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# **Role of Induction Motors in Voltage Instability and Coordinated Reactive Power Planning**

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Additional information is available at the end of the chapter

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## **1. Introduction**

As the power system is being operated in an economic and environment friendly fashion, there is more emphasis on effective resource utilization to supply the ever increasing demand. Consequently system experiences heavy power transaction, and one of the very important stability phenomena, namely voltage stability, is capturing the attention of many power system engineers, operators, researchers, and planners. Concerns for voltage instability and collapse are prompting utilities to better understand the phenomenon so as to devise effective, efficient and economic solutions to the problem.

Past studies have investigated the intricate relationship that exist between insufficient reactive power support and unreliable system operation including voltage collapses [1, 2, 3, 4], as was observed in 2003 blackout of USA [5]. It is not just the amount of reactive support, but also the quality and placement of reactive support that matters. For instance, it is found that due to the presence of electric loads that are predominantly induction motors the voltage recovery of the system following a severe disturbance is delayed due to lack of fast responding reactive support, thereby threatening to have secondary effects such as undesirable operation of protective relays, electric load disruption, and motor stalling [6, 7]. While a number of techniques have been developed in the past to address the problems of voltage instability [8], there has been little work towards a long term reactive power (VAR) planning (RPP) tool that addresses both steady state as well as dynamic stability issues.

The available reactive power devices can be classified into static and dynamic devices [9]. The static devices include mechanically switched shunt capacitors (MSCs) and series capacitors that exert discrete open-loop control action and require more time delay for correct operation. The dynamic devices are more expensive power electronics based fast-acting devices such as static VAR compensators (SVC), Static Synchronous Compensator (STATCOM), Unified Power Flow Controller (UPFCs) that exert continuous feedback

control action and have better controllability and repeatability of operation. While MSCs are able to strengthen a power system against long term voltage instability issues [1], transient voltage dip and slow voltage recovery issues (influenced by electric load dynamics) is most effectively addressed by fast responding dynamic VAR sources [10, 11, 12]. While, past methods allocate static and dynamic VAR sources sequentially for one contingency at a time, the key to solving transmission system problems in the most cost-effective way will be to coordinate reactive power requirements simultaneously under many contingencies for both static and dynamic problems. The RPP should therefore identify the right mix of VAR devices, good locations and appropriate capacities for their installation.

In this chapter, we present a long term VAR planning algorithm, which coordinates between network investments that most effectively address steady-state voltage instability and those which most effectively address transient voltage recovery problems under severe contingencies. The study takes into account the induction motor dynamic characteristics, which influence the transient voltage recovery phenomenon. The algorithm is applied on a portion of a large scale system consisting of 16173 buses representing the US eastern interconnection. The planning method is a mixed integer programming (MIP) based optimization algorithm that uses sensitivity information of performance measures with respect to reactive devices to plan for multiple contingencies simultaneously.

The remaining parts of this chapter are organized as follows. In section 2, we discuss the very important role played by induction motor loads in the voltage stability phenomena. Section 3 sheds focus on the models used to build a base case for voltage stability assessment, and on appropriate performance criteria and solution strategies used in this chapter to devise the proposed coordinated planning. A summary of proposed RPP algorithm that considers both static as well as dynamic reactive resources in a coordinated planning framework, together with the various stages of planning are presented in Section 4. Section 5 illustrates the influence of induction motors on voltage stability phenomenon under severe contingencies, and demonstrates application of the coordinated VAR planning method on a large scale system to effectively avoid induction motor trips. Section 6 presents the conclusions.

## 2. Voltage instability phenomena

Several theories have been proposed to understand the mechanism of voltage instability. Voltage instability leading to collapse is system instability in that it involves many power system components and their variables at once. There are several system changes that can contribute to voltage collapse [4] such as increase in loading, SVC reaching reactive power limits, action of tap changing transformers (LTCs), load recovery dynamics, line tripping and generator outages. Most of the above mentioned system changes have a large effect on reactive power production or transmission. To discuss voltage collapse some notion of time scales is needed that accounts for fast acting variables in time scales of the order of seconds such as induction motors, SVCs to slow acting variables having long term dynamics in hours such as LTCs, load evolution etc.

## 2.1. Role of induction motors

A major factor contributing to voltage instability is the voltage drop that occurs when active and reactive power flow through inductive reactance of the transmission network; which limits the capability of the transmission network for power transfer and voltage support [13]. The power transfer and voltage support are further limited when some of the generators hit their field or armature current time-overload capability limits. Under such stressed conditions, the driving force for voltage instability is usually the inductive loads that try to recuperate after a disturbance. For instance, in response to a disturbance, power consumed by the induction motor loads tends to be restored by the action of motor slip adjustment, distribution voltage regulators, tap-changing transformers, and thermostats [6]. Restored loads increase the stress on the high voltage network by increasing the reactive power consumption and causing further voltage reduction. A run-down situation causing voltage instability occurs when load dynamics attempt to restore power consumption beyond the capability of the transmission network and the connected generation to provide the required reactive support.

The publication [14] corroborates this voltage instability phenomenon by means of a power-voltage (PV) curve, as shown in Figure 1. For a particular system and loads considered, the normal system can be stable with both resistive and motor loads at points where load curves and system curves intersect. However, when the system becomes stressed, with increased system reactance, it can only have a stable operating point with a resistive load. Due to lack of reactive power support that limits the transfer capability or loadability of the system, there is no intersection of system and load curves for the induction motor load since there is no stable operating point.

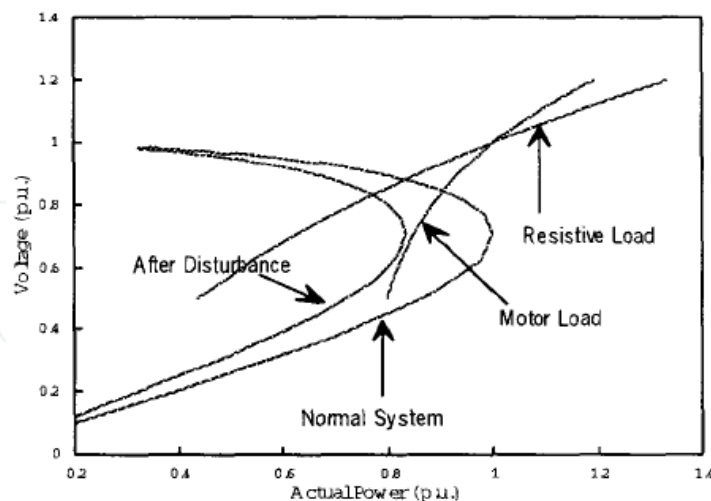
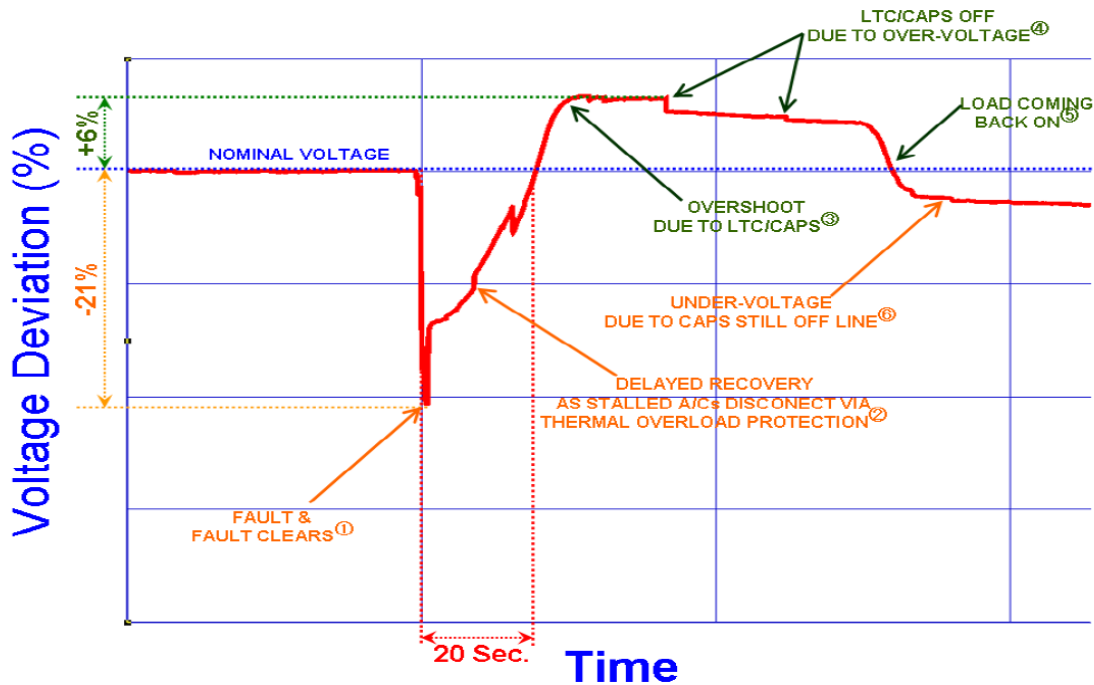


Figure 1. Stability and Load Characteristics

## 2.2. A typical scenario of slow voltage recovery leading to collapse

Heavily loaded transmission lines during low voltage conditions can result in operation of protective relays causing some transmission lines to trip in a cascading mode. A common

scenario is a large disturbance such as a multi-phase fault near a load center that decelerates induction motor loads. Following fault clearing with transmission outages, motors draw very high current while simultaneously attempting to reaccelerate, as discussed in previous section, thereby making slowing down the voltage recovery process. A typical slow voltage recovery phenomenon following a disturbance is indicated in Figure 2 [15]. While trying to recover, if the voltage drops to a very low point for a sustained duration due to system's inability to provide reactive support, some motors may stall. Such drastic stalling of motor further exacerbates the conditions by increasing the reactive power requirements quickly, and the rate of voltage decline can accelerate catastrophically [6, 7, 16]. Massive loss of load and possibly area instability and voltage collapse may follow.



**Figure 2.** Post-Fault Transient Voltage Characteristics

There are several works [10] that have documented many short-term (few seconds) voltage collapse incidents with loss of load. In all cases, adequate dynamic reactive power support was not available which resulted in a large loss of load.

All the above discussions give a physical sense of how the problem of voltage instability occurs, and shed importance on the requirement of study techniques and good models, especially of induction motor dynamics as they are particularly hazardous from the viewpoint of voltage stability.

### 3. Voltage stability assessment and criteria

Traditionally, voltage stability investigations have been based on steady-state analyses, which involve solving conventional or modified power flow equations [17]. In such studies

system P-V curve and the sensitivity information derived from the power flow jacobian are used to assess and plan for voltage instability. But the realization that voltage stability is a dynamic phenomenon has led to dynamic formulations of the problem and application of dynamic analysis tools [18] that uses time-domain simulations to solve nonlinear system differential algebraic equations.

A pre-contingency steady state base case is required for the voltage stability study to be performed, which is usually generated from real-time sequence control (State estimator solution), or via an already recorded power flow solution. In the case of a base case for dynamics study, as discussed in previous section, the dynamic phenomenon of voltage stability is largely determined by load characteristics and the available means of voltage control. The response speeds of these loads may be comparable to the speed of response of the dynamic voltage control equipment. So in such studies involving dynamic phenomena, using static models will give forth dubious results. So it is very important to properly model dynamic behavior of such large, small and trip induction motor loads; along with that of relevant voltage controls. Investigating the post-fault dynamic system response and effective planning to prevent a voltage collapse depends on inclusion of relevant system component models.

### 3.1. System component models

This section presents the various components and their relevant models that are required to build a suitable base case for comprehensive voltage stability assessment and planning.

#### 3.1.1. Static device models

1. **Transmission lines** are represented as pi-sections, possibly with unsymmetrical line charging; accompanying data include line pi-section impedances/admittances data; line thermal limit both normal and emergency.
2. **Transformers** represented as pi-sections whereby the various impedance/admittance components may be explicit functions of tap settings; three winding transformers must be properly modeled. The data also include transformer limits under normal/emergency cases. **Phase-shifting transformers** are represented by complex tap ratios, allowing both shift in angle and change in voltage magnitude;
3. **Generators** as real-power source together with a reactive power capability curve as a function of terminal voltage; The required generator static data include minimum and maximum ratings, nominal terminal voltage and reactive power capability curve as a function of terminal voltage
4. **Shunt elements** by their impedance/admittance and Static Var compensators by static gain and maximum/minimum limits
5. **Loads** by ZIP model, i.e., as a combination of constant impedance (Z), constant current (I), and constant real/reactive injection (P) components; The data necessary are default ZIP load partition ratios at nominal voltage, load limits and default power factors



### 3.1.2. Dynamic device models

1. **Machine** mechanical dynamic equation (swing with damping) and machine electrical dynamic equations; machine mechanical parameters such as inertia constant and damping co-efficient and machine electrical parameters such as transient/sub-transient reactances and time constants etc are required. Saturation model data is also very vital.
2. **Excitation systems** of various types; the data for each model available in standard power system stability analysis programs such as EPRI's ETMSP, PTI's PSS/E etc are used in most cases.
3. **Governor systems** of various types; Again the necessary data for each model are usually available in standard power system stability analysis programs.
4. **Load modeling** is very vital for performing a voltage stability study.
5. Models for selected prime mover, power system stabilizers, and control devices such as SVC etc. are also required.

Induction motor and SVC modeling will be discussed further in this section. In addition to all the system/device data, other system data include convergence parameters such as threshold and maximum iteration counts for static power flow studies, and also various other solution parameters used for the dynamic time domain simulation.

#### 3.1.2.1. Induction motor modeling

As mentioned earlier large induction motor loads generally affect the voltage recovery process after voltage sag has been incepted due to system faults, and in many occasions due to extended voltage sag secondary effects such as stalling or tripping of sensitive motors might happen leading to massive load disruption. So, it is very vital to represent large, small and trip induction motor loads in various combinations in the system, so that we capture the stalling phenomenon of induction motor load, the real and reactive power requirements in the stalled state, and the tripping caused by thermal protection.

The induction motor load must be modeled such that it is sensitive to dynamic variations in voltage and frequency, and emulates the typical characteristic of consuming more power at increased speeds. Equation (1) shows the modeling of mechanical torque ( $T_{load}$ ) as a function of speed deviation from nominal and motor load torque at synchronous speed ( $T_{nom}$ ).

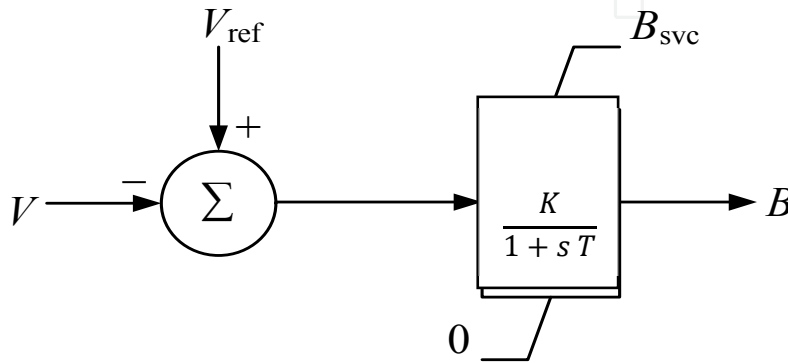
$$T_{load} = T_{nom}(1 + \Delta\omega)^D \quad (1)$$

Some of the primary model parameters include stator and rotor resistances and inductances, mutual inductance, saturation components, MVA base, inertia (H), per unit voltage level below which the relay to trip the motor will begin timing ( $V_T$ ), time in cycles for which the voltage must remain below the threshold for the relay to trip ( $C_T$ ), breaker delay time in cycles, nominal torque ( $T_{nom}$ ), load damping factor ( $D$ ) etc.

There are new composite load models developed by WECC LMTF [19] that improves the representation of induction motor load dynamics, and thereby more closely captures the critical role played by such loads in delayed voltage recovery events. These composite loads are represented by CMLD models in PSS/E and CMPLDW models in PSLF [20].

### 3.1.2.2. Static VAR compensator modeling

In the study done in this chapter we employ SVC as an effective means to mitigate transient voltage dip problem by providing fast responding dynamic reactive power support. The SVC is modeled as shown in the figure 3, with a non-windup limit  $B_{svc}$  (in MVAR) on the SVC output, which constrains the SVC output  $B$ . At its limit, SVC output is non-controllable and functions as a shunt capacitor. Hence, the ability of an SVC to provide dynamic support for mitigating the transient voltage dip problems depends on the SVC's capacitive limit (size)  $B_{svc}$ , which also increases the SVC cost. The RPP finds the optimum rating of SVC that is economical and enhances system reliability.



**Figure 3.** Static VAR compensator model

Some of the main parameters include voltage limits  $V_{MAX}$  and  $V_{MIN}$  to specify the active range of the voltage control, a time constant (of about 0.05 sec. or less) to model the delay in reactor's response, a steady-state voltage control gain  $K$  of about 100, and the time constant  $T$  of about 0.01 to provide transient gain reduction in the control loop.

Once the voltage stability base case is ready, next system analysis has to be performed to check the severity of the contingencies that need planning. So the next vital step in the planning procedure is voltage stability assessment of contingencies.

## 3.2. Post-contingency performance criteria for voltage stability assessment

In order to effectively plan against steady state and dynamic voltage stability problems under a certain set of contingencies, we need to identify proper performance criteria. Voltage stability of the power system should be assessed based on voltage security criteria of interest to, and accepted by, the utility.

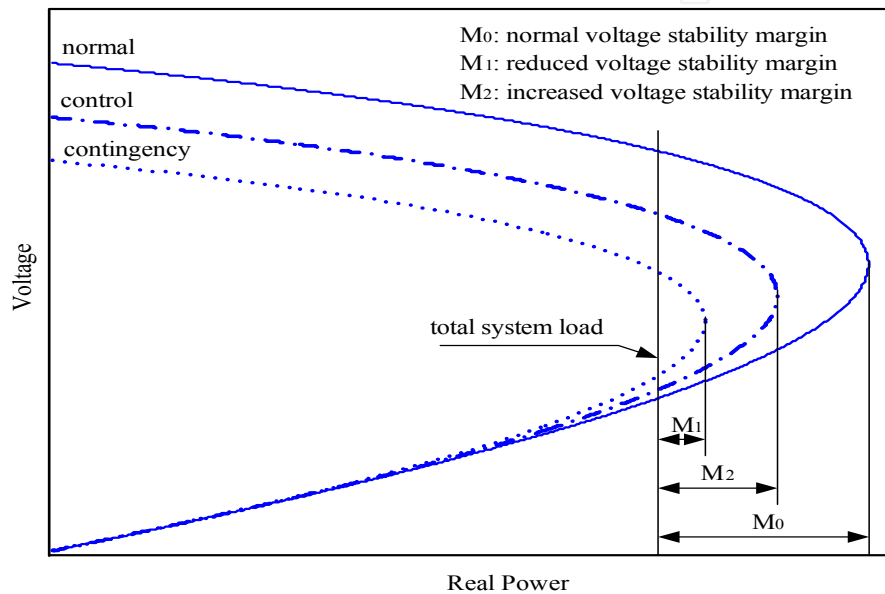
As far as the steady state performance criteria are concerned, there are many criteria such as reactive reserve in different parts of the system, post-contingency voltage, Eigenvalues, etc. that enable to quantify the severity of a contingency with respect to voltage stability. In this work we utilize the most basic and widely accepted criteria, namely, post-contingency voltage stability margin [21, 22] for steady state performance assessments. Voltage stability margin, a steady state performance criterion, is defined as the amount of additional load in a specific pattern of load increase that would cause voltage instability. Contingencies such as



unexpected component (generator, transformer, transmission line) outages often reduce the voltage stability margin, and control actions increase it [23, 24], as shown in figure 4 [25].

The disturbance performance table within the NERC (North American Electric Reliability Corporation)/WECC (Western Electricity Coordinating Council) planning standards [26] provides the minimum acceptable performance specifications for post-contingency voltage stability margin under credible events, that it should be atleast,

- greater than 5% for N-1 contingencies,
- greater than 2.5% for N-2 contingencies, and
- greater than 0% for N-3 contingencies.



**Figure 4.** Voltage stability margin

As far as the performance measure for dynamic stability phenomena is concerned, in [11] it is stated that the needs of the industry related to voltage dips/sags for power system stability fall under two main scenarios. One is the traditional transient angle stability where voltage “swing” during electromechanical oscillations is the concern. The other is “short-term” voltage stability generally involving voltage recovery following fault clearing where there is no significant oscillations, for which much greater load modeling detail is required (specifically induction motor loads) with the fault applied in the load area rather than near generation. In [25], it is stated that many planning and operating engineers are insufficiently aware of potential short-term voltage instability, or are unsure on how to analyze the phenomena. In this work we focus on planning for the transient voltage recovery after a fault is cleared. For the transient voltage analysis, the minimum planning criteria is,

- **Slow voltage recovery:** The induction motor trip relay timer is actuated when the bus voltage dips below 0.7p.u and trips if voltage doesn’t recover to 0.7p.u within next 20 cycles.

### 3.3. Sensitivities of post-contingency performance criteria

The objective of the reactive power planning problem is to satisfy these minimum performance criteria mentioned in the previous section under various credible contingencies. This is greatly dependent upon the amount, location and type of reactive power sources available in the system, which are all decision variables in the proposed coordinated RPP, as will be dealt in the next section of this chapter. If the reactive power support is far away, or insufficient in size, or too dependent on shunt capacitors (slow acting static device), a relatively normal contingency (such as a line outage or a sudden increase in load) can trigger a large system voltage drop. Hence, in order to properly allocate the reactive power support in terms of optimal type, location and amount, in this work we employ linear sensitivities of performance measure with respect to various control actions. These sensitivities enable obtaining the necessary information, i.e., the sensitivities of performance measure for the type of reactive support, the location and amount of reactive support, which are very useful to perform the proposed coordinated RPP. This section sheds light on these linear sensitivity measures.

#### 3.3.1. Voltage stability margin sensitivity

The sensitivity of voltage stability margin refers to how much the margin changes for a small change in system parameters such as real power and reactive power bus injections, regulated bus voltages, bus shunt capacitance, line series capacitance etc [2, 27]. References [28, 29] first derived these margin sensitivities for different changing parameters.

Let the steady state of the power system satisfying a set of equations in the vector form be,

$$F(x, p, \lambda) = 0 \quad (2)$$

where  $x$  is the vector of state variables,  $p$  is any parameter in the power system steady state equations such as demand and base generation or the susceptance of shunt capacitors or the reactance of series capacitors, and  $\lambda$  denotes the system load/generation level called the scalar bifurcation parameter. The system reaches a state of voltage collapse, when  $\lambda$  hits its maximum value (the nose point of the system PV curve), and the value of the bifurcation parameter is equal to  $\lambda^*$ . For this reason, the system equation at equilibrium state is parameterized by this bifurcation parameter  $\lambda$  as shown below.

$$P_{li} = (1 + K_{lpi}\lambda)P_{li0} \quad (3)$$

$$Q_{li} = (1 + K_{lqi}\lambda)Q_{li0} \quad (4)$$

$$P_{gj} = (1 + K_{gj}\lambda)P_{gj0} \quad (5)$$

where  $P_{li0}$  and  $Q_{li0}$  are the initial loading conditions at the base case corresponding to  $\lambda=0$ .  $K_{lpi}$  and  $K_{lqi}$  are factors characterizing the load increase pattern (stress direction).  $P_{gj0}$  is the real power generation at bus  $j$  at the base case.  $K_{gj}$  represents the generator load pick-up

factor. For a power system model using ordinary algebraic equations, the bifurcation point sensitivity with respect to the control variable  $p_i$  evaluated at the saddle-node bifurcation point is [27]

$$\frac{\partial \lambda^*}{\partial p_i} = -\frac{w^* F_{p_i}}{w^* F_{\lambda}} \quad (6)$$

where  $w$  is the left eigenvector corresponding to the zero eigenvalue of the system Jacobian  $F_x$ ,  $F_{\lambda}$  is the derivative of  $F$  with respect to the bifurcation parameter  $\lambda$  and  $F_{p_i}$  is the derivative of  $F$  with respect to the control variable parameter  $p_i$ .

The sensitivity of the voltage stability margin with respect to the control variable at location  $i$ ,  $S_i$ , is

$$S_i = \frac{\partial M}{\partial p_i} = \frac{\partial \lambda^*}{\partial p_i} \sum_{i=1}^n K_{lpi} P_{li0} \quad (7)$$

where  $M$  is the voltage stability margin given by

$$M = \sum_{i=1}^n P_{li}^* - \sum_{i=1}^n P_{li0} = \lambda^* \sum_{i=1}^n K_{lpi} P_{li0} \quad (8)$$

### 3.3.2. Post-fault transient voltage recovery sensitivity

The sensitivity of the voltage dip time duration to the SVC capacitive limit ( $B_{svc}$ ) can be defined as the change of the voltage dip time duration (voltage recovery time) for a given change in the SVC capacitive limit. Let  $\tau^{(1)}$  be the time at which the transient voltage dip begins after a fault is cleared, and  $\tau^{(2)}$  be the time at which the transient voltage dip ends, as shown in figure 5 [25]. Then the time duration of the transient voltage dip  $\tau_{dip}$  is given by

$$\tau_{dip} = \tau^{(2)} - \tau^{(1)} \quad (9)$$

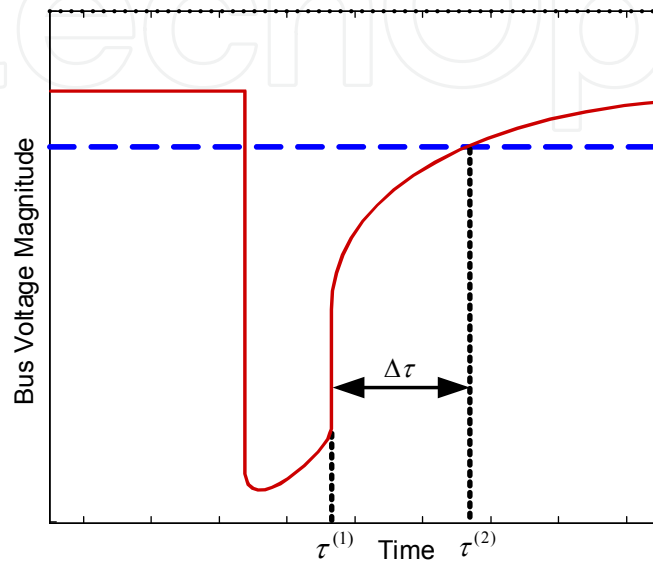
Thus, the sensitivity of the voltage dip time duration to the capacitive limit of an SVC,  $S_{\tau}$ , is

$$S_{\tau} \equiv \frac{\partial \tau_{dip}}{\partial B_{svc}} = \frac{\partial(\tau^{(2)} - \tau^{(1)})}{\partial B_{svc}} = \frac{\partial \tau^{(2)}}{\partial B_{svc}} - \frac{\partial \tau^{(1)}}{\partial B_{svc}} = \tau_{B_{svc}}^{(2)} - \tau_{B_{svc}}^{(1)} \quad (10)$$

where  $\tau_{B_{svc}}^{(1)}$  and  $\tau_{B_{svc}}^{(2)}$  are calculated based on trajectory sensitivity computations as derived in [25]. For a large power system, this method requires computing integrals of a set of high dimension differential algebraic equations. An alternative to calculate the sensitivities is using numerical approximation [25], as shown by equation (11). This procedure of sensitivity calculation by numerical approximation requires repeated simulations of the system model for the SVC capacitive limits  $B_{svc}$  and  $B_{svc} + \Delta B_{svc}$ . The sensitivities are then given by the change of the voltage recovery time divided by the SVC capacitive limit change

$\Delta B_{\text{sbc}}$ . This procedure is easy to implement for a large power system using available simulation tools.

$$S_{\tau} = \frac{\partial \tau_{\text{dip}}}{\partial B_{\text{sbc}}} \approx \frac{\Delta \tau_{\text{dip}}}{\Delta B_{\text{sbc}}} = \frac{\tau_{\text{dip}}(B_{\text{sbc}} + \Delta B_{\text{sbc}}) - \tau_{\text{dip}}(B_{\text{sbc}})}{\Delta B_{\text{sbc}}} \quad (11)$$



**Figure 5.** Post-fault clearance slow voltage recovery

#### 4. Coordinated reactive power planning

The planning algorithm addresses the problems discussed in Table 1.

Problem	Planning Objective
<b>A</b> (Steady State)	To restore post-disturbance equilibrium and increase post contingency voltage stability margin beyond the minimum criteria after a severe contingency that can cause voltage instability
<b>B</b> (Dynamic)	To improve the characteristics of post-fault transient voltage recovery phenomenon satisfying the required minimum criteria, and prevent induction motor tripping

**Table 1.** Coordinated Reactive Power Planning Objectives

Figure 6 shows the general flow of the RPP procedure developed.

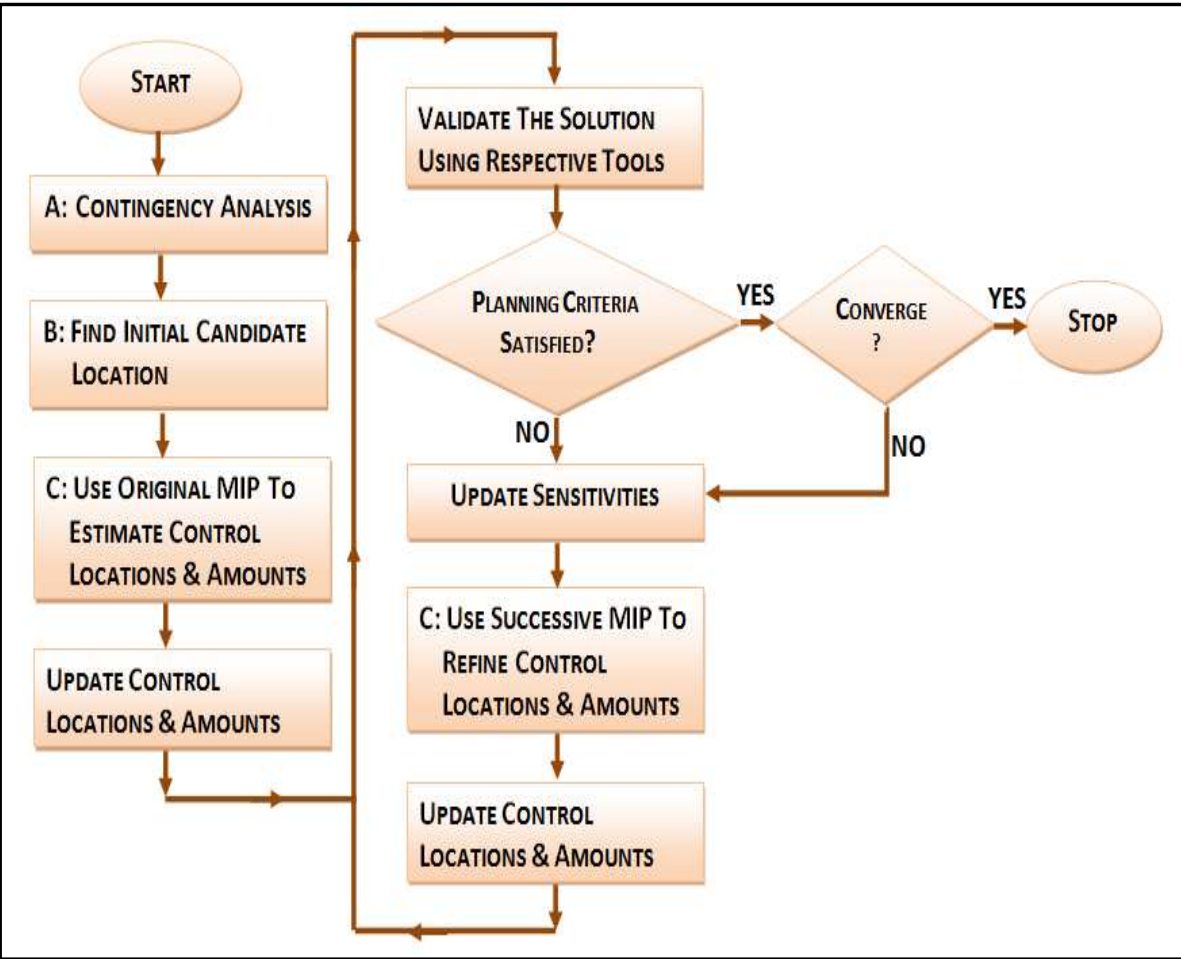


Figure 6. Flowchart of RPP procedure with successive MIP

#### 4.1. Contingency analysis

Continuation power flow (CPF) [17] based tools and time domain simulation are used to perform the contingency analysis. The post-contingency state of the system is checked for performance criteria violations, and a list of critical contingencies is formed. In the case of steady state analysis the process of contingency screening using margin sensitivity information [29] is used to reduce the computational burden [30].

#### 4.2. Formulation of coordinated control planning algorithm

This section presents the formulation of optimization developed to address the coordinated VAR planning problem for all the contingencies that have either one or both the planning problems shown in Table 1. The information required for the optimization algorithm are reactive device cost and maximum capacity limit at various voltage levels, performance measures and their sensitivities with respect to MSCs and SVCs under each critical contingencies, and an initial set of candidate locations for MSCs and SVCs. While many methodologies in the past determine static and dynamic VAR support sequentially, this method simultaneously determines the optimal allocation of static and dynamic VAR [31].

#### 4.2.1. Original mixed integer programming

The objective of mixed integer program (MIP) is to minimize the total installation cost of MSCs and SVCs while satisfying the requirements of long-term voltage stability margin and short-term post-fault transient voltage characteristics.

Minimize

$$\sum_{i \in \Omega} [C_{vi\_MSC} B_{i\_MSC} + C_{fi\_MSC} q_{i\_MSC} + C_{vi\_svc} B_{i\_svc} + C_{fi\_svc} q_{i\_svc}] \quad (12)$$

Subject to

$$\sum_{i \in \Omega} S_{M,i}^{(k)} [B_{i\_MSC}^{(k)} + B_{i\_svc}^{(k)}] + M^{(k)} \geq M_r, \forall k \quad (13)$$

$$\sum_{i \in \Omega_{svc}} S_{\tau,n,i}^{(k)} B_{i\_svc}^{(k)} + \tau_{dip,n}^{(k)} \leq \tau_{dip,n,r}, \forall n, k \quad (14)$$

$$0 \leq B_{i\_MSC}^{(k)} \leq B_{i\_MSC}, \forall k \quad (15)$$

$$0 \leq B_{i\_svc}^{(k)} \leq B_{i\_svc}, \forall k \quad (16)$$

$$B_{i\_min\_MSC} q_{i\_MSC} \leq B_{i\_MSC} \leq B_{i\_max\_MSC} q_{i\_MSC} \quad (17)$$

$$B_{i\_min\_svc} q_{i\_svc} \leq B_{i\_svc} \leq B_{i\_max\_svc} q_{i\_svc} \quad (18)$$

$$q_{i\_MSC}, q_{i\_svc} = 0, 1 \quad (19)$$

The decision variables are  $B_{i\_MSC}^{(k)}$ ,  $B_{i\_MSC}$ ,  $q_{i\_MSC}$ ,  $B_{i\_svc}^{(k)}$ ,  $B_{i\_svc}$ , and  $q_{i\_svc}$ .

$C_{f\_MSC}$  is fixed installation cost and  $C_{v\_MSC}$  is variable cost of MSCs,

$C_{f\_svc}$  is fixed installation cost and  $C_{v\_svc}$  is variable cost of SVCs,

$B_{i\_MSC}$ : size of the MSC at location  $i$ ,

$B_{i\_svc}$ : size of the SVC at location  $i$ ,

$q_{i\_MSC}=1$  if the location  $i$  is selected for installing MSCs, otherwise,  $q_{i\_MSC}=0$ ,

$q_{i\_svc}=1$  if the location  $i$  is selected for installing SVCs, otherwise,  $q_{i\_svc}=0$ ,

the superscript  $k$  represents the contingency causing insufficient voltage stability margin and/or slow voltage recovery problems,

$\Omega_{MSC}$ : set of candidate locations to install MSCs,

$\Omega_{svc}$ : set of candidate locations to install SVCs,



$\Omega$ : union of  $\Omega_{MSC}$  and  $\Omega_{svc}$ ,

$B_i^{(k)}_{MSC}$ : size of the MSC to be switched at location  $i$  under contingency  $k$ ,

$B_{i\_svc}^{(k)}$ : size of the SVC at location  $i$  under contingency  $k$ ,

$S_{M,i}^{(k)}$ : sensitivity of the voltage stability margin with respect to the shunt susceptance of MSC at location  $i$  under contingency  $k$ ,

$S_{\tau,n,i}^{(k)}$ : sensitivity of the voltage recovery time duration at bus  $n$  with respect to the size of the SVC at location  $i$  under contingency  $k$ ,

$M^{(k)}$ : voltage stability margin under contingency  $k$  and without controls,

$M_r$ : required voltage stability margin,

$\tau_{dip,n}^{(k)}$ : time duration of voltage recovery at bus  $n$  under contingency  $k$  and without controls,

$\tau_{dip,n,r}$ : maximum allowable time duration of voltage recovery at bus  $n$ ,

$B_{imin\_MSC}$ : minimum size of the MSC at location  $i$ ,

$B_{imax\_MSC}$ : maximum size of the MSC at location  $i$ ,

$B_{imin\_svc}$ : minimum size of the SVC at location  $i$ , and

$B_{imax\_svc}$ : maximum size of the SVC at location  $i$ .

Note from (13) that SVCs can also be used to increase the voltage stability margin.

#### 4.2.2. Updated successive mixed integer programming

The output of the mixed integer-programming problem in section 4.2.1 is the combined reactive compensation locations and amounts for all concerned contingencies. Now the network configuration is updated by including the identified reactive power support under each contingency. After that, the voltage stability margin is recalculated using CPF to check if sufficient margin is achieved for each concerned contingency. Also, time domain simulations are carried out to check whether the requirement of the transient voltage recovery performance is met. This step is necessary because the power system model is inherently nonlinear, and the mixed integer programming algorithm uses linear sensitivities to estimate the effect of variations of reactive support levels on the voltage stability margin and post-fault voltage recovery. So if need be, the reactive compensation locations and/or amounts can be further refined by re-computing sensitivities (with updated network configuration) under each concerned contingency, and solving a second-stage mixed integer programming problem.

This successive MIP problem based on updated sensitivity and system performance information is again formulated to minimize the total installation cost of MSCs and SVCs, subject to the constraints of the requirements of voltage stability margin and voltage

recovery. It will terminate once the post-contingency performance criteria are satisfied for all concerned contingencies, and there is no significant change in decision variables from the previous MIP solution.

## 5. Numerical illustration

The control planning method described in this paper was applied to a particular portion of US Eastern Interconnection system consisting of about 16173 buses. This subsystem belonging to a particular utility's control area, henceforth will be referred to as the "study area." The study area within this large system consisted of 2069 buses with 30065.2 MW of loading and 239 generators producing 37946.7 MW.

The contingencies considered for the study are the more probable ones, i.e., N-1 and N-G-T. For N-1 and N-G-T contingencies, according to the WECC/NERC performance table, minimum steady state performance criteria is to have a post-contingency voltage stability margin of at least 5% of the sub-system's base load. For the slow voltage recovery problem, the minimum performance should be such that it avoids the induction motor tripping. The trip relay timer of the trip induction motor is actuated when the bus voltage dips below 0.7p.u and trips if voltage doesn't recover to 0.7p.u within next 20 cycles. Therefore, the objective of the coordinated RPP is to identify a minimum cost mix of static and dynamic Var resources that results in satisfactory voltage stability and transient voltage recovery performance for all considered contingencies.

The study area was grouped into 6 Market Zones (MZ), representing the 6 different load increase (stress) directions required to perform CPF analysis. For a particular stress direction, the sink is characterized by the set of loads inside a MZ, and the source is characterized by generators outside of that MZ, but within the study area.

For the transient study, dynamic models for generators, exciter, governor systems, and appropriate load and SVC models are used. Loads in the focus area were partitioned as 50% induction motor load (dynamic) and 50% ZIP load (static). Induction motor was modeled using CIM5 model in PSS/E, which is sensitive to changing voltage and frequency. SVCs were modeled using CSVGN1 and CSVGN3 family of SVC models. The PSS/E manual presents the block diagram, parameters and detailed description for each of these models. The dynamic models of induction motor loads were further split equally into three different kinds, i.e., large, small, and trip motors. Table 2 shows some of the important parameters of each of these motor loads as defined in section 3.1.2.1. The ZIP load is modeled as 50% constant impedance and 50% constant current for real power load and 100% constant impedance for reactive power load.

The steady state contingency analysis using CPF is performed in Matlab, while the dynamic voltage stability analysis using time domain simulation is performed in PTI PSS/E [32]. As part of this study, which required Matlab using the input files in PSS/E's "raw" data format, a data conversion module was built that converted the system raw data to a format that was understandable by Matlab. The conversion module includes tasks such as careful modeling

of 3-winding transformer data, zero-impedance lines, switched shunt data, checking system topology, and checking for any islanding, so that the utility's base case is transported without any errors into Matlab.

	$H$ (p.u. motor base)	$V_T$ (p.u)	$C_T$ (cycles)	$D$	$T_{nom}$ (p.u)
<b>Large Motor</b>	1.5	0	20	2	1
<b>Small Motor</b>	0.5	0	20	2	1
<b>Trip Motor</b>	0.5	0.7	20	2	1

**Table 2.** Induction Motor Dynamic Model Parameters

Based on the pre-processing performed using these computational tools, respective sensitivity information of steady state and dynamic performance measures with respect to the reactive control device are computed, as explained in section 3.3. All the necessary input including candidate locations and control device cost information are fed as input to the coordinated planning algorithm, which is coded in Matlab and executed using CPLEX. Since candidate control locations and linear sensitivity information are used, the MIP optimization is faster even for a very large scale system. PSS/E's ability to store the results in \*.csv format is utilized in post-processing the simulation results using MS Excel.

The cost is modeled as two components [24]: fixed cost and variable cost. For SVCs the fixed and variable costs were taken to be 1.5 M\$ and 5 M\$/100MVar respectively. The maximum capacity limit for SVCs was fixed at 300 MVar. Table 3 shows the MSC cost information and the default maximum capacity constraints at every feasible location under different voltage levels. The maximum MSC limit at various voltage levels ensures avoiding over-deployment of MSCs at those voltage levels, which could degrade the voltage magnitude performance. At every solution validation step, if any bus has post-contingency voltage exceeding 1.06 p.u, a new maximum MSC (MVar) constraint is developed, and the optimization is re-run with the new constraint included.

<b>Base KV</b>	<b>Fixed Cost (Million \$)</b>	<b>Variable Cost (M \$/100MVar)</b>	<b>Maximum MSC (MVar)</b>
<b>69</b>	0.025	0.41	30
<b>100</b>	0.05	0.41	75
<b>115</b>	0.07	0.41	120
<b>138</b>	0.1	0.41	150
<b>230</b>	0.28	0.41	200
<b>345</b>	0.62	0.41	300
<b>500</b>	1.3	0.41	300

**Table 3.** MSC Cost Information and Maximum Compensation

### 5.1. Steady state reactive power planning and impact of induction motors

In this section we present only the steady state RPP, i.e., plan to satisfy post-contingency voltage stability margin only using MSCs, without including dynamic problem and SVCs in

the overall planning methodology. This in effect means we consider only constraints (13), (15), and (17) in original MIP of section 4.2.1, and also in updated successive MIP of section 4.2.2. We also analyze the impact of induction motors on the transient voltage recovery performance, and demonstrate the ineffectiveness of steady state planning approach to counter the ill-dynamics caused by induction motors in voltage stability phenomena. This strongly motivates the need for considering the short-term voltage performance and fast-responding dynamic reactive resource within a coordinated planning approach.

Steady state voltage stability analysis using CPF-based contingency screening was performed for all credible N-1 (2100 branches (T) and 168 generators (G)) and N-G-T (all possible combinations of G and T) contingencies under 6 different stress directions. The results indicated that only the stress direction corresponding to the MZ1 region was found to have post contingency steady state voltage stability problems, as indicated by the 56 critical N-G-T contingencies (28 line outages under 2 N-G base case, where  $G = \{14067, 14068\}$ ) in Table 4. The total base load in MZ1 zone (sink) is 2073 MW. The collapse point for basecase was at 2393 MW, which is equivalent to a stability margin of 15.44%. All the 56 contingencies in Table 4 have stability margin lesser than 5%, with the top 14 facing instability issues. Table 4 also shows the final optimal static VAR solution that just satisfies the minimum performance criteria under every critical contingency. The final optimal investments will be the maximum amount of MSCs required at each location. The total investment cost of MSCs to solve all the steady state voltage stability problems is 2.665 M\$.

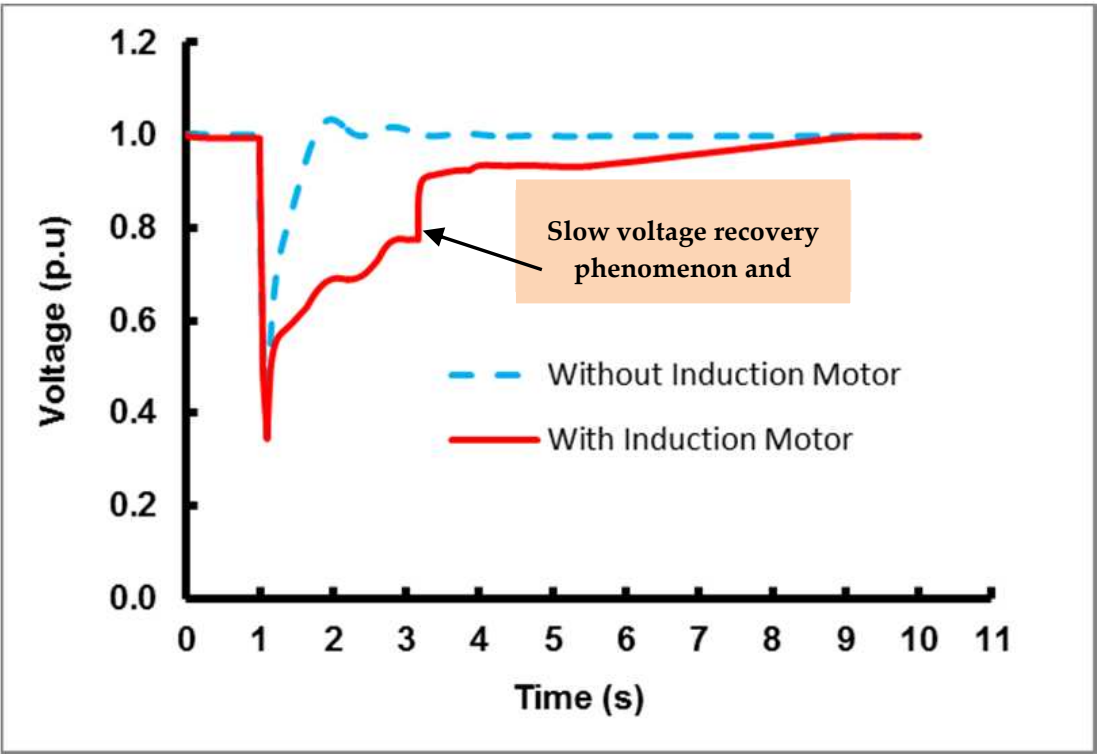
Cont. No	Transmission Line		Base KV	Stability Index (Margin in %)		Capacitor Allocation (p.u Mvar)			
	From	To		Gen#1-14067	Gen#2-14068	14073	14071	14080	14160
1	14079	14083	138	Unstable	Unstable	1.5	0	1.5	0.25
2	14094	14326	230	Unstable	Unstable	1.5	1.5	0.75	0
3	14175	14319	230	Unstable	Unstable	1.2	1.5	0	0.85
4	14296	14322	500	Unstable	Unstable	1	1.2	1.25	0
5	14319	14321	230	Unstable	Unstable	1.5	1.5	0.5	0
6	14319	14326	230	Unstable	Unstable	1.5	1.5	0.9	0
7	10983	14129	345	Unstable	Unstable	1.5	1.5	1.5	1
8	14077	14080	138	1.495417	1.784853	0.3	0	0	0
9	14130	14142	138	2.411963	2.556681	0.65	0	0	0
10	14129	14162	345	2.556681	2.701399	0.85	0	0	0
11	14294	14319	230	2.990835	3.135552	0.68	0	0	0
12	14295	14302	138	2.990835	3.135552	0.65	0	0	0
13	14071	14079	138	3.28027	3.424988	0.57	0	0	0
14	14126	14142	138	3.28027	3.424988	0.55	0	0	0
15	14109	14363	138	3.617945	3.617945	0.52	0	0	0
16	14148	14232	138	3.617945	3.762663	0.45	0	0	0

17	14302	14363	138	3.617945	3.907381	0.52	0	0	0
18	14124	14126	138	3.907381	3.907381	0.5	0	0	0
19	14148	14149	138	3.907381	3.907381	0.4	0	0	0
20	14232	14328	138	3.907381	4.052098	0.5	0	0	0
21	14237	14328	138	3.907381	4.052098	0.5	0	0	0
22	14322	14494	500	3.907381	4.052098	0.5	0	0	0
23	14106	14109	138	3.95562	4.052098	0.5	0	0	0
24	14297	14311	138	4.196816	4.341534	0.47	0	0	0
25	14134	14290	138	4.245055	4.486252	0.35	0	0	0
26	14103	14130	138	4.486252	4.63097	0.4	0	0	0
27	14106	14110	138	4.486252	4.63097	0.4	0	0	0
28	14238	14297	138	4.63097	4.823927	0.4	0	0	0

**Table 4.** Contingency List and Optimal Allocation of MSCs by Steady State Reactive Power Planning

Even though the system is planned against steady state voltage instability using MSCs, nevertheless the effect of induction motors in transient phenomena has not been investigated.

Figure 7 shows the voltage recovery phenomenon at a certain bus with and without dynamic modeling of induction motor. The figure shows the significance of modeling induction motor properly, which has important role in ascertaining post-fault-clearance short-term voltage stability of the system. Once this phenomenon is captured by appropriate modeling, we can counter it by the proposed coordinated RPP.



**Figure 7.** Voltage recovery phenomenon with and without Induction Motor modeling

With appropriate modeling of induction motor dynamics, time domain simulations were run for the top 7 contingencies by applying a 3-phase fault at  $t=0$  at one end of the transmission circuit and then clearing the fault and the circuit at 6 cycles ( $t = 0.1s$ ). All the contingencies lead to a slow voltage recovery due to the presence of induction motor loads that ultimately tripped. A list of all the buses having slow voltage recovery problem under each severe contingency was made. Table 5 shows the time domain simulation results under few of the contingencies of Table 4, wherein we notice the buses at which the minimum criteria for transient voltage recovery (more than 20 cycles below 0.7p.u) is violated. The induction motor loads connected to these buses also get tripped, in a similar manner as was shown in Figure 7. The transient voltage response under other contingencies was also very poor. For instance, under contingencies 2, 4, 5, and 6, about 76, 18, 72, 64 buses respectively violated the minimum post-fault voltage recovery criteria.

Bus Number	Contingency 1		Contingency 3	
	Recovery time	Cycles	Recovery time	Cycles
14079	0.841	50.46	0.344	20.64
14071	0.771	46.26	0.36	21.6
14084	0.694	41.64	0.36	21.6
14060	0.614	36.84	0.353	21.18

**Table 5.** Buses resulting in Slow Voltage Recovery and Induction Motor Tripping

Table 6 ranks the top 7 contingencies (under both gen. outages) shown in Table 4, based on their severity, which is quantified in terms of worst-case recovery times. It can be expected that the most severe contingencies will drive the amount of dynamic VARs needed.

Contingency No.	Bus Numbers		kV	Rank
	From	To		
1	14079	14083	138	2
2	14094	14326	230	4
3	14175	14319	230	6
4	14296	14322	500	7
5	14319	14321	230	3
6	14319	14326	230	1
7	10983	14129	345	5

**Table 6.** Contingency Ranking in terms of Worst-case Recovery Times

## 5.2. Coordinated reactive power planning

The proposed coordinated planning, as discussed in section 4.2, was performed to mitigate both transient as well as steady state problems under these contingencies. Sensitivities of post-contingency voltage stability margin and transient voltage recovery times are used as one of the inputs to find the optimal solution. The candidate locations were identified according to the following criteria:



1. Buses for which one or more contingencies result in:
  - the bus being among the top 5 worst voltage dips and
  - the bus has induction motor load that trips
2. Buses must have high steady state voltage stability margin sensitivity so that they can also efficiently address the static problems.

Table 7 shows the stage 1 MIP result, which selects two SVC locations to solve both static as well as dynamic problems. Investigation indicates the reason for this is that the transient voltage problems are so severe that the amount of SVC required to solve them is also sufficient to mitigate the steady state voltage stability problems.

N-G-T Contingencies			SVC (p.u MVAR)		Steady State Stability Margin (%)	
No.	Bus Number		KV	14071		14084
	From	To				
1	14079	14083	138	3	0.85	9.64
2	14094	14326	230	3	0.8	9.16
3	14175	14319	230	3	0.7	9.4
4	14296	14322	500	1.7	0	8.68
5	14319	14321	230	3	1.5	9.4
6	14319	14326	230	3	1.53	8.92
7	10983	14129	345	3	0.95	5.1

**Table 7.** Stage 1 MIP Result of Coordinated RPP and Steady State Stability Validation

When stage I MIP solution was validated for steady state voltage stability problems for the top 7 contingencies (under both gen. outages) of Table 4, it was found that they not only attained equilibrium, but also had post contingency voltage stability margin more than the minimum requirement of 5% as shown in Table 7. The remaining contingencies in Table 4 were also validated with this stage I MIP solution and were found to have sufficient post contingency voltage stability margin.

When time domain simulations were done to validate stage 1 MIP result, it was found that contingencies 1, 2, 5, 6, and 7 still had buses that violated the minimum recovery time requirement, resulting in tripping of some induction motors. Figures 8 and 9 show the voltage profiles at the most severely-affected buses under contingency 2 before and after stage 1 MIP result implementation, respectively.

Figure 9 also shows that the SVC placed after stage 1 MIP at buses 14071 and 14084, which are just sufficient to provide voltage recover within 20 cycles at some buses. For the contingencies having slow voltage recovery problems at some buses even after stage 1 MIP solution, a successive MIP is performed with updated sensitivity information using stage 1 SVC solution.

Table 8 shows the final operational solution of the coordinated planning problem, which chooses only SVCs due to the nature of problems under contingencies.

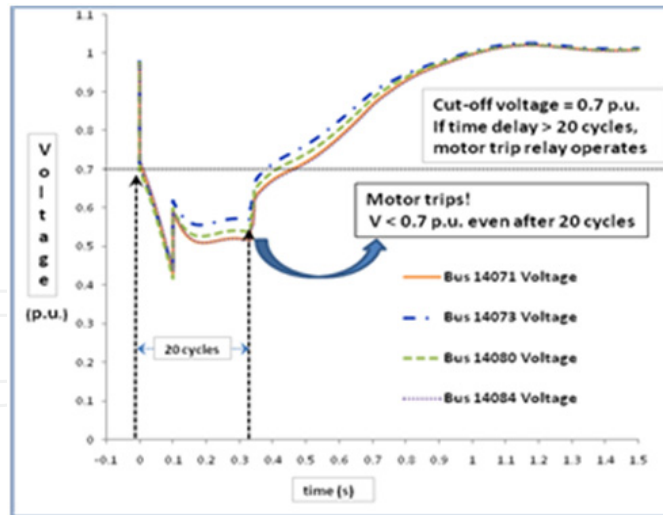


Figure 8. Voltage profiles of worst-hit buses under cont. 2 without SVC

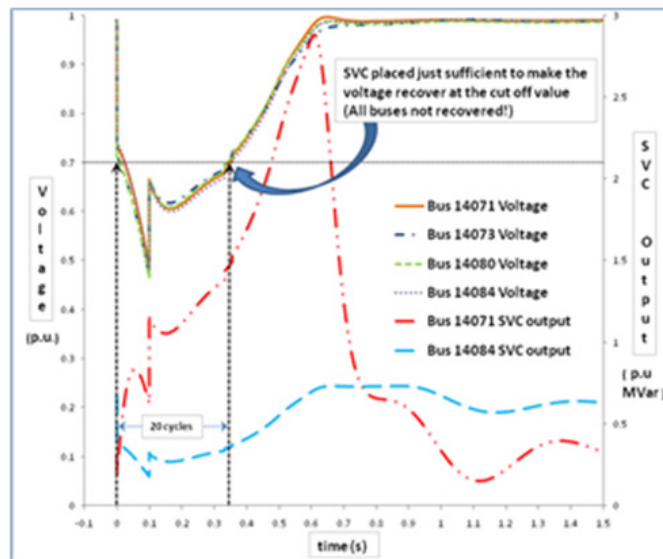
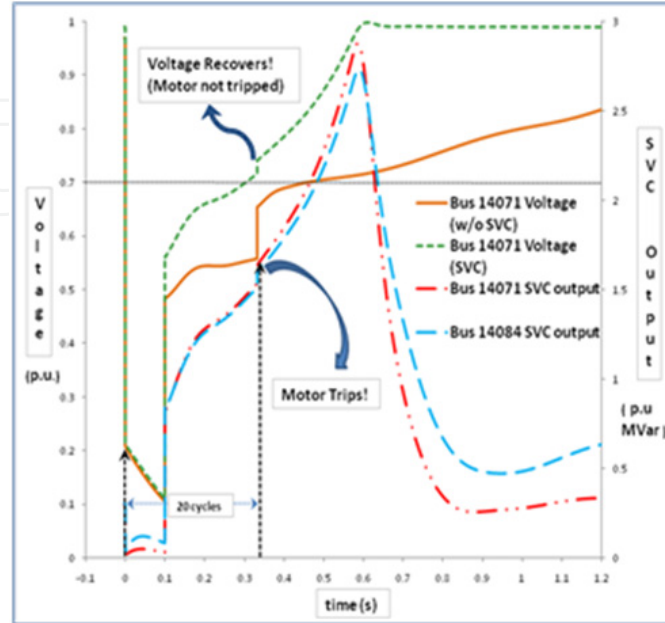


Figure 9. Voltage profiles of worst-hit buses under cont. 2 with SVC from stage 1 MIP result

N-G-T Contingencies under both the N-G				SVC (p.u MVAR)	
No.	Bus Number		KV	14071	14084
	From	To			
1	14079	14083	138	3	2.65
2	14094	14326	230	3	2.7
3	14175	14319	230	3	0.7
4	14296	14322	500	1.7	0
5	14319	14321	230	3	2.7
6	14319	14326	230	3	2.85
7	10983	14129	345	3	2.1

Table 8. Final Solution of Coordinated Optimal Planning

Figure 10 shows the improved voltage profile at Bus 14071 under contingency 1 after implementing the final SVC solution. Figure 10 also shows the output of the two SVCs. The SVC peak output at bus 14071 is 290 MVar, and that of bus 14084 is 270 MVar.



**Figure 10.** Bus 14071 voltage profile under cont. 1 with final allocation

The final investment solution for the coordinated planning problem includes SVC placements at buses 14071 and 14084 of maximum capacity 3.0 MVar and 2.85 MVar respectively<sup>1</sup>. The total cost is 32.25 M\$ under the cost assumptions used for this study.

## 6. Conclusions

This chapter presented the critical role played by induction motors in short-term voltage stability phenomenon, and emphasizes the need for including proper dynamic models of motor loads in voltage stability related planning studies. The chapter also sheds light on the need of a coordinated framework for reactive power planning considering both static and dynamic reactive control devices, which mitigates both post-contingency steady state as well transient voltage recovery and motor stalling problems. The mixed integer optimization based VAR planning algorithm is formulated to satisfy the minimum post-contingency steady state voltage stability margin and post-fault transient voltage recovery performance criteria under many critical contingencies simultaneously. The developed method was illustrated in large-scale system with 16173 buses representing US Eastern Interconnection, wherein induction motor dynamic characteristics modeling and bus voltage magnitude limit enforcement were incorporated within the overall analysis and planning respectively. The results verify that the method works satisfactorily to plan an optimal mix of static and dynamic VAR devices under

<sup>1</sup> There may be cases, when the required SVC to mitigate slow voltage recovery problems might be less, and they may still not satisfy the post-contingency steady state voltage stability margin requirements. For such cases, the MIP may choose MSCs as an effective addition to the optimal solution.

many critical contingencies simultaneously and averts steady-state and dynamic voltage instability issues, including induction motor tripping.

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