Chapter 5

Effects of Voltage Quality on Induction Motors' Efficient Energy Usage

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Additional information is available at the end of the chapter

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1. Introduction

Today, about 50% of electrical energy produced is used in electric drives. Electrical motors consume around 40% of total consumed electrical energy (Almeida et al., 2007) and of that thereof, induction motors account for 96% of energy consumption. Around 67% of this energy is used in induction motors with a rating below 75 kW and it can be shown that 85% of the energy losses are dissipated in these rating motors. Efficiency improvements of constant-speed drives, both constant-torque and variable-torque drives, is very important. It is usual that techniques for efficiency improvements of variable-torque drives are different from those of constant-speed and constant-torque applications. The latter is dealt with through optimization; it is very difficult to design and build a motor with high rating efficiency and rating power factor - it has been shown (Fei et al., 1989) that higher efficiencies are associated with lower power factor. It is especially difficult to design and build a drive operating at high efficiency and power factor over an entire range of loads, say from 25 - 100% of rated load (P_N), i.e. at partial load.

Electrical energy savings in the drive could be realized by improvements of power quality in the consumer network. Term power quality (Linders, 1972; Bonnett, 2000) mostly means quality of supply voltage that should meet the following requirements:

- voltage value (permissive variations are in the range of $U_N \pm 5\%$ of nominal voltage),
- permissible voltage asymmetry is 2% and has greater influence on accurate and economical motor operation,
- permissive total harmonics distortion of voltage is $THD_U \leq 3\text{-}8\%$, where higher values correspond to lower voltage networks.

Power losses and reactive loads depend from on voltage magnitude and they are further increased due to unbalanced voltage and (or) the presence of harmonics in supply voltage.

Unbalance voltage can occur due to the presence of larger single-phase consumers or asymmetrical capacitor banks with damage or capacitors that are switched off due to the fuse burning only in one phase. Nowadays, the presence of higher harmonics in the supply voltage is ever more frequent due to the growth of consumers who are supplied through the rectifiers and inverters: regulated actuators, electrothermical consumers and consumers alike.

The effect of a variation in supply voltage, wave-form or frequency on the motor's efficiency and power factor characteristics depends on the individual motor design. Typical variations of current, speed, power factor and efficiency with voltage for constant output power are given in Fink (1983). The usual characteristics of induction motors within the ±10% voltage band (U_n±10%) are well known. These are included in corresponding table for typical 30-100 kW, 1500 or 1800 1/min motors (Linders, 1972; Fink, 1983), but the effect of saturation has been largely neglected in these tables. However, it is the author's intent to show a correlation between motor characteristics and voltage level.

This proposed has three parts:

1. Study of the effect of voltage magnitude on motor losses and motor reactive loads,
2. analysis of the effect of unbalanced voltages, and
3. analysis of the effect of non sinusoidal voltages to the efficient energy use.

2. Effect of voltage magnitude on motor power losses and motor reactive loads

Voltage magnitude has a significant and different influence on motor loads and electric energy consumption, depending on types, nominal powers and load level ($p=P/P_N$) of motors. Data found in classic references (Fink, 1983) are almost identical to data from older references (Linders, 1972;), although they are not accurate enough for motors made after 1970. The main reason for disagreement is higher values of no-load current and more significant dependency of core losses on voltage magnitude for newer motors ("U" or "T" shape of magnetization curve). Data for loads less than 50% are not available in Fink (1983) and Hamer et al. (1997), although more than half of motors operate in these load regimes.

The influence of voltage magnitude on energy characteristics of standard induction motors (made after 1970.) is significant, as was affirmed by the author's research and verification (Kostic, 1998). It was also ascertained that the changing of energy characteristics is more significant for smaller motors. According to newer literature, efficiency (η) and power factor ($cos\varphi$) dependency on voltage value is more significant than was shown in standard literature.

The paper by Hamer et al. (1997) analyses the effects of voltage magnitude only on two motors (10HP and 100HP) and loads from 50-100%, and the results, and results are illustrated in Fig. 1. Results for standard efficiency motors, given in Fig. 1, are almost equal to the author's results (Kostic, 1998, 2010). A brief theoretical approach for determining dependency of power losses and reactive loads on voltage value will be presented, as well as proceedings for calculation and analysis of power losses and reactive loads on voltage value.

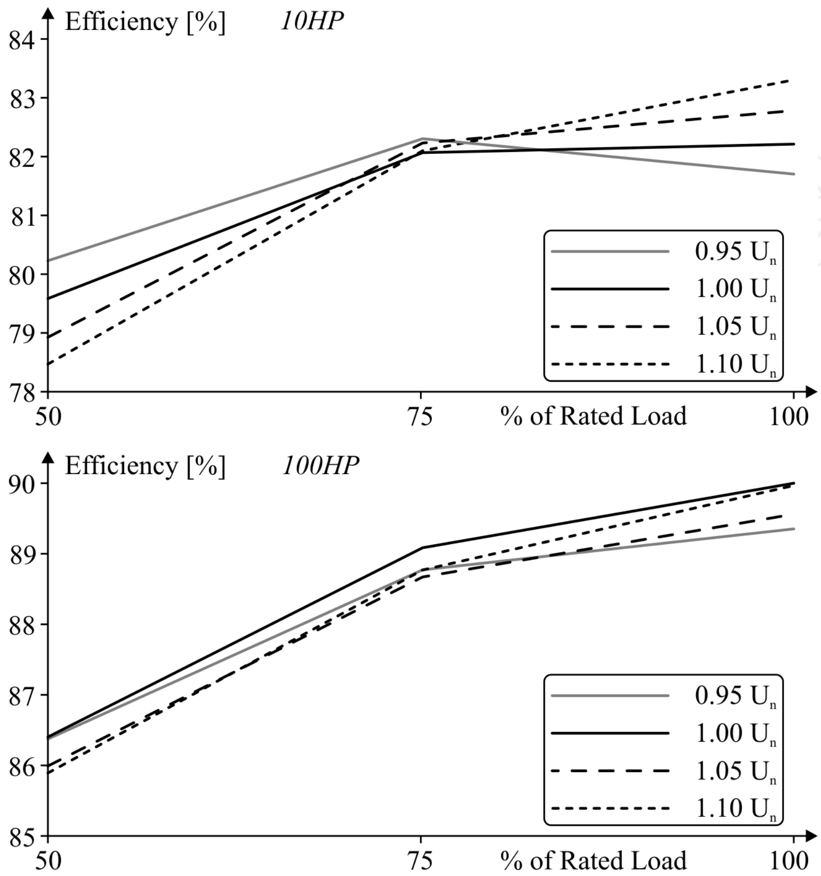

Figure 1. Efficiency versus load for various applied voltages in percent of its 460 V rating for a standard efficiency motor (10 HP and 50 HP)

2.1. Dependency of power losses and reactive loads on voltage value

In order to determine total dependence of power losses on voltage, for the load range from no-load to full load, it is necessary to determine no-load power - voltage dependency, $P_0(u)$:

$$P_0(u) = P_{Cu0}(u) + P_{Fe}(u) + P_{fw} \tag{1}$$

Where are

P_{Cu0} copper losses in no-load,

P_{Fe} core losses in no-load,

P_{fW} friction and windage losses in no-load.

Load losses component (P_{LL}) depend on relative load ($p_L=P_L/P_N$) and relative voltage values ($u=U/U_N$):

$$P_{\gamma P} = P_{LL,N} \cdot p^2 / u^2 \tag{2}$$

where are $P_{LL,N} = P_{Cu,S} + P_{Cu,R} + P_{\gamma ad}$ - a load losses in a nominal regime (P_N, U_N), and $P_{\gamma ad}$ are additional load losses. Load losses, $P_{LL,N}$, can also be calculated as a difference of full load power losses ($P_{\gamma N}$) and no-load power (P_{0N}):

$$P_{LL,N} = P_{\gamma N} - P_{0N} \tag{3}$$

or in per unit ($p_\gamma = P_\gamma / P_N$, $p_0 = P_0 / P$, and $p_{LL,N} = P_{LL,N} / P_N$) as:

$$p_{LL,N} = p_{\gamma N} - p_{0N} \tag{4}$$

Total load losses can be calculated in absolute values as:

$$P_\gamma(p,u) = P_0(u) + P_{LL,N} \cdot p^2 / u^2 \tag{5}$$

or in per unit:

$$p_\gamma(p,u) = p_0(u) + p_{LL,N} \cdot p^2 / u^2 \tag{6}$$

In order to ascertain reactive loads $Q(u)$ dependency, it is necessary to determine no-load reactive power versus voltage, in absolute values ($Q_0(u)$):

$$Q_0(u) = \sqrt{3} \cdot U_0 I_0(u) \cdot \sin\phi_0 \approx \sqrt{3} U_0 I_0(u) \tag{7}$$

Or in per unit values ($q_0\ (u) = Q_0 / P_N$), for the load range from no-load to full load

$$q_0(u) = \frac{\sqrt{3} \cdot u_0 i_0(u)}{\eta_N \cos\phi_N} \tag{8}$$

In the rated regime are: efficiency, $\eta_N = P_N / P_{1N}$, and power factor, $\cos\phi_N = P_N / (\sqrt{3} \cdot U_N I_N)$. Values of reactive power in the load branch, Q_{LN} and q_{LN} are calculated from the quotient of maximum and nominal torque T_m / T_N, as explained in Appendix, (Kostic, 1998, 2001):

$$Q_{LN} = 0.5 \cdot P_N / (T_m / T_N) \tag{9}$$

or in per unit as:

$$q_{LN} = 0.5 / (T_m / T_N) \tag{10}$$

Equations (9) and (10) are obtained by the procedure given in the Appendix, gained from the equivalent Γ-circuit of the induction machine (Kostic, 2001, 2010). Difference of nominal reactive power and no-load reactive power is a little bit less from calculated value of Q_{LN} and q_{LN} because of reactive power reduction on magnetization branch ($\Delta q_{\mu N} = (0.01\text{-}0.10) q_{0N}$). Total reactive load is calculated in absolute values as:

$$Q_1(u) = Q_0(u) + Q_{LN} \cdot p^2 / u^2. \tag{11}$$

or in per unit

$$q_1(u) = q_0(u) + q_{2n} \cdot p^2 / u^2 \tag{12}$$

For motors of nominal power ≤ 3kW, value of nominal reactive power is almost equal to no-load power ($Q_{1N} \approx Q_0$), because $Q_{LN} \approx \Delta Q_{\mu N}$, so $Q_1(u) \approx Q_0(u)$ and $q_1(u) \approx q_0(u)$, (Kostic, 1998, 2010). Expressions (11) and (12) are commonly in use. Instead of Q_{LN} and q_{LN}, ΔQ_N and Δq_N can be used if they are known or if they can be calculated ($\Delta Q_N = Q_{1N} - Q_{0N}$). For calculating the dependency $P_\gamma(u)$ and $Q_1(u)$, according to expressions (5) and (11), it is necessary to know:

- no-load characteristic $I_0(u)$, $Q_0(u)$, $P_0(u)$ and $P_{Fe}(u)$, for the analyzed voltage range,
- motor catalogue data: nominal power (P_N), nominal current (I_N), efficiency (η), power factor ($\cos\varphi$), slip (s_N) and the quotient of maximum and nominal torque (T_m/T_N), and
- $P_{\gamma N}$ and Q_{LN} are calculated.

2.2. Dependency of motor input power and reactive loads on voltage values

Dependencies of input power (P_1/P_{1N}) and reactive loads (Q_1/P_{1N}) versus relative voltage value (U/U_N), for P_{LL}/P_N = 25%, 50%, 75% and 100%, for motors of nominal powers 1 kW, 10 kW and 100 kW, have been determined by the procedure described in chapter A; results are illustrated in the Fig. 2, (Kostic, 1998, 2010). Influence of voltage on reactive loads and power loss is notable, especially for small motors and for lower loads P_{LL}/P_N.

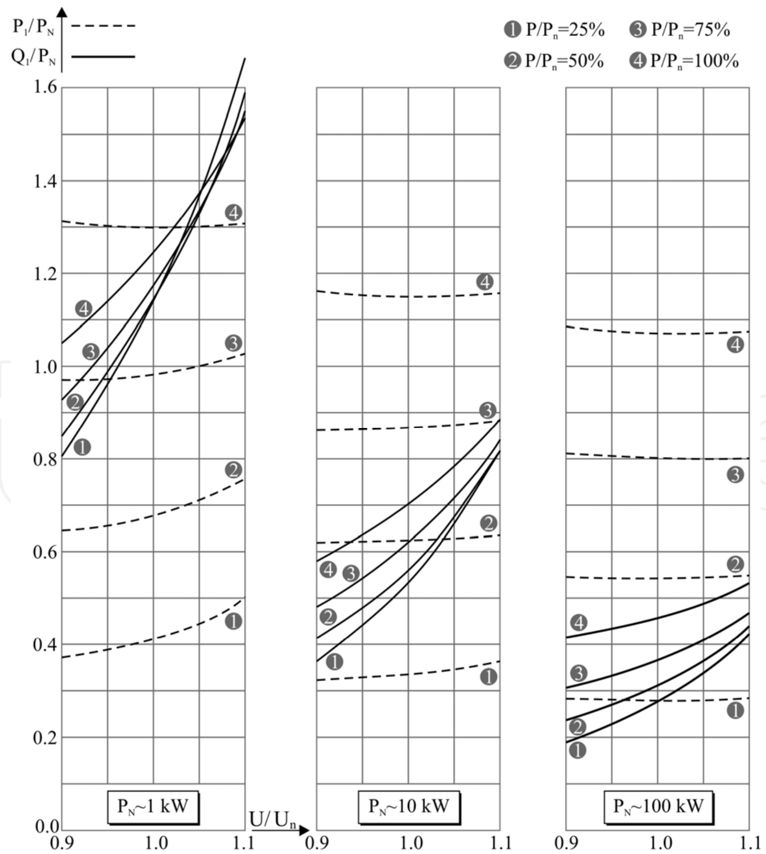

Figure 2. Dependencies of motor input power and reactive loads on supply voltage

Results of the author's research (Kostic, 1998, 2010) confirmed that there are significant possibilities for energy savings by setting voltage values within the voltage band (U_n±5%), because more then 80% of induction motors, especially small and medium power (1 - 30 kW), operate at partial load (≤70%). Dependencies of power loss $P_\gamma(u)$ and reactive loads $Q_1(u)$ for motor of nominal power 2 MW, for P_1 / P_N = 0% (no-load), 25%, 50%, 75% and 100% are given in Fig. 3, (Kostic et al., 2006 and Kostic, 2010).

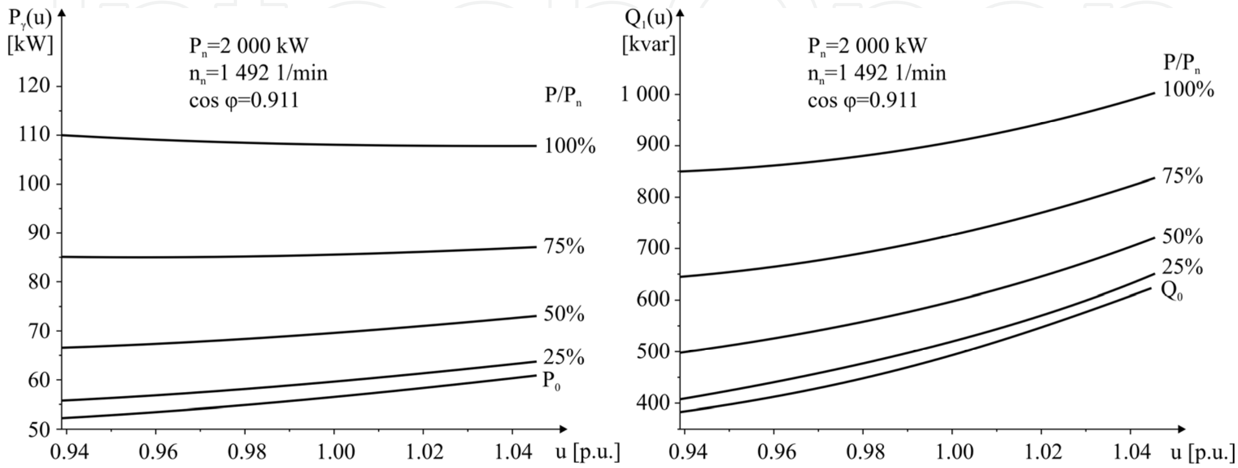

Figure 3. Dependencies of power losses and reactive loads on supply voltage

2.3. Basic reduction of electric energy own consumption of power plants by setting voltage within U_n±5%

Subject of concrete project (Kostic et al., 2006) is reduction of electric energy own consumption of thermal power plant "Nikola Tesla" B, Obrenovac (Serbia), with 2 blocks.. The own consumption of the electric energy, with motors on medium voltage (6.6 kV), is about 90% and with motors on voltage 0.4 kV is about 10%, supplied by a transformer whose primary is directly connected to the generator bus. Nominal powers of the transformers (21 kV/6.6 kV) are approximately equal to total nominal powers of all installed motors which are about 140 MW. Active and reactive loads are about 70 MW and 60 Mvar. As the load of most motors is about 35–70% of full load, reduction of electric energy own consumption could be achieved by setting the voltage magnitude in the range U_n±5%.

Application of this procedure results in reduction: of core losses (P_{Fe}), reactive loads (Q) and active power losses component $P_{CuQ}=RI_Q^2$. Thereby, both active and reactive energy consumptions are decreased. Optimal voltage values are being determined according to appropriate calculations and analysis is based on motor catalogue data and its experimental verification at actual load regimes.

According to the effects of voltage magnitude on motor power losses (P_γ) and motor reactive loads (Q_1), for overall own consumption, the following dependencies can be determined: restrictions:

- motor power losses change $\Delta P_\gamma = P_\gamma(U_i) - P_\gamma(U_N) = \Delta P_1 = P_1(U_i) - P_1(U_N)$, i.e. active loads change, and

- reactive loads change $\Delta Q_1(u) = Q_1(U) - Q_1(U_N)$,

for the voltage range U/U_N = 0.955-1.045, i.e. for U = 6300V-6900V (U_N = 6 600V), Fig. 4.

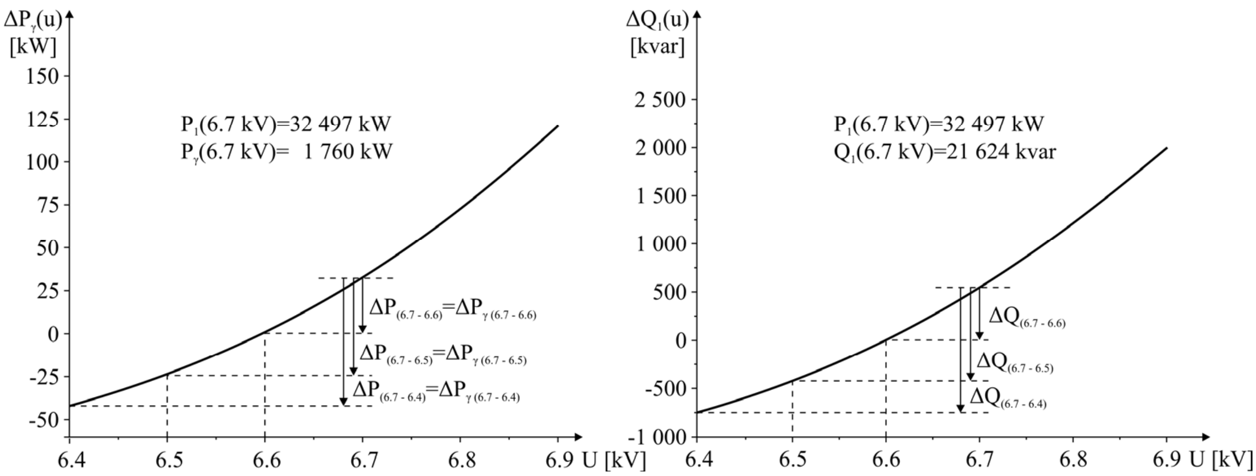

Figure 4. Dependency of total power losses and reactive loads for own consumption

As was shown in Kostic et al. (2006), application of this procedure causes reduction of the electric energy own consumption. Changing voltage value (regulation at own consumption transformers 1BT and 2BT, and at common consumption transformer OBT) from 6.8 kV (1.03 U_N) to 6.6 kV (U_N) or 6.5 kV (0.985 U_N) causes reduction of:

- Active power losses for 161 kW and 213 kW, respectively,
- Reactive loads for 3 544 knar and 4 559 knar.

Power losses addition reduction in the own consumption network for 42.4 kW and 54.7 kW, respectively, due to of the above mentioned of reactive loads' reduction in the own consumption network.

According to values of reduced power losses, reactive loads and assumed operational plan of the thermal plant (estimated 6 000 h/years), savings in active and reactive energy have been determined and given in Table I, (Kostic et al., 2006, and Kostic, 2010).

Consumption objects	Load Reduction		Consumption Energy Reduction	
	Active [kW]	Reactive [kvar]	Active Energy [kWh/year]	Reactive Energy [kvarh/year]
Block 1 Motors	161	3 544	966 000	21 264 000
Block 1 Network	42	-	252 000	-
Block 2 Motors	213	4 559	1 278 000	27 354 000
Block 2 Network	55	-	330 000	-
Total	**471**	**8 103**	**2 826 000**	**48 618 000**

Table 1. Reduction of the active and reactive energy own consumption of Power plant by voltage change, Kostic et al. (2006).

3. Motor voltage asymmetry and its influence on inefficient energy usage

Analysis of the effect of unbalanced voltages on the three-phase induction motor is presented in the paper by Bonnett (2000). Since the unbalanced voltage of 2%, 3.5% and 5% causes an increase in losses, in the same order, for 8%, 25% and 50% of nominal power losses in the motors, it is a reasonable requirement to permit voltage asymmetry ≤2%, so this is the upper limit in most national and international standards. The truth is that with less load, the motor could safely work also at higher values of unbalanced voltage. The literature (Linders, 1972) states that the information given previously is determined from measurements and that they are higher than calculated values. However, it is explained here by the fact that the rotor inverse resistance is higher by 1.41 times compared to the rotor resistance in short-circuit mode (Kostic & Nikolic, 2010), since current frequency of the negative sequence in the rotor winding is twice as high ($f_{r,NS} \approx 2f_1=2f_{r,SC}$), i.e. it is higher by 1.41 times than corresponding values given in the literature. Thus, it is increasingly convincing that the requirements which are given in the most appropriate standards are justified. Performed analysis shows that there are some considerations that should be included in current standards. Motor operation is not generally allowed when voltage asymmetry (U_{NS}/U_N) is higher than 5%, because in the (rare) case that the direct and inverse component of the stator currents in one phase are collinear, increase of the current in that phase would be ≥ 1.38 times, and the increase of the losses in the windings of that phase would be ≥ 90% (Linders, 1972; Kostic & Nikolic, 2010).

The effect of voltage asymmetry on the three-phase induction motor is equivalent to the appearance of negative sequence voltage system that creates a rotating field which rotates contrary to the rotation of the positive sequence field and motor rotating direction. The consequence is that small values of negative sequence voltage produce relatively high values of negative sequence currents. By definition, the coefficient of asymmetry (K_{NS}%) is the ratio of negative sequence voltage (U_{NS}) and positive sequence voltage (U_{DS}). For simplification, Standard NEMA use the following definition of voltage unbalance

Voltage unbalance=100· (maximum voltage deviation from average value)/average value

For instance, for measured voltages of 396V, 399V and 405V, average value is 400V. Then, the highest variation from average voltage is determined (405V – 400V = 5V). At the end, the coefficient (percent) of voltage asymmetry is calculated as the quotient of highest variation and average value: $K_{NS}\% = 100\cdot(5/400) = 1.25\%$. Since percentages of negative sequence currents are 6-10 times higher than corresponding voltage asymmetry, negative sequence currents could reach 10%. This causes additional motor heating and the appearance of inverse torque that reduces starting and maximum motor torque, and causes a small increase of motor slip. Because power losses and motor heating are increased, allowed motor loading is decreased. As the percent of asymmetry rises, the derating factor of nominal power decreases, according to NEMA MG1 (Bonnett, 2000), as shown in Fig. 5.

With an increase of voltage asymmetry coefficient, motor efficiency decreases under all load levels. Dependence of motor efficiency is given in Fig. 6 for the voltage unbalance of 0.00%, 2.50%, 5.00% and 7.50% (Bonnett, 2000).

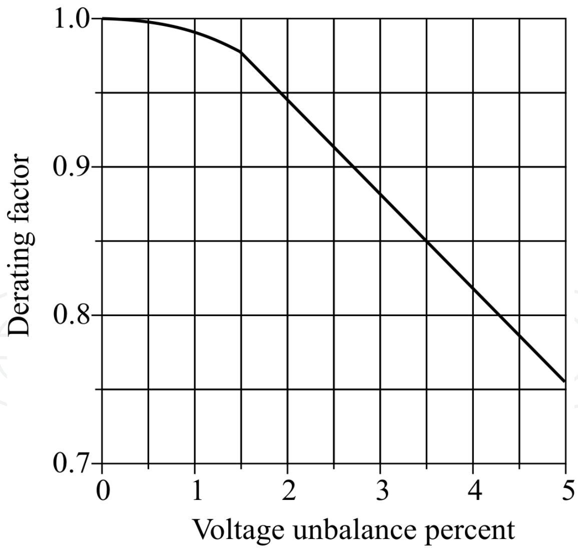

Figure 5. Relation between derating factor and voltage asymmetry

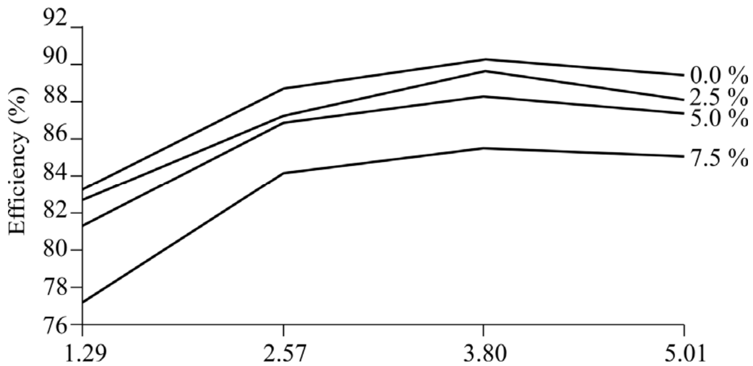

Figure 6. Motor efficiency in dependence of motor load for different voltage asymmetries

Electrical energy consumption is unnecessarily increased due to the lower motor efficiency, so maintenance of low voltage asymmetry (≤ 2%) is a measure of efficient energy usage. In that case, the influence of voltage asymmetry (negative sequence voltages) will be presented in detail in the paper as follows. At first, a procedure for calculation and analysis of negative sequence currents and corresponding power losses will be presented. Then, evaluation of voltage unbalance that could arise in the considered consumer network is presented.

3.1. Equivalent circuit of induction motor for negative sequence

When the induction motor is supplied from the network with asymmetrical voltages, then the three-phase voltage system should be decomposed to positive, negative and zero sequences. Further, using equivalent motor circuits (Boldea & Nasar, 2002; Ivanov-Smolensky, 1982) separately for positive (Fig. 7a) and for negative sequences (Fig. 7b), calculations and analyses of motor energy and operation characteristics (currents, power losses, torques) are performed. At the end, with corresponding superposition of relevance values, their overall values are obtained. Only in that way could real (overall) values of motor power losses be determined.

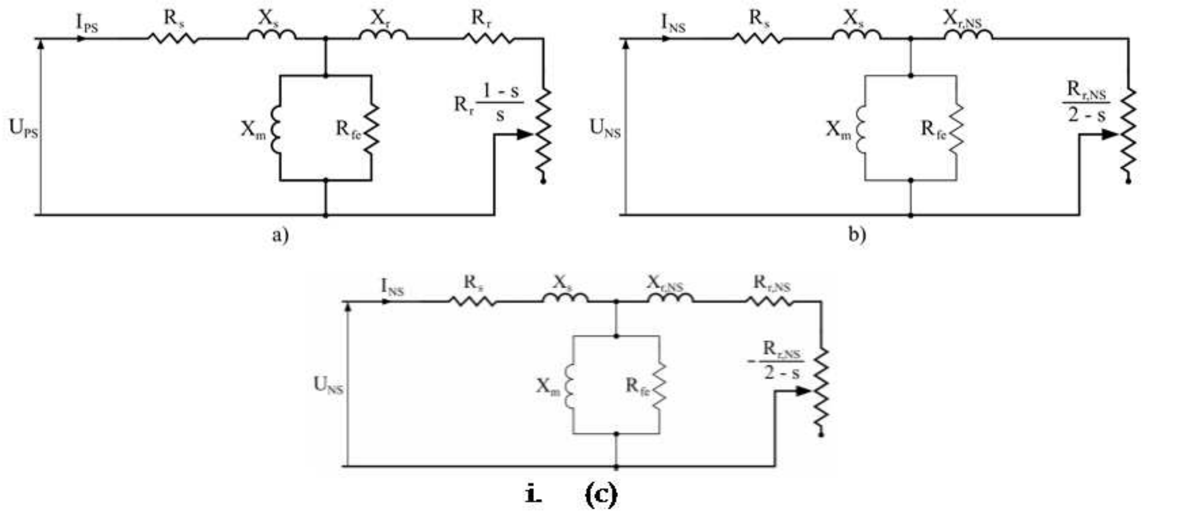

Figure 7. Equivalent circuits of induction motor a) positive and b) negative voltage sequence, and c) completely circuit for negative sequence voltage

3.2. Parameters of equivalent circuit for negative sequence

Stator winding resistance (R_s) and reactance (X_s) are almost the same for positive and negative voltage sequences. The parameters of equivalent circuits that are correlated to the rotor side and negative sequence voltage system (resistance $R_{r,NS}$ and reactance $X_{r,NS}$) are substantially different than those for positive voltage sequences, because the frequencies of the rotor currents in negative sequences are higher for 50÷100 times:

$$f_{NS} = f_1 \cdot (2 - s) \approx 2 f_1 \succ\succ s \cdot f_1 \tag{13}$$

(For example, for load slip s=0.01÷0.05: $f_{r,NS}=(1.90÷1.98)f_1 >> f_{r,PS}=s \cdot f_1=(0.01÷0.05)f_1$).

Values of resistance $R_{r,NS}$ and reactance $X_{r,NS}$ are determined from the corresponding parameters in the short-circuit regime, $R_{r,SC}$ and $X_{r,SC}$. In Boldea & Nasar (2002) and Ivanov-Smolensky (1982) it is noted that values of corresponding resistances and reactances are approximately equal to those in the short-circuit regime: $R_{r,NS} \approx R_{r,SC}$ and $X_{r,NS} \approx X_{r,SC}$.

If we look carefully, we could find that this statement is not correct. Those parameter changes are dependent on the ratio of the rotor conductor height (H_b) and field penetration depth ($\partial_{r,SC} = (2 \cdot \rho/(\mu \cdot 2 \cdot \pi \cdot f_1))^{1/2}$, i.e. the quotient $\zeta = H_b/\partial_{r,SC}$; where: f_1 – frequency of supply voltage first order harmonic, ρ - conductor specific resistance, $\mu = \mu_0$ – magnetic permeability of conductor.

By Fig. 8 (Kostic, 2010), for $\zeta_{SC} = H_b/\partial_{SC} = 1.2÷3$ are obtained: $R_{r,SC} =1÷3R_r$ and $L_{r,SC} =1÷0.50L_r$, where R_r and L_r are resistance and inductance in a low slip regime, respectively, for instance in a nominal regime. Since current frequency of the negative sequence in the rotor conductor (bar) is twice as high ($f_{r,NS} \approx 2f_1=2f_{r,SC}$), the penetration depth of those currents is lower by √2 times then in a short-circuit regime. Corresponding values of resistance and reactance are higher by √2 times:

$$R_{r,NS} = R_{r,SC} \cdot \sqrt{2} \tag{14}$$

$$X_{r,NS} = X_{r,SC} \cdot \sqrt{2} \tag{15}$$

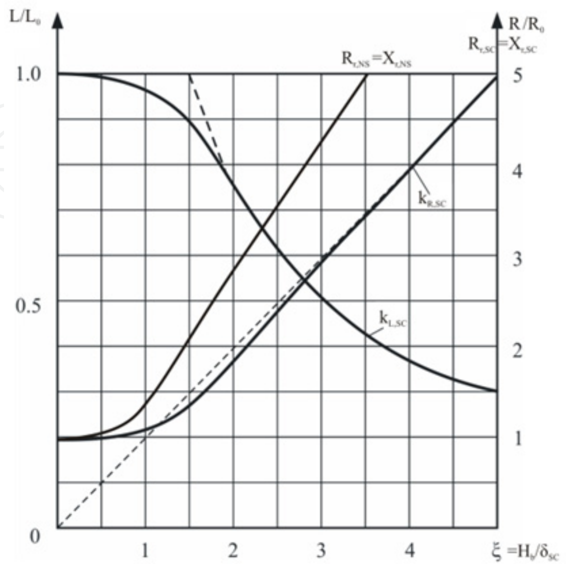

Figure 8. Dependence of rotor resistance and inductance from ratio ζ=$H_b/\partial_{r,SC}$

The explanation is the following: for motors with powers higher than 5 kW (or with relative rotor conductor height $\xi_{SC} = H_b/\partial_{SC} \geq 1.2$), already in short-circuit mode, the rotor induced currents not the entire cross section, or rotor conductor (bar) height H_b, (Kostic, 2010). From that it could be concluded that for the negative sequence currents ($I_{r,NS}$ with frequency $f_{r,NS} \approx 2f$) it is used for $\sqrt{2}$ times lower part of the section of the rotor conductor and rotor resistance and rotor reactance are higher by √2 times, Fig. 8.

Since usually values of rotor resistance (R_r) and reactance (X_r) are known in the nominal regime, then it is necessary to know values of the coefficient of rotor resistance increase (k_R>1) and the coefficient of rotor inductance decrease ($k_L < 1$), both in the short-circuit regime. From (14) and (15), the values for $R_{r,NS}$ and $X_{r,NS}$ could be calculated as:

$$R_{r,NS} = k_{R,NS} \cdot R_r = \sqrt{2} k_{R,SC} \cdot R_r \tag{16}$$

$$X_{r,NS} = k_{X,NS} \cdot X_r = \sqrt{2} k_{X,SC} \cdot X_r \tag{17}$$

where values of coefficients $k_{R,NS}$ and $k_{X,NS}$ are determined from Fig.8 for previously established ratio value $\zeta_{NS} = H_b/\partial_{NS}$, where is ∂_{NS} –penetration depth of negative sequence field in rotor conductor. In such a manner approximate values of coefficients k_R and k_X are determined, for different rotor bar height H_b (mm)= 15, 20, 30, 40, 50 and given in Tab. 2.

From the quantitative review of corresponding values for $k_{R,SC}$ and $k_{R,NS}$, and $k_{X,SC}$ and $k_{X,NS}$, it could be seen that valid relations are: $k_{R,NS} \approx 1.41 \cdot k_{R,SC}$ and $k_{X,NS} \approx k_{R,SC}/1.41$, and it could be concluded that relations (14) and (15) are correct. In that way the author's statement that **"it**

is not correct to believe that values of corresponding resistances and reactances for negative sequence currents are approximately equal to those for motor short-circuit regime" is confirmed**,** as it is quoted in the literature (Boldea & Nasar, 2002; Ivanov-Smolensky, 1982). To the contrary, it is only correct that those values are in relation by (16) and (17). It is useful to specify common values for rotor conductor (bar) height H_b (mm) and frame sizes (axial height) for standard induction motors, as given in Tab. 3. From these facts more precise calculations and analyses of negative sequence voltage (and negative sequence currents) influencing the motor operation could be performed (Kostic & Nikolic, 2010).

Rotor slot depth H [mm]	15	20	30	40	50
$K_{R,SC}$	1.30	2.00	3.00	4.00	5.00
$K_{R,NS}$	2.00	2.80	4.20	5.60	7.00
$K_{X,SC}$	0.90	0.75	0.50	0.38	0.30
$K_{X,NS}$	0.75	0.54	0.36	0.27	0.22
$\partial_{Al,SC}$ = 10 mm; $\partial_{Al,NS}$ = 7 mm					

Table 2. Coefficients k_R and k_X in short-circuit regime and negative sequence currents

Nominal power, Pn [kW]	1.1 - 2.2	3 - 7.5	11 - 18.5	22 - 45	55 - 160	200 - 355
Axial height [mm]	80 - 90	100 - 112	132 - 160	180 - 200	225 - 280	315 - 400
Rotor slot depth, H [mm]	13 - 17	18 - 22	24 - 34	35 - 44	40 - 50	40 - 50

Table 3. Usual values for rotor bar height H_b, and frame sizes for standard induction motors

3.3. Negative sequence currents and power losses

The negative effect on the motor operation due to the presence of negative sequence voltage is obvious for two reasons:

- it induces negative sequence currents that produces losses in the stator and rotor windings, i.e. on the stator (R_s) and rotor ($R_{r,NS}$) resistance, and
- inverse torque appears which is opposite to the motor operative torque.

It is useful to express the value of negative sequence current ($I_{1,NS}$) in the units of nominal positive sequence current $I_{1N,PS}$:

$$\frac{I_{1,NS}}{I_{1n}} = \frac{U_{1,NS} / Z_{M,NS}}{U_{1N} / Z_{M,n}} \approx \frac{U_{1,NS} / X_{M,NS}}{U_{1,n} / Z_{M,N}} = \frac{U_{1,NS}}{U_{1N}} \cdot \frac{1}{X_{M,NS}} \approx (6 \div 8) \cdot \frac{U_{1,NS}}{U_{1N}} \tag{18}$$

since negative sequence motor impedance $Z_{M,NS} \approx X_{M,NS} \approx$ (0.8-0.9) $X_{M,SC}$ and motor short-circuit reactance $X_{M,SC} \approx$ (0.15÷0.20) $Z_{M,N}$, where $Z_{M,N}$ is motor impedance in the nominal regime. It could be seen from (18) that negative sequence currents in stator and rotor windings are 5÷8 times higher from the values of negative sequence voltage coefficients ($U_{1,NS} / U_n$). Since negative sequence currents are not dependent on motor load and slip, and then we suggest calculating the coefficient of asymmetry in the units of nominal motor voltage. In the following calculations and analyses the value X_{NS} =0.13 (or X_{SC} = 0.16, i.e. I_{SC}/I_N=6.25) is used, so:

$$\frac{I_{1,NS}}{I_{1n}} = 7.7 \cdot \frac{U_{1,NS}}{U_{1N}} \tag{19}$$

The value of increased losses in the phase windings of stator is proportional to the square of negative sequence currents, and then from (19) it could be calculated as:

$$\frac{P_{CuS,NS}}{P_{CuS,n}} = \left(\frac{I_{1,NS}}{I_{1N}}\right)^2 = 60 \cdot \left(\frac{U_{1,NS}}{U_n}\right)^2 \tag{20}$$

The percentage of losses in rotor conductors could be higher by up to 3-6 times (2-5 times due to the higher rotor resistance for negative sequence currents and further up to 1.2 times due to the additional increase of rotor winding temperature under such a high power losses), i.e. $R_{r,NS} \approx R_{r,SC}$=(2÷6) R_r. The equation for losses calculation in the rotor conductors for $R_{r,NS} = 5R_r$, is:

$$\frac{P_{Cur,NS}}{P_{Cur,n}} = 5 \cdot \left(\frac{I_{1,NS}}{I_{1N}}\right)^2 = 300 \cdot \left(\frac{U_{1,NS}}{U_N}\right)^2 \tag{21}$$

Assuming that negative sequence impedance is $Z_{SC,NS} \approx X_{SC,NS} = 0.13$ (or I_{SC}/I_N=7.7) and negative sequence rotor resistance is $R_{r,NS} = 5\, R_r$, the power losses values are calculated for voltage asymmetry of 2.5%, 3.5% and 5%. Such calculated values from (20) and (21), in percent of nominal losses, for particular motor parts (stator, rotor) and whole motor, are given in Tab. 4. Similar data are given in Linders (1972) where it was noted that measured values are 50% higher than calculated values. This difference was explained by an additional temperature increase of the rotor conductor, i.e. an increase of rotor resistance. Although an additional increase of rotor temperature by more than 100°C is not possible. The mentioned difference of measured and calculated values in Linders (1972) can be explained only by the fact that real rotor inverse resistance is 40% higher (based on (16) then its conventional value. Calculated values for permitted motor load are similar to those provided in NEMA standards (Fig. 5).

Negative sequence voltage [%]	0.0	2.0	3.5	5.0
Negative sequence current [%]	0.0	15.0	27.0	38.0
Stator current (RMS) [%]	100.0	101.0	104.0	107.5
Increased stator winding losses [%]	0.0	2.4	7.4	15.0
Increased rotor winding losses [%]	0.0	12.0	37.0	75.0
Increased iron losses [%], Fig. 8	0.0	2.5	7.5	15.0
Increased total motor losses [%]	0.0	5.5	17.0	34.0
Permissible motor load P/P_n [%]	100	97	91	81

Table 4. Influence of negative sequence voltage on permissible motor load ($P_N \geq 100$ kW)

Given the pessimistic assumption, especially for **smaller motors' power (up to 10 kW)** when the rotor resistance is increased only by 2-3 times (up to 1.5-2.5 times due to the higher rotor resistance for negative sequence currents and even up to 1.2 times due to additional increase in temperature of the conductor rotor in such a large loss of power, i.e. $R_{r,NS} \approx 1.4\, R_{r,SC} = (2 \div 3)\, R_r$, calculation of losses in the rotor conductors, for $R_{r,NS} = 3R_r$, was conducted by the expression:

$$\frac{P_{Cur,NS}}{P_{Cur,n}} = 60 \cdot \left(\frac{I_{1,NS}}{I_{1n}} \right)^2 = 180 \cdot \left(\frac{U_{1,NS}}{U_n} \right)^2 \tag{22}$$

since the inverse of impedance $Z_{1,NS} \approx X_{NS} \approx 0.9\, X_{SC} = 0.13$ (i.e. when the motor short-circuit reactance $X_{SC} = 0.143$, or $I_{SC} / I_n = 7$). Thus obtained data are given in Tab. 5 and they are more accurate for motors with power below 10kW. Based on these calculations, it was found that the effect of unbalanced voltage on power losses is smaller for motors with nominal power ≤ 10 kW. Thus, acceptable voltage asymmetry for these motors could be 3%.

Negative sequence voltage [%]	0.0	2.0	3.0	5.0
Negative sequence current [%]	0.0	15.0	22.0	38.0
Stator current (RMS) [%]	100.0	101.0	102.0	107.5
Increased stator winding losses [%]	0.0	2.4	5.4	15.0
Increased rotor winding losses [%]	0.0	7.0	16.0	45.0
Increased iron losses [%], Fig. 8	0.0	2.5	7.5	15.0
Increased total motor losses [%]	0.0	4.0	9.0	25.0
Permissible motor load P/P_n [%]	100	98	95	87

Table 5. Influence of negative sequence voltage on permissible motor load ($P_N \leq 10$ kW)

Based on data given in Tab. 5, it is concluded that the effects of unbalance on increased power loss is less compared to the data specified in the relevant standards for motors of nominal power ≤ 10 kW. Thus, motor total losses increase is 9% (Tab. 5) for the voltage asymmetry of 3%. As it is less then permissible 10% for these motors, for the networks with motors below 10 kW may be accept a voltage asymmetry ($U_{1,NS} / U_N$) ≤ 3%.

Although the unbalanced voltage losses in the rotor conductors are much higher than the corresponding losses in the stator winding, the increase to the losses of one phase of the stator can be the greatest. Specifically, it is unfavorable in the case where the direct and inverse current component could be in phase in one phase. Current in this phase, at the voltages' unbalance of 5%, would be:

- at nominal load $I = I_{PS} + I_{NS} = I_N + 0.38\, I_N = 1.38\, I_N$, while
- at 80% of the motor load: $I = I_{PS} + I_{NS} = 0.8\, I_N + 0.38\, I_N = 1.18\, I_N$.

These corresponding power loss values, respectively, were higher than the nominal 100% $(1.38^2\text{-}1) = 90\%$ and 100% $(1.18^2\text{-}1) = 39\%$. Then the current increase in that phase would be 1.38 times at the negative sequence voltage $U_{1,NS} / U_N = 5\%$ and increase of the losses in the windings of that phase could reach 90% = 100% (1.38^2-1). Otherwise, in practice it could rarely be the case when direct and inverse components of the current matching phase angle. For example, it is necessary to stress that the asymmetry is a consequence of only one phase voltage deviation of the values (not phased by the angle) and the phase angles of the direct and inverse impedance are a little different, which is rarely filled because tan(ϕ_{NS})= 0.3 ÷ 0.4. But it is possible that the phase angle between these components is about 30^0, and a corresponding increase in this phase will be lower:

- current increase would be in the analyzed cases 1.235 I_N and 1.079 I_N, and
- corresponding increase in losses in this phase would be 52.5% and 16.4%.

respectively, for nominal load and 80% load, both for voltages' unbalance of 5%.

For these reasons the motor operation is not allowed when the values of the coefficient of negative sequence voltage is $U_{1NS} / U_N \geq 5\%$.

Experimental measurements (Boldea & Nasar, 2002) showed that the effect of unbalanced voltages on the iron losses increase more if the motor is powered with high voltage, Fig. 9:

- loss increase in iron is 25W (or 15%) and the unbalanced voltage of 5% and nominal voltage, while
- loss increase in iron is 35W (or 25%) for voltage asymmetry of 5% and the 110%voltage.

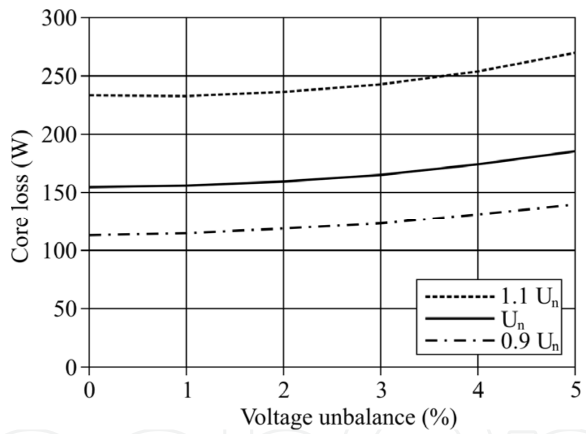

Figure 9. Dependence of iron losses on voltage asymmetry for three values of supply voltage

This additionally has an influence on reducing the coefficient of nominal power (derating factor), as well as reducing motor efficiency and increasing power consumption.

3.4. Causes and evaluation of inverse voltage values

By definition, the coefficient of asymmetry is the relationship between the inverse system voltage (U_{NS}) and direct voltage systems (U_{PS}). Thus, the percentage of unbalanced voltage is calculated using the formula:

$$K_{NS}\% = 100\frac{U_{NS}}{U_{DS}} \tag{23}$$

where the direct and inverse system voltage component are calculated using the formula

$$U_{PS} = \frac{U_{ab} + aU_{bc} + a^2U_{ca}}{3} \tag{24}$$

$$U_{NS} = \frac{U_{ab} + a^2U_{bc} + aU_{ca}}{3} \tag{25}$$

Thus, in the case of symmetrical voltages at the motor $\underline{U}_{ab} = a^2\underline{U}_{bc} = a\underline{U}_{ca}$, we get $U_{PS} = U_{ab} = U_{bc} = U_{ca}$ (24) and U_{NS}=0 (25).

Asymmetry can arise for several reasons. One is the joining of large consumers to one or two phases. Thus, if a consumer who connected to one phase, e.g. phase "a", creates a voltage drop $\Delta U = 3\%$, then it causes the asymmetry of 1% (U_{NS}= 1%), since the asymmetrical voltage system can be presented as the sum of the symmetric system voltage ($U_{ab} = U_{bc} = U_{ca}$) and the unbalanced system voltages ($U_{ab} = \Delta U = 3\%$, $U_{bc} = 0$, $U_{ca} = 0$). This second voltage system, according to (25), corresponds to unbalanced system voltage of $U_{NS} = 1\%$. If a purely inductive consumer is connected between two phases, so that the voltage levels on each phase of the impedance network is 3%, then a similar analysis leads to the conclusion that this causes asymmetry of 2% (U_{NS} = 2%), at the motor connections. These cases are possible in practice and rarely exceed the specified quantitative values.

Asymmetry may be a consequence of fuse capacitor burning. The fall out of capacitors part between two phases, concerning of the voltage reduction, is equivalent to appear of inductive loads between the two the same phases. As a consequence of that is the appearance of the inverse voltage, which is value equal to 2/3 change of the phase voltages mentioned. The general assessment is that it is almost always the asymmetry coefficient $K_{NS} < 2\%$, except for the interruption of one phase in the network, or interruption in any of the motor phase windings.

3.5. Motor inverse torque

The system of negative sequence voltages leads to the appearance of the inverse torque ($T_{em,NS}$), which is opposed to the torque that drives the motor (the torque that comes from the direct system voltages and currents, $T_{em,PS}$). Resultant driving torque is reduced, T_{em}= $T_{em,PS}$ - $T_{em,NS}$ and the direct torque must be increased to compensate that decrease. Therefore, the slip and direct current systems are increased, and also the corresponding power losses. Direct ($T_{em,PS}$) and inverse ($T_{em,NS}$) electromagnetic torques is:

$$T_{em,PS} = \frac{3U_{1PS}^2 R_r}{s\Omega\left[(R_s + R_r / s)^2 + X_{M,SC}^2\right]} \tag{26}$$

$$T_{em,NS} = \frac{3U_{1NS}^2 R_r / (2-s)}{s\Omega\left[(R_s + R_r / (2-s))^2 + X_{M,SC}^2\right]} \tag{27}$$

Where are values R_r, R_s, $Z_{M,N}$, $X_{M,SC}$, $Z_{M,SC}$ are defined in in 3.3 (page 12).

Assuming that $R_{rNS}=R_{rPS}=R_r$ and $R_r/(2\text{-}s) = R_r/2$ and with less neglect, leads to a relationship:

$$\frac{T_{em,NS}}{T_{em,PS}} \approx (\frac{U_{1NS}}{U_{1PS}})^2 \cdot \frac{Z_{PS}}{2X_{SC}} \tag{28}$$

Sometimes it is convenient to express the inverse torque in units of the nominal torque i.e.:

$$\frac{T_{em,NS}}{T_{em,N}} \approx (\frac{U_{1NS}}{U_{1N}})^2 \cdot \frac{Z_{M,N}}{2X_{SC}} \approx 3 \cdot \left(\frac{U_{1NS}}{U_{1N}}\right)^2 \tag{29}$$

This means that, for the coefficient of unbalanced voltage $U_{1,NS} / U_N = 0.04$, would be $T_{em,NS}/T_{em,N} = 3 \cdot 0.04^2 = 0.0048 = 0.48\%$. Slip and power losses will also be increased for 0.0048 times, or 0.5%. But if we assume that the inverse rotor resistance for 2-5 times higher (up to 2-4 times due to the higher rotor resistance for negative sequence currents and even up to 1.2 times due to additional increase in temperature of the conductor), then the expression for the relative values of the inverse torque is:

$$\frac{T_{em,NS}}{T_{em,N}} \approx (6 \div 15) \cdot \left(\frac{U_{1NS}}{U_{1N}}\right)^2 \tag{30}$$

The lower value of the coefficient related to the motor power up to 10 kW, and the upper value for motor power above 100 kW. When coefficient of unbalanced voltage $U_{1,NS}/ U_N = 0.05$, inverse torque in units of the nominal torque could be at $T_{em,NS}/T_{em,N} = 10 \cdot 0.05^2 = 0.025 = 2.5\%$. Slip and power losses will be increased also for 0.025 times, or 2.5% P_N. As this is a medium power motor, with $\eta_N \approx 90\%$ or with losses of $P_{\gamma N} \approx 10\%$ P_N power losses are increased by $\Delta P_\gamma = 25\%$ $P_{\gamma N}$, which is less than half of the determined value of increasing losses $\Delta P_{\gamma, NS} = 50\%$.

The explanation lies in the fact that in this way (i.e. through the power that is allocated to resistance $R_{r,NS}/2$, Fig.7c) covers only half the power losses in resistance $R_{r,,NS}$ while the remaining half of the compensated part of mechanical power (P_m), which is obtained through the axis of direct voltage system, Fig. 10. Thus, another procedure confirmed adequate accuracy of quantitative estimates given in Table 5.

4. Influence of motor non-sinusoidal voltage on efficient energy usage

Analysis of the effect of non-sinusoidal voltages on the three-phase induction motor is presented in this chapter. Power losses and reactive load are increased due to the presence of harmonics in supply voltage. Nowadays, the presence of harmonics in the supply voltage is even more frequent due to the growth of consumers which are supplied through the rectifiers and inverters.

Two interesting cases have been analyzed (Kostic M. & Kostic B., 2011).

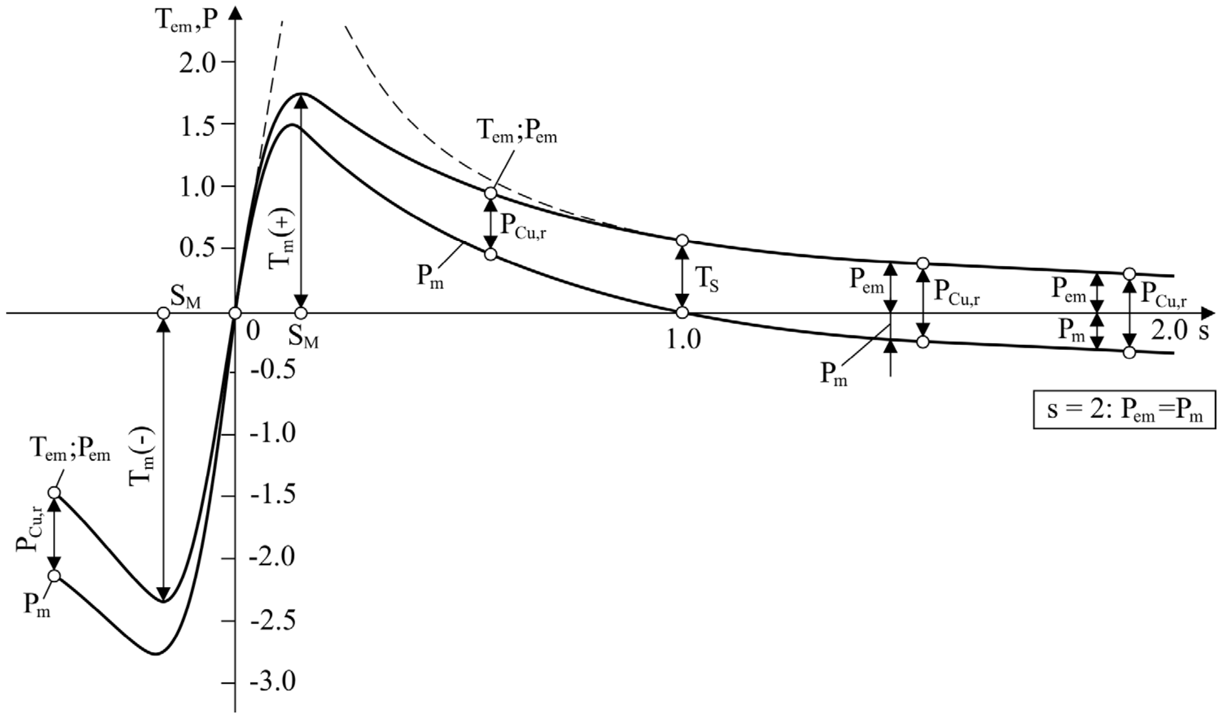

Figure 10. Electromagnetic torque characteristics (T_{em}) and the electromagnetic power (P_{em}) in the following regimes: generator ($s < 0$), motor ($0 \le s \le 1$) and braking ($s > 1$), as well as at point $s = 2$ (for the inverse voltage)

1. The case that the voltage, containing harmonics of order $h = 1, 5, 7, 11, 13, 17, ..., 37$, whose amplitudes are equal $U_h = 5\%$.
2. As the induction motors are supplied by a rectangular shape of the voltage inverter with high levels of harmonic voltage ($U_{h,i} = 1/h_i$).

The reason for this (new analysis) lies in the fact that it is (wrongly) believed that the resistance of the smaller motor's rotor does not change for higher harmonic frequencies, i.e. that is identical for all harmonics ($R_{r,h} = R_{r,1} = R_r = Const.$), which brings the difference mentioned above – and error. It is shown in Kostic M. & Kostic B. (2011) that values of the rotor resistance and the rotor reactance in a short-circuit regime could be estimated on the basis of the induction motor's catalogue date. Two important conclusions are established (probably for the first time) in Kostic (2010):

1. that rotor slot resistance and the rotor slot reactance values, $R_{r\text{-}sl,SC}$ and $X_{r\text{-}sl,SC}$, are practically equally in a short-circuit regime, and
2. that they have approximately equally values, per unit, for all motors of the same series.

4.1. Equivalent circuit of induction motors for harmonics

The equivalent circuit for the harmonics is identical to the corresponding equivalent circuit for a short-circuit regime for fundamental frequency f_1. Only, instead of rotor short-circuit resistance (R_{isk}) and rotor short-circuit reactance (R_{isk}), the two rotor resistances, rotor end (R_{ah}) and rotor slot resistance ($R_{r\text{-}sl,h}$), and two rotor reactances, rotor end ($X_{r\text{-}e,h}$) and rotor slot reactance ($X_{r\text{-}sl,h}$), are used for the harmonics order (h), Fig. 11a (Kostic M. & Kostic B., 2011).

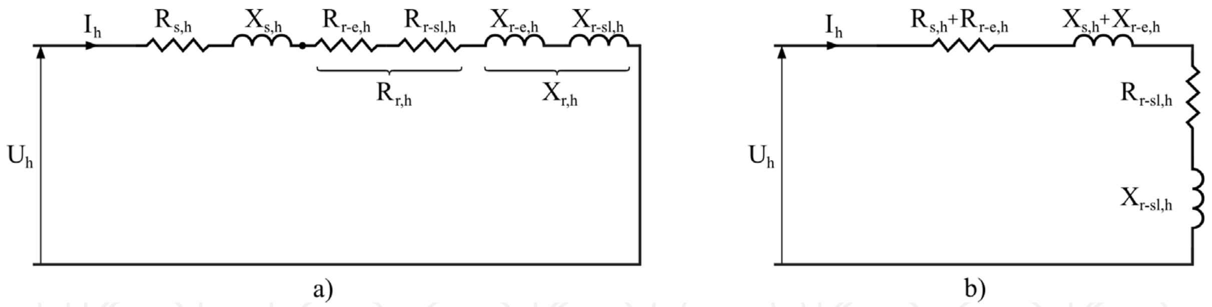

Figure 11. Motor equivalent circuit for harmonics: a) with separate rotor resistances and rotor reactances and b) with grouped motor resistances and motor reactances

Increasing the order of harmonic causes increased frequency of induced currents in rotor conductors, compared to the one in the short–circuit regime. The skin effect is practically present only in the part of the conductor in the slot of the rotor, i.e. it leads only to an increase of rotor slot resistance ($R_{r\text{-}sl,h}$) and a reduction of rotor slot inductance ($L_{r\text{-}sl,h}$). As the depth of penetration is $\partial_{Al,h=5}$ = 4.5 mm, already for the fifth harmonic, $R_{r\text{-}sl,h}$ is always equal to $X_{r\text{-}sl,h}$, Fig. 12. For this reason, similar to the corresponding scheme for short-circuit mode (Kostic, 2010), the rotor reactance ($X_{r,h}$) and resistance ($R_{r,h}$) are separated into two components in the equivalent circuit for the harmonics, Fig. 11a, i.e.:

$$R_{r,h} = R_{r-sl,h} + R_{r-e,h} \tag{31}$$

$$X_{r,h} = X_{r-sl,h} + X_{r-e,h} \tag{32}$$

where $R_{r\text{-}e,h}$ is rotor winding end resistance and $X_{r\text{-}e,h}$ is rotor winding end reactance ("e" in the index comes from the abbreviation of the word "end").

Finally, resistance and reactance of stator windings, and resistance and reactance of rotor conductors outside of slots are grouped (Fig. 11b), for which the influence of skin effects can be neglected from the nominal regime to the short–circuit regime. These are:

- grouped resistance $R_{s,h} + R_{r\text{-}e,h}$ and
- summary reactance, $X_{s,h} + X_{r\text{-}e,h}$.

The remaining resistance $R_{r\text{-}sl,h}$ and reactance $X_{r\text{-}sl,h}$, are separated (Fig 11).

4.2. Parameters of equivalent circuit for harmonics

In the paper by Caustic (2010) it is shown that values of rotor slot resistance ($R_{r\text{-}sl,SC}$) and rotor slot reactance ($X_{r\text{-}sl,SC}$) in the short-circuit mode are approximately the same for motors of all powers in a series and they are approximately equal to each other, i.e.:

$$R_{r-sl,sc} = X_{r,sc} \approx 0.030 \text{ (p.u.)} \tag{33}$$

Their values are within the narrow range of $X_{r\text{-}sl,SC} = R_{r\text{-}sl,SC} \approx 0.027 \div 0.033$ p.u., respectively for the motors of large (> 100 kW), medium (11 - 50 kW) and low power (1 - 7.5 kW).

The explanation is the following: for motors with powers higher than 5 kW (or with relative rotor conductor height $\xi_{SC} = H_b/\partial_{SC} \geq 1.5$), already in short-circuit mode, the rotor induced currents not the entire cross section, or height of rotor bars H, (Kostic, 2010). Current frequencies of individual harmonics in the rotor winding are h times higher ($f_{r,h} \approx h \cdot f_1 = h \cdot f_{r,SC}$), so the actual depth of penetration of individual harmonic currents (∂_h) is $\sqrt{h}$ times lower. Therefore, the corresponding cross section of rotor conductor is $\sqrt{h}$ times lower, so the values of rotor slot resistance are $\sqrt{h}$ times higher and the values of rotor slot inductance are $\sqrt{h}$ times lower as compared to those values for the fundamental harmonic in the short-circuit regime. In general, rotor slot resistance ($R_{r\text{-}sl,h}$), rotor slot inductance ($L_{r\text{-}sl,h}$) and rotor slot reactance ($X_{r\text{-}sl,h}$), in the function of harmonic order $h = f/f_1$, are shown in Fig. 12.

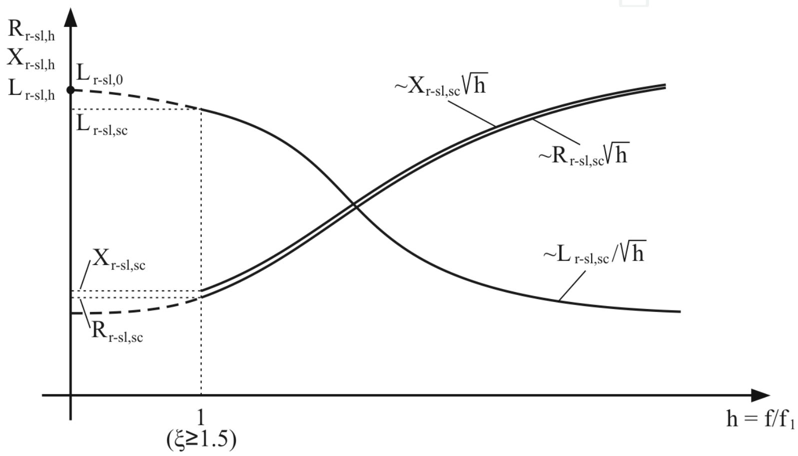

Figure 12. Dependencies of rotor slot resistance, inductance and reactance on harmonic order

If the ratio $\xi = H_b/\delta_{r,SC} \geq 1.5$, then equality $X_{r\text{-}sl,SC} = R_{r\text{-}sl,SC}$ is already true for the fundamental harmonic. As for the harmonics of order $h \geq 5$, the relative rotor conductor height is always equal to $\xi_h = H_b/\delta_h \geq 2$, then for motors of all powers is shown in Fig. 12:

$$R_{r-sl}(hf_1) / R_{r-sl,SC}(f_1) = \sqrt{h} \tag{34}$$

$$L_{r-sl}(hf_1) / L_{r-sl,SC}(f_1) = 1 / \sqrt{h} \tag{35}$$

According to this, it is concluded that the following equations can be written:

$$R_{r-sl,h} = R_{r-sl,SC} \cdot \sqrt{h} \tag{36}$$

$$L_{r-sl,h} = L_{r-sl,SC} \Big/ \sqrt{h} \tag{37}$$

$$X_{r-sl,h} = X_{r-sl,SC} \cdot \sqrt{h} \tag{38}$$

This means that, based on given values of rotor slot resistance ($R_{r\text{-}sl,SC}$), rotor slot inductance ($L_{r\text{-}sl,SC}$) and rotor slot reactance ($X_{r\text{-}sl,SC}$) for fundamental frequency f_1 in short-circuit mode, the corresponding parameter values for harmonics $h = f_h/f_1$ can be calculated, i.e. values: resistance ($R_{r\text{-}sl,h}$), inductance ($L_{r\text{-}sl,h}$) and reactance ($X_{r\text{-}sl,h}$).

The values of penetration depth in the stator copper conductors are $\delta_{Cu,} \geq 1.6$ mm for frequencies $f \leq 2000$ Hz. As a rule, since the diameter of the stator winding conductors is $d_{Cu} \leq 2$ mm, it can be assumed that the value of stator windings' resistance keeps almost the same value for all harmonics of order $h \leq 40$ (or 2000 Hz/50 Hz), i.e. the following equation is valid:

$$R_{s,h} \approx R_{s,1} = R_s, \ \ for \ \ h \leq 40 \tag{39}$$

The same assumption approximately applies to the rotor end resistance ($R_{r\text{-}e,h}$) and to the rotor end inductance ($L_{r\text{-}e,h}$). On this basis, the following equations can be written:

$$L_{r-e,h} \approx L_{r,e} \tag{40}$$

$$X_{s,h} + X_{r-e,h} = h \cdot \left(X_s + X_{r-e} \right) \tag{41}$$

$$R_{s,h} + R_{r-e,h} = R_s + R_{r-e} \tag{42}$$

The total value of motor resistance ($R_{M,h}$), motor reactance ($X_{M,h}$) and motor impedance ($Z_{M,h}$), for current harmonics, are given in the following expressions:

$$R_{M,h} = \left(R_s + R_{r-e} \right) + R_{r-sl,SC} \cdot \sqrt{h} \tag{43}$$

$$X_{M,h} = \left(X_s + X_{r-e} \right) \cdot h + X_{r-sl,SC} \cdot \sqrt{h} \tag{44}$$

$$Z_{M,h} = \sqrt{R_{M,h}^2 + X_{M,h}^2} \tag{45}$$

The summary value of resistance ($R_s + R_{r\text{-}e}$) ≈ *Const* and the summary value of inductance ($L_s + L_{r\text{-}e}$) ≈ *Const*, retain approximately the same value in all modes: operating regime, the short-circuit regime and for regimes with harmonics. For calculation values of $R_{M,h}$ and $X_{M,h}$ by equations (43) and (44), values (R_s+$R_{r\text{-}e}$) and (X_s+$X_{r\text{-}e}$) ought to be determined from a locked rotor test, and assumed value $R_{r\text{-}sl,\,SC}$= $R_{r\text{-}sl,SC}$=*0.030pu.*

Harmonics currents ($I_{M,h}$), due to the existence of the corresponding harmonics voltages ($U_{M,h}$), are calculated using the formula (as a percentage of nominal current, %I_N):

$$I_{M,h} = 100 \cdot U_{M,h} / Z_{M,h} \ \ \left(\% I_N \right) \tag{46}$$

Harmonic losses in the motor, which are caused by harmonic currents through the windings of stator and rotor, are calculated using the formula (as a percentage of nominal motor power, %P_N):

$$P_{Cu,h} = 100 \cdot \frac{R_{M,h} \cdot I_{M,h}^2}{\eta \cdot \cos\phi} \ \ \left(\% P_N \right) \tag{47}$$

Commonly, these power losses are calculated as a percentage of nominal power losses in motor windings ($\%P_{CuN}$). Thus, assuming that losses P_{CuN} make up one half of the total power losses in the motor, their value can be determined from the formula:

$$P_{Cu,h} = 100 \cdot \frac{R_{M,h} \cdot I_{M,h}^2}{\eta \cdot \cos\phi} \cdot \frac{2\eta}{1-\eta} \left(\% P_{CuN} \right) \tag{48}$$

4.3. Current harmonics and harmonic losses in motors supplied from the network with voltage harmonics

The harmonic fields induce a current in the rotor and, as a result of interaction, are given corresponding asynchronous torques. The direction of these harmonic torques coincides with the fundamental torque direction, when h=6n+1, and harmonic torque is the opposite to the fundamental torque direction when h=6n-1. In a motor regime with slip equal to s = 0.01–0.06, in relation to the rotating fields of harmonics, the slip is approximately equal to 1, $s_h = 1 \pm 1/h \approx 1$.

In Radin et al. (1989), the following assumptions (partly wrong ones) are often listed:

- for motors of lower powers (≤ 20 kW), stator and rotor coil resistances and inductances practically do not depend on frequency, so the only values that become increased are the values of the stator and rotor leakage reactance ($X_{sh} = hX_s$, $X_{rh} = hX_r$),
- for motors of medium power (30-100 kW), rotor resistance is increased according to formula $R_{rh} \approx hR_r$, and
- for motors of high power (≥ 110 kW), both stator and rotor resistance are increased according to formula $R_{sh} \approx hR_s$ and $R_{rh} \approx hR_r$,

while the reactance values are somewhat reduced, $X_{sh} \leq hX_s$, $X_{rh} \leq hX_r$, since the values of corresponding inductances are decreased, $L_r(hf) < L_r$ and $L_s(hf) \leq L_s$.

The truth however is slightly different (Kostic, 2010):

- values of the stator resistance and inductance are practically unchanged in all low-voltage motor powers of 100–300 kW,
- the value of the slot resistance of the rotor in short-circuit mode (in relative units) is slightly changed with motor power, so it could be considered $R_{r\text{-}sl,sc} \approx 0.030$ p.u., as it is shown in Kostic (2010),
- rotor resistance on the part outside the slot is approximately given by the expression $R_{r\text{-}e\text{-}} \approx R_r/3 \approx R_s/3$.

By this and (43), an expression for determining the value of the motor resistance $R_{M,h}$ for harmonic order $h \geq 5$ is obtained:

$$R_{Mh} = \frac{4}{3} R_s + 0.03 \cdot \sqrt{h} \tag{49}$$

Example 1

For motors with power 5 kW - 400 kW, in the same order, the ranges of parameter values are given (Kravcik, 1982) for:

- efficiency factor $\eta = 0.85$ - 0.95 and power factor $cos\phi = 0.85$ - 0.92, i.e. $\eta \cdot cos\phi = 0.72$ - 0.875,
- stator resistance (R_s), for example from $R_s = 0.045Z_N$ to $R_s = 0.015Z_N$,

and the

- the corresponding values of stator harmonic resistances $R_{s,h} = R_s = 0.015Z_N \div 0.050Z_N = Const.$,
- the corresponding value of rotor resistance $R_{r,h} = 0.03 \sqrt{h}$,
- the corresponding values of motor resistance $R_{M,h}$, according to (43)
- the corresponding values of motor reactance $X_{M,h}$, according to (44),
- the corresponding values of motor impedance $Z_{M,h}$, according to (45),
- harmonic currents, according to (46), and
- the value of the harmonic losses, as a percentage of nominal motor power $P_{M,h}$ [%P_N], according to (47), and as a percentage of nominal power losses in the windings $P_{M,h}$ [%$P_{Cu,N}$] according to (48).

Application of the suggested method is illustrated by Tab. 6. The results show the amounts of increase in power losses due to the presence of harmonics in a given amount ($U_i = 5\%$, $i = 5, 7, .., 37$) in the supply voltage.

$h=f/f_1$	$U_{h,i}$	R_s	$R_{r,h}$	$R_{M,h}$	$X_{M,h}$	$Z_{M,h}$	$I_{M,h}$ [%I_n]	$P_{M,h}$ [%P_n]	$P_{M,h}$ [%P_{Cun}]
1	1.00	0.015 - 0.05	0.030	0.045 - 0.080	0.161	0.167 - 0.180			
5	0.05	0.015 - 0.05	0.067	0.072 - 0.117	0.735	0.739 - 0.744	6.748	0.039 - 0.074	1.482 - 0.839
7	0.05	0.015 - 0.05	0.079	0.094 - 0.129	1.018	1.022 - 1.053	4.822	0.026 - 0.042	0.988 - 0.476
11	0.05	0.015 - 0.05	0.099	0.114 - 0.144	1.579	1.583 - 1.586	3.157	0.014 - 0.020	0.532 - 0.227
13	0.05	0.015 - 0.05	0.108	0.123 - 0.158	2.416	2.419 - 2.421	2.066	0.006 - 0.010	0.228 - 0.113
17	0.05	0.015 - 0.05	0.124	0.129 - 0.174	2.694	2.643 - 2.646	1.891	0.006 - 0.009	0.228 - 0.102
19	0.05	0.015 - 0.05	0.131	0.146 - 0.181	3.249	3.252 - 3.254	1.537	0.004 - 0.006	0.152 - 0.068
23	0.05	0.015 - 0.05	0.144	0.159 - 0.194	3.526	3.534 - 3.536	1.414	0.004 - 0.005	0.152 - 0.057
25	0.05	0.015 - 0.05	0.150	0.165 - 0.200	3.833	3.836 - 3.838	1.303	0.003 - 0.005	0.114 - 0.057
29	0.05	0.015 - 0.05	0.212	0.177 - 0.212	4.080	4.084 - 4.086	1.224	0.003 - 0.005	0.114 - 0.057
31	0.05	0.015 - 0.05	0.167	0.182 - 0.217	4.357	4.361 - 4.362	1.147	0.003 - 0.004	0.114 - 0.045
35	0.05	0.015 - 0.05	0.177	0.192 - 0.227	4.910	4.919 - 4.920	1.016	0.002 - 0.003	0.076 - 0.034
37	0.05	0.015 - 0.05	0.182	0.197 - 0.232	5.187	5.191 - 5.192	0.963	0.002 - 0.003	0.076 - 0.034
Total	THD_u = 7.3%						THD_i =9.87%	$\Sigma P_{M,h}$ 0.112 - 0.186	$\Sigma P_{M,h}$ 4.256 - 2.109

Table 6. Values of harmonic resistances ($R_{M,h}$), reactances ($X_{M,h}$) and impedances ($Z_{M,h}$) and corresponding currents and harmonic losses for motors > 100 kW (left) and < 5 kW (right), for the given values of voltage harmonics $U_{h,i}$ (p.u.) = 5%

The results in Tab. 6 show that, at the maximum permitted content of harmonics in supply voltage ($U_{h,i}$ = 5%, i = 1–37), the percentage of harmonic losses, (in units of the nominal motor power $P_{M,h}$ [%P_N]), is relatively low:

- for motors of lower power (< 5 kW), an increase of losses is by about 0.186%P_N, so a decrease in efficiency is by about 0.2%, which is slightly less compared to 0.25% in Radin et al. (1989),
- for motors of greater power (> 100 kW), an increase of losses is by about 0.112%P_N, so a corresponding decrease in efficiency is by about 0.12%.

Increments of harmonic losses are relatively small, as a percentage of nominal power losses $P_{M,h}$ [%$P_{Cu,N}$]. Apparently:

- increase of losses is about 2.109% $P_{Cu,N}$, for motors of lower power (< 5 kW),
- increase of losses is about 4.256%$P_{Cu,N}$, for motors of greater power (> 100 kW).

By the results in Tab. 6, for $U_{h,i}$ = Const (example $U_{h,i}$ = 5%, h_i = 1–37, as in Tab. 6), the following approximate equations is confirmed:

$$\frac{P_{Cu,h2}}{P_{Cuh1}} \approx \frac{h_1}{h_2}\sqrt{\frac{h_1}{h_2}}, \ for\ U_{h5} = U_7 = U_{hi} = Const, forh_i \le 40 \tag{50}$$

Equation (50) is derived by the following approximate assumptions: $Z_{Mh} \approx X_{Mh} \approx hX_{M,SC}$ and $R_{Mh} \approx R_{r,h} \approx R_{r,SC} \cdot \sqrt{h}$.

4.4. Harmonic losses when the motor is operating with rectangular shaped voltage

When a motor is supplied by rectangular shaped voltage $U_{h,i}$ (p.u.) = $1/h_i$, h_i = 1 to 37, it is required to calculate the correspondent approximate values of harmonic losses for motors with nominal powers from 5 kW to 400 kW (i.e. for the values of stator resistance from $R_s = 0.045Z_N$ to $R_s = 0.015Z_N$ and correspondent approximate values in a short-circuit regime, $R_{r,SC} \approx 0.03$, for each motor. For motors within the power range 3 kW-400 kW, for which parameters are given in chapter 4.3, power losses are determined.

The given results in Tab. 7 show that, in the specified harmonic content h = 1, 5, 7, 11, 13, 17, 19 ...35 and 37, the percentage of additional power losses, $P_{M,h}$ [%P_N], is relatively high:

- for motors of greater power (>100 kW), an increase of losses is by about 0.94%P_N, so a decrease in efficiency is by 1%,
- for motors of lower power (3-10 kW), an increase of losses is by about 1.68%P_N, so a decrease in efficiency is by 1.7%.

The literature (Radin et al., 1989) often states that the percentage of increase of power losses in the windings of stator and rotor is due to the harmonics in $P_{M,h}$ [%$P_{Cu,N}$]. Data from Tab. 6, column $P_{M,h}$ [%$P_{Cu,N}$], show that:

- an increase of losses is by 19.05%$P_{Cu,N}$, for motors of power (3–10 kW),
- an increase of losses is by 34.92%$P_{Cu,N}$, for motors of greater power (>100 kW).

This last figure corresponds to the values that are found in the literature, while the value of 19.05%$P_{Cu,N}$, for motors of lower power (3–10 kW), is much higher than the figure which is referred to in the literature (by about 5–10%). The reason for this lies in the fact that it is (wrongly) believed that the resistance of the rotor does not change for harmonic frequencies, i.e. that is identical for all harmonics ($R_{r,h} = R_{r,1} = R_r = Const$), which brings the difference mentioned above - and error. However, things are different because the rotor resistance is variable: $R_{r,h} > R_{r,SC} > R_{r,1}$, (Kostic, 2010; Kostic M. & Kostic B., 2011). To be precise, the values of rotor slot resistance are higher and the values of rotor slot inductance are √h times lower as compared to those values for the fundamental harmonic in short-circuit mode.

Some examples from the literature can be used as proof of the view that the rotor resistance changes for low power motors. Specifically in Vukic (1985), the influence of harmonics on the motor of low power (1.6 kW) was tested. The calculation results, which were carried out assuming that $R_r = Const$, gave an increase in power losses of 12.6%, while the experimental measurements showed that the actual increase in losses was 18.5%. Our calculations give rise to losses of 19%, which slightly differs from the measured values. The accuracy of our calculations has been increased with respect to the fact that slot reactance of the rotor increases √h times, for the harmonics of order *h*.

$h=f/f_1$	$U_{h,i}$	R_s	$R_{r,h}$	$R_{M,h}$	$X_{M,h}$	$Z_{M,h}$	$I_{M,h}$ [% I_n]	$P_{M,h}$ [% P_n]	$P_{M,h}$ [% P_{Cun}]
1	1.00	0.015-0.05	0.030	0.045-0.080	0.161	0.167-0.180			
5	0.20	0.015-0.05	0.067	0.072-0.117	0.735	0.739-0.744	26.990	0.618-1.184	23.447-13.418
7	0.14	0.015-0.05	0.079	0.094-0.129	1.018	1.022-1.053	13.790	0.213-0.341	7.668-3.864
11	0.11	0.015-0.05	0.099	0.114-0.144	1.579	1.583-1.586	7.010	0.069-0.099	2.504-1.122
13	0.08	0.015-0.05	0.108	0.123-0.158	2.416	2.419-2.421	3.180	0.015-0.022	0.556-0.252
17	0.06	0.015-0.05	0.124	0.129-0.174	2.694	2.643-2.646	2.230	0.007-0.012	0.244-0.136
19	0.05	0.015-0.05	0.131	0.146-0.181	3.249	3.252-3.254	1.600	0.005-0.007	0.180-0.079
23	0.04	0.015-0.05	0.144	0.159-0.194	3.526	3.534-3.536	1.220	0.003-0.004	0.106-0.046
25	0.04	0.015-0.05	0.150	0.165-0.200	3.833	3.836-3.838	1.040	0.002-0.003	0.075-0.036
29	0.03	0.015-0.05	0.162	0.177-0.212	4.080	4.084-4.086	0.830	0.001-0.002	0.036-0.025
31	0.03	0.015-0.05	0.167	0.182-0.217	4.357	4.361-4.362	0.730	0.001-0.002	0.036-0.025
35	0.03	0.015-0.05	0.177	0.192-0.227	4.910	4.919-4.920	0.590	0.001-0.002	0.036-0.025
37	0.03	0.015-0.05	0.182	0.197-0.232	5.187	5.191-5.192	0.520	0.001-0.002	0.036-0.025
Total	THD_u = 30.3%						THD_i =31.5%	$\Sigma P_{M,h}$ 0.94 -1.68	$\Sigma P_{M,h}$ 34.92-19.05

Table 7. Values of harmonic resistances ($R_{M,h}$), reactances ($X_{M,h}$) and impedances ($Z_{M,h}$); as harmonic currents ($I_{M,h}$) and harmonic losses ($P_{M,h}$) for motors with power > 100 kW (left) and lower power, 3–10 kW (right), when the motor is supplied by the rectangular voltage, i.e. by voltage with harmonics $U_{hi} = 1/h_i$.

As R_s=0.050÷0.015, respectively, for motors of power 3÷200 kW, the given results are useful for the evaluation of harmonic currents ($I_{M,h}$) and harmonic losses ($P_{M,h}$) for all mentioned motors, by extrapolation.

5. Summary

The results of the analysis presented in this chapter are summarised in the following.

A) Effect of voltage magnitude on motor power losses and motor reactive loads

The results of the research show dependencies of the input power and reactive load on voltage magnitude, as given in Fig. 2.

1. Decreasing voltage magnitude by 1%, by setting voltages in range of U_n±5%, leads to:
   a. reactive loads decreasing
      - from 1% to 2%, at loads from 100% to 75%, respectively, for motors above100 kW,
      - up to 3% to 4%, at loads from 75% to 25%, respectively, for motors below 10 kW;
   b. power losses (and active input powers) decreasing/increasing (sign „ - ")
      - from (-0.1%) to 0.1%%, respectively at loads from 100% to 50%, for motors above100 kW
      - from 0% to 0.6%, respectively at loads from 100% to 25%, for motors about 10 kW,
      - from 0% to 1.6%, respectively, at loads from 100% to 25%, for motors below 1 kW.
2. On the basis of the investigation presented in this paper, it is confirmed that there are significant possibilities for energy savings by means of voltage magnitude setting, within values U_n±5%, in networks with induction motors which are light loaded (<70%).
3. Setting voltage within band 0.9 U_n - 0.95 U_n is not recommended, even if it leads to reduced power losses and reactive loads, because starting and maximal torque are decreased and it can also cause motor operation instability.

B) The most important conclusions regarding motor operation with the unbalanced voltage

4. It is explained that the rotor inverse resistance and rotor inverse reactance are higher for √2 times compared to the rotor resistance and rotor reactance in short-circuit mode, since current frequency of the negative sequence in the rotor winding is twice as high ($f_{r,NS} \approx 2f_1 = 2f_{r,SC}$), i.e. they are higher by 1.41 times than the corresponding values given in the literature.
5. Voltage unbalance causes increase of the motor heating, occurrence of inverse torques and a small increase in motor slip. Thus, for the voltage asymmetry of 2%, 3%, 4% and 5%, this causes an increase in power losses of 5.5%, 12%, 22% and 34% of motor nominal losses. Corresponding values of derating factors are 0.97, 0.94, 0.88 and 0.81, respectively, as noted in NEMA standards, so the acceptable voltage asymmetry is 2%.
6. Based on the actual calculation and analysis, it was found that the effects of an unbalance on power loss are smaller for motors of nominal power ≤ 10 kW. Thus, acceptable voltage asymmetry for these motors could be 3%.
7. Generally, motor operation is not allowed when voltage asymmetry is greater than 5%, because, in some cases, current and losses in one phase could be increased for 38% and 90%, respectively.

C) The most important conclusions regarding motor operation with the non-sinusoidal voltage

8. The given results show that, at the maximum permitted content of harmonics in supply voltage, $U_{h,i}$ = 5%, i = 1 to 37), the percentage of harmonic losses is relatively small:
   - about 2.109% $P_{Cu,N}$, for motors of lower power (< 5 kW), and
   - about 4.256%$P_{Cu,N}$, for motors of greater power (> 100 kW).
9. When the induction motors are supplied by rectangular shaped voltage with high levels of harmonic voltages, an increase in harmonic losses in stator and rotor windings are:
   - around 30-35% $P_{Cu,N}$, for high power motors (> 100 kW), and
   - around 15-20% $P_{Cu,N}$, for lower power motors (3–10 kW).

The increase in harmonic power losses for lower power motors is much higher than it was noted in the literature (5-10%) because it is (wrongly) believed that the resistance of the rotor does not change for higher harmonic frequencies.

Appendix

For deriving equations for electromagnetic torque and power, the equivalent Γ-circuit, shown in Fig. 13, is used (Kostic, 2010):

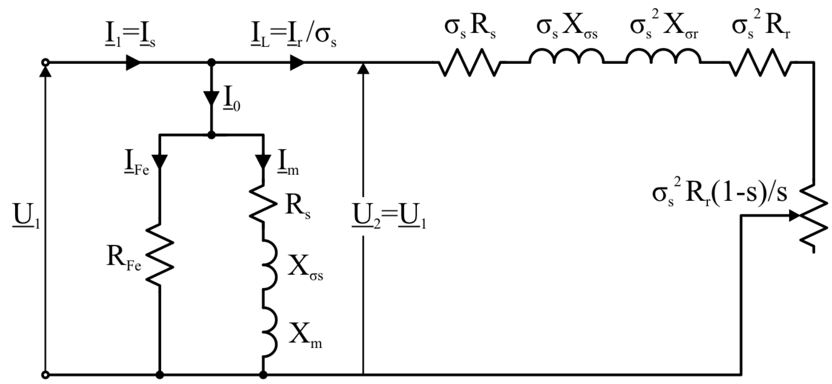

Figure 13. Equivalent Γ-circuit of induction machine

Equation (9) is completely derived in this Appendix.

1. Electromagnetic power ($P_{em,N}$) **at rated load**, i.e. at slip $s=s_N$, can be expressed as following:

$$P_{em,N} = T_{em,N} \cdot \Omega_1 = \frac{I_L^2 \cdot \sigma_s{}^2 R_r}{s_N} = \frac{U_1^2 \cdot \sigma_s{}^2 R_r / s_N}{(\sigma_s R_s + \sigma_s{}^2 R_r)^2 + (\sigma_s X_s + \sigma_s{}^2 R_r / s_N)^2} \tag{51}$$

For motors with power within the range of 1÷200kW, values for s_N are 0.05÷0.01, respectively, and therefore: $\sigma_s^2 R_r/s_N = (20\div100)\cdot\sigma_s R_s$ and $\sigma_s X_s + \sigma_s^2 X_r) \approx 0.20\cdot\sigma_s^2 R_r/s_m$.

$$P_{em,N} = T_{em,N} \cdot \Omega_1 = \frac{I_L^2 \cdot \sigma_s{}^2 R_r}{s_N} \approx \frac{U_1^2 \cdot \sigma_s{}^2 R_r / s_N}{(1.15 \div 1.05)^2 (\sigma_s{}^2 R_r / s_N)^2} = \frac{U_1^2}{(1.15 \div 1.05)(\sigma_s^2 R_r / s_N)} \tag{52}$$

2. **Regime with maximum input power,** i.e. at $s=s_{Pm}$, accrues when resistance ($\sigma_s X_{\sigma s}+\sigma_s^2 X_{\sigma r}$) and reactance in load branch ($\sigma_s R_s+\sigma_s^2 R_r/s_m$) are equal, i.e., and when the load branch impedance is $Z_{2,m}=\sqrt{2}(\sigma_s X_{\sigma s}+\sigma_s^2 X_{\sigma r})$. Corresponding electromagnetic power ($P_{em,Pm}$) on the resistance $\sigma_s^2 R_r/s_m$ is:

$$P_{em,Pm}=T_{em,Pm}\Omega_1=I_L^2\cdot\sigma_s{}^2\frac{R_r}{s_{Pm}}=\frac{U_1^2\cdot\sigma_s{}^2 R_r/s_{Pm}}{2(\sigma_\sigma X_{\sigma s}+\sigma_\sigma{}^2 X_{\sigma r})^2} \tag{53}$$

Since for motors with power within the range of 1÷200kW, values for corresponding slip are s_{Pm} = 0.25÷0.05, respectively, the skin effect in the bars of the squirrel-cage is minor (the depth of penetration $\delta_r(s_m f_1)\geq H_b$-the bar (conductor rotor) height), so it is $\sigma_s^2 R_r/s_m$ = (5÷20) $\sigma_s R_s$. Consequently, it is:

$$\sigma_s^2 R_r s_{Pm}=(0.8\div 0.95)\cdot(\sigma_s R_s+\sigma_s^2 R_r/s_{Pm})=(0.8\div 0.95)\cdot(\sigma_s X_{\sigma s}+\sigma_s^2 X_{\sigma r}) \tag{54}$$

and the electromagnetic power ($P_{em,m}$), in the regime with maximum input power, is:

$$P_{em,Pm}=T_{em,Pm}\cdot\Omega_1=\frac{I_L^2\cdot\sigma_s{}^2 R_r}{s_{Pm}}\approx\frac{U_1^2\cdot(0.8\div 0.95)(\sigma_s X_{\sigma s}+\sigma_s{}^2 X_{\sigma r})}{2\cdot(\sigma_s X_{\sigma s}+\sigma_s{}^2 X_{\varsigma r})^2}=\frac{U_1^2\cdot(0.8\div 0.95)}{2\cdot(\sigma_s X_{\sigma s}+\sigma_s^2 X_{\sigma r})} \tag{55}$$

3. If $\sigma_s^2 R_r/s_N$ is expressed from (A-4), and ($\sigma_s X_{\sigma s}+\sigma_s^2 X_{\sigma r}$) is expressed from (A-5), then it is:

$$\sigma_s{}^2 R_s+\sigma_s{}^2 R_r/s_N=\frac{U_1^2}{T_{em,N}\cdot\Omega_1(1.15\div 1.05)} \tag{56}$$

$$\sigma_s X_{\sigma s}+\sigma_s{}^2 X\sigma_r=\frac{U_1^2\cdot(0.8\div 0.95)}{2T_{em,Pn}\cdot\Omega_1} \tag{57}$$

On the base of (A-6) and (A-7), it is obtained:

$$\frac{\sigma_s X_{\sigma s}+\sigma_s{}^2 X_{\sigma r}}{\sigma_s{}^2 R_s+\sigma_s{}^2 R_r/s_N}=\frac{T_{em,N}}{2T_{em,Pm}}\cdot(0.8\div 0.95)\cdot(1.15\div 1.05) \tag{58}$$

Reactive power in the load branch of Γ-circuit, under rated condition, $Q_{2N}\approx Q_{LN}$ (Q_{LN} – load component of reactive power), can be expressed in terms of the electromagnetic power, $P_{em,N}$

$$Q_{2N}=P_{em,N}\cdot\frac{\sigma_s X_{\sigma s}+\sigma_s{}^2 X\sigma_r}{\sigma_s{}^2 R_s+\sigma_s{}^2 R_r/s_N}\approx Q_{LN} \tag{59}$$

Since the relation between the electromagnetic power ($P_{em,N}$) and the rating power (P_N) is:

$$P_{em,N}=P_N\cdot\frac{\sigma_s^2 R_s+\sigma_s^2 R_r/s_N}{(\sigma_s^2 R_r/s_N)}\cdot\frac{1}{1-s_N} \tag{60}$$

then, based on equations (A-7), (A-9) and (A-10), it follows:

$$Q_{LN} = P_N \cdot \frac{T_{em,N}}{2T_{em,Pm}} \cdot \frac{(0.08 \div 0.95) \cdot (1.15 \div 1.05)}{0.95 \div 0.99} = \frac{T_{em,N}}{2T_{em,Pm}} \cdot (0.98 \div 1.01) \approx 0.5 P_N / (T_m / T_N) \quad (61)$$

$$Q_{LN} \approx P_N \cdot \frac{T_{em,N}}{2T_{em,Pm}} = 0.5 \cdot \frac{T_{em,N}}{T_{em,Pm}} \quad (62)$$

Since the maximum torque ($T_m \approx T_{em,m}$), which is catalogue data, is greater up to 2% from mentioned torque ($T_{em,Pm}$) in the regime with maximum input power, i.e. $T_{em,Pm} \leq 1.02\, T_m$, it might be concluded that the equation (9) sufficiently accurate for calculating the rating component of reactive power in load branch, $Q_{LN}=0.5 \cdot P_N/(T_m/T_N)$.

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