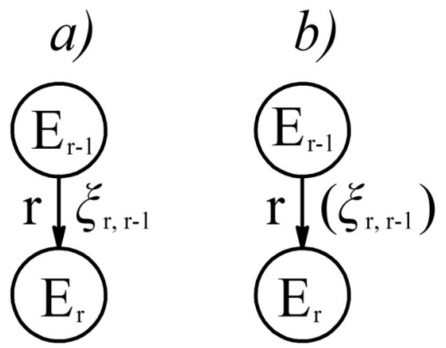

Figure 1. Mathematical model of a technological operation r of transformation of one (a) or several (b) property indices of a product from technological state E_{r-1} into technological state E_r with transformation coefficients ξr,r-1 or (ξr,r-1), respectively.

For each PI achieved by TS E_r, it is convenient to split the combined random error $\omega^2_{r_\Sigma}$ into three components: inherent error ω_r, extrinsic error ψ_r (carried from the previous TO or TOs), and check error κ_r, with the following equation valid for the variances of these errors [2–4]:

$$\omega^2_{r_\Sigma} = \omega^2_r + \psi^2_r + \kappa^2_r. \tag{3}$$

1 Initial letter of the word «Existence» – state (French)

Neglecting the infinitely small quantities of higher orders, formula (3) allows transition to the product properties transformation coefficient

$$\xi_{r,r\text{-}1_\Sigma} = \psi_r \,/\, \omega_{r\text{-}1_\Sigma}. \tag{4}$$

However, it should be noted that in some cases, where functional connection between coefficient $\xi_{r,r\text{-}1}$ and PI exists in some or other form, it is not possible to neglect these infinitely small quantities of higher orders[2]. This coefficient is considered here as "weight" of edge r, fig. 1.

In case of several PIs, formulas (3) and (4) may be written in vectorial-matrix form:

$$(\omega^2_{r_\Sigma}) = (\omega^2_r) + (\psi^2_r) + (\kappa^2_r), \tag{5}$$

where round brackets denote vectorial form of the relevant errors, and

$$(\xi_{r,r\text{-}1_\Sigma}) = (\psi_r) / (\omega_{r\text{-}1_\Sigma}), \tag{6}$$

where $(\xi_{r,r\text{-}1\,\Sigma})$ is the matrix of transformation of PI from TS r-1 to TS r.

Passing to the nonrandom component Δ_{r_Σ} of PI combined error, it is necessary to tie its center of grouping to zero reference point which corresponds to PI nominal value. Depending on accepted normalization method, such point may be either the middle of PI tolerance zone, or one of the limits (left or right) of PI tolerance zone. These limits represent the so-called functional (if related to E_s) thresholds or technological (in this case) thresholds [4–6], left $x_{\ulcorner}$ and right $x_{\urcorner}$.

Hence, the requirements to PI may be represented for each of these thresholds by semi-open intervals

$$x \geq x_{\ulcorner},\ [x_{\ulcorner}, \infty) \quad \text{and} \quad x \leq x_{\urcorner}, (0,\, x_{\urcorner}], \tag{7}$$

respectively, and for the tolerance zone – by segment

$$x_{\ulcorner} \leq x \leq x_{\urcorner}, [x_{\ulcorner}, x_{\urcorner}], \tag{8}$$

allowing to place PI values on x number axis.

2 E.g., in case of assembling fuel-regulating components of gas turbine engines.

If TS E_r contains several non-random combined errors (Δ_{r_Σ}), they may be united, similar to random errors, into the common vector of displacement of their centers of grouping. Therefore, the non-random analog of formula (5) will be:

$$(\Delta_{r_\Sigma}) = (\Delta\omega_r) + (\Delta\psi_r) + (\Delta\kappa_r), \tag{9}$$

where $\Delta\kappa r$ is set to zero because of assumed centrality of measurement errors distribution (systematic error of measurements must be close to zero due to timely certification and calibration of measuring instruments).

Then formula (9) will take the form

$$(\Delta_{r_\Sigma}) = (\Delta\omega_r) + (\Delta\psi_r), \tag{10}$$

Then it is necessary to reveal the inversion of PI errors, showing how the errors from the previous TSs migrate to subsequent TSs, and to perform, so to say, their mathematical convolution, uniting them into appropriate mathematical expressions [2–4, 6]. Let us start from consecutive transformation of errors of random PI components.

Thus, as mentioned earlier, blank parts or raw materials are received to the enterprise warehouse. Naturally, their PI has a combined error ω_{1_Σ} specified by delivery terms (at first, let us consider the simplest case of inversion of a single PI). In this case inversion starts from TS E_1 with combined technological error ω_{1_Σ}, and its first step is: transition from TS E_1 to TS E_2, with quadratic transformation of error variances corresponding to this step

$$\omega_{2_\Sigma}^2 = \omega_2^2 + \psi_2^2 + \kappa_2^2 = \omega_2^2 + \xi_{21}^2\omega_{1_\Sigma}^2 + \kappa_2^2. \tag{11}$$

The second step performs transition from TS E_2 to TS E_3, which is characterized by two quadratic transformations:

$$\begin{aligned} \omega_{3_\Sigma}^2 &= \omega_3^2 + \psi_3^2 + \kappa_3^2 = \omega_3^2 + \xi_{32}^2\omega_{2_\Sigma}^2 + \kappa_3^2 = \\ &= \omega_3^2 + \xi_{32}^2(\omega_2^2 + \xi_{21}^2\omega_{1_\Sigma}^2 + \kappa_2^2) + \kappa_3^2 = \\ &= \omega_3^2 + \xi_{32}^2\omega_2^2 + \xi_{32}^2\xi_{21}^2\omega_{1_\Sigma}^2 + \xi_{32}^2\kappa_2^2 + \kappa_3^2. \end{aligned} \tag{12}$$

Structure of formula (12) contains the forming, so to say, nucleus of inversion of manufacturing errors, or the inversion nucleus:

$$\xi_{32}^2\omega_2^2 + \xi_{32}^2\xi_{21}^2\omega_{1_\Sigma}^2 \tag{13}$$

Using the method of mathematical induction, let us try to find out the tendencies of subsequent evolution of this nucleus in course of approaching to the final TS. For this purpose, let us perform similar quadratic transformations on the third step of inversion

$$\begin{aligned}
&\omega_{4_\Sigma}^2 = \omega_4^2 + \psi_4^2 + \kappa_4^2 = \omega_4^2 + \xi_{43}^2\omega_{3_\Sigma}^2 + \kappa_4^2 = \omega_4^2 + \xi_{43}^2(\omega_3^2 + \xi_{32}^2\omega_2^2 + \\
&\xi_{32}^2\xi_{21}^2\omega_{1_\Sigma}^2 + \xi_{32}^2\kappa_2^2 + \kappa_3^2) + \kappa_4^2 = \omega_4^2 + \xi_{43}^2\omega_3^2 + \xi_{43}^2\xi_{32}^2\omega_2^2 + \\
&+\xi_{43}^2\xi_{32}^2\xi_{21}^2\omega_{1_\Sigma}^2 + \xi_{43}^2\xi_{32}^2\kappa_2^2 + \xi_{43}^2\kappa_3^2) + \kappa_4^2.
\end{aligned} \tag{14}$$

and on the fourth step of inversion

$$\begin{aligned}
&\omega_{5_\Sigma}^2 = \omega_5^2 + \psi_5^2 + \kappa_5^2 = \omega_5^2 + \xi_{54}^2\omega_{4_\Sigma}^2 + \kappa_5^2 = \\
&= \omega_5^2 + \xi_{54}^2(\omega_4^2 + \xi_{43}^2\omega_3^2 + \xi_{43}^2\xi_{32}^2\omega_2^2 + \xi_{43}^2\xi_{32}^2\xi_{21}^2\omega_{1_\Sigma}^2 + \xi_{43}^2\xi_{32}^2\kappa_2^2 + \\
&+\xi_{43}^2\kappa_3^2 + \kappa_4^2) + \kappa_5^2 = \\
&= \omega_5^2 + \xi_{54}^2\omega_4^2 + \xi_{54}^2\xi_{43}^2\omega_3^2 + \xi_{54}^2\xi_{43}^2\xi_{32}^2\omega_2^2 + \xi_{54}^2\xi_{43}^2\xi_{32}^2\xi_{21}^2\omega_{1_\Sigma}^2 + \\
&+\xi_{54}^2\xi_{43}^2\xi_{32}^2\kappa_2^2 + \xi_{54}^2\xi_{43}^2\kappa_3^2 + \xi_{54}^2\kappa_4^2 + \kappa_5^2.
\end{aligned} \tag{15}$$

Formula (14) shows quite evidently the general tendencies of increase of inversion nucleus components and increase of the inversion structure as a whole. This allows making the first steps for generalization and more convenient perception of the results obtained.

To improve visual appearance of formula (14), let us introduce the generalizing coefficient Ξ_{s1}, denoting it as multiplicative coefficient of PI transformation. For s-1 linear transformations of PI, this coefficient is the product:

$$\Xi_{s1} = \xi_{21}\xi_{32}...\xi_{r,r\text{-}1}...\xi_{s,s\text{-}1} = \prod_{r=2}^{s} \xi_{r,r\text{-}1}. \tag{16}$$

Similarly, for quadratic transformation of errors characterized by$\xi_{r,r\text{-}1}^2$:

$$\Xi_{s1}^2 = \xi_{21}^2\xi_{32}^2...\xi_{r,r\text{-}1}^2...\xi_{s,s\text{-}1}^2 = \prod_{r=2}^{s} \xi_{r,r\text{-}1}^2. \tag{17}$$

Now formula (14) may be rewritten in a simpler manner:

$$\begin{aligned}
&\omega_{5\Sigma}^2 = \omega_5^2 + \xi_{54}^2\omega_4^2 + \Xi_{53}^2\omega_3^2 + \Xi_{52}^2\omega_2^2 + \Xi_{51}^2\omega_{1_\Sigma}^2 + \\
&+\Xi_{52}^2\kappa_2^2 + \Xi_{53}^2\kappa_3^2 + \Xi_{54}^2\kappa_4^2 + \kappa_5^2.
\end{aligned} \tag{18}$$

Then let us generalize formula (17) for arbitrary number s of TSs, with parallel combining of similar terms:

$$\omega_{s_\Sigma}^2 = \omega_s^2 + \Xi_{s,s\text{-}1}^2(\omega_{s\text{-}1}^2 + \kappa_{s\text{-}1}^2) + \Xi_{s,s\text{-}2}^2(\omega_{s\text{-}2}^2 + \kappa_{s\text{-}2}^2) + \ldots + \\ + \Xi_{s,r}^2(\omega_r^2 + \kappa_r^2) + \ldots \Xi_{s,2}^2(\omega_2^2 + \kappa_2^2) + \Xi_{s,1_\Sigma}^2 \omega_{1_\Sigma}^2 + \kappa_s^2. \tag{19}$$

The following step for generalization of the results obtained will be introduction in formula (18) of the operator $\sum\limits_{r=3}^{s}$ for summing multiplicative coefficients $\Xi_{r,r\text{-}1}$ of transformation for the current index r which is the number of TSs, i.e. $\sum\limits_{r=3}^{s} \Xi_{r,r\text{-}1}^2$:

$$\omega_{s_\Sigma}^2 = \omega_s^2 + \sum_{r=3}^{s} \Xi_{r,r\text{-}1}^2(\omega_{r\text{-}1}^2 + \kappa_{r\text{-}1}^2) + \Xi_{s,1}^2 \omega_{1_\Sigma}^2 + \kappa_s^2, \tag{20}$$

representing the mathematical convolution of combined limiting error ω_{s_Σ} in the technological process containing s TOs performed consecutively.

In case of parallel execution of TOs, as mentioned above, the mathematical convolution on the basis of formula (2) will be

$$\omega_{s_\Sigma}^2 = \xi_{s,1_\Sigma}^2 \omega_{1_\Sigma}^2 + \xi_{s,2_\Sigma}^2 \omega_{2_\Sigma}^2 + \ldots + \xi_{s,r_\Sigma}^2 \omega_{r_\Sigma}^2 + \ldots + \xi_{s,s\text{-}1_\Sigma}^2 \omega_{s\text{-}1_\Sigma}^2 \tag{21}$$

or in concise form

$$\omega_{s_\Sigma}^2 = \sum_{r=1}^{s\text{-}1} \xi_{s,r}^2 \omega_{r_\Sigma}^2 \tag{22}$$

Now it is possible to consider in detail the structure of formulas (19) and (20). Formula (19) contains two inversion nuclei: the main nucleus

$$\sum_{r=3}^{s} \Xi_{r,r\text{-}1}^2(\omega_{r\text{-}1}^2 + \kappa_{r\text{-}1}^2) \tag{23}$$

and additional nucleus

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$$\Xi_{s1}^{2}(\omega_{1_{\Sigma}}^{2}) \tag{24}$$

The additional inversion nucleus shows that the error of blank part PI or raw material PI at TS E_1 directly affects PI of the resulting TS E_S, regardless of other TSs. Once again this demonstrates that special diligence is required for checking incoming blank parts, materials and supplies received from exterior enterprises for reprocessing. Both nuclei are circumposed by intrinsic errors $\omega_S{}^2$ and $\kappa_S{}^2$ of the final, S-th TO; these errors also deserve close attention.

It should be noted that the extrinsic (introduced) error $\psi_{r,}$ is not present in formulas (19) and (20). It may be compared to a sewing needle which does not remain in the fabric sewn by it. As for the parallel transformation of PI errors given by formula (20) is concerned, the inversion of PI errors is performed here in the manner formally identical for all and every TS.

The resulting formula for the non-random component of PI error and consequently performed TOs will look like the linear analog of formula (19):

$$\Delta_{s_{\Sigma}} = \Delta_{s} + \sum_{r=3}^{s} \Xi_{r,r\text{-}1}\Delta_{r\text{-}1} + \Xi_{s,1_{\Sigma}}\Delta_{1_{\Sigma}} \tag{25}$$

and for TOs performed in parallel – like the linear analog of formula (20):

$$\Delta_{s_{\Sigma}} = \sum_{r=1}^{s\text{-}1} \xi_{s_{s_{\Sigma}},r}\Delta_{r}. \tag{26}$$

For several PIs, according to formulas (5) and

(6), expressions (19) and (20) will become vectorial-matrix expressions, i.e.

$$(\omega^{2}{}_{s_{\Sigma}}) = (\omega_{s}^{2}) + \sum_{r=3}^{s}(\Xi_{r,r\text{-}1}^{2})\Big[(\omega_{r\text{-}1}^{2}) + (\kappa_{r\text{-}1}^{2})\Big] + (\Xi_{s1}^{2})(\omega_{1_{\Sigma}}^{2}) + (\kappa_{s}^{2}), \tag{27}$$

and

$$(\omega^{2}{}_{s_{\Sigma}}) = \sum_{r=1}^{s\text{-}1}\Big(\xi_{s,r_{\Sigma}}^{2}\Big)(\omega_{r_{\Sigma}}^{2}), \tag{28}$$

respectively.

The same relates to expressions (21) and (22):

$$(\Delta_{s_\Sigma}) = (\Delta_s)+\sum_{r=3}^{s}(\varXi_{r,r-1})(\Delta_{r-1})+(\varXi_{s,1})(\Delta_{1_\Sigma}) \tag{29}$$

and

$$(\Delta_{s_\Sigma})=\sum_{r=1}^{s-1}(\xi_{s_\Sigma,r})(\Delta_r). \tag{30}$$

In formulas (23) – (26), the round brackets indicate vectorial nature of the relevant component, excluding multiplicative transformation coefficients ($\varXi_{s1}$). These coefficients here are the product of matrices, either linear matrices

$$(\Xi_{s1}) = (\xi_{21})(\xi_{32})...(\xi_{r,r-1})...(\xi_{s-1}) = \prod_{r=2}^{s}(\xi_{r,r-1}) \tag{31}$$

or quadratic matrices

$$(\Xi_{s1}^2) = (\xi_{21}^2)(\xi_{32}^2)...(\xi_{r,r-1}^2)...(\xi_{s,s-1}^2) = \prod_{r=2}^{s}(\xi_{r,r-1}^2) \tag{32}$$

If we consider the consequently performed TOs, then the combined measurement error κ_{s_Σ} accumulated for one PI during the entire technological process in the resultant TS E_S may be obtained from formula (19) in the form

$$(\kappa_{s_\Sigma}^2) = (\kappa_s^2)+\sum_{r=3}^{s}(\Xi_{r,r-1}^2)(\kappa_{r-1}^2) \tag{33}$$

In case of TOs performed in parallel, this error may be expressed according to formula (20) as:

$$(\kappa_{s_\Sigma}^2) = \sum_{r=1}^{s-1}(\xi_{sr_\Sigma}^2)(\kappa_{r_\Sigma}^2). \tag{34}$$

When several PIs are checked, the formulas (29) and (30) will take vectorial-matrix form, i.e.

$$(\kappa_{s_\Sigma}^2) = (\kappa_s^2)+\sum_{r=3}^{s}(\Xi_{r,r-1}^2)(\kappa_{r-1}^2) \tag{35}$$

and

$$(\kappa_{s_\Sigma}^2) = \sum_{r=1}^{s-1} (\xi_{s_r}^2)(\kappa_{r_\Sigma}^2) \tag{36}$$

Formulas (29) – (32) allow determining the share of measurement errors $\kappa_{s_\Sigma}^2$ in the combined error $\omega_{s_\Sigma}^2$ for a single PI as well as for several PIs, $(\kappa_{s_\Sigma}^2)$ in $(\omega_{s_\Sigma}^2)$, respectively, i.e. $\kappa_{s_\Sigma}/\omega_{s_\Sigma}$ in the resulting TS E_s [6]. For the current, intermediate TSs E_r this relation will have a similar form $\kappa_{r_\Sigma}/\omega_{r_\Sigma}$.

The described above method of mathematical convolution of errors, including measurement errors, in a multi-operational technological process has been applied to production of aggregates for shipbuilding and aerospace industry [3,4,7]. It allows not only revealing, performing mathematical convolution and determining the relationship between PI errors and measurement errors, but also creates prerequisites for comprehensive optimization of measurement errors and selection of measuring instruments at all TOs of a technological process [5].

In connection with broadening introduction of mathematically fuzzy (MF) methods in technological practice [8], it is interesting to know, at least as a first approximation, how the described above may be interpreted in MF form. In the aspect under consideration it is quite often caused by complexity or practical impossibility of actual determination of the value $\xi_{r,r\text{-}1}$ or values $(\xi_{r,r\text{-}1})$ for transformation coefficients of product PIs using analytical or, so to say, mathematically unfuzzy (MUF) methods. First of all, we are interested in MUF results of forming product PIs in a multi-operational technological process represented by formulas (21 – 26) obtained above.

Let us regard fig. 4, which is a MF analog of fig. 1 for MUF transformation, as the first step in solving this problem. As before, TO here is represented by the oriented graph of transforming PI from TS $E_{r\text{-}1}$ into E_r, which edge now symbolizes MF coefficient $\xi_{r,r\text{-}1}$ of this transformation.

Formally this coefficient may be supposed to exist as a MF analog of formula (4) – the ratio of dividing two MF numbers in the symbolic notation

$$\xi_{r,r\text{-}1} = \boldsymbol{\psi}_r / \boldsymbol{\nu}_{r_{\Sigma\text{-}1}}, \tag{37}$$

where $\xi_{r,r\text{-}1,}$ $\boldsymbol{\psi}_r$ and $\boldsymbol{\nu}_{r_{\Sigma\text{-}1}}$ are the components of formula (4) expressed in MF form, highlighted hereinafter by bold type to distinguish from MUF form.

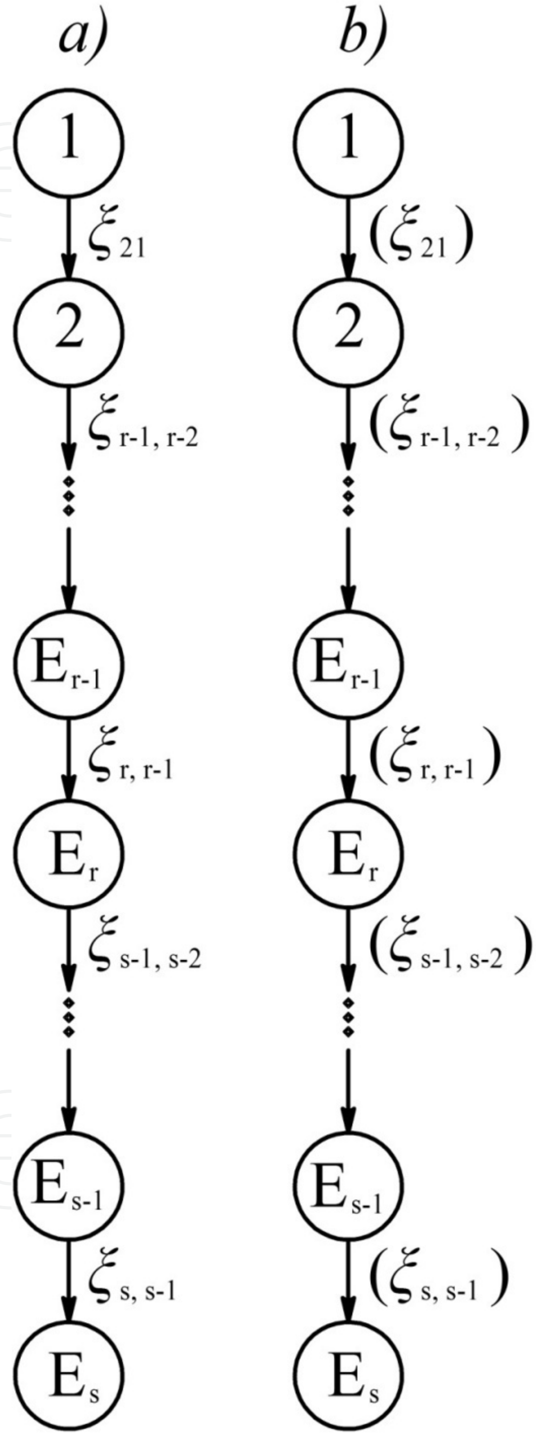

Figure 2. Mathematical model of a technological process with sequential transformation of one (a) or several (b) property indices from the first technological state E_1 into resulting technological state Es with ig

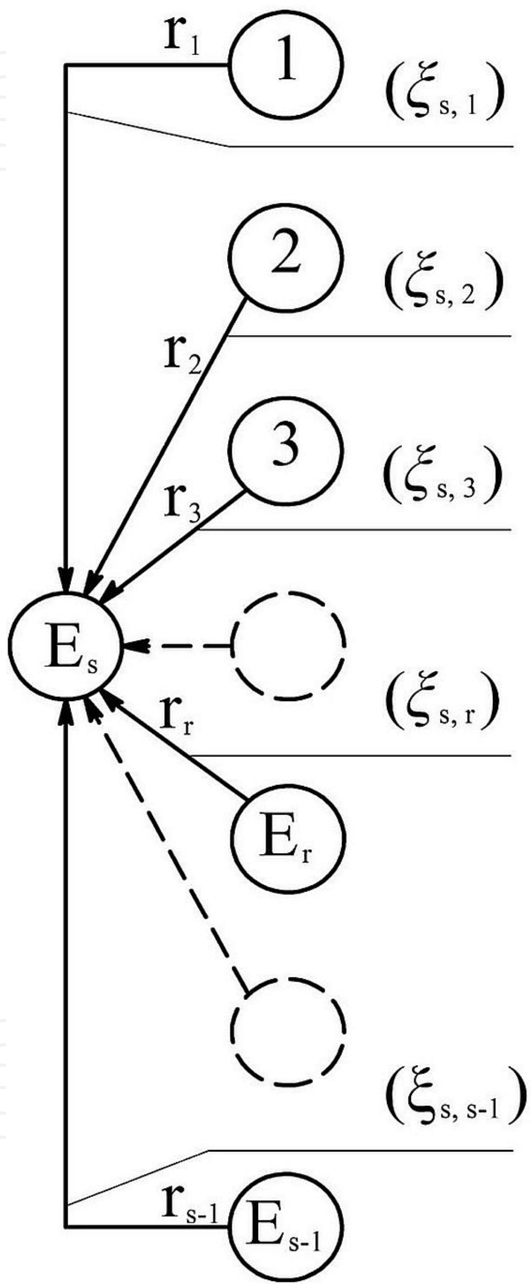

Figure 3. Mathematical model of a technological process with parallel transformation of several property indices from S-1 preceding technological states into one resulting technological state ES by performing r-1 technological operations with transformation coefficients (ξr,r-1).

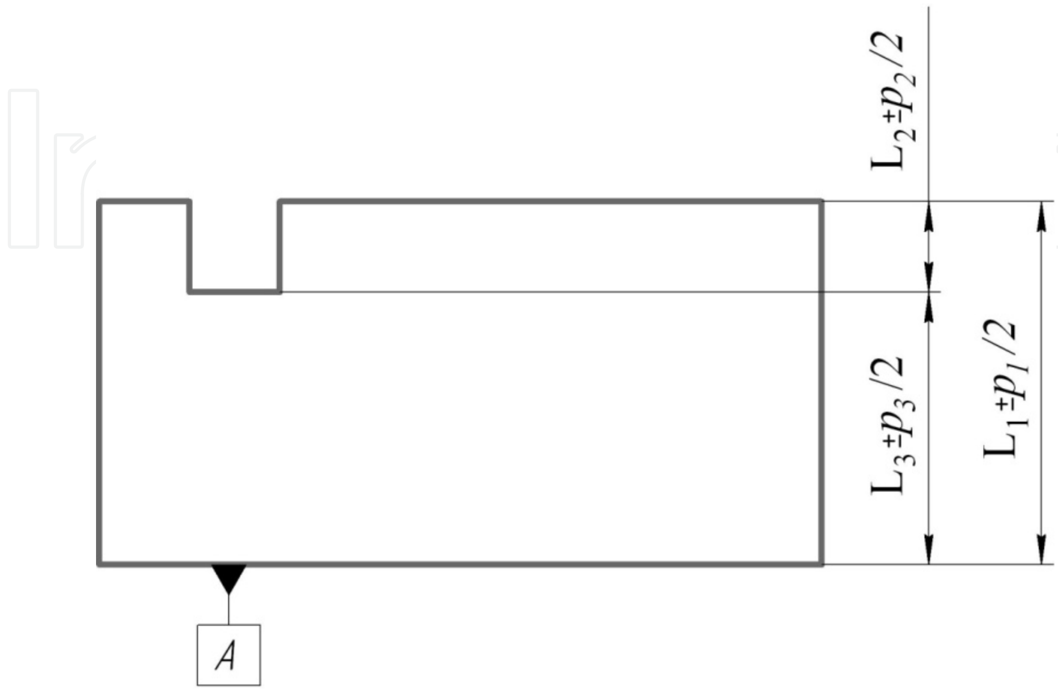

Figure 4. The outline of a component to be measured. ρ_2 – total limiting technologic spread of L_2 dimension, obtained in the process of elaboration of the component production technology,$\rho_{1,\ 3}$ – desired total limiting technologic spreads of L_1 and L_3 dimensions, required for the selection of an appropriate measuring tool.

However, here this ratio in general case is not applicable in the form of transformation coefficient, because MF operations of multiplying and dividing of MF numbers are not inverse to each other. This means that if X and Y are MN numbers, then X • Y / X ≠ Y. Regrettably, this also holds for operations of algebraic addition and deduction: (X+Y) - Y ≠ X.

Therefore, in MF case, MUF coefficient $\xi_{r,r-1}$ may be applied for its direct purpose only in the special case when determined relation exists between MF PIs of adjacent TSs E_r and E_{r-1}. The MF PIs obtained by some or other method shall be brought to mathematical unfuzziness (mathematically cleared)[1)] or defuzzied.

Then the following relationships will be true:

$$\bar{\xi}_{r,r\text{-}1} = \mathrm{B}\psi_r \,/\, \mathrm{B}\nu_{r_{\Sigma\text{-}1}} = \bar{\psi}_r \,/\, \bar{\nu}_{r_{\Sigma\text{-}1}}, \tag{38}$$

where:

$B\psi_r$ and $B\nu_{r_{\Sigma-1}}$ are carriers (or bases) of MF numbers ψ_r and $\nu_{r_{\Sigma-1}}$,

$\bar{\psi}_r$ and $\bar{\nu}_{r_{\Sigma-1}}$ are mathematically cleared (defuzzied) values of MF errors ψ_r and $\nu_{\Sigma_{r-1}}$, respectively,

« – » is the superscript of mathematical clearing of MF number.

It may be noted that $B\psi_r$ and $B\nu_{r_{\Sigma-1}}$ are analogs of ψ_r and $\nu_{\Sigma_{r-1}}$, while $\bar{\psi}_r$ and $\bar{\nu}_{r_{\Sigma-1}}$ are analogs of $\Delta\psi_r$ and $\Delta\nu_{r_{\Sigma-1}}$, respectively. This means that both MF data and MUF data are combined in one and the same MF number, allowing to present MF convolution for PI formation by one expression, rather than by two expressions, as in MUF case and in this transitional case.

For this purpose we will have to refer to MF binary relations on classical sets. The latter are a special case of MF sets defined on Cartesian product [9]. In the case under consideration, as shown in [10], for PI of TS E_{r-1} and E_r, there is a fuzzy binary relation of R –order of P_{r-1} and P_r, respectively[3]:

$$P_{r-1} R P_{r,} \tag{39}$$

which is a fuzzy set with membership function on unfuzzy Cartesian product of two universals P_{r-1} and P_r..

Now let us determine appearance of PI quality check by measurement in MF case. For a single PI x it consists of the following [10]:

- actual value of PI x is determined;
- using inequalities (7) or (8), it is compared with PI value(s) specified in the act on production delivery and acceptance, i. e. with PI functional thresholds x ┌ and x┐ ;
- basing on these inequalities, either presence or absence of the relevant property P_x with the product is revealed;
- if property Px is present, the product quality is considered as complying with the requirement imposed on it;
- if property Px is absent, the product quality is considered as non-complying with the requirement imposed on it

In this connection, when MF approach is used, measurement errors on the left $x_{\ulcorner}$ and right $x_{\urcorner}$ functional thresholds and the influence of these errors on the results of product quality control are of interest.

The measurement errors here have the form of the so-called function of membership (FM)

3 "And I saw mathematically clear…" (N.V.Gogol)

$$\eta_x(\theta) = \langle \eta_1 + \eta_2 + \ldots + \eta_\theta + \ldots \eta_\Theta \rangle, \tag{40}$$

where x means the PI measured, ___

θ means current (sequential) number of the term ($\theta = 1,\Theta$),

Θ means overall number of terms,

η means grade of membership (GM) of the term in respect of the measurement result ($0 \le \eta \le 1$),

+ means summation sign, considered as logical only inside angle brackets “< “ and “ >”.

A priori, when knowledge base (in the form of expert estimates, experimental data or some other precedents) is not available, it is reasonable to use the probabilistic FM composed basing on Gaussian normal differential distribution law normalized in regard of mean square deviations. For this purpose, MF unitary normalization of probabilities of this law is additionally used by means of dividing these probabilities by modal value. This value here is assumed equaling to 0.3989. Then these, now Gaussian, FM will look as follows for different Θ:

$$\Theta = 3 \quad \langle 0{,}0110_{-3{,}0\sigma} + 1{,}0000 + 0{,}0110^{+3{,}0\sigma} \rangle, \tag{41}$$

$$\Theta = 5 \quad \langle 0{,}0110_{-3{,}0\sigma} + 0{,}3246_{-1{,}5\sigma} + 1{,}0000 + +0{,}3246^{+1{,}5\sigma} + 0{,}0110^{+3{,}0\sigma} \rangle, \tag{42}$$

$$\Theta = 7 \quad \langle 0{,}0110_{-3{,}0\sigma} + 0{,}1354_{-2{,}0\sigma} + 0{,}6067_{-1{,}0\sigma} + 1{,}0000 + 0{,}6067^{+1{,}0\sigma} + 0{,}1354^{+2{,}0\sigma} + 0{,}0110^{+3{,}0\sigma} \rangle. \tag{43}$$

FM (38) and (39) in graphic form are shown in fig.6 and 7, respectively.

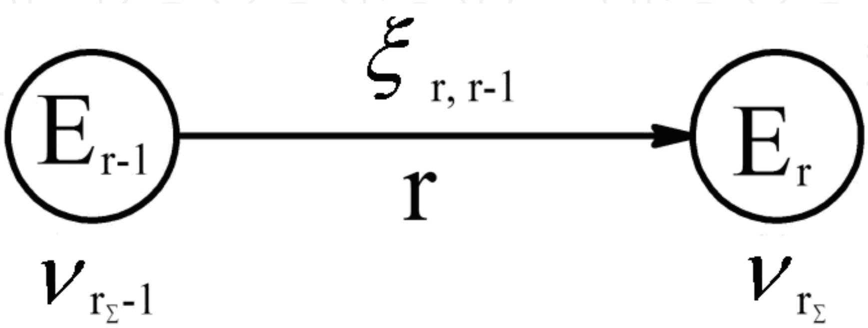

Figure 5. Mathematically fuzzy model of a technological operation r of transformation of one of property indices of a product from technological state E_{r-1} into technological state E_r. $\boldsymbol{v}_{r_\Sigma}$ and $\boldsymbol{v}_{r_{\Sigma-1}}$ – functions of appurtenance of property indices in the technological states of E_r and E_{r-1}.

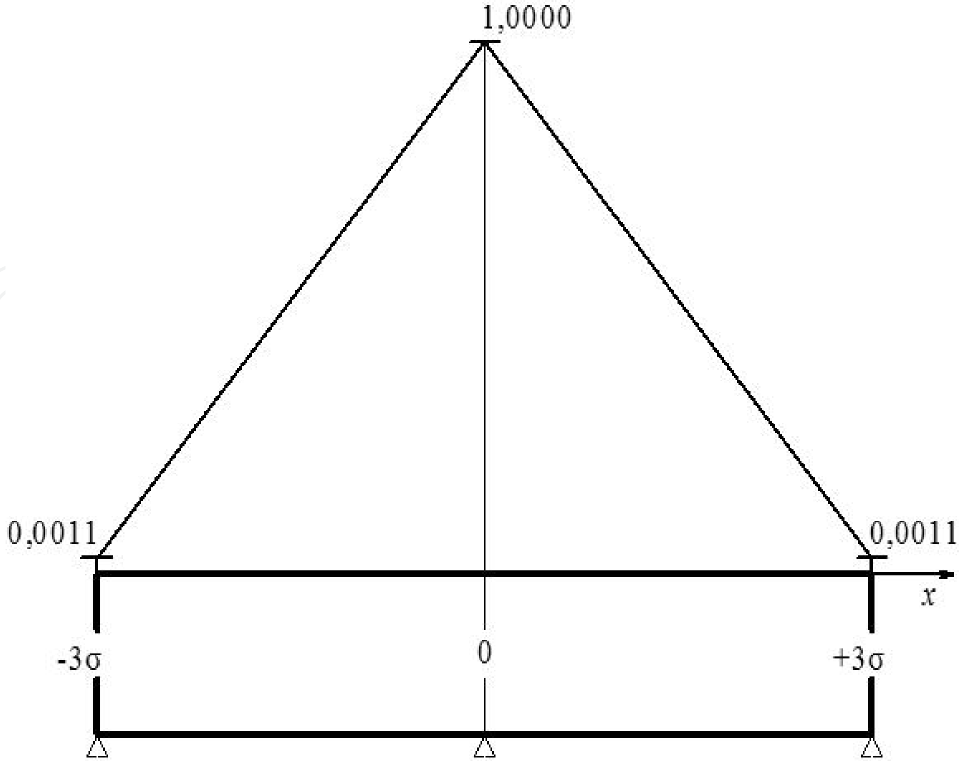

Figure 6. Three-term Gaussian function of membership (Θ = 3).

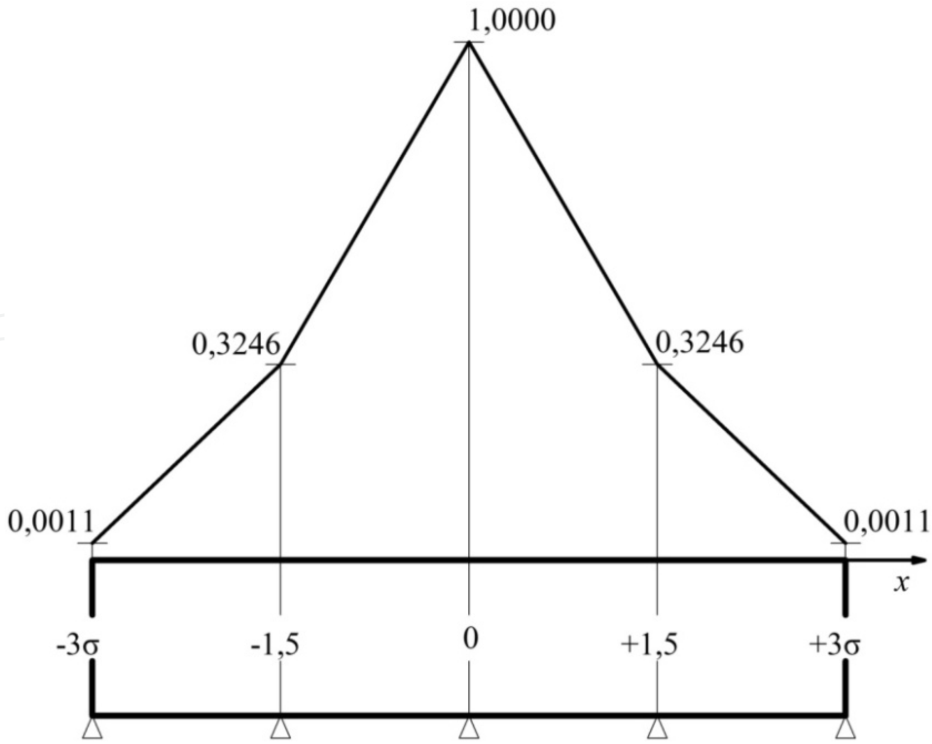

Figure 7. Five-term Gaussian function of membership (Θ = 5).

It is important to note that though fig. 6 in appearance resembles the so-called MF triangular number, but in no case should be confused with it, because of "eine grosse Kleinigkeit" (German) – zero GM value at its left and right edges.

Logical summands of FM (37) – (39) are the GM of terms provided with subscripts or superscripts, except the modal term, which GM always equals to 1. These subscripts and superscripts indicate the number of root-mean-square deviations σ along PI x axis of current terms from the modal term, with relevant sign. Positive deviations are contained in superscripts, negative deviations – in subscripts.

For the majority of practical measurements, it is quite sufficient to evaluate the combined limiting measurement error κ_{r} using three-term FM (37). Combined limiting spread of PI x is most conveniently represented by five-term FM (38) and by seven-term FM (39).

Let us assume that the dimension of the component is checked by a checking measurement system employing a double-limit electric contact sensor, and has FM (37) for the limiting spread of sensor contacts triggering.

$$\mathbf{v}_{sensor} = \left\langle 0.01_{-1} + 1.00 + 0.01^{+1} \right\rangle, \tag{44}$$

figure 8 a, where values −1,0 μm of subscript and +1,0 μm of superscript of GM 0,01 for two utmost terms correspond to combined limiting error ±1 μm of sensor contacts triggering.

Let us assume a priori, in the first approximation, that the spread of the dimension of a component corresponds to FM (39) in the form

$$\mathbf{v}_{comp} = \left\langle 0.01_{-3} + 0.14_{-2} + 0.61_{-1} + 1.00 + 0.61^{+1} + 0.14^{+2} + 0.01^{+3} \right\rangle \tag{45}$$

graphically presented in fig. 8 b.

As seen from FM (39), the width of its carrier in the units of measurement of subscripts and superscripts equals to 6 μm. GM values in formulas (40) and (41) are given with accuracy of two digits after decimal point, which is practically sufficient for performing logical operations (algebraic operations using GM values will not be given here at all).

As a result of this, FM (41) is "fuzzified", creating the combined FM determined by MF summing shown in figure 8.

Then let us proceed with check by measurement. From MF point of view, check operation means alignment of the left ($x_{\ulcorner}$) or, as the case may be, right ($x_{\urcorner}$) thresholds – limits of tolerance zone of component dimension, i.e. FM carrier (38), with the appropriate position of sensor contacts triggering adjusted for each of these thresholds. This alignment causes triggering of sensor contacts, in this case – at the low limit of sensor adjustment, introducing into FM (38) the check error characterized by FM (39). As the result, FM (38) is "fuzzified",

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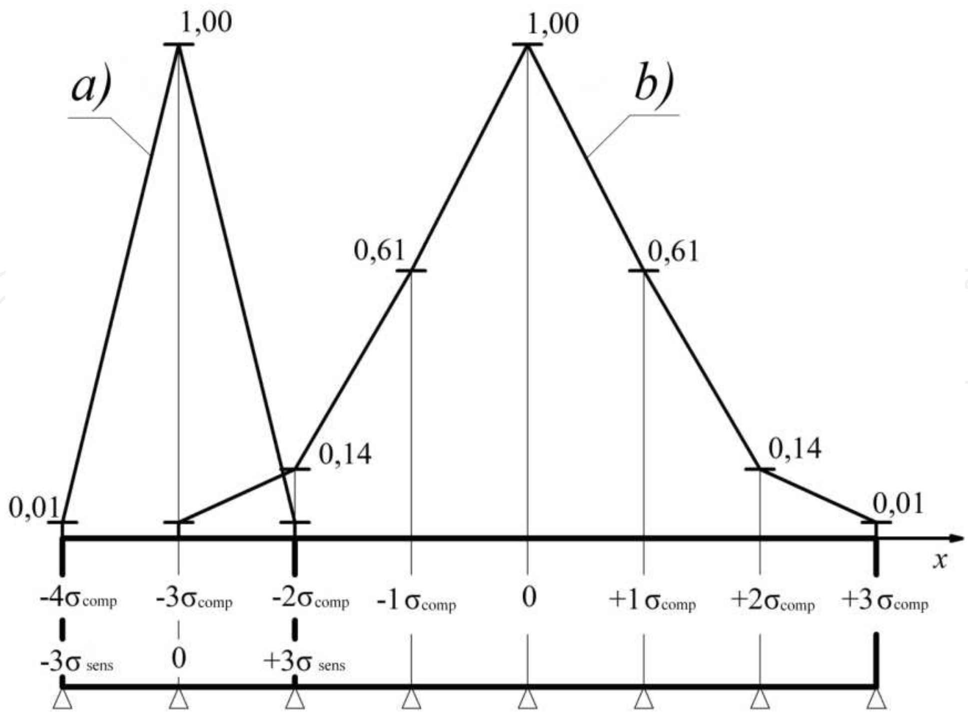

Figure 8. Mathematically fuzzy relationships during check by measurement of component dimensions using electrical contact sensor at the lower limit of tolerance zone. *a* i *b* – functions of appurtenance of electric contact check errors and controlled component dimensions, respectively.

creating the combined FM determined by MF summing shown in figure 8.

Eventually, we get the required sum

$$\left\langle 0{,}01_{-4} + 0{,}01_{-3} + 0{,}14_{-2} + 0{,}61_{-1} + 1{,}00 + 0{,}61^{+1} + 0{,}14^{+2} + 0{,}01^{+3} + 0{,}01^{+4} \right\rangle, \tag{46}$$

which is the seven-term FM (39),"fuzzified" by two terms up to nine-term FM.

This leads to the following conclusions related to quality check by measurement:

1. Adjustment of triggering of any threshold checking device to one of the limits of the specified tolerance zone of PI x of the product causes additional error ω_{sensor}, located symmetrically to the left and to the right of this zone as $\omega_{\text{sensor/2}}$ with MF normalized GM η, which is not over 0,01 (more precisely, 0,0110) for a priori assumed Gaussian FM;
2. If PI x of a product is given as a functional or technological threshold, then the error ω_{sensor} introduced by threshold checking device is located symmetrically to the left and to the right from this threshold, with the same MF indices of precision as for the tolerance zone mentioned above;
3. Manufacturing of a product which quality corresponds to PI x specified by some or other method may be guaranteed by symmetrical respective narrowing of its tolerance zone.

4. In order to increase the accuracy of the results of checking PI of the product $\omega_{\text{sensor/2}}$ at the left side and at the right side, or by the same displacement to the right and to the left of the left threshold $x_{\ulcorner}$ or right threshold $x_{\urcorner}$ specified instead of it, it is necessary to reduce the error ω_{sensor} to reasonable technical-economic limits, while MF normalized GM shall be not over 0.01.

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