Chapter 2

Robust Control of Distributed Parameter Systems with Demonstration in Casting Technology and MATLAB/Simulink/DPS Blockset Software Support

Cyril Belavý, Gabriel Hulkó and Karol Ondrejkovič

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/46460

1. Introduction

Most of the dynamical systems analysed in engineering practice have the dynamics, which depends on both position and time. Such systems are classified as distributed parameter systems (DPS). The time-space coupled nature of the DPS is usually mathematically described by partial differential equations (PDE) as infinite-dimensional systems. However, from point of view of implementation of DPS control in technological practice, where a finite number of sensors and actuators for practical sensing and control is at disposal, such infinite-dimensional systems need to be approximated by finite-dimensional systems. There are many dimension reduction methods, which can be used to solve this problem.

In the first mathematical foundations of DPS control, analytical solutions of the underlying PDE have been used (Butkovskij, 1965; Lions, 1971; Wang, 1964). That is the decomposition of dynamics into time and space components based on the eigenfunctions of the PDE. Continuous and approximation theories aimed to control of parabolic systems presents monograph (Lasiecka & Triggiani, 2000). Methodical approach from the view of time-space separation with model reduction is presented in (Li & Qi, 2010). Variety of transfer functions for systems described by PDE are illustrated by means of several examples in (Curtain & Morris, 2009). Well-known reduction methods based on finite difference method (FDM), or finite element method (FEM), spectral method require an accurate nominal PDE model and usually lead to a high-order model, which requires unpractical high-order controller.

An engineering approach for the control of DPS is being developed since the eighties of the last century (Hulkó et al., 1981, 1987, 1998, 2009a, 2009b). In the field of lumped parameters system (LPS) control, where the state/output quantities $x(t)/y(t)$ – parameters are given as finite dimensional vectors, the actuator together with the controlled plant make up a controlled LPS. In this sense the actuators and the controlled plant as a DPS create a controlled lumped-input and distributed-parameter-output system (LDS).

In this chapter the decomposition of dynamics of controlled LDS into time and space components is introduced. Based on this decomposition a methodical framework of control synthesis decomposition into space and time tasks will be presented. In the space domain, approximation problems are solved. In the time domain, synthesis of control is performed by lumped parameter control loops, where robust controllers are used.

The casting technology is a typical case of the DPS. There in order to obtain the desired solidification structure, the casting process requires a specific temperature field of the mould, which is defined on complex-shape 3D definition domain. Modelling, simulation and evaluation of real-time experiments in this area is now widely accepted as an important tool in product design and process development to improve productivity and casting quality. For analysis of the casting process dynamics as DPS, especially temperature fields in the casting mould and control synthesis purposes, the benchmark casting plant with steel mould of complex-shape was designed at Faculty of Mechanical Engineering STU in Bratislava.

The main emphasis of this chapter is to present an engineering approach for the robust control of DPS with demonstration in the casting technology along with software support in the MATLAB & Simulink programming environment. This approach opens a wide space for novel applications of the toolboxes and blocksets of the MATLAB & Simulink software environment. For the software support of modelling, control and design of DPS, given on 1D-3D definition domains, the **Distributed Parameter Systems Blockset for MATLAB & Simulink (DPS Blockset),** a Third-Party Product of The MathWorks Company - www.mathworks.com/products/connections/ has been developed at the Institute of Automation, Measurement and Applied Informatics, Faculty of Mechanical Engineering STU, (Hulkó et al. 2003-2010). Also a web portal named **Distributed Parameter Systems Control** - www.dpscontrol.sk has been created for those, who are interested in solving problems of DPS control (Hulkó et al., 2003-2007). This web portal contains application examples from different areas of engineering practice, such as the control of technological and manufacturing processes, mechatronic structures, groundwater remediation, etc. In addition, this web portal offers the demo version of the **DPS Blockset** with the **Tutorial, Show, Demos** and **DPS Wizard** for download, along with the **Interactive Control** service for the interactive solution of model control problems via the Internet.

In this chapter, for the control synthesis purpose, LDS models of temperature fields in the casting mould were created by means of evaluation of real-time experiments. Robust control synthesis based on internal model control (IMC) structure in the time domain has been done. Designed robust controllers were used for the robust control of preheating casting mould in the real-time experiment in accordance to casting technology requirements. Identification, uncertainty analysis of the models, robust control synthesis and experiments were performed with the software support of DPS Blockset and toolboxes of the MATLAB & Simulink, especially the System Identification Toolbox, Control Systems Toolbox, Robust

Control Toolbox, Optimization Toolbox, System Identification Toolbox, Real-Time Windows Target and Simulink Design Optimization.

2. LDS/HLDS representation of DPS

In general, DPS are systems whose state or output quantities, $X(x,y,z,t)/Y(x,y,z,t)$ are distributed quantities or fields of quantities, where (x,y,z) are spatial coordinates in 3D. These systems are often considered as systems whose dynamics is described by PDE. In the input-output relation, PDE define distributed-input/distributed-output systems (DDS) between distributed input, $U(x,y,z,t)$ and distributed output quantities, $Y(x,y,z,t)$, at initial and boundary conditions given. Distributed parameter systems are very frequently found in various technical and non-technical branches with limited number of manipulated input quantities, or actuators. These lumped input quantities by means of interaction of fields and quantities generate distributed output of real DPS. Representation of such DPS is either in the form of LDS, Fig. 1 a), or in the form of LDS with zero-order hold unit H (HLDS), when discrete-time lumped input quantities are used, Fig. 1. b) , (Hulkó et al., 1981, 1987, 1998).

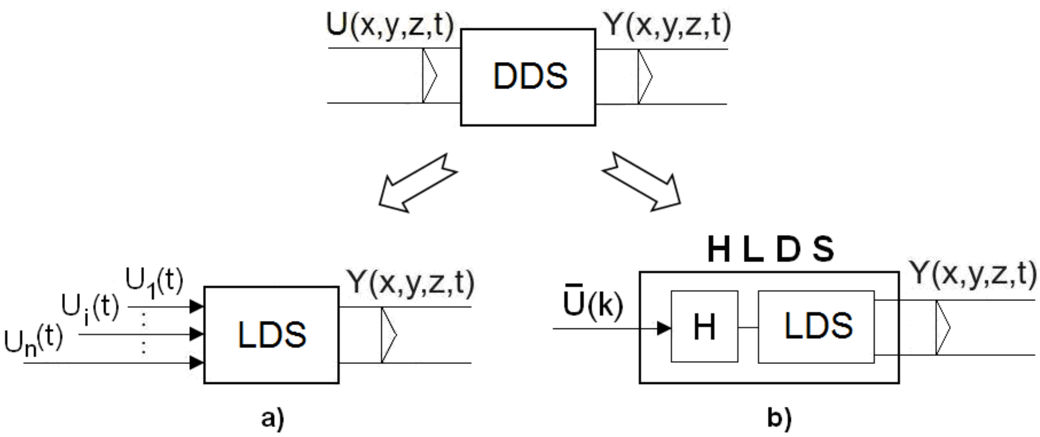


Figure 1. Representation of DPS: a) LDS - lumped-input/distributed-output system, b) HLDS –LDS with block of zero-order hold units H, $U(x,y,z,t)$ - distributed input quantity, $Y(x,y,z,t)$ - distributed output quantity, $\{U_i(t)\}_{i=1,n}$ - lumped input quantities, $\bar{U}(k)$ - vector of discrete-time lumped input quantities

2.1. Dynamics of LDS

Distributed output of the linear LDS from zero initial conditions either in continuous, or in discrete-time (DT) is in the form:

$$Y(\bar{x},t)=\sum_{i=1}^{n}Y_i(\bar{x},t)=\sum_{i=1}^{n}\mathcal{G}_i(\bar{x},t)\otimes U_i(t) \tag{1}$$

$$Y(\bar{x},k)=\sum_{i=1}^{n}Y_i(\bar{x},k)=\sum_{i=1}^{n}\mathcal{G}H_i(\bar{x},k)\oplus U_i(k) \tag{2}$$

where $\otimes$ denotes convolution product and $\oplus$ denotes convolution sum, $\overline{x}=(x,y,z)$ is position vector in 3D, $\mathcal{G}_i(\overline{x},t)$ - distributed parameter impulse response of LDS to the i-th input, $\mathcal{G}H_i(\overline{x},k)$ - DT distributed parameter impulse response of LDS with zero-order hold units H (HLDS) to the i-th input, $Y_i(\overline{x},t)$ - distributed parameter output quantity of LDS to the i-th input, $Y_i(\overline{x},k)$ - DT distributed parameter output quantity of HLDS to the i-th input, $U_i(t)$ - lumped input quantity, $U_i(k)$ - DT lumped input quantity, (Hulkó et al. 1998).

When $U_i(t)$ is a unit-step (Heaviside) function, $Y_i(\overline{x},t)$ is in the form of distributed step response function $\mathcal{H}_i(\overline{x},t)$. Similarly, for the unit-step function $U_i(k)$: $Y_i(\overline{x},k)\to\mathcal{H}H_i(\overline{x},k)$. For simplicity in this chapter distributed quantities are considered mostly as continuous scalar quantity fields with unit sampling interval in the time domain. Whereas DT distributed parameter step responses $\{\mathcal{H}H_i(\overline{x},k)\}_i$ of HLDS can be computed by common analytical or numerical methods, then DT distributed parameter impulse responses can be obtained as

$$\{\mathcal{G}H_i(\overline{x},k)=\mathcal{H}H_i(\overline{x},k)-\mathcal{H}H_i(\overline{x},k-1)\}_i \tag{3}$$

For points $\{\overline{x}_i=(x_i,y_i,z_i)\}_i$ located in surroundings of lumped input quantities $\{U_i(t)\}_i$, where partial distributed transient responses $\{\mathcal{H}_i(\overline{x}_i,t)\}_i$ attains maximal amplitudes, partial distributed output quantities are obtained in time-domain and next either continuous $\{S_i(\overline{x}_i,s)\}_i$, or discrete transfer functions $\{SH_i(\overline{x}_i,z)\}_i$ with sampling period T are identified.

$$\{Y_i(\overline{x}_i,t)=\mathcal{G}_i(\overline{x}_i,t)\otimes U_i(t)\}_i \quad\to\quad \{Y_i(\overline{x}_i,s)=S_i(\overline{x}_i,s)U_i(s)\}_i \tag{4}$$

$$\{Y_i(\overline{x}_i,k)=\mathcal{G}H_i(\overline{x}_i,k)\oplus U_i(k)\}_i \quad\to\quad \{Y_i(\overline{x}_i,z)=SH_i(\overline{x}_i,z)U_i(z)\}_i \tag{5}$$

For the space dependency and in the steady-state we can define reduced transient step responses between i-th input quantity at point $\overline{x}_i=(x_i,y_i,z_i)$ and corresponding partial distributed output quantity in the steady-state:

$$\left\{\mathcal{H}HR_i(\overline{x},\infty)=\frac{\mathcal{H}H_i(\overline{x},\infty)}{\mathcal{H}H_i(\overline{x}_i,\infty)}\right\}_i \tag{6}$$

for $\{\mathcal{H}H_i(\overline{x}_i,\infty)\}_i\neq 0$.

Dynamics of LDS/HLDS is decomposed to the time and space components:

Time Components of Dynamics $\{S_i(\overline{x}_i,s)\}_i$, or $\{SH_i(\overline{x}_i,z)\}_i$ - for given i and chosen $\overline{x}_i$

Space Components of Dynamics $\{\mathcal{H}HR_i(\overline{x},\infty)\}_i$ - for given i in ∞

2.2. Feedback control loop based on HLDS dynamics

Decomposition of dynamics enables also to decompose the control synthesis (CS) to time synthesis (TS) and space synthesis (SS) tasks in the feedback control loop of the distributed parameter system, Fig. 2.

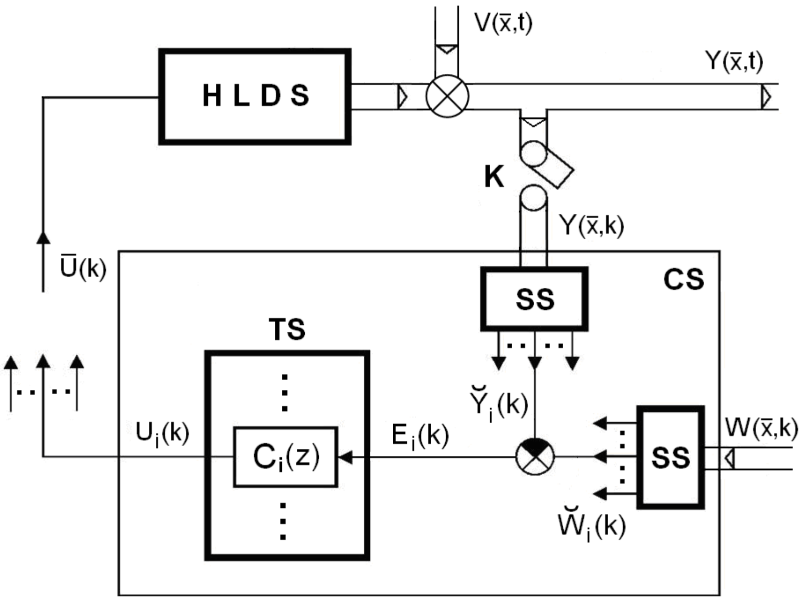


Figure 2. Distributed parameter feedback control loop: HLDS - LDS with zero-order holds $\{H_i\}_i$ on the input, CS - control synthesis, TS - control synthesis in time domain, SS - control synthesis in space domain, K - time/space sampling, $V(\bar{x},t)$ - disturbance quantity, $Y(\bar{x},t)$ - distributed controlled quantity, $Y(\bar{x},k)$ - sampled distributed controlled quantity, $\{\breve{Y}_i(k)\}_i$ - approximation parameters of controlled quantity, $W(\bar{x},k)$ - reference quantity, $\{\breve{W}_i(k)\}_i$ - approximation parameters of reference quantity, $\{E_i(k)\}_i$ - control errors, $\{C_i(z)\}_i$ - lumped parameter controllers, $\{U_i(k)\}_i$ - lumped control quantities

Let us consider a step change of distributed parameter control quantity $W(\bar{x},k)=W(\bar{x},\infty)$ and $V(\bar{x},t)=0$. The goal of the control synthesis is to generate a sequence of control inputs $\bar{U}(k)$ in such manner, that in the steady-state, for $k\to\infty$, the control error $E(\bar{x},k)$ will approach its minimal value $\left\|\breve{E}(\bar{x},\infty)\right\|$ in the quadratic norm:

$$\min\left\|E(\bar{x},\infty)\right\| = \min\left\|W(\bar{x},\infty)-Y(\bar{x},\infty)\right\| = \left\|\breve{E}(\bar{x},\infty)\right\| \tag{7}$$

First, in the SS blocks, the approximation both of sampled distributed controlled quantity $Y(\bar{x},k)$ and reference quantity $W(\bar{x},\infty)$, on the set of reduced steady-state distributed step responses $\{\mathcal{H}HR_i(\bar{x},\infty)\}_i$, are solved in following form:

$$\min_{Y_i}\left\|Y(\bar{x},k)-\sum_{i=1}^{n}Y_i(\bar{x}_i,k)\mathcal{H}HR_i(\bar{x},\infty)\right\| = \left\|Y(\bar{x},k)-\sum_{i=1}^{n}\breve{Y}_i(k)\mathcal{H}HR_i(\bar{x},\infty)\right\| \tag{8}$$

$$\min_{W_i}\left\|W(\bar{x},\infty)-\sum_{i=1}^{n}W_i(\bar{x}_i,\infty)\mathcal{H}HR_i(\bar{x},\infty)\right\| = \left\|W(\bar{x},\infty)-\sum_{i=1}^{n}\breve{W}_i\,\mathcal{H}HR_i(\bar{x},\infty)\right\| \tag{9}$$

Basis functions $\{\mathcal{H}HR_i(\bar{x},\infty)\}_i$ form a finite-dimensional subspace of approximation functions in the strictly convex normed linear space of distributed parameter quantities with quadratic norm, where the approximation problem is solved. From approximation theory involves, that solution of the approximation problems (8), (9) is guaranteed as a unique the best approximation in the form $\sum_{i=1}^{n} \breve{Y}_i(k)\mathcal{H}HR_i(\bar{x},\infty)$ with the vector of optimal approximation parameters $\{\breve{Y}_i(k)\}_i$ in task (8) and the best approximation in the form $\sum_{i=1}^{n} \breve{W}_i\,\mathcal{H}HR_i(\bar{x},\infty)$ with the vector of optimal approximation parameters $\{\breve{W}_i\}_i$ for approximation task (9).

Let us formulate a DPS control problem for the distributed reference quantity $W(\bar{x},\infty)$. When $W(\bar{x},k)$ is assumed, the space control synthesis is performed in each time step k, which gives $\{\breve{W}_i(k)\}_i$ parameters. Graphical interpretation of the approximation problem (9) for HLDS defined on 1D space $\bar{x}$ is on Fig. 3.

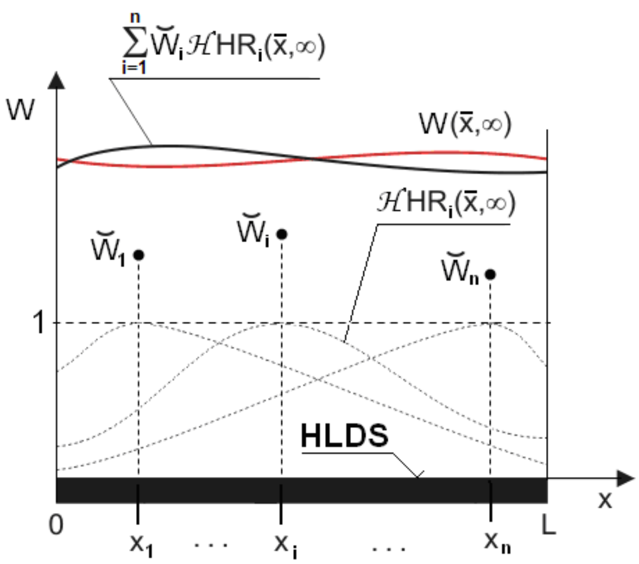


Figure 3. Solution of the approximation problem for the distributed reference quantity $W(\bar{x},\infty)$

Next, based on the solution of approximation problem, the vector of control error is created:

$$\bar{E}(k) = \{E_i(k)\}_i = \{\breve{W}_i - \breve{Y}_i(k)\}_i \tag{10}$$

The control errors vector $\bar{E}(k) = \{E_i(k)\}_i$ enters into the block TS, where the vector of control quantities, $\bar{U}(k) = \{U_i(k)\}_i$ is generated by controllers $\{C_i(z)\}_i$ in single-parameter control loops. During the control process, for $k \to \infty$ the control task (7) is accomplished.

Finally, we may state as a summary, that in the feedback control of DPS with dynamics represented in the form of HLDS, the control synthesis is performed as:

Space Tasks of Control Synthesis – as approximation tasks.
Time Tasks of Control Synthesis – on the level of lumped parameter control loops.

3. Robust control system

In general, a mathematical model for the plant dynamics is the basis for analysis and design of control systems. Also for LDS representation of DPS lumped and distributed models are used. However, in practice, no mathematical model describes exactly a physical process. It is obvious, that although no model represents the process exactly, some of them will do so with greater accuracy than others.

The theory of the robust control represents one of the possible approaches to the control system design in the presence of uncertainty. The goal of the robust system design is to retain a good quality of system performance in spite of model inaccuracies and changes. For the design techniques, the following requirements are supposed to be fulfilled: formulation of nominal plant model, different plant uncertainty models and requirements for both, robust stability and performance.

3.1. Sources of uncertainties in the LDS structure

LDS representation of DPS means decomposition of dynamics to space and time components. Uncertainties may occur in both, time and space components.

In distributed parameter control system, according to Fig. 2, single-input, single-output control loops in the block TS are tuned as closed feedback control loops using usual methods. In these loops, as models of the controlled system, transfer functions $\left\{S_i\left(\overline{x}_i, s\right)\right\}_i$ and/or $\left\{SH_i\left(\overline{x}_i, z\right)\right\}_i$ in the z-domain are used. These transfer functions describe the dynamics between sequences $\left\{U_i\left(k\right)\right\}_i$ and $\left\{Y_i\left(\overline{x}_i, k\right)\right\}_i$.

In this case, the sources of uncertainties are given by:

- procedure of dynamics modelling and possible change of parameters in models (4), (5)
- solution of approximation problem (8), (9), where lumped quantities are obtained

In order to treat uncertainties, it will be further assumed that the dynamic behaviour of a plant is described not by a single linear time invariant model, but by a family of linear time invariant models, Ψ_i . This family in the frequency domain, e.g. for models $\left\{S_i\left(\overline{x}_i, s\right)\right\}_i$, takes the following form:

$$\Psi_i = \left\{S_i : \left| \; S_i\left(\overline{x}_i, j\omega\right) \; - \; \tilde{S}_i\left(\overline{x}_i, j\omega\right) \; \right| \le \overline{L}_{ai}\left(\omega\right)\right\} \tag{11}$$

where $\tilde{S}_i\left(\overline{x}_i, j\omega\right)$ is the nominal plant model. Any member of the family Ψ_i fulfils the conditions:

$$S_i\left(\overline{x}_i, j\omega\right) = \tilde{S}_i\left(\overline{x}_i, j\omega\right) + L_{ai}\left(j\omega\right) \tag{12}$$

$$\left|L_{ai}\left(j\omega\right)\right| \le \overline{L}_{ai}\left(\omega\right) \quad , \; \forall \; S_i \in \Psi_i \tag{13}$$

where $L_{ai}\left(j\omega\right)$ is an additive uncertainty and $\overline{L}_{ai}\left(\omega\right)$ is the bound of additive uncertainty. If we wish to work with multiplicative uncertainties, we define the relations:

$$L_{mi}(j\omega) = \frac{L_{ai}(j\omega)}{\tilde{S}_i(\bar{x}_i, j\omega)} \; ; \qquad \bar{L}_{mi}(\omega) = \frac{\bar{L}_{ai}(\omega)}{\left|\tilde{S}_i(\bar{x}_i, j\omega)\right|} \tag{14}$$

where $L_{mi}(j\omega)$ is a multiplicative uncertainty and $\bar{L}_{mi}(\omega)$ is the bound of multiplicative uncertainty.

3.2. Design of IMC robust controllers

A robust control system for HLDS will be designed using the Internal Model Control (IMC) strategy (Morari & Zafiriou, 1989) with the general structure depicted in Fig. 4. a). It is possible to transform this structure to the classical feedback control loop, Fig. 4. b) and to incorporate it into the TS block of the DPS feedback control system. The relationship between the classical feedback controller C and the IMC controller Q for the nominal model $\tilde{P}$ of the controlled process P is as follows, and vice-versa:

$$Q = \frac{C}{1 + \tilde{P}C} \; ; \qquad C = \frac{Q}{1 - \tilde{P}Q} \tag{15}$$

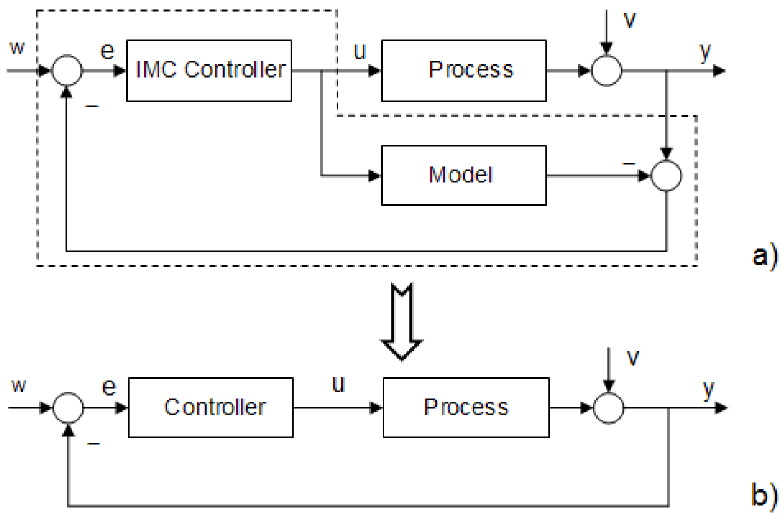


Figure 4. a) Internal Model Control structure, b) equivalent classical feedback control loop

It is well known that IMC strategy has the following properties:

1. *Dual stability*: Assume a perfect model, $(\tilde{P} = P)$ and if the controller and process are stable, then the IMC structure guarantees the closed-loop stability.
2. *Perfect control*: Assume a perfect model, $(\tilde{P} = P)$ and the closed-loop system is stable, while $Q = \tilde{P}^{-1}$, then there is no output steady-state error for set-point variance and disturbances.

The IMC structure thus provides the following benefits with respect to classical feedback: better dynamic response, system stability and robustness. One can search for Q instead of C without any loss of generality.

Structure of the distributed parameter feedback robust control system DPS with HLDS dynamics and IMC controllers is on Fig. 5.

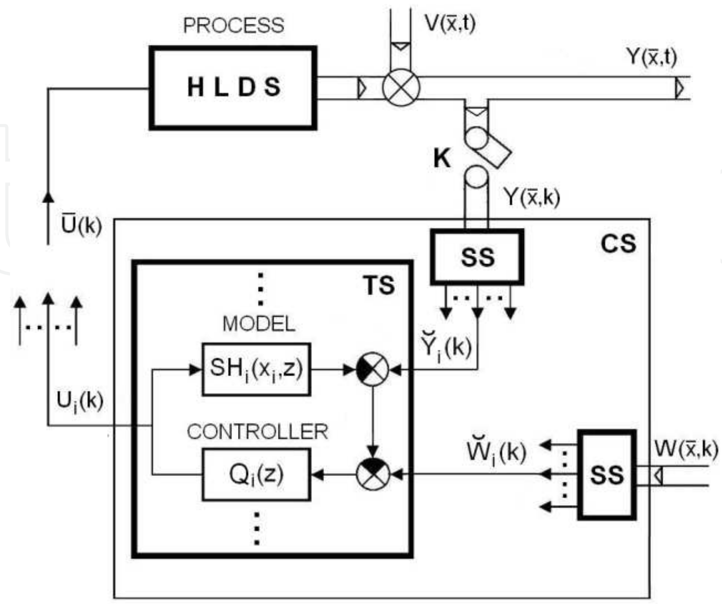


Figure 5. Distributed parameter feedback robust control system: HLDS - LDS with zero-order holds $\{H_i\}_i$ on the input, CS - control synthesis, TS - control synthesis in time domain, SS - control synthesis in space domain, K - time/space sampling, $V(\overline{x},t)$ - disturbance quantity, $Y(\overline{x},t)$ - distributed controlled quantity, $Y(\overline{x},k)$ - sampled distributed controlled quantity, $\{\breve{Y}_i(k)\}_i$ - approximation parameters of controlled quantity, $W(\overline{x},k)$ - reference quantity, $\{\breve{W}_i(k)\}_i$ - approximation parameters of reference quantity, $\{E_i(k)\}_i$ - control errors, $\{U_i(k)\}_i$ - lumped control quantities, $\{SH_i(x_i,z)\}_i$ - models of lumped controlled systems, $\{Q_i(z)\}_i$ - lumped parameter IMC controllers

H_2 optimal IMC controllers $\{Q_i(z)\}_i$ for inputs $\{\breve{W}_i(k)\}_i$ in the form unit-step function $\gamma(z)=\dfrac{z}{z-1}$ are obtained from solution of the following minimization problem:

$$\min_{Q_i(z)}\left\|e_i(z)\right\|_2=\min_{Q_i(z)}\left\|\left(1-SH_i(\overline{x}_i,z)Q_i(z)\right)\gamma_i(z)\right\|_2 \tag{16}$$

subject to the constraint $Q_i(z)$ to be stable and causal.

First, factorize the nominal stable transfer function $SH_i(x_i,z)$:

$$SH_i(\overline{x}_i,z)=SH_{Ni}(\overline{x}_i,z)\,SH_{Mi}(\overline{x}_i,z) \tag{17}$$

where $SH_{Ni}(\overline{x}_i,z)$ includes positive zeros or time-delays of the transfer function $SH_i(x_i,z)$. After this, optimal IMC controller $Q_i(z)$ is given by:

$$Q_i(z)=SH_{Mi}(\overline{x}_i,z)^{-1} \tag{18}$$

Finally, controller $Q_i(z)$ is augmented by low-pass filter $F_i(z)$ with parameter $0<\alpha_i<1$:

$$F_i(z) = \frac{1-\alpha_i}{z-\alpha_i} \tag{19}$$

Resulting IMC controller with filter $Q_{Fi}(z)$ is in following form:

$$Q_{Fi}(z) = Q_i(z)F_i(z) = Q_i(z)\frac{(1-\alpha_i)}{z-\alpha_i} \tag{20}$$

Parameter of the filter α_i is the only one tuning parameter to be selected by the user to achieve the appropriate compromise between performance and robustness and to keep the action of the manipulated variable within bounds. It must be chosen with respect to both, robust stability and robust performance condition:

$$\left|F_i\left(e^{j\omega T}\right)\right| < \left[\left|S_i(\overline{x}_i, j\omega)Q_i\left(e^{j\omega T}\right)\right|\overline{L}_{mi}(\omega)\right]^{-1}, \quad 0 \le \omega \le \frac{\pi}{T} \tag{21}$$

$$\left| Q_i(j\omega) \right| \overline{L}_{ai}(\omega) + \left|1 - S_i(\overline{x}_i, j\omega)Q_i(j\omega)\right| G_{wi}(\omega) < 1, \quad 0 \le \omega \le \frac{\pi}{T} \tag{22}$$

where $G_{wi}(\omega)$ is weighting function

For $SH_i(x_i, z) = SH_{Mi}(\overline{x}_i, z)$ and low-pass filter (19), robust controller $C_i(z)$ in equivalent classical feedback control loop takes form:

$$C_i(z) = \frac{SH_{Mi}(\overline{x}_i, z)^{-1} F_i(z)}{1 - SH_i(\overline{x}_i, z)\, SH_{Mi}(\overline{x}_i, z)^{-1} F_i(z)} = \frac{1}{SH_{Mi}(\overline{x}_i, z)} \cdot \frac{F_i(z)}{1 - F_i(z)} \tag{23}$$

4. Benchmark casting plant

The casting mould is one of the key components of a casting. It is well known, that the quality of the castings is affected strongly by the surface quality and the distribution of temperature in the mould, which has both time and space dependence. For study of the physical phenomena occurring during the casting solidification, from a DPS control point of view, control system development as well as mathematical model validation, a benchmark of the casting processes was designed (Belavý et al., 2009). At the study of casting processes and design of experimental plant, simulation studies in virtual software environments ProCAST and COMSOL Multiphysics were used.

4.1. Construction of the benchmark casting plant

Scheme of the benchmark casting plant is depicted in Fig. 6. The core item is the two-part steel mould of a complex-shape mounted in the frame of the ejector mechanism, Fig. 7. This mechanism is hinge-mounted to the main frame, to enable tilting of the mould for optimal filling and metal flow. Further, a hydraulic cooling circuit, which consists of an array of induction motor driven roller vane pumps, a bunch of hoses, a flow divider, a collector with built-in check valves, a plate heat exchanger and an expansion tank. The main cooling circuit is divided into five independent circuits thus enabling the control of heat extraction from the casting via the chills. The coolant flow is controlled by means of frequency converters, since volumetric pumps are used. The main heating circuit is also divided into five independent circuits.

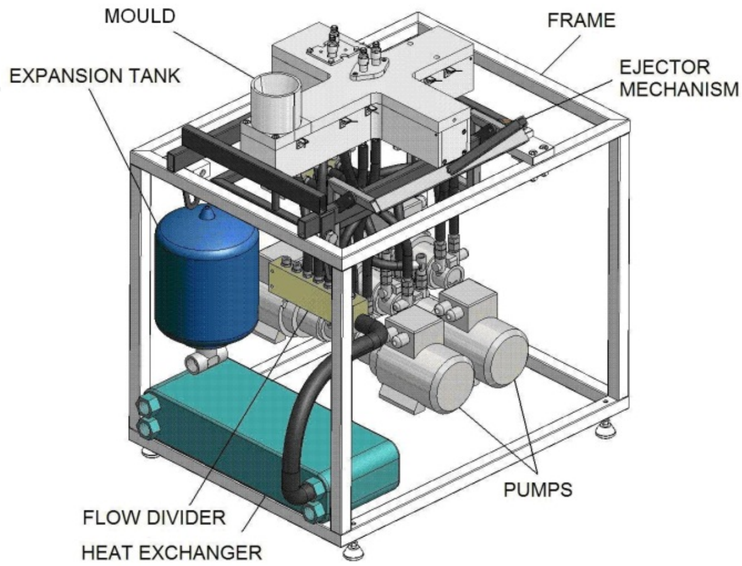


Figure 6. Scheme of the benchmark casting plant

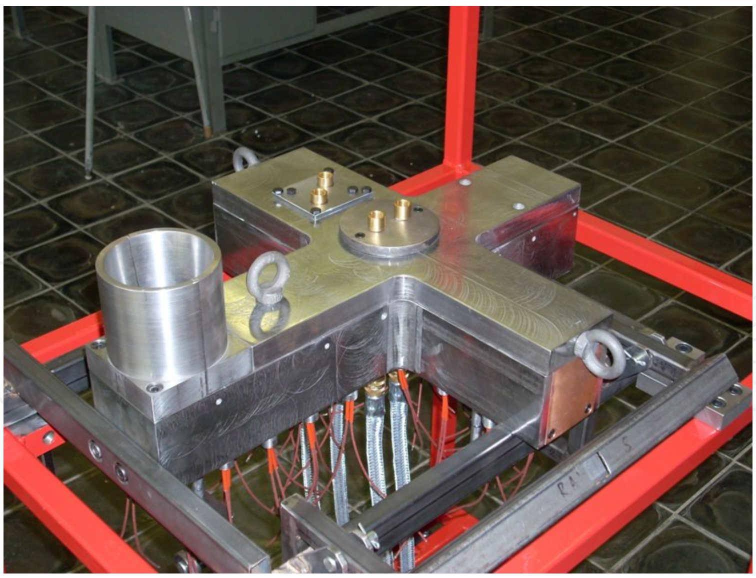


Figure 7. Two-part steel mould in detail

Inside of the casting mould are built-in 26 electric heating elements, each with maximal heating power 400 W. Heating elements are grouped to 5 zones and their heating power is actuated by the input voltage range of (0 – 10) V. In the body of the mould is also placed 7 water-cooled copper chills and 11 thermocouples, Fig. 8., Fig 9. Coordinates of measuring points of thermocouples in x-y plane are given in Tab. 1 and z - coordinate is -0.05 m. Location of built-in elements has been carefully designed based on simulation studies in software environments ProCAST and COMSOL Multiphysics, in order to have the possibility of preheating the mould in 5 zones achieving desired temperature profile as well as directional solidification of the casting by means of active heat removal. The temperature field in the mould-casting system is possible to estimate through interpolation of data, measured by thermocouples.

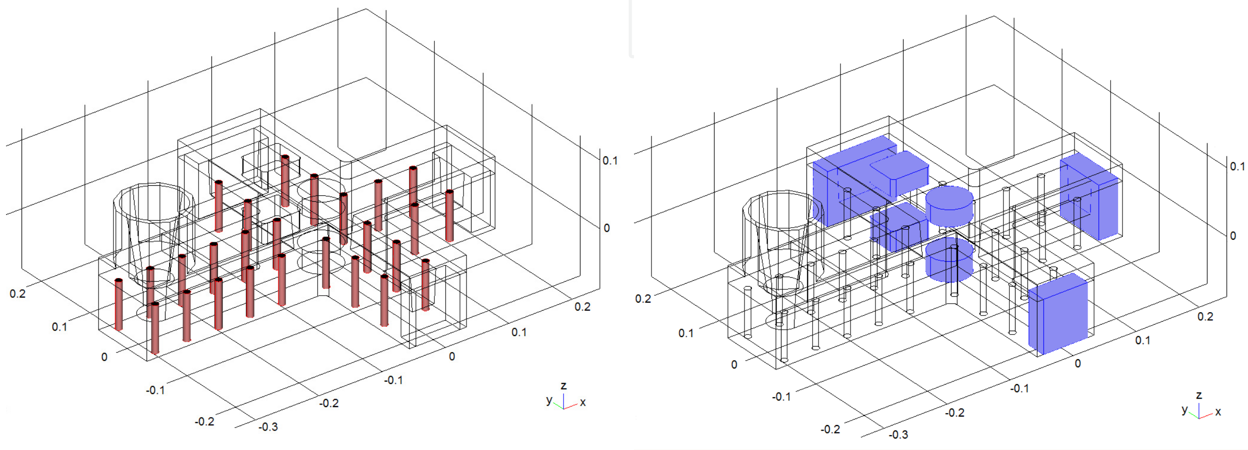


Figure 8. Location of heating elements (red) and copper chills (blue) in the mould

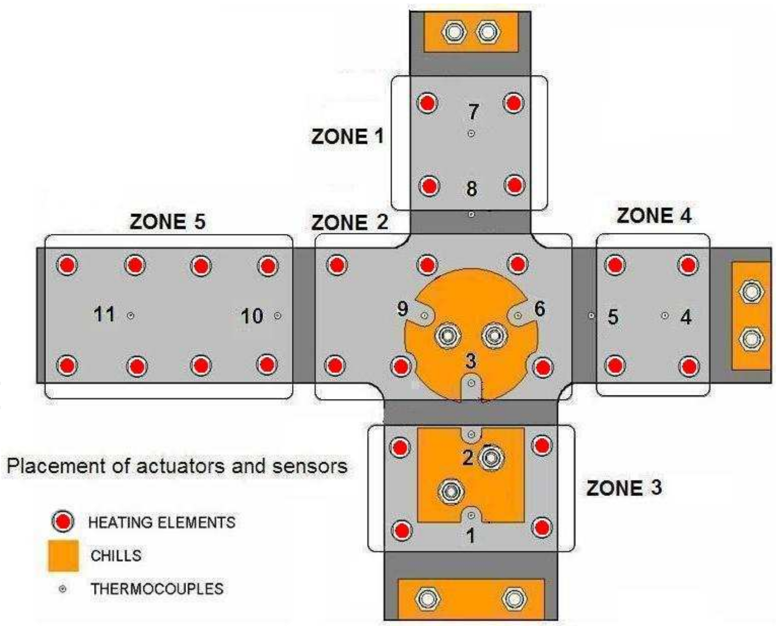


Figure 9. Bottom side of the steel casting mould

Position	1	2	3	4	5	6	7	8	9	10	11
x (m)	0	0	0	0.145	0.090	0.035	0	0	-0.035	-0.145	-0.255
y (m)	0.1585	0.0985	0.050	0	0	0	-0.135	-0.075	0	0	0

Table 1. Coordinates of thermocouples in x-y plane

4.2. Measurement and control scheme in MATLAB & Simulink

The measurement and control task of temperature field in the permanent casting mould was performed in the MATLAB & Simulink environment, where a *mould_exp_robust.mdl* scheme was setup, Fig. 10.

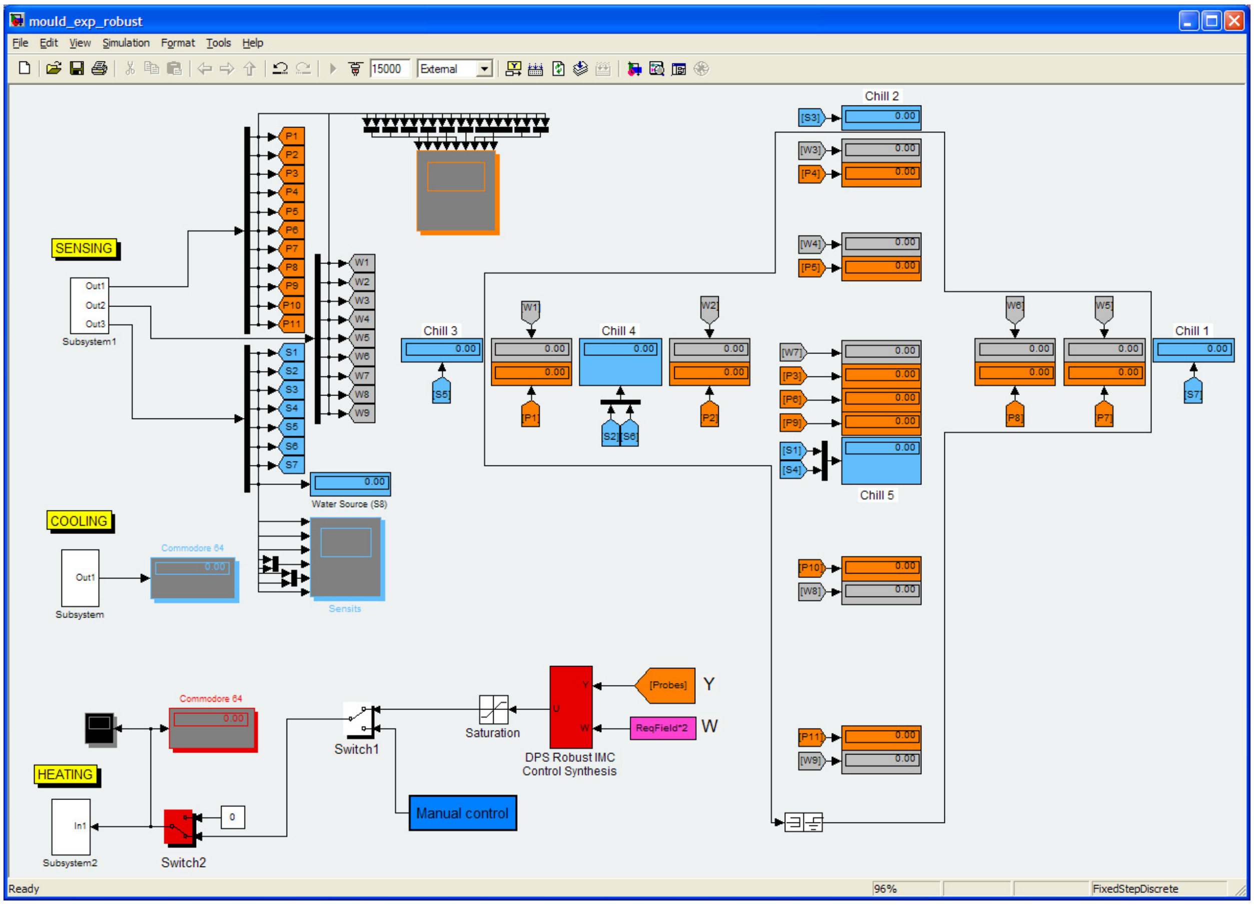


Figure 10. Measurement and control scheme in the MATLAB & Simulink

The scheme is composed of three main subsystems, namely: SENSING, HEATING and COOLING. Utilizing these subsystems and communication interface between process and computer, it is possible to measure dynamical characteristics of temperature field at zone heating. It is also possible to execute the experiment of controlled preheating of permanent mould before casting operation and controlled cooling during casting solidification. The above mentioned communication interface consists of data acquisition cards Advantech PCI-1710 [Ah], PCI-1710 [Ch] as analog input and Humusoft AD622 [Eh] as digital input.

Communication interface is performed by means of Simulink Real-Time Windows Target Toolbox and the communication is triggered by *Connect to Target* icon.

The SENSING subsystem enables the temperature measurement by thermocouples No. 1 to 11, which are permanently located in the bottom part of the steel mould, Fig. 9. These are marked on the scheme as P1 to P11. The subsystem also enables temperature measurements during casting solidification by temporal thermocouples W1 to W9, located in the casting domain. Thermocouple sensing junctions are located in the middle of the arm cross section, right above permanent thermocouples "P", except the node of the casting, where one temporal thermocouple points to the center of the node. Sensors S1 to S7 measure the temperature of cooling water in embedded chills. *Display* blocks of the sensors "P" and "W" in the scheme correspond to the real positions of sensors. Measured temperatures are saved to the data file and continuously displayed on the *Scope* block.

The HEATING subsystem has two basic operational regimes, which are activated by *Switch2* block. In the manual control regime, it is possible to set heating performance in range (0÷10) Volt and measure the temperature transient characteristics with P1 to P11 thermocouples in given locations of the mould. The second regime activated by *Switch2* block enables to perform controlled preheating of the casting mould to desired temperature profile W defined in 11 points, where thermocouples P1 to P11 are located. The control task is performed by *DPS Robust IMC Control Synthesis* block.

5. Distributed Parameter Systems Blockset for MATLAB & Simulink

For the MATLAB & Simulink based software support of modelling, control and design of Distributed Parameter Systems given on complex 3D domains of definition, the programming environment **Distributed Parameter Systems Blockset for MATLAB & Simulink** (**DPS Blockset**) as Third-Party MathWorks Product has been developed by the Institute of Automation, Measurement and Applied Informatics, Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, within the program CONNECTIONS of The MathWorks Corporation, Fig. 11. , (Hulkó et al., 2003-2010).

The library of DPS Blockset shows Fig. 12. Blocks **HLDS** and **RHLDS** serve for modelling of distributed parameter systems as lumped-input/distributed-output systems with zero-order hold units. The block **DPS Control Synthesis** provides feedback to distributed parameter controlled systems in control loops with blocks for discrete-time **PID**, **Algebraic**, **State-Space** and **Robust Control**. The block **DPS Input** generates distributed quantities which can be used as distributed control quantities or distributed disturbances, etc. **DPS Display** presents distributed quantities with many options including export to AVI files. The block **DPS Space Synthesis** performs space synthesis as an approximation problem.

The block **DPS Wizard** in step-by-step operation, by means of several model examples with default parameters on 1D-3D definition domains, gives an automatized guide for arrangement and setting distributed parameter control loops. The block **Demos** contains examples oriented to methodology of modelling and control synthesis. The block **Show** contains motivation examples such as: *Control of temperature field of 3D metal body* (the controlled system was modelled in the virtual software environment COMSOL Multiphysics)**;** *Control of 3D beam of „smart" structure* (the controlled system was modelled in the virtual software environment ANSYS); *Adaptive control of glass furnace* (the controlled system was modelled by Partial Differential Equations Toolbox of the MATLAB), and *Groundwater remediation control* (the controlled system was modelled in the virtual software environment MODFLOW)**.** The block **Tutorial** presents methodological framework both for formulation and solution of control tasks for distributed parameter systems.

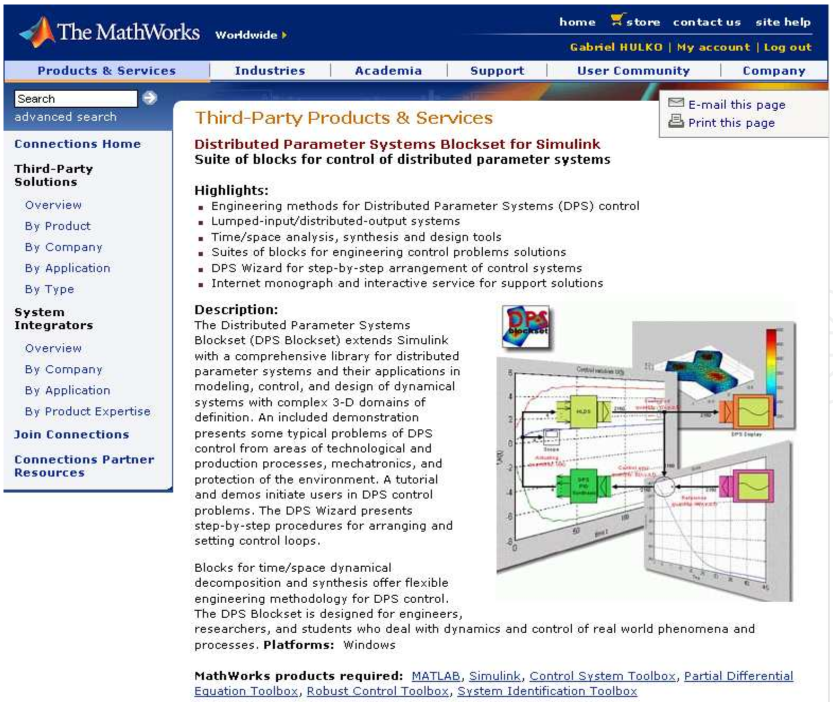


Figure 11. DPS Blockset on the web portal of The MathWorks Corporation

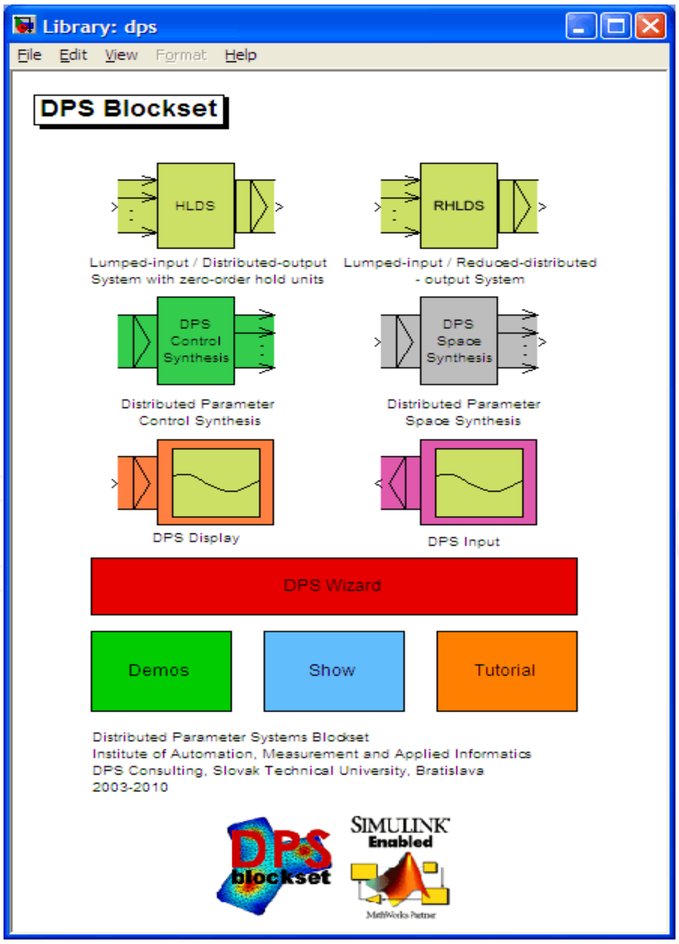


Figure 12. The library of DPS Blockset for MATLAB & Simulink

6. Dynamics of the temperature field in the casting mould

For control synthesis purpose, LDS/HLDS models of temperature fields in the casting mould have been created by means of evaluation of real-time experiments. The measurement of temperatures fields in the casting mould was performed by MATLAB & Simulink scheme *mould_exp_robust.mdl*, Fig. 10.

6.1. Experimental identification of transfer functions

In the casting mould lumped inputs $\{U_i\}_{i=1,5}$ are heating elements which act on sub-domains $\{\Omega_i\}_{i=1,5}$, (Zone 1 - 5). Distributed output is the temperature field of the casting mould.

Temperatures in the casting mould were measured by 11 thermocouples as a time-response to the step change of heating power, which was activated by the input voltage step from 0 to 2,5 V, for heating elements separately in each zone. Results of measurements for Zone #1, #3 and #5 are depicted in Fig. 13, where are time and *x-y* space dependences of temperatures and Fig. 14 presents temperature profiles in steady-states.

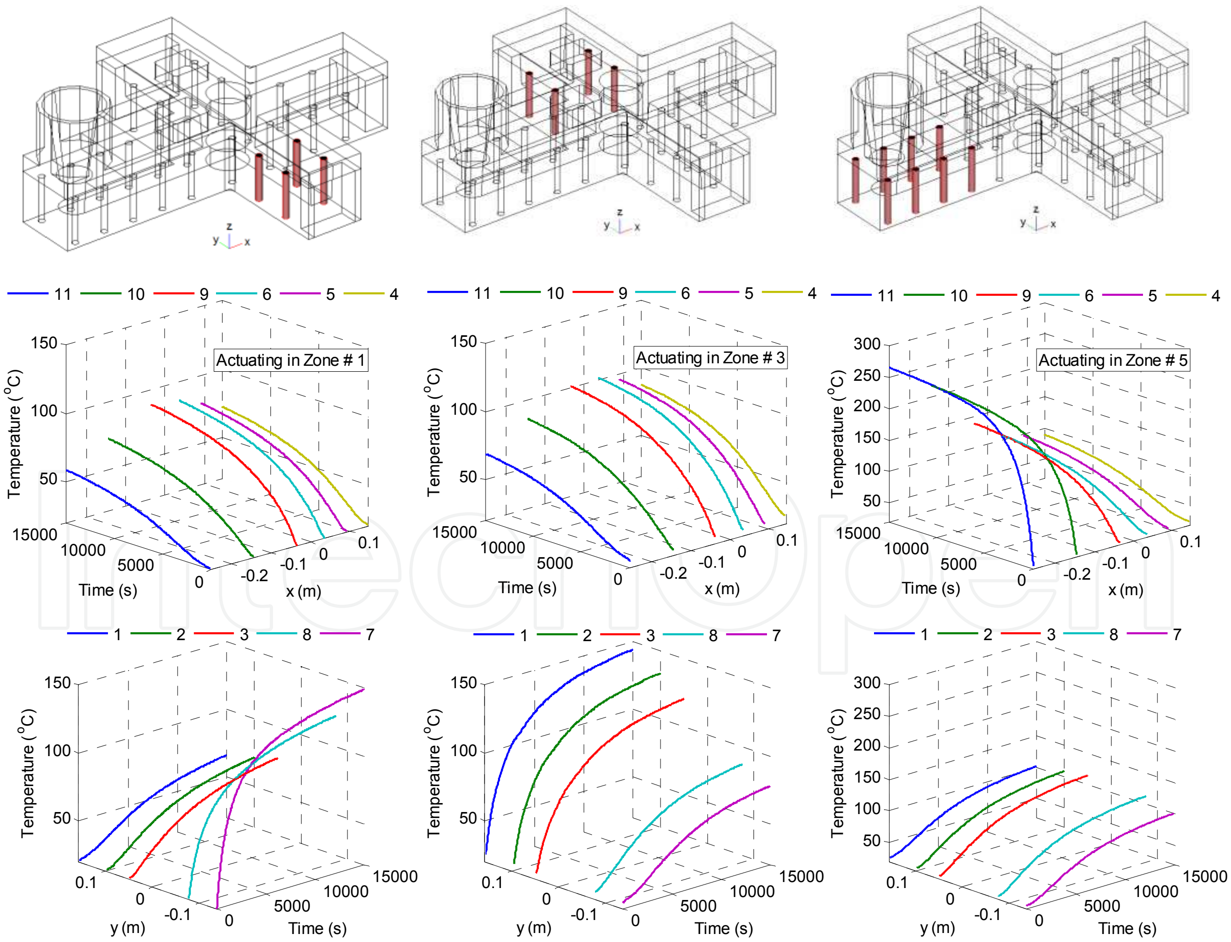


Figure 13. Time course of temperatures measured by thermocouples No. 1 - 11 after step change of the heating power separately in Zone #1, #3 and #5

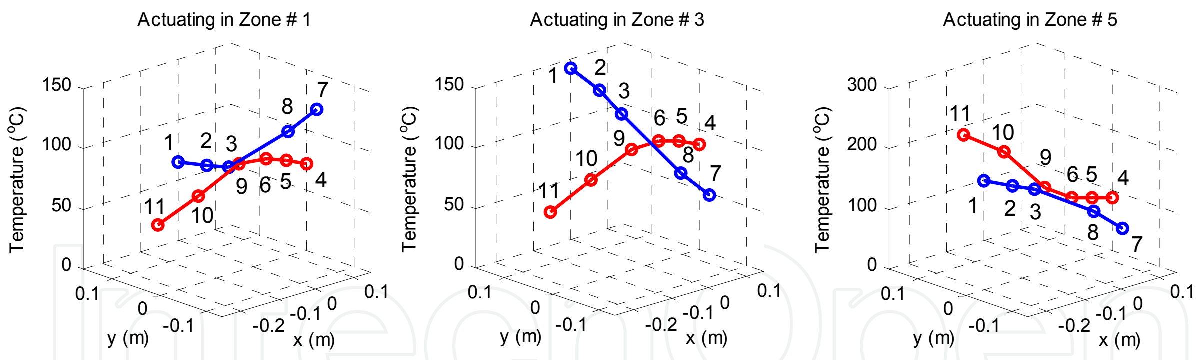


Figure 14. Temperature profiles in the steady-state measured by thermocouples No. 1 - 11 after step change of the heating power separately in Zone #1, #3 and #5

For identification of i-th transfer functions $\left\{S_i\left(\overline{x}_i,s\right)\right\}_{i=1,5}$, were determined points $\left\{\overline{x}_i=\left(x_i,y_i\right)\right\}_{i=1,5}$, located in each zone closely of lumped input quantities $\left\{U_i\right\}_{i=1,5}$, where temperatures attain maximal amplitudes by actuating the heating power in each zone separately. These points for actuating of the heating power in each zone are represented by positions of thermocouples given in Table 2.

Zone No.	1	2	3	4	5
Thermocouple No.	7	9	1	4	11

Table 2. Position of thermocouples for identification of measured temperatures

Identification of measured dynamical characteristics of temperatures was performed in the MATLAB software environment, where graphical user interface (GUI) *ident* from System Identification Toolbox was activated. There after importing the time domain input/output data, from the pop-up menu *Process models* transfer function in the form (24) for identification has been chosen, see Fig. 15., where are also results of identification from actuating in zone #1. Comparison of measured and identified model output in zone #1 is presented in Fig. 16. Identified parameters of transfer functions $\left\{S_i\left(\overline{x}_i,s\right)\right\}_{i=1,5}$ are in Table 3. Continuous transfer functions with structure (24) are converted by means of function *zpk* to zero-pole-gain format (ZPK). Then, for control synthesis purposes, they are transformed to discrete transfer functions $\left\{SH_i\left(\overline{x}_i,z\right)\right\}_{i=1,5}$ with sample time T= 10 s.

$$\frac{K\left(Tz\,s+1\right)}{\left(Tp_1\,s+1\right)\left(Tp_2\,s+1\right)} \tag{24}$$

Zone No.	1	2	3	4	5
K	53.40	58.16	49.36	62.56	97.87
Tp_1	5881.68	4945.00	5489.58	5340.97	4578.71
Tp_2	939.66	1958.20	1398.20	973.90	1214.73
Tz	3616.81	3301.80	3201.10	3195.90	3410.50

Table 3. Identified parameters of transfer functions $\left\{S_i\left(\overline{x}_i,s\right)\right\}_{i=1,5}$ for actuating in each zone

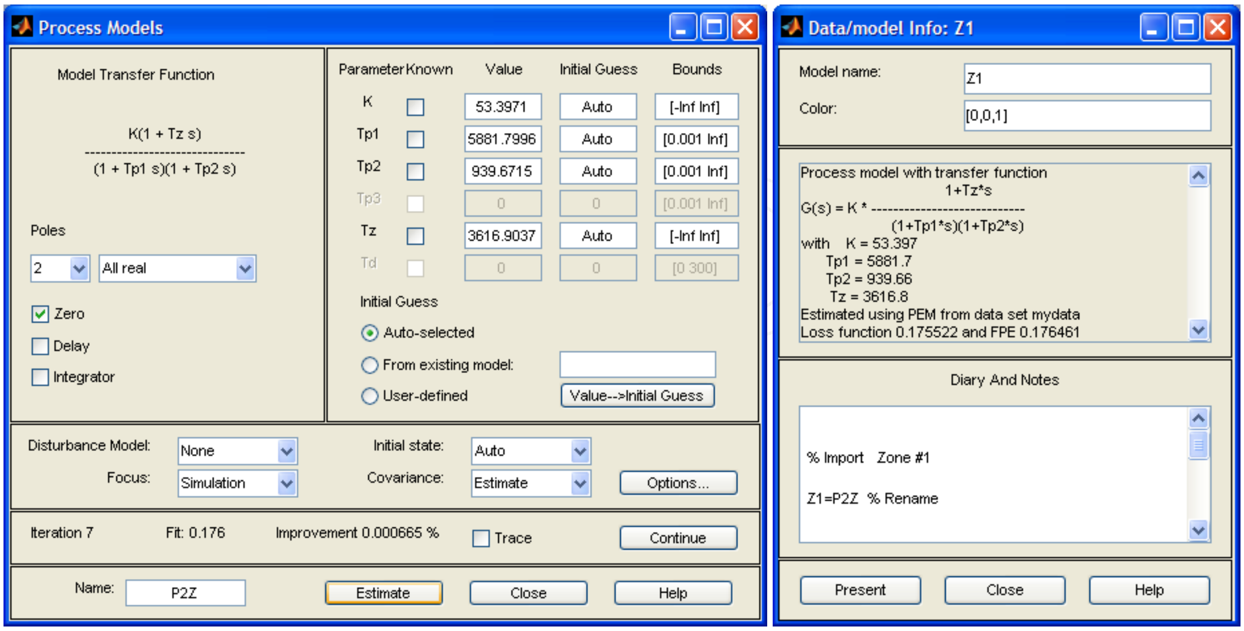


Figure 15. GUI Process Models menu and results of identification $S_i(\overline{x}_i, s)$ in Zone #1

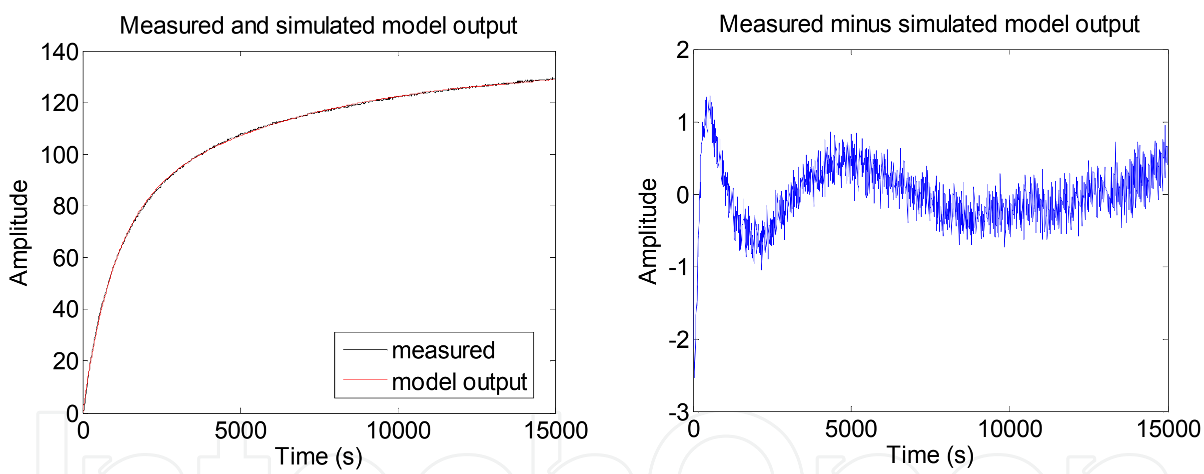


Figure 16. Comparison of measured and identified model output in Zone #1

6.2. Uncertainty analysis in the space and time domain

The proposed structure of the DPS control systems, Fig. 2, Fig. 5., are significant by decomposition of the control synthesis into the space and time subtasks. In the SS blocks, the approximation both of distributed controlled quantity $Y(\overline{x},k)$ and reference quantity $W(\overline{x},\infty)$, formulated as (8), (9) is solved. As the best approximation of controlled quantity $Y(\overline{x},k)$ vector of optimal approximation parameters $\{\breve{Y}_i(k)\}_i$ is obtained. Dynamics of these lumped quantities is different in compare with lumped quantities, $\{Y_i(\overline{x}_i,k)\}_i$, given by transfer functions $\{SH_i(\overline{x}_i,z)\}_i$ thus is created an uncertainty region in the time domain.

For uncertainty analysis, which takes place during the approximation problem solution, scheme from both, DPS Blockset blocks and Simulink blocks was arranged, see Fig. 17. There were obtained both, step responses $\{Y_i(\overline{x}_i,k)\}_i$ from each lumped input, $\{U_i\}_{i=1,5}$, to the corresponding output and approximated quantities corresponding to lumped output $\{\breve{Y}_i(k)\}_i$ from the block SS. Characteristics with uncertainty regions for actuating in each zone are depicted in Fig. 18.

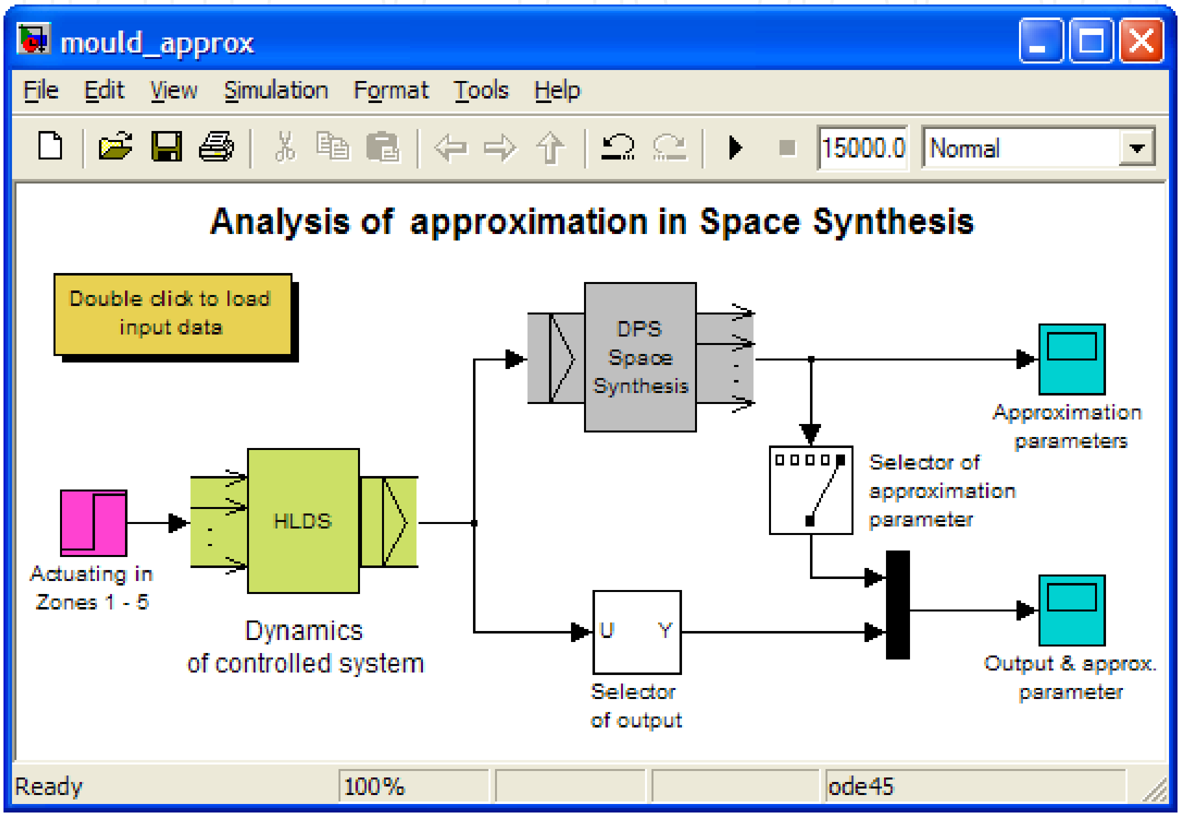


Figure 17. Block scheme in the MATLAB & Simulink for the analysis of approximation in the space synthesis and step responses in Zones 1 – 5

Identified transfer function $\{S_i(\overline{x}_i,s)\}_{i=1,5}$ in the structure (24) were also analysed in terms of uncertainties of their parameters. Analysis was performed in MATLAB environment, where functions from Robust Control Toolbox were used. Using functions *ureal* and *gridureal* were generated families of step responses with defined percentage variability of parameters in the nominal transfer functions $\{S_i(\overline{x}_i,s)\}_{i=1,5}$. Results for zone 1 are depicted in Fig. 19.

The gain variability K strongly affects to the value of the transient response in steady-state. The variability of the time constants Tp1 and Tp2 influences dynamics of transient response. Uncertainty region is caused by solution of the approximation task in the space synthesis. It would be appropriate to cover it by variability of nominal transfer function parameters. Cover of the uncertainty region in the time domain by variability of nominal parameters of the transfer function $S_i(\overline{x}_i,s)$ in Zone #1, #3 and #5 is presented in Fig. 20.

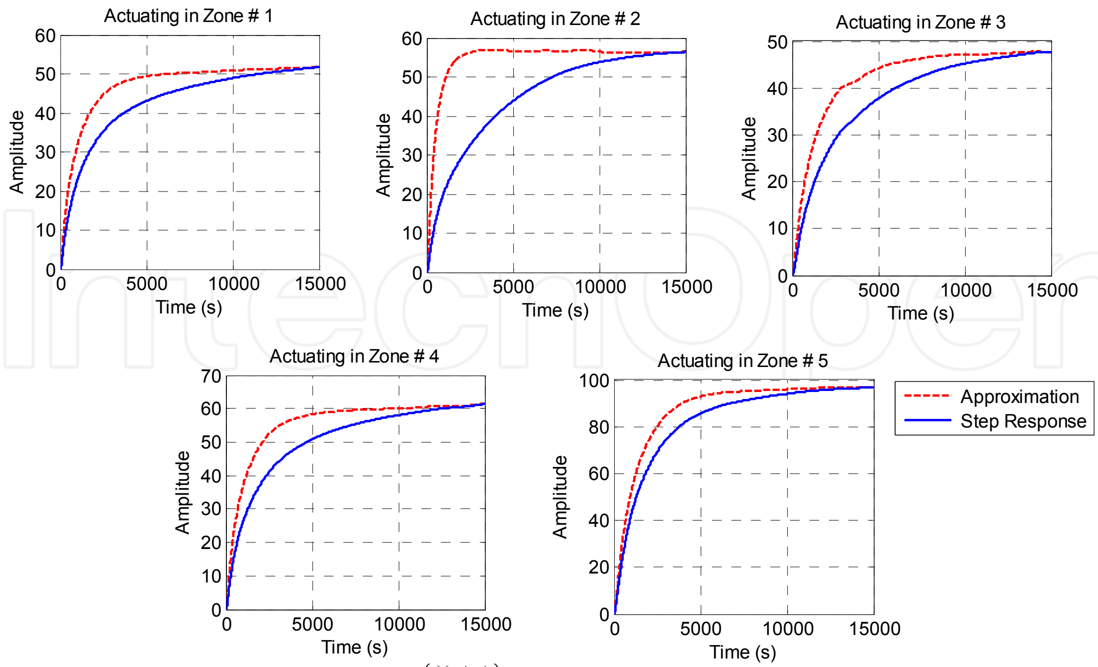


Figure 18. Approximation parameters $\{\breve{Y}_i(k)\}_i$ as a result of DPS space synthesis and step responses $\{\mathcal{H}H_i(\overline{x}_i,k)\}_i$ for actuating in Zones 1 – 5

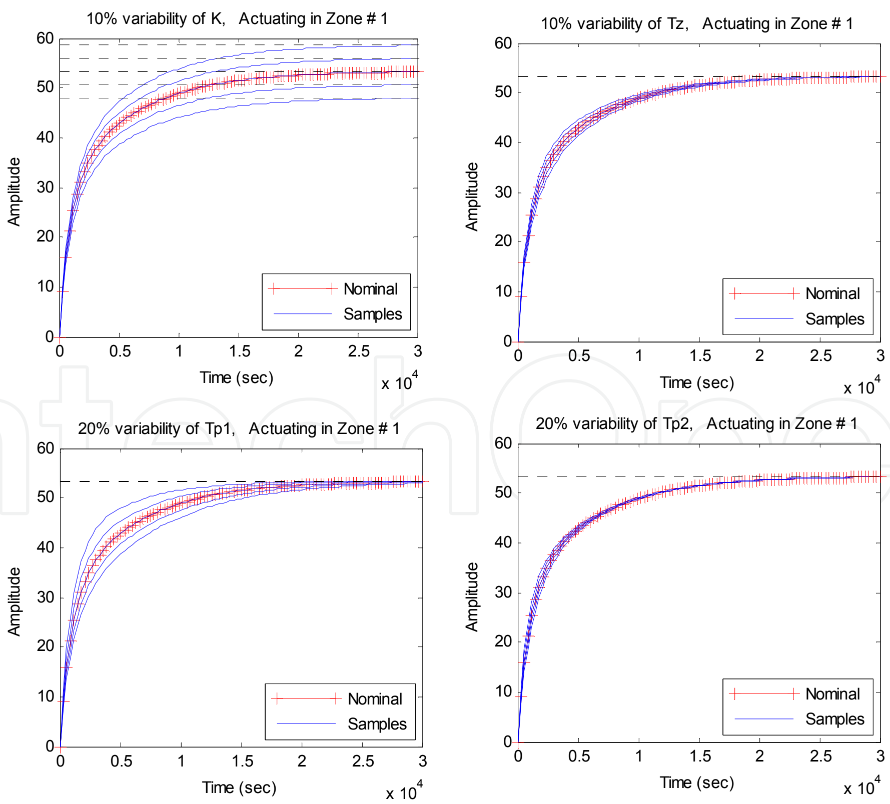


Figure 19. Family (Samples) of step responses for the defined percentage variability of parameters in the nominal transfer function $S_1(\overline{x}_i,s)$

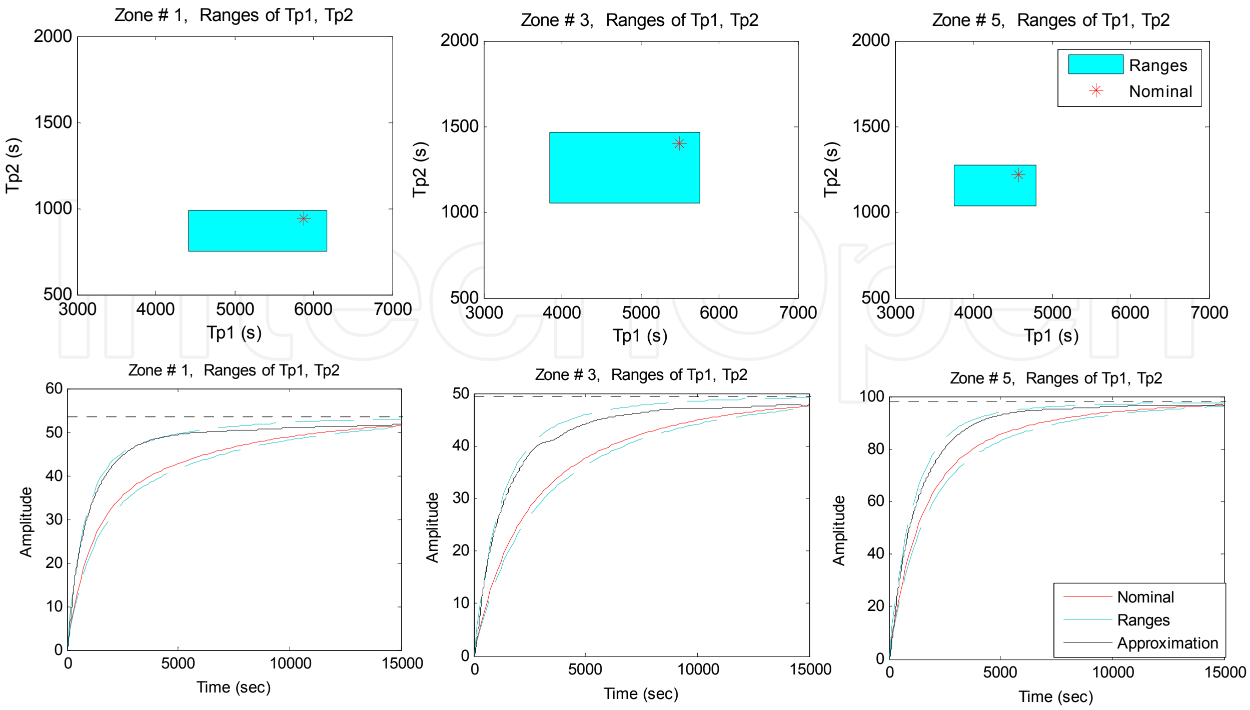


Figure 20. Cover of the uncertainty region in the time domain by variability of the nominal parameters Tp1 and Tp2 of the transfer function $S_i(\overline{x}_i, s)$ in Zone #1, #3 and #5

7. Robust control process

Robust control of temperature fields of the casting mould as distributed parameter system was performed with MATLAB & Simulink and DPS Blockset software support. Robust controllers designed by the IMC control strategy were first optimised through their tuning parameters $\{\alpha_i\}_i$ and then implemented to the control scheme for the real-time control of temperature fields of the casting mould in the benchmark casting plant.

7.1. Optimization of tuning parameters

In the MATLAB & Simulink environment, by means of the DPS Blockset, distributed parameter system of robust control, *mould_robust_DZPK.mdl* was arranged, see Fig. 21. It is DPS feedback control loop, where the *DPS Robust Control Synthesis* block includes both, time and spatial part of the control synthesis, see Fig. 22 a). In this case, the control system consists of five single parameter control loops, each for one zone of the mould, where discrete robust controllers based on IMC structure are used.

DPS Robust Control Synthesis contains two blocks named *DPS Space Synthesis*, where approximation of distributed controlled quantity $Y(\overline{x}, k)$ and reference quantity, $W(\overline{x}, \infty)$ is executed. The time control synthesis is performed by *Robust controllers based on IMC* block with five discrete controllers given by ZPK transfer function and filters with parameters $\{\alpha_i\}_{i=1,5}$, see Fig. 22 b).

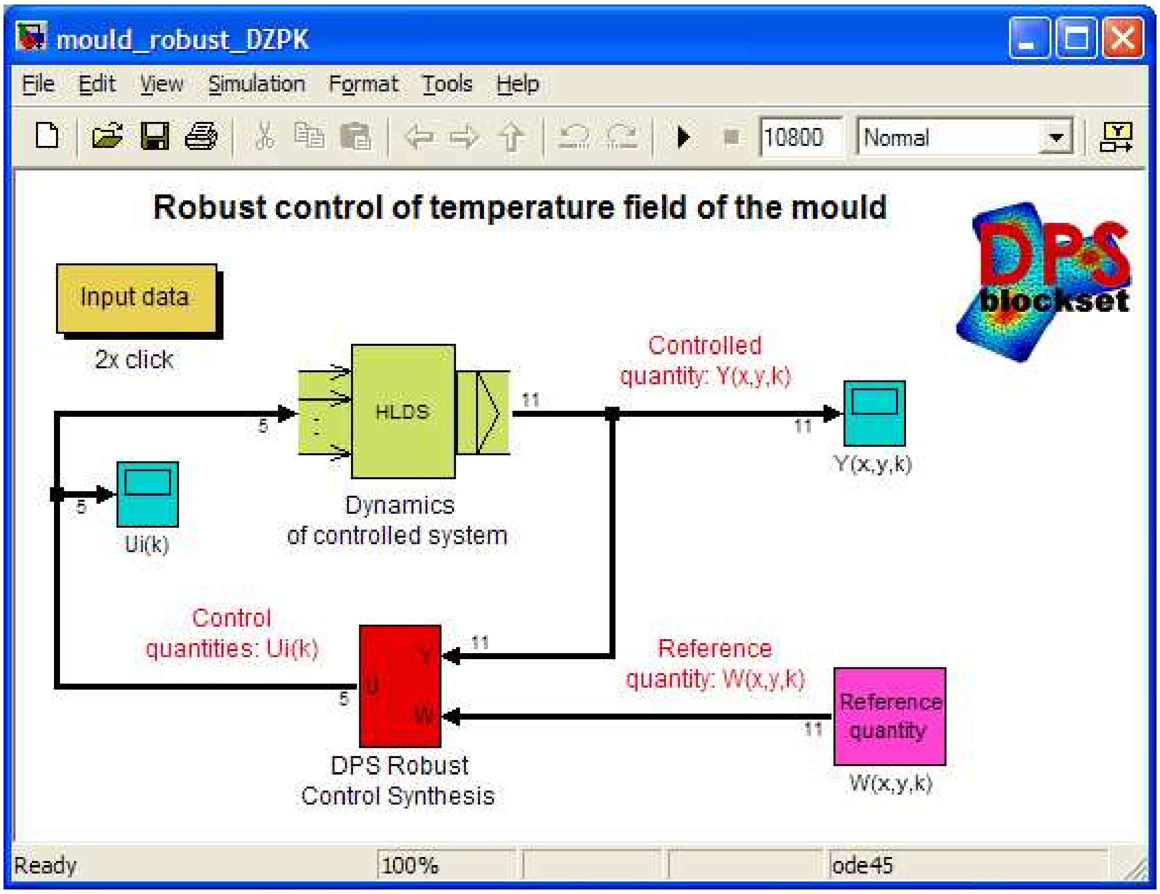


Figure 21. DPS feedback robust control system in the DPS Blockset for simulation of robust control of the temperature fields in the casting mould

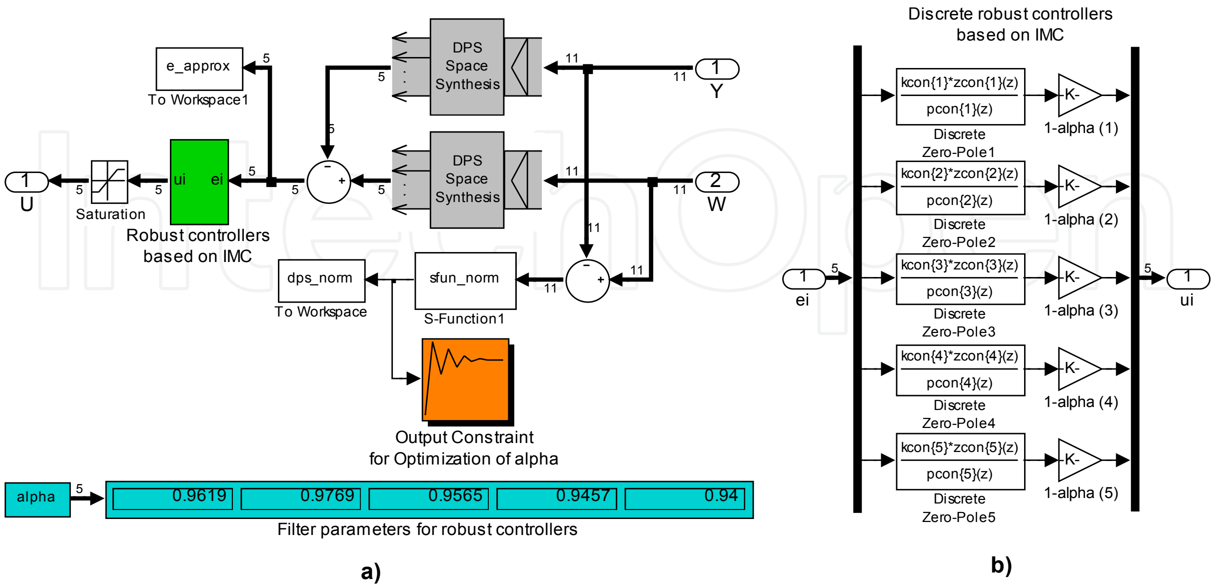


Figure 22. Structure of blocks in the scheme mould_robust_DZPK.mdl: a) DPS Robust Control Synthesis, b) Robust controllers based on IMC

The block *DPS Robust Control Synthesis* also contains block *Output Constraint for optimization of alpha*, where optimization of parameters $\{\alpha_i\}_{i=1,5}$ according to criterion function (25) is performed.

$$J = \min_{\alpha_i} \sum_{k=0}^{N} \left\| W(\overline{x},k) - Y(\overline{x},k) \right\| \tag{25}$$

Parameters of filters were optimized in the presence of constraints of the criterion function in order to assure nearly aperiodic course of the quadratic norm of the distributed control error with respect to the robust stability and robust performance conditions, see Fig. 23 and optimization progress presents Fig. 24. Control process was simulated for the reference quantities - temperatures on given 11 positions, where thermocouples are embedded. Robust stability was tested for ranges around the nominal parameters Tp1 and Tp2, of the transfer function $S_i(\overline{x}_i, s)$, see Fig. 20, with MATLAB function *robuststab*, from the Robust Control Toolbox, e.g. in the following is the printout of the robust stability testing for the control loop in zone 1.

Robust stability testing in Zone #1
StabilityMargin = UpperBound: 1.3333
LowerBound: 1.3296
DestabilizingFrequency: 8.0745e-004
DestbUnc = Tp1: 1.7210
Tp2: 438.5898
STABreport = Uncertain System is robustly stable to modeled uncertainty.
- It can tolerate up to 133% of the modeled uncertainty.

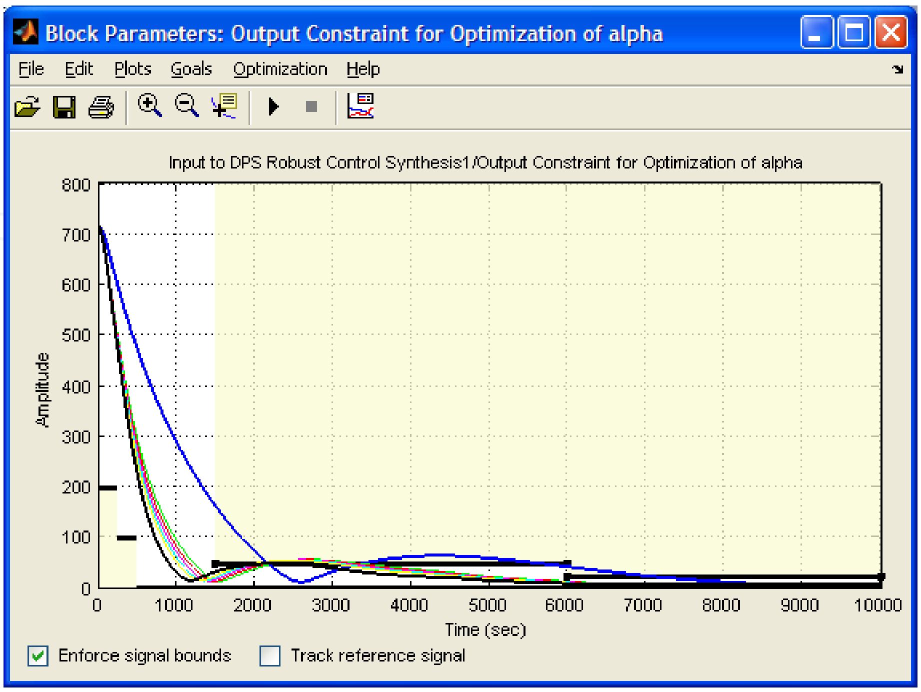
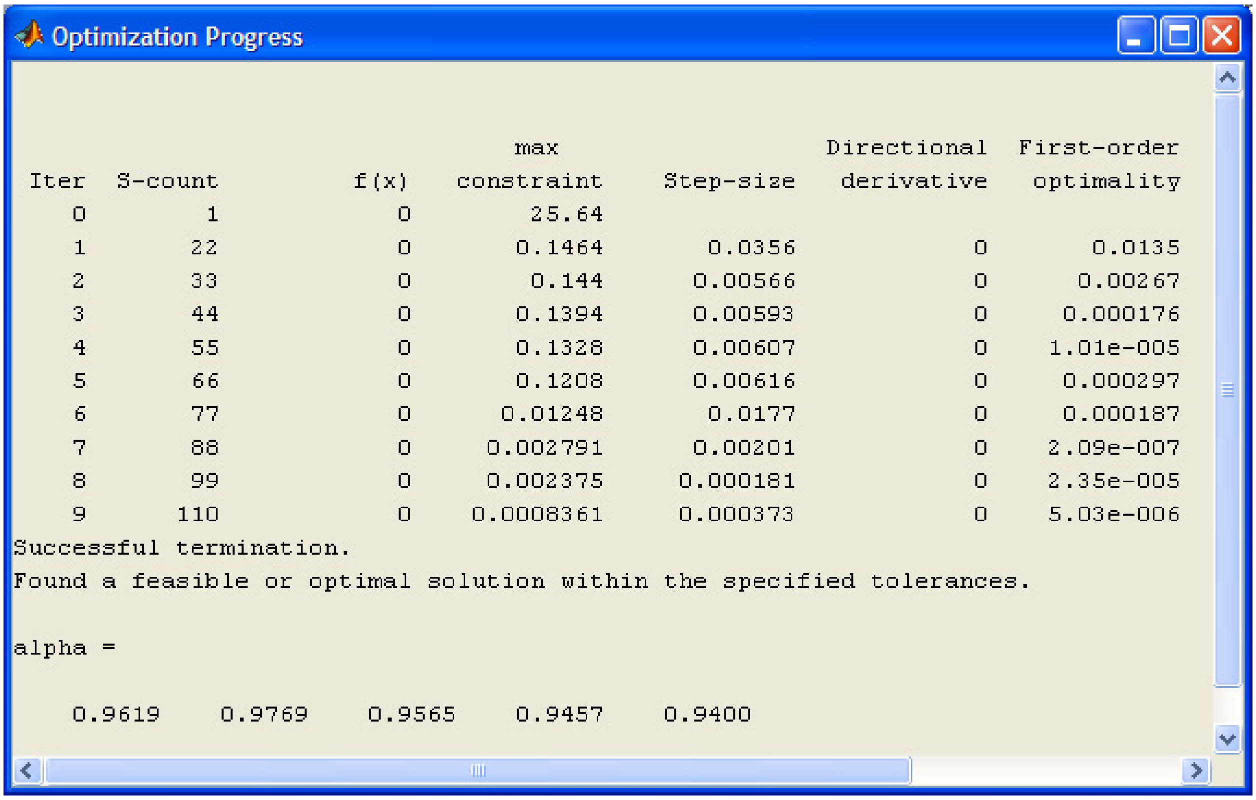


Figure 23. Minimization of the criterion function in presence of output constraints



Optimization Progress

Iter	S-count	f(x)	max constraint	Step-size	Directional derivative	First-order optimality
0	1	0	25.64			
1	22	0	0.1464	0.0356	0	0.0135
2	33	0	0.144	0.00566	0	0.00267
3	44	0	0.1394	0.00593	0	0.000176
4	55	0	0.1328	0.00607	0	1.01e-005
5	66	0	0.1208	0.00616	0	0.000297
6	77	0	0.01248	0.0177	0	0.000187
7	88	0	0.002791	0.00201	0	2.09e-007
8	99	0	0.002375	0.000181	0	2.35e-005
9	110	0	0.0008361	0.000373	0	5.03e-006

```
Successful termination.
Found a feasible or optimal solution within the specified tolerances.

alpha =

    0.9619    0.9769    0.9565    0.9457    0.9400
```

Figure 24. Optimization progress for tuning parameters $\{\alpha_i\}_{i=1,5}$ of robust controllers

- A destabilizing combination of 133% of the modeled uncertainty exists, causing an instability at 0.000807 rad/s.
- Sensitivity with respect to uncertain element ...
  'Tp1' is 100%. Increasing 'Tp1' by 25% leads to a 25% decrease in the margin.
  'Tp2' is 54%. Increasing 'Tp2' by 25% leads to a 14% decrease in the margin.

7.2. Real-time robust control of preheating in the casting mould

Real-time robust control of the temperature fields of the casting mould in the benchmark casting plant was performed by means of Simulink designed block scheme *mould_exp_robust.mdl*. The structure of the block *DPS Robust IMC Control Synthesis* is depicted in Fig. 25, where are blocks *Space Synthesis Y* and *Space Synthesis W*, and block *Robust controllers based on IMC* with the same structure as in Fig. 22 b).

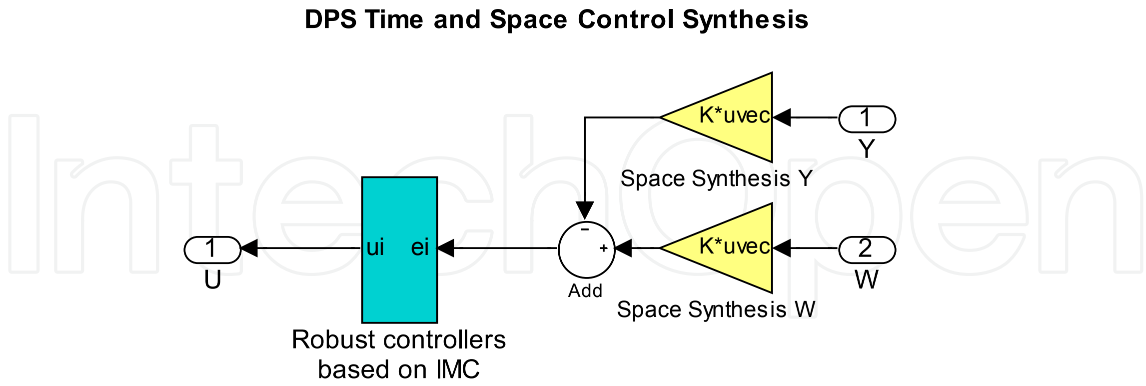


Figure 25. Structure of the block DPS Robust IMC Control Synthesis in the scheme mould_exp_robust.mdl for real-time control

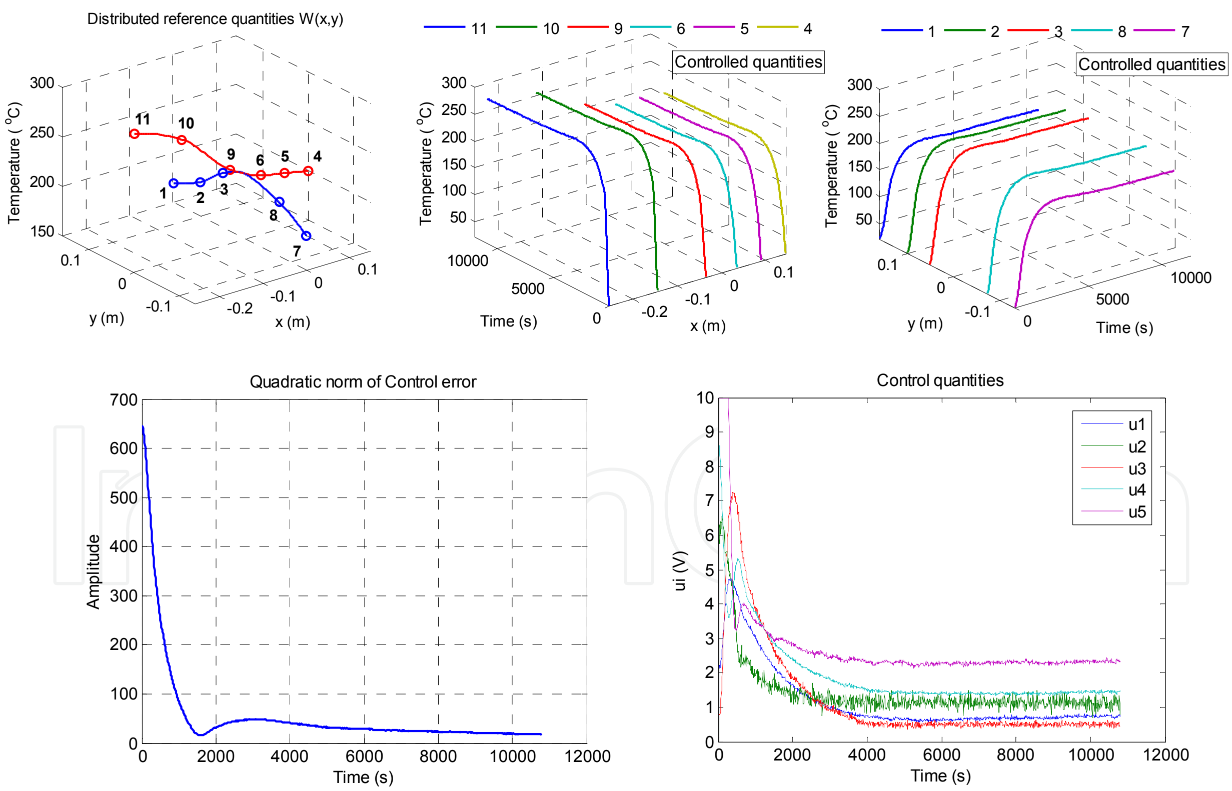


Figure 26. Robust control of the temperature field of the casting mould: distributed reference quantities, controlled quantities at given position of the mould, quadratic norm of distributed control error $\left\| E(\bar{x},k) \right\|$ and control quantities $\{U_i(k)\}_{i=1,5}$ in Zones 1-5

Control process was performed for the reference variable - temperature profile given in 11 positions, where thermocouples are embedded. Results of the real-time robust control process are on Fig. 26. The quality of control both in the time and space domain is given by the quadratic norm of the distributed control error.

8. Conclusion

The aim of this chapter was to present the engineering approach for the robust control of DPS, which opens a wide space for novel applications of the toolboxes and blocksets of the MATLAB & Simulink software environment. This approach is based on the general decomposition of controlled DPS dynamics, represented by transient and impulse characteristics, into time and space components. Starting out from this dynamics decomposition a methodical framework was presented for the decomposition of control synthesis into the space and time subtasks. In the space domain an approximation problems were solved, while in the time domain the control synthesis was performed by lumped parameter SISO control loops, where various well-known methods for design of controller is possible to utilize. The advantage of this approach is the relatively simple LDS model of DPS, which is directly suitable for control purposes and can be easily identified from input-output data by means of classical techniques.

Currently, it is interesting to formulate and solve tasks of control in various engineering branches, including the casting technology, by means of methods and tools of distributed parameter systems. Methodical approach presented in this chapter demonstrates simple possibilities, how to exploit the distributed dynamical characteristics on complex definition domains, obtained by evaluation of measured data for robust IMC control synthesis of DPS with respect to uncertainty of models and the real-time control according to technological requirements.

Author details

Cyril Belavý, Gabriel Hulkó and Karol Ondrejkovič
Institute of Automation, Measurement and Applied Informatics,
Faculty of Mechanical Engineering, Center for Control of Distributed Parameter Systems,
Slovak University of Technology in Bratislava, Slovak Republic

Acknowledgement

This work was supported by the Slovak Scientific Grant Agency VEGA under the contract No. 1/0138/11 and the Slovak Research and Development Agency under contracts No. APVV-0131-10 and No. APVV-0090-10 and also by the European Union with the co-financing of the European Social Fund, grant TAMOP-4.2.1.B-11/2/KMR-2011-0001.

9. References

Belavý, C. et al. (2009). Uncertainty Analysis and Robust Control of Temperature Fields of Casting Mould as Distributed Parameter Systems, *INCOM ´09: Preprints of the 13th IFAC symposium on Information control problems in manufacturing,* Moscow, June 3-5, 2009

Butkovskij, A. G. (1965). *Optimal Control of Distributed Parameter Systems,* Nauka, Moscow

Curtain, R. & Morris K. (2009). Transfer Functions of Distributed Parameter Systems: A tutorial. *Automatica,* Vol. 45, (2009), pp. 1101-1116

Hulkó, G. et al. (1981). On Adaptive Control of Distributed Parameter Systems, *Proceedings of 8-th World Congress of IFAC*, Kyoto, 1981

Hulkó, G. et al. (1987). Control of Distributed Parameter Systems by means of Multi-Input and Multi-Distributed-Output Systems, *Proceedings of 10-th World Congress of IFAC,* Munich, 1987

Hulkó, G. et al. (1998). *Modeling, Control and Design of Distributed Parameter Systems with Demonstrations in MATLAB,* Publishing House STU, ISBN 80-227-1083-0, Bratislava

Hulkó, G. et al (2003-2007). Distributed Parameter Systems. Web portal [Online]. Available: www.dpscontrol.sk

Hulkó, G. et al. (2009a). Engineering Methods and Software Support for Modelling and Design of Discrete-time Control of Distributed Parameter Systems. *European Journal of Control,* Vol. 15, No. Iss. 3-4, *Fundamental Issues in Control,* (May-August 2009), pp. 407-417, ISSN 0947-3580

Hulkó, G. et al. (2009b). Engineering Methods and Software Support for Control of Distributed Parameter Systems, *ASC 2009: Proceedings of the 7th Asian Control Conference,* Honk Kong, China, August 27-29, 2009

Hulkó, G. et al. (2003-2010). *Distributed Parameter Systems Blockset for MATLAB & Simulink*, www.mathworks.com/products/connections/ - Third-Party Product of The MathWorks, Bratislava-Natick, Available from: www.dpscontrol.sk

Morari, M. & Zafirou, E. (1989). *Robust Process Control,* Prentice Hall, Englewood Cliffs

Lasiecka, I. & Triggiani, R. (2000). *Control Theory for Partial Differential Equations: Continuous and Approximation Theories I. Abstract Parabolic Systems,* Cambridge University Press, ISBN 0-521-43408-4, Cambridge, UK

Li, H. X. & Qi, Ch. (2010). Modeling of Distributed Parameter Systems for Application – A Synthesized Review from Time-Space Separation. *Journal of Process Control*, No 20, (2010), pp. 891-901

Lions, J. L. (1971). *Optimal Control of Systems Governed by Partial Differential Equations,* Springer - Verlag, Berlin - Heidelberg - New York

Wang, P. K. C. (1964). *Control of Distributed Parameter Systems* (Advances in Control Systems: Theory and Applications, 1.), Academic Press, New York