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# Non Linear Viscoelastic Model Applied on Compressed Plastic Films for Light-Weight Embankment

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Additional information is available at the end of the chapter

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## 1. Introduction

This present study aims to identify the behavior of a new cellular material, including a biphasic structure (air + solid plastic films) which consists of a flexible plastic film. The finished product is in the form of compressed and attached cubic blocks whose volume is about 1m<sup>3</sup> and whose density is ranging from 0.3 to 0.6.

This material has diverse applications in Civil Engineering and Public Works. It can be used as a light weight embankment on poor mechanical characteristics soils, to minimize the loads on buried pipelines (Marston effect) and to make soundproof wall or dams to protect from falling stones.

This research was initiated by a thorough understanding of the physical reality of the material. Then, the behavior of this material has been identified by targeted experiments, and finally a theoretical model is developed.

The first form of this model is nonlinear which was modified to be able to take into account the hardening of material while loading. Finally, this model was linearized to be applied in special cases in a field of stresses which can reach 200 kPa and covers the main domain of real applications.

## 2. Manufacturing mode

The raw materials used in the manufacture of flexible films are mainly made out of polyethylene.



As the volume and the density are fixed, we put the necessary quantity of plastic in a parallelepiped shape press. After being compressed under high pressure (about 7 bar), and legated the resulting block stayed in shape after it is released. [1].(Fig.1)

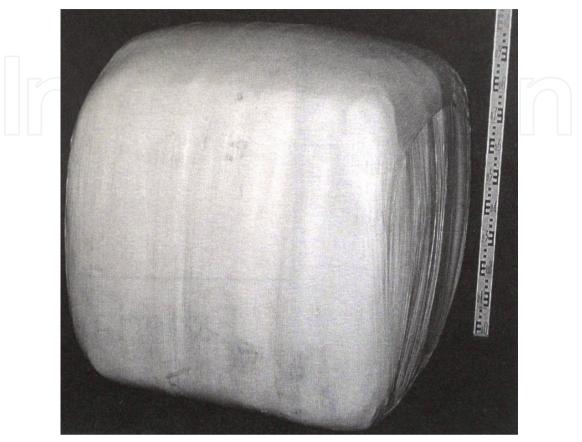


Figure 1. Photo of a block

## 3. Physical reality of material

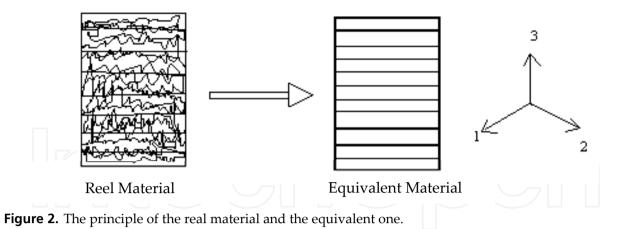
After the manufacturing process which is at unidirectional pressing, the material is in the form of stacked almost identical layers. A certain volume of air is trapped inside the plastics sheets and forms more or less closed cells. This action gives lightness and makes the material alveolar.

In reality, the material is anisotropic and discontinuous, but macroscopically, it may be considered an orthotropic, continuous material with a revolution case whose axis coincides with the direction of pressing during manufacturing. (fig.2). In this case, we will be able to apply the principles of the Continum Mechanics

On the other hand, the internal structure of the material is formed by a network of cells which are filled with air and have irregular geometric shapes. The flexible Walls of cells are not airtight.

The global deformation of loaded material is the superposition of the proper deformation of the plastic sheets and the deformation due to the changing of the volume of air including into the cells, by compressibility or and by fleeing.

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## 4. Experimental studies

The experimental studies of the behavior of the material began by compression and creep tests on full size blocks and completed by several kinds of tests on reduced size samples.

In the purpose to obtain a reduced size sample, tow apparatus were conceived and built. The first one enabled us to make cylindrical samples and the second one is conceived to produce cubic samples.

The dimensions of real blocks or samples are carefully chosen in order to neutralize the microscopic details and to be able to apply the Continum Mechanics principles.

In addition, two experimental machines are designed and constructed specifically for this type of material.

A first apparatus is intended to perform simple compression tests (Fig. 3). The load applied by a bar, was transmitted by a rigid plate that moves while remaining horizontal. This system that prevents the differential deformation is in the form of three rod lubricated to facilitate sliding.[6].

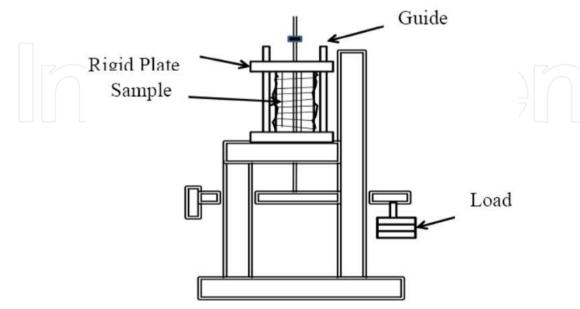
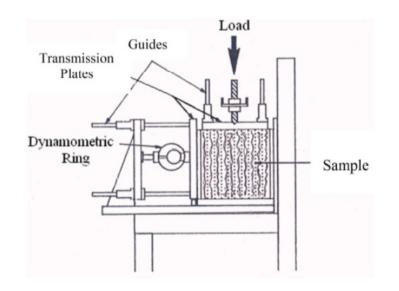


Figure 3. Apparatus for simple compression test

Secondly, a biaxial apparatus is developed (Fig. 4). In a direction, the applied load using the weight is transmitted through a rigid plate. In the other direction, the stress is measured by a dynamometer attached to another plate of the same nature. Its movement has approximately the same deformation of the ring. Other strains are measured by conventional means (comparator). The guidance system consists of four lubricated rigid rods.

The choice of biaxial testing was justified by the inability to perform simple compression tests, at the same time in the directions (1) and (2). In fact, the material is in the form of a stack of sheets clamped by metal wires. The stress in the direction, parallel to the layers causes an increase in tension in the wires and the behavior of material will be managed by their rigidity and their number. From a mechanical point of view, the sample will be charged on certain places on time (in contact with the wire) and the deformation will not be uniform on both open sides [6].



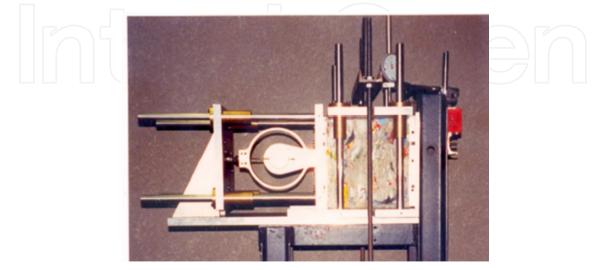


Figure 4. Biaxial apparatus

## 4.1. Testing

After testing and verifying the proper functioning of both apparatus, wide open test campaigns are made.

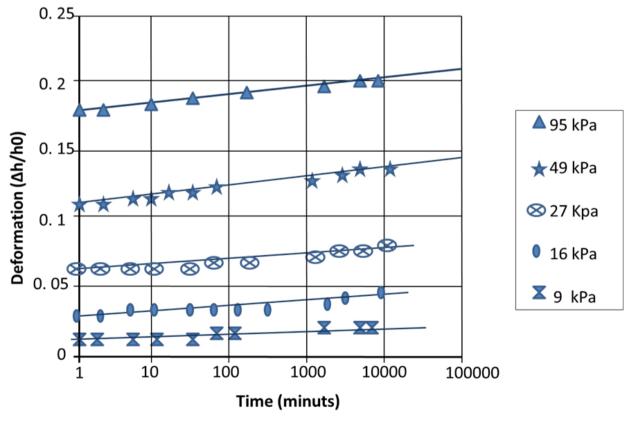
The carried out tests are :

- Compression and creep tests corresponding to the revolution axis of material.
- Cyclical and monodirectionnal tests with constante deformation velocity.
- Biaxial tests.

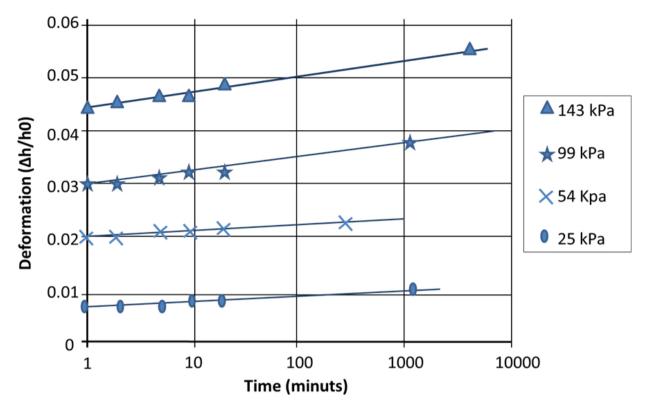
For all kind of test, real blocks and reduced samples of the same or different densities are tested according to a path of applied stresses step by step. This approach helps us to understand more the behavior of this discontinuous and anisotropic material.

## 4.2. Simple compression test (creep)

Simple compression tests along the axis of revolution of the material are carried out on real size blocks as on samples of small size with different densities (0.4, 0.5 or 0.6). The stress is applied through stages. Each strain is maintained long enough to highlight the phenomenon of creep (Fig. 5, 6, 7) [2].



**Figure 5.** Deformation- time curve in logarithmic scale, under constant stress, initial density 0.4, loading by steps. Real blocks size.



**Figure 6.** Deformation- time curve in logarithmic scale, under constant stress, initial density 0.5, loading by steps. Reduced size sample.

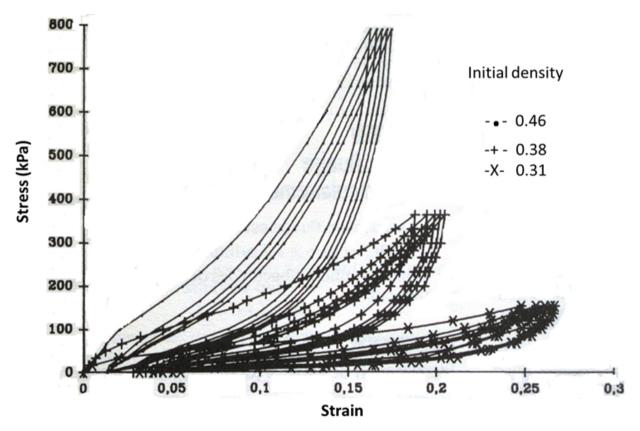
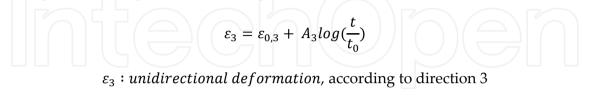


Figure 7. Stress – strain curves under constant deformation velocity (0.0003/s)

The examination and analysis of the experimental results enabled us to identify the behavior of the material and to find the necessary characteristics and parameters in order to begin the theoretical modeling:

The material behavior is a viscoelastic and nonlinear one. The deformation variation according to compression and creep tests under constant stress can be considered linear according to time in logarithmic scale (Fig. 6, 7):

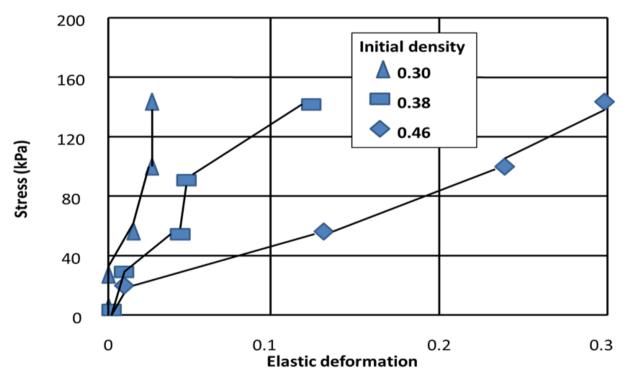


 $\varepsilon_{o,3}$ : *instantaneous deformation*, according to direction 3

 $A_3$ : slope of the deformation straight line according to time logarithm

#### $t_o: time unit$

In this scale, the slope and the ordinate at origin (instantaneous deformation) are variable according to the initial density and applied stress (fig. 8, 9).



**Figure 8.** Stress – elastic strain curves for different initial densities

The behavior depends too much on the initial density (fig.7).

Many complex rheological phenomena are observed (hardening, aging, recovery....) (fig.7).

The proper plastic deformation can be neglected in comparison with the total one (fig.10). This result was confirmed by a test realized on solid block of plastic (polyethylene whose

density is 0.94). Then, the deformation is the direct result of air volume variation by compressibility or expulsion or by both.

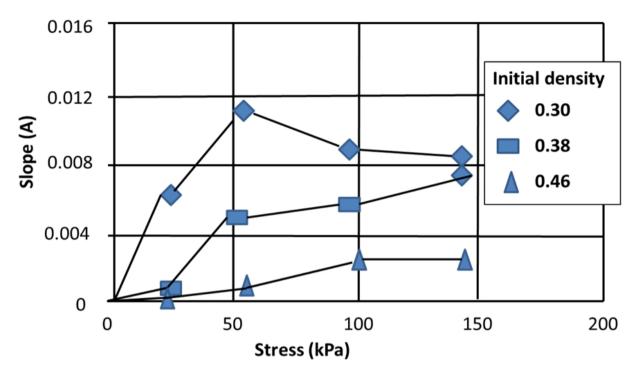
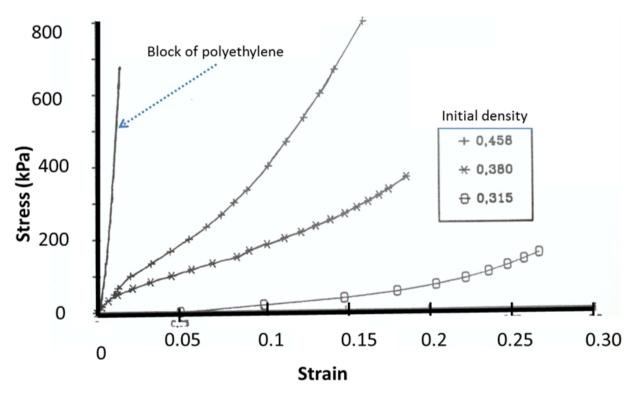


Figure 9. Slope (A) - Stress curves for different initial densities



**Figure 10.** Stress – strain curves with constant velocity deformation for the material) in comparison with the curve of solid block of polyethylene.

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#### 4.3. Biaxial tests

This test campaign is initiated by long test duration. Then, eighteen samples of four different initial densities are tested. The same stress path of the unidirectional tests is adopted. The residual stress is measured at the time of manufacture. The variation of the stress perpendicular to the layers according to the stress in the parallel direction and the time is not significant (Fig. 11, 12).

The variation of the deformation parallel to the layers can be considered as linear with the logarithm of time (Fig.13):

$$\varepsilon_2 = \varepsilon_{0,2} + A_2 log(\frac{t}{t_0})$$

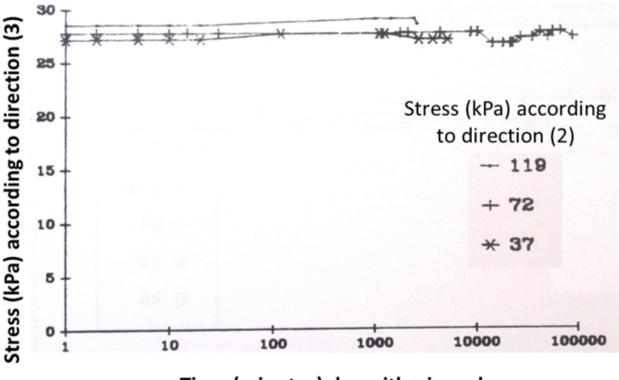
 $\varepsilon_2$  : *unidirectional deformation,* according to direction 2

 $\varepsilon_{o,2}$ : *instantaneous deformation*, according to direction 2

A<sub>2</sub>: slope of the deformation straight line according to time logarithm

#### $t_o: time unit$

In this scale, the slope A<sub>2</sub> and the ordinate at origin  $\varepsilon_{0,2}$  (instantaneous deformation) are variable according to the initial density and applied stress (fig. 14, 15, 16, 17).



## Time (minutes), logarithmic scale

**Figure 11.** Stress ( $\sigma$ 3) – time curve in logarithmic scale for initial density 0.41

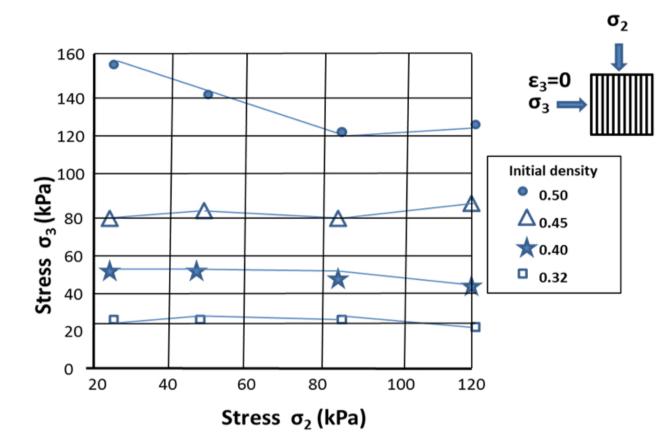
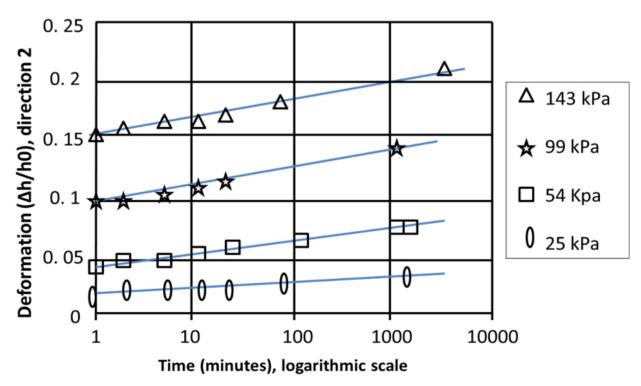


Figure 12. Stress ( $\sigma_3$ ) – Stress ( $\sigma_2$ ) for different densities



**Figure 13.** Deformation- time curve in logarithmic scale, under constant stress according to direction 2, initial density 0.32

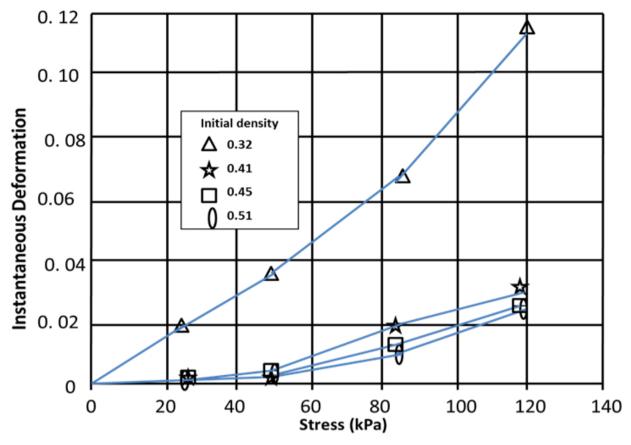


Figure 14. Instantaneous deformation (direction 2) – stress curves at constant initial density

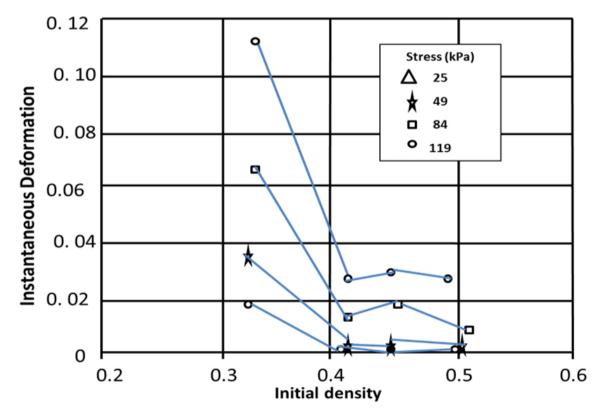


Figure 15. Instantaneous deformation (direction 2) – initial density curves at constant stress

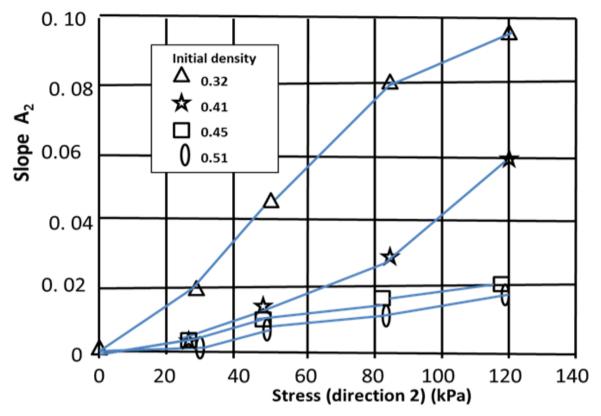


Figure 16. Slope A<sub>2</sub> (direction 2) – stress curves at constant initial density

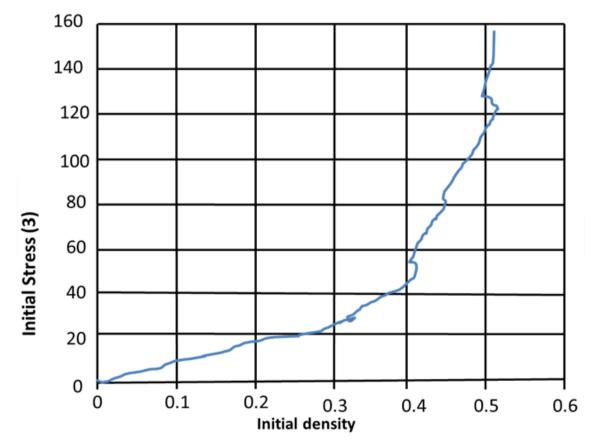


Figure 17. Initial residual stress (3) - initial density curve

# 5. Rheological Behavior and Modeling in the case of mono dimensional loading according to direction (3), perpendicular to the layers

Test campaigns on real blocks and small size samples [2] are able to show that the viscoelastic behavior is nonlinear (Fig. 5). Moreover, it is the subject of several rheological complex phenomena (hardening, aging, accommodation...) (Fig. 7) [3].

Initial density is an important parameter which influences behavior. In fact, the more great it is the more the behavior improves. The same density changes during the solicitation resulting hardening.

At first glance, the complex reality of the material makes it difficult to apply principles of mechanics of continuous materials. This is made possible by the adoption of some simplifying assumptions.

The assumptions are:

First, we assume that there is no slippage or detachment between the elementary leaves and the volume used is large enough to be able to erase the influence of microscopic details. Then, it is considered that the material is continuous, homogeneous and orthotropic revolution.

Finally, the aging process can be ignored.

On the other hand, specific plastic deformation is negligible compared to the overall deformation of material due to the change in volume of air trapped into the cells, or by compressibility and evacuation (fig.10).

Because the material is assimilated to the alveolar one, the behavior is represented by the following model (fig. 18) [8]:

The material is the subject of superposition of tow behaviors:

- Compression of the gas into closed cells
- Deformation of the walls of cells

In the case of alveolar material with open cells, the behavior of gas does not take place.

When the stress is applied quickly (case of variable stress or imposed deformation velocity), the material can be considered as alveolar with closed cells. But, when it is maintained constant for a long time (case of creep), the material can be considered as alveolar with open cells.

In conclusion, the alveolar material is characterized by an elastic limit  $\sigma_{e.}$ 

For the material under study, the elastic limit was identified by simple compression tests with constant deformation velocity (fig. 19).

The variation of elastic limit ( $\sigma_e$ ) of this material was identified experimentally and can be assimilated by a straight line whose equation is:

 $\sigma e = Pa (4.49 d0 + 1.06)$ Pa : atmospheric pressure d0 : initial density

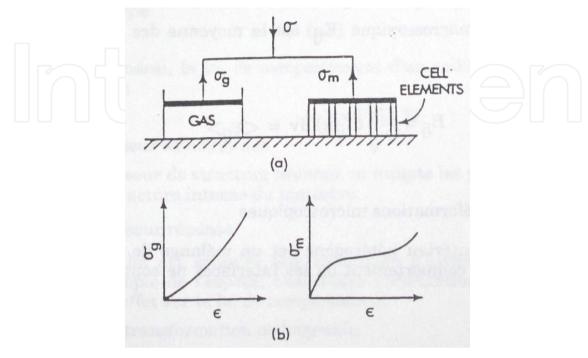


Figure 18. Rheological model of alveolar material with closed cells

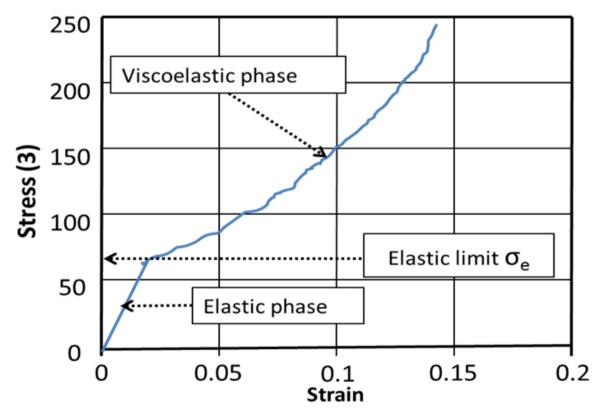
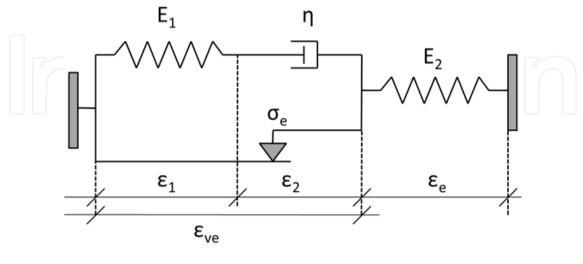


Figure 19. Stress-strain curve of material in case of closed cells

## 5.1. Nonlinear model with no hardening

Based on a simple analogical model (spring + dash-pot + skate), the behavior may be in the form:



 $\varepsilon = \varepsilon_1 + \varepsilon_2 + \varepsilon_e = \varepsilon_{ve} + \varepsilon_e$ 

The skate limits the elastic behavior on maximum value ( $\sigma_e$ ). Above this value the viscoelastic phase is activated and the behavior is described by Maxwell analogical model characterized by the Young modulus E1 and the viscosity  $\eta$ [2].

The nonlinear behavior is taken into account by the nonlinear form of  $E_1$  and  $\eta$  [2].

Then

$$\varepsilon = \varepsilon_1 + A \log(\frac{t}{t_0})$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2$$

$$\sigma = E_1 \varepsilon_1 = \eta \varepsilon_2$$

$$\varepsilon^{\bullet} = \frac{d\varepsilon}{dt} \quad \& \quad \overset{\bullet}{\sigma} = \frac{d\sigma}{dt}$$

All analysis is done, the model takes the form of a differential equation of first degree:

$$\varepsilon^{\bullet} = \left(\frac{\mathrm{d}_{\mathrm{p}} \cdot \mathrm{d}_{0}}{\mathrm{d}_{0}^{n}}\right) \left(\frac{1}{P}\right) \sigma^{\bullet} + A \exp\left[\frac{-(\varepsilon \cdot \varepsilon_{0})}{\mathrm{A}}\right]$$

With

$$\varepsilon_0 = \left(\frac{\mathbf{d}_{\mathbf{p}} - d_0}{d_0^n}\right) \left(\frac{1}{P}\right) \sigma$$

$$A = \left(\frac{d_p - d_0}{d_0^m}\right) \left(\frac{1}{P}\right) \sigma$$

$$n = a \times d_0 + b$$

$$m = a^0 \times d_0 + b^0$$

$$a, b, a^0, b^0: \text{ parameters to be identify by experience}$$

$$P: \text{ atmpspheric pressure}$$

$$d_P: \text{ proper density of plastic}$$

$$d_0: \text{ initial density}$$

#### 5.2. Nonlinear model with hardening

Until now, the initial density is taken as constant. To be able to take into account the hardening phenomenon, we will introduce the density as a function according to the strain. Necessary analysis and calculation are done. The new version of the model is presented a differential equation as the following [1]:

$$\varepsilon^{\bullet} = \left(\frac{d_{p} - \omega}{\omega^{a\omega + b}}\right) \left(\frac{1}{P}\right) \sigma^{\bullet} + \left(\frac{d_{p} - \omega}{\omega^{a_{1}\omega + b_{1}}}\right) \exp\left[\frac{-(\varepsilon - \varepsilon_{0})}{A}\right] \qquad \varepsilon_{0} = \left(\frac{d_{p} - \omega}{\omega^{a\omega + b}}\right) \left(\frac{1}{P}\right) \sigma$$
$$A = \left(\frac{d_{p} - \omega}{\omega^{a_{1}\omega + b_{1}}}\right) \left(\frac{1}{P}\right) \sigma \quad ; \quad \omega = \frac{d_{0}}{1 - \varepsilon}$$
$$dp : \text{proper density of plastic}$$

do: initial density

#### 5.3. Linearized model with no hardening

The simplification of the nonlinear model is carried out in two ways [7]:

Adoption of linear forms of the elastic and the delayed deformations according to the stress.

Adoption of linear forms of the elastic and a constant delayed one according to the stress. In another word, the linear viscoelasticity is applied.

In any case, the initial density is taken as a parameter and the model is presented as the following [5]:

$$\varepsilon^{\bullet} = \left(\frac{\mathrm{d}_{\mathrm{p}} - \mathrm{d}_{0}}{\mathrm{d}_{\mathrm{p}}^{n}}\right) \left(\frac{1}{P}\right) \sigma^{\bullet} + A \exp\left[\frac{-\left(\varepsilon - \varepsilon_{0}\right)}{A}\right]$$

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$$\varepsilon_0 = \left(\frac{\mathbf{d}_p \cdot \mathbf{d}_0}{\mathbf{d}_p^n}\right) \left(\frac{1}{P}\right) \sigma$$

 $\varepsilon 0$ : instantaneous deformation

 $\varepsilon^{\bullet}$ : unidirectional deformation velocity

A,n,m : parameters to be identified

d0: initial density

The experimental results enabled us to identify all necessary parameters.

### 5.4. Validation

The comparison of theoretical and experimental results becomes possible by numerical resolution of the differential equation in its three forms. The concordance between reality and theory is good and the simplification gives us reasons of satisfaction.(fig.20, fig.21, fig.22, fig.23) [1],[3],[7].

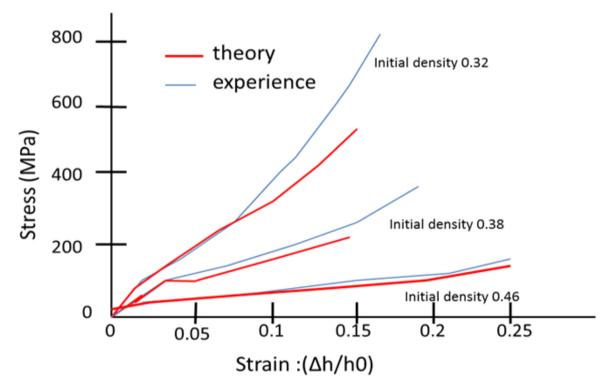
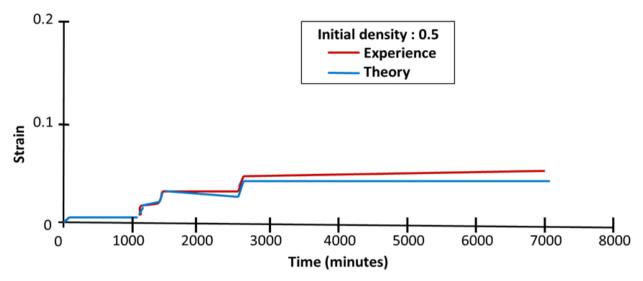


Figure 20. Non linear model, compression with constant deformation velocity

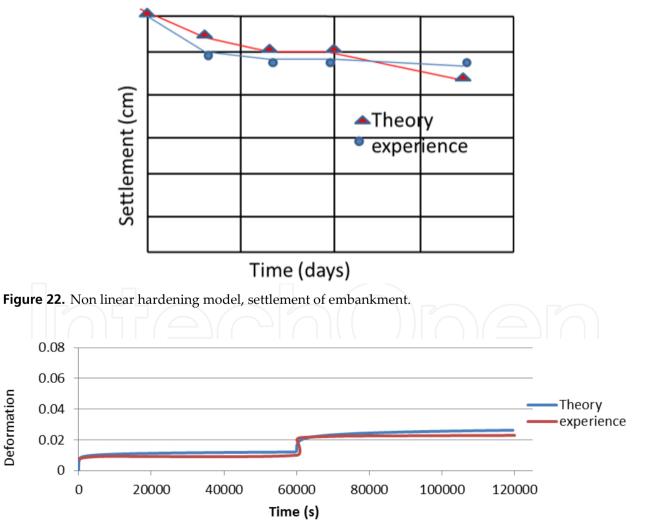
## 6. Three-dimensional model, constant stress

The hypothesis of non-aging behavior is adopted. The relation "stress-strain" in case of constant stress and small strain is represented by:

$$\overrightarrow{\sigma} = F \otimes \overrightarrow{\varepsilon}$$

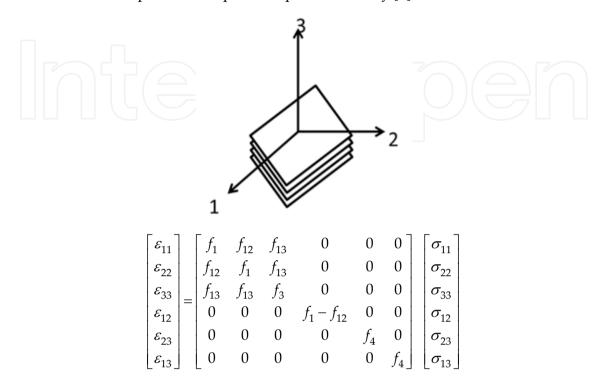


**Figure 21.** Non linear model, creep tests by stages



**Figure 23.** Linearized model. Measured and calculated deformations for an initial density: 0.493, creep tests by stages.

F is a matrix composed by  $f_1$ ,  $f_{12}$ ,  $f_3$ ,  $f_{13}$ ,  $f_4$ , five functions to be identified. They are a function of time, stress and initial density. The role of the stress perpendicular to the layers ( $\sigma_3$ ) is paramount. It marks the nonlinearity of the behavior. For the other stresses, the linearization of the behavior is adopted with a quite acceptable accuracy [6].



The identification of the functions is carried out from experimental results:

f1 and f13 identified by performing tests of simple compression along the axis of revolution.

f<sub>2</sub> and f<sub>12</sub> are determined by simple compression tests according to the two other axes or by biaxial testing.

 $f_4$  is equal to  $\frac{1}{G_{13}}$  where  $G_{13}$  is the shear modulus around the directions (1) and (2). It is identifiable by a distortion test. Without this test, it can be approached by assuming that the material is isotropic respectively with the characteristics specified in the directions (1) and (3)....

After necessary calculation, the characteristic functions take the following forms [6]:

$$f_{3} = \frac{1 - \frac{d_{0}\left(\frac{\sigma_{33}}{Pa} + B\right)}{d_{p}\left(\frac{\sigma_{33}}{Pa} + B\right) - A} + \frac{d_{p} - d_{0}}{d_{0}}\left[\left(\frac{\sigma_{33}}{Pa} + 1\right)^{m} - 1\right]\left[\ln\left(\frac{t}{t_{0}}\right)\right]}{\frac{\sigma_{33}}{Pa}}$$

 $\mathbf{f}_{13} = \alpha_3 f_3$ 

$$\begin{split} f_{1} &= \left(d_{p} - d_{0}\right) \left[\frac{1}{d_{0}^{k}} + \frac{1}{d_{0}^{l}} \ln\left(\frac{t}{t_{0}}\right)\right] + \\ &\alpha_{3}^{2} f_{3}. \ f_{12} &= \alpha_{1} \left(d_{p} - d_{0}\right) \left[\frac{1}{d_{0}^{k}} + \frac{1}{d_{0}^{l}} \ln\left(\frac{t}{t_{0}}\right)\right] + \\ &\alpha_{3}^{2} f_{3}. f_{4} &= \frac{f_{3} \left(1 - \alpha_{3}\right) + K \left(1 - \alpha_{3}\right)}{2} ; \\ &K &= \left(d_{p} - d_{0}\right) \left[\frac{1}{d_{0}^{k}} + \frac{1}{d_{0}^{l}} \ln\left(\frac{t}{t_{0}}\right)\right] \end{split}$$

Given the approximation of  $f_4$ , we can neglect  $\alpha_1$  and  $\alpha_3$  by comparison to (1) and  $f_4$  becomes:

$$f_4 = \frac{\left(f_3 + K\right)}{2},$$

t: time,
to: initial time,
d<sub>P</sub>: density of plastic
do: initial density
A, B, k, l: parameters depending on the initial density



Figure 24. One of realized projects in France

## 7. Conclusion

The development of mono axial and biaxial testing, the design and implementation of an appropriate apparatus helped us to perform many tests. Therefore, the characteristic functions of the material were determined satisfactorily. This allowed us to identify the essential parameters to the theoretical models developed.

In a particular field of stress up to 200 kPa, the simplified model gives good satisfaction. The results are acceptable and the difference between theoretical and experimental ones does not overstep the bounds of uncertainty both in physical measurement and caracteristics variability due to the material anisotropy.

However, in several cases of application, the model in its many forms helps us to conceive and design structures by using this technique. Indeed, many projects were conceived and realized in France (fig.24).

## Author details

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