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Chapter 4

A Type-2 Fuzzy Model Based on Three Dimensional Membership Functions for Smart Thresholding in Control Systems

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Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/47798

1. Introduction

This chapter focuses on the basic concepts of novel fuzzy sets, three dimensional (3D) memberships and how they are applied in the design of type-1 and type-2 fuzzy thresholding in control systems. Automatic fuzzification and membership functions shape selection play a crucial role in fuzzy thresholding design and finally determination of outputs via defuzzification. The related methodology and theoretical base will be discussed in depth, using real examples in automatic control (Pavement distress detection and classification). In spatial domain, selection of membership functions is a difficult task. It should be noted that selection of a supper membership function is a golden key. This is one of the major aims of this chapter to introduce a robust method to consider the uncertainty of membership values by using flexible thresholding for controller problems.

In direct approach to fuzzy modeling, deep knowledge of expert plays a key role for membership functions generation. In application, ambiguity of membership function assignment is the main problem with fuzzy sets and systems. So, different fuzzy membership functions may have various impacts on the systems and, then, different thresholds in control problems.

To solve this problem, type II fuzzy thresholding is recommended. The upper and lower membership functions promote this dilemma; however the figure of uncertainty (FOU) has a fixed value that is equal to one, in upper and lower membership function. So, Type-2 fuzzy logic can effectively improve the control characteristics using FOU of the membership functions.

In this chapter, a smart thresholding technique with its application will be presented, which processes threshold as flexible type-2 fuzzy sets. The concept of ultra-fuzziness aims at

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capturing/eliminating the uncertainties within fuzzy systems using regular (type I) fuzzy sets. A measure of 3D ultra-fuzziness is also presented. Several Experimental results are provided in order to demonstrate the usefulness of the proposed approach.

We start with a real problem in control. The simplest method is to visually inspect the pavements and evaluate them by subjective human experts. This approach, however, involves high labor costs and produces unreliable and inconsistent results. Furthermore, it exposes the inspectors to dangerous working conditions on highways. Destructive Testing (DT) and Non Destructive Testing (NDT) are both costly and time consuming. To overcome the limitations of the subjective visual evaluation process; several attempts have been made to develop an automatic procedure (Moghadas Nejad and Zakeri, 2011,a,b,c) and (Daqi et al, 2009).

Most current systems use computer vision and image processing technologies to automate the process. However, due to the irregularities of pavement surfaces, there has been a limited success inaccurately detecting cracks and classifying crack types. In addition, most systems require complex algorithm with high levels of computing power. While many attempts have been made to automatically collect pavement crack data, better approaches are needed to evaluate these automated crack measurement systems (Moghadas Nejad and Zakeri, 2011,a,b,c) and (Daqi et al,2009)

A Hybrid Automatic Expert System (HAES) for automatic distress detection developed, based on complex AI methods (Expert system, Polar Fuzzy Logic) and image processing methods (Wavelet Transform, Inverse Wavelet Transform, 3D Radon Transform, Fast Fourier transform, EH, etc). Fuzzy logic methods are one among favorite and overwhelming architect that used for uncertainty simulations. Type-1 fuzzy sets (T1 FSs) have been successfully used many area such as image processing, pattern recognition, machin learning. (Choi and Rhee, 2009), (Hagras,2004), (Hwang. Rhee, 2004), (Hwang. Rhee, 2007), (John, 2000), (Karnik, J. Mendel, 1999), (Liang et al. 2000), (Liang, J. Mendel, 2001), (Makrehchi, et al. 2003), (Rhee, 2007), (Rhee, Choi, 2007), (Rhee, Hwang, 2001), (Rhee, Hwang, 2002) and (Rhee, Hwang, 2003). Automatic generation of T1 FMFs classified as a interesting and hot research area. many T1 FMF generation models have been tested and various degree of successes achieved (Choi and Rhee, 2009), (Makrehchi, et al.2003), (Medasani et al,1998), (Rhee, and Krishnapuram, 1993), (Wang, 1994) and (Yang and Bose, 2006). Heuristics, histograms, probability, and entropy are good tools to automate the T1 FMFs generation. Several methods under title of AI have been implemented to data sets to generate T1 FMFs. A good classification proposed for T1 FMFs by Choi and Rhee, (2009). Based on this classification, algorithms based on the fuzzy nearest neighbor, back-propagation neural network, fuzzy C-means (FCM), robust agglomerative Gaussian mixture decomposition (RAGMD), and self-organizing feature map (SOFM) were used to generate T1 FMFs must be a considered as FMFs generator. (Choi and Rhee, 2009).

Uncertain meaning, uncertain measurement and noisy data are main causes that we cannot obtain satisfactory results using T1 FSs, therefore in this mode employment of type-2 fuzzy sets (T2 FSs) for managing uncertainty solved the problems (Ensafi & Tizhoosh, 2005), (Choi and Rhee, 2009). Choi and Rhee (2009) stated that, because of the extra degree of freedom (DOF), T2 FSs can control the blurring better than T1 FSs. However, undesirable amount of computations stand in front of extension T2 FSs in vast scale applications. Interval type-2 fuzzy sets (IT2 FSs) are proposed to reduce the complexity (Choi and Rhee, 2009). Many algorithms based on the T2 FMF have been proposed. (Choi and Rhee, 2009), (Hagras,2004), (Hwang. Rhee, 2004), (Hwang. Rhee, 2007), (John, 2000), (Karnik, J. Mendel, 1999), (Liang et al. 2000), (Liang, J. Mendel, 2001), (Makrehchi, et al. 2003), (Rhee, 2007), (Rhee, Choi, 2007), (Rhee, Hwang, 2001), (Rhee, Hwang, 2002) and (Rhee, Hwang, 2003).

In this chapter, we focus on the generation of 3D Polar fuzzy Memberships functions to use in hybrid expert system for systematic pavement distress detection and classification. In particular, we consider 3D polar type-1 fuzzy membership functions (3D T1 PMFs) that are generated from sample images and then developed to 3D polar type-2 fuzzy membership functions (3D T2 PMFs). First, we review three methods based on heuristics, histograms, and interval type-1 fuzzy C-means (IT1 FCM). For each method, the footprint of uncertainty (FOU) is only required to be obtained, since the FOU can completely describe a T1 PMF. We proposed two methods based on 3D domain and then 3D polar under the theory of type 2 fuzzy sets.

This paper is organized as follows.

In Section 2, we briefly review basic concepts and existing methods and background. In Section 3, we managed the IT2 FMF generation methods. In Section 4, concepts of polar fuzzy are discussed and we explain how our proposed IT2 PMF generation methods can be implemented. Section 5 approximate reasoning and fuzzy inference discussed. Finally, Section 6 gives the summary and conclusions.

2. Background

The extension of T1 FSs to T2 FSs can be used to effectively describe uncertainties in situations where the available information is uncertain. T2 FSs consider as a blurred membership function. The blurring used to model the uncertainty of crisp T1 FSs. A T2 FS can be formulated as follow:

$$\overset{=}{A} = \int_{x\in X} \frac{\mu_{\overset{=}{A}(x)}}{x} = \int_{x\in X} \frac{\left[\int_{x\in X} \frac{f_x(u)}{u}\right]}{XJ_x}, XJ_x \subseteq \left[0,1\right] \tag{1}$$

where $f_x\left(u\right)$ is the blurred membership function and J_x is the original membership (Mendel, 2001). Footprint of uncertainty (FOU) is a region between the blurred membership function. The FOU of Ǎ can be expressed by as

$$FOU\left(\overset{=}{A}\right) = \bigcup_{\forall x \epsilon X} J_x = \left\{\left(x,u\right): u\epsilon J_x \subseteq [0,1]\right\} \overline{\mu_{\tilde{A}}} = \overline{FOU}\left(\overset{=}{A}\right) and \text{ by } \underline{\mu_{\tilde{A}}} = \underline{FOU}\left(\overset{=}{A}\right) \tag{2}$$

FOU constructed form upper membership function (UMF) and lower membership function (LMF). (Choi and Rhee, 2009)

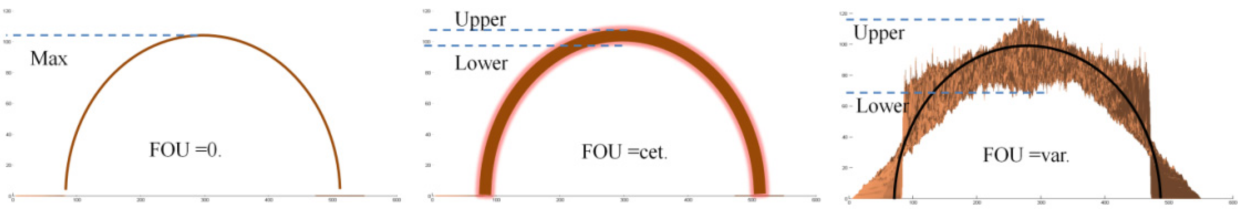

Figure 1. A possible way to construct type II fuzzy sets. The interval between lower and upper membership values (shaded region) should capture the footprint of uncertainty (FOU).

Although T2 FSs may be useful in modeling uncertainty, where T1 FSs cannot, the operations of T2 FSs involve numerous embedded T2 FSs which consider all possible combinations of secondary membership values. Therefore, undesirably large amount of computations may be required. An effectively method to reduce the computational complexity is interval type-2 fuzzy sets (IT2 FSs).

In General, FOU ($\overset{=}{A}$) can be expressed as: (Choi and Rhee, 2009)

$$\mathrm{FOU}\left(\overset{=}{A}\right)=\bigcup\left[\underline{FOU}\left(\overset{=}{A}\right),\overline{FOU}\left(\overset{=}{A}\right)\right]_{\forall x\epsilon X} \tag{3}$$

As a result, IT2 FSs requires only simple interval arithmetic for computing.

3. Automatic MF generators (AMFG)

In this section, we introduce a method for effectively crating IPT-1 FMF automatically from images data. Several methods such as heuristics, histograms, and interval type-2 fuzzy C-means (IT2 FCM) are proposed by (Choi & Rhee, 2009) for generating IT2 FMF automatically from pattern data. Using scaling factor and heuristic T1 FMFs, IT2 FMF simply can be generating. The histogram based method uses suitable parameterized functions chosen to model the smoothed histogram for each class and feature extracted from sample data (Choi and Rhee, 2009),(Hagras,2004), (Hwang and Rhee, 2004), (Hwang and Rhee, 2007), (John, 2000), (Karnik, J. Mendel, 1999), (Liang et al. 2000), (Liang, J. Mendel, 2001), (Makrehchi, et al. 2003), (Rhee, 2007), (Rhee and Choi, 2007), (Hwang and Rhee, 2001), (Rhee, Hwang, 2002) and (Rhee and Hwang, 2003). The IT2 FCM based method uses the derived formulas of the IT2 FMFs in the IT2 FCM algorithm (Hwang and Rhee, 2002). A detailed description of each method is discussed. The heuristic method simply uses an appropriate predefined T1 FMF function, such as triangular, trapezoidal, Gaussian, S, or p function, to name a few, to initially represent the distribution of the pattern data. The following are some frequently used heuristic membership functions. (Choi and Rhee, 2009) Membership functions for fuzzy sets can be constructed by any method exact, heuristic and Meta heuristic, such as triangular, trapezoidal, Gaussian, S, or p function in the domain. Two most important constraints must be considered for selecting a membership functions first, A membership function must be restricted between [0 1] and the next $\mu_A(x)$ must be unique. Four possible membership functions are presented in Fig.2. Where type III and polar are new generation of fuzzy membership function that can be used in several application in the control and classification domains. In the field of pavement management system this new generation of MF play a powerful link between several tools such as multi-resolution methods (wavelet and beyond the wavelet methods), image processing, NN and expert system.

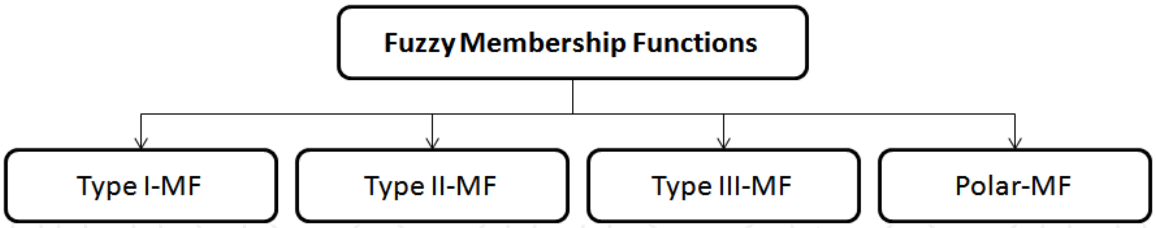

Figure 2. Possible membership functions

A possible membership function can be defined for every category by expert with any tools. For example using image processing techniques and Radon transform, several membership function generated and shows in Fig.3 for pavement cracking distress.

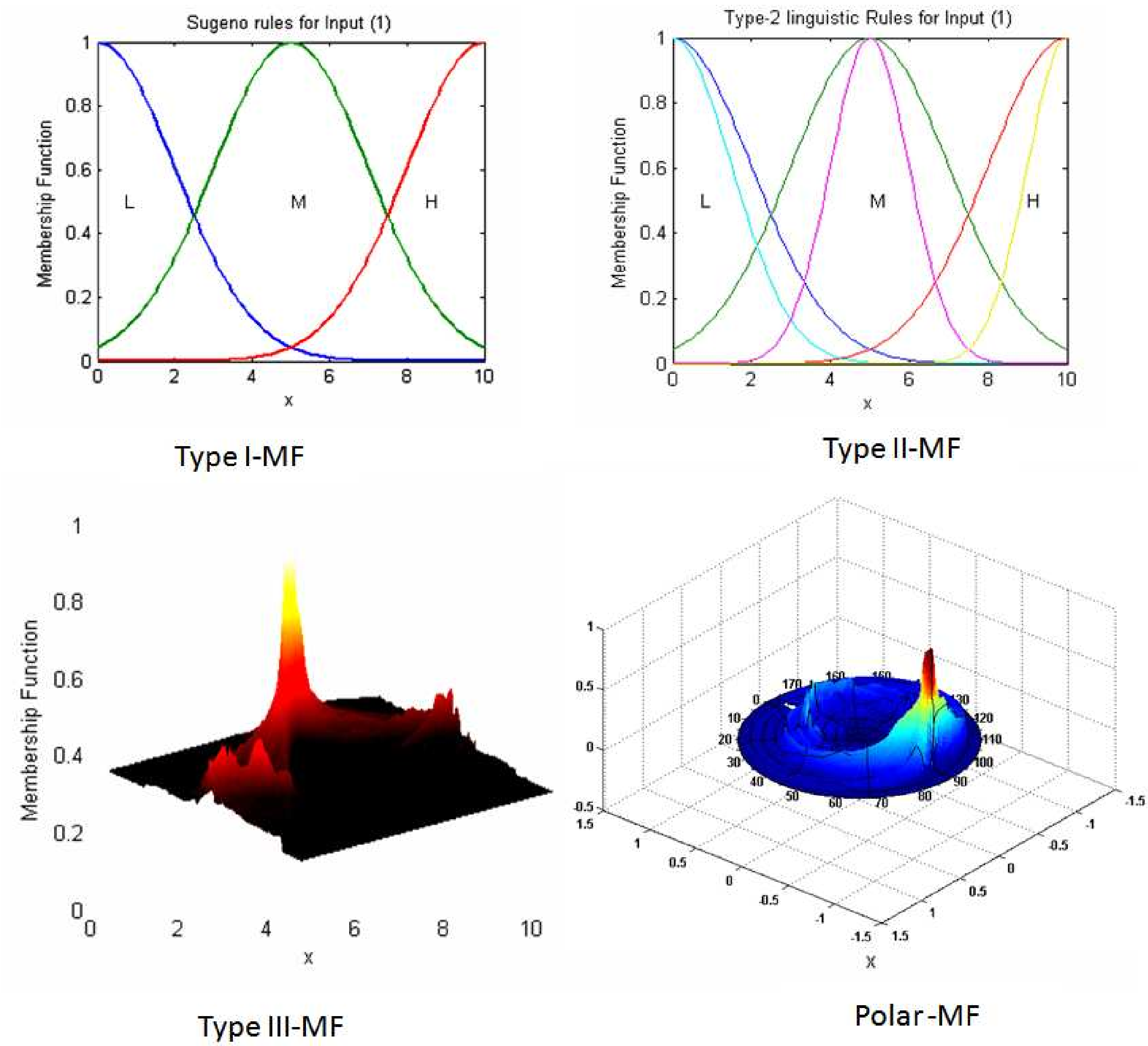

Figure 3. Variety of membership functions

More simple and complex functions can be used under the form of discrete and continues. Generally the ordinary functions categorized in Triangular, Trapezoidal, Γ-membership, S-membership, Logistic, Exponential-like and Gaussian function. Additionally several more advanced membership function which generate by automatic generator introduced. More applications in image processing frequently used heuristic membership functions that can be generally categorized in Table.1.

• Triangular function	$IF\ x \le a, then\ \mu = 0$ $IF\ a \le x \le b, Then\ \mu = (x-a)/(b-a)$ $IF\ a \le x \le b, Then\ \mu = (c-x)/(c-b)$ $IF\ x \ge c, then\ \mu = 0$
• Trapezoidal function	$IF\ x \le a, then\ \mu = 0$ $IF\ a \le x \le b, Then\ \mu = (x-a)/(b-a)$ $IF\ b \le x \le c, Then\ \mu = 1$ $IF\ c \le x \le d, Then\ \mu = (c-x)/(d-c)$ $IF\ x \ge d, then\ \mu = 0$
• Gaussian function	$\mu(x) = \left(e^{-(x-c)^2/2\sigma^2}\right)$
• S-function • $b = (a+b)/2$	$IF\ x \le a, then\ \mu = 0$ $IF\ a \le x \le b, Then\ \mu = 2 \times \left((x-a)/(b-a)\right)^2$ $IF\ a \le x \le b, Then\ \mu = 1 - 2 \times \left((x-a)/(c-a)\right)^2$ $IF\ x \ge c, then\ \mu = 1$
• P-function	$IF\ x \le c, then\ \pi(x) = s(x; c-b, c-\frac{b}{2}, c)$ $IF\ x > c, then\ \pi(x) = s(x; c-b, c-\frac{b}{2}, c)$

Table 1. Heuristic membership functions, (Choi & Rhee,2009)

By control parameters, one can select a various interval pattern. Theses parameters usually trained and learned by experts. Under the title of Control Parameter (α), the UMF of the IT2 FMF and LMF can be designed. The LMF and UMF determined by scaling α *and* β between 0 and 1, which can be also tuned in supervised and unsupervised manner or provided by an expert. Choi & Rhee, (2009) proposed a simple definition for FOU, which categorized in heuristic methods. For feature i

$$\begin{aligned} \mathrm{FOU}\left(\overset{=}{A}\right) &= \bigcup\left[\underline{FOU}\left(\overset{=}{A}\right), \overline{FOU}\left(\overset{=}{A}\right)\right]_{\forall x \epsilon X} \\ &= \bigcup\left[\underline{FOU}\left(\overset{=}{A}\right), \alpha.\underline{FOU}\left(\overset{=}{A}\right)\right]_{\forall x \epsilon X} \quad or \bigcup\left[\beta.\overline{FOU}\left(\overset{=}{A}\right), \overline{FOU}\left(\overset{=}{A}\right)\right]_{\forall x \epsilon X} \end{aligned} \tag{4}$$

$$0 < \alpha , \beta < 1$$

to generalization, Choi & Rhee, (2009) choose the min operation as intersection for obtain the overall FOU by taking intersections of all upper and lower memberships.

$$\begin{aligned} \mathrm{FOU}\left(\bar{\bar{A}}\right) &= \bigcup \left[\underline{FOU}\left(\bar{\bar{A}}\right), \overline{FOU}\left(\bar{\bar{A}}\right) \right]_{\forall x \epsilon X} \\ &= \bigcup \left[\underline{\min\left\{ FOU\left(\bar{\bar{A}}\right)\right\}}, \min\left\{ \alpha . \underline{FOU}\left(\bar{\bar{A}}\right)\right\} \right]_{\forall x \epsilon X} \; or \bigcup \left[min \left\{ \beta . \overline{FOU}\left(\bar{\bar{A}}\right)\right\}, \min\left\{ \overline{FOU}\left(\bar{\bar{A}}\right)\right\} \right]_{\forall x \epsilon X} \end{aligned} \tag{5}$$

where $\overline{FOU}\left(\bar{\bar{A}}\right)$ and $\underline{FOU}\left(\bar{\bar{A}}\right)$ are the minimum UMF and LMF among all UMFs and LMFs, respectively. Heuristic method which proposed Choi & Rhee (2009) is summarized in Fig.4.

Histogram based method (HBM) for membership function generation is another method which is more flexible than heuristic methods. In HBM, distribution of the feature values, have a crucial role in T1 FMF determination elements. Choi & Rhee, (2009) clearly stated that,"membership functions generated from HBM may be considered more suitable for arbitrary distributed data than from heuristics". Based on this theory Choi & Rhee (2009) propose a new method for generation IT2 FMFs. Using smoothed histograms which generated by hyper-cube or triangular window and then normalized, the upper and lower membership function flourished and mapped to real data. Selection a well trained parameters function to model the smoothed histograms has a tangible ramification on performances of MF generator system. To avoid over fitting lowest, the suitable degree of the polynomial function (PF) is stood out as the knee point of error. As a result, HBM FSs requires good estimation of PF.

In our case, as a real example in control, Type, severity and extents of cracking in pavement surface transform in a transform realm to generate a simple features. Simple features can use for generation of T1 FMF. Approximate parameter values such as the number, height, and location of peaks which related to cracking used to determine the optimal parameter values of the function.

Choi & Rhee (2009) considered Gaussian functions as suitable to model the IT2 FMFs (Rhee and Krishnapuram, 1993). They used a heuristic approach (Choi & Rhee, 2009) to obtain the initial parameters. Choi & Rhee (2009) ignored the ones that have small peaks. This means that we have a threshold that it considers as a crisp threshold. The $\underline{FOU}\left(\bar{\bar{A}}\right)$ and $\overline{FOU}\left(\bar{\bar{A}}\right)$ are obtained by again fitting PFs to the smoothed histograms. New again histograms crystallized upper and lower MFs fitted to PF. As dimensional parameters or overall size of problem increase, undesirably become more and more. These complexities arise due to the high process in smoothing and fitting. This is a challenging point that set in motion to

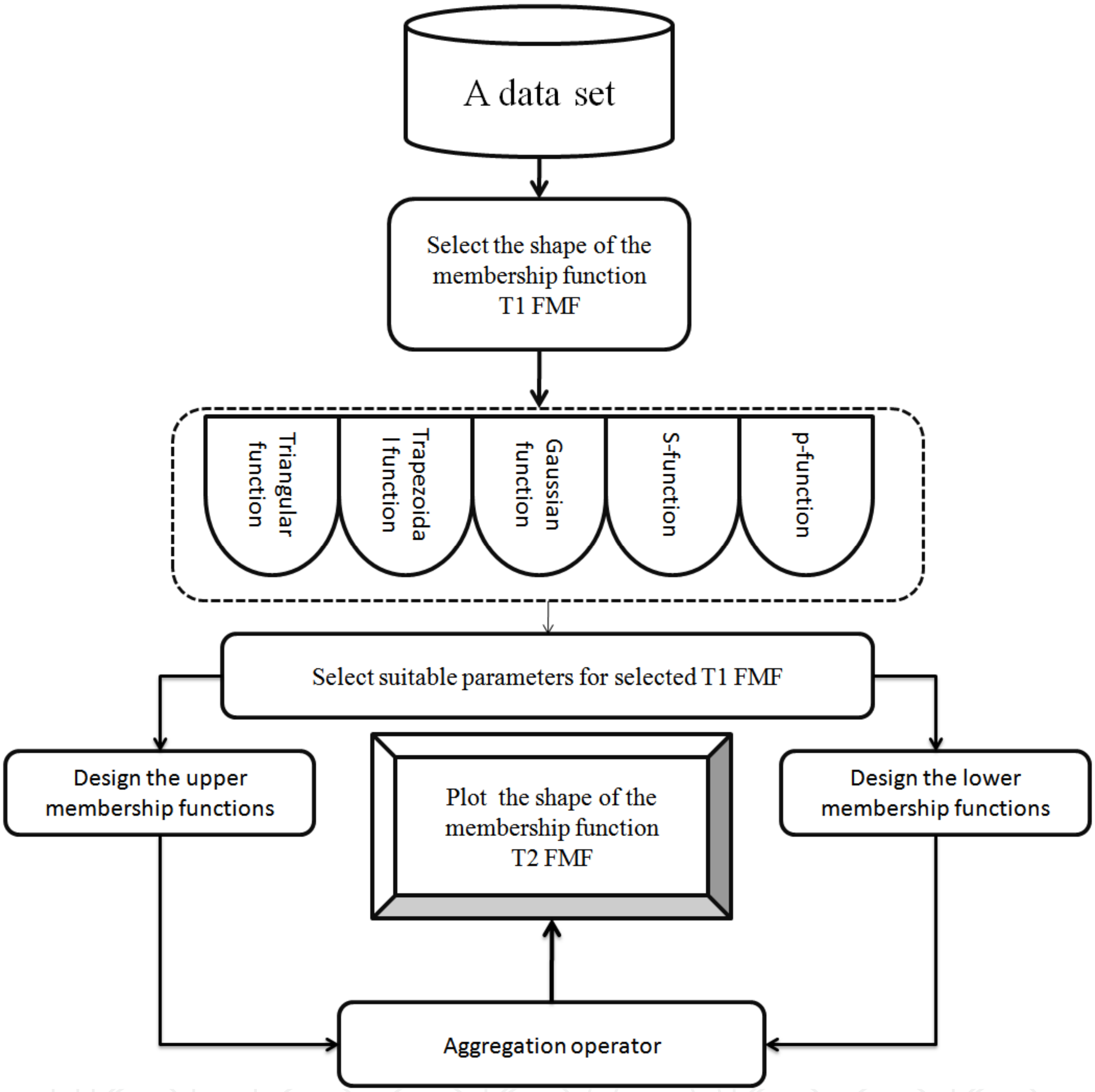

Figure 4. Heuristic based IT2 FMF generation method

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product a new heuristic to handle computational load. Choi & Rhee (2009) proposed two steep methods **1)** calculate one-dimensional 1DUMF and 1DLMF for HBM. **2)** Obtain the overall $\underline{FOU}\left(\overset{=}{A}\right)$ and $\overline{FOU}\left(\overset{=}{A}\right)$ by Intersections operation. Intersections operation which proposed for this aggregation expressed as

$$\mathrm{FOU}\left(\overset{=}{A}\right)=\bigcup\left[\underline{FOU}\left(\overset{=}{A}\right),\overline{FOU}\left(\overset{=}{A}\right)\right]_{\forall x\epsilon X}=\left[\underline{min_i\left\{FOU\left(\overset{=}{A}\right)\right\}},min_i\left\{\overline{FOU}\left(\overset{=}{A}\right)\right\}\right]_{\forall x\epsilon X} \qquad (6)$$

where $\overline{FOU}\left(\overset{=}{A}\right)$ is the UMF, $\underline{FOU\left(\overset{=}{A}\right)}$ is the LMF, and i is the feature's number. From our points of view, the main contributions of Choi & Rhee's methods are developing in membership's generation. These methods enable them to transfer the knowledge when expert facing with N dimensional features. These methods are applicable for images realm. We assert that this contribution is valuable. Nevertheless we would like to highlight that high process in discrete smoothing and fitting(first 1DUMF and 1DLMF calculation and then aggregation) faced us to problem to products an effective MF generator. Heuristic method to generate T2 FMF's, which proposed Choi & Rhee (2009) is summarized in Fig.5.

Choi & Rhee (2009) considered **fuzzy C-means (FCM)** functions to model the IT2 FMFs (Hwang and Rhee, 2007) (Choi & Rhee, 2009). The fuzzifier m in FCM, can be fired as a membership generator. IT2 FCM based method proposed by Choi & Rhee (2009). They stated that, "*Due to the constraint on the memberships we cannot design this region with any particular single value of fuzzifier m to be used in the FCM*". IT2 FCM algorithm was proposed to solving this problem (Hwang and Rhee, 2007). Indeed they products a simple dynamic fuzzifuyer AMFG to generating the Membership function. According to IT2 FCM, two fuzzifier m_1, m_2 are employed to control the blurring area in fuzzy domain. The proposed IT2 FMF in IT2 FCM expressed as (Choi & Rhee, 2009).

$$J_x = \left\{ \left(x,u\right) : u \epsilon [\underline{FOU}\left(\overset{=}{A}\right), \overline{FOU}\left(\overset{=}{A}\right)] \right\} \tag{7}$$

$$\overline{FOU}\left(\overset{=}{A}\right) = \left\{ \begin{array}{l} IF \dfrac{1}{\sum_{K=1}^{C}\left(\dfrac{d_{ij}}{d_{ik}}\right)^{\frac{2}{(m_1-1)}}} > \dfrac{1}{\sum_{K=1}^{C}\left(\dfrac{d_{ij}}{d_{ik}}\right)^{\frac{2}{(m_2-1)}}} \\ THEN\ \overline{FOU}\left(\overset{=}{A}\right) = \dfrac{1}{\sum_{K=1}^{C}\left(\dfrac{d_{ij}}{d_{ik}}\right)^{\frac{2}{(m_1-1)}}} \\ ELSE\ \overline{FOU}\left(\overset{=}{A}\right) = \dfrac{1}{\sum_{K=1}^{C}\left(\dfrac{d_{ij}}{d_{ik}}\right)^{\frac{2}{(m_2-1)}}} \end{array} \right\}, \tag{8}$$

and

$$\underline{FOU}\left(\overset{=}{A}\right)=\left\{\begin{array}{l} IF \frac{1}{\sum_{K=1}^{C}\left(\frac{d_{ij}}{d_{ik}}\right)^{\frac{2}{(m_1-1)}}} \leq \frac{1}{\sum_{K=1}^{C}\left(\frac{d_{ij}}{d_{ik}}\right)^{\frac{2}{(m_2-1)}}} \\ THEN\ \overline{FOU}\left(\overset{=}{A}\right)=\frac{1}{\sum_{K=1}^{C}\left(\frac{d_{ij}}{d_{ik}}\right)^{\frac{2}{(m_1-1)}}} \\ ELSE\ \overline{FOU}\left(\overset{=}{A}\right)=\frac{1}{\sum_{K=1}^{C}\left(\frac{d_{ij}}{d_{ik}}\right)^{\frac{2}{(m_2-1)}}} \end{array}\right\} \tag{9}$$

However IT2 FCM for updating cluster prototypes requires type-reduction. Using type-2 fuzzy operations therefore is essential. The crisp center obtained mean of centers of defuzzification as the centroid obtained by the type-reduction according Eq.10

$$V_{\tilde{x}}=\left[\underline{C},\overline{C}\right]=\frac{\sum_{\overline{FOU}\left(\overset{=}{A}\right)\in J_{x1}}\cdots\sum_{\overline{FOU}\left(\overset{=}{A}\right)\in J_{x1}}1}{\frac{\sum_{i=1}^{N}x_i\overline{FOU}\left(\overset{=}{A}\right)_i^m}{\sum_{i=1}^{N}\overline{FOU}\left(\overset{=}{A}\right)_i^m}} \tag{10}$$

The UMF and LMF for class k and input pattern xj can be expressed by modifying

$$d_{kj}^{*}=min_p\left\{d(x_j,V_{\tilde{x}}{}^{k})\right\},p\epsilon\left\{n_k\right\} \tag{11}$$

Based on Choi & Rhee's (2009) method the membership values for the UMFs and LMFs are based on m_1 and m_2 and they are highly dependent on value selection of threshold which is itself considered crisp. Choi & Rhee's (2009) stated that IT2 FCM can desirably control the uncertainty that is quite simple handle all features of high dimensional problems. Their heuristic method summarized in Fig 6.

The accuracy of IT2 FCM highly dependent on fuzzifiers selection. These parameters have significant role in designing the FOU for a data set. In general, select unsuitable fuzzifier worth poor clustering. (Choi & Rhee's, 2009)

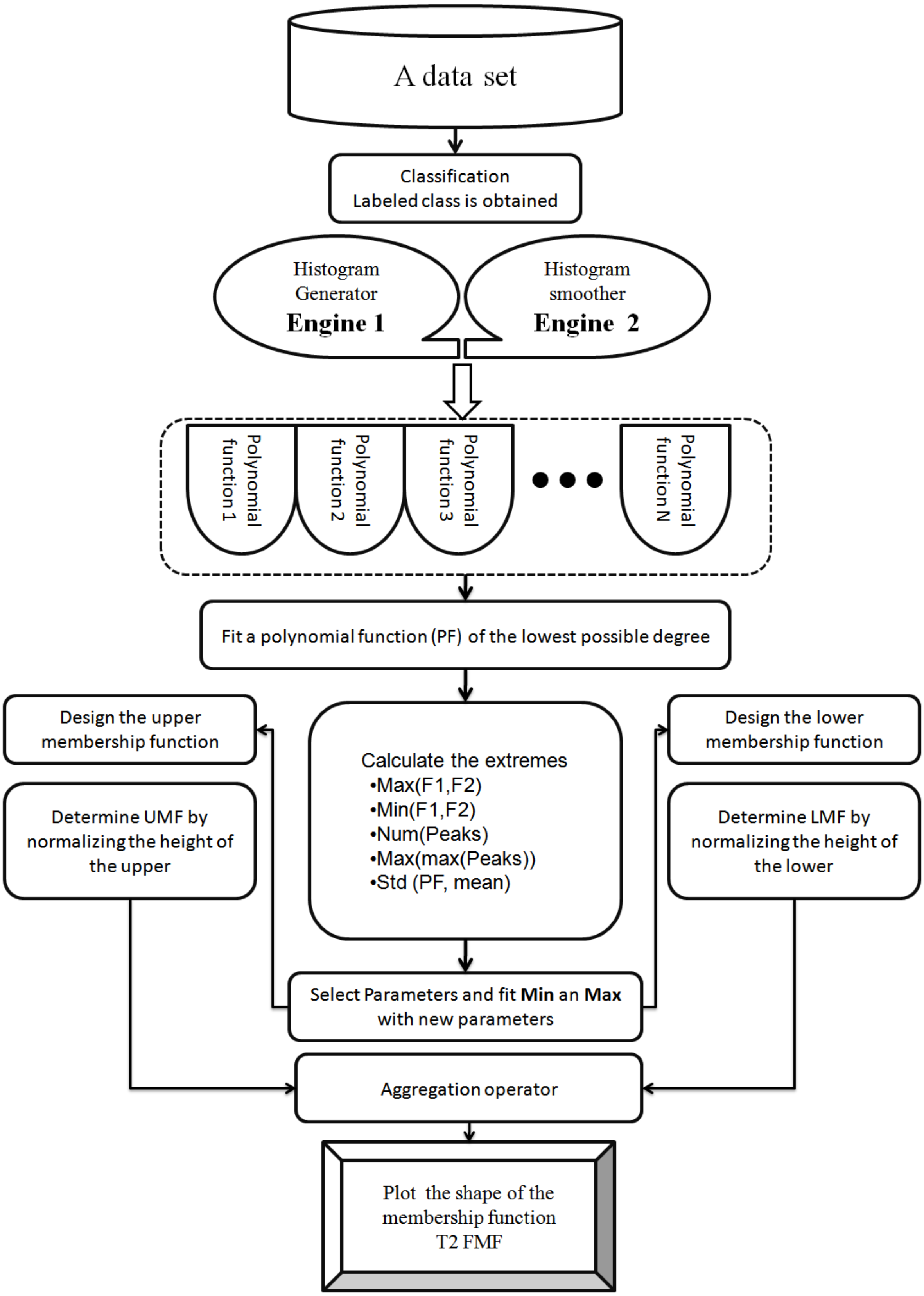

Figure 5. Heuristic based IT2 FMF generation method

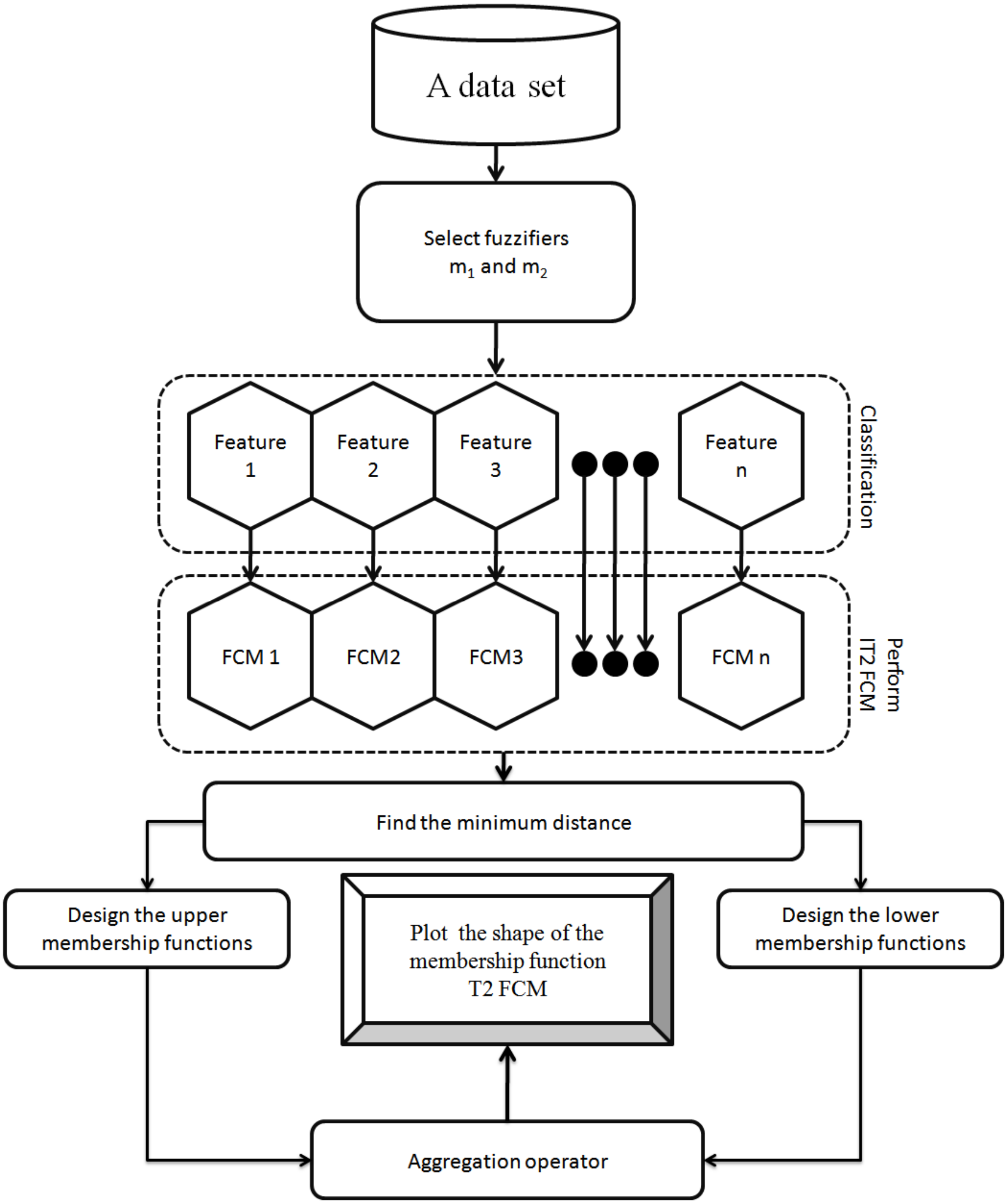

Figure 6. Heuristic based IT2 FCM generation method

4. Interval type-2 Polar Fuzzy Method (IT2 PFM)

4.1. Type III-MF

The Interval type-2 Polar Fuzzy Method (IT2 PFM) algorithm was proposed to automatically control the uncertainty. In this section, we proposed an intelligent IT2 FMF generator agent. First, the IT2 FMF algorithm introduced, and then our IT2 FPM based method are described. We selected Cubic Smoothing Spline (CSS) for generate the upper and lower membership functions because of non-uniform illumination of the Three Dimensional Memebership Functions (3DMFs). In the Type-2 domain, the estimation of the $3DMF_u$ and $3DMF_L$ are exanimate from the fitting of a cubic smoothing Spline,(Mora et al.,2011) to the 3DMF*(x,y)*. The select CSS is a special class of Spline that can capture the low 3DMF value that limited the non-uniformity of the 3DMF (Culpin, 1986). The fitting objective is to minimize the equation.

$$M = P.\sum_{y=1}^{m}\sum_{x=1}^{n}(f(x,y)-s(x,y))^2+(1-p)\iint(D^2S(x,y))^2dxdy, \tag{12}$$

where, this equation include two parts:

- *Compactness:* measures how close the spline is to the data that reflect to the summation term which weighed by the smoothing factor p,
- *Smoothness:* measures the spline smoothness using its second derivative that reflect to the integral term weighed by (1 - p).

The smoothing factor p, controls the balance between being an interpolating spline crossing all data points (with p = 1) and being a strictly smooth Spline (with p = 0). The smoothing spline *f* minimizes when

$$\left[\left[P\sum_{j=1}^{n}\left(\omega(j)\left|y(:,j)-f(x(j))\right|^2\right)\right]+\left[(1-p)\int\lambda(t)\left|D^2f(t)\right|^2dt.\right]\right] \tag{13}$$

where, $|z|_2$ represent for the sum of the squares of all the entries of *n*, N and M is the number of entries of x and y, and the integral is over the smallest interval containing all the entries of x and y. The default value for the weight vector w in the error measure is ones (size(x)). The default value for the piecewise constant weight function λ in the roughness measure is the constant function 1. Further, D_2f denotes the second derivative of the function f. The default value for the smoothing parameter, p, is chosen in dependence on the given data sites x and y (Pal and Bezdek, 1994). The smoothing parameter determines the relative weight to place on the contradictory demands of having f be smooth vs having f be close to the data. For p = 0, f is the least-squares straight line fit to the data, while, at the other extreme, i.e., for p = 1, f is the variational, or 'natural' cubic spline interpolant. As p moves from 0 to 1, the smoothing spline changes from one extreme to the other. (See Fig. 7)

The interesting range for p is often near $1/\left(1 + \left(min(N,M)\right)^3/600\right)$, with *h* the average spacing of the data sites, and it is in this range that the default value for *p* is chosen. For uniformly spaced data, one would expect a close following of the data for $p = 1/(1 + (min(N,M))^3/6000)$ and some satisfactory smoothing for

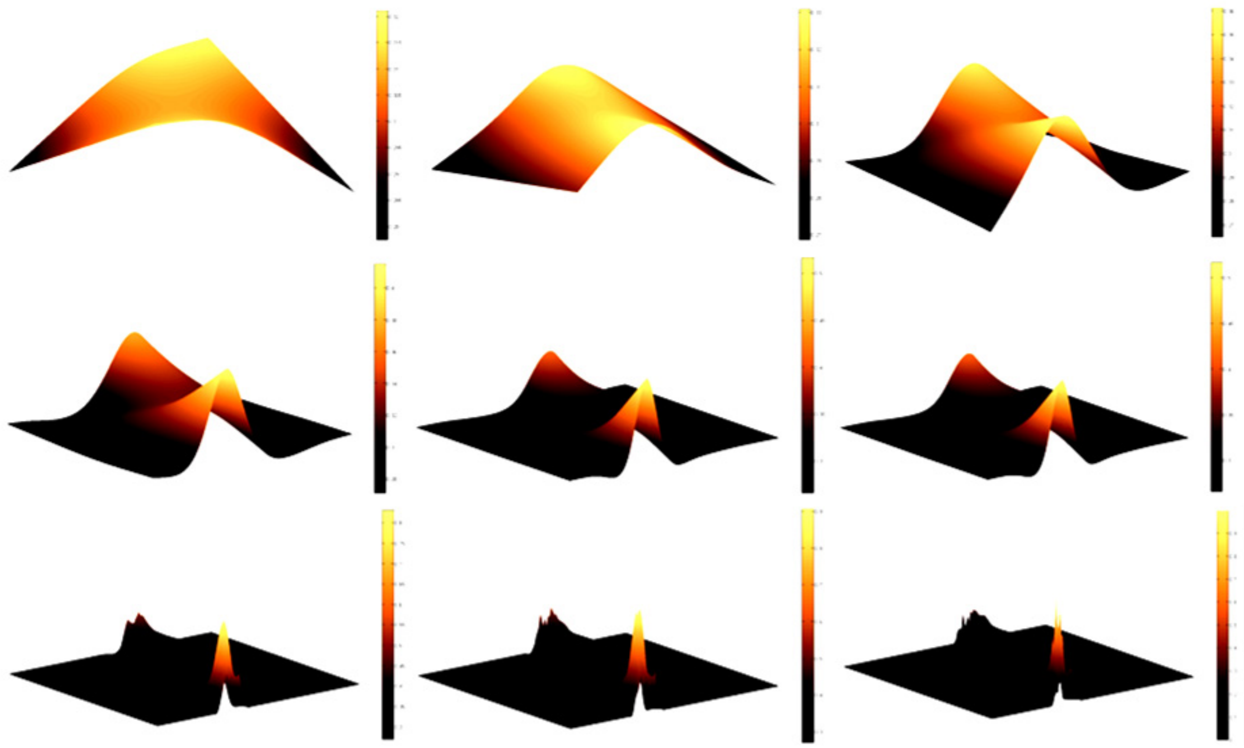

Figure 7. As p moves from 0 to 1, the smoothing spline changes from one extreme to the other.

$$p = 1/\left(1 + \left(min\left(N,M\right)\right)^3 / 60\right).p > 1 \tag{14}$$

can be input, but this leads to a smoothing spline even rougher than the variational cubic spline interpolate (Pal and Bezdek,1994).

$$p = \left(\frac{1}{1+\frac{(min(N,M))^3}{600}}\right), p_U = \left(\frac{1}{1+\frac{(min(N,M))^3}{\alpha \times 600}}\right)_U, p_L = \left(\frac{1}{1+\frac{(min(N,M))^3}{\beta \times 600}}\right)_L \tag{15}$$

A reference smoothing factor $\left(p = 1e-4\right)$ was obtained empirically for constructed MF in upper bound and $\left(p = 0.93e-5\right)$ for constructed MF in lower bound. For example, in the case of image thresholding, After testing several thresholds, the general rule can be extract from 3DRT thresholds for upper and lower bounds by good selection of α and β.

4.1.1. A measure of ultrafuzziness

Using a simple method, we turned ultrafuzzy to the 3DRT fuzzy set. According a type II membership function, MF must be in [0,1]. One can be taking out the normalization form 3DMF using division every point by max 3DRT.

$$RTMF_{(i,j)} = \left(\left[v_{(3DRT)(i,j)}^{\;h} \right] \Big/ max\left[v_{(3DRT)(i,j)}^{\;h} \right] \right)^{\frac{1}{h}}, \tag{16}$$

$$\mu_{(i,j)} = \left(\left[v_{(3DRT)(i,j)}^{\;h} \right] \Big/ max\left[v_{(3DRT)(i,j)}^{\;h} \right] \right)^{\frac{1}{h}} + H, \tag{17}$$

and

$$GR_{(i,j)} = \frac{1}{(MN)^h} \sum_{j=1}^{N} \sum_{i=1}^{M} \left(\left[v_{(3DRT)(i,j)}^{\;h} \right] \Big/ max\left[v_{(3DRT)(i,j)}^{\;h} \right] \right)^{\frac{1}{h}}, \tag{18}$$

where, M and N denotes the size of 3DMF platform,H is high platform, $h\epsilon(1,\infty)$ and $v_{(3DRT)(i,j)}^{\;h}$ is 3DRT value in the position *i* and *j*. Select a bigger h is worth a more enhanced distress for example in pavement distress detection and classification problem and smoother noisy background (see Fig.8). In order to define a type II fuzzy set, one can define a type I fuzzy set and assign upper and lower membership degrees to each element to (re)construct the footprint of uncertainty (Fig. 9) (Tizhoosh,2005). For example, when Radon Transform is applied to wavelet modulus, a distress (crack) is transformed into a peak in radon domain. Originally, every distress reflects to RT and has different intensity in 3DMF histograms. For example mean of 3DRT have variety range. According to the above Eq. 18 the max GR must be equal 1. To extend the fuzzy membership to type II fuzzy sets, ultrafuzziness should be zero, if the MF can be selected without any ambiguous such as type I. The amount of ultrafuzziness will increase by rising uncertainly bound.

The extreme case of maximal ultrafuzziness, equal 1, is worth to completely vagueness. pal and bezdek (1994) had extensive reviewed well known fuzziness index, two general classes proposed by them was additive and multiplicative class (Pal and Bezdek,1994). Based on kufmann's Index of fuzziness for a set $A \in r_n(x)$,

$$H_{ka}(A) = \left(\frac{2}{n^k} \right) d(A, A^{near}) \tag{19}$$

Where, $k \in R^+$, d is a metric, and A^{near} is the crisp set close to the A. generally, based on d, weigh of k determined. The $d(A, A^{near})$ and linear or quadratic $H_{ka}(A)$ cab be determined by q-norms,

$$d(A,A^{near}) = \left(\sum_{i=1}^{n}\left|\mu_i - \mu_{A^{near},i}\right|^q\right)^{1/q} \quad H_{ka}(q,A) = \left(\frac{2}{n^{\frac{1}{q}}}\right)\left(\sum_{i=1}^{n}\left|\mu_i - \mu_{A^{near},i}\right|^q\right)^{1/q} \tag{20}$$

Where $q \in [1,\infty)$. On the other side, Tizhoosh developed a simple ultrafuzziness index for the special case as fallow (Tizhoosh, 2005),

$$\tilde{\gamma}\left(\tilde{A}\right) = \frac{1}{MN}\sum_{i=1}^{M-1}\sum_{j=1}^{N-1}\left[\mu_U(g_{ij}) - \mu_L(g_{ij})\right] \tag{21}$$

where $\mu_U\left(g\right) = \left[\mu_A\left(g\right)\right]^{1/\alpha}$ and $\mu_U\left(g\right) = \left[\mu_A\left(g\right)\right]^{1/\alpha}$, $\alpha \in (1,2]$. and in general term it present as follow,

$$\tilde{\gamma}\left(\tilde{A}\right) = \frac{1}{MN}\sum_{g=0}^{L-1}\left[\mu_U(g_{ij}) - \mu_L(g_{ij})\right] \times h(g) \tag{22}$$

Based on these theory and with respect to Tizhoosh's method (Tizhoosh, 2005),for developing ultrafuzziness on 2D data, a measure of ultrafuzziness $\tilde{\gamma}$ for a platform 3DMF with M*N sets, surf 3DMF and the membership function $\mu_{\tilde{A}(i,j)}$ can be developed as follows:

$$\tilde{\gamma}\left(A\right) = \left(\frac{1}{\left(MN\right)^{\frac{1}{q}}}\right)\left[\sum_{j=1}^{M-1}\sum_{i=1}^{N-1}\left|\mu_{u(i,j)} - \mu_{L(i,j)}\right|^q\right]^{1/q}$$

$$\frac{\partial}{\partial T}\tilde{\gamma}\left(RT_{(i,j)}\right) = \frac{\partial}{\partial T}\left(\frac{1}{\left(MN\right)^{\frac{1}{q}}}\right)\left[\sum_{j=1}^{M-1}\sum_{i=1}^{N-1}\left|\mu_{u((i,j),T)} - \mu_{L((i,j),T)}\right|^q\right]^{1/q} = 0 \tag{23}$$

This basic definition relies on the assumption that the singletons sitting on the FOU are all equal in height (which is the reason why the interval-based type II is used), (Tizhoosh, 2005). The variation in the space can be measured by this method, therefore the new Index introduced in three dimensional domain of FOU for 3D fuzzy sets, (3DFOU). This method can resolve the problems about the ultrafuzziness index -"uncertainty (FOU) has a constant value, that equals one, in all the intervals of the universe of discourse" (Ioannis et al., 2008) - using introducing flexible membership function across the intervals path (see Fig 9, 10).

Similarly, We are evaluated, proposed method, based four conditions *Minimum ultrafuzziness, Maximum ultrafuzziness, Equal ultrafuzziness* and *Reduced ultrafuzziness* that every measure of fuzziness should satisfy, which introduced by Kaufmann (Kaufmann, 1975). In a similar way, we established that the new index is qualified for measure of ultrafuzziness in 3D domain with these conditions.

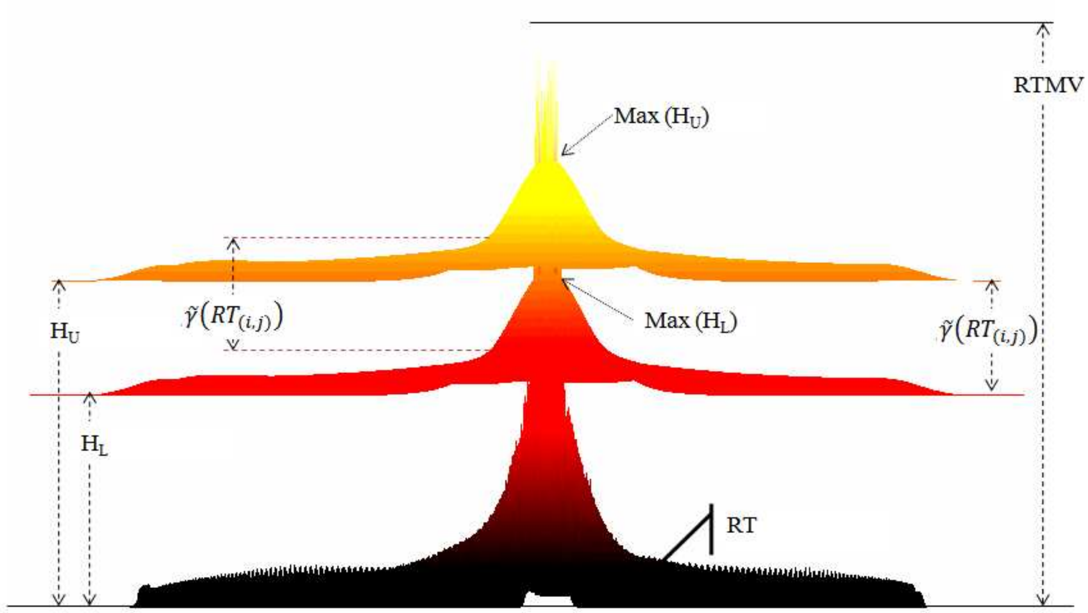

Figure 8. Basic rules for construction 3D fuzzy type II memebership function.

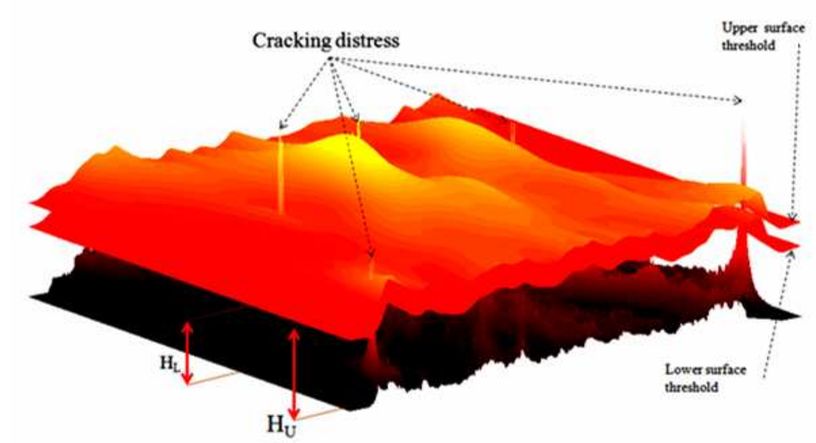

Figure 9. Three dimensional domain of FOU for 3D fuzzy sets, (3DFOU)

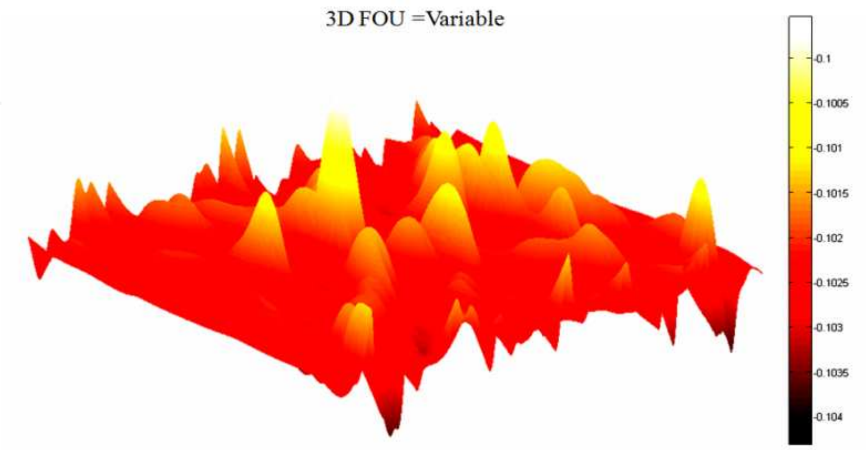

Figure 10. Example of Three FOU for 3D fuzzy sets, (3DFOU) using proposed algorithm

1. ***IF*** $\mu_{(i,j)}$ consider as a type I fuzzy set ***Then*** $\mu_{u(i,j)} = \mu_{L(i,j)}$ ***AND*** $\tilde{\gamma}(A) = 0$

$$\tilde{\gamma}(A) = \left(\frac{1}{(MN)^{\frac{1}{q}}}\right)\left[\sum_{j=1}^{M-1}\sum_{i=1}^{N-1}\left|\mu_{u(i,j)} - \mu_{U(i,j)}\right|^{q}\right]^{1/q} = 0 \; for\, q \in [1,\infty). \tag{24}$$

2. ***IF*** $\left|\mu_{u(i,j)} - \mu_{L(i,j)}\right| = 1$ (high ambiguity) ***Then*** $\tilde{\gamma}(\tilde{A}) = 1.$

$$\tilde{\gamma}(A) = \left(\frac{1}{(MN)^{\frac{1}{q}}}\right)\left[\sum_{j=1}^{M-1}\sum_{i=1}^{N-1}\left|\mu_{u(i,j)} - \mu_{U(i,j)}\right|^{q}\right]^{1/q} = 1 \; for\, q \in [1,\infty). \tag{25}$$

3. $\tilde{\gamma}(\tilde{A}) = \tilde{\gamma}(\tilde{\bar{A}})$ Where $(\tilde{\bar{A}})$is type II fuzzy set and it's complement set can be determined by 1- $\mu_{u(i,j)}$ and 1- $\mu_{L(i,j)}$, therefore complement set defined as follow

   For the complement set, the ultrafuzziness $\tilde{\gamma}$ is equal:

$$\tilde{\gamma}(\tilde{\bar{A}}) = \left(\frac{1}{(MN)^{\frac{1}{q}}}\right)\left[\sum_{j=1}^{M-1}\sum_{i=1}^{N-1}\left|1 - \mu_{u(i,j)} - 1 - \mu_{L(i,j)}\right|^{q}\right]^{1/q} = \tilde{\gamma}(\tilde{A}) \; for\, q \in [1,\infty) \tag{26}$$

4. ***IF*** $3DFOU_{(i,j)} < 3DFOU_{(d,c)}$ ***Then*** $\tilde{\gamma}(\tilde{A}_{(i,j)}) < \tilde{\gamma}(\tilde{A}_{(d,c)})$.

4.1.2. Finding the optimum interval 3DMF

The general approach for 3DMF based on upper and lower MF is equal:

$$\xi = \left[1 - \frac{\min \tilde{\gamma}(i,j)}{\max \tilde{\gamma}(i,j)}\right], SURF(i,j) = \mu_L(g_{ij})[\xi + 1], OR\, \mu_U(g_{ij})[1 - \xi] \tag{27}$$

Where (ξ) is ultra fuzzy coefficient and $\tilde{\gamma}(i,j)$ *is* ultra fuzzy value for $\mu_{u(i,j)}$ and $\mu_{L(i,j)}$, in upper and lower threshold.

4.2. Interval type-2 polar based method

Image processing is one among interesting applications of 3DMF. Instead of type reduce from Type-2 to type-1, we used a polar transform to make uniformity by same scale in [0,2π]. The RT of a two-dimensional function $f(x,y)$ in (r,θ) plane is defined as:

$$P(r,\theta) = R(r,\theta)\left[f(x,y)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,\delta(r - x\cos\theta - y\sin\theta)\,dx\,dy, \tag{28}$$

where $f(x, y)$ represents an image, $P(r, \theta)$ is the radon transform of $f(x, y)$, θ represents the line direction, and r is the distance away from the origin of coordinates. (Radon, 1919), (Miao et al., 2012) Where $\delta(\cdot)$ is the Dirac function, r∈[−∞,∞] is the perpendicular distance of a line from the origin and $\theta \in [0,\pi]$ is the angle formed by the distance vector. For the spatial case such as 3DMF, the fuzziness can be calculated as follows (Tizhoosh, 2005);

$$\gamma_l(A) = \left(\frac{2}{(MN)}\right)\left[\sum_{j=1}^{M-1}\sum_{i=1}^{N-1} min\left[\mu_{A(r_{i,j})}, 1-\mu_{A\,(r_{i,j})}\right]\right], \tag{29}$$

where $M \times N$ is subset $A \subseteq X$ with L radon transform value, $r \in [0, L-1]$, the histogram $h(RT)$ and the membership function $\mu x(RT)$, the linear index of fuzziness γ_l can be defined as follows (see Fig.6):

$$\gamma_l(A) = \left(\frac{2}{(MN)}\right)\left[\sum_{r=0}^{L-1} h(r) \times min\left[\mu_{A(r)}, 1-\mu_{A\,(r)}\right]\right], \tag{30}$$

To quantify the object fuzziness, a suitable membership function $\mu_A(r)$ should be determined. Tizhoosh present different functions, such as the standard S-function, the Huang and Wang function, LR-type fuzzy number (Tizhoosh et al, 1998; Huang and Wang, 1995; Pal and Bezdek, 1994; Pal and Murthy, 1990). Similar 3DMF presented in section 4.1, to generation of polar MF, CSS is used. The estimation of the MF also exanimate from the fitting of a cubic smoothing Spline, (Mora et al.,2011) to the 3DPMF*(r,θ)*. The fitting objective is to minimize the equation.

$$M = P.\sum_{y=1}^{m}\sum_{x=1}^{n}(f(r,\theta) - s(r,\theta))^2 + (1-p)\iint (D^2S\,(r,\theta))^2\,drd\theta, \tag{31}$$

where, this equation include two parts: *Compactness* and *Smoothness*. The smoothing factor p, controls the balance between being an interpolating spline crossing all data points (with p = 1) and being a strictly smooth Spline (with p = 0). In the polar transform, as p moves from 0 to 1, the smoothing spline changes from one extreme to the other. (See Fig. 11)

Using Radon transform for MF generation have several benefits such as Translation, Rotation and Scaling in IT2 FPM. (Miao et al., 2012).

$$R(\rho,\theta)\{f(x-x_0, y-y_0\} = P(r-r_0,\theta), \tag{32}$$

$$R(\rho,\theta)\{f(xcos\varphi + ysin\varphi, -xcos\varphi + ysin\varphi\} = P(\rho,\theta+\varphi) \tag{33}$$

$$R(\rho,\theta)\left\{f\left(x/_{\gamma}, y/_{\gamma}\right)\right\} = \gamma P\left(r/_{\gamma},\theta\right), \gamma \neq 0 \tag{34}$$

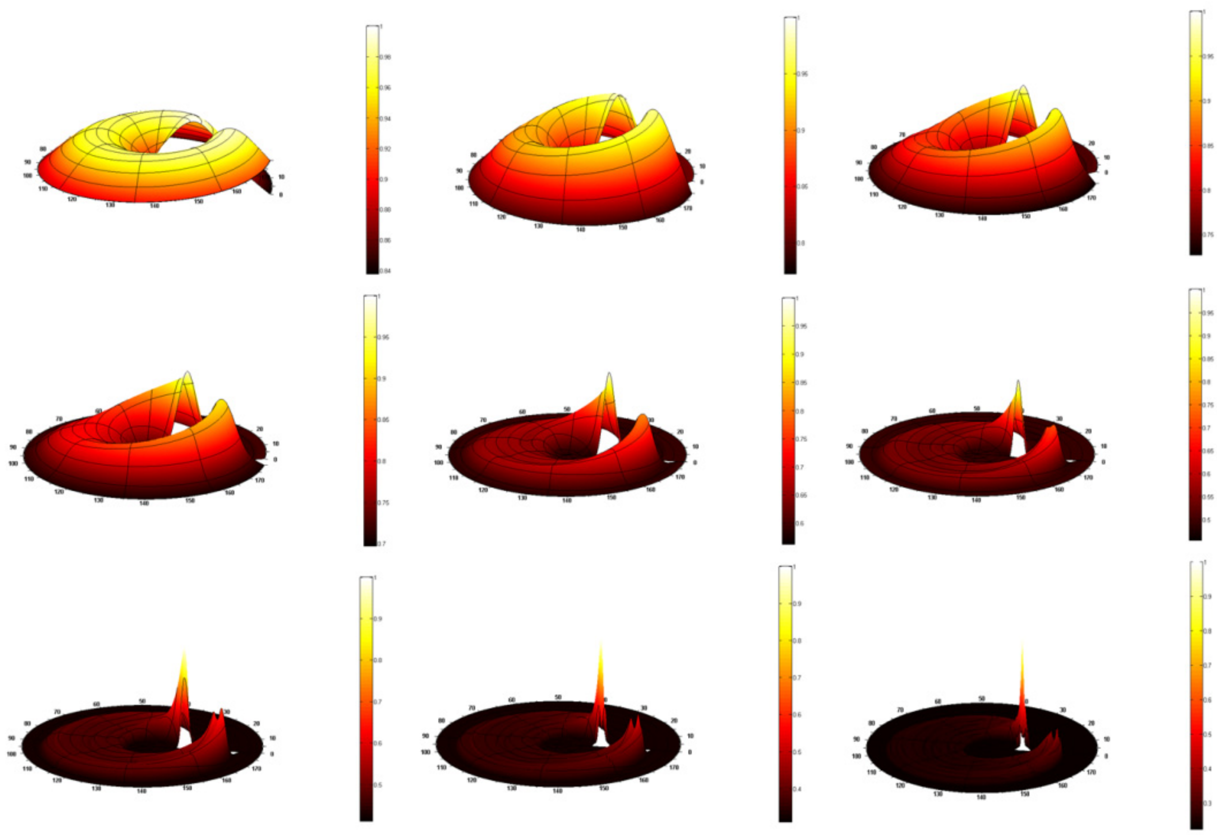

Figure 11. A sample of polar memberships function, As p moves from 0 to 1, the smoothing spline changes from one extreme to the other.

Where $r_0 = x_0 \cos\theta + y_0 \sin\ \theta$, γ is the scaling factor and φ is the rotation angle. A rotation of $f(x, y)$ by angle φ leads to a translation of $P(\rho,\theta)$ in the variable θ. A scaling of $f(x, y)$ results in a scaling in the ρ coordinate, as well as an intensity scaling of $P(\rho,\theta)$. (Miao et al., 2012). For the Fuzzy Polar based Method, we proposed use the following heuristic approach. This method consists of seven steps to obtain the 3D membership function in the polar domain.

Step 1. Three Dimensional Surface (**3D Data)**, Using Radon transform generate the 3D surface from image and construct 3D data surface.

Step 2. Three Dimensional Polar Surface (**3D Polar)**, Transfer data to the polar domain and uniform data in multi-scale.

Step 3. Polar Histogram Generator (**PHG)**, generates polar histogram in all direction using polar histogram generator.

Step 4. Approximate Smoother fitting parameters (**SF)**, Perform SF parameter to obtain the approximate parameter value (p).

Step 5. Polar Smooth Generator (**PSG)** smooths the histogram of the overall polar surface.

Step 6. Perform **PSG** fitting for the upper and lower histogram values.

Step 7. Determine PFMF, by normalizing the height of the upper PSG and LMF by the lower PSG.

On advantages of T2 PFM method is decrease on computational load in comparison with histogram based IT2 FMF. According our proposed method computational load can decrease, due to the stimulatory dimension in muli-scale surface and decrease computational load because of modified histogram smoothing process and fitting. Instead finding the one-dimensional UMF and LMF for each class label and feature which used by histogram based method, we fired all points in polar system with a cubic-spline. Next, we obtain the overall UMF and LMF Simultaneously. To obtain the generated IT2 PMF, it essential the three polar FOU (3D PFOU) be calculated. The UMF and LMF are designed by refitting cubic-spline. According proposed method the smoothed histograms have values that are above or below the mother fitted surface. Fig. 12a shows the one example constructed by polar upper and lower cubic-spline functions.

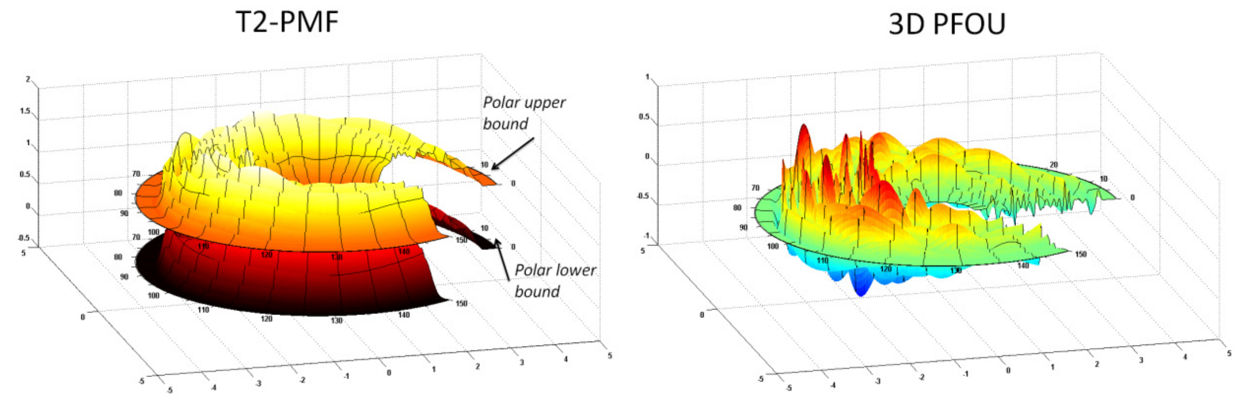

Figure 12. A)Three dimensional domain polar upper bound and lower bound, b) FOU for 3D polar fuzzy sets, (3D PFOU)

Using the upper and lower mother surface, to obtain the 3D PFOU, the PUMFs and PLMFs surface are designed. The UMF and LMFs surface normalized. Fig. 12,b. shows the IT2 PMF obtained by our proposed method. The shaded region between the 3D UMF and 3D LMF indicates the 3D PFOU. As shown in the Fig.13, our proposed method can effectively design IT2 FMFs based on the distribution of the input 3D data. The 3D PFOU can be expressed as

$$\mathrm{PFOU}\left(\overset{=}{A}\right)=\bigcup\left[\underline{PFOU}\left(\overset{=}{A}\right),\overline{PFOU}\left(\overset{=}{A}\right)\right]_{\forall x \epsilon X}=\bigcup\left[\underline{\min\left\{\overline{PFOU}\left(\overset{=}{A}\right)\right\}},\min\left\{\underline{PFOU}\left(\overset{=}{A}\right)\right\}\right]_{\forall x \epsilon X} \quad (35)$$

where $\overline{PFOU}\left(\overset{=}{A}\right)$ is the UMF, $\underline{PFOU}\left(\overset{=}{A}\right)$ s the LMF, and i is the feature number.

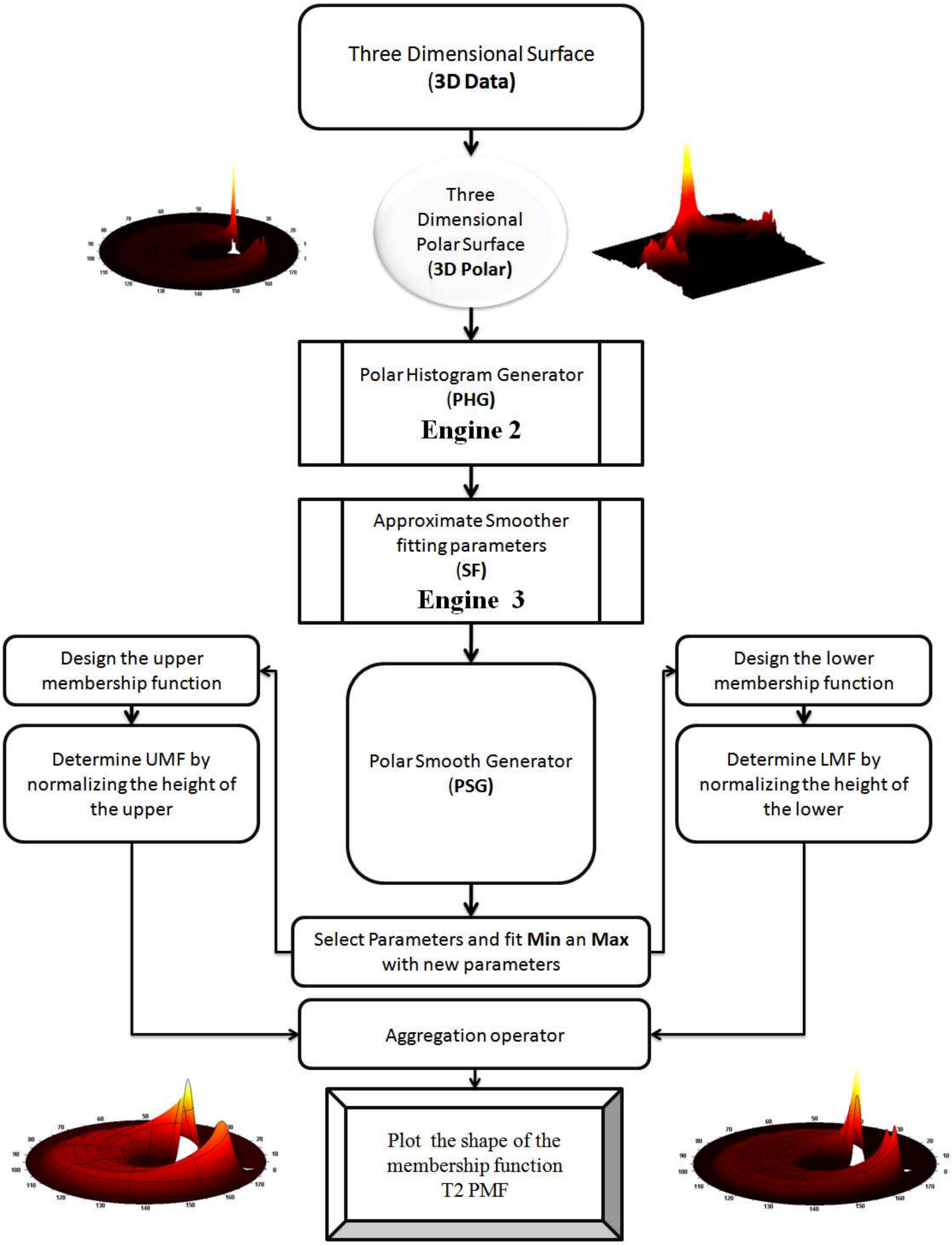

Figure 13. Heuristic based IT2 FPM generation method

4.2.1. A measure of polar ultrafuzziness

Polar ultrafuzzy can be calculated based on 3DMF fuzzy set. Such as defuzzifcation method proposed in measure of surface ultrafuzziness in section 4.1.1., type II membership function must be in [0,1]. Similaraway, normalization must be used for 3D PMF generation by division every point at $(\rho\ ,\theta)$ by max 3D PRT.

$$PN_{(\rho,\theta)}=\left(\left[v_{(3D\,PMF)(\rho,\theta)}^{\ h}\right]\Big/max\left[v_{(3D\,PMF)(\rho,\theta)}^{\ h}\right]\right)^{\frac{1}{h}}, \tag{36}$$

$$\mu_{(\rho,\theta)}=\left(\left[v_{(3D\,MF)(\rho,\theta)}^{\ h}\right]\Big/max\left[v_{(3DRT)(\rho,\theta)}^{\ h}\right]\right)^{\frac{1}{h}}+H \tag{37}$$

and

$$GR_{(i,j)}=\frac{1}{2\left(2\pi r\right)^{h}}\int_{0}^{2\pi}\int_{0}^{r}\left(\left[v_{(3DMF)(\rho,\theta)}^{\ h}\right]\Big/max\left[v_{(3DRT)(\rho,\theta)}^{\ h}\right]\right)^{\frac{1}{h}}d\rho d\theta, \tag{38}$$

where, $2\pi r$ denotes the size of 3DPMF platform,H is high polar platform $H\epsilon[0,1]$, $h\epsilon(1,\infty)$ and $v_{(3DRT)(\rho,\theta)}^{\ h}$ is 3DMF value in the position ρ and θ. In thresholding, selection H controller can use for select an optimum threshold based on type II fuzzy. Select a bigger H is worth a more enhanced maximum value. In order to define a type II fuzzy set in polar domain, first we develop a type I fuzzy set and assign upper and lower membership degrees to each element to (re)construct the footprint of uncertainty in polar system (Fig. 14). Hear we select H=0 to calculate the real 3D PMF. In polar system the definition for uncertainty is slightly deferent 3D FMF. Uncertainty can present in ring and height which reflect to polar memberships function. (Fig. 16) For example, when Radon Transform is applied to wavelet modulus, a distress (crack) is transformed into a peak in radon domain. Originally, every distress reflects to RT and has different intensity in 3DMF histograms. For example mean of 3D PRT have variety range. According to the above Eq. 38 the max GR must be equal 1. Similaty 3D FMF method, in 3D PMF, the amount of ultrafuzziness will increase by rising uncertainly bound.

The extreme case of maximal ultrafuzziness in polar system, equal 1, is worth to completely vagueness. Based on Pal and Bezdek (1994) research on several fuzziness index, two general classes proposed by them was additive and multiplicative class (Pal and Bezdek, 1994). Based on Kufmann's Index of fuzziness for a set $P \in r_n(x)$,

$$H_{ka}\left(A\right)=\left(\frac{2}{n^{k}}\right)d(P,P^{near}) \tag{39}$$

Where, $k \in R^{+}$,d is a metric, and A^{near} is the crisp set close to the P. generally, based on *d*,weigh of k determined. The $d(P,P^{near})$ and linear or quadratic $H_{ka}(P)$ can be determined by q-norms such as 3D FMF,

$$d(P,P^{near})=\left(\sum_{i=1}^{n}\left|\mu_{(\rho,\theta)}-\mu_{A^{near},(\rho,\theta)}\right|^{q}\right)^{1/q},H_{ka}(q,P)=\left(\frac{2}{n^{\frac{1}{q}}}\right)\left(\sum_{i=1}^{n}\left|\mu_{i}-\mu_{P^{near},i}\right|^{q}\right)^{1/q} \tag{40}$$

Where $q \in \left[1,\infty\right)$. Based on Tizhoosh ultrafuzziness index, we developed a new index in continues polar domain (Tizhoosh, 2005),

$$\tilde{\gamma}\left(\tilde{P}\right)=\frac{1}{4\pi\rho^{2}}\int_{0}^{2\pi}\int_{0}^{r}\left(\mu_{U}(g_{(\rho,\theta)})-\mu_{L}(g_{(\rho,\theta)})\right)d\rho d\theta, \tag{41}$$

Where $\mu_{U}\left(g\right)=\left[\mu_{A}\left(g\right)\right]^{1/\alpha}$ and $\mu_{L}\left(g\right)=\left[\mu_{A}\left(g\right)\right]^{1/\beta}$, and in general term it present as follow,

$$\tilde{\gamma}\left(\tilde{P}\right)=\frac{1}{4\pi\rho^{2}}\int_{0}^{2\pi}\int_{0}^{r}\int_{0}^{1}\left(\mu_{U}(g_{(\rho,\theta)})-\mu_{L}(g_{(\rho,\theta)})\right)\times h(g)\ d\rho.d\theta.dh \tag{42}$$

A measure of ultrafuzziness $\tilde{\gamma}$ for a polar 3D FMF in $4\pi\rho^{2}$, polar 3D FMF and the membership function $\mu_{\tilde{P}(\rho,\theta)}$ can be developed as follows:

$$\tilde{\gamma}\left(P\right)=\left(\frac{1}{\left(Area\right)^{\frac{1}{q}}}\right)\left[\int_{0}^{2\pi}\int_{0}^{r}\left|\mu_{U}(g_{(\rho,\theta)})-\mu_{L}(g_{(\rho,\theta)})\right|^{q}\right]^{1/q},$$

$$\frac{\partial}{\partial T}\tilde{\gamma}\left(PMF_{(\rho,\theta)}\right)=\frac{\partial}{\partial T}\left(\frac{1}{\left(Area\right)^{\frac{1}{q}}}\right)\left[\int_{0}^{2\pi}\int_{0}^{r}\left|\mu_{U}(g_{(\rho,\theta)})-\mu_{L}(g_{(\rho,\theta)})\right|^{q}\right]^{1/q}d\rho d\theta=0 \tag{43}$$

The variation in the polar space can be measured by this method, therefore the new Index introduced in polar dimensional domain of FOU for 3D polar fuzzy sets, (3D PFOU). This method can resolve the problems about the discontinues domain and in a same time reduce on dimension by using polar transform. (see Fig 15, 16).

Similarly, Kaufmann conditions consists of *Minimum ultrafuzziness, Maximum ultrafuzziness, Equal ultrafuzziness* and *Reduced ultrafuzziness* evaluated for polar method (Kaufmann, 1975). Polar Index is qualified for measure of ultrafuzziness in 3D polar domain with these conditions.

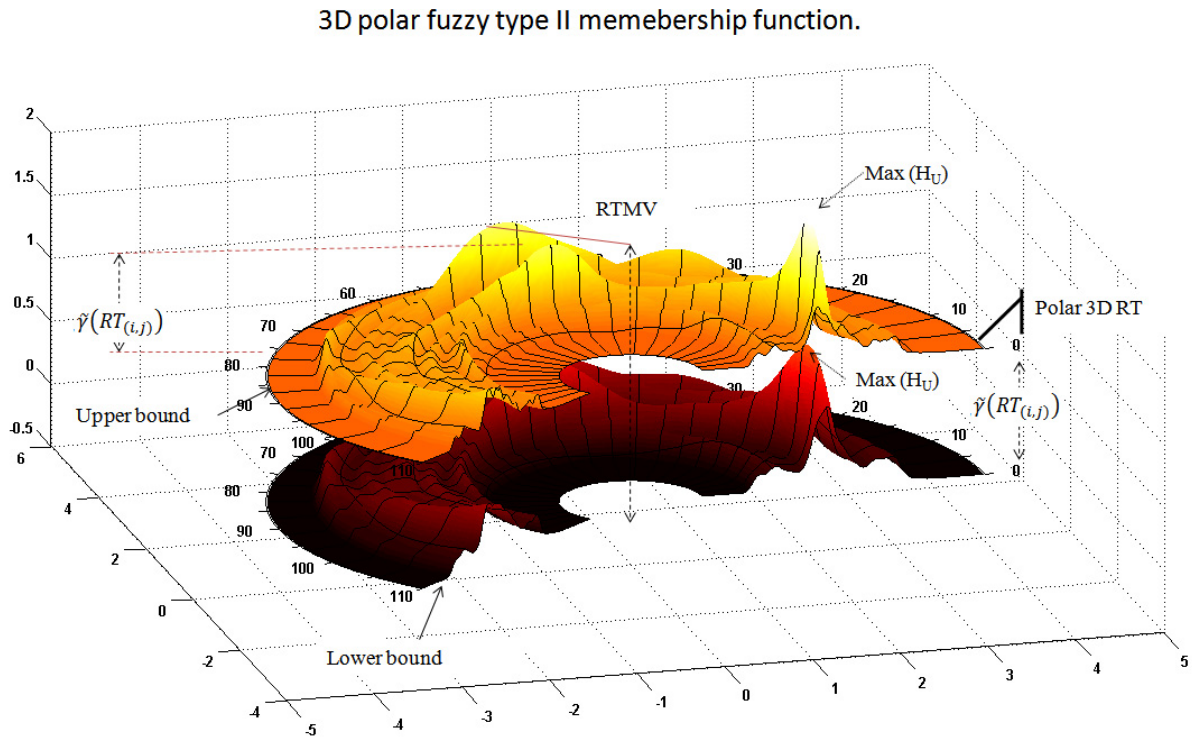

Figure 14. Basic rules for construction 3D polar fuzzy type II memebership function.

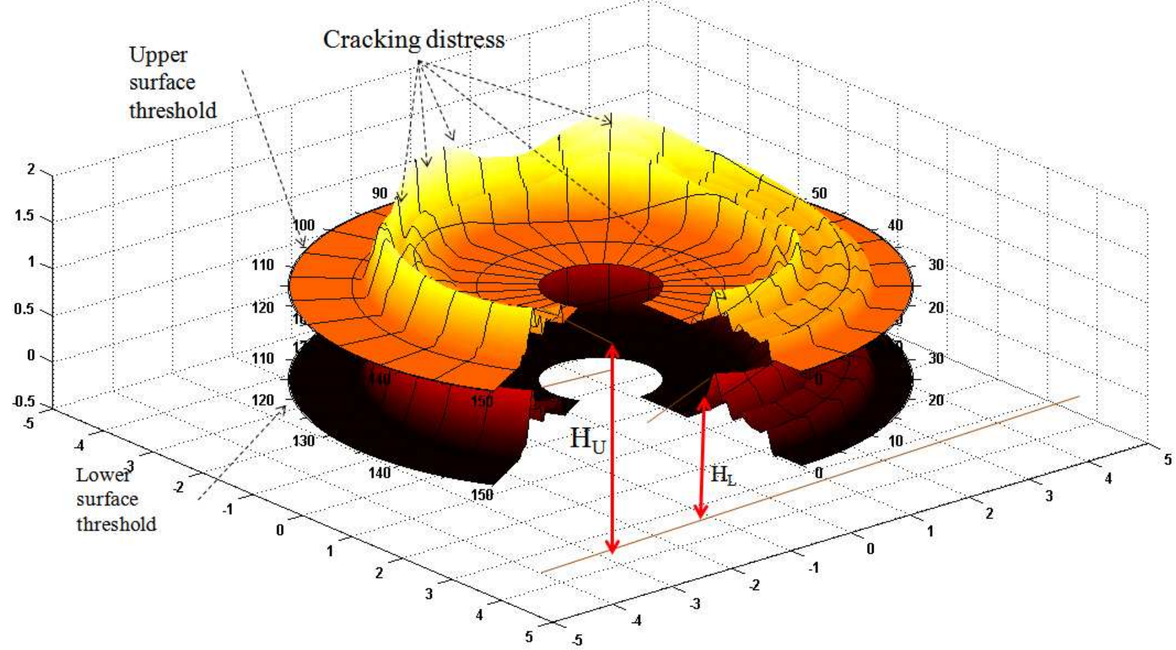

Figure 15. Three dimensional polar domain of FOU for 3D fuzzy sets, (3D PFOU)

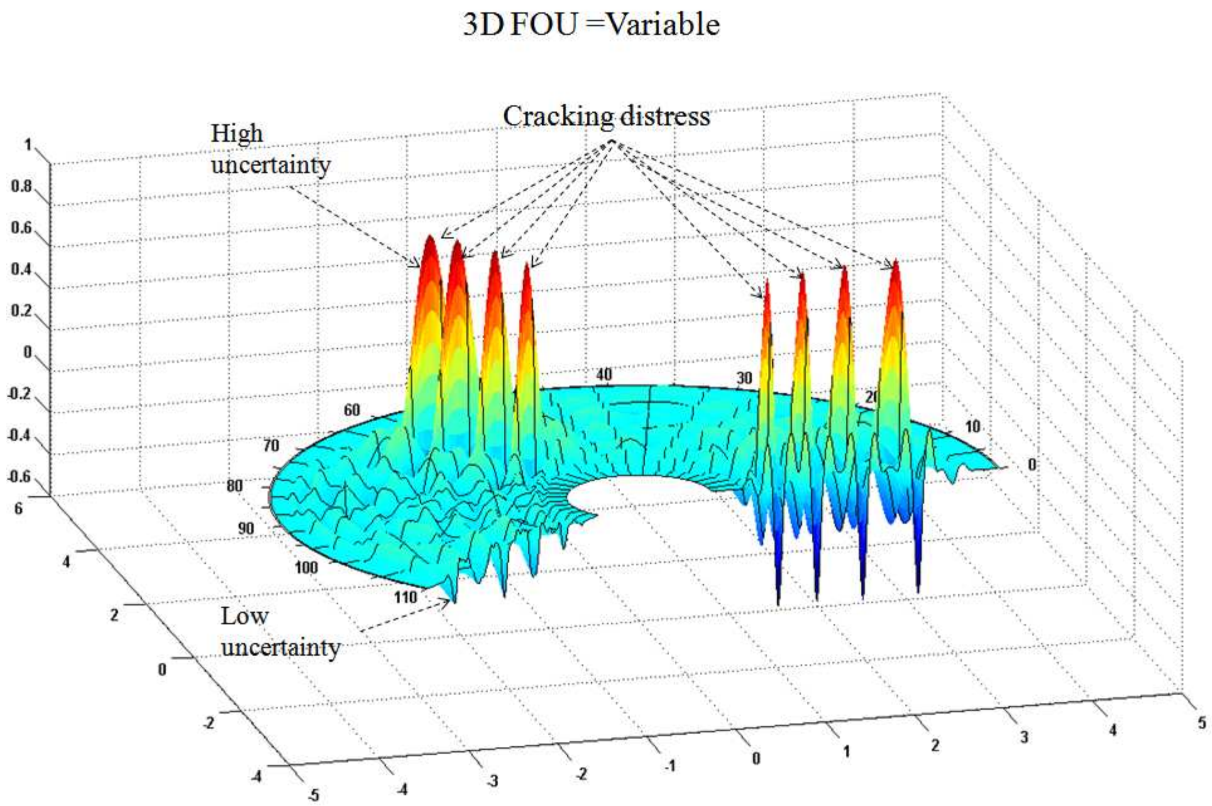

Figure 16. Example of Three polar FOU for 3D fuzzy sets, (3D PFOU) using proposed algorithm

1. **IF** $\mu_{(\rho,\theta)}$ consider as a type I polar fuzzy set **Then** $\mu_{u(\rho,\theta)} = \mu_{L(\rho,\theta)}$ **AND** $\tilde{\gamma}(P) = 0$

$$\tilde{\gamma}(P) = \left(\frac{1}{(Area)^{\frac{1}{q}}}\right)\left[\int_0^{2\pi}\int_0^{r}\left|\mu_U(g_{(\rho,\theta)}) - \mu_L(g_{(\rho,\theta)})\right|^q\right]^{1/q} = 0 \; for \, q \in [1,\infty). \tag{44}$$

2. ***IF*** $\left|\mu_{u(\rho,\theta)} - \mu_{L(\rho,\theta)}\right| = 1$ (high ambiguity) ***Then*** $\tilde{\gamma}(\tilde{P}) = 1$

$$\tilde{\gamma}(P) = \left(\frac{1}{(Area)^{\frac{1}{q}}}\right)\left[\int_0^{2\pi}\int_0^{r}\left|\mu_U(g_{(\rho,\theta)}) - \mu_L(g_{(\rho,\theta)})\right|^q\right]^{1/q} = 1 \; for \, q \in [1,\infty). \tag{45}$$

3. $\tilde{\gamma}(\tilde{P}) = \tilde{\gamma}(\tilde{\tilde{P}})$. Where $(\tilde{\tilde{P}})$ is type II fuzzy set and it's complement set can be determined by 1- $\mu_{u(\rho,\theta)}$ and 1- $\mu_{L(\rho,\theta)}$, therefore complement set defined as follow

$$\tilde{\tilde{P}} = \left\{ x, \mu_{u(\rho,\theta)}, \mu_{u(\rho,\theta)} \middle| \begin{matrix} 1 - \left(\left[v_{(3DPRT)(\rho,\theta)}^{h} \right] \Big/ \max\left[v_{(3DPRT)(\rho,\theta)}^{h} \right] \right)^{\frac{1}{h}} + H_U \\ , \\ 1 - \left(\left[v_{(3DPRT)(\rho,\theta)}^{h} \right] \Big/ \max\left[v_{(3DPRT)(\rho,\theta)}^{h} \right] \right)^{\frac{1}{h}} + H_L \end{matrix} \right\} \quad (46)$$

Where $H_U = H_L = 0$, for the complement set, the ultrafuzziness $\tilde{\gamma}$ is equal:

$$\tilde{\gamma}\left(\tilde{\tilde{P}}\right) = \left(\frac{1}{(Area)^{\frac{1}{q}}} \right) \left[\int_0^{2\pi} \int_0^{r} \left| 1 - \mu_U(g_{(\rho,\theta)}) - 1 + \mu_L(g_{(\rho,\theta)}) \right|^q \right]^{1/q} = \tilde{\gamma}\left(\tilde{P}\right) \, for \, q \in [1, \infty) \quad (47)$$

4. ***IF*** $3DPFOU_{(\rho,\theta)} < 3DPFOU_{(d,c)}$ ***Then*** $\tilde{\gamma}\left(\tilde{P}_{(\rho,\theta)}\right) < \tilde{\gamma}\left(\tilde{P}_{(d,c)}\right)$.

4.2.2. Finding the optimum interval 3D PMF

The general approach for 3DMF based on upper and lower MF is equal:

$$\xi_P = \left[-\frac{\tilde{\gamma}(\rho,\theta)}{2} \right], IT2\,PMF(\rho,\theta) = \left[\mu_L\left(g_{\rho,\theta}\right) - \xi_P \right], OR\left[\mu_U\left(g_{\rho,\theta}\right) + \xi_P \right] \quad (48)$$

Where (ξ_P) is polar ultra fuzzy coefficient and $\tilde{\gamma}(\rho,\theta)$ *is* ultra fuzzy value for $\mu_{u(\rho,\theta)}$ and $\mu_{L(\rho,\theta)}$, in upper and lower bound. For example, Fig.16 presents the principle polar memberships function of the interval type 2 polar fuzzy sets in position (ρ,θ).

5. Polar fuzzy type-2 approximate reasoning

In this part, basic theory of fuzzy polar rule interpolation in fuzzy rule based will be presented for polar membership's function of type -2 fuzzy sets. Lets us show polar fuzzy rules with multiple antecedent and single consequent based on T2 PMF rules, Multi Input Single Output (MISO):

Rule 1: **If** X_1 is $(\overset{=}{P}A)_{11}$ & X_2 is $(\overset{=}{P}A)_{12}$ & … & X_m is $(\overset{=}{P}A)_{1m}$ **Then** Y is $(\overset{=}{P}C)_1$

Rule 2: **If** X_1 is $(\overset{=}{P}A)_{21}$ & X_2 is $(\overset{=}{P}A)_{22}$ & … & X_m is $(\overline{\overline{PA}})_{2m}$ **Then** Y is $(\overset{=}{P}C)_2$

Rule n: **If** X_1 is $(\overset{=}{PA})_{n1}$ & X_2 is $(\overset{=}{PA})_{n2}$ & … & X_m is $(\overset{=}{PA})_{nm}$ **Then** Y is $(\overset{=}{PC})_n$

Where X_p denotes the pth T2-PFM antecedent and Y denote the T2- PFM consequence. $(\overset{=}{PA})_{nm}$ is the *n,m* th consequence of T2-PFM fuzzy set of Rule n. According polar MISO method, fuzzy interpolative polar reasoning result which denoted by $\left(\overset{=}{P}\right)$ can be extracted based on observation polar fuzzy set $\left(\overset{=}{O}\right)$.

Observation: **If** X_1 is $(\overset{=}{O})_1$ & X_2 is $(\overset{=}{O})_2$ & … & X_m is $(\overset{=}{O})_m$

Conclusions: $\overset{=}{O}$ is $\overset{=}{P}$.

This method at the first glance, is similar to method which proposed by Chen & Chang (2011) under the title of "fuzzy rule interpolation based on principle membership functions and uncertainty grad function of interval type-2 fuzzy sets"; however, the main difference between our proposed method with their method relay in type decreasing (Chen & Chang, 2011). Based on Chen & Chang (2011) method first type -2 reduced by type-1, then MISO applied, but in our method first MISO applied separately on upper and lower. After that the type-2 polar reasoning result which denoted by $\left(\overset{=}{P}\right)$ extracted and interval 3D PMF calculated based on section 4.2 theories. This method can be expanded for Multi Input Multi Output (MIMO) systems based on polar T2 PMF's. For example lets us show polar fuzzy rules with multiple antecedents and multiple consequent based on T2 PMF rules, Multi Input Multi Output (MIMO):

Rule 1: **If** X_1 is $(\overset{=}{PA})_{11}$ & … & X_m is $(\overset{=}{PA})_{1m}$ **Then** Y is $(\overset{=}{PC})_{11}$ & $(\overset{=}{PC})_{12}$ & …& $(\overset{=}{PC})_{1r}$

Rule 2: **If** X_1 is $(\overset{=}{PA})_{21}$ & … & X_m is $(\overset{=}{PA})_{2m}$ **Then** Y is $(\overset{=}{PC})_{21}$ & $(\overset{=}{PC})_{22}$ & …& $(\overset{=}{PC})_{2r}$

Rule n: **If** X_1 is $(\overset{=}{PA})_{n1}$ & … & X_m is $(\overset{=}{PA})_{nm}$ **Then** Y is $(\overset{=}{PC})_{n1}$ & $(\overset{=}{PC})_{n2}$ & …& $(\overset{=}{PC})_{nr}$

Observation: **If** X_1 is $(\overset{=}{O})_1$ & … & X_m is $(\overset{=}{O})_r$

Conclusions: $\overset{=}{O}$ is $\overset{=}{P}_1$ & $\overset{=}{P}_2$ & …& $\overset{=}{P}_r$.

where, such as polar MISO, X_p denotes the pth T2-PFM antecedent and Y denote the T2- PFM consequence which expanded in $(\overset{=}{PC})_{nr}$, r denote the number of consequence in rule n.

$(\overset{=}{PA})_{nm}$ is the *n,m* th consequence of T2-PFM fuzzy set of Rule n. According polar MIMO method, fuzzy interpolative polar reasoning result which denoted by $\left(\overset{=}{P_r}\right)$ can be extracted based on observation polar fuzzy set $\left(\overset{=}{O}\right)$ in window r for conscience *r*.

Logical and Mamdani, in the linguistic models considered as prepositions. The general form of these linguistic rules shows as:

Mamdani:

$$\textbf{If } X_1 \text{ is } \left(\overset{=}{PA}\right)_{n1} \;\&\ldots\&\; X_m \text{ is } \left(\overset{=}{PA}\right)_{nm} \;\textbf{Then } \mathrm{Y} \text{ is } \left(\overset{=}{PC}\right)_{n1} \;\&\; \left(\overset{=}{PC}\right)_{n2} \;\&\ldots\&\; \left(\overset{=}{PC}\right)_{nr}$$

Logical:

$$\textbf{If } X_1 \text{ is } \left\| \widetilde{\left(\overline{\overline{PA}}\right)_{n1}} \right\| \;\&\ldots\&\; X_m \text{ is } \left\| \widetilde{\left(\overline{\overline{PA}}\right)_{nm}} \right\| \;\textbf{Then } \mathrm{Y} \text{ is } \left(\overset{=}{PC}\right)_{n1} \;\&\; \left(\overset{=}{PC}\right)_{n2} \;\&\ldots\&\; \left(\overset{=}{PC}\right)_{nr}$$

Where $\left(\overset{=}{PA}\right)_{nm}$ is the T2 PFM as an antecedent variable in polar system $\left(\overset{=}{PC}\right)_{nr}$ is the consequent variable, n denotes to number of rule and r denoted to *r* denote the number of consequences and $\left\| \widetilde{\left(\overline{\overline{PA}}\right)_{n1}} \right\|$ is complement of the T2 PFM as an antecedent variable in polar system. Concrete effective model proposed by Yager, is a complex method for combination of these two models, presented as follow:

$$\overset{=}{P}_y = \beta(\overset{=}{P}_L) + (1-\beta)(\overset{=}{P}_M) \tag{49}$$

Where $\overset{=}{P}_y$ is the Yager result(complex model), $(\overset{=}{P}_L)$ is result of polar type-2 fuzzy under logical model, $\overset{=}{P}_M$ is consequents of mamdani model and β is control factor which move from 0 to 1. (Fazel et al. 2009). The defuzzication method for every θ must be computed based on theory which presented in section 4.2. Defuzzification agent in polar system constructed as three steps,

Step 1. Calculate the center of area inθ, in which θ defined from 0, to 2π (Gold Veins Root)
Step 2. Plot the fuzzy Gold veins determined from first step.
Step 3. Calculate the center of Gold veins and consider it as defuzzify conclusion result.

In step 1, center of area or center of gravity can be calculated in polar system using eq.38, which is a most common model used :

$$C(\theta)=\left(\frac{1}{Area(\theta)}\right)\left[\int_0^r\left|\mu_U\left(g_{(\rho,\theta)}\right).\rho-\mu_L\left(g_{(\rho,\theta)}\right).\rho\right|\right]d\rho. \tag{50}$$

and

$$C(\theta)_m=\left(\frac{1}{Area(\theta)^q}\right)\left[\int_0^r\left|\left(\mu_U\left(g_{(\rho,\theta)}\right)\right)^q.\rho-\left(\mu_L\left(g_{(\rho,\theta)}\right)\right)^q.\rho\right|\right]d\rho \tag{51}$$

Gold Veins Root is a vector consist of paired $(\theta, C(\theta))$, which is shows the direction center of gravity and it present a useful information from membership function variance and crisp result without type reduction. $C(\theta)_m$ is a modified Gold Veins Root that can be control the final defuzzication result, in which q play a defuzzifer role. An example of Gold Veins Root extracted from polar T2 PMF is depicted in Fig.17. Two result from original and modified center of gravity by q=3, present in Fig 17.

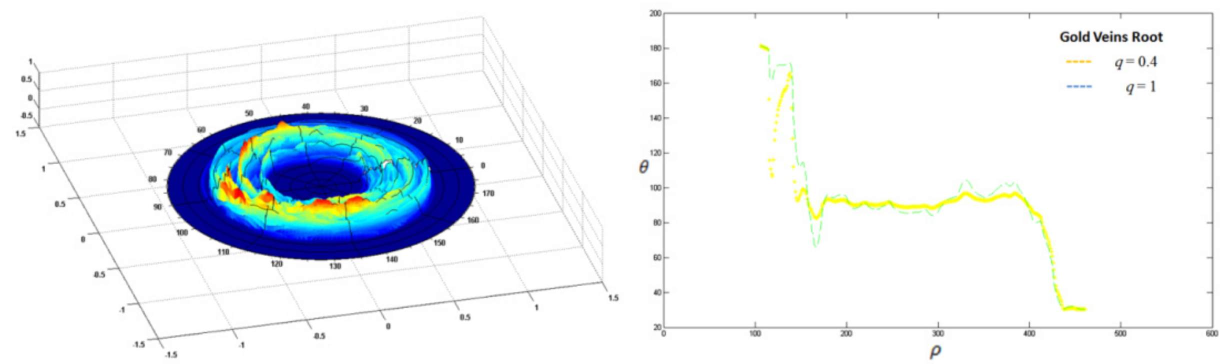

Figure 17. Gold Veins Root in T2 PMF

In order to effectively implementation and test proposed model in some applications such as prediction problem and pattern recognition, such as the other models in fuzzy, we need set in motion to product a crisp result. We chose a heuristic function to generate a best crisp defuuzy value from Gold Veins Root, based on Eq.51

$$C=\left[\frac{\max(GVR)-\min(GVR)}{2}\right] \tag{52}$$

For the case Fig.18, C=96.33 worth in blurred section from $\rho=200,400$, provided good prediction orientation and radios for extension of T2 PFM. Now we present a logical method for type -2 in 3D techniques. Remove type reduction; turn T2 PMF as faster than existing techniques and to be more accurate model for type-2 inference techniques because of combination Logical & Mamdani models.

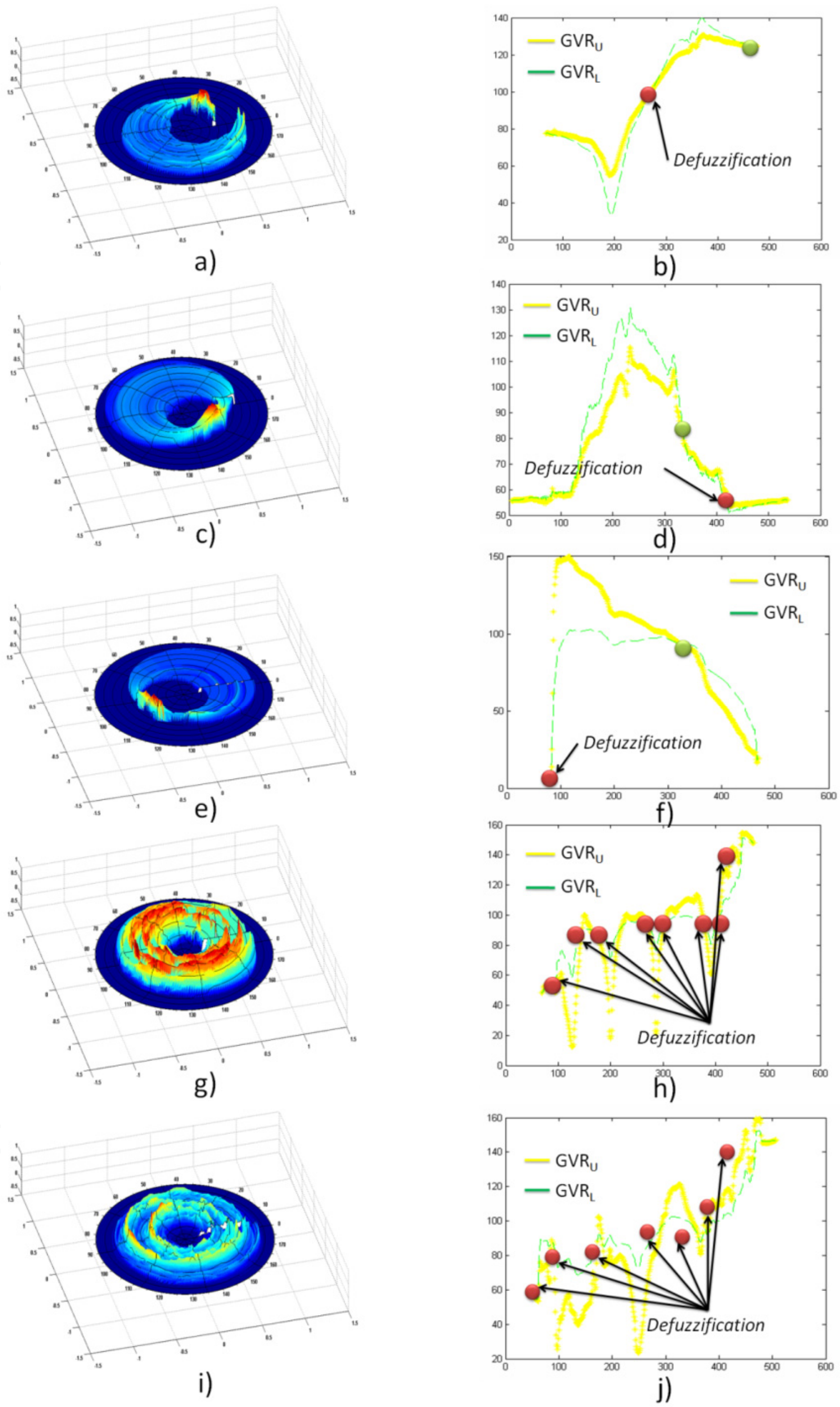

Figure 18. a) Input T2 PMF longitudinal cracking, b)Final difuzzification points longitudinal Cracking, c) Input T2 PMF transverse cracking, d)Final difuzzification points transverse Cracking, e) Input T2PMF diagonal cracking, f)Final difuzzification points diagonal Cracking, g) Input T2 PMF block cracking, h)Final difuzzification points block Cracking, i) Input T2 PMF aligator cracking, j)Final difuzzification points alligator Cracking.

6. Conclusions

In this chapter the basic concepts of new fuzzy sets, three dimensional (3D) memberships and how they are applied in the design of type-1 and type-2 fuzzy thresholding in control systems are presented. The robustness of a system highly depends on automatic fuzzification and membership functions shape and defuzzification. The related methodology and theoretical base are discussed, using real examples in automatic control in civil engineering. Selection of a supper membership function is a golden key in fuzzy controls. A robust method to consider the uncertainty of membership values by using flexible thresholding for controller problems proposed in the special a polar domain presented in this chapter. Different fuzzy membership functions may have various impacts on the systems and, then, different thresholds in control problems. To solve this problem, type II fuzzy thresholding is recommended. The upper and lower membership functions promote this dilemma; however the figure of uncertainty (FOU) has a fixed value that is equal to one, in all the upper and lower membership function. Type-2 fuzzy logic can effectively improve the control characteristic by using FOU of the membership functions.

A new fuzzy thresholding (flexible thresholding) technique developed, which processes threshold as a flexible type-2 fuzzy sets. Experimental results are provided in order to demonstrate the usefulness of the proposed approach. A review of types of fuzzy threshold methods in control problems provided and their algorithms presented. In type-2 thresholding method, measurement of fuzziness gives a quantitative index to vagueness. To quantify the object fuzziness, a suitable membership function based on thresholding for control problems introduced. A measure for ultra-fuzziness in 3D fuzzy model is proposed. A new method for thresholding algorithm based on 3D type-2 fuzzy and selection the optimum thresholding in 3D surface are addressed. By an example the validity of novel fuzzy algorithm in control systems, based on three dimensional membership functions demonstrated.

This paper presents a new type of fuzzy membership functions and uncertainly grade in the frame of polar systems. The proposed method can be used and generalized for several problems; however in this paper we present implementation of polar fuzzy type-2(PFT2) as a part of Hybrid expert system for pavement distress detection and classification.

Vast applications are predicted this fuzzy reasoning. The central idea of this work was to introduce the application of polar type II fuzzy sets.

The most important aspect of the proposed model is the ability of self-organization of the membership function and initial height platform without requiring programming.

Additional experiments reinforced this conclusion. More extensive investigations on other measures of ultrafuzziness and the effect of parameters influencing the width/length of FOU should certainly be conducted.

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