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Spatio-Temporal Feature in Two-Wave and Multi-Wave Propagations

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Additional information is available at the end of the chapter

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1. Introduction

It is well known that the wave propagation depends mainly on its velocity and frequency in one direction for a single wave[1-3]. There are many literatures devoted researches of single wave propagation such as solitary wave, periodic wave, chirped wave, rational wave etc [4-6]. However, what can be happen when two and even more waves with different features propagate together along different directions? In the past decades, many methods have been proposed for seeking two waves and multi-wave solutions to nonlinear models in modern physics. Recently, some effective and straight methods have been proposed such as homoclinic test approach(HTA)[7-8], extended homoclinic test approach(EHTA)[9-10] and three wave method [11-12]. These methods were applied to many nonlinear models. Several exact waves with different properties have been found out, such as periodic solitary wave, breather solitary wave, breather homoclinic wave, breather heteroclinic wave, cross kink wave, kinky kink wave, periodic kink wave, two-solitary wave, doubly periodic wave, doubly breather solitary wave, and so on. Because of interaction between waves with different features in propagation process of two-wave or multi-wave, some new phenomena have been discovered and numerically simulated, for example, resonance and non resonance, fission and fusion, bifurcation and deflexion etc. Furthermore, similar to the bifurcation theory of differential dynamical system, constant equilibrium solution of nonlinear evolution equation and propagation velocity of a wave as parameters are introduced to original equation, and then by using the small perturbation of parameter at a special value, two-wave or multi-wave propagation occurs new spatiotemporal change such as bifurcation of breather multi-soliton, periodic bifurcation and soliton degeneracy and so on.

This chapter mainly focus on explanation of different test methods and comprehensive applications to two-wave or multi-wave propagation. New methods will be described such as HTA, EHTA, Three-wave method and parameter small perturbation method. The spatiotemporal variety in exact two-wave and multi-wave propagation will be investigated and numerically simulated. In this chapter, some important models such as shallow water wave propagation models under the transverse long-wave disturbance Potential

Kadomtsev-Petviashvili equation [13-17], Kadomtsev-Petviashvili [18-27] equation, and specially, Kadomtsev-Petviashvili with positive dispersion equation are considered. Applied new methods to these equations, some new two-wave and multi-wave are obtained and spatiotemporal variety in multi-wave propagation is investigated and numerically simulated.

This chapter consists of four sections. In section 1, we introduce some methods including homoclinic test approach (HTA), extended homoclinic test approach (EHTA), three-wave method and parameter small perturbation method. In section 2, we consider the potential Kadomtsev-Petviashvili equation and investigate the exact periodic kink-wave and degeneracy of soliton. In section 3, we consider the Kadomtsev-Petviashvili equation and investigate periodic bifurcation, deflexion of two solitary waves. In section 4, we consider the Kadomtsev-Petviashvili equation with positive dispersion. By using three-wave method, we obtain some breather kinds of multi-solitary wave solutions, and investigate the fission and fusion of multi-wave.

2. Some methods for seeking two-wave and multi-wave

2.1. Homoclinic test approach

Consider a (2+1) dimensional nonlinear evolution equation of the general form

$$F(u, u_t, u_x, u_y, \dots) = 0 \quad (1)$$

where F is a polynomial of $u(x, y, t)$ and its derivatives, t represents time variable and x, y represent spatial variables. Assume that there exists a transformation of unknown function such that Eq.(1) becomes a bilinear equation in the following form

$$G(D_t, D_x, D_y, \dots) f \cdot f = 0 \quad (2)$$

where G is a general polynomial in $D_t, D_x, D_y, \dots$, where the Hirota's bilinear operator D -operator is defined by [2]

$$D_x^m D_t^n f(x, t) \cdot g(x, t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^n [f(x, t) g(x', t')] \Big|_{x'=x, t'=t} \quad (3)$$

Traditionally, one obtains two soliton solutions with the assumption

$$f = 1 + e^{\eta_1} + e^{\eta_2} + d e^{\eta_1 + \eta_2} \quad (4)$$

where d is a constant, $\eta_1 = k_1 x + l_1 y + c_1 t$, $\eta_2 = k_2 x + l_2 y + c_2 t$, and $k_j, l_j, c_j, j = 1, 2$ are real numbers. If we treat k_1 and k_2 as complex numbers by taking $k_1 = -k_2 = ik, i^2 = -1$ and assuming $l_1 = l_2 = l$ and $c_1 = c_2 = c$, Eq.(4) can be convert into the following form

$$f = 1 + \cos(kx) e^{ly+ct} + d e^{2ly+2ct} \quad (5)$$

In order to get more forms solution of Eq.(1), we use a more general Ansatz which contains complete variables of Eq.(2) replacing Eq.(5):

$$f = 1 + \cos(\eta_1) e^{\eta_2} + d e^{2\eta_2} \quad (6)$$

where

$$\begin{aligned} \eta_1 &= k_1 x + l_1 y + c_1 t \\ \eta_2 &= k_2 x + l_2 y + c_2 t \end{aligned} \quad (7)$$

where all of $d, k_j, l_j, c_j, j = 1, 2$ may be real numbers or complex numbers. This process is similar to the procedure that one can obtain the homoclinic orbit of the defocusing nonlinear Schrödinger equation from its dark-hole soliton solutions[1]. As a result, we call this skill as "Homoclinic test approach".

To derive analytic expression, we can take the following procedure in detail: inserting Eq.(6) into Eq.(2), then equating the coefficients of the same kind terms to zero, subsequently, solving the resulting algebraic equations to determine the relationship between variables $k_j, l_j, j = 1, 2, \dots$ with the help of symbolic computation software such as Maple. In Eq.(7), noting the cos functions is meaningful because we often take into account periodic effect in real physical background. Indeed, we can observe this solution is periodic breathing from their profile.

2.2. Extended homoclinic test approach

After substituting Eq.(6) into Eq.(2), we can get that whether the Eq.(2) has the nonzero solution. Furthermore, we do some mathematical simplicity. Rewrite Eq.(6) as follows:

$$f = e^{\eta_2} (e^{-\eta_2} + \cos(\eta_1) + de^{\eta_2}) \quad (8)$$

or

$$f = e^{\eta_2} [\sqrt{d} \cosh(\eta_2 + \ln(\sqrt{d})) + \cos(\eta_1)] \quad (9)$$

We replace Eq.(9) with

$$f = \sqrt{d} \cosh(\eta_2 + \ln(\sqrt{d})) + b_1 \cos(\eta_1) \quad (10)$$

where b_1, d are constants. Now, factoring out the exponentials produces: Exploiting this Ansatz to obtain the new solutions of nonlinear evolution equation is called "Extended Homoclinic test approach". To derive analytic expression, we can take the following procedure in detail: inserting Eq.(10) into Eq.(2), then equating the coefficients of the same kind terms to zero, subsequently, solving the resulting algebraic equations to determine the relationship between variables $k_j, l_j, j = 1, 2, \dots$ with the help of symbolic computation software such as Maple.

2.3. Three-wave method

Multi-wave solutions are important because they reveal the interactions between the inner-waves and the various frequency and velocity components. The whole multi-wave solution, for instance, may sometimes be converted into a single soliton of very high energy that propagates over large regions of space without dispersing and an extremely destructive wave is therefore produced of which the tsunami is a good example. Since all double-wave solutions can be found by using the exp-function method proposed by Fu and Dai [18], we propose an extension of the three-soliton method [6] in this section (called the three-wave method) for finding coupled wave solutions. Consider a high dimensional nonlinear evolution equation of the general form

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, \dots) = 0 \quad (11)$$

where $u = u(x, y, z, t)$ and F is a polynomial u of and its derivatives, t represents time variable and x, y, z represent spatial variables. The three-wave method operates as follows.

Step 1: By Painleve analysis, a transformation

$$u = T(f) \quad (12)$$

is made for some new and unknown function f .

Step 2: Convert Eq. (11) into Hirota bilinear form:

$$H(D_t, D_x, D_y, D_z, \dots) f \cdot f = 0 \quad (13)$$

where D is identical to the Eq.(3).

Step 3: Traditionally, we taking the following Ansätz to obtain the three soliton solution

$$f = 1 + e^{\xi_1} + e^{\xi_2} + e^{\xi_3} + a_{12}e^{\xi_1+\xi_2} + a_{13}e^{\xi_1+\xi_3} + a_{23}e^{\xi_2+\xi_3} + a_{123}e^{\xi_1+\xi_2+\xi_3} \quad (14)$$

where

$$\xi_j = a_j x + b_j y + c_j z + d_j t, \quad j = 1, 2, 3 \quad (15)$$

a_{13}, a_{23}, a_{123} are the constants. Eq.(14) can be rewritten as

$$f = e^{\frac{\eta_1}{2}} (e^{-\eta_1} + e^{\eta_2} + e^{\eta_3} + e^{\eta_4} + a_{12}e^{\eta_4} + a_{13}e^{\eta_3} + a_{23}e^{-\eta_2} + a_{123}e^{\eta_1}) \quad (16)$$

where

$$\eta_1 = \frac{\xi_1+\xi_2+\xi_3}{2}, \quad \eta_2 = \frac{\xi_1-\xi_2-\xi_3}{2}, \quad \eta_3 = \frac{-\xi_1+\xi_2-\xi_3}{2}, \quad \eta_4 = \frac{-\xi_1-\xi_2+\xi_3}{2} \quad (17)$$

Thus, this three soliton Ansätz contains four variables η_1, η_2, η_3 and η_4 . Here, we treat it in a different way. We factor out the $e^{\frac{\eta_1}{2}}$ in Eq.(16) and decrease the numbers of variables to three terms. On the other hand, we set some parameters in a complex way. At last, the above analysis allows us to construct the following assumptions:

$$f = e^{-\xi_1} + \delta_1 \cos(\xi_2) + \delta_2 \cosh(\xi_3) + \delta_3 e^{\xi_1} \quad (18)$$

or

$$f = e^{-\xi_1} + \delta_1 \cos(\xi_2) + \delta_2 \sinh(\xi_3) + \delta_3 e^{\xi_1} \quad (19)$$

where $\delta_j, j = 1, 2, 3$ are constants. In fact, from Eq.(18) or Eq.(19), it is easily seen that it only contains three wave variables. As a result, we call this method "three wave method". It is obvious that three-wave method is the extension and improvement of the traditional three-soliton method.

Step 4: Substitute Eq.(18) (or Eq.(19)) into Eq.(13), and collect the coefficients of $e^{-\xi_1}$, e^{ξ_1} , $\sin(\xi_2)$, $\cos(\xi_2)$, $\cosh(\xi_3)$ and $\sinh(\xi_3)$. Then equate the coefficients of these terms to zero and obtain a set of over-determined algebraic equations in a_j, b_j, c_j and $d_j, j = 1, 2, 3$.

Step 5: Solve the set of algebraic equations in Step 4 using Maple and solve for a_j, b_j, c_j, d_j and $\delta_j, j = 1, 2, 3$.

Step 6: Substituting the identified values into Eq.(12) and Eq.(13). Thus, we can deduce the exact multi-wave solutions of Eq.(11).

2.4. Introducing parameters and small perturbation method

In this section, we still consider a high dimensional nonlinear evolution equation of the general form

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, \dots) = 0 \quad (20)$$

where $u = u(x, y, z, t)$ and F is a polynomial u of and its derivatives, t represents time variable and x, y, z represent spatial variables. Here, we introduce a parameter to Eq.(20) in two ways.

(i). Initial solution as a parameter is introduced to Eq.(20). We assume that the u_0 is an initial solution of Eq.(20), then we use the transformation

$$u = u_0 + T(f) \quad (21)$$

to convert nonlinear evolution equation (20) into Hirota bilinear form which contains the small perturbation parameters u_0 :

$$H(u_0, D_t, D_x, D_y, D_z, \dots) f \cdot f = 0 \quad (22)$$

where D is also the Hirota D -operator. The next step is to exploit the existent method to solving the Eq.(22). The perturbation parameter u_0 plays an important role to the resulting solution, where the spatiotemporal feature in multi-wave propagation including velocity and direction even the shape will change as u_0 makes small perturbation.

(ii). The velocity of a wave variable as a parameter is introduced to Eq.(20). We can assume that $\xi = z - \alpha t$ in Eq.(20), where ξ is a new wave variable and α is its velocity. Then Eq.(20) can be became to

$$F_1(u, \alpha, u_\xi, u_x, u_y, u_{xx}, \dots) = 0 \quad (23)$$

Then solving Eq.(23). In this case, the spatiotemporal feature in multi-wave propagation for Eq.(20) may happen outstanding change as α makes a small perturbation.

3. Potential Kadomtsev-Petviashvili equation

Potential Kadomtsev-Petviashvili (PKP) equation studied in this section is described as

$$u_{xt} + 6u_x u_{xx} + u_{xxx} + u_{yy} = 0 \quad (24)$$

where $u : R_x \times R_y \times R_t^+ \rightarrow R$. It is well known that the PKP equation arose in number of remarkable non-linear problems both in physics and mathematics, the solutions of PKP equation have been studied extensively in various aspects. By applying various methods and techniques to PKP equation, exact traveling wave solutions, linearly solitary wave solutions, soliton-like solutions and some numerical solutions have been obtained[1-3,5-7]. Recently, two soliton and periodic soliton solutions were presented, resonance and non-resonance interactions between periodic soliton and different line solitons were investigated[6].

In this section, we use homoclinic test method and extended homoclinic test technique to seek periodic soliton solution, exact periodic kink-wave solution, periodic soliton solution and doubly periodic solution of PKP equation. Furthermore, it is explicitly exhibited that the feature of solution is different varying with direction of wave propagation on the x -axis, periodic soliton is degenerated into doubly periodic wave when the direction of a wave propagation changes from progressing to the left into right.

3.1. Degenerative periodic solitary wave

In this section, the periodic soliton solution is constructed by homoclinic test technique and bilinear form method and the degeneracy of solitary wave is investigated.

Setting $\xi = x - \alpha t$ in Eq.(24) yields

$$u_{yy} - \alpha u_{\xi\xi} + 6u_{\xi}u_{\xi\xi} + u_{\xi\xi\xi\xi} = 0 \quad (25)$$

where α is a wave velocity. Let $u = (\ln F)_{\xi}$, then Eq.(25) can be reduced into the following bilinear form

$$[D_y^2 - \alpha D_{\xi}^2 + D_{\xi}^4 - A]F \cdot F = 0 \quad (26)$$

where A is an integration constant, $D_x^m D_t^k$ is defined in Eq.(3). With regard to Eq.(26), using the homoclinic test technique, we can seek the solution in the form

$$F = 1 + b_1(e^{ip\xi} + e^{-ip\xi})e^{\Omega y + \gamma} + b_2 e^{2\Omega y + 2\gamma} \quad (27)$$

where $A, p, \Omega, \gamma, b_1$ and b_2 are all real to be determined below.

Substituting Eq.(27) into Eq.(26) yields the exact solution of Eq.(24) in the form

$$u = \frac{-2b_1 p e^{\Omega y + \gamma} \sin(p\xi)}{1 + 2b_1 \cos(p\xi) e^{\Omega y + \gamma} + b_2 e^{2\Omega y + 2\gamma}} \quad (28)$$

where parameters A, b_1, b_2, Ω, p and γ satisfy dispersive relations

$$A = 0 \quad \Omega^2 = -p^4 - \alpha p^2 \quad b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 - 3p^4} \quad (29)$$

Obviously, $\alpha < 0$ is required so that the conditions $\Omega^2 > 0, b_1^2 > 0$ and $0 < p^2 < -\alpha$ can be satisfied. Taking $\xi = x - \alpha t, b_2 = 1, \gamma = 0$ in Eq.(28), then $b_1 = \sqrt{\frac{\Omega^2}{\Omega^2 - 3p^4}} > 1$, the exact solution to PKP equation can be expressed as

$$u = \frac{-b_1 p \sin(p(x - \alpha t))}{b_1 \cos(p(x - \alpha t)) + \cosh(\Omega y)} \quad (30)$$

where

$$\begin{cases} \alpha < 0 \\ 0 < p^2 < -\frac{\alpha}{4} \\ \Omega^2 = -p^4 - \alpha p^2 \\ b_1^2 = \frac{\Omega^2}{\Omega^2 - 3p^4} \end{cases} \quad (31)$$

Here, we choice $0 < p^2 < -\frac{\alpha}{4}$ such that $b_1^2 > 0$, then Eq.(30) is the periodic soliton solution of PKP equation which is a periodic traveling wave progressing to the left with velocity $|\alpha|$ on the x -axis, and meanwhile is a soliton on the y -axis (see Fig.1).

Making a variable transformation $\xi = x - \alpha t, \eta = iy$ in Eq.(24), then it can be transformed into the following form

$$u_{\eta\eta} - (-\alpha)u_{\xi\xi} - 6u_{\xi}u_{\xi\xi} - u_{\xi\xi\xi\xi} = 0 \quad (32)$$

Letting $u = (\ln F)_{\xi}$, being similar to the way of dealing with PKP equation in above, we take

$$F = 1 + b_3(e^{ip_1\xi} + e^{-ip_1\xi})e^{\Omega_1\eta + \gamma_1} + b_4 e^{2\Omega_1\eta + 2\gamma_1}$$

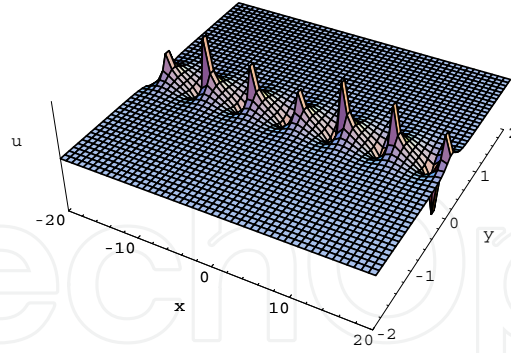


Figure 1. The periodic soliton solution which is a periodic wave progressing to the left with velocity $|\alpha| > 0$ on the x -axis, and meanwhile is a soliton on the y -axis

By computing, the exact solution of Eq.(32) is given by

$$u = -\frac{2b_3p_1 \sin(p_1\tilde{\zeta})e^{\Omega_1\eta+\gamma_1}}{1 + 2b_3 \cos(p_1\tilde{\zeta})e^{\Omega_1\eta+\gamma_1} + b_4e^{2\Omega_1\eta+2\gamma_1}} \quad (33)$$

where parameters b_3, b_4, Ω_1, p_1 satisfy following dispersive relations

$$\Omega_1^2 = p_1^4 + \alpha p_1^2 \quad b_3^2 = \frac{\Omega_1^2 b_4}{\Omega_1^2 + 3p_1^4} \quad (34)$$

We can see that $\Omega_1^2 \geq 0$ always holds for every real p_1 as long as $\alpha \geq 0$.

Taking $\tilde{\zeta} = x - \alpha t, \eta = iy$ in Eq.(33), the exact solution to PKP equation ($\alpha \geq 0$) is expressed as

$$u = -\frac{2b_3p \sin(p_1(x - \alpha t))}{2b_3 \cos(p_1(x - \alpha t)) + (e^{-i\Omega_1 y - \gamma_1} + b_4 e^{i\Omega_1 y + \gamma_1})} \quad (35)$$

where

$$\begin{cases} \alpha \geq 0, & p_1^2 \geq 0 \\ \Omega_1^2 = p_1^4 + \alpha p_1^2 \\ b_3^2 = \frac{\Omega_1^2 b_4}{\Omega_1^2 + 3p_1^4} \end{cases} \quad (36)$$

In particularly, taking $\gamma_1 = 0, b_4 = 1$ in Eq.(35) yields

$$u = -\frac{b_3p_1 \sin(p_1(x - \alpha t))}{b_3 \cos(p_1(x - \alpha t)) + \cos(\Omega_1 y)} \quad (37)$$

Obviously, both $\cos p_1(x - \alpha t)$ and $\cos(\Omega_1 y)$ are periodic, so the solution given by Eq.(37) is an doubly periodic solution which is a periodic traveling wave progressing to the right with velocity α on the x -axis, and meanwhile is a periodic standing wave on the y -axis (see Fig.2).

It is important that we can take the same p and p_1 such that $p^2 = p_1^2 = |\frac{\alpha}{k}| = p_0^2$ in Eq.(30) and Eq.(37) from the constraint condition Eq.(31) and Eq.(36) respectively, where $k > 4$ is an

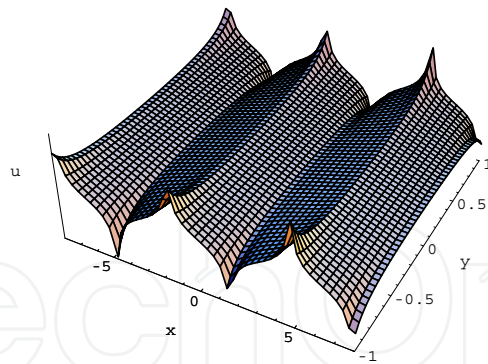


Figure 2. The doubly periodic solution which is a periodic wave progressing to the right with velocity α on the x -axis, and meanwhile is a periodic standing wave on the y -axis.

arbitrary real number. Therefore, we obtain a periodic soliton solution and an doubly periodic solution which have the same period with x -direction as follows

$$\begin{cases} u = \frac{-b_1 p_0 \sin(p_0(x - \alpha t))}{b_1 \cos(p_0(x - \alpha t)) + \cosh(\Omega y)}, & \alpha < 0 \\ u = -\frac{b_3 p_0 \sin(p_0(x - \alpha t))}{b_3 \cos(p_0(x - \alpha t)) + \cos(\Omega_1 y)}, & \alpha \geq 0 \end{cases} \quad (38)$$

It is easy to find that the feature of solution of Eq.(24) is different from Eq.(38) on the arbitrary small neighborhood of velocity $\alpha = 0$. There exists a periodic soliton solution which is a periodic with x -direction, and a soliton with y -direction as well as $\alpha < 0$. When $\alpha \geq 0$, this periodic soliton degenerates into a doubly periodic wave which are periodic with both x -direction and y -direction. However, period with x -direction is preserved identically.

3.2. Exact periodic kink-wave solution

In this section, a new type of periodic kink-wave solution for PKP equation is obtained using extended homoclinic test technique.

Taking a transformation

$$u = (\ln F)_x \quad (39)$$

Then Eq.(24) can be transformed as

$$[D_y^2 + D_x D_t + D_x^4] F \cdot F = 0 \quad (40)$$

In this case, we let

$$F = e^{-\Omega(x+\alpha t)-\beta y} + b_1 \cos(p(x - \alpha t) + \beta y) + b_2 e^{\Omega(x+\alpha t)} \quad (41)$$

Following the procedure of extended homoclinic test technique, we derive the periodic kink-wave solution of PKP equation(see Fig.3)

$$u = \frac{p[-e^{-p(x+11p^2t)} - b_1 \sin(p(x - 11p^2t \pm 3py)) + \frac{b_1^2}{20} e^{p(x+11p^2t)}]}{e^{-p(x+11p^2t)} + b_1 \cos(p(x - 11p^2t \pm 3py)) + \frac{b_1^2}{20} e^{p(x+11p^2t)}} \quad (42)$$

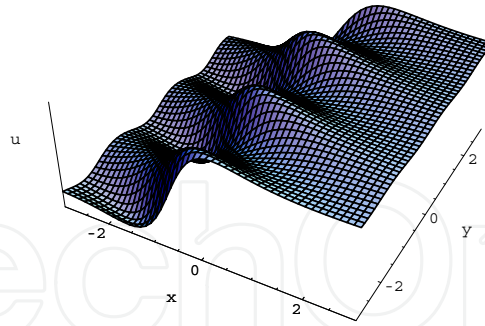


Figure 3. The periodic kink-wave solution

It can be rewritten as

$$u = \frac{p[\frac{b_1}{2\sqrt{5}}e^{p(x+11p^2t)} - 2\sqrt{5}\sin(p(x - 11p^2t \pm 3py)) - \frac{2\sqrt{5}}{b_1}e^{-p(x+11p^2t)}]}{\frac{b_1}{2\sqrt{5}}e^{p(x+11p^2t)} + 2\sqrt{5}\cos(p(x - 11p^2t \pm 3py)) + \frac{2\sqrt{5}}{b_1}e^{-p(x+11p^2t)}} \quad (43)$$

Specially, taking $b_1 = 2\sqrt{5}k$ and $\gamma = \ln k, k > 1$, yields a periodic kink solution as follows

$$u(x, y, t) = \frac{p[2\sinh(p(x + 11p^2t) + \gamma) - \sin(p(x \pm 3py - 11p^2t))]}{2\cosh(p(x + 11p^2t) + \gamma) + \cos(p(x \pm 3py - 11p^2t))} \quad (44)$$

It shows a periodic kink-wave whose speed is $11p^2$ and period is $2\pi/3p$ of space variable y . It exhibits elastic interaction between a solitary wave and periodic wave with the same speed in opposed direction each other. It is an interesting phenomenon in fluid mechanics (see Fig.3).

4. Kadomtsev-Petviashvili equation

The (2+1)-dimensional (two spatial and one temporal) generalization of Korteweg-de Vries equation (KdV) was given by Kadomtsev and Petviashvili to discuss the stability of (1+1)-dimensional soliton to the transverse long-wave disturbances, which is known as KP equation or the (2+1)-dimensional KdV equation. There are two distinct versions of KP equation, which can be written in normalized form as follows[4]:

$$u_t - 3(u^2)_x - u_{xxx} + s^2\partial_x^{-1}u_{yy} = 0 \quad (*)$$

with the operator ∂_x^{-1} defined by

$$\partial_x^{-1}f(x) = \int_{-\infty}^x f(\xi)d\xi$$

The propagation property of solitons depends essentially on the sign of s^2 in equation. The coefficient is defined as follows: $s = \pm i, i^2 = -1$ for negative dispersion and $s = \pm 1$ for positive dispersion. Here $u = u(x, y, t)$ is a scalar function, x and y are respectively the longitudinal and transverse spatial coordinates, the subscripts x, y, t denote partial derivatives. When $s = \pm i$, it is usually called KPI, while for $s = \pm 1$, it is called KP II. KP equation is the natural generalization of the well known KdV equation ($u_t - 6uu_x - u_{xxx} = 0$)

from one to two spatial dimensions. It arises naturally in many other applications, particularly in plasma physics, gas dynamics, and elsewhere. Both KPI and KP II are exactly integrable via the Inverse Scattering Transformation. Kadomtsev and Petviashvili have shown that the line soliton of the KP equation is unstable in the case of positive dispersion and is stable for the negative dispersion[3,22]. The solutions of KP equation have been studied extensively in various aspects. The inclined periodic soliton solution and the lattice soliton solution were expressed as exact imbricate series of rational soliton solutions to KP equation with positive dispersion[22]. Resonant interaction of two-soliton among three obliquely oriented solitons in higher dimension was first studied by Miles[26]. Y.Kodama and Dai have proved that the KP equation provides line solitons in shallow water and these solitons can be of resonance[7, 27].

In this section, spatial-temporal bifurcation for KP equation is considered, several types of exact solutions to KP equation were constructed by bilinear form and homoclinic test approach. It is explicitly analyzed that the feature of the solitary wave is different on the both sides of equilibrium solution $u_0 = -\frac{1}{6}$, which is a unique periodic bifurcation point for KPI and deflexion point of soliton for KP II. As for KPI, when the equilibrium u_0 varies from one side of $-\frac{1}{6}$ to another side, two-solitary wave changes into doubly periodic wave. Whereas, the y -periodic solitary wave changes into $x - t$ -periodic solitary wave for KP II, the propagation direction of periodic solitary wave occurs outstanding deflexion. In addition, some new type of multi-wave solutions are obtained using three-wave method.

4.1. Spatiotemporal bifurcation and deflexion of the soliton

Kadomtsev-Petviashvili (KP) equation in normalized variable $u(x, y, t)$ reads

$$u_{xt} - u_{xxxx} - 3(u^2)_{xx} - s^2 u_{yy} = 0 \quad (45)$$

where $u : R_x \times R_y \times R_t \rightarrow R$. It is easily to note that $u = u_0$ is an equilibrium solution of KP equation, where u_0 is an arbitrary constant.

Now, we consider KPI equation

$$u_{xt} - u_{xxxx} - 3(u^2)_{xx} - u_{yy} = 0 \quad (46)$$

Setting $\xi = i(x - t)$ gives

$$u_{yy} - u_{\xi\xi} - 3(u^2)_{\xi\xi} + u_{\xi\xi\xi\xi} = 0 \quad (47)$$

Let $u = u_0 - 2(\ln F)_{\xi\xi}$, Eq.(46) can be transformed into the following bilinear form

$$[D_y^2 - (1 + 6u_0)D_\xi^2 + D_\xi^4 - A]F \cdot F = 0 \quad (48)$$

Using "homoclinic test technique", we are going to seek the solution of the form

$$F = 1 + b_1(e^{ip\xi} + e^{-ip\xi})e^{\Omega y + \gamma} + b_2 e^{2\Omega y + 2\gamma} \quad (49)$$

where $A, p, \Omega, \gamma, b_1$ and b_2 are all real.

Substituting Eq.(49) into Eq.(48) with Eq.(47) yields the exact solution of Eq.(47) of the form

$$u = u_0 + \frac{2p^2[4b_1^2 + b_1(e^{ip\xi} + e^{-ip\xi})(b_2 e^{\Omega y + \gamma} + e^{-\Omega y - \gamma})]}{[b_1(e^{ip\xi} + e^{-ip\xi}) + (b_2 e^{\Omega y + \gamma} + e^{-\Omega y - \gamma})]^2} \quad (50)$$

where parameters satisfy

$$A = 0 \quad \Omega^2 = -p^4 - (1 + 6u_0)p^2 \quad b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 - 3p^4} \quad (51)$$

It is obviously that $u_0 < -\frac{1}{6}$ is required so that the conditions $\Omega^2 > 0$, $b_1^2 > 0$ and $p^2 < -\frac{1+6u_0}{4}$ can be satisfied and u_0 is a free parameter.

In Eq.(50), taking $\xi = i(x - t)$, we obtain the two-soliton solution of KPI equation (see Fig.4)

$$u_1(x, y, t) = u_0 + \frac{2p^2[4b_1^2 + b_1(e^{p(x-t)} + e^{-p(x-t)})(b_2 e^{\Omega y + \gamma} + e^{-\Omega y - \gamma})]}{[b_1(e^{p(x-t)} + e^{-p(x-t)}) + (b_2 e^{\Omega y + \gamma} + e^{-\Omega y - \gamma})]^2} \quad (52)$$

where

$$\begin{cases} u_0 < -\frac{1}{6} \\ p^2 < -\frac{1+6u_0}{4} \\ \Omega^2 = -p^4 - (1 + 6u_0)p^2 \\ b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 - 3p^4} \end{cases} \quad (53)$$

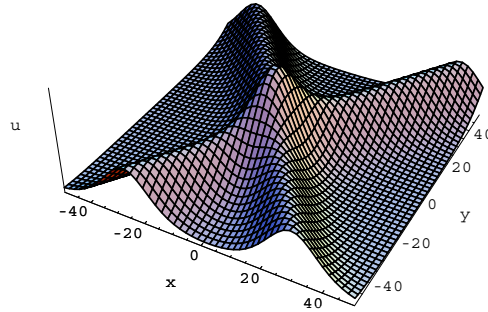


Figure 4. The two-soliton solution for KPI equation as $u_0 = -\frac{1}{4}$

KPII equation is given by

$$u_{xt} - u_{xxxx} - 3(u^2)_{xx} + u_{yy} = 0 \quad (54)$$

Making a variable transformation $\xi = x - t$ in Eq.(54), it can be transformed into the following form

$$u_{yy} - u_{\xi\xi} - 3(u^2)_{\xi\xi} - u_{\xi\xi\xi\xi} = 0 \quad (55)$$

Letting $u = u_0 + 2(\ln F)_{\xi\xi}$ and using a similar way dealing with KPI, we take

$$F = 1 + b_1(e^{ip\xi} + e^{-ip\xi})e^{\Omega y + \gamma} + b_2 e^{2\Omega y + 2\gamma}$$

By computing, the exact solution of Eq.(55) is given by

$$u = u_0 - \frac{2p^2[4b_1^2 + b_1(e^{ip\xi} + e^{-ip\xi})(b_2 e^{\Omega y + \gamma} + e^{-\Omega y - \gamma})]}{[b_1(e^{ip\xi} + e^{-ip\xi}) + (b_2 e^{\Omega y + \gamma} + e^{-\Omega y - \gamma})]^2} \quad (56)$$

where parameters satisfy

$$\Omega^2 = p^4 - (1 + 6u_0)p^2 \quad b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 + 3p^4} \quad (57)$$

It is easily to see that $u_0 \geq -\frac{1}{6}$ is available as long as $p^2 \geq 1 + 6u_0$.

Taking $\xi = x - t$ into Eq.(56), the exact solution to KPII equation is expressed as

$$u_2(x, y, t) = u_0 - \frac{2p^2[4b_1^2 + b_1 \cos(p(x - t))(b_2 e^{\Omega y + \gamma} + e^{-\Omega y - \gamma})]}{[b_1 \cos(p(x - t)) + (b_2 e^{\Omega y + \gamma} + e^{-\Omega y - \gamma})]^2} \quad (58)$$

where

$$\begin{cases} u_0 \geq -\frac{1}{6} \\ p^2 \geq 1 + 6u_0 \\ \Omega^2 = p^4 - (1 + 6u_0)p^2 \\ b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 + 3p^4} \end{cases} \quad (59)$$

Obviously, $\cos p(x - t)$ is periodic, so the solution given by Eq.(58) is a periodic soliton solution with $x - t$ -direction (see Fig.5).

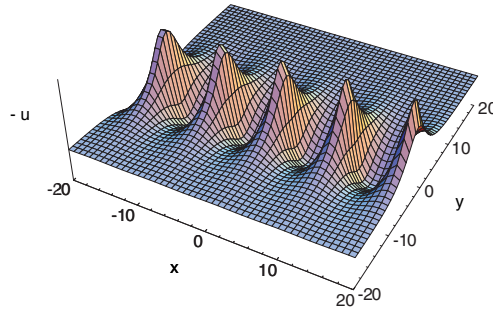


Figure 5. The $x - t$ -periodic soliton solution for KPII equation as $u_0 = -\frac{1}{8}$

Comparing Eq.(47) with Eq.(55), it is easily to find that the Eq.(47) may be changed into Eq.(55) and vice versa by using the temporal and spatial transformation $(\xi, y) \rightarrow (i\xi, iy)$. Because the solutions of KP equation are real functions, it is naturally to take specially $b_2 = 1, \gamma = 0$. Making variable transformation $\xi \rightarrow i\xi, y \rightarrow iy, i^2 = -1$ in Eq.(50) and Eq.(56) yields

$$u_3(x, y, t) = u_0 - \frac{2p^2[b_1^2 + b_1 \cos(p(x - t)) \cos(\Omega y)]}{[b_1 \cos(p(x - t)) + \cos(\Omega y)]^2} \quad (60)$$

It is noted that the solution given by Eq.(59) is a singular periodic solution to KPI equation. In order to avoid the singularity, we set $\cos(p(x - t)) > 0$ and $\cos(\Omega y) > 0$ (see Fig.6).

Besides, the y -periodic soliton solution to KPII is also given by

$$u_4(x, y, t) = u_0 + \frac{2p^2[4b_1^2 + b_1(e^{p(x-t)} + e^{-p(x-t)})(e^{i\Omega y} + e^{-i\Omega y})]}{[b_1(e^{p(x-t)} + e^{-p(x-t)}) + (e^{i\Omega y} + e^{-i\Omega y})]^2} \quad (61)$$

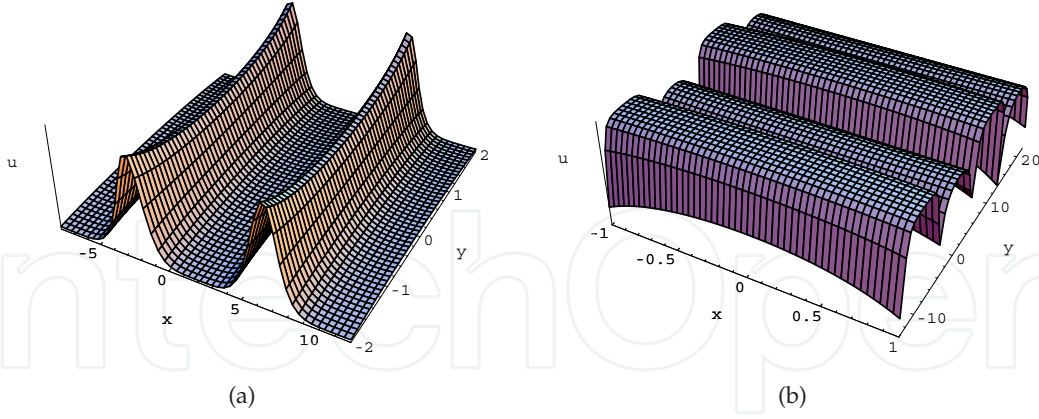


Figure 6. (a) The doubly periodic solution for KPI equation with $x - t$ direction as $u_0 = -\frac{1}{8}$
 (b) The doubly periodic solution for KPI equation with y direction as $u_0 = -\frac{1}{8}$

where

$$\begin{cases} u_0 < -\frac{1}{6} \\ p^2 < -\frac{1+6u_0}{4} \\ \Omega^2 = -p^4 - (1+6u_0)p^2 \\ b_1^2 = \frac{\Omega^2 b_2}{\Omega^2 - 3p^4} \end{cases} \quad (62)$$

The solution given by Eq.(60) represents periodic soliton with y -direction (see Fig.7).

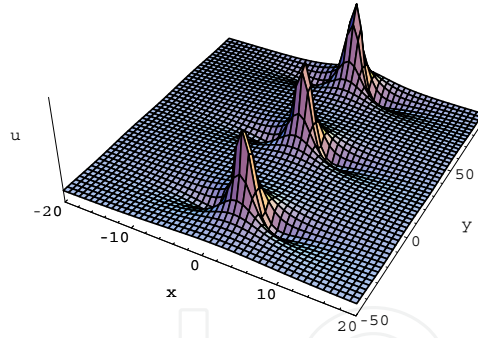


Figure 7. The y -periodic soliton solution for KP II equation as $u_0 = -\frac{1}{4}$

According to above discussion, we get that $u_0 = -\frac{1}{6}$ is a unique periodic bifurcation point for KPI and deflexion of soliton for KP II. Around the both sides at u_0 , the property of solutions to KPI and KP II is all changed. As for KPI, when the equilibrium u_0 varies from one side of $-\frac{1}{6}$ to another side, two-soliton solution changes into doubly periodic solution. Whereas, the y -periodic soliton changes into x -periodic soliton for KP II. The double-soliton waves and doubly periodic soliton waves of KPI, periodic soliton waves on different spatial variable of KP II are interchanged around u_0 .

4.2. Exact multi-wave solution

Let

$$u = 2(\ln f)_{xx} \quad (63)$$

in Eq.(45), where $f = f(x, y, t)$ is an unknown real function. Substituting Eq.(63) into Eq.(45), we can reduce Eq.(45) into the bilinear form

$$(D_x D_t - D_x^4 - p^2 D_y^2) f \cdot f = 0 \quad (64)$$

In order to obtain three wave solution of KP equation, we provide that

$$f = \cos(\gamma_1) + a_{-1} \exp(-\delta_1) + a_1 \exp(\delta_1) + a_2 \sinh(\xi_1) \quad (65)$$

where $\gamma_1 = p_1(x - \alpha_1 t)$, $\delta_1 = p_3(x + \beta_3 y + \alpha_3 t)$ and $\xi_1 = p_2(x + \beta_2 y + \alpha_2 t)$. Substituting Eq.(65) into Eq.(64) and equating the coefficients of all powers of $\sinh(\xi_1)$, $\cos(\gamma_1)$, $\sinh(\xi_1)$, $\exp(\delta_1)$, $\sinh(\xi_1) \exp(-\delta_1)$, $\cosh(\xi_1)$, $\sin(\gamma_1)$, $\cosh(\xi_1) \exp(\delta_1)$, $\cosh(\xi_1) \exp(-\delta_1)$, $\sin(\gamma_1) \exp(\delta_1)$, $\sin(\gamma_1) \exp(-\delta_1)$, $\cos(\gamma_1) \exp(\delta_1)$, $\cos(\gamma_1) \exp(-\delta_1)$ to zero, we can obtain a set of algebraic equations for a_{-1} , a_2 , a_1 , α_1 , α_2 , α_3 , β_2 , β_3 , p_1 , p_2 and p_3 , then by solving these set of algebraic equations and let $p_1 = p_2$, we obtain the breather two-solitary wave (three wave) solution of KPI as follows:

$$u_1(x, y, t) = 2 \left(\frac{a_2 \sqrt{a_1 a_{-1}} ((A_1 - p_1)^2 \sinh(\delta_2 + \xi_2) - (A_1 + p_1)^2 \sinh(\delta_2 - \xi_2))}{(\cos(\gamma_2) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_2) + a_2 \sinh(\xi_2))^2} \right. \\ + \frac{4a_1 a_{-1} A_1^2 - a_2^2 p_1^2 + 2\sqrt{a_1 a_{-1}} (A_1^2 - p_1^2) \cosh(\delta_2) \cos(\gamma_2)}{(\cos(\gamma_2) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_2) + a_2 \sinh(\xi_2))^2} \\ \left. + \frac{2p_1 (a_2 p_1 \cosh(\xi_2) + 2\sqrt{a_1 a_{-1}} A_1 \sinh(\delta_2)) \sin(\gamma_2) - p_1^2}{(\cos(\gamma_2) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_2) + a_2 \sinh(\xi_2))^2} \right)$$

where

$$a_1 a_{-1} > 0, \quad \theta_1 = \ln\left(\sqrt{\frac{a_1}{a_{-1}}}\right), \quad \gamma_2 = p_1(x - C_1 t) \\ \delta_2 = A_1(x + D_1 y + B_1 t) + \theta_1, \quad \xi_2 = p_1(x + \beta_2 y + C_1 t) \\ A_1 = \frac{\sqrt{2p_1^2 \beta_2^2 + 12p_1^4}}{\beta_2}, \quad B_1 = -\frac{(4p_1^2 + \beta_2^2)\beta_2^2 + 96p_1^4}{2\beta_2^2} \\ C_1 = -\frac{4p_1^2 - \beta_2^2}{2}, \quad D_1 = -\frac{12p_1^4 - \beta_2^2}{2\beta_2}$$

Here, a_{-1} , a_2 , p_1 and β_2 are free parameters.

In the case of $a_1 a_{-1} < 0$, the breather two-solitary wave solution of KPI can be expressed as

$$u_2(x, y, t) = 2 \left(\frac{a_2 \sqrt{\gamma} ((A_1 - p_1)^2 \cosh(\delta_2 + \xi_2) - (A_1 + p_1)^2 \cosh(\delta_2 - \xi_2)) + 4\gamma A_1^2 - a_2^2 p_1^2}{(\cos(\gamma_2) + \sqrt{\gamma} \sinh(\delta_2) + a_2 \sinh(\xi_2))^2} \right. \\ \left. + \frac{2\sqrt{\gamma} (A_1^2 - p_1^2) \sinh(\delta_2) \cos(\gamma_2) + 2p_1 (a_2 p_1 \cosh(\xi_2) + 2\sqrt{\gamma} A_1 \cosh(\delta_2)) \sin(\gamma_2) - p_1^2}{(\cos(\gamma_2) + \sqrt{\gamma} \sinh(\delta_2) + a_2 \sinh(\xi_2))^2} \right)$$

where $\gamma = -a_1 a_{-1}$.

Similarity, the breather two-solitary wave solutions of KPII equation also obtained as follows

$$u_3(x, y, t) = 2 \left(\frac{a_2 \sqrt{a_1 a_{-1}} ((A_2 - p_1)^2 \sinh(\delta_3 + \xi_3) - (A_2 + p_1)^2 \sinh(\delta_3 - \xi_3))}{(\cos(\gamma_3) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_3) + a_2 \sinh(\xi_3))^2} \right. \\ \left. + \frac{a_1 a_{-1} A_2^2 - a_2^2 p_1^2 + 2\sqrt{a_1 a_{-1}} (A_2^2 - p_1^2) \cosh(\delta_3) \cos(\gamma_3)}{(\cos(\gamma_3) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_3) + a_2 \sinh(\xi_3))^2} \right. \\ \left. + \frac{2p_1 (a_2 p_1 \cosh(\xi_3) + 2\sqrt{a_1 a_{-1}} A_2 \sinh(\delta_3)) \sin(\gamma_3) - p_1^2}{(\cos(\gamma_3) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_3) + a_2 \sinh(\xi_3))^2} \right)$$

where $\gamma_3 = p_1(x - C_2 t)$, $\delta_3 = A_2(x + D_2 y + B_2 t) + \theta_3$ and $\xi_3 = p_1(x + \beta_2 y + C_2 t)$ as $a_1 a_{-1} > 0$. Here

$$A_2 = \frac{\sqrt{2p_1^2 \beta_2^2 - 12p_1^4}}{\beta_2}, \quad B_2 = -\frac{(\beta_2^2 - 4p_1^2)\beta_2^2 + 96p_1^4}{2\beta_2^2}, \quad C_2 = -\frac{4p_1^2 + \beta_2^2}{2}, \quad D_2 = \frac{12p_1^4 + \beta_2^2}{2\beta_2}$$

a_{-1}, a_2, p_1 and β_2 are free parameters. And

$$u_4(x, y, t) = 2 \left(\frac{a_2 \sqrt{a_1 a_{-1}} ((p_3 - p_1)^2 \sinh(\delta_4 + \xi_4) - (p_3 + p_1)^2 \sinh(\delta_4 - \xi_4))}{(\cos(\gamma_4) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_4) + a_2 \sinh(\xi_4))^2} \right. \\ \left. + \frac{4a_1 a_{-1} p_3^2 - a_2^2 p_1^2 + 2\sqrt{a_1 a_{-1}} (p_3^2 - p_1^2) \cosh(\delta_4) \cos(\gamma_4)}{(\cos(\gamma_4) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_4) + a_2 \sinh(\xi_4))^2} \right. \\ \left. + \frac{2p_1 (a_2 p_1 \cosh(\xi_4) + 2\sqrt{a_1 a_{-1}} p_3 \sinh(\delta_4)) \sin(\gamma_4) - p_1^2}{(\cos(\gamma_4) + 2\sqrt{a_1 a_{-1}} \cosh(\delta_4) + a_2 \sinh(\xi_4))^2} \right)$$

where $\gamma_4 = p_1(x - C_3 t)$, $\delta_4 = p_3(x + D_3 y + B_3 t) + \theta_3$, $\xi_4 = p_1(x + A_3 y + C_3 t)$, with

$$A_3 = 2\sqrt{3}p_3, \quad B_3 = -2(3p_1^2 + p_3^2), \quad C_3 = -2(3p_3^2 + p_1^2), \quad D_3 = \frac{\sqrt{3}(p_3^2 + p_1^2)}{p_3}, \quad \theta_3 = \ln\left(\sqrt{\frac{a_1}{a_{-1}}}\right)$$

a_{-1}, a_2, p_1 and p_3 are free parameters.

5. Kadomtsev-Petviashvili equation with positive dispersion

The purpose of this section is to investigate the fission and fusion interactions of the breather-type multi-solitary waves solutions to the KP equation with positive dispersion.

By transformation of independent variable $t \rightarrow -t, y \rightarrow \sqrt{3}y$ in (*), KP equation with positive dispersion can be written as

$$u_t + 6uu_x + u_{xxx} - 3\partial_x^{-1}u_{yy} = 0 \quad (66)$$

By the transformation of a dependent variable u

$$u = 2(\ln f)_{xx} \quad (67)$$

Eq.(66) can be transformed into the bilinear form

$$F(D_x, D_y, D_t)f \cdot f = (D_x D_t + D_x^4 - 3D_y^2)f \cdot f = 0 \quad (68)$$

where $f(x, y, t)$ is a real function.

5.1. Fission and fusion of multi-wave

In the following, by using generalized three-wave type of Ansatz approach, we study the interaction and spatiotemporal feature of three-wave of KP equation with positive dispersion. Now we suppose the solution of Eq.(68) as

$$f(x, y, t) = e^{\xi_1} + \delta_1 \cos \xi_2 + \delta_2 \cosh \xi_3 + \delta_3 e^{-\xi_1} \quad (69)$$

where $\xi_1 = a_1x + b_1y + c_1t + \theta_1$, $\xi_2 = a_2x + b_2y + c_2t + \theta_2$, $\xi_3 = a_3x + b_3y + c_3t + \theta_3$ and $a_j, b_j, c_j, \delta_j, j = 1, 2, 3$ are some constants to be determined. Substituting the Ansatz Eq.(69) into Eq. (68) will produce the following relations:

$$\begin{aligned} a_3 &= -\frac{b_2a_1 - a_2b_1}{a_2^2 + a_1^2}, \quad c_3 = \frac{\Delta_1}{(a_2^2 + a_1^2)^3} \\ c_1 &= -\frac{a_1^5 - 2a_1^3a_2^2 + 3b_2^2a_1 - 3b_1^2a_1 - 3a_2^4a_1 - 6a_2b_1b_2}{a_2^2 + a_1^2} \\ b_3 &= \frac{2a_1^3a_2^3 + a_2^2b_1b_2 + a_1^5a_2 - a_1^2b_1b_2 + a_2^5a_1 + b_1^2a_1a_2 - b_2^2a_1a_2}{(a_2^2 + a_1^2)^2} \\ \delta_3 &= \frac{4(a_2^2 + a_1^2)^3(a_1^3 + a_2b_1 + a_1a_2^2 - b_2a_1)(a_1^3 + b_2a_1 + a_1a_2^2 - a_2b_1)}{3a_1^4a_2 + 2a_1^2a_2^3 - 3b_2^2a_2 + 3b_1^2a_2 - a_2^5 - 6b_1b_2a_1} \\ c_2 &= -\frac{a_2^2 + a_1^2}{\Delta_2} \\ \Delta_1 &= 3b_2a_1^7 + 3a_2b_1a_1^6 + 9a_2^2b_2a_1^5 + 9a_2^3b_1a_1^4 + 9a_1^3a_2^4b_2 - 3a_1^3b_1^2b_2 \\ &\quad + a_1^3b_2^3 + 3a_1^2a_2b_1^3 - 9a_1^2a_2b_1b_2^2 + 9a_1^2a_2^5b_1 + 3a_1a_2^6b_2 - 3a_1a_2^2b_2^3 \\ &\quad + 9a_1a_2^2b_1^2b_2 + 3a_2^3b_1b_2^2 - a_2^3b_1^3 + 3a_2^7b_1 \\ \Delta_2 &= -\left(b_2^2a_1^2 - 2a_2b_1b_2a_1 + a_2^2b_1^2 + a_1^4a_2^2 + 2a_2^4a_1^2 + a_2^6\right) \\ &\quad \left((a_2^2 + a_1^2)^3\delta_1^2 - (a_2b_1 - b_2a_1 + a_1^3 + a_1a_2^2)(a_1^3 - a_2b_1 + b_2a_1 + a_1a_2^2)\delta_2^2\right) \end{aligned} \quad (70)$$

where $a_1, \delta_1, \delta_2, a_2, b_1, b_2$ are some free constants and $a_1 \neq 0, a_2 \neq 0$. Now we can explicitly write down the three-wave solutions using

$$\begin{aligned} u(x, y, t) &= 2 \frac{2a_1^2 \sqrt{\delta_3} \cosh(\xi_1 - \ln \sqrt{\delta_3}) - \delta_1 a_2^2 \cos \xi_2 + \delta_2 a_3^2 \cosh \xi_3}{2 \sqrt{\delta_3} \cosh(\xi_1 - \ln \sqrt{\delta_3}) + \delta_1 \cos \xi_2 + \delta_2 \cosh \xi_3} \\ &\quad - 2 \frac{(2 \sqrt{\delta_3} a_1 \sinh(\xi_1 - \ln \sqrt{\delta_3}) - \delta_1 a_2 \sin \xi_2 + \delta_2 a_3 \sinh \xi_3)^2}{(2 \sqrt{\delta_3} \cosh(\xi_1 - \ln \sqrt{\delta_3}) + \delta_1 \cos \xi_2 + \delta_2 \cosh \xi_3)^2} \end{aligned} \quad (71)$$

where a_j, b_j, c_j, δ_j ($j = 1, 2, 3$) satisfy Eq.(70). It is called the breather type multi-solitary waves solutions. Fig.8 is the plot of the spatial structure with the parameters selected as

$$\begin{pmatrix} a_1 & b_1 & \delta_1 \\ a_2 & b_2 & \delta_2 \end{pmatrix} = \begin{pmatrix} 0.1 & 1 & 0.5 \\ 1.1 & 0.1 & 10^{-4} \end{pmatrix}$$

Fig.8 is the process of interaction for two solitons solutions with the evolution of time, where s_1 and s_2 are the different two solitons, and s_{12} represents the interaction between s_1, s_2 and the periodic wave $\cos(\xi_2)$. The phenomena of soliton interaction are clearly presented. It shows that the two soliton experience interaction, they will fusion with few oscillations and later travel ahead continuously. The value of δ_2 will determine the length of resonant soliton. Obviously, the solitons s_1 and s_2 can not approach each other closely from picture. They interact produce the breather wave.

Usually, the interactions between solitons for a lot of integrable or non-integrable system are considered to be completely elastic. That is to say, the amplitude, velocity and wave shape of a soliton do not change after the nonlinear collisions. However, for several nonlinear partial differential equations, completely nonelastic interactions will occur. On the other hand, two or more solitons will fusion to one soliton. These two types of phenomena was called soliton fission and soliton fusion, respectively. Now we demonstrate soliton fission and fusion of the Kadomtsev-Petviashvili equation with positive dispersion.

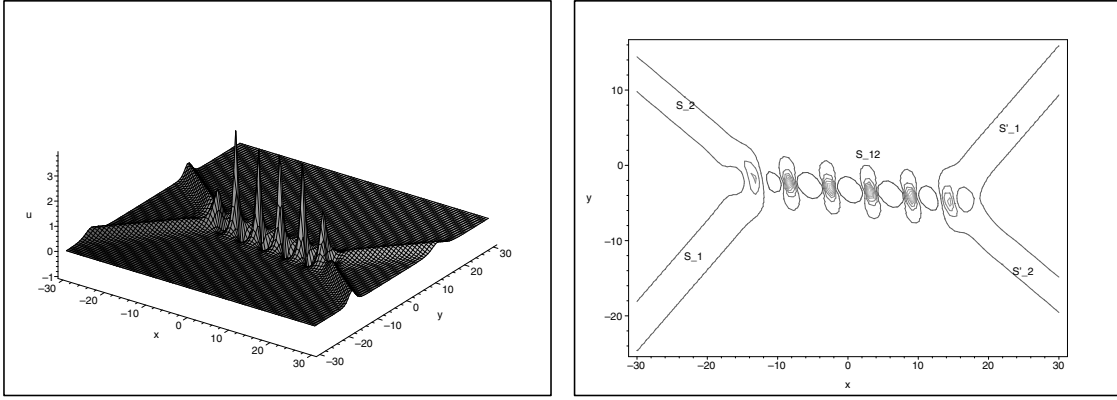


Figure 8. The plot of the space structure of the breather-type multi-solitary waves solutions and contour plot map of u in (x, y) -plane.

In Eq.(70), let $\delta_3 = 0$, then

$$\frac{\Delta ((a_1^3 + a_2 b_1 + a_1 a_2^2 - a_1 b_2)(a_1^3 + a_1 b_2 + a_1 a_2^2 - a_2 b_1) \delta_2^2 - (a_1^2 + a_2^2)^3 \delta_1^2)}{4(a_1^2 + a_2^2)^3 (a_1^3 + a_1 b_2 + a_1 a_2^2 - a_2 b_1)(a_1^3 + a_2 b_1 + a_1 a_2^2 - a_1 b_2)} = 0$$

where $\Delta = a_2^2 (a_2^2 + a_1^2)^2 + (a_2 b_1 - b_2 a_1)^2$, $a_1 \neq 0$ and $a_2 \neq 0$.

For $a_2^2 (a_2^2 + a_1^2)^2 + (a_2 b_1 - b_2 a_1)^2$ can not be zero, then the condition must be satisfied:

$$(a_1(a_1^2 + a_2^2 - b_2) + a_2 b_1)(a_1(a_1^2 + a_2^2 + b_2) - a_2 b_1) \delta_2^2 - (a_1^2 + a_2^2)^3 \delta_1^2 = 0$$

So, we obtain the following solution:

$$u(x, y, t) = \frac{2(a_1^2 e^{\xi_1} - \delta_1 a_2^2 \cos \xi_2 + \delta_2 a_3^2 \cosh \xi_3)}{e^{\xi_1} + \delta_1 \cos \xi_2 + \delta_2 \cosh \xi_3} - \frac{2(a_1 e^{\xi_1} - \delta_1 a_2 \sin \xi_2 + \delta_2 a_3 \sinh \xi_3)^2}{(e^{\xi_1} + \delta_1 \cos \xi_2 + \delta_2 \cosh \xi_3)^2} \quad (72)$$

where $\xi_1 = a_1x + b_1y + c_1t$, $\xi_2 = a_2x + b_2y + c_2t$, $\xi_3 = a_3x + b_3y + c_3t$ and $a_j, b_j, c_j, j = 1, 2, 3$, and δ_1, δ_2 with the following relations:

$$c_2 = -\frac{3b_1^2a_2 - a_2^5 - 3b_2^2a_2 + 2a_1^2a_2^3 + 3a_1^4a_2 - 6a_1b_1b_2}{a_1^2 + a_2^2}$$

$$c_1 = -\frac{a_1^5 - 3a_2^4a_1 - 6a_2b_1b_2 - 2a_1^3a_2^2 - 3b_1^2a_1 + 3b_2^2a_1}{a_1^2 + a_2^2}$$

$$c_3 = (3b_2a_1^7 + 3a_2b_1a_1^6 + 9a_2^2b_2a_1^5 + 9a_2^3b_1a_1^4 + 9a_1^3a_2^4b_2 + a_1^3b_2^3 - 3a_1^3b_1^2b_2 - 9a_1^2a_2b_1b_2^2 + 9a_1^2a_2^5b_1 + 3a_1^2b_1^3a_2 + 3a_1a_2^6b_2 - 3a_1a_2^2b_2^3 + 9a_1a_2^2b_1^2b_2 + 3a_2^3b_1b_2^2 - b_1^3a_2^3 + 3a_2^7b_1)((a_1^2 + a_2^2)^3)^{-1}$$

$$a_3 = \frac{a_2b_1 - b_2a_1}{a_1^2 + a_2^2}$$

$$b_3 = \frac{a_2a_1^5 + 2a_2^3a_1^3 - a_1^2b_1b_2 + a_1b_1^2a_2 + a_1a_2^5 - a_1b_2^2a_2 + b_1b_2a_2^2}{(a_1^2 + a_2^2)^2}$$

$$\delta_2^2 = \frac{(a_1^2 + a_2^2)^3}{(a_1^3 + a_2b_1 + a_1a_2^2 - b_2a_1)(a_1^3 + b_2a_1 + a_1a_2^2 - a_2b_1)}\delta_1^2$$

Fig.9 shows the plot of two kinds of interaction behavior between two single solitons with different parameters and a breather wave, where

$$\begin{pmatrix} a_1 & b_1 & \delta_1 \\ a_2 & b_2 & \delta_2 \end{pmatrix} = \begin{pmatrix} -1.5 & 1 & 0.05 \\ -1.1 & 1.2 & 2 \end{pmatrix} \text{ and } \begin{pmatrix} a_1 & b_1 & \delta_1 \\ a_2 & b_2 & \delta_2 \end{pmatrix} = \begin{pmatrix} 1.5 & -1 & 0.05 \\ -1.1 & 1.2 & 2 \end{pmatrix}$$

respectively. From the first picture of Fig.9, we can see that two single solitons interact strongly to make a resonance breather-wave solution from a point at which two incident solitons meet

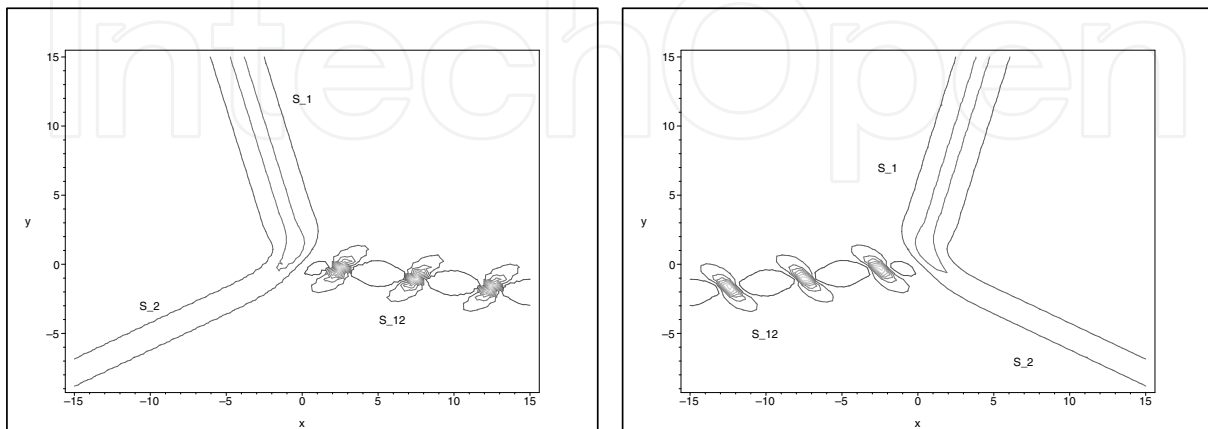


Figure 9. Contourplot of the breather-type multi-solitary waves solutions with the different parameters.

together. This phenomena is called soliton fusion. However, in the right figure it is found that the breather wave with the period oscillation can split up into two smaller line solitons with different directions. This phenomena is called the soliton fission.

6. Conclusion

Using Homoclinic test approach, Extend Homoclinic test approach, Three-wave method and Introducing parameters and small perturbation method, we obtain novel solutions of Potential Kadomtsev-Petviashvili equation and Kadomtsev-Petviashvili equation such as periodic solitary wave, breather solitary wave, breather homoclinic wave, breather heteroclinic wave, cross kink wave, kinky kink wave, periodic kink wave, two-solitary wave, doubly periodic wave, doubly breather solitary wave. Moreover, we observed that there were differently spatiotemporal features in two-wave and multi-wave propagations including the degeneracy of soliton, periodic bifurcation and soliton deflexion of two-wave, fission and fusion of breather two-wave and so on. In future, we intend to study the stability and the interactions patterns of N -wave solutions in KP equation. What's more, can we obtain similar results to another integrable or non-integrable system? How can one use the soliton fission and fusion of models to study the practically observed soliton fission and fusion in the experiments?

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7. References

- [1] Zabusky N, Kruskal M (1965) Interaction of "Solitons" in a Collisionless Plasma and the Recurrence of Initial States. *Phys.Rev.lett.* 15:240-243.
- [2] Ablowitz M, Clarkson P (1991) Soliton, nonlinear evolution equations and inverse scattering. Cambridge University Press.
- [3] Weiss J, Tabor M, Carnevale G (1983) The Painlevé property for partial differential equations. *J. Math. Phys.* 24:522-527.
- [4] Kadomtsev B, Petviashvili V (1970) On the stability of solitary waves in weakly dispersive media. *Soviet.Phys.Dokl.* 15:539-541.
- [5] Chow K (2001) Periodic solutions for a system of four coupled nonlinear Schrödinger equations. *Phys. Lett. A* 285:319.

- [6] Zeng X, Dai Z, Li D, Han S(2008) Some exact periodic soliton solutions and resonance for the potential Kadomtsev-Petviashvili equation. J. of Physics. Conference series 96: 012149.
- [7] Dai Z, Li S, Dai Q, Huang J(2007) Singular periodic soliton solutions and resonance for the Kadomtsev-Petviashvili equation. Chaos, Solitons and Fractals :1148-1153.
- [8] Dai Z, Liu J, Liu Z (2010) Exact periodic kink-wave and degenerative soliton solutions for potential Kadomtsev -petviashvili equation. Commun. Nonlinear. Sci. Numer. Simulat. 15: 2331-2336.
- [9] Dai Z, Li S, Li D, Zhu A (2007) Periodic Bifurcation and Soliton deflexion for Kadomtsev-Petviashvili Equation, Chin. Phys. Lett. 24 :1420-1424.
- [10] Dai Z, Liu J, Li D (2009) Applications of HTA and EHTA to YTSF Equation. Applied Mathematics and Computation. 207:360-364.
- [11] Dai Z, Lin S, Fu H, Zeng X (2010) Exact three-wave solutions for the KP equation. Applied Mathematics and Computation. 216:1599-1604.
- [12] Dai Z, Wang C, Lin S, Li D, Mu G (2010) The Three-wave Method for Nonlinear Evolution Equations. Nonl. Sci. Lett. A, 1:77-82.
- [13] Wang C, Dai Z, Fission and fusion of breather multi-solitary waves to the Kadomtsev-Petviashvili equation with positive dispersion(to appear).
- [14] Li D, Zhang H (2003) New soliton-like solutions to the potential Kadomtsev-Petviashvili equation. Appl. Math. Comput. 146:381-384.
- [15] Li D, Zhang H (2003) Symbolic computation and various exact solutions of potential Kadomtsev- Petviashvili equation. Appl. Math. Comput. 145:351-359.
- [16] Kaya D, El-Sayed S (2003) Numerical soliton-like solutions of the potential Kadomtsev-Petviashvili equation by the decomposition method. Phys. Lett. A 320:192-199.
- [17] Inan E, Kaya D (2003) Some exact solutions to the potential Kadomtsev-Petviashvili equation and to a system of shallow water equation. Phys. Lett. A 355:314-318.
- [18] Fu H, Dai Z (2009) Double Exp-function method and application. Int. J. Nonlinear Sci. Num. Simul. 10(7):927-933.
- [19] Ukai S (1989) Local solutions of the Kadomtsev-Petviashvili equation. J. Fac. Sci. Univ. Tokyo, Sect. IA Math. 36:193-209.
- [20] Cheng Y, Li Y(1991) The constraint of the Kadomtsev-Petviashvili equation and its special solutions. Phys. Lett. A 157:22-26.
- [21] Isaza P, Mejia J, Stallbohm V (1995) local solution for the Kadomtsev-Petviashvili equation in R^2 . Math. Anal. Appl. 196:566-587.
- [22] Tajiri M, Watanabe Y (1997) Periodic soliton solutions as imbricate series of rational solitons:solutions to the Kadomtsev-Petviashvili equation with positive dispersion. Nonlinear Math. Phys. 4:350-357.
- [23] Zhu J (2007) Multisoliton excitations for the Kadomtsev-Petviashvili equation and the coupled Burgers equation. Chaos, Solitons and Fractals. 31: 648-657.
- [24] Yomba E (2004) Construction of new soliton-like solutions for the $(2 + 1)$ dimensional Kadomtsev-Petviashvili equation. Chaos, Solitons and Fractals. 22:321-325.
- [25] Tagumi Y (1989) Soliton-like solutions to a $(2+1)$ dimensional generalization of the KdV equation. Phys. Lett. A 141: 116-120.
- [26] Miles J (1977) Resonantly interacting solitary waves. J. Fluid Mech. 79:171-179.
- [27] Kodama Y (2004) Young diagrams and N-soliton solutions of the KP equation. J. Phys. A: Math. Gen. 37:11169-11190.