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Homotopy Perturbation Method to Solve Heat Conduction Equation

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Additional information is available at the end of the chapter

1. Introduction

Fins are extensively used to enhance the heat transfer between a solid surface and its convective, radiative, or convective radiative surface. Finned surfaces are widely used, for instance, for cooling electric transformers, the cylinders of air-craft engines, and other heat transfer equipment. In many applications various heat transfer modes, such as convection, nucleate boiling, transition boiling, and film boiling, the heat transfer coefficient is no longer uniform. A fin with an insulated end has been studied by many investigators [Sen, S. Trinh(1986)]; and [Unal (1987)]. Most of them are immersed in the investigation of single boiling mode on an extended surface. Under these circumstances very recently, [Chang (2005)] applied standard Adomian decomposition method for all possible types of heat transfer modes to investigate a straight fin governed by a power-law-type temperature dependent heat transfer coefficient using 13 terms. [Liu (1995)] found that Adomian method could not always satisfy all its boundaries conditions leading to boundaries errors.

The governing equations for the temperature distribution along the surfaces are nonlinear. In consequence, exact analytic solutions of such nonlinear problems are not available in general and scientists use some approximation techniques to approximate the solutions of nonlinear equations as a series solution such as perturbation method; see [Van Dyke M. (1975)], and Nayfeh A.H. (1973)], and homotopy perturbation method; see [He J. H. (1999, (2000), and(2003)].

In this chapter, we applied HPM to solve the linear and nonlinear equations of heat transfer by conduction in one-dimensional in two slabs of different material and thickness L.

2. The perturbation method

Many physics and engineering problems can be modelled by differential equations. However, it is difficult to obtain closed-form solutions for them, especially for nonlinear

ones. In most cases, only approximate solutions (either analytical ones or numerical ones) can be expected. Perturbation method is one of the well-known methods for solving nonlinear problems analytically.

In general, the perturbation method is valid only for weakly nonlinear problems[Nayfeh (2000)]. For example, consider the following heat transfer problem governed by the nonlinear ordinary differential equation, see [Abbasbandy (2006)]:

$$(1 + \varepsilon u)u' + u = 0, \qquad u(0) = 1 \tag{1}$$

where $\varepsilon > 0$ is a physical parameter, the prime denotes differentiation with respect to the time t. Although the closed-form solution of u(t) is unknown, it is easy to get the exact result $u'(0) = -1/(1+\varepsilon)$, as mentioned by [Abbasbandy (2006)]. Regard that ε as a perturbation quantity, one can write *u(t)* into such a perturbation series

$$u(t) = u_0(t) + \varepsilon u_1(t) + \varepsilon^2 u_2(t) + \varepsilon^3 u_3(t) + \dots\dots\dots\dots \tag{2}$$

Substituting the above expression into (1) and equating the coefficients of the like powers of ε, to get the following linear differential equations

$$u_0' + u_0 = 0, \qquad u_0(0) = 1, \tag{3}$$

$$u_1' + u_1 = -u_0 u_0', \qquad u_1(0) = 0, \tag{4}$$

$$u_2' + u_2 = -(u_0 u_1' + u_1 u_0'), \qquad u_2(0) = 0, \tag{5}$$

$$u_3' + u_3 = -(u_0 u_2' + u_1 u_1' + u_2 u_0'), \qquad u_3(0) = 0, \tag{6}$$

Solving the above equations one by one, one has

$$u_0(t) = e^{-t}$$

$$u_1(t) = e^{-t} - e^{-2t}$$

$$u_2(t) = \frac{1}{2}e^{-t} - 2e^{-2t} + \frac{3}{2}e^{-3t} \tag{7}$$

Thus, we obtain $u(t)$ as a perturbation series

$$u(t) = e^{-t} + \varepsilon(e^{-t} - e^{-2t}) + \varepsilon^2(\frac{1}{2}e^{-t} - 2e^{-2t} + \frac{3}{2}e^{-3t}) + \dots\dots\dots \tag{8}$$

which gives at t = 0 the derivative

$$u'(0) = -1 + \varepsilon - \varepsilon^2 + \varepsilon^3 - \varepsilon^4 + \varepsilon^5 - \varepsilon^6 + \varepsilon^7 - \varepsilon^8 + \varepsilon^9 - \varepsilon^{10} + \dots\dots\dots \tag{9}$$

Obviously, the above series is divergent for $\varepsilon \geq 1$, as shown in Fig. 1. This typical example illustrates that perturbation approximations are valid only for weakly nonlinear problems in general. In view of the work by [Abbasbandy (2006)], the HAM extends a series approximation beyond its initial radius of convergence.
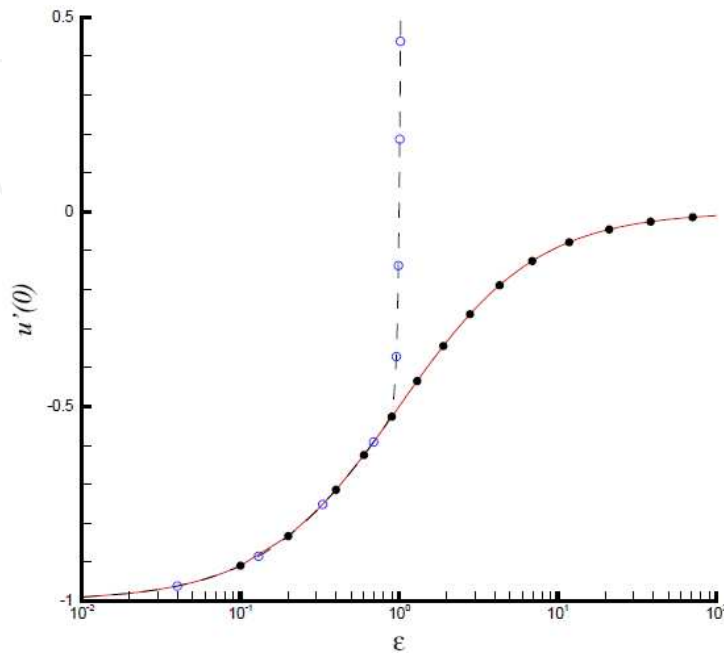

Figure 1. Comparison of the exact and approximate solutions of (1). Solid line: exact solution $u'(0) = -1/(1+\varepsilon)$; Dashed-line: 31th-order perturbation approximation; Hollow symbols: 15th-order approximation given by the HPM; Filled symbols: 15th-order approximation given by the HAM when $h = -1/(1+2\varepsilon)$.

To overcome the restrictions of perturbation techniques, some non-perturbation techniques are proposed, such as the Lyapunov's artificial small parameter method [Lyapunov A.M. (1992)], the δ-expansion method [Karmishin et al(1990)] , the homotopy perturbation method [He H., J.,(1998)], and the variational iteration method (VIM), [He H., J.,(1999)],. Using these non-perturbation methods, one can indeed obtain approximations even if there are no small/large physical parameters. However, the convergence of solution series is not guaranteed. For example, by means of the HPM, we obtain the same and exact approximation of Eq.(1), as the perturbation result in Eq.(9), that is divergent for ε > 1, as shown in Fig.1. ; For details, see [Abbasbandy (2006)]. This example shows the importance of the convergence of solution series for all possible physical parameters. From physical points of view, the convergence of solution series is much more important than whether or not the used analytic method itself is independent of small/large physical parameters. If one does not keep this in mind, some useless results might be obtained. For example, let us consider the following linear differential equation [Ganji et al (2007)]:

$$u_t + u_x = 2u_{xxt}, \qquad x \in R, t > 0 \tag{10}$$

$$u(x,0) = e^{-x} \tag{11}$$

Its exact solution reads

$$u_{exact}(x,t) = e^{-x-t} \tag{12}$$

By means of the homotopy perturbation method, [Ganji et al (2007)] wrote the original equation in the following form:

$$(1-p)\frac{\partial \varphi(x,t:p)}{\partial t} + p[\frac{\partial \varphi(x,t:p)}{\partial t} + \frac{\partial \varphi(x,t:p)}{\partial x} - \frac{\partial^3 \varphi(x,t:p)}{\partial x^2 \partial t}] = 0 \tag{13}$$

subject to the initial condition

$$\varphi(x,0:p) = e^{-x} \tag{14}$$

where $p \in [0;1]$ is an embedding parameter. Then, regarding p as a small parameter, [Ganji et al (2007)] expanded $\varphi(x,t:p)$ in a power series

$$\varphi(x,t:p) = u_0(x,t) + \sum_{m=1}^{+\infty} u_m(x,t).p^m \tag{15}$$

which gives the solution. For $p = 1$, and substitute (15) into the original equation (13) and initial condition in (14), then equating the coefficients of the like powers of p, one can get governing equations and the initial conditions for $u_m(x,t)$. In this way, [Ganji et al (2007)] obtained the mth-order approximation

$$u(x,t) \approx u_0(x,t) + \sum_{k=1}^{m} u_k(x,t) \tag{16}$$

and the 5th-order approximation reads

$$u_{HPM}(x,t) \approx \frac{e^{-x}}{720}[t^6 + 66t^5 + 1470t^4 + 13320t^3 + 47440t^2 + 45360t + 720] \tag{17}$$

However, for any given $x \geq 0$, the above approximation enlarges monotonously to the positive infinity as the time t increases, as shown in Fig.2. Unfortunately, the exact solution monotonously decreases to zero! Let

$$\delta(t) = \left| \frac{u_{exact} - u_{HAM}}{u_{exact}} \right| \tag{18}$$

where $\delta(t)$ denotes the relative error of the HPM approximation (17). As shown in Fig. 2, the relative error $\delta(t)$ monotonously increases very quickly:

In fact, it is easy to find that the HPM series solution (16) is divergent for all x and t except t = 0 which however corresponds to the given initial condition in (11). In other words, the convergence radius of the HPM solution series (17) is zero. It should be emphasized that, the variational iteration method (VIM) obtained exactly the same result as (17) by the 6th

iteration see; [He H. J., (1999)], and [Ganji et al (2007)]. This example illustrates that both of the HPM and the VIM might give divergent approximations. Thus, it is very important to ensure the convergence of solution series obtained.
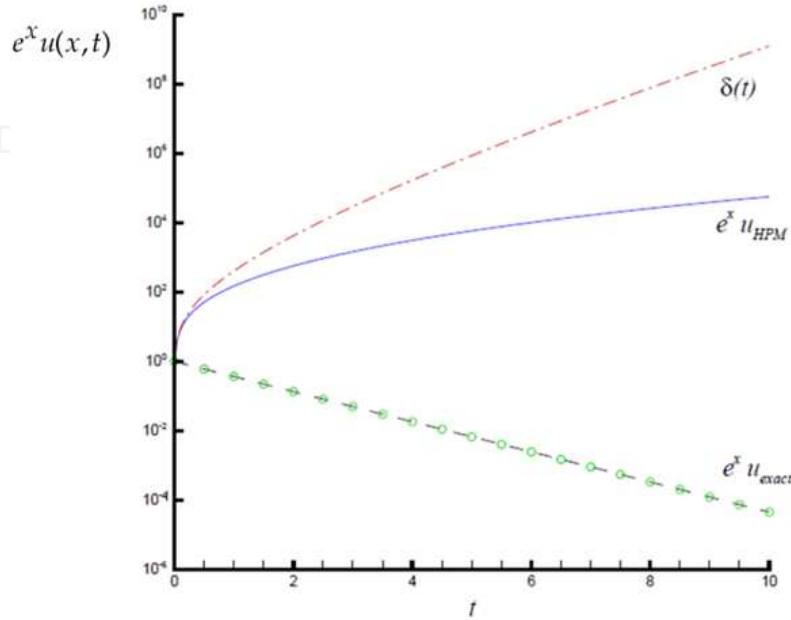

Figure 2. Approximations of (10) given by the homotopy perturbation method. Dashed-line: exact solution(12); Solid line: the 5th-order HPM approximation(17); Dash-dotted line: the relative error δ(t) defined by(18).

3. Outline of Homotopy Perturbation Method (HPM)

The homotopy analysis method (HAM) has been proposed by Liao in his PhD dissertation in [Liao (1992)]. Liao introduced the so-called auxiliary parameter in [Liao (1997a)] to construct the following two-parameter family of equation:

$$(1-p)L(u-u_0) = hpN(u) \tag{19}$$

where u_0 is an initial guess. [Liao (1997a)] pointed out that the convergence of the solution series given by the HAM is determined by h, and thus one can always get a convergent series solution by means of choosing a proper value of h. Using the definition of Taylor series with respect to the embedding parameter p (which is a power series of p), [Liao (1997b)] gave general equations for high-order approximations.

[He J. H. (1999)] followed Dr. Liao's early idea of Homotopy Perturbation Method (HPM) when he constructed the one-parameter family of equation:

$$(1-p)L(u) + pN(u) = 0 \tag{20}$$

where Eq.(20) represented special case of Eq.(19) for convergent solution of (HAM) at $h = -1$. To illustrate the basic ideas of this method, consider the following general nonlinear differential equation [see Ghasemi et al (2010)].

$$A(u) - f(r) = 0, \qquad r \in \Omega \tag{21}$$

With boundary conditions

$$B(u, \frac{\partial u}{\partial n}) = 0, \qquad r \subset \Gamma \tag{22}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, and is the boundary of the domain .

The operator A can be generally divided into linear and nonlinear parts, say L and N. Therefore (21) can be

written as

$$L(u) + N(u) - f(r) = 0 \tag{23}$$

[He (1999)] constructed a homotopy $v(r, p) : \Omega \times [0,1] \rightarrow R$ which satisfies:

$$H(v, p) = (1 - p)[L(v) - L(v_0)] + p[A(v) - f(r)] = 0 \tag{24}$$

where $r \in \Omega$, $p \in [0,1]$ that is called homotopy parameter, and v_0 is an initial approximation of (19). Hence, it is obvious that:

$$H(v, 0) = L(v) - L(v_0) = 0 \tag{25}$$

and

$$H(v, 1) = [A(v) - f(r)] = 0 \tag{26}$$

In topology, $L(v) - L(v_0)$ is called deformation, and $[A(v) - f(r)]$ is called homotopic. The embedding parameter p monotonically increases from zero to unit as the trivial problem $H(v, 0) = 0$ in (25) is continuously deforms the original problem in (26), $H(v, 1) = 0$. The embedding parameter $p \in [0,1]$ can be considered as an expanding parameter. [Nayfeh A.H. (1985)] Apply the perturbation technique due to the fact that $0 \le p \le 1$, can be considered as a small parameter, the solution of (21) or (23) can be assumed as a series in p, as follows:

$$v = v_0 + p v_1 + p^2 v_2 + p^3 v_3 + ... \tag{27}$$

when $p \rightarrow 1$, the approximate solution, i.e.,

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + v_3 + ... \tag{28}$$

The series (28) is convergent for most cases, and the rate of convergence depends on $A(v)$, [He, L. (1999)].

4. Application of Homotopy Perturbation Method HPM

An analytic method for strongly nonlinear problems, namely the homotopy analysis method (HAM) was proposed by Liao in 1992, six years earlier than the homotopy perturbation method by [He H., J.,(1998)], and the variational iteration method by [He H., J.,(1999)]. Different from perturbation techniques, the HAM is valid if a nonlinear problem contains small/large physical parameters.

More importantly, unlike all other analytic techniques, the HAM provides us with a simple way to adjust and control the convergence radius of solution series. Thus, one can always get accurate approximations by means of the HAM. In the next section, HPM is applied to solve the linear and nonlinear equations of heat transfer by conduction in one-dimensional in a slab of thickness (L). [Anwar (2010)] solved the linear and non-linear heat transfer equations by means of HPM.

4.1. Non-Linear Heat transfer equation

Consider the heat transfer equation by conduction in one-dimensional in a slab of thickness L. The governing equation describing the temperature distribution is:

$$\frac{d}{dx}(k\frac{dT}{dx}) = 0, \qquad x \in [0, L] \tag{29}$$

Where the two faces are maintained at uniform temperatures T_1 and T_2 with $T_1 > T_2$ the slab make of a material with temperature dependent thermal conductivity $k = k(T)$; see [Rajabi A.(2007)]. The thermal conductivity k is assumed to vary linearly with temperature, that is:

$$k = k_2[1 + \varepsilon\frac{T - T_2}{T_1 - T_2}] \qquad T(0) = T_1, \qquad T(L) = T_2 \tag{30}$$

where ε is a constant and k_2 is the thermal conductivity at temperature T_2. Introducing the dimensionless quantities

$$\theta = \frac{T - T_2}{T_1 - T_2}, \quad \varepsilon = \frac{k_1 - k_2}{k_2} \quad X = \frac{x}{L} \quad X \in [0, 1]$$

where k_1 is the thermal conductivity at temperature T_1, then (29) reduces to

$$\frac{d^2\theta}{dX^2} + \varepsilon\frac{(\frac{d\theta}{dX})^2}{(1 + \varepsilon\theta)} = 0, \qquad X \in [0, 1] \tag{31}$$

$$\theta(0) = 1, \qquad \theta(1) = 0$$

The problem is formulated by using (19) as:

$$(1 - p)L[\hat{\theta}(X, p) - \theta_0(X)] + pN[\hat{\theta}(X, p)] = 0 \tag{32}$$

Where the Linear operator:

$$L(\theta) = \theta'' \tag{33}$$

and,

$$\theta_{0XX} = 0 \tag{34}$$

from Eq.(31), the initial guess is:

$$\theta_0(X) = 1 - X \tag{35}$$

and the linear operator:

$$L[C_1 X + C_2] = 0 \tag{36}$$

and the nonlinear operator of $\overset{\wedge}{\theta}(X,p)$ is:

$$N[\overset{\wedge}{\theta}(X,p)] = \overset{\wedge}{\theta}(X,p)_{XX} + \varepsilon[\overset{\wedge}{\theta}(X,p).\overset{\wedge}{\theta}(X,p)_{XX} + (\overset{\wedge}{\theta}(X,p)_X)^2] = 0 \tag{37}$$

and:

$$\overset{\wedge}{\theta}(0,p) = 1, \qquad \overset{\wedge}{\theta}(1,p) = 0 \tag{38}$$

where $p \in [0,1]$ is an embedding parameter. For p = 0 and 1, we have

$$\overset{\wedge}{\theta}(X,0) = \theta_0(X) \qquad \overset{\wedge}{\theta}(X,1) = \theta(X) \tag{39}$$

$\theta_0(X)$ tends to $\theta(X)$ as p varies from 0 to 1. Due to Taylor's series expansion:

$$\overset{\wedge}{\theta}(X,p) = \theta_o(X) + \sum_{s=1}^{\infty} \frac{1}{s!} \frac{\partial^s \overset{\wedge}{\theta}(X,p)}{\partial p^s}\Bigg|_{p=0} \tag{40}$$

and the convergence of series (40) is convergent at $p = 1$. Then by using (35) and (36) one obtains

$$\theta(X) = \theta_0(X) + \sum_{s=0}^{\infty} \theta_s(X) \tag{41}$$

For the s-th- order problems, if we first differentiate Eq.(32) s times with respect to p then divide by s! and setting p = 0 we obtain:

$$L[\theta_s(X) - u_s \theta_{s-1}(X)] + [\theta''_{s-1}(X) + \varepsilon \sum_{n=0}^{s-1} (\theta'_{s-1-n}.\theta'_n + \theta_{s-1-n}\theta''_n)] = 0 \tag{42}$$

Where:

$$\theta_s(0) = \theta_s(1) = 0 \tag{43}$$

$$u_s = \begin{cases} 0, & s \leq 1 \\ 1, & s > 1 \end{cases} \tag{44}$$

The general solutions of (42) can be written as:

$$\theta_s(X) = \theta_0(X) + \theta_s^*(X) \tag{45}$$

where $\theta_s^*(X)$ is the particular solution.

The linear non-homogeneous (42) is solved for the order s = 1, 2, 3,..., for s=1, (42) becomes:

$$\frac{d^2\theta_1}{dX^2} + \varepsilon(\frac{d\theta_0}{dX}\frac{d\theta_0}{dX} + \theta_0\frac{d^2\theta_0}{dX^2}) = 0 \qquad \theta_1(0) = 0, \qquad \theta_1(1) = 0 \tag{46}$$

Then

$$\frac{d^2\theta_1}{dX^2} + \varepsilon = 0 \tag{47}$$

the solution of (47) gives :

$$\theta_1 = \frac{\varepsilon}{2}(X - X^2) \tag{48}$$

For s = 2, Eq.(42) becomes:

$$\frac{d^2\theta_2}{dX^2} + \varepsilon(2\frac{d\theta_1}{dX}\frac{d\theta_0}{dX} + \theta_1\frac{d^2\theta_0}{dX^2} + \theta_0\frac{d^2\theta_1}{dX^2}) = 0, \qquad \theta_2(0) = 0, \qquad \theta_2(1) = 0 \tag{49}$$

Solution of (49) gives :

$$\theta_2 = \frac{\varepsilon^2}{2}[2X^2 - X^3 - X] \tag{50}$$

Then, solution of (31) is:

$$\theta(X) = (1 - X) + \frac{\varepsilon}{2}(X - X^2) + \frac{\varepsilon^2}{2}[2X^2 - X^3 - X] \tag{51}$$

Results of θ obtained for different values of ε are presented in Table 1 and Fig. 3. Clearly, for small value for $0 \leq \varepsilon \leq 1$ then (51) is a good approximation to the solution. That means for $\varepsilon = 0$, then $k_1 = k_2$, for $\varepsilon = 1$, then $k_1 = 2k_2$. However, as ε increases, (51) produces

inaccurate divergent results. The results obtained via HPM are compared to those via General Approximation Method GAM obtained by Khan R. A.(2009). For this problem, it is found that HPM produces agreed results compared to GAM.

X	ε =.5	ε =.8	ε =1.0	ε =1.5	ε =2
0.1	0.912375	0.91008	0.9045	0.876375	0.828
0.2	0.824	0.82304	0.816	0.776	0.704
0.3	0.734125	0.73696	0.7315	0.692125	0.616
0.4	0.642	0.64992	0.648	0.618	0.552
0.5	0.546875	0.56	0.5625	0.546875	0.5
0.6	0.448	0.46528	0.472	0.472	0.448
0.7	0.3446249	0.36384	0.3735	0.386625	0.384
0.8	0.2359999	0.2537599	0.2639999	0.2839999	0.2959999
0.9	0.1213749	0.1331199	0.1404999	0.1573749	0.1719999

Table 1. Solutions θ via X for different values of ε.

Special computer program was used as special case, the temperature distribution along a road of length (L = 1 m) when $T_1 = 100\ C^0$ and $T_2 = 50\ C^0$, are presented in Table 2 and Fig.4.

X	ε =.5	ε =.8	ε =1.0	ε =1.5	ε =2
0.0	100	100	100	100	100
0.1	95.61875	95.504	95.225	93.81875	91.4
0.2	91.2	91.152	90.8	88.8	85.2
0.3	86.70625	86.848	86.575	84.60625	80.8
0.4	82.1	82.496	82.4	80.9	77.6
0.5	77.34375	78	78.125	77.34375	75
0.6	72.4	73.264	73.6	73.6	72.4
0.7	67.23125	68.192	68.675	69.33125	69.2
0.8	61.8	62.688	63.2	64.2	64.8
0.9	56.06874	56.656	57.025	57.86874	58.6
1.0	50	50	50	50	50

Table 2. Solutions T via X for different values of ε

4.2. Linear Heat transfer equation

In this section we consider the linear one-dimensional equation of heat transfer by conduction (diffusion equation) [Anderson (1984)]:

$$\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2} = 0 \qquad 0 \le x \le 1,\ t > 0 \tag{52}$$
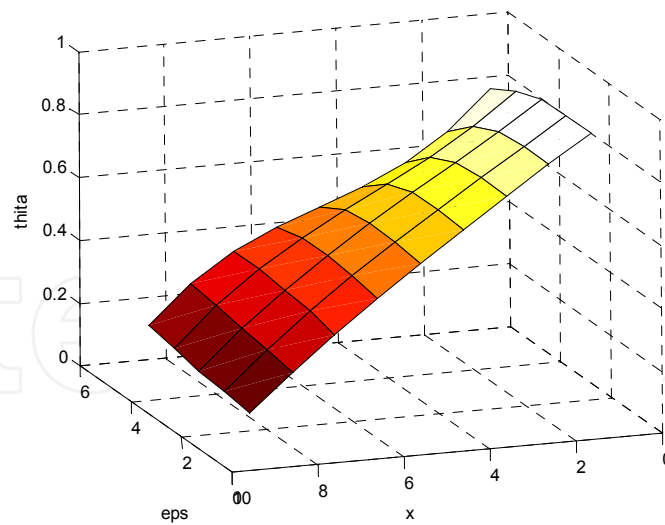
for initial condition

Figure 3. Graphical results θ via X obtained by HPM for different values of ε.
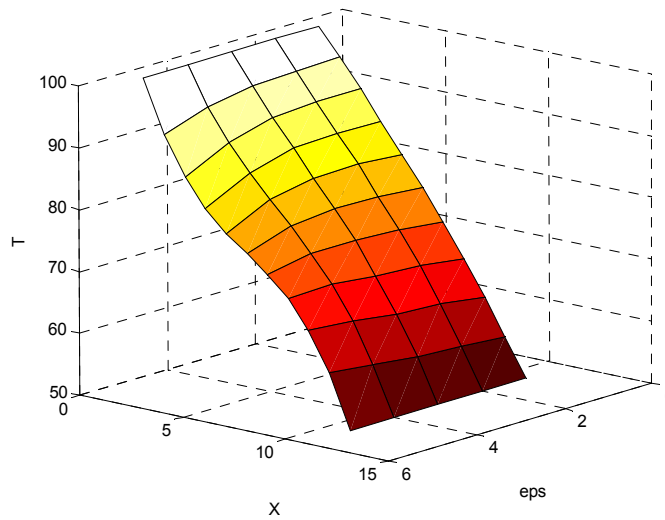

Figure 4. Graphical results of Temperature via X obtained by HPM for different values of ε.

$$T(x,0) = g(x) = \sin(2\pi.x) \tag{53}$$

and boundary condition

$$T(0,t) = T(1,t) = 0 \tag{54}$$

α is thermal conductivity that is assumed constant with temperature. To solve the parabolic partial differential equation (52) using HPM, we consider a correction functional equation as:

$$(1-p)[\frac{\partial T}{\partial t} - \frac{\partial u_0}{\partial t}] + p[\frac{\partial T}{\partial t} - \alpha \frac{\partial^2 T}{\partial x^2}] = 0 \tag{55}$$

Then:

$$\frac{\partial T}{\partial t} - \frac{\partial u_0}{\partial t} - p\frac{\partial u_0}{\partial t} - \alpha p\frac{\partial^2 T}{\partial x^2} = 0 \tag{56}$$

$$\frac{\partial T}{\partial t} - \alpha p\frac{\partial^2 T}{\partial x^2} = 0 \tag{57}$$

$$\frac{\partial(T_0 + pT_1 + p^2T_2 + p^3T_3 + ...)}{\partial t} - \alpha p\frac{\partial^2(T_0 + pT_1 + p^2T_2 + p^3T_3 + ..)}{\partial x^2} = 0 \tag{58}$$

For zeroth order of p:

$$\frac{\partial T_0}{\partial t} = 0 \tag{59}$$

Then $T_0(x,t) = \sin(2\pi.x)$

For first order of p:

$$\frac{\partial T_1}{\partial t} - \alpha\frac{\partial^2 T_0}{\partial x^2} = 0 \tag{60}$$

$$\frac{\partial T_1}{\partial t} + 4\pi^2\alpha\sin(2\pi.x) = 0 \tag{61}$$

$$T_1(x,t) = \sin(2\pi.x) - 4\pi^2\alpha\sin(2\pi.x)t \tag{62}$$

For second order of p:

$$\frac{\partial T_2}{\partial t} - \alpha\frac{\partial^2 T_1}{\partial x^2} = 0 \tag{63}$$

$$T_2(x,t) = \sin(2\pi.x) - 4\pi^2\alpha\sin(2\pi.x)t + 8\pi^4\alpha^2\sin(2\pi.x)t^2 \tag{64}$$

Using equation (56) for other orders of p, we can obtain the following results:

$$T(x,t) = \sin(2\pi.x)[1 - (4\pi^2\alpha.t) + \frac{1}{2}(4\pi^2\alpha.t)^2 - .....] \tag{65}$$

It is obvious that $T(x,t)$ converge to the exact solution as increasing orders of p:

$$T(x,t) = \sin(2\pi.x).\exp(-4\pi^2\alpha.t) \tag{66}$$

Fig.5 and Fig.6 represent the HPM solution T(x, t) for $\alpha = 0.05$, and $\alpha = 0.1$ respectively for $0 \le x \le 1$, $0 \le t \le 0.4$.

X	t=0	t = 0.1	t = 0.2	t = 0.3	t = 0.4
0	0	0	0	0	0
0.05	0.3091373	0.2082383	0.1402717	9.448861E-2	6.364859E-2
0.1	0.5879898	0.3960766	0.2668017	0.1797206	0.1210618
0.15	0.8092399	0.5451131	0.3671944	0.2473463	0.1666153
0.2	0.9512127	0.6407476	0.4316148	0.2907406	0.1958462
0.25	0.9999998	0.6736112	0.4537521	0.3056525	0.205891
0.3	0.9508218	0.6404843	0.4314375	0.2906211	0.1957657
0.35	0.8084964	0.5446123	0.366857	0.247119	0.1664622
0.4	0.5869664	0.3953872	0.2663373	0.1794078	0.1208511
0.45	0.3079342	0.207428	0.1397258	0.0941209	0.0634009
0.5	0	0	0	0	0
0.55	-0.3103399	-0.2090485	-0.1408174	-0.0948562	-6.389621E-2
0.60	-0.5890125	-0.3967655	-0.2672657	-0.1800332	-0.1212724
0.65	-0.8099824	-0.5456133	-0.3675313	-0.2475732	-0.1667681
0.70	-0.9516023	-0.64101	-0.4317916	-0.2908597	-0.1959264
0.75	-0.9999982	-0.6736101	-0.4537514	-0.3056521	-0.2058907
0.80	-0.9504291	-0.6402198	-0.4312593	-0.2905011	-0.1956849
0.85	-0.8077511	-0.5441103	-0.3665189	-0.2468912	-0.1663087
0.90	-0.5859417	-0.3946969	-0.2658723	-0.1790946	-0.1206401
0.95	-0.3067303	-0.206617	-0.1391795	-9.37529E-2	-6.315302E-2
1	0	0	0	0	0

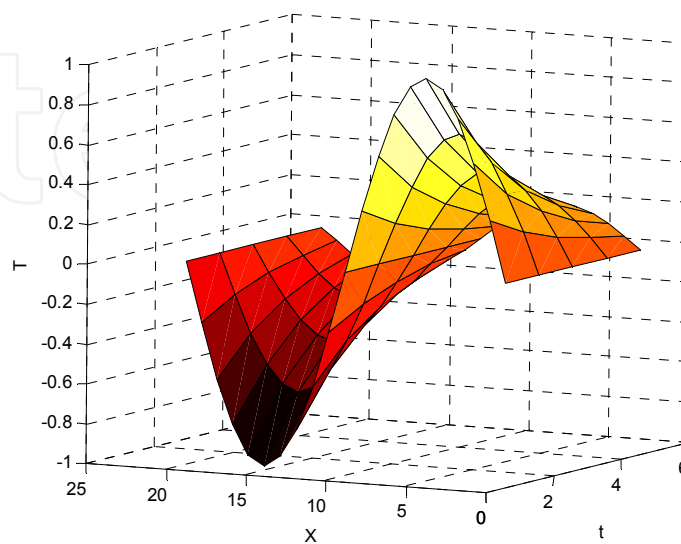**Table 3.** Solution $T(x, t)$ for $0 \le x \le 1$, $0 \le t \le 0.4$ at $\alpha = 0.05$.

Figure 5. Solution $T(x, t)$ for $0 \le x \le 1$, $0 \le t \le 0.4$ and $\alpha = 0.05$.

X	t=0	t = 0.1	t = 0.2	t = 0.3	t = 0.4
0	0	0	0	0	0
0.05	0.3091373	0.2537208	0.2082383	0.1709092	0.1402717
0.1	0.5879898	0.4825858	0.3960766	0.3250752	0.2668017
0.15	0.8092399	0.6641742	0.5451131	0.4473952	0.3671944
0.2	0.9512127	0.7806966	0.6407476	0.5258861	0.4316148
0.25	0.9999998	0.8207381	0.6736112	0.5528585	0.4537521
0.3	0.9508218	0.7803758	0.6404843	0.52567	0.4314375
0.35	0.8084964	0.6635639	0.5446123	0.4469841	0.366857
0.4	0.5869664	0.4817458	0.3953872	0.3245094	0.2663373
0.45	0.3079342	0.2527334	0.207428	0.1702441	0.1397258
0.5	0	0	0	0	0
0.55	-0.3103399	-0.2547078	-0.2090485	-0.1715741	-0.1408174
0.60	-0.5890125	-0.4834251	-0.3967655	-0.3256406	-0.2672657
0.65	-0.8099824	-0.6647836	-0.5456133	-0.4478057	-0.3675313
0.70	-0.9516023	-0.7810164	-0.64101	-0.5261015	-0.4317916
0.75	-0.9999982	-0.8207368	-0.6736101	-0.5528576	-0.4537514
0.80	-0.9504291	-0.7800536	-0.6402198	-0.5254529	-0.4312593
0.85	-0.8077511	-0.6629522	-0.5441103	-0.4465721	-0.3665189
0.90	-0.5859417	-0.4809047	-0.3946969	-0.3239429	-0.2658723
0.95	-0.3067303	-0.2517453	-0.206617	-0.1695785	-0.1391795
1	0	0	0	0	0

Table 4. Solution $T(x, t)$ for $0 \le x \le 1$, $0 \le t \le 0.4$ at $\alpha = 0.1$.
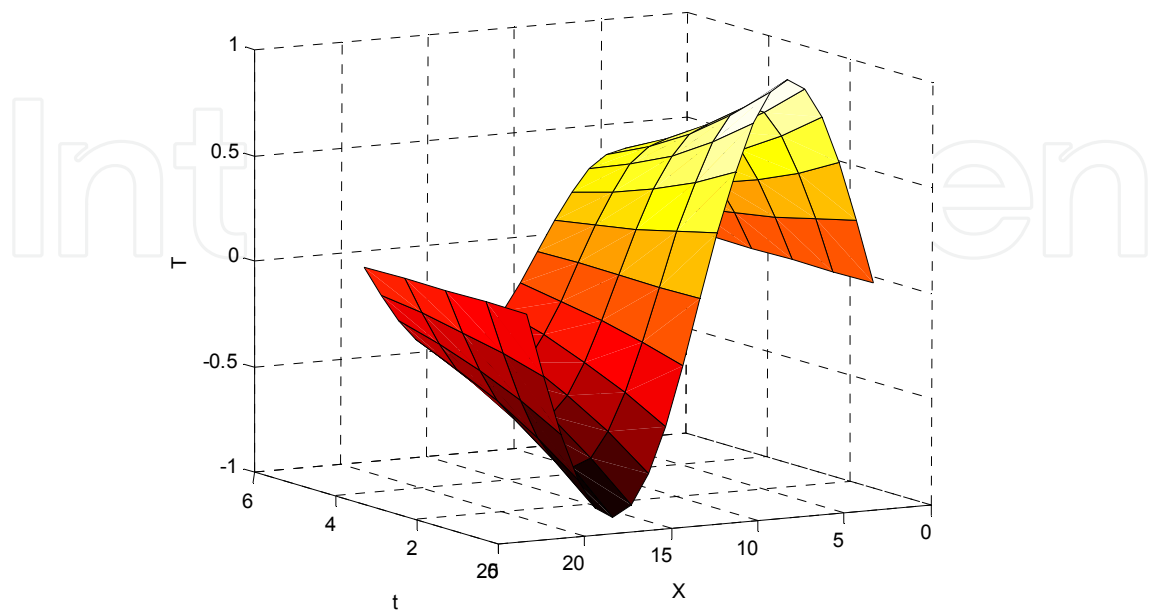

Figure 6. Solution $T(x, t)$ for $0 \le x \le 1$, $0 \le t \le 0.4$ and $\alpha = 0.1$

5. Discussion

For example.1, clearly, for $0 \leq \varepsilon \leq 1$, (51) is a good approximation to the solution. That means for $\varepsilon = 0$, then $k_1 = k_2$, and for $\varepsilon = 1$, then $k_1 = 2k_2$. However, as ε increases, (51) produces inaccurate divergent results. For example 2, (66) is a good approximation to the solution as α values decreased. That means as α increase, (66) produces inaccurate divergent results.

6. Conclusion

Homotopy Perturbation Method HPM is applied to solve the linear and nonlinear partial differential equation. Two numerical simulations are presented to illustrate and confirm the theoretical results. The two problems are about heat transfer by conduction in two slabs. Results obtained by the homotopy perturbation method are presented in tables and figures. Results are compared with those studied by the generalized approximation method by [Sajida et al (2008)]. Homotopy Perturbation Method is considered as effective method in solving partial differential equation.

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