Chapter 5

# Measurement of Work-in-Process and Manufacturing Lead Time by Petri Nets Modeling and Throughput Diagram

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Additional information is available at the end of the chapter

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## 1. Introduction

A proper planning and the search for better results in the production processes are important for the competitiveness that manufacturing can add to business operations. However, changes in manufacturing involve risks and uncertainties that may affect the company's operations. In this case, modeling and simulation of the production line can assist the decision-making process, avoiding unnecessary expenses and risks before making a decision. A model that can be simulated in the computer is a mechanism that turns input parameters, known and associated requirements of the process, into output parameters and performance metrics that have not yet happened in the real world (Law; Kelton, 1991).

Thereby, a line production model, which can be used in a computer simulation, can be a tool for decision support, because, before the results will crystallize in the real world manufacturing, it can be predicted, with a given reliability, in virtual simulation.

Inventory in process and throughput time that a production plan will generate are quantities that may be useful in decision making in manufacturing and can be predicted by computer simulation. The inventory process (work in process or WIP) consists of materials that have already been released for manufacture (have already left the warehouse or have been received from suppliers), but their orders still not been completed. Lead time is the time between release manufacture order and the product availability for shipment to the customer (Antunes et al., 2007). Some decisions in internal logistics of manufacturing may be related to these quantities: choosing alternatives for compliance with scheduled delivery dates, intermediate storage areas for processing of applications, equipment for internal movement; resources for tool changes and machinery preparation. The most important decision that can be supported by the proposed method is the definition of in-process inventory level that will be allowed in manufacturing. This should not be so low as to generate idle nor so high as to increase the throughput time.

In the first two chapters will be presented basic concepts for modelling the proposed system using Petri Nets and throughput diagram, these methods will be applied in a real manufacturing and the results compared with the real manufacturing outputs.

The aim of this paper is to measure in advance in-process inventory and lead time in manufacturing that a production plan will generate. Knowing the magnitudes of the plan prior to release, a manager can predict and possibly prevent problems, changing the plan. The specific objectives were: i) mapping manufacturing, ii) model building for PN, refining and validated by field data, iii) with the results simulated by throughput diagram, calculate the inventory in process and expected lead time; and iv) discuss the application. Computer simulation is the research method. Delimitation is that made in a single application in shoe manufacturing, in a period of two weeks. The working method includes two operations research techniques, Petri nets (PN) and the throughput diagram and was tested in a production plan already performed, whose results served to refine and validate the model, which can be used in plans not yet released for manufacturing.

The main contribution of this paper is the method of working, replicable to other applications: simulation PN, validated by data field and use the throughput diagram results to calculate the performance metric. The method can be useful in ill-structured problems, as may occur in manufacturing.

## 2. Petri Nets

The PN describes the system structure as a directed graph and can capture precedence relations and structural links of real systems with graphical expressiveness to model conflicts and queues. Formally, it can be defined as a sixfold (P, T, A, $M_0$, W, K) in wich: P is a set of states/places, T is a set of transitions, A is a set of arcs subject to the constraint that arcs do not connect directly two positions or transitions, $M_0$ is the initial state, which tells how many marks/tokens there are in each position to the beginning of the processing, W is a set of arc weights, which tells, for each arc, how many marks are required for a place by the transition or how many are placed in a place after the respective transition; and K is a set of capacity constraints, which reports to each position, the maximum number of marks which may occupy the place (Castrucci; Moraes, 2001). Applying the definition in the PN of Figure 1, P = [$p_0$, $p_1$]; T = [$t_0$]; A = [($p_0$, $t_0$), ($t_0$, $p_1$)]; W: w ($p_0$, $t_0$) = 1, w ($t_0$, $p_1$) = 1 e $M_0$ = [1; 0]. The token in $p_0$ enables the transition $t_0$. After firing, M = [0; 1].

The transitions correspond to changes of states and places correspond to state variables of the system. In the firing of a transition, the tokens move across the network in two phases: enabling and firing transition. A transition $t_j \in T$ is enabled by a token m if $\forall\ p_i \in P$, m ($p_i$) ≥ w($p_i$, $t_j$), i.e., the token in place pi is greater than or equal to the arc weight that connects $p_i$ to $t_j$.

Some variations are allowed in Petri Nets and were used for modeling, for example, the use of inhibitor arcs.


**Figure 1.** Symbolic representation of Petri Nets

## 3. Throughput diagram in manufacturing

In manufacturing, a queue arises when, for variability, at a given instant, the number of orders to be implemented is greater than the available job centers. The manufacturing arrives at the work position (or center), waiting its turn, is processed and proceeds. The sequence is subject to change priorities and interruptions for maintenance or lack of materials (Silva; Morabito, 2007; Papadopoulos et al., 1993).

A work center (machine, production line or manufacturing plant) can be compared to a funnel, in which orders arrive (input), waiting for service (inventory) and leave the system (output). When the work center is observed for a continuous period, the reference period, the cumulative results can be plotted. In Figure 2, it is possible to observe strokes representing the accumulated input and output, measured in amount of work (Wiendahl, 1995). This quantity may be in parts, numbers of hours or another unit value which represents a significant manufacturing effort (Sellitto, 2005).

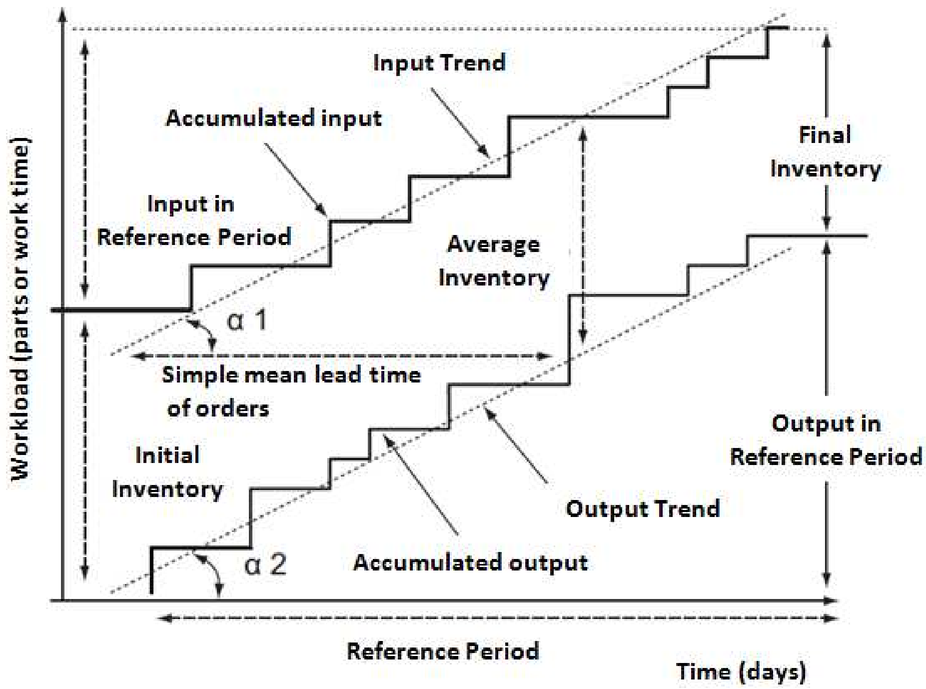


**Figure 2.** Throughput diagram of a work center

To obtain the line that represents input is necessary knowing the amount of work waiting in the initial inventory at the beginning of the reference period and the output is plotted summing the completed work orders. Wiendahl (1995) presents an analytical development related to the throughput diagram and calculates various quantities of interest to workload control (WLC) such as: lead time, average performance, autonomy, work progress and delays in delivery of orders. For this study case, the funnel formula will be applied:

$$TL_m = I_m / P_m \quad (1)$$

Where, $TL_m$ = simple mean lead time of orders (days); $I_m$ = mean inventory (parts); and $P_m$ = mean performance (parts per day).

For the demonstration, the author uses the figure and considers steady state, i.e., the balance between input and outputs ($\alpha 1 = \alpha 2$) and tan $\alpha 1$ = Im / TLm e tan $\alpha 2$ = Pm. Wiendahl (1995) suggests that the equation can be used for measurement and control of manufacturing.

## 4. Research - characterization

The research method was a computer simulation. Simplifications have been admitted, but without losing the replicability. The work method was: i) choice of manufacturing,

process mapping and data collection; ii) model construction by Petri Nets; iii) feeding the model with the initial situation of load in a production plan already executed, run and use the results to fit the model; iv) with the results, calculate the mean lead time of orders; and v) discussing and refining the method, analysis of the implications of its use in manufacturing management.

### 4.1. Manufacturing process mapping

The production process consists of modeling, cutting, sewing, assembling, packing and delivery (dispatching process). Some works use multitasking labour, that moves between closer stages. The manufacturing was divided in three different process: i) Process 1 (cutting, splitting and chamfer); ii) Process 2 (preparation and sewing); and iii) Process 3 (assembling and dispatching).

Basically, the first process consists in cutting operations of the pre-fabricated (insole and sole) and the upper part of the shoes (leather upper). In pre-fabricated, the model was simplified grouping sequential operations. The input of the system is the place "INPUT – m1", all the orders should be loaded in this place, the transition Separation Table (after place m41) is only enabled when all the parts arrive at the place "Separation Table – m41", and is guaranteed by the auxiliary places (m78, m82, m79, m28, m83, m84, m85). The cutting of leather upper was detailed, this process is sequential because the parts should be cutted in different parts of the leather. The input of this process is the place "m42", observe that some operations are performed by the same operators, which explains the use of inhibitor arcs. The third process includes the assembling and dispatching, after this, the shoe process will be finished and ready to leave the factory. The output of the system is the place "BOX OUTPUT – m16".

### 4.2. Transition time assignments

To assign time to the transitions, all the processes were timed. With the orientation of the production supervisor, the start/end time of each task was defined. It was considered a confidence level of 95% and used the calculation model suggested by Vaz (1993) and AEP (2003). As an example, for the sole cutting operation the average time was 18.13 seconds and the standard deviation was 2 seconds. The minimum number of samples to ensure the confidence level was 19.5, adopting 20 samples. The time for each transition is the average values collected.

### 4.3. Simulation, inventory and lead time calculations

To test and refine the model was chosen a plan already done, two weeks and nine production orders. It was informed the load for each place, resulting from earlier orders, at the moment of the first evaluated order will enter the system. A new order is queued of previous processing orders, which explains why the lead time in manufacturing is much higher than the standard manufacturing time. The queuing discipline adopted was FIFO (First-In-First-Out). Table 1 shows data from nine manufacturing orders contained in the production plan (dates are considering working days – 8h40m/day = 3.200s/day). In the last column, there is the order lead time, calculated by simulation, and their average.

| Order (pairs) | Real Input Date (days) | Real Output Date (days) | Real Input Date (s) | Real Output Date (s) | Simulated Output Date (s) | Simulated Lead Time of Order (s) |
|---|---|---|---|---|---|---|
| 1,000 | 0 | 2.5 | 0 | 78,000 | 66,480 | 66,480 |
| 500 | 1 | 3.5 | 31,200 | 109,200 | 104,450 | 73,250 |
| 1,500 | 2 | 6.5 | 62,400 | 202,800 | 174,860 | 112,460 |
| 800 | 4 | 7.5 | 124,800 | 234,000 | 233,872 | 109,072 |
| 800 | 5 | 9 | 156,000 | 280,800 | 280,704 | 124,704 |
| 400 | 7 | 10 | 218,400 | 312,000 | 313,763 | 95,363 |
| 1,000 | 10.2 | 12.5 | 318,240 | 390,000 | 361,404 | 43,164 |
| 500 | 11.2 | 13 | 349,440 | 405,600 | 399,304 | 49,864 |
| 500 | 11.7 | 14 | 365,040 | 436,800 | 428,504 | 63,464 |
| TOTAL 7,000 Pairs | | | AVERAGE 81,980 s = 2,627 days | | | |

**Table 1.** Information for inventory and mean lead time calculation in manufacturing

Wiendahl (1995) presents a method that considers the size of the order Qi. By this method, TLm = [ ΣQi x TLorder I ] / ΣQi = 2.73 days, close to the calculated 2.63 days. The correlation between real and simulated outputs (column 5 and 6) is 0.99 and the absolute error | real - simulated | average is 9,821s (2.27% of the largest real value). Figure 3 shows the comparison of information from real and simulated outputs, order to order.

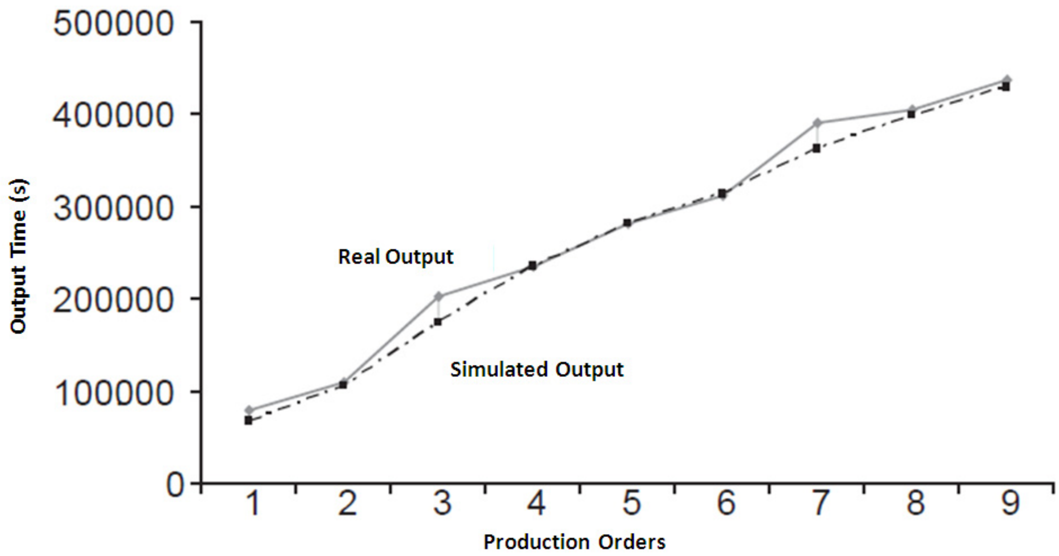


**Figure 3.** Comparison of information from real and simulated outputs

The simulated mean performance is Pm = 31,200 x [ 7,000 / (428,504 – 66,480 ) ] = 583 pairs of shoes per day. The calculation basis is: in (428,504 - 66,480) seconds, were delivered 7,000 pairs. The real mean performance is Pm = 31,200 x [ 7,000 / (428,504 – 78,000 ) ] = 608 pairs of shoes per day.

The time interval between simulated outputs is Δt = [(428,504 – 66,480) / 7,000] = 51.7s and the real is Δt = [(436,800 – 78,000) / 7,000] = 51,25s. The expected mean inventory is Im =Pm.TLm = 583 pairs / day x 2.627 days = 1,531 pairs.

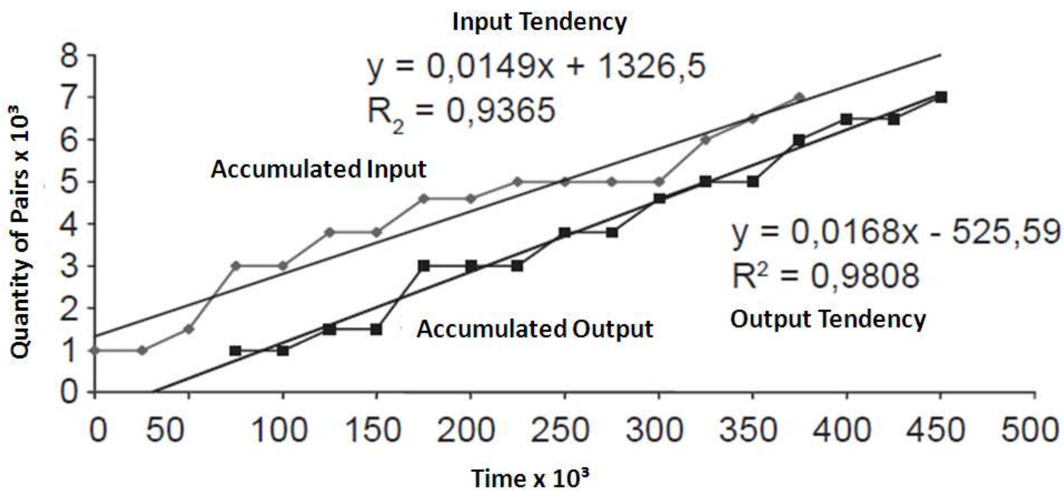


**Figure 4.** Throughput diagram for real inputs and simulated outputs (at intervals of 25,000s)

The instantaneous numbers of pairs in the system is N(t) = I(t) – O(t). The average, an indicator of mean inventory, calculated by this method is close than the one calculated by the funnel method (1,546 and 1,531 pairs respectively).

| Time x $10^3$ (s) | Accumulated Inputs I(t) | Accumulated Outputs O(t) | Number of Pairs in Manufacturing N(t) |
|---|---|---|---|
| 0 | 1,000 | - | - |
| 25 | 1,000 | - | - |
| 50 | 1,500 | - | - |
| 75 | 3,000 | 1,000 | 2,000 |
| 100 | 3,000 | 1,000 | 2,000 |
| 125 | 3,800 | 1,500 | 2,300 |
| 150 | 3,800 | 1,500 | 2,300 |
| 175 | 4,600 | 3,000 | 1,600 |
| 200 | 4,600 | 3,000 | 1,600 |
| 225 | 5,000 | 3,000 | 2,000 |
| 250 | 5,000 | 3,800 | 1,200 |
| 275 | 5,000 | 3,800 | 1,200 |
| 300 | 5,000 | 4,600 | 400 |
| 325 | 6,000 | 5,000 | 1,000 |
| 350 | 6,500 | 5,000 | 1,500 |
| 375 | 7,000 | 6,000 | 1,000 |
| 400 | - | 6,500 | - |
| 425 | - | 6,500 | - |
| 450 | - | 7,000 | - |
| AVERAGE: 1,546 Pairs | | | |

**Table 2.** Accumulated inputs and outputs of each order presented in Table 1 (at the same interval)

## 5. Applications in manufacturing management – results discussion

The simulation can generate data for all processes, individually. For instance, Figure 5 shows the results in place INPUT Sewing Process – m44, the operator at this place is overloaded, also observed in the real process. An alternative would be a redistribution of tasks, adopting parallelisms, without overloading the following posts.

A different situation is shown in Figure 6, the time that the operator is idle in this place is low, and there are no accumulations of tasks over time. This represents that, for this place, the tasks are well distributed.

Other screens allow similar analyzes in all manufacturing places. It is important to analyze the changes in the manufacturing and the impacts that an action causes in each process (for instance, allocate more operators to develop a specific task).

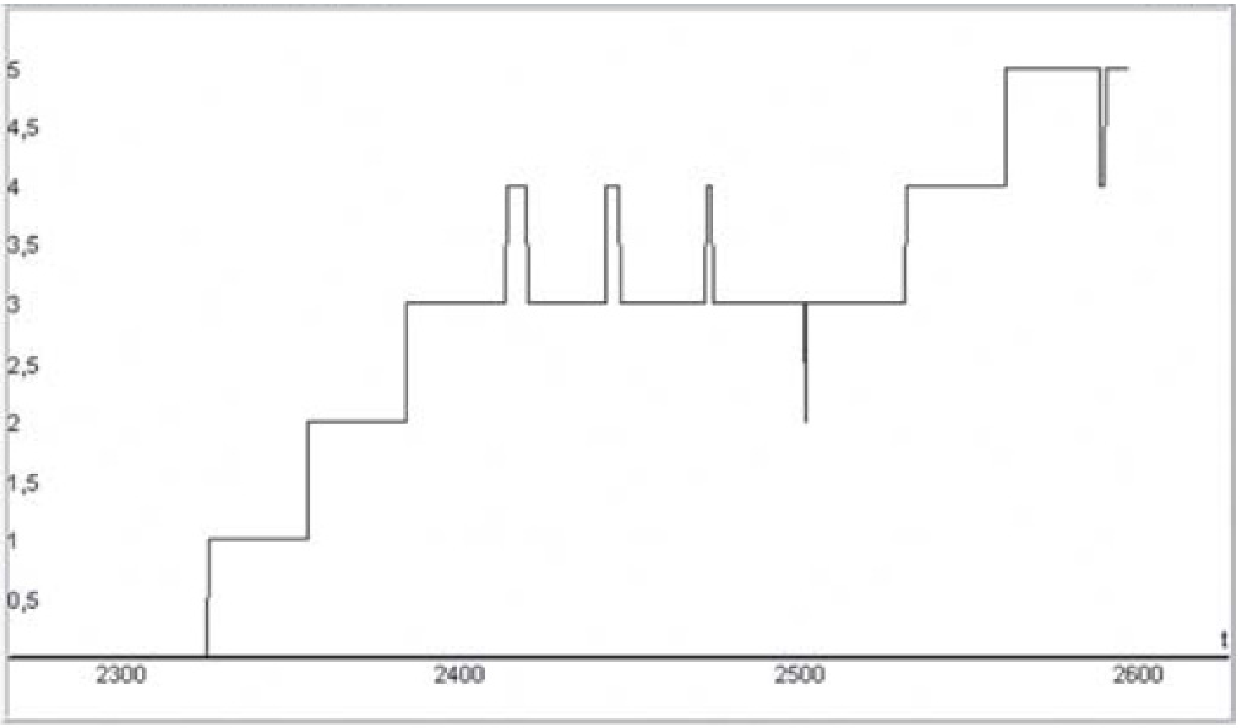


**Figure 5.** Place: "INPUT Sewing Process – m44"– Results obtained with the simulation

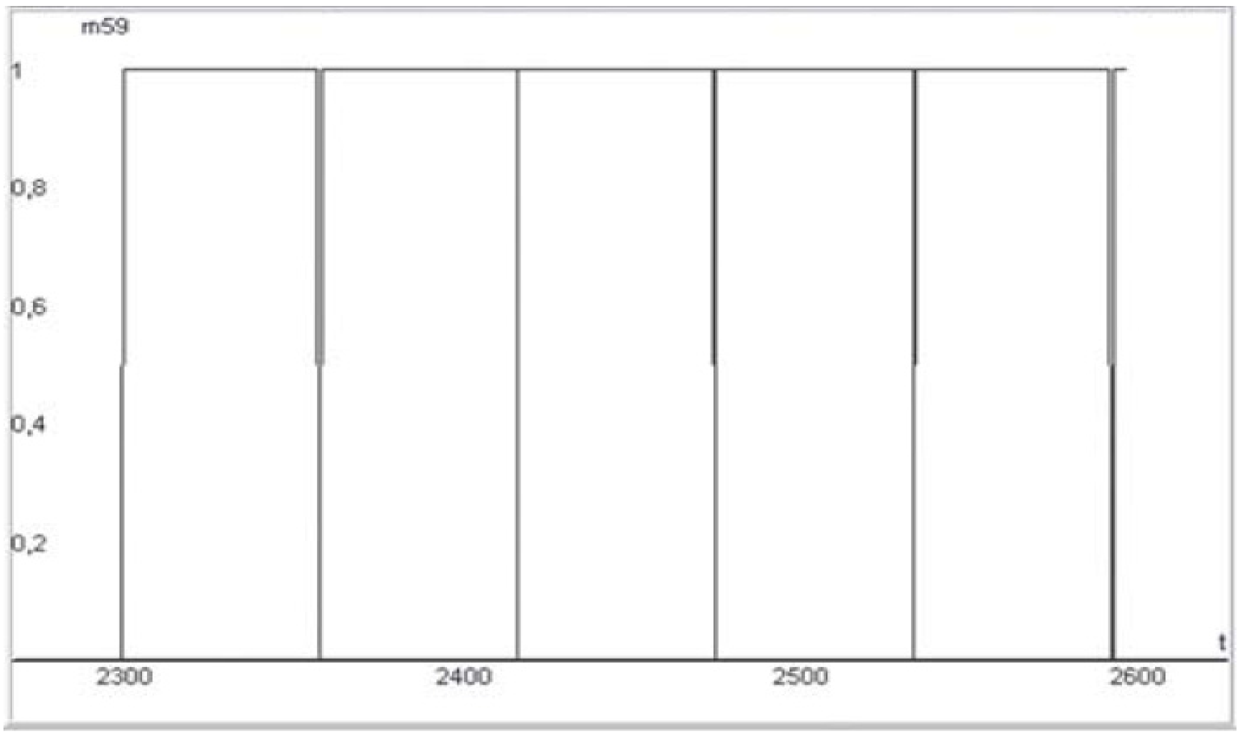


**Figure 6.** Place: Eyelets verification – m59– Results obtained with the simulation

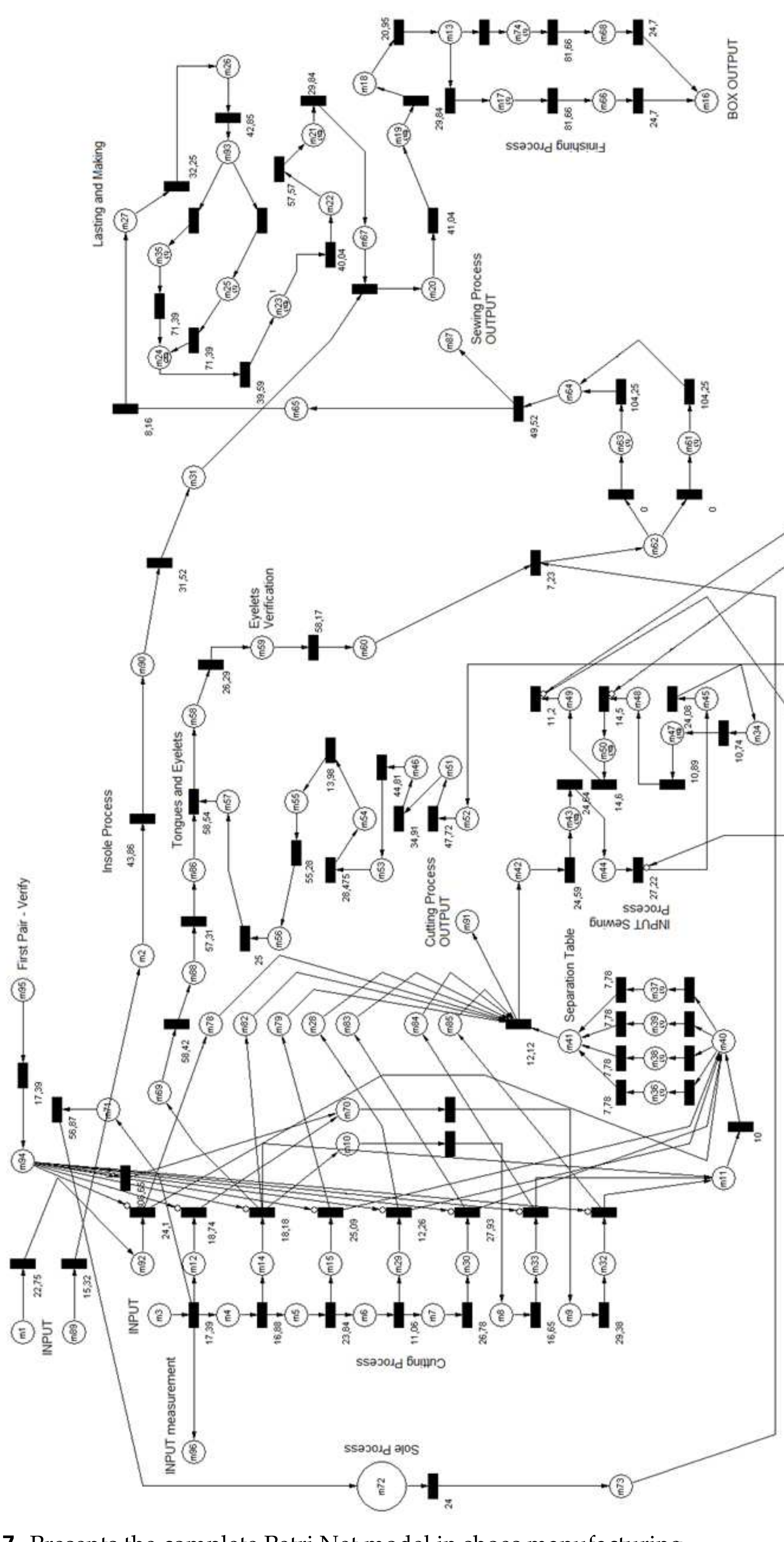


**Figure 7.** Presents the complete Petri Net model in shoes manufacturing.

## 6. Final considerations

It was presented and tested a method based on modelling an simulation by Petri Nets and Throughput Diagram for the calculation of two important indicators in manufacturing management: in process inventory and lead time. With the simulation results (provided by the Petri Net model outputs and the throughput diagram) the manufacturing process can be predicted, as well as some modification can be measured and analyzed to optimize the production. As well as save money on alterations that could produce losses in production processes and often, in the real world, are hard to be perceived.

## Author details

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