We are IntechOpen, the world's leading publisher of Open Access books
Built by scientists, for scientists

6,900
Open access books available

185,000
International authors and editors

200M
Downloads

154
Countries delivered to

Our authors are among the

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

CLARIVATE ANALYTICS
BOOK CITATION INDEX
INDEXED

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

The Cherenkov Effect in Graphene-Like Structures

Miroslav Pardy

Additional information is available at the end of the chapter

1. Introduction

More than 70 years ago, Peierls [29] and Landau [16] performed a proof that the 2-dimensional crystal is not thermodynamically stable and cannot exist. They argued that the thermodynamical fluctuations of such crystal leads to such displacements of atoms that these displacements are of the same size as the distances between atoms at the any finite temperature. The argument was extended by Mermin [21] and it seemed that many experimental observations supported the Landau-Peierls-Mermin theory. So, the "impossibility" of the existence of graphene was established.

In 2004, Andre Geim, Kostia Novoselov [13, 22, 23] and co-workers at the University of Manchester in the UK by delicately cleaving a sample of graphite with sticky tape produced a sheet of crystalline carbon just one atom thick, known as graphene. Geims group was able to isolate graphene, and was able to visualize the new crystal using a simple optical microscope. Nevertheless, Landau-Peierls-Mermin proof remained of the permanent historical and pedagogical meaning.

At present time, there are novel methods how to create graphene sheet. For instance, Dato et al. [5] used the plasma reactor, where the graphene sheets were synthesized by passing liquid ethanol droplets into an argon plasma.

Graphene is the benzene ring (C_6H_6) stripped out from their H-atoms. It is allotrope of carbon because carbon can be in the crystalline form of graphite, diamond, fullerene (C_{60}), carbon nanotube and glassy carbon (also called vitreous carbon).

Graphene unique properties arise from the collective behavior of so called pseudoelectrons with pseudospins, which are governed by the Dirac equation in the hexagonal lattice.

The Dirac fermions in graphene carry one unit of electric charge and so can be manipulated using electromagnetic fields. Strong interactions between the electrons and the honeycomb lattice of carbon atoms mean that the dispersion relation is linear and given by $E = vp$, where v is so called the Fermi-Dirac velocity, p is momentum of a pseudoelectron.

The linear dispersion relation follows from the relativistic energy relation for small mass together with approximation that the Fermi velocity is approximately only about 300 times less than the speed of light.

The pseudospin of the pseudoelectron follows from the graphene structure. The graphene is composed of the system of hexagonal cells and it means that graphene is composed from the systems of two equilateral triangles. If the wave function of the first triangle sublattice system is φ_1 and the wave function of the second triangle sublattice system is φ_2, then the total wave function of the electron moving in the hexagonal system is superposition $\psi = c_1\varphi_1 + c_2\varphi_2$, where c_1 and are c_2 appropriate functions of coordinate $\mathbf{x}$ and functions φ_1, φ_2 are functions of wave vector $\mathbf{k}$ and coordinate $\mathbf{x}$. The next crucial step is the new spinor function defined as [19].

$$\chi = \left\{ \begin{array}{c} \varphi_1 \\ \varphi_2 \end{array} \right\} \tag{1}$$

and it is possible to prove that this spinor function is solution of the Pauli equation in the nonrelativistic situation and Dirac equation of the generalized case. The corresponding mass of such effective electron is proved to be zero.

The introduction of the Dirac relativistic Hamiltonian in graphene physics is the description of the graphene physics by means of electron-hole medium. It is the analogue of the description of the electron-positron vacuum by the Dirac theory of quantum electrodynamics. The pseudoelectron and pseudospin are not an electron and the spin of quantum electrodynamics (QED), because QED is the quantum theory of the interaction of real electrons and photons where mass of an electron is the mass defined by classical mechanics and not by collective behavior in hexagonal sheet called graphene.

The graphene can be considered as the special form the 2-dimensional graphene-like structures, where for instance silicene has the analogue structure as graphene [8]. The band structure of a free silicene layer resembles the band structure of graphene. The Fermi velocity v of electrons in silicene is lower than that in graphene.

If we switch on an electric field, the symmetry between the A and B sublattices of silicene's honeycomb structure breaks and a gap Δ is open in the band structure at the hexagonal Brillouin zone (BZ) points K and K'. In the framework of a simple nearest-neighbor tight-binding model, this manifests itself in the form of an energy correction to the on-site energies that is positive for sublattice A and negative for B. This difference in on-site energies $\Delta = E_A - E_B$ leads to a spectrum with a gap for electrons in the vicinity of the corners of the BZ with $E_\pm = \pm\sqrt{(\Delta/2)^2 + |v\mathbf{p}|^2}$, where $\mathbf{p}$ is the electron momentum relative to the BZ corner. Opening a gap in graphene by these means would be impossible because the A and B sublattices lie in the same plane [7].

So, silicene consists of a honeycomb lattice of silicon atoms with two sublattices made of A sites and B sites. The states near the Fermi energy are orbitals residing near the K and K' points at opposite corners of the hexagonal Brillouin zone. While silicon is dielectric medium, silicene is the conductive medium with Hall effect and it is possible to study the Mach cone generated by motion of a charged particle through the silicene sheet.

On the other side, there are amorphous solids - glasses, the atomic structure of which lack any long range translational periodicity. However, due to chemical bonding characteristics, glasses do possess a high degree of short-range order with respect to local atomic polyhedra.

In other words such structures can be considered as the graphene-like structures with the appropriate index of refraction, which is necessary for the the existence of Čerenkov effect.

The last but not least graphene-like structure can be represented by graphene-based polaritonic crystal sheet [2] which can be used to study the Čerenkov effect.

We derive in this chapter the power spectrum of photons generated by charged particle moving in parallel direction and perpendicular direction to the graphene-like structure with index of refraction n. The Graphene sheet is conductive contrary to some graphene-like structures, for instance graphene with implanted ions, which are dielectric media and it means that it enables the experimental realization of the Čerenkov radiation. We calculate it from the viewpoint of the Schwinger theory of sources [24–27, 30–32].

To be pedagogically clear we introduce the quantum theory of the index of refraction (where the dipole polarization of matter is the necessary condition for its existence), the classical and quantum theory of Čerenkov radiation and elements of the Schwinger source theory formalism for electrodynamic effect in dielectric medium. We involve also the Čerenkov effect with massive photons.

2. The quantum theory of index of refraction

The quantum theory of dispersion can be derived in the framework of the nonrelativistic Schrödinger equation [33] for an electron moving in dielectric medium and in the field with the periodic force

$$F_x = -eE_0 \cos \omega t, \quad F_y = F_z = 0. \tag{2}$$

Then, the corresponding potential energy is

$$V' = -exE_0 \cos \omega t \tag{3}$$

and this potential energy is the perturbation energy in the Schrödinger equation

$$\left(i\hbar \frac{\partial}{\partial t} - H_0 - V'\right) \psi_k(t) = 0, \tag{4}$$

where for $V' = 0$ $\psi_k(t) \rightarrow \psi_k^0(t)$ and

$$\psi_k^0(t) = \psi_k^0 e^{-\frac{i}{\hbar} E_k t} = \psi_k^0 e^{-i\omega_k t}, \tag{5}$$

where ψ_k^0 is the solution of the Schrödinger equation without perturbation, or,

$$\left(i\hbar \frac{\partial}{\partial t} - H_0\right) \psi_k^0(t) = 0. \tag{6}$$

We are looking for the solution of the Schrödinger equation involving the perturbation potential in the form

$$\psi_k(t) = \psi_k^0(t) + \psi_k^1(t), \tag{7}$$

where $\psi_k^1(t)$, is the perturbation wave function correction to the non-perturbation wave function.

After insertion of formula (7) to eq. (4), we get

$$\left(i\hbar\frac{\partial}{\partial t} - H_0\right)\psi_k^1(t) = \frac{1}{2}exE_0\psi_k^0\left(e^{-it(\omega_k-\omega)} + e^{-it(\omega_k+\omega)}\right). \tag{8}$$

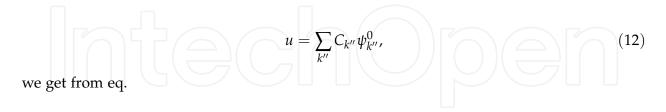Let us look for the solution of eq. (8) in the form:

$$\psi_k^1(t) = ue^{-it(\omega_k-\omega)} + ve^{-it(\omega_k+\omega)}. \tag{9}$$

After insertion of (9) into (8), we get two equations for u and v:

$$(\hbar(\omega_k - \omega) - H_0)u = \frac{1}{2}exE_0\psi_k^0, \tag{10}$$

$$(\hbar(\omega_k + \omega) - H_0)v = \frac{1}{2}exE_0\psi_k^0. \tag{11}$$

Then, using the formal expansion

$$u = \sum_{k''}C_{k''}\psi_{k''}^0, \tag{12}$$

we get from eq.

$$(E_{k''} - H_0)\psi_{k''}^0 = 0 \tag{13}$$

the following equation

$$\hbar\sum_{k''}C_{k''}(\omega_{kk''} - \omega)\psi_{k''}^0 = \frac{exE_0}{2}\psi_k^0 \tag{14}$$

with

$$\omega_{kk''} = \frac{E_k - E_{k''}}{\hbar}. \tag{15}$$

Using the orthogonal relation

$$\int \psi_{k'}^{0*} \psi_{k''}^0 d^3x = \delta_{k'k''}, \tag{16}$$

we get the following relation for C_k and u as follows:

$$C_k = -\frac{eE_0}{2\hbar} \cdot \frac{x_{k'k}}{\omega_{k'k} + \omega}, \tag{17}$$

$$u = \sum_{k'} \left(-\frac{eE_0}{2\hbar}\right) \cdot \frac{x_{k'k}}{\omega_{k'k} + \omega} \psi_{k'}^0 \tag{18}$$

and $v = u(-\omega)$, or

$$v = \sum_{k'} \left(-\frac{eE_0}{2\hbar}\right) \cdot \frac{x_{k'k}}{\omega_{k'k} - \omega} \psi_{k'}^0 \tag{19}$$

and

$$x_{k'k} = \int \psi_{k'}^{0*} x \psi_k^0 d^3x. \tag{20}$$

The general wave function can be obtained from eqs. (7), (9), (18) and (19) in the form:

$$\psi_k(t) = e^{-i\omega_k t} \left\{ \psi_k^0 - \frac{eE_0}{\hbar} \sum_{k'} \frac{x_{k'k}}{\omega_{k'k}^2 - \omega^2} \psi_{k'}^0 [\omega_{k'k} \cos \omega t - i\omega \sin \omega t] \right\}. \tag{21}$$

The classical polarization of a medium is given by the well known formula

$$P = Np = -Nex, \tag{22}$$

where N is the number of atom in the unite volume of dielectric medium. So we are able to define the quantum analogue form of the polarization as it follows:

$$P = N\bar{p} = -Ne \int \psi_k^*(t) x \psi_k(t) d^3x, \tag{23}$$

or, with

$$\int \psi_k^{0*} x \psi_k^0 d^3x = 0, \tag{24}$$

we have

$$P = \sum_{k'} \left(2 \frac{Ne^2 E_0}{\hbar} \right) \cdot \frac{\omega_{k'k} |x_{k'k}|^2}{\omega_{k'k}^2 - \omega^2} \cos \omega t. \tag{25}$$

Using the classical formula for polarization P,

$$\mathbf{P} = \frac{n^2 - 1}{4\pi} \mathbf{E}, \tag{26}$$

we get for the quantum model of polarization

$$\frac{n^2 - 1}{4\pi} = \sum_{k'} \left(2 \frac{Ne^2}{\hbar} \right) \cdot \frac{\omega_{k'k} |x_{k'k}|^2}{\omega_{k'k}^2 - \omega^2}. \tag{27}$$

Using the definition of the coefficients $f_{k'k}$ by relation

$$f_{k'k} = \frac{2m}{\hbar} \omega_{k'k} |x_{k'k}|^2, \tag{28}$$

we get the modified equation (27) as follows:

$$\frac{n^2 - 1}{4\pi} = \frac{Ne^2}{m} \sum_{k'} \frac{f_{k'k}}{\omega_{k'k}^2 - \omega^2}. \tag{29}$$

The last formula should be compared with the classical one:

$$\frac{n^2 - 1}{4\pi} = \frac{e^2}{m} \sum_{k} \frac{N_k}{\omega_k^2 - \omega^2}, \tag{30}$$

where N_k is number of electrons moving with frequency ω_k in the unit volume.

3. The classical description of the Čerenkov radiation

In electrodynamics, a fast moving charged particle in a medium when its speed is faster than the speed of light in this medium produces electromagnetic radiation which is called the Čerenkov radiation. This radiation was first observed experimentally by Čerenkov [3, 4] and theoretically interpreted by Tamm and Frank [34], in the framework of the classical electrodynamics [9].

The charge and current density of electron moving with the velocity **v** and charge e is as it is well known:

$$\varrho = e\delta(\mathbf{x} - \mathbf{v}t) \tag{31}$$

$$\mathbf{j} = e\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t). \tag{32}$$

The equations for the potentials **A**, φ are given by equations [17, 18]

$$\Delta\mathbf{A} - \frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}e\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t) \tag{33}$$

and

$$\Delta' - \frac{\varepsilon}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -4\pi\frac{e}{\varepsilon}\delta(\mathbf{x} - \mathbf{v}t) \tag{34}$$

with the additional Lorentz calibration condition:

$$\mathrm{div}\mathbf{A} + \frac{\varepsilon}{c}\frac{\partial\varphi}{\partial t} = 0, \tag{35}$$

where magnetic permeability $\mu = 1$ and ε is dielectric constant of medium.

After the Fourier transformation the vector potential

$$\frac{1}{(2\pi)^{3/2}}\int \mathbf{A}e^{-i\mathbf{k}\mathbf{x}}d^3x = \mathbf{A_k}; \quad \mathbf{A}(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}}\int \mathbf{A_k}e^{i\mathbf{k}\mathbf{x}}d^3k, \tag{36}$$

we get

$$\Delta\mathbf{A_k} - \frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{A_k}}{\partial t^2} = \frac{4\pi e}{c(2\pi)^{3/2}}\mathbf{v}\int e^{-i\mathbf{k}\mathbf{x}}\delta(\mathbf{x} - \mathbf{v}t)d^3x, \tag{37}$$

or,

$$\Delta\mathbf{A_k} - \frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{A_k}}{\partial t^2} = -\frac{4\pi e\mathbf{v}}{c(2\pi)^{3/2}}e^{-i\mathbf{kv}t}. \tag{38}$$

On the other hand, we have:

$$\Delta\mathbf{A} = \Delta\int\mathbf{A_k}e^{i\mathbf{kx}}d^3k = -\int k^2\mathbf{A_k}e^{i\mathbf{kr}}d^3k, \tag{39}$$

from which we have

$$\Delta\mathbf{A_k} = k^2\mathbf{A_k} \tag{40}$$
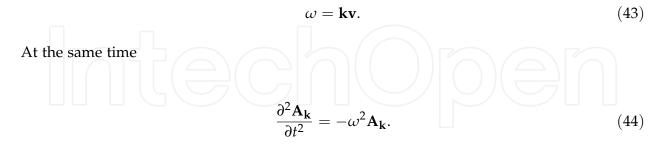
and

$$-k^2\mathbf{A_k} - \frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{A_k}}{\partial t^2} = -\frac{4\pi e\mathbf{v}}{c(2\pi)^{3/2}}e^{-i\mathbf{kv}t}. \tag{41}$$

Formula (41) shows that the dependence $\mathbf{A_k}$ on time is of the form:

$$\mathbf{A_k} \sim e^{-i\mathbf{kv}t} = e^{-i\omega t}, \tag{42}$$

where

$$\omega = \mathbf{kv}. \tag{43}$$

At the same time

$$\frac{\partial^2\mathbf{A_k}}{\partial t^2} = -\omega^2\mathbf{A_k}. \tag{44}$$

We can transcribe eq. (41) in the following form:

$$\mathbf{A_k} = \frac{4\pi e}{c(2\pi)^{3/2}}\frac{\mathbf{v}}{k^2 - \frac{\omega^2\varepsilon}{c^2}}e^{-i\omega t}. \tag{45}$$

By analogy with the formula (45) we can derive the formula concerning the Fourier transform of φ. Or,

$$\varphi_{\mathbf{k}} = \frac{4\pi e}{\varepsilon(2\pi)^{3/2}} \frac{1}{k^2 - \frac{\omega^2 \varepsilon}{c^2}} e^{-i\omega t}. \tag{46}$$

The intensity of the electric field has the Fourier Component as follows:

$$\mathbf{E_k} = -\frac{1}{c}\frac{\partial \mathbf{A_k}}{\partial t} - \operatorname{grad}\varphi_{\mathbf{k}} = \frac{i\omega}{c}\mathbf{A_k} - i\mathbf{k}\varphi_{\mathbf{k}} \quad =$$

$$\frac{4\pi e}{(2\pi)^{3/2}}\left(\frac{\omega \mathbf{v}}{c^2} - \frac{\mathbf{k}}{\varepsilon}\right)\frac{i}{k^2 - \frac{\omega^2\varepsilon}{c^2}}e^{-i\omega t}, \tag{47}$$

from which follows that the intensity of the electric field induced in the dielectric medium is:

$$\mathbf{E} = \frac{ie}{2\pi^2}\int\left(\frac{\omega\mathbf{v}}{c^2} - \frac{\mathbf{k}}{\varepsilon}\right)\frac{e^{i(\mathbf{kx}-\omega t)}}{k^2 - \frac{\omega^2\varepsilon}{c^2}}dk_x dk_y dk_z. \tag{48}$$

The formula (48) gives $\mathbf{E}$ in the form of the moving plane wave in case that we can write

$$\omega = vk_z = vk\cos\Theta \equiv \frac{kc}{n(\omega)}. \tag{49}$$

From the last equation we have:

$$\cos\Theta = \frac{c}{n(\omega)v}. \tag{50}$$

Now, let us chose the direction of the particle motion along the z-axis and let us introduce the cylindrical coordinates putting

$$k^2 = k_x^2 + k_y^2 + k_z^2 = k_z^2 + q^2. \tag{51}$$

At the same time

$$dk_x dk_y dk_z = q\, dq\, d\varphi\, dk_z. \tag{52}$$

Further

$$dk_z = \frac{d\omega}{v} \tag{53}$$

and therefore

$$dk_x dk_y dk_y = q\, dq\, d\varphi\, \frac{d\omega}{v}. \tag{54}$$

In such a way we have for the intensity of the electrical field:

$$\mathbf{E} = \frac{ie}{\pi} \int q\, dq\, d\omega \left(\frac{\omega \mathbf{v}}{c^2} - \frac{\mathbf{k}}{\varepsilon}\right) \frac{e^{i(\mathbf{kx}-\omega t)}}{v\left[q^2 + \omega^2\left(\frac{1}{v^2} - \frac{\varepsilon}{c^2}\right)\right]}, \tag{55}$$

where the φ-integration was already performed. The ω-integration involves both positive and negative frequencies.

The quantity, which is experimentally meaningful, is the energy loss of the moving particle per unit length, or, dW/dz in the prescribed frequency interval $d\omega$. This energy loss is in the relation with the work of force which acts on the particle by the induced electromagnetic field. The work is expressed by the formula:

$$dW = -F_z dz = -e(E_z)_{\mathbf{x}=\mathbf{v}t} dz, \tag{56}$$

where $(E_z)_{\mathbf{x}=\mathbf{v}t}$ is the z-component of the electric intensity at the point where the particle is. The sign minus denotes the physical fact that the force acts against the vector of velocity, or, in the negative direction of the z-axis.

Thus we have:

$$\frac{dW}{dz} = -e\,(E_z)_{\mathbf{x}=\mathbf{v}t} = -\frac{ie^2}{\pi} \int q\, dq\, d\omega \left(e^{i(\mathbf{kx}-\omega t)}\right)_{\mathbf{x}=\mathbf{v}t} \frac{\left(\frac{\omega \mathbf{v}}{c^2} - \frac{k_z}{\varepsilon}\right)}{v\left[q^2 + \omega^2\left(\frac{1}{v^2} - \frac{\varepsilon}{c^2}\right)\right]}$$

$$= \frac{ie^2}{\pi} \int q\, dq\, \omega d\omega\, \frac{1}{\varepsilon}\, \frac{\left(\frac{1}{v^2} - \frac{\varepsilon}{c^2}\right)}{q^2 + \omega^2\left(\frac{1}{v^2} - \frac{\varepsilon}{c^2}\right)}. \tag{57}$$

The energy loss of particle per unit length and in the frequency interval $\omega, \omega + d\omega$ is obviously given as

$$\frac{d^2W}{dzd\omega} = \frac{ie^2\omega}{\pi} \int qdq \frac{\left(\frac{1}{v^2\varepsilon_+} - \frac{1}{c^2}\right)}{q^2 + \omega^2\left(\frac{1}{v^2} - \frac{\varepsilon_+}{c^2}\right)} - \frac{ie^2|\omega|}{\pi} \int qdq \frac{\left(\frac{1}{v^2\varepsilon_-} - \frac{1}{c^2}\right)}{q^2 + \omega^2\left(\frac{1}{v^2} - \frac{\varepsilon_-}{c^2}\right)}. \qquad (58)$$

We introduced in the last formula notation $\varepsilon_+ = \varepsilon(\omega)$ and $\varepsilon_- = \varepsilon(-|\omega|)$ for ε at positive and negative values of ω. We know that in the absorptive dielectric media ε has the imaginary component. This imaginary component of ε is positive for $\omega < 0$ and negative for $\omega > 0$. In fact, the absorption in medium is real effect and it means that $\exp\{-ikx\}$ must correspond to the absorption for $x > 0$ for the arbitrary sign of ω. This experimental requirement determines the sign of the imaginary part of the permitivity of medium.

In formula (58) we can neglect in the numerator the imaginary part of the permitivity and write:

$$\left(\frac{1}{v^2\varepsilon_+} - \frac{1}{c^2}\right) = \left(\frac{1}{v^2\varepsilon_-} - \frac{1}{c^2}\right) = \left(\frac{1}{v^2n^2} - \frac{1}{c^2}\right). \qquad (59)$$

On the other hand, such operation cannot be performed in the denominator which follows from the next text. Let us introduce the new complex quantity:

$$u = q^2 + \omega^2\left(\frac{1}{v^2} - \frac{\varepsilon}{c^2}\right). \qquad (60)$$

Then we have

$$\frac{d^2W}{dxd\omega} = \frac{ie^2\omega}{2\pi}\left(\frac{1}{v^2n^2} - \frac{1}{c^2}\right)\left\{\int_{C_1}\frac{du}{u} - \int_{C_2}\frac{du}{u}\right\}. \qquad (61)$$

In case of the absence of absorption in medium i.e. $\Im\varepsilon = 0$, the formula (61) gives meaningless zero result. However ε and therefore also u has the nonzero imaginary part. It means that the integrals in (61) is considered in the complex plane. The contour of integration is chosen in such a way that it is going in parallel to the real axis above this axis in case of $\Im u > 0$ (it corresponds to $\omega > 0$) and it corresponds to the curve C_1, and under the axis at $\Im u < 0$ (i.e. $\omega < 0$) which corresponds to the curve C_2. The singular point $u = 0$ is avoided along the infinitesimal semi-circles above and under the axis. Thus evidently:

$$\int_{C_1}\frac{du}{u} - \int_{C_2}\frac{du}{u} = \oint\frac{du}{u} = 2\pi i. \qquad (62)$$

The integration was performed as a limiting procedure along the infinitesimal circle with the center in the origin of the coordinate system.

Using eq. (62) we can write the energy loss formula formula (61) in the following simple form:

$$\frac{d^2W}{dxd\omega} = \frac{e^2\omega}{c^2}\left(1 - \frac{c^2}{n^2v^2}\right)\omega.\tag{63}$$

This formula was derived for the first time by Tamm and Frank in year 1937. The fundamental features of the Čerenkov radiation are as follows:

1. The radiation arises only for particle velocity greater than the velocity of light in the dielectric medium is.

2. It depends only on the charge and not on mass of the moving particles

3. The radiation is produced in the visible interval of the light frequencies, i. e. in the ultraviolet part of the frequency spectrum. The radiation does not exists for very short waves.

4. The spectral dependency on the frequency is linear for the homogeneous medium.

5. The radiation generated in the given point of the trajectory spreads on the surface of the cone with the vertex in this point and with the axis identical with the direction of motion of the particle. The vertex angle of the cone is given by the relation $\cos\Theta = c/nv$.

Let us remark that the energy loss of a particle caused by the Čerenkov radiation are approximately equal to 1 % of all energy losses in the condensed matter such as the bremsstrahlung and so on. The fundamental importance of the Čerenkov radiation is in its use for the modern detectors of very speed charged particles in the high energy physics. The detection of the Čerenkov radiation enables to detect not only the existence of the particle, however also the direction of motion and its velocity and according to eq. (63) also its charge.

4. The quantum theory of the Čerenkov effect

Let us start with energetic consideration. So, let us suppose that the initial momentum and energy of electron is $\mathbf{p}$ and E and the final momentum and energy of electron is $\mathbf{p}'$ and E'. The momentum of the emitted photon let be $\hbar\mathbf{k}$. Then, after emission of photon the energy conservation laws are as follows:

$$\sqrt{p^2c^2 + m^2c^4} - \hbar\omega = \sqrt{p'^2c^2 + m^2c^4}\tag{64}$$

$$\mathbf{p} - \hbar\mathbf{k} = \mathbf{p}',\tag{65}$$

where $\mathbf{p}'$ is the momentum of electron after emission of photon. Let us make the quadratical operation of both equations and let us eliminate p'. Then we have:

$$2\hbar p k c^2 \cos \Theta = c^2 \hbar^2 k^2 - (\hbar \omega)^2 + 2\hbar \omega \sqrt{p^2 c^2 + m^2 c^4}. \tag{66}$$

where Θ is the angle between the direction of electron motion and the emission of photon. Putting $\omega = ck/n$ and expressing the momentum of electron in dependence of its velocity

$$\mathbf{v} = \frac{\mathbf{p} c^2}{E}, \tag{67}$$

we get with $\beta = v/c$

$$\cos \Theta = \frac{1}{n\beta} + \frac{\hbar k}{2p}\left(1 - \frac{1}{n^2}\right). \tag{68}$$

Now, following Harris [12], we show, using the second quantization method how to derive the Čerenkov effect in a dielectric medium characterized by its dielectric constant $\varepsilon(\omega)$ and its index of refraction n, which is given by relation $\sqrt{\varepsilon \mu}$ where μ is the magnetic permeability.

The relation between frequency and the wave number in a dielectric medium is

$$\omega = \frac{c}{n}k = \frac{c}{\sqrt{\varepsilon \mu}}k. \tag{69}$$

It was shown [17] that in such dielectric medium the energy of the electromagnetic field is given by the relation

$$U = \int d^3x \frac{1}{8\pi}\left\{|\mathbf{E}|^2 \frac{\partial}{\partial \omega}\omega \varepsilon(\omega) + |\mathbf{B}|^2\right\}. \tag{70}$$

Since

$$\text{rot } \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{71}$$

$$i\mathbf{k} \times \mathbf{E} = \frac{i\omega}{c}\mathbf{B} \tag{72}$$

$$|\mathbf{B}|^2 = \frac{c^2}{\omega^2}|\mathbf{k} \times \mathbf{E}|^2 = \frac{c^2 k^2}{\omega^2}|\mathbf{E}|^2 = \varepsilon|\mathbf{E}|^2, \tag{73}$$

we have:

$$U = \int d^3x \frac{1}{8\pi} |\mathbf{E}|^2 \left[\frac{\partial}{\partial \omega} \omega \varepsilon + \varepsilon \right] = \int d^3x \frac{1}{4\pi} |\mathbf{E}|^2 \frac{1}{2\omega} \frac{\partial \omega^2 \varepsilon}{\partial \omega}. \tag{74}$$

We see that the energy density that a vacuum would have in a vacuum must be corrected by the factor

$$\frac{1}{2\omega} \frac{\partial}{\partial \omega} \omega^2 \varepsilon(\omega) \tag{75}$$

when it moves in a medium of dielectric constant $\varepsilon(\omega)$.

Now, let us consider the Fourier transformation of the electromagnetic potential $\mathbf{A}$:

$$\mathbf{A} = \sum_{\mathbf{k}} \sum_{\sigma=1,2} \left(\frac{2\pi\hbar c^2}{\Omega \omega_k} \right)^{1/2} \mathbf{u}_{\mathbf{k}\sigma} \left\{ a_{\mathbf{k}\sigma}(t) e^{i\mathbf{k}\mathbf{x}} + a_{\mathbf{k}\sigma}^+(t) e^{-i\mathbf{k}\mathbf{x}} \right\}, \tag{76}$$

where the factor

$$\left(\frac{2\pi\hbar c^2}{\Omega \omega_k} \right)^{1/2} \tag{77}$$

is a normalization factor chosen for later convenience. In other words it is chosen in order the energy of the el. magnetic field to be interpreted as the sum of energies of the free harmonic oscillators, or,

$$H = \sum_{k,\sigma} \hbar\omega_k a_{k\sigma}^+ a_{k\sigma}, \tag{78}$$

where a^+, a are creation and annihilation operators fulfilling commutation relations

$$[a_{\mathbf{k}\sigma}, a_{\mathbf{k}'\sigma'}^+] = \delta_{\mathbf{k},\mathbf{k}'} \delta_{\sigma,\sigma'}. \tag{79}$$

We want the total energy rather than just the el.mag. field energy to have the form of eq. (78). And it means we are forced to replace the normalization factor by

$$\left(\frac{2\pi\hbar c^2}{\Omega [\frac{1}{2\omega} \frac{\partial}{\partial \omega} \omega^2 \varepsilon(\omega)]_{\omega_k}} \right)^{1/2}, \tag{80}$$

and it leads to renormalized $\mathbf{A}(\mathbf{x}, t)$ as follows:

$$\mathbf{A}(\mathbf{x}, t) = \sum_{\mathbf{k}, \sigma} \left(\frac{2\pi\hbar c^2}{\Omega[\frac{1}{2\omega} \frac{\partial}{\partial\omega} \omega^2 \varepsilon(\omega)]_{\omega_k}} \right)^{1/2} \mathbf{u}_{\mathbf{k}\sigma} \left\{ a_{\mathbf{k}\sigma} e^{i\mathbf{k}\mathbf{x}} + a_{\mathbf{k}\sigma}^+ e^{-i\mathbf{k}\mathbf{x}} \right\}. \tag{81}$$

The interaction Hamiltonian H' is unchanged except for the change in the normalization factor, or

$$H' = -\frac{e}{mc} \sum_{\mathbf{k}, \sigma} \left(\frac{2\pi\hbar c^2}{\Omega[\frac{1}{2\omega} \frac{\partial}{\partial\omega} \omega^2 \varepsilon(\omega)]_{\omega_k}} \right)^{1/2} \mathbf{p} \cdot \mathbf{u}_{\mathbf{k}\sigma} \left\{ a_{\mathbf{k}\sigma} e^{i\mathbf{k}\mathbf{x}} + a_{\mathbf{k}\sigma}^+ e^{-i\mathbf{k}\mathbf{x}} \right\}. \tag{82}$$

Now, let be the initial state $|i\rangle$ and the final state $|f\rangle$. The transition probability per unit time is given by the first order perturbation term, or

$$\left(\frac{\text{trans prob}}{\text{time}} \right) = \frac{2\pi}{\hbar} |\langle f|i \rangle|^2 \delta(E_f - E_i). \tag{83}$$

We use the last formula to calculate the transition probability per unit time for a free electron of momentum $\hbar\mathbf{q}$ to emit a photon of momentum $\hbar\mathbf{k}$ thereby changing its momentum to $\hbar(\mathbf{q} - \mathbf{k})$.

We find:

$$\left(\frac{\text{trans prob}}{\text{time}} \right)_{\mathbf{q} \to \mathbf{q} - \mathbf{k}} = \frac{2\pi}{\hbar} \left(\frac{e}{mc} \right)^2 \left(\frac{2\pi\hbar c^2}{\Omega[\frac{1}{2\omega} \frac{\partial}{\partial\omega} \omega^2 \varepsilon(\omega)]_{\omega_k}} \right) \quad \times$$

$$|\langle \mathbf{q} - \mathbf{k}|\mathbf{p} \cdot \mathbf{u}_{\mathbf{k}\sigma} e^{-i\mathbf{k}\mathbf{x}}|\mathbf{q} \rangle|^2 \delta \left[\frac{\hbar^2 q^2}{2m} - \frac{\hbar^2}{2m} |\mathbf{q} - \mathbf{k}|^2 - \hbar\omega_k \right]. \tag{84}$$

The matrix element in (84) is just equal to $\hbar\mathbf{q} u_{\mathbf{k}\sigma}$. Letting Θ to be the angle between $\mathbf{q}$ and $\mathbf{k}$ and writing $\mathbf{v} = \hbar\mathbf{q}/m$ be the particle velocity, we find:

$$\left(\frac{\text{trans prob}}{\text{time}} \right)_{\mathbf{q} \to \mathbf{q} - \mathbf{k}} = \left(\frac{4\pi^2 e^2 \hbar^2 |\mathbf{q} \cdot \mathbf{u}_{\mathbf{k}\sigma}|^2}{m^2 \Omega \hbar v k[\frac{1}{2\omega} \frac{\partial}{\partial\omega} \omega^2 \varepsilon(\omega)]_{\omega_k}} \right) \quad \times$$

$$\delta \left[\cos\Theta - \frac{c}{nv} - \frac{\hbar\omega n}{2mcv} \right]. \tag{85}$$

Note that the photon is emitted at an angle to the path of the electron given by

$$\cos\Theta = \frac{c}{nv}\left[1 + \frac{\hbar\omega n^2}{2mc^2}\right].$$ (86)

If the energy of the photon $\hbar\omega$ is much less than the rest mass of the electron mc^2 then $\cos\Theta \approx c/nv$ which gives the classical Čerenkov angle. This can only be satisfied if the velocity of the particle is greater than c/n which is the velocity of the electromagnetic wave in medium. In vacuum where $n = 1$, v can never exceed c and so emission cannot occur.

The quantity of physical interest is the loss of energy per unit length of path of the electron. It is given by the formula:

$$\frac{dW}{dx} = \frac{1}{v}\frac{dW}{dt} = \frac{1}{v}\sum_{\mathbf{k},\sigma}\hbar\omega_{\mathbf{k}}\left(\frac{\text{trans prob}}{\text{time}}\right)_{\mathbf{q}\to\mathbf{q}-\mathbf{k}}.$$ (87)

Using

$$\sum_{\sigma}|\mathbf{q}\cdot\mathbf{u}_{\mathbf{k}}\sigma|^2 = q^2(1 - \cos^2\Theta) = \frac{m^2v^2}{\hbar^2}(1 - \cos^2\Theta)$$ (88)

and (for infinite Ω)

$$\lim_{\Omega\to\infty}\sum_{\mathbf{k}} \rightarrow \frac{\Omega}{(2\pi)^3}\int d^3k$$ (89)

and introducing spherical coordinates in k-space, we find:

$$\frac{dW}{dx} = e^2\int_0^{\infty}kdk\int_{-1}^{1}d(\cos\Theta)\frac{(1 - \cos^2\Theta)\delta\left[\cos\Theta - (\frac{c}{nv}) - \frac{\hbar\omega n}{2mcv}\right]}{\left[\frac{1}{2\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon(\omega))\right]_{nkc}} =$$

$$\frac{e^2}{c^2}\int\frac{\varepsilon(\omega)\omega^2 d\omega}{\left[\frac{1}{2}\frac{\partial}{\partial\omega}(\omega^2\varepsilon(\omega))\right]}\left[1 - \frac{c^2}{n^2v^2}\left(1 + \frac{\hbar\omega n^2}{2mc^2}\right)^2\right].$$ (90)

It is clear from this derivation that the integration over ω is only over those frequencies for that eq. (86) can be satisfied. Since

$$\lim_{\omega\to\infty}n(\omega) \longrightarrow 1,$$ (91)

the range of integration does not go to infinity and the integral is convergent.

It is possible to show that the Čerenkov angle relation for the relativistic particles with spin zero particles is given by the relation:

$$\cos \Theta = \frac{c}{nv} \left[1 + \frac{\hbar \omega}{2mc^2} (n^2 - 1) \sqrt{1 - \frac{v^2}{c^2}} \right].$$ (92)

This expression can be also derived using the so called Duffin-Kemmer equation for particles with spin 0 or 1.

4.1. The Dirac electron

We have seen that in the nonrelativistic situation the appropriate Hamiltonian involved the renormalization term

$$\left(\frac{1}{2} \frac{\partial}{\partial \omega} \omega^2 \varepsilon(\omega) \right),$$ (93)

must be the same also in case of the relativistic situation.

Let us consider the process where an electron of momentum $\hbar(\mathbf{p} + \mathbf{k})$ emits a photon of momentum $\hbar \mathbf{k}$ and polarization σ. The interaction Hamiltonian in case of Dirac electron is

$$H_I = -e \int d^3 \psi^+ \boldsymbol{\alpha} \cdot \mathbf{A} \psi,$$ (94)

where $\boldsymbol{\alpha}$ are the Dirac matrices, $\mathbf{A}$ is the electromagnetic potential [12].

Expanding ψ^+, ψ and $\mathbf{A}$ by the second quantization method, we have for the interacting potential:

$$H_I = \sum_{\mathbf{k},\sigma} \sum_{\mathbf{p},\lambda} \sum_{\lambda'} \left[\frac{2\pi \hbar c^2}{\Omega \frac{1}{2} \frac{\partial}{\partial \omega} (\omega^2 \varepsilon)} \right] \left\{ u^+_{\mathbf{p}+\mathbf{k},\lambda} (\boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma} u_{\mathbf{p}\lambda}) b^+_{\mathbf{p}+\mathbf{k},\lambda'} b_{\mathbf{p}\lambda} a_{\mathbf{k},\lambda} + h.c. \right\},$$ (95)

where h.c. denotes operation of Hermite conjugation.

Then, the transition probability per unit time is:

$$\left(\frac{\text{trans prob}}{\text{time}} \right)_{\mathbf{p}+\mathbf{k},\lambda' \to \mathbf{p},\lambda} = \frac{2\pi}{\hbar} e^2 \left[\frac{2\pi \hbar c^2}{\Omega \frac{1}{2} \frac{\partial}{\partial \omega} (\omega^2 \varepsilon)} \right] |u^+_{\mathbf{p}+\mathbf{k},\lambda} (\boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma} u_{\mathbf{p}\lambda})|^2 \quad \times$$

$$\delta \left[\sqrt{\hbar^2 c^2 |\mathbf{p} + \mathbf{k}|^2 + m^2 c^4} - \sqrt{\hbar^2 c^2 p^2 + m^2 c^4} - \hbar\omega \right] \tag{96}$$

and we may proceed to calculate the energy loss per length as we did in the nonrelativistic case.

There is one modification in this calculation. The sum over final states must include a sum over the final spin states of the electron $\lambda = 1, 2$. We also average over the initial spin states. Thus the general formula is of the form:

$$\frac{dW}{dx} = \frac{1}{v} \frac{1}{2} \sum_{\lambda'=1}^{2} \sum_{\lambda=1}^{2} \sum_{\mathbf{k},\sigma} \hbar\omega_k \left(\frac{\text{trans prob}}{\text{time}} \right). \tag{97}$$

So, we must evaluate

$$\frac{1}{2} \sum_{\lambda'=1}^{2} \sum_{\lambda=1}^{2} (u_{\mathbf{p}+\mathbf{k},\lambda}^{+} \boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma} u_{\mathbf{p}\lambda})(u_{\mathbf{p},\lambda}^{+} \boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma} u_{\mathbf{p}+\mathbf{k}\lambda'}). \tag{98}$$

Let us demonstrate the easy way of calculation of the sums. The first step is to extend the sums over λ' and λ to include all four values. We can do this by noting that

$$\frac{H_{\mathbf{p}} + |E_{\mathbf{p}}|}{2|E_{\mathbf{p}}|} u_{\mathbf{p},\lambda} = \begin{cases} u_{\mathbf{p},\lambda}, & \lambda = 1, 2 \\ 0, & \lambda = 3, 4 \end{cases}, \tag{99}$$

where

$$H_{\mathbf{p}} = \boldsymbol{\alpha} \cdot \mathbf{p} + \beta m c^2. \tag{100}$$

We can use the relation (99) and the similar relation with $u_{\mathbf{p}+\mathbf{k},\lambda'}$ to write eq. (98) as follows:

$$\frac{1}{2} \sum_{\lambda'=1}^{2} \sum_{\lambda=1}^{2} (u_{\mathbf{p}+\mathbf{k},\lambda'}^{+} \boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma}(H_{\mathbf{p}} + |E_{\mathbf{p}}|) u_{\mathbf{p}\lambda}) \quad \times$$

$$\left[u_{\mathbf{p},\lambda}^{+} \boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma}(H_{\mathbf{p}+\mathbf{k}} + |E_{\mathbf{p}+\mathbf{k}}|) u_{\mathbf{p}+\mathbf{k}\lambda'} \right] \frac{1}{4|E_{\mathbf{p}}||E_{\mathbf{p}+\mathbf{k}}|}. \tag{101}$$

Now, let us consider

$$\sum_{\lambda=1}^{4} u_{\mathbf{p},\lambda} u_{\mathbf{p},\lambda}^{+}. \tag{102}$$

Using the relation of completeness, the eq. (102) is just the 4×4 unit matrix. Therefore eq. (101) becomes:

$$\frac{1}{2} \sum_{\lambda'=1}^{4} \left[u_{\mathbf{p}+\mathbf{k},\lambda'}^{+} \boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma}(H_{\mathbf{p}} + |E_{\mathbf{p}}|)\boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma}(H_{\mathbf{p}+\mathbf{k}} + |E_{\mathbf{p}+\mathbf{k}}|)u_{\mathbf{p}+\mathbf{k}\lambda'} \right] \frac{1}{4|E_{\mathbf{p}}||E_{\mathbf{p}+\mathbf{k}}|} =$$

$$\frac{1}{8|E_{\mathbf{p}}||E_{\mathbf{p}+\mathbf{k}}|} \text{Tr} \left[\boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma}(H_{\mathbf{p}} + |E_{\mathbf{p}}|)\boldsymbol{\alpha} \cdot \mathbf{u}_{\mathbf{k}\sigma}(H_{\mathbf{p}+\mathbf{k}} + |E_{\mathbf{p}+\mathbf{k}}|) \right], \tag{103}$$

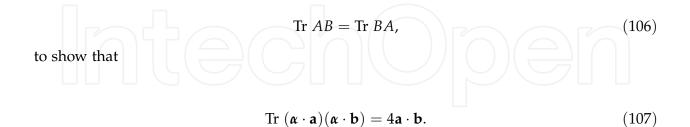where Trace can be evaluated with the certain difficulties.

First, we note that

$$\text{Tr } \alpha_i = \text{Tr } \beta. \tag{104}$$

The trace of a product of any odd number of the matrices α_x, α_y, α_z and β is zero. We may use the following identity:

$$(\boldsymbol{\alpha} \cdot \mathbf{a})(\boldsymbol{\alpha} \cdot \mathbf{b}) = 2(\mathbf{a} \cdot \mathbf{b})1 - (\boldsymbol{\alpha} \cdot \mathbf{b})(\boldsymbol{\alpha} \cdot \mathbf{a}), \tag{105}$$

where $\mathbf{a}$ and $\mathbf{b}$ are arbitrary vectors and

$$\text{Tr } AB = \text{Tr } BA, \tag{106}$$

to show that

$$\text{Tr } (\boldsymbol{\alpha} \cdot \mathbf{a})(\boldsymbol{\alpha} \cdot \mathbf{b}) = 4\mathbf{a} \cdot \mathbf{b}. \tag{107}$$

We can show also that

$$\text{Tr } (\boldsymbol{\alpha} \cdot \mathbf{a})\beta(\boldsymbol{\alpha} \cdot \mathbf{b})\beta = -4\mathbf{a} \cdot \mathbf{b} \tag{108}$$

and

$$\text{Tr } (\boldsymbol{\alpha} \cdot \mathbf{a})(\boldsymbol{\alpha} \cdot \mathbf{b})(\boldsymbol{\alpha} \cdot \mathbf{c})(\boldsymbol{\alpha} \cdot \mathbf{d}) \quad =$$

$$4(\mathbf{a} \cdot \mathbf{b})(\mathbf{c} \cdot \mathbf{d}) - 4(\mathbf{a} \cdot \mathbf{c})4(\mathbf{b} \cdot \mathbf{d}) + 4(\mathbf{a} \cdot \mathbf{d})4(\mathbf{b} \cdot \mathbf{c}) \tag{109}$$

for any three vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$.

Using the formulas with operation Trace, we can evaluate eq. (103). We find:

$$\frac{1}{2} \left\{ 1 - \frac{m^2 c^4}{|E_\mathbf{p}||E_{\mathbf{p+k}}|} + 2 \frac{(\mathbf{u}_{\mathbf{k}\sigma} \cdot \mathbf{v}_1)^2}{c^2} - \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{c^2} \right\}, \tag{110}$$

where we have used

$$\mathbf{v} = \frac{c^2 \mathbf{p}}{E}, \tag{111}$$

and where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities before and after emission of photon. The sum over polarizations can be carried out as was done in eq. (102). The result is that eq. (103) summed over polarization is:

$$\frac{v_1^2}{c^2}(1 - \cos^2 \Theta) + \frac{1}{2} \left\{ 1 - \sqrt{\left(1 - \frac{v_1^2}{c^2}\right)\left(1 - \frac{v_2^2}{c^2}\right)} - \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{c^2} \right\}, \tag{112}$$

where again Θ is the angle between $\mathbf{p}$ and $\mathbf{k}$ and it is given by the formula:

$$\cos \Theta = \frac{c}{nv} \left[1 + \frac{\hbar \omega}{2mc^2}(n^2 - 1)\sqrt{1 - \frac{v^2}{c^2}} \right]. \tag{113}$$

We have used

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{114}$$

to obtain eq. (113) from eq. (110). The second term in eq. (113) is a small correction to the result formed in the spin 0 case.

The momentum of photon is negligible in comparison with the momentum of electron. Then $(\mathbf{v}_1 \approx \mathbf{v}_2)$ and the term in braces vanishes. This will be true in both the classical limit $\hbar \to 0$

and the extremal relativistic limit ($\mathbf{v} \rightarrow c$). We neglect this term in the remainder of the calculation. The rest of the calculation is the similar to the case with the spin 0. The only differences is that eq. (113) must be used instead of eq. (112). The result is

$$\frac{dW}{dx} = \frac{e^2}{c^2} \int \frac{\varepsilon(\omega)\omega^2 d\omega}{\frac{1}{2}\left(\frac{\partial}{\partial\omega}\right)\omega^2\varepsilon(\omega)} \left[1 - \frac{c^2}{n^2 v^2}\left(1 + \frac{\hbar\omega}{2mc^2}(n^2-1)\sqrt{1-\frac{v^2}{c^2}}\right)^2\right]. \tag{115}$$

5. The source theory of the Čerenkov effect

Source theory [6, 30–32] is the theoretical construction which uses quantum-mechanical particle language. Initially it was constructed for description of the particle physics situations occurring in the high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity where the interactions are mediated by photon or graviton respectively.

The basic formula in the source theory is the vacuum to vacuum amplitude [30]:

$$< 0_+|0_- > = e^{\frac{i}{\hbar}W(S)}, \tag{116}$$

where the minus and plus tags on the vacuum symbol are causal labels, referring to any time before and after space-time region where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add.

The electromagnetic field is described by the amplitude (116) with the action

$$W(J) = \frac{1}{2c^2} \int (dx)(dx') J^\mu(x) D_{+\mu\nu}(x - x') J^\nu(x'), \tag{117}$$

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is the charge and current densities. The symbol $D_{+\mu\nu}(x - x')$, is the photon propagator and its explicit form will be determined later.

It may be easy to show that the probability of the persistence of vacuum is given by the following formula [30]:

$$|< 0_+|0_- >|^2 = \exp\{-\frac{2}{\hbar}\text{Im}\,W\} \stackrel{d}{=} \exp\{-\int dt d\omega \frac{P(\omega,t)}{\hbar\omega}\}, \tag{118}$$

where we have introduced the so called power spectral function $P(\omega, t)$. In order to extract this spectral function from Im W, it is necessary to know the explicit form of the photon propagator $D_{+\mu\nu}(x - x')$.

The electromagnetic field is described by the four-potentials $A^\mu(\varphi, \mathbf{A})$ and it is generated by the four-current $J^\mu(c\varrho, \mathbf{J})$ according to the differential equation [30]:

$$(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2})A^\mu = \frac{\mu}{c}(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu)J_\nu \tag{119}$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_+^{\mu\nu} = \frac{\mu}{c}(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu)D_+(x-x'), \tag{120}$$

where $\eta^\mu \equiv (1, \mathbf{0})$, μ is the magnetic permeability of the dielectric medium with the dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_+(x-x')$ was derived by Schwinger et al. [30] in the following form:

$$D_+(x-x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}e^{-i\omega|t-t'|}. \tag{121}$$

Using formulas (117), (118), (120) and (121), we get for the power spectral formula the following expression [30]:

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt'\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos[\omega(t-t')]\times$$

$$\times \left\{\varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x}, t)\cdot\mathbf{J}(\mathbf{x}', t')\right\}. \tag{122}$$

Now, we are prepared to apply the last formula to the situations of the charge moving in the dielectric medium.

The charge and current density of electron moving with the velocity $\mathbf{v}$ and charge e is

$$\varrho = e\delta(\mathbf{x} - \mathbf{v}t) \tag{123}$$

$$\mathbf{J} = e\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t). \tag{124}$$

After insertion of eqs. (123) and (124) in equation for spectral density (122), we find:

$$P(\omega, t) = \frac{e^2}{4\pi c^2} \mu \omega v (1 - \frac{1}{n^2 \beta^2}); \quad n\beta > 1 \tag{125}$$

$$P(\omega, t) = 0; \quad n\beta < 1, \tag{126}$$

where $\beta = v/c$. Relations (125) and (126) determine the Čerenkov spectrum and the threshold condition for the existence of the Čerenkov effect $n\beta = 1$.

6. The Čerenkov effect in the dielectric 2D hexagonal structure

In case of the two dimension situation, the form of equations (119) and (120) is the same with the difference that $\eta^\mu \equiv (1, \mathbf{0})$ has two space components, or $\eta^\mu \equiv (1, 0, 0)$, and the Green function D_+ as the propagator must be determined by the two-dimensional procedure. I other words, the Fourier form of this propagator is with $(dk) = dk^0 d\mathbf{k} = dk^0 dk^1 dk^2 = dk^0 k \, dk \, d\theta$

$$D_+(x - x') = \int \frac{(dk)}{(2\pi)^3} \frac{1}{\mathbf{k}^2 - n^2(k)^2} e^{ik(x - x')}, \tag{127}$$

or, with $R = |\mathbf{x} - \mathbf{x}'|$

$$D_+(x - x') = \frac{1}{(2\pi)^3} \int_0^{2\pi} d\theta \int_0^\infty k \, dk \int_{-\infty}^\infty \frac{d\omega}{c} \frac{e^{ikR\cos\theta - i\omega(t - t')}}{k^2 - \frac{n^2\omega^2}{c^2} - i\varepsilon}. \tag{128}$$

Using $\exp(ikR\cos\theta) = \cos(kR\cos\theta) + i\sin(kR\cos\theta)$ and $(z = kR)$

$$\cos(z\cos\theta) = J_0(z) + 2\sum_{n=1}^\infty (-1)^n J_{2n}(z)\cos 2n\theta \tag{129}$$

and

$$\sin(z\cos\theta) = \sum_{n=1}^\infty (-1)^n J_{2n-1}(z)\cos(2n - 1)\theta, \tag{130}$$

where $J_n(z)$ are the Bessel functions [15], we get after integration over θ:

$$D_+(x - x') = \frac{1}{(2\pi)^2} \int_0^\infty k \, dk \int_{-\infty}^\infty \frac{d\omega}{c} \frac{J_0(kR)}{k^2 - \frac{n^2\omega^2}{c^2} - i\varepsilon} e^{-i\omega(t - t')}, \tag{131}$$

where the Bessel function $J_0(z)$ has the following expansion [15]:

$$J_0(z) = \sum_{s=0}^{\infty} \frac{(-1)^s z^{2s}}{s!s!2^{2s}} \tag{132}$$

The ω-integral in (131) can be performed using the residuum theorem after integration in the complex half ω-plane.

The result of such integration is the propagator D_+ in the following form:

$$D_+(x - x') = \frac{i}{2\pi c} \int_0^{\infty} d\omega J_0\left(\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|\right) e^{-i\omega|t-t'|}. \tag{133}$$

The initial terms in the expansion of the Bessel function with exponent zero is as follows:

$$J_0(z) = 1 - \frac{z^2}{2^2} + \frac{z^4}{2^2 4^2} - \frac{z^6}{2^2 4^2 6^2} + \frac{z^8}{2^2 4^2 6^2 8^2} - \cdots \quad . \tag{134}$$

The spectral formula for the two dimensional Čerenkov radiation is of the analogue of the formula (122), or,

$$P(\omega, t) = -\frac{\omega}{2\pi} \frac{\mu}{n^2} \int d\mathbf{x} d\mathbf{x}' dt' J_0\left(\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|\right) \cos[\omega(t - t')] \times$$

$$\times \left\{ \varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') - \frac{n^2}{c^2} \mathbf{J}(\mathbf{x}, t) \cdot \mathbf{J}(\mathbf{x}', t') \right\}, \tag{135}$$

where the charge density and current involves only two-dimensional velocities and integration is also only two-dimensional with two-dimensional $d\mathbf{x}, d\mathbf{x}'$.

The difference is in the replacing mathematical formulas as follows:

$$\frac{\sin \frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \quad \longrightarrow \quad J_0\left(\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|\right). \tag{136}$$

So, After insertion the quantities (123) and (124) into (135), we get:

$$P(\omega, t) = \frac{e^2}{2\pi} \frac{\mu\omega v}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right) \int dt' J_0\left(\frac{nv\omega}{c}|t - t'|\right) \cos[\omega(t - t')], \quad \beta = v/c, \tag{137}$$

where the t'-integration must be performed. Putting $\tau = t' - t$, we get the final formula:

$$P(\omega, t) = \frac{e^2}{2\pi} \frac{\mu \omega v}{c^2} \left(1 - \frac{1}{n^2 \beta^2}\right) \int_{-\infty}^{\infty} d\tau J_0 \left(n\beta\omega\tau\right) \cos(\omega\tau), \quad \beta = v/c. \tag{138}$$

The integral in formula (138) is involved in the tables of integrals [11] on page 745, number 8. Or,

$$J = \int_0^{\infty} dx J_0 \left(ax\right) \cos(bx) = \frac{1}{\sqrt{a^2 - b^2}}; \quad 0 < b < a,$$

$$J = \infty; \quad a = b, \quad J = 0; \quad 0 < a < b. \tag{139}$$

In our case we have $a = n\beta\omega$ and $b = \omega$. So, the power spectrum of in eq. (138) is as follows with $J_0(-z) = J_0(z)$:

$$P(\omega, t) = \frac{e^2}{\pi} \frac{\mu \omega v}{c^2} \left(1 - \frac{1}{n^2 \beta^2}\right) \frac{2}{\omega \sqrt{n^2 \beta^2 - 1}}, \quad n\beta > 1, \quad \beta = v/c. \tag{140}$$

and

$$P(\omega, t) = 0; \quad n\beta < 1, \tag{141}$$

where condition $n\beta = 1$ is the threshold of the existence of the two-dimensional form of the Čerenkov radiation.

7. The Čerenkov radiation in two-dimensional structure generated by a pulse

Let us consider the electron moving perpendicularly to the 2D sheet in the pane $y - z$ with the index of refraction n and the magnetic permeability μ. Then, the charge density and current density for the charge moving along the axis is ($v > 0$)

$$\varrho = e\delta(vt)\delta(\mathbf{x}) = \frac{e}{v}\delta(t)\delta(\mathbf{x}) \tag{142}$$

$$\mathbf{J} = 0. \tag{143}$$

After insertion of the last formulas into the spectral formula for the Čerenkov radiation (135) with regard to (136), we get

$$P(\omega, t) = \frac{e^2}{2\pi} \frac{\mu\omega}{n^2 v^2} \int dt' \delta(t)\delta(t') J_0(0) \cos[\omega(t - t')], \tag{144}$$

After performing the t and t' integration we get

$$\int dt P(\omega, t) = \frac{e^2}{2\pi} \frac{\mu\omega}{n^2 v^2} J_0(0). \tag{145}$$

The derived formula does not involve the Čerenkov radiation threshold. At the same time the formula does not involve the transition radiation which is generated by the charge when it is moving outside of the sheet. Nevertheless, such radiation can be easily determined by the Ginzburg method [10].

8. The Čerenkov effect with massive photons

The massive electrodynamics in medium can be constructed by generalization of massless electrodynamics to the case with massive photon. In our case it means that we replace only eq. (119) by the following one:

$$\left(\Delta - \frac{\mu\epsilon}{c^2} \frac{\partial^2}{\partial t^2} + \frac{m^2 c^2}{\hbar^2} \right) A^\mu = \frac{\mu}{c} \left(g^{\mu\nu} + \frac{n^2 - 1}{n^2} \eta^\mu \eta^\nu \right) J_\nu, \tag{146}$$

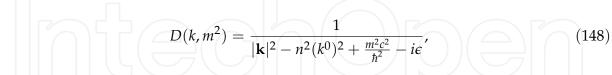where m is mass of photon. The Lorentz gauge of massless photons is conserved also in the massive situation.

In superconductiviy photon is a massive spin 1 particle as a consequence of a broken symmetry of the Landau-Ginzburg Lagrangian. The Meissner effect can be used as a experimental demonstration that photon in a superconductor is a massive particle. In particle physics the situation is analogous to the situation in superconductivity. The masses of particles are also generated by the broken symmetry or in other words by the Higgs mechanism. Massive particles with spin 1 form the analogue of the massive photon.

Kirzhnits and Linde [14] proposed a qualitative analysis wherein they indicated that, as in the Ginzburg-Landau theory of superconductivity, the Meissner effect can also be realized in the Weinberg model. Later, it was shown that the Meissner effect is realizable in renormalizable gauge fields and also in the Weinberg model [35].

We will investigate how the spectrum of the Čerenkov radiation is modified if we suppose the massive photons are generated instead of massless photons. The derived results form an analogue of the situation with massless photons. According to author Pardy [25–27] and Dittrich [6] with the analogy of the massless photon propagator $D(k)$ in the momentum representation

$$D(k) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 - i\epsilon}, \tag{147}$$

the massive photon propagator is of the form (here we introduce $\hbar$ and c):

$$D(k, m^2) = \frac{1}{|\mathbf{k}|^2 - n^2(k^0)^2 + \frac{m^2 c^2}{\hbar^2} - i\epsilon}, \tag{148}$$

where this propagator is derived from an assumption that the photon energetic equation is

$$|\mathbf{k}|^2 - n^2(k^0)^2 = -\frac{m^2 c^2}{\hbar^2}, \tag{149}$$

where n is the parameter of the medium and m is mass of photon in this medium.

From eq. (149) the dispersion law for the massive photons follows:

$$\omega = \frac{c}{n}\sqrt{k^2 + \frac{m^2 c^2}{\hbar^2}}. \tag{150}$$

Let us remark here that such dispersion law is valid not only for the massive photon but also for electromagnetic field in waveguides and electromagnetic field in ionosphere. It means that the corresponding photons are also massive and the theory of massive photons is physically meaningful. It means that also the Čerenkov radiation of massive photons is physically meaningful and it is meaningful to study it.

The validity of eq. (149) can be verified using very simple idea that for $n = 1$ the Einstein equation for mass and energy has to follow. Putting $\mathbf{p} = \hbar \mathbf{k}$, $\hbar k^0 = \hbar(\omega/c) = (E/c)$, we get the Einstein energetic equation

$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4. \tag{151}$$

The propagator for the massive photon is then derived as

$$D_+(x - x', m^2) = \frac{i}{c}\frac{1}{4\pi^2}\int_0^\infty d\omega \, \frac{\sin\left[\frac{n^2\omega^2}{c^2} - \frac{m^2 c^2}{\hbar^2}\right]^{1/2}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|t - t'|}. \tag{152}$$

The function (152) differs from the the original function D_+ by the factor

$$\left(\frac{\omega^2 n^2}{c^2} - \frac{m^2 c^2}{\hbar^2}\right)^{1/2}. \tag{153}$$

From eq. (152) the potentials generated by the massless or massive photons respectively follow. In case of the massless photon, the potential is according to Schwinger defined by the formula:

$$V(\mathbf{x} - \mathbf{x}') = \int_{-\infty}^{\infty} d\tau D_+(\mathbf{x} - \mathbf{x}', \tau) = \int_{-\infty}^{\infty} d\tau \left\{ \frac{i}{c} \frac{1}{4\pi^2} \int_0^{\infty} d\omega \frac{\sin \frac{n\omega}{c} |\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|\tau|} \right\}. \tag{154}$$

The τ-integral can be evaluated using the mathematical formula

$$\int_{-\infty}^{\infty} d\tau \, e^{-i\omega|\tau|} = \frac{2}{i\omega} \tag{155}$$

and the ω-integral can be evaluated using the formula

$$\int_0^{\infty} \frac{\sin ax}{x} dx = \frac{\pi}{2}, \quad \text{for} \quad a > 0. \tag{156}$$

After using eqs. (155) and (156), we get

$$V(\mathbf{x} - \mathbf{x}') = \frac{1}{c} \frac{1}{4\pi} \frac{1}{|\mathbf{x} - \mathbf{x}'|}. \tag{157}$$

In case of the massive photon, the mathematical determination of potential is the analogical to the massless situation only with the difference we use the propagator (152) and the tables of integrals [11]:

$$\int_0^{\infty} \frac{dx}{x} \sin\left(p\sqrt{x^2 - u^2}\right) = \frac{\pi}{2} e^{-pu}. \tag{158}$$

Using this integral we get that the potential generated by the massive photons is

$$V(\mathbf{x} - \mathbf{x}', m^2) = \frac{1}{c} \frac{1}{4\pi} \frac{\exp\left\{-\frac{mcn}{\hbar} |\mathbf{x} - \mathbf{x}'|\right\}}{|\mathbf{x} - \mathbf{x}'|}. \tag{159}$$

If we compare the potentials concerning massive and massless photons, we can deduce that also Čerenkov radiation with massive photons can be generated. So, the determination of the Čerenkov effect with massive photons is physically meaningful.

In case of the massive electromagnetic field in the medium, the action W is given by the following formula:

$$W = \frac{1}{2c^2} \int (dx)(dx') J^\mu(x) D_{+\mu\nu}(x - x', m^2) J^\nu(x'), \tag{160}$$

where

$$D_+^{\mu\nu} = \frac{\mu}{c}[g^{\mu\nu} + (1 - n^{-2})\eta^\mu\eta^\nu] D_+(x - x', m^2), \tag{161}$$

where $\eta^\mu \equiv (1, \mathbf{0})$, $J^\mu \equiv (c\varrho, \mathbf{J})$ is the conserved current, μ is the magnetic permeability of the medium, ϵ is the dielectric constant od the medium and $n = \sqrt{\epsilon\mu}$ is the index of refraction of the medium.

The probability of the persistence of vacuum is of the following form:

$$|\langle 0_+ | 0_- \rangle|^2 = e^{-\frac{2}{\hbar}\mathrm{Im}W}, \tag{162}$$

where Im W is the basis for the definition of the spectral function $P(\omega, t)$ as follows:

$$-\frac{2}{\hbar}\mathrm{Im}W \overset{d}{=} -\int dt d\omega \frac{P(\omega, t)}{\hbar\omega}. \tag{163}$$

Now, if we insert eq. (161) into eq. (160), we get after extracting $P(\omega, t)$ the following general expression for this spectral function:

$$P(\omega, t) = -\frac{\omega}{4\pi^2} \frac{\mu}{n^2} \int d\mathbf{x} d\mathbf{x}' dt' \left[\frac{\sin[\frac{n^2\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2}]^{1/2}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} \right] \times$$

$$\cos[\omega(t - t')][\varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x}, t) \cdot \mathbf{J}(\mathbf{x}', t')]. \tag{164}$$

Now, let us apply the formula (164) in order to get the Čerenkov distribution of massive photons. let as consider a particle of charge Q moving at a constant velocity $\mathbf{v}$. In such a way we can write for the charge density and for the current density:

$$\varrho = Q\delta(\mathbf{x} - \mathbf{v}t), \qquad \mathbf{J} = Q\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t). \tag{165}$$

After insertion of eq. (165) into eq. (164), we get ($v = |\mathbf{v}|$).

$$P(\omega, t) = \frac{Q^2}{4\pi^2}\frac{v\mu\omega}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right)\int_\infty^\infty \frac{d\tau}{\tau}\sin\left(\left[\frac{n^2\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2}\right]^{1/2}v\tau\right)\cos\omega\tau, \tag{166}$$

where we have put $\tau = t' - t, \beta = v/c$.

For $P(\omega, t)$, the situation leads to evaluation of the τ-integral. For this integral we have:

$$\int_{-\infty}^\infty \frac{d\tau}{\tau}\sin\left(\left[\frac{n^2\omega^2}{c^2} - \frac{c^2}{m^2}\right]^{1/2}v\tau\right)\cos\omega\tau = \begin{cases} \pi, \; 0 < m^2 < \frac{\omega^2}{c^2v^2}(n^2\beta^2 - 1) \\ 0, \; m^2 > \frac{\omega^2}{c^2v^2}(n^2\beta^2 - 1). \end{cases} \tag{167}$$

From eq. (167) immediately follows that $m^2 > 0$ implies the Čerenkov threshold $n\beta > 1$. From eq. (166) and (167) we get the spectral formula of the Čerenkov radiation of massive photons in the form:

$$P(\omega, t) = \frac{Q^2}{4\pi}\frac{v\omega\mu}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right) \tag{168}$$

for

$$\omega > \frac{mcv}{\hbar}\frac{1}{\sqrt{n^2\beta^2 - 1}} > 0, \tag{169}$$

and $P(\omega, t) = 0$ for

$$\omega < \frac{mcv}{\hbar}\frac{1}{\sqrt{n^2\beta^2 - 1}}. \tag{170}$$

Using the dispersion law (150) we can write the power spectrum $P(\omega)$ as a function dependent on k^2. Then,

$$P(k^2) = \frac{Q^2}{4\pi}\frac{v\mu}{nc}\sqrt{k^2 + \frac{m^2c^2}{\hbar^2}}\left(1 - \frac{1}{n^2\beta^2}\right); \quad k^2 > \frac{m^2c^2}{\hbar^2}\frac{1}{n^2\beta^2 - 1} \tag{171}$$

and $P(\omega, t) = 0$ for $k^2 < (m^2c^2/\hbar^2)(n^2\beta^2 - 1)^{-1}$.

The most simple way how to get the angle Θ between vectors $\mathbf{k}$ and $\mathbf{p}$ is the use the conservation laws for an energy and momentum.

$$E - \hbar\omega = E' \tag{172}$$

$$\mathbf{p} - \hbar\mathbf{k} = \mathbf{p}', \tag{173}$$

where E and E' are energies of a moving particle before and after act of emission of a photon with energy $\hbar\omega$ and momentum $\hbar k$, and $\mathbf{p}$ and $\mathbf{p}'$ are momenta of the particle before and after emission of the same photon.

If we raise the equations (172) and (173) to the second power and take the difference of these quadratic equations, we can extract the $\cos\Theta$ in the form:

$$\cos\Theta = \frac{1}{n\beta}\left(1 + \frac{m^2c^2}{\hbar^2k^2}\right)^{1/2} + \frac{\hbar k}{2p}\left(1 - \frac{1}{n^2}\right) - \frac{m^2c^2}{2n^2p\hbar k}, \tag{174}$$

which has the correct massless limit. The massless limit also gives the sense of the parameter n which is introduced in the massive situation. We also observe that while in the massless situation the angle of emission depends only on $n\beta$, in case of massive situation it depends also on the wave vector k. It means that the emission of the massive photons are emitted by the Čerenkov mechanism in all space directions.

So, in experiment the Čerenkov production of massive photons can be strictly distinguished from the Čerenkov production of massless photons, or, from the hard production of spin 1 massive particles.

9. Perspective

The article is in some sense the preamble to the any conferences of ideas related to the Čerenkov effect in the graphene-like dielectric structures. At present time, the most attention is devoted in graphene physics with a goal to construct the computers with the artificial intelligence. However, we do not know, a priori, how many discoveries are involved in the investigation of the Čerenkov effect in graphene-like structures.

The information on the Čerenkov effect in graphene-like structures and also the elementary particle interaction with graphene-like structures is necessary not only in the solid state physics, but also in the elementary particle physics in the big laboratories where graphene can form the substantial components of the particle detectors. We hope that these possibilities will be consider in the physical laboratories.

The monolithic structures can be also built into graphene-like structures by addition and re-arrangement of deposit atoms [20]. The repeating patterns can be created to form new carbon allotropes called haeckelites. The introducing such architectonic defects modifies mechanical, electrical, optical and chemical properties of graphene-like structures and it

is not excluded that special haeckelites are superconductive at high temperatures. The unconventional graphene-like materials can be prepared by special technique in order to do revolution in the solid state physics.

While the last century economy growth was based on the inventions in the Edison-Tesla electricity, the economy growth in this century will be obviously based on the graphene-like structures physics. We hope that these perspective ideas will be considered at the universities and in the physical laboratories.

Author details

Miroslav Pardy

⋆ Address all correspondence to: pamir@physics.muni.cz

Department of Physical Electronics, Masaryk University, Brno, Czech Republic

10. References

[1] Berestetzkii, V. B.; Lifshitz, E. M. and Pitaevskii, L. P. (1989). Quantum electrodynamics (Moscow, Nauka). (in Russian).

[2] Yu. V. Bludov, N. M. R. and Peres, M. I. Vasilevskiy, Graphene-based polaritonic crystal Yu. V.,[cond-mat.mes-hall] 17 Apr. 2012

[3] Čerenkov. P. A. (1934), The visible radiation of pure liquids caused by γ-rays, *Comptes Rendus Hebdomaclaires des Scances de l' Academic des Sciences* USSR 2, 451.

[4] Čerenkov, P. A., *C. R. Acad. Sci.* (USSR) 3 (1936), 413.

[5] Dato, A.; Radmilovic, V.; Lee, Z.; Philips, J. and Frenklach, M. (2008). Substrate-free gas phase synthesis of graphene sheets, *Nano Letters*, 8 (7), 2012.

[6] Dittrich, W.(1978). Source methods in quantum field theory *Fortschritte der Physik* 26, 289.

[7] Drummond, N. D.; Zťolyomi, V. and Falko, V. I. (2011). Electrically Tunable band gap in silicene, arXiv:1112.4792v1 [cond-mat.mes-hall] 20 Dec 2011.

[8] Ezawa, M. (2012). Topological insulator and helical zero mode in silicene under inhomogeneous electric field, arXiv:1201.3687v1 [cond-mat.mes-hall] 18 Jan 2012.

[9] Frank, I. M. (1988). The Vavilov-Čerenkov radiation, (Nauka). (in Russian).

[10] Ginzburg, V. L. and Tsytovich, V. N. (1984). Transition radiation and transition scattering, (Moscow, Nauka). (in Russian).

[11] Gradshteyn, S. and Ryzhik, I. M.(1962). Tables of integrals, sums, series and products, (Moscow). (in Russian).

[12] Harris, E. G. (1972). A pedestrian approach to quantum field theory, (John Willey and Sons, Inc., New York, London).

[13] Kane, Ch. L. (2005). Erasing electron mass, *Nature*, 438, November 205, 168.

[14] Kirzhnits, A. D. and Linde, A. D. (1972). Macroscopic consequences of the Weinberg model, *Physics Letters* 42 B, No. 4, 471.

[15] Kuznetsov, D. S. (1962). The special functions, (Moscow). (in Russian).

[16] Landau, L. D. (1937). Zur Teorie der Phasenumwandlungen II., *Phys. Z. Sowjetunion* 11, 26.

[17] Landau, L. D. and Lifshitz, E. M. (1984). Electrodynamics of continuous media, Second revised edition, (Pergamon Press, Oxford, ..).

[18] Levich, V. G.; Vdovin, Yu. A. and Miamlin, V. A. (1962). Course of theoretical Physics, II., (GIFML, Moscow). (in Russsian).

[19] Lozovik, Yu. E.; Merkulov, S. P. and Sokolik, A. A. (2008). Collective electron phenomena in graphene, *Uspekhi Fiz. Nauk*, 178, (7), 758.

[20] Lusk, M. T.; Carr, L. D. (2008). Creation of graphene allotropes using patterned defects, cond-mat.mtrl-sci/0809.3160v2.

[21] Mermin, N. D. (1968). Crystalline order in two dimensions, *Phys. Rev.*, 176.

[22] Novoselov, K. S.; Geim, A.K.; Morozov, S. V.; Jiang, D.; Zhang. Y.; Dubonos, S. V.; Grigorieva, I.V. and Firsov, A.A. (2004). Electric field effect in atomically thin Carbon films, *Science*, 306, 666-669.

[23] Novoselov, K.S.; Geim, A. K.; Morozov, S.V. et al. (2005). Two-dimensional gas of massless Dirac fermions in graphene, *Nature*, 438, 197.

[24] Pardy, M. (1983). Particle production by the Čerenkov mechanism, *Phys. Lett.* 94 A , 30.

[25] Pardy, M. (1989). Finite temperature Čerenkov radiation, *Phys. Lett.* A 134, No.6, 357.

[26] Pardy, M. (1994). The Čerenkov effect with radiative corrections, *Phys. Lett.* B 325, 517.

[27] Pardy, M. (2002). Čerenkov effect with massive photons, *Int. Journal of Theoretical Physics* 41 No. 5, 887.

[28] Pardy, M. (2004). Massive photons and the Volkov solution, *International Journal of Theoretical Physics*, 43(1), 127.

[29] Peierls, R. E. (1934). Bemerkungen über Umwandlungstemperaturen. *Helvetica Physica Acta* 7, 81; ibid. (1935). Quelques proprietes typiques des corpses solides. *Ann. Inst. Henri Poincaré 5*, 177.

[30] Schwinger, J., Tsai, W. Y. and Erber, T. (1976). Classical and quantum theory of synergic synchrotron-Čerenkov radiation, *Annals of Physics* (NY) 96, 303.

[31] Schwinger, J. (1970).Particles, Sources and Fields Vol. I (Addison-Wesley, Reading, Mass.).

[32] Schwinger, J. (1988). Particles, sources and fields, (Addison-Wesley Publishing Company, Inc.).

[33] Sokolov, A. A.; Loskutov, Yu. M. and Ternov, I.M. (1962). Quantum mechanics, (Moscow). (in Russian).

[34] Tamm, I. E. and Frank, I. M. (1937). The coherent radiation of a fast electron in a medium, *Dokl. Akad. Nauk SSSR* 14, 109.

[35] Yildiz, A. (1977). Meissner effect in gauge fields *Phys. Rev. D* 16, No. 12 , 3450.