Chapter 2

Robust Control Design of Uncertain Discrete-Time Descriptor Systems with Delays

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Additional information is available at the end of the chapter

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1. Introduction

A descriptor system describes a natural representation for physical systems. In general, the continuous-time descriptor representation consists of differential and algebraic equations, and the discrete-time descriptor system has difference and algebraic equations. Hence, the descriptor system is a generalized representation of the state-space system. This system appears in various physical systems. In fact, descriptor systems can be found in electrical circuits, moving robots and many other practical systems which are modeled with additional algebraic constraints. The descriptor system is also referred to as singular system, implicit system, generalized state-space system, differential-algebraic system, or semistate system. System analysis and control design of descriptor systems have been extensively investigated in the past years due to their potential representation ([4], [6], [7], [17], [23]). An important characteristic of continuous-time descriptor systems is the possible impulse modes, which are harmful to physical systems and are undesirable in system control. The discrete-time descriptor system may not have causality, which leads to no solution of the system states. In [4], [32], such descriptor system behaviors are described and notion of regularity, non-impulse, causality, and admissibility are given. In [1] and [22], quadratic stability for continuous-time descriptor systems was considered. Its discrete-time system counterpart was investigated in [31] and [32].

When we make a mathematical model for a physical system, time-delay is another phenomenon. We often see time-delay in the process of control algorithms and the transmission of information. Time-delay often appear in many practical systems and mathematical formulations such as electrical system, mechanical system, biological system, and transportation system. Hence, a system with time-delay is also a natural representation, and its analysis and synthesis are of theoretical and practical importance. In the past decades, research on continuous-time delay systems has been active. Difficulty that arises in continuous time-delay systems is that the system is infinite dimensional and a corresponding controller can be a memory feedback. This class of a controller may minimize a certain performance index, but it is difficult to implement it to practical systems because it feeds back past information of the system. To overcome such a difficulty, a memoryless controller

is used for time-delay systems. In the last decade, sufficient stability conditions for time-delay systems have been given via linear matrix inequalities (LMIs), and stabilization methods by memoryless controllers have been investigated by many researchers. Since Li and de Souza considered robust stability and stabilization problems in [18], less conservative stability conditions for continuous time-delay systems have been obtained in [14] and [26]. Recently, H_∞ disturbance attenuation conditions have also been given ([25], [34], [35]). The results in [10], [27], [33], [36] considered discrete-time systems with time-invariant delays. Gao and Chen [11], Hara and Yoneyama [12], [13] gave robust stability conditions. Fridman and Shaked [8] solved a guaranteed cost control problem. Fridman and Shaked [9], Zhang and Han [37] considered the H_∞ disturbance attenuation. The results have been extended to a class of discrete-time descriptor delay systems in [2], [3], [24].

In general, control systems are designed not only for the stability, but also for robustness with respect to system parameters. In addition, they are designed for the optimization of multiple control performance measures. Most designed control systems require accurate controllers. Thus, when a desired controller is implemented, all of the controller coefficients are required to be the exact values as those to be designed. However, it is not always possible in practical applications since actuators may be of malfunction, and round-off errors in numerical computations by calculations are possibly encountered. Therefore, it is necessary that the designed controller should be able to tolerate some uncertainty in its control gains. Since controller fragility problem has to be considered when implementing a designed controller in practical applications, the non-fragile control design problem has been investigated in [5], [15], [16], [19], [20], [21]. For state-space systems, several recent research works have been devoted to the design problem of non-fragile robust control ([5], [19], [20], [21]). Most of these are derived via either Riccati matrix equation approach or linear matrix inequality (LMI) approach. The design problem of non-fragile robust controllers of continuous-time descriptor systems was investigated in [15] and [16]. The discrete-time counterpart was given in [28].

In this chapter, the robust non-fragile control design problem and the robust H_∞ non-fragile control design problem for uncertain discrete-time descriptor systems are considered. The controller gain uncertainties and uncertain system parameters under consideration are supposed to be time-varying but norm-bounded. The problem to be addressed is the control design problem of state feedback controller, which is subject to norm-bounded uncertainty, such that the resulting closed-loop system is regular, causal and robustly admissible with H_∞ disturbance attenuation for all admissible uncertainties. Sufficient conditions for the solvability of the robust H_∞ non-fragile control design problem for descriptor systems are obtained, for the cases with multiplicative controller uncertainties. The results are developed for a class of uncertain discrete-time descriptor systems with time-delay. Finally, some numerical examples are shown to illustrate our proposed controller design methods.

2. Descriptor systems

Consider the discrete-time descriptor system

$$Ex(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $x(k) \in \Re^n$ is the state and $u(k) \in \Re^m$ is the control. E, A and B are system matrices with appropriate dimensions. E satisfies $rankE = r \leq n$. Unforced descriptor system (1) with $u(k) = 0$ is denoted by the pair $(E,\ A)$.

Definition 2.1. *(Dai [4]) (i) The pair $(E,\ A)$ is said to be regular if $det(zE - A)$ is not identically zero.*
(ii) The pair $(E,\ A)$ is said to be causal if it is regular and $deg(det(zE - A)) = rank(E)$.
(iii) Define the generalized spectral radius as $\rho(E,\ A) = \max_{\lambda \in \{det(zE-A)=0\}} |\lambda|$. The pair $(E,\ A)$ is said to be stable if $\rho(E,\ A) < 1$.
(iv) The pair $(E,\ A)$ is said to be admissible if it is regular, causal and stable.

Lemma 2.2. *(Dai [4]) (i) The descriptor system*

$$Ex(k+1) = Ax(k)$$

where

$$(E,\ A) = \left(\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},\ \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \right)$$

is regular and causal if and only if A_4 is invertible.
(ii) The pair $(E,\ A)$ is admissible if and only if A_4 is nonsingular and $\rho(A_1 - A_2 A_4^{-1} A_3) < 1$.

Unlike the standard state-space system, a descriptor system may not be regular and causal. These unique characteristics lead to no solution of the system (1), and thus should be taken care of.

Next, we consider the uncertain descriptor system

$$Ex(k+1) = (A + \Delta A)x(k) + Bu(k) \tag{2}$$

where an uncertain matrix is of the form

$$\Delta A = HF(k)G \tag{3}$$

where $F(k) \in \Re^{l \times j}$ is an unknown time-varying matrix satisfying $F^T(k)F(k) \leq I$ and H and G are constant matrices of appropriate dimensions.

3. Non-fragile control

This section provides system analysis and control design for uncertain descriptor systems. First, the non-fragile controller is discussed in Section 3.1, and then the system analysis of the closed-loop system with a non-fragile controller is investigated in Section 3.2. Finally, Section 3.3 proposes non-fragile control design methods.

3.1. Form of controller and preliminary results

The ideal form of a feedback controller is given by

$$u(k) = Kx(k)$$

where K is a feedback gain to be determined. In practical situations where malfunction in the actuator and uncertain calculation of control gain may occur, the actual controller is assumed to be of the form

$$u(k) = [I + \alpha\Phi(k)]\, Kx(k) \tag{4}$$

where $\alpha\Phi(k)K$ shows uncertainty in the control gain. $\Phi(k)$ is an unknown time-varying matrix satisfying

$$\Phi^T(k)\Phi(k) \leq I, \tag{5}$$

and α is a known positive constant which indicates the measure of non-fragility against controller gain variation. Applying the controller (4) to the system (2), we have the closed-loop system

$$Ex(k+1) = (A + BK + H_c F_c(k) G_c)x(k) \tag{6}$$

where

$$H_c = [H \;\; \alpha B],\; F_c(k) = \mathrm{diag}[F(k) \;\; \Phi(k)],\; G_c = \begin{bmatrix} G \\ K \end{bmatrix}.$$

Definition 3.1. *The system (6) is said to be robustly admissible if it is admissible for all admissible uncertainties (3) and (5).*

The problem is to find a controller (4) which makes the system (2) robustly admissible. In the following, we consider the robust admissibility of the closed-loop system (6). The following lemmas are useful to prove our results.

Lemma 3.2. *(Xie [29]) Given matrices $Q = Q^T$, H, G and $R = R^T > 0$ with appropriate dimensions.*

$$Q + HF(k)G + G^T F^T(k) H^T < 0$$

for all $F(k)$ satisfying $F^T(k)F(k) \leq R$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Q + \frac{1}{\varepsilon} HH^T + \varepsilon G^T RG < 0.$$

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Lemma 3.3. *(Xu & Lam [32]) (i) The descriptor system (1) is admissible if and only if there exist matrices $P > 0$ and Q such that*

$$A^T PA - E^T PE + A^T SQ^T + QS^T A < 0 \tag{7}$$

where $S \in \Re^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^T S = 0$.
(ii) The descriptor system (1) is admissible if and only if there exists a matrix P such that

$$\begin{aligned} E^T PE &\geq 0, \\ A^T PA - E^T PE &< 0. \end{aligned}$$

Lemma 3.4. *(Xie and de Souza [30]) Given matrices X, $Y > 0$ and Z with appropriate dimensions, we have*

$$X^T Z + Z^T X + X^T YX \geq -Z^T Y^{-1} Z.$$

3.2. Robust admissibility analysis

The following two theorems give a necessary and sufficient condition for the closed-loop system (6) to be robustly admissible.

Theorem 3.5. *Given K, the descriptor system (6) is robustly admissible if and only if there exist matrices $P > 0$, Q and scalar $\varepsilon > 0$ such that*

$$\begin{bmatrix} \begin{pmatrix} QS^T(A+BK) + (A+BK)^T SQ^T \\ -E^T PE + \varepsilon(G^T G + K^T K) \end{pmatrix} & (A+BK)^T P & QS^T H & \alpha QS^T B \\ P(A+BK) & -P & PH & \alpha PB \\ H^T SQ^T & H^T P & -\varepsilon I & 0 \\ \alpha B^T SQ^T & \alpha B^T P & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{8}$$

where $S \in \Re^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^T S = 0$.

Proof: (Sufficiency) Suppose that there exist matrices $P > 0$, Q and scalar $\varepsilon > 0$ such that the condition (8) holds. Then, by Schur complement formula, we have

$$\begin{aligned} &\begin{bmatrix} QS^T(A+BK) + (A+BK)^T SQ^T - E^T PE & (A+BK)^T P \\ P(A+BK) & -P \end{bmatrix} \\ &+\varepsilon^{-1} \begin{bmatrix} QS^T H_c \\ PH_c \end{bmatrix} \begin{bmatrix} H_c^T SQ^T & H_c^T P \end{bmatrix} + \varepsilon \begin{bmatrix} G_c^T \\ 0 \end{bmatrix} \begin{bmatrix} G_c & 0 \end{bmatrix} < 0. \end{aligned} \tag{9}$$

Now, using Lemma 3.2, we have

$$\begin{aligned}
&\begin{bmatrix} \begin{pmatrix} QS^T(A+BK+H_cF_c(k)G_c) \\ +(A+BK+H_cF_c(k)G_c)^TSQ^T - E^TPE \end{pmatrix} & (A+BK+H_cF_c(k)G_c)^TP \\ P(A+BK+H_cF_c(k)G_c) & -P \end{bmatrix} \\
&= \begin{bmatrix} QS^T(A+BK)+(A+BK)^TSQ^T-E^TPE & (A+BK)^TP \\ P(A+BK) & -P \end{bmatrix} \\
&\quad + \begin{bmatrix} QS^TH_c \\ PH_c \end{bmatrix} \Phi(k) \begin{bmatrix} G_c & 0 \end{bmatrix} + \begin{bmatrix} G_c^T \\ 0 \end{bmatrix} \Phi(k) \begin{bmatrix} H_c^TSQ^T & H_c^TP \end{bmatrix} \\
&\leq \begin{bmatrix} QS^T(A+BK)+(A+BK)^TSQ^T-E^TPE & (A+BK)^TP \\ P(A+BK) & -P \end{bmatrix} \\
&\quad \varepsilon^{-1} \begin{bmatrix} QS^TH_c \\ PH_c \end{bmatrix} \begin{bmatrix} H_c^TSQ^T & H_c^TP \end{bmatrix} + \varepsilon \begin{bmatrix} G_c^T \\ 0 \end{bmatrix} \begin{bmatrix} G_c & 0 \end{bmatrix}.
\end{aligned}$$

It follows from (9) that

$$\begin{bmatrix} \begin{pmatrix} QS^T(A+BK+H_c\Phi(k)G_c) \\ +(A+BK+H_c\Phi(k)G_c)^TSQ^T - E^TPE \end{pmatrix} & (A+BK+H_c\Phi(k)G_c)^TP \\ P(A+BK+H_c\Phi(k)G_c) & -P \end{bmatrix} < 0.$$

This implies by Schur complement formula and Lemma 3.3(i) that the descriptor system (6) is robustly admissible.

(Necessity) Assume that the descriptor system (6) is robustly admissible. Then, it follows from Definition 3.1 and Lemma 3.3 that there exist matrices $P > 0$, Q and scalar $\varepsilon > 0$ such that (7) with A replaced by $A + BK + H_c\Phi(k)G_c$ holds. Thus, for all admissible $F(k)$ and $\Phi(k)$, the following inequality holds:

$$\begin{bmatrix} \begin{pmatrix} QS^T(A+BK+H_cF_c(k)G_c) \\ +(A+BK+H_c\Phi(k)_cG_c)^TSQ^T - E^TPE \end{pmatrix} & (A+BK+H_cF_c(k)G_c)^TP \\ P(A+BK+H_cF_c(k)G_c) & -P \end{bmatrix} < 0.$$

That is,

$$\begin{aligned}
&\begin{bmatrix} QS^T(A+BK)+(A+BK)^TSQ^T-E^TPE & (A+BK)^TP \\ P(A+BK) & -P \end{bmatrix} \\
&+ \begin{bmatrix} QS^TH_c \\ PH_c \end{bmatrix} \Phi(k) \begin{bmatrix} G_c & 0 \end{bmatrix} + \begin{bmatrix} G_c^T \\ 0 \end{bmatrix} \Phi(k) \begin{bmatrix} H_c^TSQ^T & H_c^TP \end{bmatrix} < 0
\end{aligned}$$

is satisfied for all admissible $F(k)$ and $\Phi(k)$. It follows from Lemma 3.2 that

$$\begin{aligned}
&\begin{bmatrix} QS^T(A+BK)+(A+BK)^TSQ^T-E^TPE & (A+BK)^TP \\ P(A+BK) & -P \end{bmatrix} \\
&+\varepsilon^{-1} \begin{bmatrix} QS^TH_c \\ PH_c \end{bmatrix} \begin{bmatrix} H_c^TSQ^T & H_c^TP \end{bmatrix} + \varepsilon \begin{bmatrix} G_c^T \\ 0 \end{bmatrix} \begin{bmatrix} G_c & 0 \end{bmatrix} < 0,
\end{aligned}$$

which leads to the condition (8) by Schur complement formula. This completes the proof.

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Similarly, using Lemma 3.3(ii), we obtain the following result.

Theorem 3.6. *Given K, the descriptor system (6) is robustly admissible if and only if there exist matrix P and scalar $\varepsilon > 0$ such that*

$$E^T P E \geq 0, \tag{10}$$

$$\begin{bmatrix} -E^T P E + \varepsilon(G^T G + K^T K) & (A+BK)^T P & 0 & 0 \\ P(A+BK) & -P & PH & \alpha PB \\ 0 & H^T P & -\varepsilon I & 0 \\ 0 & \alpha B^T P & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{11}$$

3.3. Robust control design

In the previous section, we have obtained robust admissibility conditions of the closed-loop system (6). Based on those conditions, we now seek how to calculate a feedback gain K in the controller (4).

Theorem 3.7. *There exists a controller (4) that makes the descriptor system (2) robustly admissible if there exist matrices $P > 0$, Q and scalar $\varepsilon > 0$ such that*

$$\Gamma = P^{-1} - \varepsilon^{-1} H_c H_c^T > 0, \tag{12}$$

$$QS^T A + A^T S Q^T - E^T P E + \varepsilon G^T G + \varepsilon^{-1} Q S^T H_c H_c^T S Q^T + \Theta^T \Gamma^{-1} \Theta - \Psi \Lambda^{-1} \Psi^T < 0 \tag{13}$$

where $S \in \Re^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E^T S = 0$, and

$$\begin{aligned} \Psi &= QS^T B + \Theta^T \Gamma^{-1} B, \\ \Lambda &= B^T \Gamma^{-1} B + \varepsilon I, \\ \Theta &= A + \varepsilon^{-1} H_c H_c^T S Q^T. \end{aligned}$$

In this case, a feedback gain in the controller (4) is given

$$K = -\Lambda^{-1} \Psi^T. \tag{14}$$

Proof: (Sufficiency) The closed-loop system (6) with the feedback gain (14) is given by

$$Ex(k+1) = (A - B\Lambda^{-1}\Gamma^T + H_c F_c(k) G_c) x(k).$$

where $G_c = \begin{bmatrix} G^T & -\Psi\Lambda^{-1} \end{bmatrix}^T$. Then, by some mathematical manipulation and (13), it can be verified that

$$
\begin{aligned}
& QS^T(A - B\Lambda^{-1}\Psi^T) + (A - B\Lambda^{-1}\Psi^T)^T SQ^T - E^T PE + \varepsilon G_c^T G_c + \varepsilon^{-1} QS^T H_c H_c^T SQ^T \\
& +(A - B\Lambda^{-1}\Psi^T + \varepsilon^{-1} H_c H_c^T SQ^T)^T \Gamma^{-1}(A - B\Lambda^{-1}\Psi^T + \varepsilon^{-1} H_c H_c^T SQ^T) \\
= & QS^T A + A^T SQ^T - E^T PE + (A + \varepsilon^{-1} H_c H_c^T SQ^T)^T \Gamma^{-1}(A + \varepsilon^{-1} H_c H_c^T SQ^T) \\
& +\varepsilon G^T G - QS^T B\Lambda^{-1}\Psi^T - (A + \varepsilon^{-1} H_c H_c^T SQ^T)^T \Gamma^{-1} B\Lambda^{-1}\Psi^T - \Psi\Lambda^{-1} B^T SQ^T \\
& -\Psi\Lambda^{-1} B^T \Gamma^{-1}(A + \varepsilon^{-1} H_c H_c^T SQ^T) + \Psi\Lambda^{-1} B^T \Gamma^{-1} B\Lambda^{-1}\Gamma^T + \varepsilon\Psi\Lambda^{-1}\Lambda^{-1}\Psi^T \\
& +\varepsilon^{-1} QS^T H_c H_c^T SQ^T \\
= & QS^T A + A^T SQ^T - E^T PE + (A + \varepsilon^{-1} H_c H_c^T SQ^T)^T \Gamma^{-1}(A + \varepsilon^{-1} H_c H_c^T SQ^T) \\
& +\varepsilon G^T G - [QS^T + (A + \varepsilon^{-1} H_c H_c^T SQ^T)^T \Gamma^{-1}] B\Lambda^{-1}\Psi^T \\
& -\Psi\Lambda^{-1} B^T [SQ^T + \Gamma^{-1}(A + \varepsilon^{-1} H_c H_c^T SQ^T)] + \Psi\Lambda^{-1}\Psi^T + \varepsilon^{-1} QS^T H_c H_c^T SQ^T \\
= & QS^T A + A^T SQ^T - E^T PE + \varepsilon G^T G + \varepsilon^{-1} QS^T H_c H_c^T SQ^T + \Theta^T \Gamma^{-1}\Theta - \Psi\Lambda^{-1}\Psi^T < 0.
\end{aligned}
$$

By Schur complement formula, we obtain

$$
\begin{bmatrix}
\begin{pmatrix} QS^T(A - B\Lambda^{-1}\Psi^T) + (A - B\Lambda^{-1}\Psi^T)^T SQ^T \\ -E^T PE + \varepsilon G^T G + \varepsilon\Psi\Lambda^{-1}\Lambda^{-1}\Psi^T \end{pmatrix} & (A - B\Lambda^{-1}\Psi^T)^T P & QS^T H_c \\
P(A - B\Lambda^{-1}\Psi^T) & -P & PH_c \\
H_c^T SQ^T & H_c^T P & -\varepsilon I
\end{bmatrix} < 0.
$$

Hence, by Theorem 3.5 we can show that the closed-loop system (6) is robustly admissible. (Necessity) Assume there exists a feedback control of the form (4) which makes the descriptor system (1) robustly admissible. Then, it follows from Theorem 3.5 that there exists a scalar $\varepsilon > 0$ such that

$$
\begin{bmatrix}
QS^T(A + BK) + (A + BK)^T SQ^T - E^T PE + \varepsilon G_c^T G_c & (A + BK)^T P & QS^T H_c \\
P(A + BK) & -P & PH_c \\
H_c^T SQ^T & H_c^T P & -\varepsilon I
\end{bmatrix} < 0.
$$

It follows from Schur complement formula that

$$
QS^T A + A^T SQ^T - E^T PE + \varepsilon^{-1} QS^T H_c H_c^T SQ^T + \Theta^T \Gamma^{-1}\Theta + K^T\Psi^T + \Psi K + K^T\Lambda K < 0.
$$

By Lemma 3.4, we have

$$
K^T\Psi^T + \Psi K + K^T\Lambda K \geq -\Psi\Lambda^{-1}\Psi^T.
$$

Therefore, we obtain the conditions (12) and (13), and the feedback gain K is calculated as in (14).

Similarly, we can prove the following theorem by using Theorem 3.6.

Theorem 3.8. *There exists a controller (4) that makes the descriptor system (1) robustly admissible if there exist matrix P and scalar $\varepsilon > 0$ such that (10), (12), and*

$$
-E^T PE + \varepsilon G^T G + A^T \Gamma^{-1} A - A^T \Gamma^{-1} B\Lambda^{-1} B^T \Gamma^{-1} A < 0 \tag{15}
$$

where Γ, Λ are given in Theorem 3.7. In this case, the feedback gain in the controller (4) is given as in (14).

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4. H_∞ non-fragile control

In this section, we consider the robust admissibility with H_∞ disturbance attenuation. H_∞ disturbance attenuation problem plays an important role in control systems. Section 4.1 discusses the analysis of H_∞ disturbance attenuation, and Section 4.2 gives design methods of the H_∞ non-fragile controllers.

4.1. Robust H_∞ disturbance attenuation for uncertain systems

First, we consider the robust H_∞ disturbance attenuation γ for the following uncertain descriptor system

$$\begin{aligned} Ex(k+1) &= (A+\Delta A)x(k) + B_1 w(k) + B_2 u(k), \\ z(k) &= Cx(k) + Du(k) \end{aligned} \tag{16}$$

where $w(k) \in \Re^{m_1}$ is the disturbances and $z(k) \in \Re^p$ is the controlled output. A, B_1, B_2, C and D are system matrices with appropriate dimensions. Uncertain matrix ΔA is assumed to be of the form (3).

The controller is assumed to be of the form (4). Applying the controller (4) to the system (16), we have the closed-loop system

$$\begin{aligned} Ex(k+1) &= (A + B_2K + H_cF_c(k)G_c)x(k) + B_1 w(k), \\ z(k) &= (C + DK + \alpha D\Phi(k)K)x(k) \end{aligned} \tag{17}$$

where

$$H_c = [H \;\; \alpha B_2],\; F_c(k) = \mathrm{diag}[F(k) \;\; \Phi(k)],\; G_c = \begin{bmatrix} G \\ K \end{bmatrix}.$$

Define the cost function

$$J = \sum_{k=0}^{\infty} (z^T(k)z(k) - \gamma^2 w^T(k)w(k)). \tag{18}$$

The problem is to find a controller (4) which makes the system (16) with $w(k) = 0$ robustly admissible, and makes it satisfy $J < 0$ in (18). If there exists such a controller, it is said to be an H_∞ non-fragile controller and the closed-loop system is said to be robustly admissible with H_∞ disturbance attenuation γ.

The following is a well-known result for the admissibility with H_∞ disturbance attenuation γ of linear descriptor systems.

Lemma 4.1. *(Xu & Lam [32]) Consider the system*

$$\begin{aligned} Ex(k+1) &= Ax(k) + Bw(k), \\ z(k) &= Cx(k) + Dw(k) \end{aligned} \tag{19}$$

where A, B, C and D are matrices of appropriate dimensions.
(i) Given a scalar $\gamma > 0$. The descriptor system (19) is robustly admissible with H_∞ disturbance attenuation γ if and only if there exist matrices $P > 0$, Q such that

$$\begin{bmatrix} A^TPA - E^TPE + C^TC & A^TPB + C^TD \\ B^TPA + D^TC & B^TPB + D^TD - \gamma^2 I \end{bmatrix} + \begin{bmatrix} A^T \\ B^T \end{bmatrix} SQ^T + QS^T \begin{bmatrix} A & B \end{bmatrix} < 0 \tag{20}$$

where $S \in \Re^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TS = 0$.
(ii) Given a scalar $\gamma > 0$. The descriptor system (19) is robustly admissible with H_∞ disturbance attenuation γ if and only if there exist matrix P such that

$$E^TPE \geq 0, \tag{21}$$

$$\begin{bmatrix} A^TPA - E^TPE + C^TC & A^TPB + C^TD \\ B^TPA + D^TC & B^TPB + D^TD - \gamma^2 I \end{bmatrix} < 0. \tag{22}$$

The following theorem provides a necessary and sufficient condition for the robust admissibility with H_∞ disturbance attenuation of (17).

Theorem 4.2. *Given γ and K, the descriptor system (17) is robustly admissible with H_∞ disturbance attenuation γ if and only if there exist matrices $P > 0$, $Q = [Q_1^T \; Q_2^T]^T$ and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$ such that*

$$\begin{bmatrix} \Pi_{11} & A_K^TSQ_2^T + Q_1S^TB_1 & A_K^TP & (C+DK)^T & Q_1S^TH_c & 0 \\ Q_2S^TA_K + B_1^TSQ^T & \Pi_{22} & B_1^TP & 0 & Q_2S^TH_c & 0 \\ PA_K & PB_1 & -P & 0 & PH_c & 0 \\ C+DK & 0 & 0 & -I & 0 & \alpha D \\ H_c^TSQ_1^T & H_c^TSQ_2^T & H_c^TP & 0 & -\varepsilon_1 I & 0 \\ 0 & 0 & 0 & \alpha D^T & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{23}$$

where $A_K = A + B_2K$, $S \in \Re^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^TS = 0$, and

$$\begin{aligned} \Pi_{11} &= Q_1S^TA_K - E^TPE + A_K^TSQ_1^T + \varepsilon_1 G_c^TG_c + \varepsilon_2 K^TK, \\ \Pi_{22} &= -\gamma^2 I + Q_2S^TB_1 + B_1^TSQ_2^T. \end{aligned}$$

Proof: (Sufficiency) Suppose that there exist matrices $P > 0$, $Q = [Q_1^T \; Q_2^T]^T$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that the condition (23) holds. Then, by Schur complement formula, we have

$$\bar{Q} + \varepsilon_1^{-1}\bar{H}_1\bar{H}_1^T + \varepsilon_1\bar{G}_2^T\bar{G}_1 + \varepsilon_2^{-1}\bar{H}_2\bar{H}_2^T + \varepsilon_2\bar{G}_2^T\bar{G}_2 < 0. \tag{24}$$

where

$$\bar{Q} = \begin{bmatrix} Q_1S^TA_K - E^TPE + A_K^TSQ_1^T & A_K^TSQ_2^T + Q_1S^TB_1 & A_K^TP & (C+DK)^T \\ Q_2S^TA_K + B_1^TSQ^T & -\gamma^2 I + Q_2S^TB_1 + B_1^TSQ_2^T & B_1^TP & 0 \\ PA_K & PB_1 & -P & 0 \\ C+DK & 0 & 0 & -I \end{bmatrix},$$
$$\bar{H}_1 = \begin{bmatrix} H_c^TSQ_1^T & H_c^TSQ_2^T & H_c^TP & 0 \end{bmatrix}^T,$$
$$\bar{H}_2 = \begin{bmatrix} 0 & 0 & 0 & \alpha D^T \end{bmatrix}^T,$$
$$\bar{G}_1 = \begin{bmatrix} G_c & 0 & 0 & 0 \end{bmatrix},$$
$$\bar{G}_2 = \begin{bmatrix} K & 0 & 0 & 0 \end{bmatrix}.$$

Now, using Lemma 3.2, we have

$$\begin{aligned} &\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & (C+DK+D\Phi(k)K)^T \\ \Delta_{12}^T & \Delta_{22} & B_1^TP & 0 \\ \Delta_{13}^T & PB_1 & -P & 0 \\ C+DK+D\Phi(k)K & 0 & 0 & -I \end{bmatrix} \\ &= \bar{Q} + \bar{H}_1F_c(k)\bar{G}_1 + \bar{G}_1^TF^T(k)\bar{H}_1^T + \bar{H}_2\Phi(k)\bar{G}_2 + \bar{G}_2^T\Phi^T(k)\bar{H}_2^T \\ &\le \bar{Q} + \varepsilon_1^{-1}\bar{H}_1H_1^T + \varepsilon_1\bar{G}_1^T\bar{G}_1 + \varepsilon_2^{-1}\bar{H}_2H_2^T + \varepsilon_2\bar{G}_2^T\bar{G}_2 \end{aligned}$$

where

$$\begin{aligned} \Delta_{11} &= Q_1S^T(A + B_2K + H_cF_c(k)G_c) + (A + B_2K + H_cF_c(k)G_c)^TSQ_1^T - E^TPE, \\ \Delta_{12} &= (A + B_2K + H_cF_c(k)G_c)^TSQ_2^T + Q_1S^TB_1, \\ \Delta_{13} &= (A + B_2K + H_cF_c(k)G_c)^TP, \\ \Delta_{22} &= -\gamma^2 I + Q_2S^TB_1 + B_1^TSQ_2^T. \end{aligned}$$

It follows from (24) that

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & (C+DK+D\Phi(k)K)^T \\ \Delta_{12}^T & \Delta_{22} & B_1^TP & 0 \\ \Delta_{13}^T & PB_1 & -P & 0 \\ C+DK+D\Phi(k)K & 0 & 0 & -I \end{bmatrix} < 0.$$

This implies by Schur complement formula and Lemma 4.1(i) that the descriptor system (17) is robustly admissible with H_∞ disturbance attenuation γ.

(Necessity) Assume that the descriptor system (17) is robustly admissible with H_∞ disturbance attenuation γ. Then, it follows from Lemma 4.1 that there exist matrices $P > 0$, Q and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that (20) with A replaced by $A + BK + H_cF_c(k)G_c$ and C by $C + DK + \alpha\Phi(k)K$ holds. Thus, for all admissible $F(k)$ and $\Phi(k)$, the following inequality holds:

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & (C+DK+D\Phi(k)K)^T \\ \Delta_{12}^T & \Delta_{22} & B_1^TP & 0 \\ \Delta_{13}^T & PB_1 & -P & 0 \\ C+DK+D\Phi(k)K & 0 & 0 & -I \end{bmatrix} < 0.$$

That is,

$$\bar{Q} + \bar{H}_1 F_c(k) \bar{G}_1 + \bar{G}_1^T F_c^T(k) \bar{H}_1^T + \bar{H}_2 \Phi(k) \bar{G}_2 + \bar{G}_2^T \Phi^T(k) \bar{H}_2^T < 0$$

is satisfied for all admissible $F(k)$ and $\Phi(k)$. It follows from Lemma 3.2 that

$$\bar{Q} + \varepsilon_1^{-1} \bar{H}_1 \bar{H}_1^T + \varepsilon_1 \bar{G}_1^T \bar{G}_1 + \varepsilon_2^{-1} \bar{H}_2 \bar{H}_2^T + \varepsilon_2 \bar{G}_2^T \bar{G}_2 < 0,$$

which leads to the condition (23). This completes the proof.

Based on Lemma 4.1(ii), we have the following theorem. The proof is similar to that of Theorem 4.2, and is thus omitted.

Theorem 4.3. *Given γ and K, the descriptor system (17) is robustly admissible with H_∞ disturbance attenuation γ if and only if there exist matrix P and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$ such that (21), and*

$$\begin{bmatrix} -E^T P E + \varepsilon_1 G_c^T G_c + \varepsilon_2 K^T K & 0 & (A + B_2 K)^T P & (C + DK)^T & 0 & 0 \\ 0 & -\gamma^2 I & B_1^T P & 0 & 0 & 0 \\ P(A + B_2 K) & P B_1 & -P & 0 & P H_c & 0 \\ C + DK & 0 & 0 & -I & 0 & \alpha D \\ 0 & 0 & H_c^T P & 0 & -\varepsilon_1 I & 0 \\ 0 & 0 & 0 & \alpha D^T & 0 & -\varepsilon_2 I \end{bmatrix} < 0. \tag{25}$$

4.2. H_∞ non-fragile control design for uncertain systems

Now, we are at the position to propose design methods of H_∞ non-fragile controller for uncertain descriptor systems.

Theorem 4.4. *There exists a controller (4) that makes the descriptor system (16) robustly admissible with H_∞ disturbance attenuation γ if there exist matrices $P > 0$, $Q = [Q_1^T \; Q_2^T]^T$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that*

$$\Gamma = P^{-1} - \varepsilon_1^{-1} H_c H_c^T > 0, \tag{26}$$

$$Z = I - \varepsilon_2^{-1} \alpha^2 D D^T > 0, \tag{27}$$

$$W = \gamma^2 I - Q_2 S^T B_1 - B_1^T S Q_2^T - \varepsilon_1^{-1} Q_2 S^T H_c H_c^T S Q_2^T - \Theta_B^T \Gamma^{-1} \Theta_B > 0, \tag{28}$$

$$\begin{aligned} Q_1 S^T A + A^T S Q_1^T - E^T P E + \varepsilon_1 G^T G + \varepsilon_1^{-1} Q_1 S^T H_c H_c^T S Q_1^T \\ + C^T Z^{-1} C + Y W^{-1} Y^T + \Theta_A^T \Gamma^{-1} \Theta_A - \Psi \Lambda^{-1} \Psi^T < 0 \end{aligned} \tag{29}$$

where $S \in \Re^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E^T S = 0$, and

$$\begin{aligned} &\Psi = (Q_1 S^T + \Theta_A^T \Gamma^{-1}) B_2 + C^T Z^{-1} D + Y W^{-1} (Q_2 S^T + \Theta_B^T \Gamma^{-1}) B_2, \\ &\Lambda = B_2^T \Gamma^{-1} B_2 + D^T Z^{-1} D + (\varepsilon_1 + \varepsilon_2) I + B_2^T (S Q_2^T + \Gamma^{-1} \Theta_B) W^{-1} (Q_2 S^T + \Theta_B^T \Gamma^{-1}) B_2, \\ &\Theta_A = A + \varepsilon_1^{-1} H_c H_c^T S Q_1^T, \\ &\Theta_B = B_1 + \varepsilon_1^{-1} H_c H_c^T S Q_2^T, \\ &Y = A^T S Q_2^T + (Q_1 S^T + \Theta_A^T \Gamma^{-1}) \Theta_B. \end{aligned}$$

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In this case, a feedback gain in the controller (4) is given by

$$K = -\Lambda^{-1}\Psi^T. \tag{30}$$

Proof: (Sufficiency) The closed-loop system (17) with the feedback gain (30) is given by

$$\begin{aligned} Ex(k+1) &= (A - B_2\Lambda^{-1}\Psi^T + H_cF_c(k)G_c)x(k) + B_1w(k), \\ z(k) &= (C - D\Lambda^{-1}\Psi^T - \alpha D\Phi(k)\Lambda^{-1}\Psi^T)x(k) \end{aligned} \tag{31}$$

where $G_c = \begin{bmatrix} G^T & -\Psi\Lambda^{-1} \end{bmatrix}^T$. Then, by some mathematical manipulation, we have

$$\begin{aligned} &Q_1S^T(A - B_2\Lambda^{-1}\Psi^T) + (A - B_2\Lambda^{-1}\Psi^T)^TSQ_1^T + \varepsilon_1G_c^TG_c + \varepsilon_2\Psi\Lambda^{-1}\Lambda^{-1}\Psi^T \\ &= Q_1S^TA - Q_1S^TB_2\Lambda^{-1}\Psi^T + A^TSQ_1^T - \Psi\Lambda^{-1}B_2^TSQ_1^T + \varepsilon_1G^TG \\ &\quad +(\varepsilon_1+\varepsilon_2)\Psi\Lambda^{-1}\Lambda^{-1}\Psi^T, \end{aligned}$$

$$\begin{aligned} &(C - D\Lambda^{-1}\Psi^T)^TZ^{-1}(C - D\Lambda^{-1}\Psi^T) \\ &= C^TZ^{-1}C - \Psi\Lambda^{-1}DZ^{-1}C - C^TZ^{-1}D\Lambda^{-1}\Psi^T + \Psi\Lambda^{-1}DZ^{-1}D\Lambda^{-1}\Psi^T, \end{aligned}$$

$$\begin{aligned} &(A - B_2\Lambda^{-1}\Psi^T + \varepsilon_1^{-1}H_cH_c^TSQ_1^T)^T\Gamma^{-1}(A - B_2\Lambda^{-1}\Psi^T + \varepsilon_1^{-1}H_cH_c^TSQ_1^T) \\ &= \Theta_A^T\Gamma^{-1}\Theta_A - \Psi\Lambda^{-1}B_2^T\Gamma^{-1}\Theta_A - \Theta_A^T\Gamma^{-1}B_2\Lambda^{-1}\Psi^T + \Psi\Lambda^{-1}B_2^T\Gamma^{-1}B_2\Lambda^{-1}\Psi^T, \end{aligned}$$

$$\begin{aligned} &[(A - B_2\Lambda^{-1}\Psi^T)^TSQ_2^T + (Q_1S^T + (A - B_2\Lambda^{-1}\Psi^T + \varepsilon_1^{-1}H_cH_c^TSQ_1^T)^T\Gamma^{-1})\Theta_B]W^{-1} \\ &\quad\times[(A - B_2\Lambda^{-1}\Psi^T)^TSQ_2^T + (Q_1S^T + (A - B_2\Lambda^{-1}\Psi^T + \varepsilon_1^{-1}H_cH_c^TSQ_1^T)^T\Gamma^{-1})\Theta_B)]^T \\ &= YW^{-1}Y^T - \Psi\Lambda^{-1}B_2^T(SQ_2^T + \Gamma^{-1}\Theta_B)^TW^{-1}Y^T - YW^{-1}(SQ_2^T + \Gamma^{-1}\Theta_B)^TB_2\Lambda^{-1}\Psi^T \\ &\quad +\Psi\Lambda^{-1}B_2^T(SQ_2^T + \Gamma^{-1}\Theta_B)W^{-1}(SQ_2^T + \Gamma^{-1}\Theta_B)^TB_2\Lambda^{-1}\Psi^T. \end{aligned}$$

Thus, it can be verified with (29) that

$$\begin{aligned} &Q_1S^T(A - B_2\Lambda^{-1}\Psi^T) + (A - B_2\Lambda^{-1}\Psi^T)^TSQ_1^T + \varepsilon_1G_c^TG_c + \varepsilon_2\Psi\Lambda^{-1}\Lambda^{-1}\Psi^T \\ &+(C - D\Lambda^{-1}\Psi^T)^TZ^{-1}(C - D\Lambda^{-1}\Psi^T) - E^TPE + \varepsilon_1^{-1}Q_1S^TH_cH_c^TSQ_1^T \\ &+(A - B_2\Lambda^{-1}\Psi^T + \varepsilon_1^{-1}H_cH_c^TSQ_1^T)^T\Gamma^{-1}(A - B_2\Lambda^{-1}\Psi^T + \varepsilon_1^{-1}H_cH_c^TSQ_1^T) \\ &+[(A - B_2\Lambda^{-1}\Psi^T)^TSQ_2^T + (Q_1S^T + (A - B_2\Lambda^{-1}\Psi^T + \varepsilon_1^{-1}H_cH_c^TSQ_1^T)^T\Gamma^{-1})\Theta_B]W^{-1} \\ &\quad\times[(A - B_2\Lambda^{-1}\Psi^T)^TSQ_2^T + (Q_1S^T + (A - B_2\Lambda^{-1}\Psi^T + \varepsilon_1^{-1}H_cH_c^TSQ_1^T)^T\Gamma^{-1})\Theta_B)]^T \\ &= Q_1S^TA + A^TSQ_1^T + \varepsilon_1G^TG - E^TPE + \varepsilon_1^{-1}Q_1S^TH_cH_c^TSQ_1^T + C^TZ^{-1}C + \Theta_A^T\Gamma^{-1}\Theta_A \\ &\quad +YW^{-1}Y^T - \Psi\Lambda^{-1}\Psi^T < 0. \end{aligned}$$

By Schur complement formula, we obtain

$$\begin{bmatrix} \begin{pmatrix} Q_1S^TA_K + A_K^TSQ_1^T \\ -E^TPE + \varepsilon_1 G_c^T G_c \\ +\varepsilon_2 \Psi\Lambda^{-1}\Lambda^{-1}\Psi^T \end{pmatrix} & A_K^TSQ_2^T + Q_1S^TB_1 & A_K^TP & C_K^T & Q_1S^TH_c & 0 \\ Q_2S^TA_K + B_1^TSQ_1^T & \begin{pmatrix} -\gamma^2 I + Q_2S^TB_1 \\ +B_1^TSQ_2^T \end{pmatrix} & B_1^TP & 0 & Q_2S^TH_c & 0 \\ PA_K & PB_1 & -P & 0 & PH_c & 0 \\ A_K & 0 & 0 & -I & 0 & \alpha D \\ H_c^TSQ_1^T & H_c^TSQ_2^T & H_c^TP & 0 & -\varepsilon_1 I & 0 \\ 0 & 0 & 0 & \alpha D^T & 0 & -\varepsilon_2 I \end{bmatrix} < 0.$$

where $A_K = A - B_2\Lambda^{-1}\Psi^T$ and $C_K = C - D\Lambda^{-1}\Psi^T$. Hence, by Theorem 4.2 we can show that the closed-loop system (17) is robustly admissible with H_∞ disturbance attenuation γ. (Necessity) Assume there exists a feedback control of the form (4) which makes the descriptor system (16) robustly admissible with H_∞ disturbance attenuation γ. Then, it follows from Theorem 4.2 that there exist scalars $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that

$$\begin{bmatrix} \Pi_{11} & A_K^TSQ_2^T + Q_1S^TB_1 & A_K^TP & (C+DK)^T & Q_1S^TH_c & 0 \\ Q_2S^TA_K + B_1^TSQ^T & \Pi_{22} & B_1^TP & 0 & Q_2S^TH_c & 0 \\ PA_K & PB_1 & -P & 0 & PH_c & 0 \\ C + DK & 0 & 0 & -I & 0 & \alpha D \\ H_c^TSQ_1^T & H_c^TSQ_2^T & H_c^TP & 0 & -\varepsilon_1 I & 0 \\ 0 & 0 & 0 & \alpha D^T & 0 & -\varepsilon_2 I \end{bmatrix} < 0$$

where

$$\begin{aligned} \Pi_{11} &= Q_1S^TA_K + A_K^TSQ_1^T - E^TPE + \varepsilon_1 G_c^T G_c + \varepsilon_2 K^TK, \\ \Pi_{22} &= -\gamma^2 I + Q_2S^TB_1 + B_1^TSQ_2^T. \end{aligned}$$

It follows from Schur complement formula that we obtain the conditions (26), (27), (28), and

$$\begin{aligned} &Q_1S^TA + A^TSQ_1^T - E^TPE + \varepsilon_1 G^TG + \varepsilon_1^{-1} QS^TH_cH_c^TSQ^T + C^TZ^{-1}C + YW^{-1}Y^T \\ &+\Theta_A^T\Gamma^{-1}\Theta_A + K^T\Psi^T + \Psi K + K^T\Lambda K < 0. \end{aligned}$$

By Lemma 3.4, we have

$$K^T\Psi^T + \Psi K + K^T\Lambda K \geq -\Psi\Lambda^{-1}\Psi^T.$$

Therefore, we finally obtain the condition (29), and the feedback gain K as in (30).

Based on Theorem 4.3, we can deduce the following theorem.

Theorem 4.5. *There exists a controller (4) that makes the descriptor system (16) robustly admissible with H_∞ disturbance attenuation γ if there exist matrix P and scalars $\varepsilon_1 > 0,\ \varepsilon_2 > 0$ such that (21), (26), (27) and*

$$W = \gamma^2 I - B_1^T \Gamma^{-1} B_1 > 0, \quad (32)$$

$$-E^T P E + \varepsilon_1 G^T G + C^T Z^{-1} C + A^T \Gamma^{-1} B_1 W^{-1} B_1^T \Gamma^{-1} A + A^T \Gamma^{-1} A - \Psi \Lambda^{-1} \Psi^T < 0 \quad (33)$$

where

$$\Psi = A^T \Gamma^{-1} B_2 + C^T Z^{-1} D + A^T \Gamma^{-1} B_1 W^{-1} B_1^T \Gamma^{-1} B_2,$$
$$\Lambda = B_2^T \Gamma^{-1} B_2 + D^T Z^{-1} D + (\varepsilon_1 + \varepsilon_2) I + B_2^T \Gamma^{-1} B_1 W^{-1} B_1^T \Gamma^{-1} B_2.$$

In this case, a feedback gain in the controller (4) is given as in (30).

5. Time-delay systems

This section investigates the robust admissibility of uncertain descriptor delay systems and provides a non-fragile control design method for such systems. Section 5.1 gives a robust admissibility condition for uncertain descriptor delay systems, and Section 5.2 proposes non-fragile control design methods.

5.1. Robust admissibility for uncertain systems

Consider the following descriptor system with time-delay and uncertainties:

$$Ex(k+1) = (A + \Delta A)x(k) + (A_d + \Delta A_d)x(k-d) + Bu(k) \quad (34)$$

where $x(k) \in \Re^n$ is the state and $u(k) \in \Re^m$ is the control. A, A_d and B are system matrices with appropriate dimensions. d is a constant delay and may be unknown. Uncertain matrices are given by

$$\left[\Delta A \ \Delta A_d\right] = HF(k)\left[G \ G_d\right]$$

where $F(k) \in \Re^{l \times j}$ is an unknown time-varying matrix satisfying $F^T(k)F(k) \leq I$ and H, G and G_d are constant matrices of appropriate dimensions. Unforced nominal descriptor system (34) with $u(k) = 0$ and $\Delta A = \Delta A_d = 0$ is denoted by the triplet $(E,\ A,\ A_d)$.

Definition 5.1. *(i) The triplet $(E,\ A,\ A_d)$ is said to be regular if $det(z^{d+1}E - z^d A - A_d)$ is not identically zero.*
(ii) The triplet $(E,\ A,\ A_d)$ is said to be causal if it is regular and $deg(z^{nd} det(zE - A - z^d A_d)) = nd + rank(E)$.
(iii) Define the generalized spectral radius as $\rho(E,\ A,\ A_d) = \max_{z|\lambda \in \{det(z^{d+1}E - z^d A - A_d) = 0\}} |\lambda|$. The triplet $(E,\ A,\ A_d)$ is said to be stable if $\rho(E,\ A,\ A_d) < 1$.
(iv) The triplet $(E,\ A,\ A_d)$ is said to be admissible if it is regular, causal and stable.

Applying the controller (4) to the system (34), we have the closed-loop system

$$Ex(k+1) = (A + BK + H_c F_c(k) G_c)x(k) + (A_d + HF(k)G_d)x(k-d) \tag{35}$$

where

$$H_c = [H \;\; \alpha B],\; F_c(k) = \mathrm{diag}[F(k) \;\; \Phi(k)],\; G_c = \begin{bmatrix} G \\ K \end{bmatrix}.$$

First, we consider the robust admissibility of the closed-loop system (35). In order to show a robust admissibility condition, we need the following theorem.

Theorem 5.2. *(Xu & Lam [32]) (i) The descriptor delay system (34) with $u(k) = 0$ and $\Delta A = \Delta A_d = 0$ is admissible if and only if there exist matrices $P > 0,\; Q > 0,\; X$ such that*

$$\begin{bmatrix} A^T P A - E^T P E + Q & A^T P A_d \\ A_d^T P A & A_d^T P A_d - Q \end{bmatrix} + \begin{bmatrix} A^T \\ A_d^T \end{bmatrix} S X^T + X S^T \begin{bmatrix} A^T \\ A_d^T \end{bmatrix}^T < 0 \tag{36}$$

where $S \in \Re^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E^T S = 0$.
(ii) The descriptor delay system (34) with $u(k) = 0$ and $\Delta A = \Delta A_d = 0$ is admissible if and only if there exist matrices $P,\; Q > 0$ such that

$$E^T P E \geq 0, \tag{37}$$

$$\begin{bmatrix} A^T P A - E^T P E + Q & A^T P A_d \\ A_d^T P A & A_d^T P A_d - Q \end{bmatrix} < 0. \tag{38}$$

Robust admissibility conditions for uncertain descriptor delay system (35) are given in the following theorems.

Theorem 5.3. *Given γ and K, the descriptor system (35) is robustly admissible if there exist matrices $P > 0,\; Q > 0,\; X = [X_1^T \;\; X_2^T]^T$ and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$ such that*

$$\begin{bmatrix} \begin{pmatrix} X_1 S^T A_K - E^T P E + Q \\ + A_K^T S X_1^T + \varepsilon_1 G_c^T G_c \end{pmatrix} & A_K^T S X_2^T + X_1 S^T A_d & A_K^T P & X_1 S^T H_c & X_1 S^T H \\ X_2 S^T A_K + A_d^T S X^T & \begin{pmatrix} -Q + X_2 S^T A_d \\ + A_d^T S X_2^T + \varepsilon_2 G_d^T G_d \end{pmatrix} & A_d^T P & X_2 S^T H_c & X_2 S^T H \\ P A_K & P A_d & -P & P H_c & P H \\ H_c^T S X_1^T & H_c^T S X_2^T & H_c^T P & -\varepsilon_1 I & 0 \\ H^T S X_1^T & H^T S X_2^T & H^T P & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{39}$$

where $A_K = A + BK$ and $S \in \Re^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E^T S = 0$.

Proof: Suppose that there exist matrices $P > 0,\ Q > 0,\ X = [X_1^T\ X_2^T]^T$ and scalars $\varepsilon_1 > 0,\ \varepsilon_2 > 0$ such that the condition (39) holds. Then, by Schur complement formula, we have

$$\begin{bmatrix} X_1S^TA_K - E^TPE + A_K^TSX_1^T + Q & A_K^TSX_2^T + X_1S^TA_d & A_K^TP \\ X_2S^TA_K + A_d^TSX^T & -Q + X_2S^TA_d + A_d^TSX_2^T & A_d^TP \\ PA_K & PA_d & -P \end{bmatrix}$$
$$+\varepsilon_1^{-1}\begin{bmatrix} X_1S^TH_c \\ X_2S^TH_c \\ PH_c \end{bmatrix}\begin{bmatrix} H_c^TSX_1^T & H_c^TSX_2^T & H_c^TP \end{bmatrix} + \varepsilon_1\begin{bmatrix} G_c^T \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} G_c & 0 & 0 \end{bmatrix} \tag{40}$$
$$+\varepsilon_2^{-1}\begin{bmatrix} X_1S^TH \\ X_2S^TH \\ PH \end{bmatrix}\begin{bmatrix} H^TSX_1^T & H^TSX_2^T & H^TP \end{bmatrix} + \varepsilon_2\begin{bmatrix} 0 \\ G_d^T \\ 0 \end{bmatrix}\begin{bmatrix} 0 & G_d & 0 \end{bmatrix} < 0.$$

Now, using Lemma 3.2, we have

$$\begin{aligned}
&\begin{bmatrix} \Delta_{11} & \Delta_{12} & (A_K + H_cF_c(k)G_c)^TP \\ \Delta_{12}^T & \Delta_{22} & (A_d + HF(k)G_d)^TP \\ P(A_K + H_cF_c(k)G_c) & P(A_d + HF(k)G_d) & -P \end{bmatrix} \\
&= \begin{bmatrix} X_1S^TA_K - E^TPE + A_K^TSX_1^T + Q & A_K^TSX_2^T + X_1S^TA_d & A_K^TP \\ X_2S^TA_K + A_d^TSX^T & -Q + X_2S^TA_d + A_d^TSX_2^T & A_d^TP \\ PA_K & PA_d & -P \end{bmatrix} \\
&\quad + \begin{bmatrix} X_1S^TH_c \\ X_2S^TH_c \\ PH_c \end{bmatrix} F_c(k)\begin{bmatrix} G_c & 0 & 0 \end{bmatrix} + \begin{bmatrix} G_c^T \\ 0 \\ 0 \end{bmatrix} F_c^T(k)\begin{bmatrix} H_c^TSX_1^T & H_c^TSX_2^T & H_c^TP \end{bmatrix} \\
&\quad + \begin{bmatrix} X_1S^TH_c \\ X_2S^TH_c \\ PH_c \end{bmatrix} F(k)\begin{bmatrix} 0 & G_d & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ G_d^T \\ 0 \end{bmatrix} F^T(k)\begin{bmatrix} H_c^TSX_1^T & H_c^TSX_2^T & H_c^TP \end{bmatrix} \\
&\le \begin{bmatrix} X_1S^TA_K - E^TPE + A_K^TSX_1^T + Q & A_K^TSX_2^T + X_1S^TA_d & A_K^TP \\ X_2S^TA_K + A_d^TSX^T & -Q + X_2S^TA_d + A_d^TSX_2^T & A_d^TP \\ PA_K & PA_d & -P \end{bmatrix} \\
&\quad + \varepsilon_1^{-1}\begin{bmatrix} X_1S^TH_c \\ X_2S^TH_c \\ PH_c \\ 0 \end{bmatrix}\begin{bmatrix} H_c^TSX_1^T & H_c^TSX_2^T & H_c^TP \end{bmatrix} + \varepsilon_1\begin{bmatrix} G_c^T \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} G_c & 0 & 0 \end{bmatrix} \\
&\quad + \varepsilon_2^{-1}\begin{bmatrix} X_1S^TH_c \\ X_2S^TH_c \\ PH_c \\ 0 \end{bmatrix}\begin{bmatrix} H_c^TSX_1^T & H_c^TSX_2^T & H_c^TP \end{bmatrix} + \varepsilon_2\begin{bmatrix} 0 \\ G_d^T \\ 0 \end{bmatrix}\begin{bmatrix} 0 & G_d & 0 \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
\Delta_{11} &= X_1S^T(A + BK + H_cF_c(k)G_c) + (A + BK + H_cF_c(k)G_c)^TSX_1^T - E^TPE + Q, \\
\Delta_{12} &= (A_K + H_cF_c(k)G_c)^TSX_2^T + X_1S^T(A_d + HF(k)G_d), \\
\Delta_{22} &= -Q + X_2S^T(A_d + HF(k)G_d) + (A_d + HF(k)G_d)^TSX_2^T.
\end{aligned}$$

It follows from (40) that

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & (A_K + H_c F_c(k) G_c)^T P \\ \Delta_{12}^T & \Delta_{22} & (A_d + HF(k) G_d)^T P \\ P(A_K + H_c F_c(k) G_c) & P(A_d + HF(k) G_d) & -P \end{bmatrix} < 0.$$

This implies by Schur complement formula and Theorem 5.2(i) that the descriptor system (35) is robustly admissible.

Similarly, we can prove the following theorem by using Theorem 5.2(ii).

Theorem 5.4. *Given γ and K, the descriptor system (35) is robustly admissible if there exist matrices $P > 0,\ Q > 0$ and scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$ such that (37), and*

$$\begin{bmatrix} -E^T P E + Q + \varepsilon_1 G_c^T G_c & 0 & A_K^T P & 0 & 0 \\ 0 & -Q + \varepsilon_2 G_d^T G_d & A_d^T P & 0 & 0 \\ P A_K & P A_d & -P & P H_c & P H \\ 0 & 0 & H_c^T P & -\varepsilon_1 I & 0 \\ 0 & 0 & H^T P & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{41}$$

where $A_K = A + BK$.

5.2. Control design for time-delay systems

Now, we are ready to propose control design methods for uncertain descriptor delay systems. The following theorems propose design methods of a non-fragile controller that makes the system (34) robustly admissible.

Theorem 5.5. *There exists a controller (4) that makes the descriptor system (34) robustly admissible if there exist matrices $P > 0,\ Q > 0,\ X = [X_1^T\ X_2^T]^T$ and scalars $\varepsilon_1 > 0,\ \varepsilon_2 > 0$ such that*

$$\Gamma = P^{-1} - \varepsilon_1^{-1} H_c H_c^T - \varepsilon_2^{-1} H H^T > 0, \tag{42}$$

$$W = Q - X_2 S^T A_d - A_d^T S X_2^T - \varepsilon_2 G_d^T G_d - \Theta_D^T \Gamma^{-1} \Theta_D - X_2 S^T (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} H H^T) S X_2^T > 0, \tag{43}$$

$$X_1 S^T A + A^T S X_1^T - E^T P E + Q + \varepsilon_1 G^T G + X_1 S^T (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} H H^T) S X_1^T + \Theta_A^T \Gamma^{-1} \Theta_A + Y W^{-1} Y^T - \Psi \Lambda^{-1} \Psi^T < 0 \tag{44}$$

where $S \in \Re^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E^T S = 0$, and

$$\begin{aligned} \Psi &= (X_1 S^T + \Theta_A^T \Gamma^{-1}) B + Y W^{-1} (S X_2^T + \Theta_D^T \Gamma^{-1}) B, \\ \Lambda &= B^T \Gamma^{-1} B + \varepsilon_1 I + B^T (S X_2^T + \Gamma^{-1} \Theta_D) W^{-1} (X_2 S^T + \Theta_D^T \Gamma^{-1}) B, \\ Y &= A^T S X_2^T + (X_1 S^T + \Theta_A^T \Gamma^{-1}) \Theta_D, \\ \Theta_A &= A + (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} H H^T) S X_1^T, \\ \Theta_D &= A_d + (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} H H^T) S X_2^T. \end{aligned}$$

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In this case, a feedback gain in the controller (4) is given by

$$K = -\Lambda^{-1}\Psi^T. \tag{45}$$

Proof: The closed-loop system (35) with the feedback gain (45) is given by

$$Ex(k+1) = (A - B\Lambda^{-1}\Psi^T + H_c F_c(k) G_c)x(k) + (A_d + HF(k)G_d)x(k-d) \tag{46}$$

where $G_c = \begin{bmatrix} G^T & -\Psi\Lambda^{-1} \end{bmatrix}^T$. Then, by some mathematical manipulation, we have

$$\begin{aligned}
&X_1 S^T (A - B\Lambda^{-1}\Psi^T) + (A - B\Lambda^{-1}\Psi^T)^T S X_1^T + \varepsilon_1 G_c^T G_c \\
&= X_1 S^T A - X_1 S^T B\Lambda^{-1}\Psi^T + A^T S X_1^T - \Psi\Lambda^{-1} B^T S X_1^T + \varepsilon_1 G^T G + \varepsilon_1 \Psi\Lambda^{-1}\Lambda^{-1}\Psi^T,
\end{aligned}$$

$$\begin{aligned}
&[A - B\Lambda^{-1}\Psi^T + (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} HH^T) S X_1^T]^T \Gamma^{-1} \\
&\qquad \times [A - B\Lambda^{-1}\Psi^T + (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} HH^T) S X_1^T] \\
&= \Theta_A^T \Gamma^{-1} \Theta_A - \Psi\Lambda^{-1} B^T \Gamma^{-1} \Theta_A - \Theta_A^T \Gamma^{-1} B\Lambda^{-1}\Psi^T + \Psi\Lambda^{-1} B^T \Gamma^{-1} B\Lambda^{-1}\Psi^T,
\end{aligned}$$

$$\begin{aligned}
&[(A - B\Lambda^{-1}\Psi^T)^T S X_2^T + (X_1 S^T + (A - B\Lambda^{-1}\Psi^T + \varepsilon_1^{-1} H_c H_c^T S X_1^T)^T \Gamma^{-1})\Theta_D] W^{-1} \\
&\quad \times [(A - B\Lambda^{-1}\Psi^T)^T S X_2^T + (X_1 S^T + (A - B\Lambda^{-1}\Psi^T + \varepsilon_1^{-1} H_c H_c^T S X_1^T)^T \Gamma^{-1})\Theta_D)]^T \\
&= Y W^{-1} Y^T - \Psi\Lambda^{-1} B^T (S X_2^T + \Gamma^{-1}\Theta_D)^T W^{-1} Y^T - Y W^{-1} (S X_2^T + \Gamma^{-1}\Theta_D)^T B\Lambda^{-1}\Psi^T \\
&\quad + \Psi\Lambda^{-1} B^T (S X_2^T + \Gamma^{-1}\Theta_D) W^{-1} (S X_2^T + \Gamma^{-1}\Theta_D)^T B\Lambda^{-1}\Psi^T.
\end{aligned}$$

Thus, it can be verified with (44) that

$$\begin{aligned}
&X_1 S^T (A - B\Lambda^{-1}\Psi^T) + (A - B\Lambda^{-1}\Psi^T)^T S X_1^T + \varepsilon_1 G_c^T G_c - E^T P E + Q \\
&+ X_1 S^T (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} HH^T) S X_1^T + [A - B\Lambda^{-1}\Psi^T + (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} HH^T) S X_1^T]^T \\
&\quad \times \Gamma^{-1} [A - B\Lambda^{-1}\Psi^T + (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} HH^T) S X_1^T] \\
&+ [(A - B\Lambda^{-1}\Psi^T)^T S X_2^T + (X_1 S^T + (A - B\Lambda^{-1}\Psi^T + \varepsilon_1^{-1} H_c H_c^T S X_1^T)^T \Gamma^{-1})\Theta_D] W^{-1} \\
&\quad \times [(A - B\Lambda^{-1}\Psi^T)^T S X_2^T + (X_1 S^T + (A - B\Lambda^{-1}\Psi^T + \varepsilon_1^{-1} H_c H_c^T S X_1^T)^T \Gamma^{-1})\Theta_D)]^T \\
&= X_1 S^T A + A^T S X_1^T + \varepsilon_1 G^T G - E^T P E + Q + X_1 S^T (\varepsilon_1^{-1} H_c H_c^T + \varepsilon_2^{-1} HH^T) S X_1^T \\
&+ \Theta_A^T \Gamma^{-1} \Theta_A + Y W^{-1} Y^T - \Psi\Lambda^{-1}\Psi^T < 0.
\end{aligned}$$

By Schur complement formula, we obtain

$$\begin{bmatrix}
\begin{pmatrix} X_1 S^T A_K - E^T P E + Q \\ + A_K^T S X_1^T + \varepsilon_1 G_c^T G_c \end{pmatrix} & A_K^T S X_2^T + X_1 S^T A_d & A_K^T P & X_1 S^T H_c & X_1 S^T H \\
X_2 S^T A_K + A_d^T S X^T & \begin{pmatrix} -Q + X_2 S^T A_d \\ + A_d^T S X_2^T + \varepsilon_2 G_d^T G_d \end{pmatrix} & A_d^T P & X_2 S^T H_c & X_2 S^T H \\
P A_K & P A_d & -P & P H_c & P H \\
H_c^T S X_1^T & H_c^T S X_2^T & H_c^T P & -\varepsilon_1 I & 0 \\
H^T S X_1^T & H^T S X_2^T & H^T P & 0 & -\varepsilon_2 I
\end{bmatrix} < 0$$

where $A_K = A - B\Lambda^{-1}\Psi^T$. Hence, by Theorem 5.3 we can show that the closed-loop system (35) is robustly admissible.

The following theorem can similarly be obtained from Theorem 5.4.

Theorem 5.6. *There exists a controller (4) that makes the descriptor system (34) robustly admissible if there exist matrices $P > 0,\ Q > 0$ and scalars $\varepsilon_1 > 0,\ \varepsilon_2 > 0$ such that (37), (42),*

$$W = Q - \varepsilon_2 G_d^T G_d - A_d^T \Gamma^{-1} A_d > 0, \tag{47}$$

$$-E^T P E + Q + \varepsilon_1 G^T G + A^T \Gamma^{-1} A + A^T \Gamma^{-1} A_d W^{-1} A_d^T \Gamma^{-1} A - \Psi \Lambda^{-1} \Psi^T < 0 \tag{48}$$

where Γ is given in (42), and

$$\Psi = A^T \Gamma^{-1} B + A^T \Gamma^{-1} A_d W^{-1} A_d^T \Gamma^{-1} B,$$
$$\Lambda = B^T \Gamma^{-1} B + \varepsilon_1 I + B^T \Gamma^{-1} A_d W^{-1} A_d^T \Gamma^{-1} B.$$

In this case, a feedback gain in the controller (4) is given as in (45).

6. Numerical examples

In order to illustrate our control design methods, we consider the following two examples. The first one shows a non-fragile controller design method for an uncertain system, and the second gives the same class of a controller design method for a time-delay counterpart.

Consider an uncertain system:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(k+1) = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0.2 \end{bmatrix} + \begin{bmatrix} 0.06 \\ 0.08 \end{bmatrix} F(k) \begin{bmatrix} 0.2 & 0 \end{bmatrix} \right) x(k) + \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix} u(k).$$

Assuming the measure of non-fragility $\alpha = 0.3$, we apply Theorem 3.7 or 3.8, which gives a non-fragile control gain K in (4):

$$K = \begin{bmatrix} -4.0650 & -0.8131 \end{bmatrix}.$$

Next, we consider a uncertain time-delay system

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(k+1) = \left(\begin{bmatrix} 1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix} + \begin{bmatrix} 0.06 \\ 0.08 \end{bmatrix} F(k) \begin{bmatrix} 0.1 & 0 \end{bmatrix} \right) x(k) + \begin{bmatrix} 1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix} x(k-3) + \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} u(k).$$

Assuming the measure of non-fragility $\alpha = 0.35$, we apply Theorem 5.5 or 5.6, which gives a non-fragile control gain K in (4):

$$K = \begin{bmatrix} -0.6349 & -1.9048 \end{bmatrix}.$$

7. Conclusions

In this chapter, we investigated non-fragile control system analysis and design for uncertain discrete-time descriptor systems when the controller has some uncertainty in gain matrix. The controller is assumed to have multiplicative uncertainty in gain matrix. First, the robust admissibility of uncertain descriptor systems was discussed and the non-fragile control design methods were proposed. Then, theory was developed to the robust admissibility with H_∞ disturbance attenuation. Necessary and Sufficient conditions for the robust admissibility with H_∞ disturbance attenuation were obtained. Based on such conditions, the H_∞ non-fragile controller design methods were proposed. Next, uncertain descriptor systems with delay were considered. Based on system analysis of such systems, the non-fragile control design methods were proposed. Numerical examples were finally given to illustrate our controller design methods.

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