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Control of Redundant Robotic Manipulators with State Constraints

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1. Introduction

Recently, redundant manipulators have attracted much attention due to their potential abilities which are interesting from both a theoretical and practical point of view. Redundant degrees of freedom make it possible to perform some useful objectives such as collision avoidance in the work space with both static and moving obstacles, joint limit avoidance, and/or avoidance of singular configurations when the manipulator moves. Most of practical tasks, for example, inserting a shaft into co-operating elements (bearing, sleeve, or ratchet-wheel), require the knowledge of geometric paths (given in the work space) and a proper tolerance of matching that specifies the corresponding accuracy of the path following. In many other industrial tasks such as laser cutting or arc welding, the accuracy of path following is vital, and it is reasonable to assume that designers and manufacturers will specify precision using an absolute tolerance on tracking error. The application of redundant manipulators to such tasks complicates their performance, since these manipulators in general do not provide unique solutions. Consequently, some objective criteria should be specified to solve the robot tasks uniquely. The minimization of performance time is mostly considered in the literature. Several approaches may be distinguished in this context. Using the concept of a regular trajectory and the extended state space, the structure of path-constrained time-optimal controls has been studied in the works (Galicki, 1998b; Galicki, 2000) for kinematically redundant manipulators. Moreover, the efficient numerical procedures able to find such controls were also proposed in the works (Galicki, 1998a; Galicki & Pajak, 1999). Nevertheless, these algorithms require full knowledge of manipulator Jacobian matrix and robot dynamic equations, too.

Although all the aforementioned algorithms produce optimal solutions, they are not suitable to real-time computations due to their computational complexity. Therefore, it is natural to attempt other techniques in order to control the robot in real-time. Using on-line trajectory time scaling, a dynamic and computed torque laws respectively, a nearly time-optimal path tracking control for

non-redundant robotic manipulators with partially uncertain dynamics has been presented in works (Dahl, 1994; Kiefer et al., 1997). However, these algorithms require the solution of inverse kinematic problem along the path. A technique which avoids solving an inverse of robot kinematic equations and uses the exact Jacobian matrix, has been offered in (Galicki, 2001; Galicki 2004) for determining a collision-free trajectory of redundant manipulators operating in both a static environment (Galicki, 2001) and in a dynamic one (Galicki, 2004). Recently, a generalized transpose Jacobian controller with gravity compensation and a non-linear (saturating) derivative term has been introduced in (Galicki, 2006a) to generate robot controls subject to geometric path and actuator constraints.

As is known, many robotic controllers have been proposed to solve both a set point control problem (a regulation task) (Takegaki & Arimoto, 1981; Arimoto, 1996; Canudas de Wit et al., 1996; Sciavicco & Siciliano, 1996; Arimoto, 1990; Kelly, 1999; Galicki, 2002; Galicki, 2005) and the trajectory tracking (Slotine & Li, 1987; Slotine & Li, 1991; Feng & Palaniswami, 1992; Berghuis et al., 1993; Lewis et al., 1993; Tomei, 2000), respectively. However, most of these controllers have assumed full knowledge of manipulator kinematic equations. Recently, several approximate Jacobian setpoint controllers have been proposed (Cheach et al., 1999; Yazarel & Cheach, 2002; Cheach et al., 2003) to tackle uncertainties in both robot kinematics and dynamics. The controllers proposed do not require the exact knowledge of Jacobian matrix and dynamic equations. However, the results in (Cheach et al., 1999; Yazarel & Cheach, 2002; Cheach et al., 2003) are applicable only to a setpoint control of a robot.

This paper, which is based on our recent work (Galicki, 2006b), introduces a new class of adaptive path following controllers not requiring the full knowledge of both kinematic and dynamic equations in the control laws. Consequently, they are suitable for controlling uncertain robotic manipulators. Motivated in part by the dissipativity and adaptivity methodology (Slotine & Li, 1987), we develop path following controllers whose structure is composed of transpose adaptive Jacobian controller plus a non-linear term including an estimated control. Under the assumption of the full rank adaptive Jacobian matrix, the proposed control scheme has been derived based on the Lyapunov stability theory. By using sensory feedback of the end-effector position, it is also shown that the end-effector is able to follow a prescribed geometric path for robots with both uncertain kinematics and dynamics. Furthermore, new adaptive laws extending the adaptive algorithm from (Slotine & Li, 1987) to tackle kinematic uncertainties too, are proposed. It is to notice that approximate Jacobian setpoint controllers from (Cheach et al., 1999; Yazarel & Cheach, 2002; Cheach et al., 2003) can not be directly applicable to our task. The reason is that approximate Jacobian matrix in (Cheach et al., 1999; Yazarel & Cheach, 2002; Cheach et al., 2003) does not include kinematic parameters to be adapted and the error of approximation is a priori bounded. On the other

hand, the controller proposed in our work adaptively varies both kinematic parameters of the Jacobian matrix and the dynamic ones in such a way as to stably follow by the end-effector a geometric path. The paper is organized as follows. Section 2 formulates the robotic task to be accomplished in terms of a control problem. Section 3 describes how to employ the Lyapunov stability theory to determine controls (if they exist). Section 4 provides us with a computer example of generating the robot controls in a two dimensional task space for a planar redundant manipulator comprising three revolute kinematic pairs. Finally, some conclusions are drawn.

2. Formulation of the adaptive control problem

The control scheme designed in the next section is applicable to holonomic mechanical

systems comprising both non-redundant and redundant manipulators considered here which are described, in general, by the following dynamic equations, expressed in generalized co-ordinates (joint co-ordinates) $q = (q_1, \dots, q_n)^T \in R^n$

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u \quad (1)$$

where $M(q)$ denotes the $n \times n$ inertia matrix; $C(q, \dot{q})\dot{q}$ is the n -dimensional vector representing centrifugal and Coriolis forces; $G(q)$ stands for the n -dimensional vector of gravity forces; $u = (u_1, \dots, u_n)^T$ is the vector of controls (torques/forces); n denotes the number of kinematic pairs of the V-th class. In most applications of robotic manipulators, a desired path for the end-effector is specified in task space such as visual space or Cartesian space. The aim is to follow by the end-effector a prescribed geometric path (given in the m -dimensional task space) described by the following equations

$$p(q) - \Theta(s) = 0 \quad (2)$$

where $p: R^n \rightarrow R^m$ denotes an m -dimensional, non-linear (with respect to vector q) mapping constructed from the kinematic equations of the manipulator; $p(q) = (p_1(q), \dots, p_m(q))^T$; $\Theta(s) = (\Theta_1(s), \dots, \Theta_m(s))^T$ stands for a given geometric path; s is the current parameter of the path (e.g. its length); $s \in [0, s_{\max}]$; s_{\max} is the maximal path length. The mapping Θ is assumed to be bounded together with the first and second derivatives and not degenerated, i.e. $\left\| \frac{d\Theta}{ds} \right\| > 0$.

It should be: The kinematic equations of a manipulator are independent $\text{rank}\left(\frac{\mathbb{J}p}{\mathbb{J}q}\right) = m$. In general (i.e. when $\text{rank}\left(\frac{\mathbb{J}p}{\mathbb{J}q}\right) \neq m$, we should require that $\text{rank}\left(\frac{\mathbb{J}p}{\mathbb{J}q}\right) = \text{rank}\left(\left[\frac{\mathbb{J}p}{\mathbb{J}q} \frac{d\Theta}{ds}\right]\right)$ in order to guarantee consistency of the robotic task (2).

The problem is to determine control u which generates manipulator trajectory $q = q(t)$ and path parameterization $s = s(t)$ satisfying the equation (2) for each $t \in [0, T]$, where T denotes an (unknown) time horizon of task performance. It is natural to assume that at the initial moment $t = 0$, for which $s(0) = 0$, a given (by definition) initial configuration $q(0) = q_0$ satisfies (2), i.e.

$$p(q_0) - \Theta(0) = 0 \quad (3)$$

Final path parameterization fulfils the equality

$$s(T) - s_{\max} = 0 \quad (4)$$

Furthermore, at the initial and the final time moment, the manipulator and path velocities equal zero, i.e.

$$\dot{q}(0) = \dot{q}(T) = 0 \quad (5)$$

and

$$\dot{s}(0) = \dot{s}(T) = 0 \quad (6)$$

As is known, task space velocity \dot{p} is related to joint space velocity \dot{q} as follows

$$\dot{p} = J(q, Y)\dot{q} \quad (7)$$

where $J(q, Y) = \frac{\partial p}{\partial q}$ is the $m \times n$ Jacobian matrix; Y stands for an ordered set of kinematic parameters $Y = (Y_1, \dots, Y_k)^T$ such as link lengths, joint offsets; k denotes the number of kinematic parameters. Several important properties of dynamic equations (1) may be derived (Spong & Vidyasagar, 1989)

1. The inertia matrix $M(q)$ is symmetric and positive definite for all $q \in R^n$
2. Matrix $\frac{\dot{M}(q)}{2} - C(q, \dot{q})$ is skew-symmetric so that

$$\forall v, q, \dot{q} \in R^n \left\langle v, \left(\frac{\dot{M}(q)}{2} - C(q, \dot{q}) \right) v \right\rangle = 0 \quad (8)$$

where \langle, \rangle is the scalar product of vectors.

3. The dynamic equations (1) are linear with respect to an ordered set of physical parameters $X = (X_1, \dots, X_d)^T$, i.e.

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = D(q, \dot{q}, \ddot{q})X \quad (9)$$

where $D(q, \dot{q}, \ddot{q})$ is called the $(n \times d)$ dynamic regressor matrix; d stands for the number of the physical parameters such as link masses and inertias.

Differential equation (7) has the following property.

4. The right hand side of (7) is linear with respect to $e Y$. Consequently, equation (7) can be expressed as follows

$$\dot{p} = J(q, Y)\dot{q} = K(q, \dot{q})Y \quad (10)$$

where $K(q, \dot{q})$ is called the $(m \times k)$ kinematic regressor matrix.

In order to simplify further computations s_{max} is assumed to be equal to 1, i.e. $s_{max} = 1$. Let us define errors e and e_{m+1} of path following (task errors) as

$$e = (e_1, \dots, e_m)^T = p(q) - \Theta(s) = (p_1 - \Theta_1, \dots, p_m - \Theta_m)^T \quad (11)$$

$$e_{m+1} = s - 1.$$

Many commercial sensors are available for measurement of end-effector position p , such as vision systems, electromagnetic measurement systems, position sensitive detectors or laser tracking systems. Hence, path following error e in (11) is also assumed to be available (for a given s) from measurement. For revolute kinematic pairs, considered here, mapping $p(\cdot)$ is bounded. Consequently, we have the following property.

5. Boundedness of mappings $p(\cdot), \Theta(\cdot)$, implies that task error e is bounded.

Expressions (1)-(6) formulate the robot task as a control problem. The fact that there exist state equality constraints makes the solution of this problem difficult. The next section will present an approach that renders it possible to solve the control problem (1)-(6) making use of the Lyapunov stability theory.

3. Adaptive path control of the manipulator

Our aim is to control the manipulator such that the end-effector fulfils (2)-(6). Therefore, we propose adaptive Jacobian path following controllers for robots with both uncertain kinematics and dynamics. In our approach, the exact knowledge of both robot dynamic equations and Jacobian matrix is not required in updating the uncertain parameters.

In the presence of kinematic uncertainty, the parameters of the Jacobian matrix are uncertain and hence equality (10) can be expressed as follows

$$\hat{J}\dot{q} = K(q, \dot{q})\hat{Y} \quad (12)$$

where $\hat{J} = J(q, \hat{Y}) \in R^{m \times n}$ is an adaptive Jacobian matrix and \hat{Y} stands for the vector of estimated kinematic parameters. In order to show the stability of the path following control system in the presence of both kinematic and dynamic uncertainties, we define an adaptive joint-space sliding vector z as

$$z = \lambda \hat{J}^T e + \dot{q} \quad (13)$$

where λ is a positive scalar coefficient. The adaptivity of vector z is understood in the sense that the parameters of the adaptive Jacobian matrix will be updated by a parameter update law, defined later. Differentiating equation (13) with respect to time yields

$$\dot{z} = \lambda \dot{\hat{J}}^T e + \lambda \hat{J}^T \dot{e} + \ddot{q} \quad (14)$$

Based on equations (1) and (13)-(14), we can easily express robot dynamic equations by means of vector z and its time derivative, as

$$M(q)\dot{z} + C(q, \dot{q})z + M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) = u \quad (15)$$

Where $\ddot{q}_r = (\ddot{q}_{r,1}, \dots, \ddot{q}_{r,n})^T = -\lambda \hat{J}^T e - \lambda \hat{J}^T \dot{e}$ and $\dot{q}_r = (\dot{q}_{r,1}, \dots, \dot{q}_{r,n})^T = -\lambda \hat{J}^T e$. Moreover, we know from property 3., that the last three terms of equation (15) become linear with respect to vector X and hence they can be written as follows

$$M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) = D(q, \dot{q}, \dot{q}_r, \ddot{q}_r)X. \quad (16)$$

Inserting the right hand side of (16) into (15), robot dynamic equations (15) take the following form

$$M(q)\dot{z} + C(q, \dot{q})z + D(q, \dot{q}, \dot{q}_r, \ddot{q}_r)X = u \quad (17)$$

Based on (13), (14) and (17), let's propose the following adaptive Jacobian controller

$$u = -kz - k_p \hat{J}^T e + D(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \hat{X} \quad (18)$$

where \hat{X} stands for the estimated physical parameter vector defined below; k and k_p are some positive scalars which could be replaced by diagonal matrices of positive constants without affecting the stability results obtained further on (this should lead to improved performance). Here, scalar constants are chosen for simplicity of presentation. In order to measure error e , path parameterization $s = s(t)$ is required which is computed by solving the following scalar differential equation

$$\ddot{s} = -k_s \dot{s} - k_p \left(\left\langle e, -\frac{d\Theta(s)}{ds} \right\rangle + \gamma(s-1) + \frac{1}{2} \frac{d\gamma}{ds} (s-1)^2 \right) \quad (19)$$

where k_s denotes a positive coefficient; γ is assumed to be a strictly positive function ($\inf\{\gamma\} > 0$) of s with bounded first and second derivatives for any $|s| \leq 1 + \sqrt{\frac{\gamma_0}{\inf\{\gamma\}}}$, where $\gamma_0 = \gamma(0)$ (as will be seen further on, γ_0 determines the upper bound on the accuracy of the path following and may be specified by the user).

The choice of function γ is crucial for computational effectiveness of control scheme (18), (19). One possibility is $\gamma = \gamma(s)$ with $\frac{d\gamma}{ds} > 0$. An alternative choi-

ce could be $\gamma = \gamma(e^2)$, where γ attains its maximum for $e = 0$ and smoothly decreases as $\|e\|$ increases. For simplicity of further considerations, we take the first form of γ .

Assumption 1. Function γ is required not to satisfy differential equation

$$\gamma + \frac{1}{2} \frac{d\gamma}{ds} (s-1) = 0$$

As will be seen further on, Assumption 1. results in an asymptotic convergence of s to 1.

Let us note, that the first two terms from (18) present an adaptive transpose Jacobian controller with adaptively varying kinematic parameter vector \hat{Y} . The last term in dependence (18) is an estimated control based on equation (16). Estimated kinematic parameters \hat{Y} of the adaptive Jacobian matrix $\hat{J} = J(q, \hat{Y})$ are updated according to the following law

$$\dot{\hat{Y}} = w_k k_p K^T(q, \dot{q})e \quad (20)$$

and estimated physical parameters \hat{X} of the dynamic equations are updated by

$$\dot{\hat{X}} = -w_d D^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)z \quad (21)$$

where w_k, w_d , are, similarly as before, positive gains (scalars) which could be replaced by diagonal matrices of positive constants without affecting the stability of controller (18). Although some kinematic parameters appear in \hat{X} , we should adapt on them separately in \hat{Y} to preserve linearity.

Estimated kinematic parameters \hat{Y} (updated according to rule (20)) are then used to compute adaptive Jacobian \hat{J} and its time derivative $\dot{\hat{J}}$ using for this purpose the right hand side of (20) which is only a mapping of q, e , and \dot{q} . Having obtained the adaptive Jacobian matrix and its time derivative, we determine quantities \dot{q}_r and \ddot{q}_r . It is worth noticing, that their computation does not require any pseudoinverse of matrix \hat{J} which results in numerical stability of controller (18). Finally, based on q, \dot{q}, \dot{q}_r , and \ddot{q}_r , we may determine dynamic regressor matrix $D(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$, which is then used to update estimated physical parameter vector \hat{X} .

Let us note, that setpoint controllers proposed in works (Cheach et al., 1999; Yazarel & Cheach, 2002; Cheach et al., 2003) , which are computationally somewhat simpler, can not be applicable to our task. The reason is that the error of approximation in (Cheach et al., 1999; Yazarel & Cheach, 2002; Cheach et al., 2003) is bounded by a constant and approximate Jacobian matrix does not include parameters to be adapted. Due to adaptation law (20), one can not guarantee in our task to satisfy assumption regarding the approximation error made in works (Cheach et al., 1999; Yazarel & Cheach, 2002; Cheach et al., 2003) .

The closed-loop error dynamics is obtained by inserting the right hand side of equation (18) into equation (17)

$$\begin{aligned}
 M(q)\dot{z} &= -C(q, \dot{q})z + D(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{X} - kz - k_p \hat{J}^T e \\
 \dot{e} &= J(q, Y)\dot{q} - \frac{d\Theta(s)}{ds} \dot{e}_{m+1} \\
 \ddot{e}_{m+1} &= -k_s \dot{e}_{m+1} - k_p \left(\left\langle e, -\frac{d\Theta(s)}{ds} \right\rangle + \mathcal{J}e_{m+1} + \frac{1}{2} \frac{d\gamma}{ds} e_{m+1}^2 \right) \\
 \dot{\tilde{Y}} &= w_k k_p K^T(q, \dot{q})e \\
 \dot{\tilde{X}} &= -w_d D^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)z
 \end{aligned} \tag{22}$$

where $\tilde{Y} = \hat{Y} - Y$; $\tilde{X} = \hat{X} - X$. Applying the Lyapunov stability theory, we derive the following result.

Theorem 1. If there exists a solution to the problem (1)-(6) and adaptive Jacobian matrix \hat{J} is non-singular along end-effector path (2) and function γ fulfils Assumption 1., then control scheme (18) generates manipulator trajectory whose limit point $(\dot{q}(\infty), \dot{e}_{m+1}(\infty), e(\infty), e_{m+1}(\infty)) = (0, 0, 0, 0)$, i.e. satisfying state constraints (2)-(6), is asymptotically stable.

Proof. Consider a Lyapunov function candidate

$$V = \frac{1}{2} \left(\langle Mz, z \rangle + \frac{1}{w_d} \tilde{X}^2 + \frac{1}{w_k} \tilde{Y}^2 + \dot{e}_{m+1}^2 + k_p \mathcal{J}e_{m+1}^2 + k_p e^2 \right). \tag{23}$$

The time derivative of V is given by

$$\begin{aligned} \dot{V} = & \langle z, M\dot{z} \rangle + \left\langle \frac{\dot{M}}{2} z, z \right\rangle + k_P \langle J^T e, \dot{q} \rangle + \frac{1}{w_d} \langle \tilde{X}, \dot{\tilde{X}} \rangle + \frac{1}{w_k} \langle \tilde{Y}, \dot{\tilde{Y}} \rangle + \\ & \dot{e}_{m+1} \ddot{e}_{m+1} + k_P \left\langle e, -\frac{d\Theta}{ds} \right\rangle \dot{e}_{m+1} + k_P \mathcal{J}_{m+1} \dot{e}_{m+1} + k_P \frac{1}{2} \frac{d\gamma}{ds} e_{m+1}^2 \dot{e}_{m+1}. \end{aligned} \quad (24)$$

Substituting $M\dot{z}, \dot{e}, \ddot{e}_{m+1}, \dot{\tilde{Y}}$ and $\dot{\tilde{X}}$ from \dot{V} for the right-hand sides of closed-loop error dynamics (22) and using the skew-symmetric property of matrix $\frac{\dot{M}}{2} - C$ [property 2. eqn. (8)], we obtain after simple calculations, that

$$\dot{V} = -k \langle z, z \rangle - k_P \langle \hat{J} \hat{J}^T e, e \rangle - k_s \dot{e}_{m+1}^2.$$

Since $\dot{V} \leq 0$, function V is bounded. Therefore, z, \tilde{X} , and \tilde{Y} are bounded vectors. This implies that \hat{X} and \tilde{Y} are bounded, too. Consequently, it follows from (13), that \dot{q} is also bounded. Moreover, s and \dot{s} are bounded, too. As can be seen, \dot{V} is negative for all $(z, e, \dot{e}_{m+1}) \neq 0$ and is zero only when $(z, e, \dot{e}_{m+1}) = 0$, which implies (using LaSalle-Yoshizawa invariant theorem (Krstic et al., 1995) that (z, e, \dot{e}_{m+1}) tends asymptotically to zero, i.e. $z(T) \rightarrow 0, e(T) \rightarrow 0$, and $\dot{e}_{m+1} \rightarrow 0, as. T \rightarrow \infty$, as. By differentiating \ddot{e}_{m+1} in (22) with respect to time, it is also easy to see, that $\frac{d^3 e_{m+1}}{dt^3}$ is bounded function by assumptions regarding Θ and γ . This means, that \ddot{e}_{m+1} is uniformly continuous. Hence, $\ddot{e}_{m+1}(T) \rightarrow 0$, as $T \rightarrow \infty$, too. The convergence of path velocity and acceleration yields the following equation

$$\mathcal{J}_{m+1}(\infty) + \frac{1}{2} \frac{d\gamma}{ds} e_{m+1}^2(\infty) = 0. \quad (25)$$

Consequently, $e_{m+1}(\infty) = 0$ or $\gamma + \frac{1}{2} \frac{d\gamma}{ds} e_{m+1}(\infty) = 0$. On account of Assumption 1, the second equality is not fulfilled. Thus, $e_{m+1}(\infty) = 0$ (or equivalently $s(\infty) = 1$). On account of (13), $\dot{q}(T) \rightarrow 0$, as $T \rightarrow \infty$, too. Consequently, boundary conditions (4)-(6) are (asymptotically) fulfilled and limit point $(\dot{q}(\infty), \dot{e}_{m+1}(\infty), e(\infty), e_{m+1}(\infty)) = (0, 0, 0, 0)$ is asymptotically stable. Finally, it should be emphasized, that the chosen Lyapunov function does not guarantee convergence of parameter estimations \hat{X} and \hat{Y} to their true values.

On account of (3)-(6), we have $V_{t=0} = \frac{\tilde{X}_{t=0}^2}{2w_d} + \frac{\tilde{Y}_{t=0}^2}{2w_k} + \frac{k_p\gamma_0}{2}$.

For sufficiently large w_d and w_k , the first two terms in this equality may be omitted. Hence, we obtain $V_{t=0} \cong \frac{k_p\gamma_0}{2}$.

Since \dot{V} is not positive, function V fulfils the inequality $V \leq \frac{k_p\gamma_0}{2}$.

Consequently, the following bound on $\|e\|$ may easily be obtained, based on (23) and the last dependence

$$\|e\| \leq \sqrt{\gamma_0} \quad (26)$$

An important remark may be derived from the proof carried out. Namely, Inequality (26) presents an upper bound (path independent) on the accuracy of path following by the end-effector according to the control law (18). Let us note that estimation of the upper bound on path following error (26) is very conservative. Consequently, control gains w_d and w_k do not require large values to achieve a good path following accuracy, as the numerical simulations (given in the next section) show.

Moreover, several observations can be made regarding the control strategy (18). First note, that the proposed control law requires, in fact no information concerning the robot kinematic and dynamic equations. Second, the choice of controller parameters k, k_p, k_s, w_d and w_k according to dependencies (18)-(21) guarantees asymptotic stability of the closed-loop error dynamics (22) during the manipulator movement.

Moreover, the transpose of \hat{J} (instead of a pseudoinverse) in control scheme (18) does not result in numerical instabilities due to (possible) kinematic singularities met on the robot trajectory. Nevertheless, (18) has been derived under the assumption of full-rank adaptive Jacobian matrix along the path. Furthermore, controller (18) does not require the knowledge of task space velocity. Due to conservative estimation of the path following accuracy, control algorithm (18) results in a better accuracy of the path following as compared to upper bound given from (26), as the numerical computations carried out in the next section show. In order to prevent control (torque) oscillations at the very beginning of time histories (caused by e.g. the non-zero initial path following error) k_s from (19) should be a bounded, quickly decreasing time dependent function as $t \rightarrow \infty$ (see the next section).

Due to real-time nature of robot controller (18), we shall try to estimate the number of arithmetic operations required to implement the algorithm presented in this section. The dimension of the robot task space is assumed in estimation to be constant. Operations required for computation of \sin, \cos , and $\Theta(\cdot)$ functions are not taken into account. Furthermore, matrices \hat{J} , K and D are assumed in estimation to be given. Moreover, estimations are carried out at any time instant of the robot task accomplishment. It follows from (13) and (18) that terms $kz, k_p \hat{J}^T e$, require $O(n)$ operations. Computation of the right hand sides (19) and (20) involves $O(1)$ and $O(n)$ operations, respectively assuming that $k = O(n)$. Computational complexity for the right hand side of (21) equals $O(n^2)$ by assumption that $d = O(n)$. Computation of estimated control $D(q, \dot{q}, \ddot{q}, \ddot{q}_r) \hat{X}$ requires also the same order of complexity, i.e. $O(n^2)$ operations. Finally, computational complexity of the whole robot controller (18) is of the order $O(n^2)$.

4. A numerical example

The aim of this section is to illustrate the performance of the proposed adaptive

control algorithm using a dynamic three-joint direct-drive arm ($n = 3$) of SCARA-type robotic manipulator operating in a two-dimensional ($m = 2$) task space. Kinematic scheme of this manipulator and the task to be accomplished is shown in Fig. 1. In the simulations, SI units are used. The components of dynamic equations of this manipulator are as follows (Spong & Vidyasagar, 1989):

$$M = [M_{ij}]_{1 \leq i, j \leq 3}$$

where $M_{11} = X_1 + 2X_4c23 + X_6c3$; $ci = \cos(q_i)$, $si = \sin(q_i)$, $cij = \cos(q_i + q_j)$; $sij = \sin(q_i + q_j)$; $cijk = \cos(q_i + q_j + q_k)$; $sijk = \sin(q_i + q_j + q_k)$; $M_{21} = X_2 + X_4c2 + X_5c23$; $M_{31} = X_3$; $M_{22} = X_2 + 2X_6c3$; $M_{32} = X_3 + X_6c3$; $M_{33} = X_3$; $M_{12} = M_{21}$; $M_{13} = M_{31}$; $M_{23} = M_{32}$.

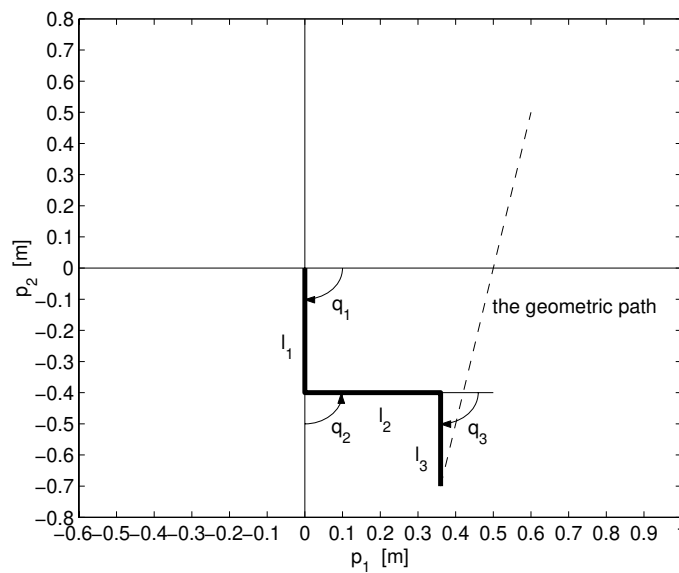


Figure.1 A kinematic scheme of the manipulator and the task to be accomplished

$$C = [C_{ij}]_{1 \leq i, j \leq 3}$$

where

$$\begin{aligned} C_{11} &= -(X_4 s_2 + X_5 s_{12})\dot{q}_2 - (X_5 s_{13} + X_6 s_{13})\dot{q}_3; \\ C_{12} &= -(X_4 s_2 + X_5 s_{12})(\dot{q}_1 + \dot{q}_2) - (X_5 s_{12} + X_6 s_{12})\dot{q}_3; \\ C_{13} &= (X_5 s_{12} + X_6 s_{12})(-\dot{q}_1 + \dot{q}_2 + \dot{q}_3); \\ C_{21} &= (X_5 s_2 + X_5 s_{12})\dot{q}_1 + X_6 s_3 \dot{q}_3; \\ C_{22} &= -(X_5 s_{12} + X_6 s_{12})\dot{q}_3; \\ C_{23} &= -X_6 s_3(3\dot{q}_1 + \dot{q}_2 + \dot{q}_3); \\ C_{31} &= (X_4 s_2 + X_5 s_{12})\dot{q}_1 - X_6 s_3 \dot{q}_2; \\ C_{32} &= X_6 s_3(\dot{q}_1 + \dot{q}_2); \\ C_{33} &= 0. \end{aligned}$$

$$G = (G_1, G_2, G_3)^T$$

where

$$G_1 = X_7 c_1 + X_8 c_{12} + X_9 c_{123}; \quad G_2 = X_8 c_{12} + X_9 c_{123}; \quad G_3 = X_9 c_{123}.$$

Parameters X_i , $i = 1 : 9$ take the following nominal values:

$$\begin{aligned}
X_1 &= 1.1956, \\
X_2 &= 0.3946, \\
X_3 &= 0.0512, \\
X_4 &= 0.4752, \\
X_5 &= 0.1280, \\
X_6 &= 0.1152, \\
X_7 &= (m_1 l_{c1} + m_2 l_1 + m_3 l_1)g, \\
X_8 &= (m_2 l_{c2} + m_3 l_2)g, \\
X_9 &= m_3 l_{c3}g,
\end{aligned}$$

where g stands for the gravity acceleration; m_i, l_i , and l_{ci} , $i = 1:3$ denote link mass, length and location of the mass center which is assumed to be equal to

$$l_{ci} = l_i / 2; l_1 = 0.4; l_2 = 0.36; l_3 = 0.3; m_1 = 3.6; m_2 = 2.6; m_3 = 2.$$

Jacobian matrix $J(q, Y)$ equals

$$J = \begin{bmatrix} -Y_1 s_1 - Y_2 s_{12} - Y_3 s_{123} & -Y_2 s_{12} - Y_3 s_{123} & -Y_3 s_{123} \\ Y_1 c_1 + Y_2 c_{12} + Y_3 c_{123} & Y_2 c_{12} + Y_3 c_{123} & Y_3 c_{123} \end{bmatrix}$$

Where $Y_i = l_i; i = 1:3$;

and the kinematic regressor matrix takes the following form

$$K = \begin{bmatrix} -s_1 \dot{q}_1 & -s_{12}(\dot{q}_1 + \dot{q}_2) & -s_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \\ c_1 \dot{q}_1 & c_{12}(\dot{q}_1 + \dot{q}_2) & c_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \end{bmatrix}$$

The end-effector position $p = (p_1 p_2)^T$ (see Fig. 1) represents in the simulations the task space coordinates ($m = 2$). The upper bound on the accuracy of the path following in all the computer simulations, is assumed to be equal to $\sqrt{\gamma_0} = 0.06$, where $\gamma(s) = 0.002 + 0.002e^{2.7s}$. Let us introduce path following errors

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} p_1 - \Theta_1 \\ p_2 - \Theta_2 \\ s - 1 \end{pmatrix}$$

to evaluate the performance of the robot controller (18). In order to examine the effects of both kinematic and dynamic uncertainties, initial values for vectors \hat{X} and \hat{Y} were set in the simulations as $\hat{X}(0) = (32110.80.5302010)^T$, $\hat{Y}(0) = (0.550.450.4)^T$. The task of the robot is to transfer the end-effector along the geometric path (the dotted line in Fig. 1), expressed by the following equations

$$\Theta_1(s) = 0.36 + 0.24s$$

$$\Theta_2(s) = -0.7 + 1.2s$$

where, $s \in [0,1]$. The initial configuration q_0 equals $q_0 = (-\pi / 2 \pi / 2 - \pi / 2)^T$. Parameters $k = 10, k_p = 3000, \lambda = 1$ and $k_s = 6,9(1 + 100e^{-10t}) + 30$ have been chosen experimentally to achieve practically reasonable time horizon of task performance and relatively small controls with $w_k = \text{diag}(101010)$ and $w_d = \text{diag}(4.854.854.854.854.854.858.554.76)$. The results of computer simulation are presented in Figs 2-20.

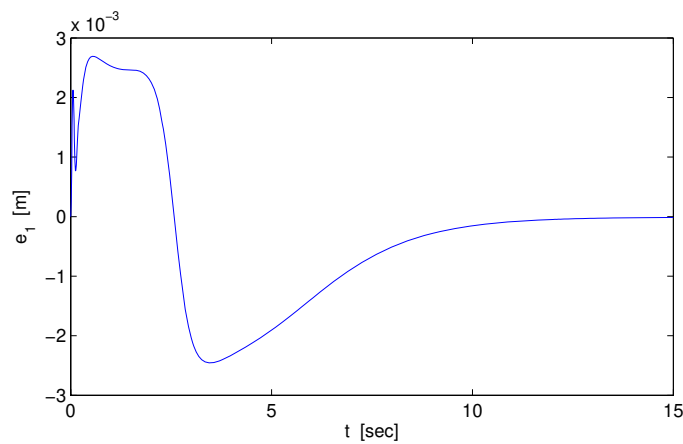


Figure 2. Path following error e_1 vs. time

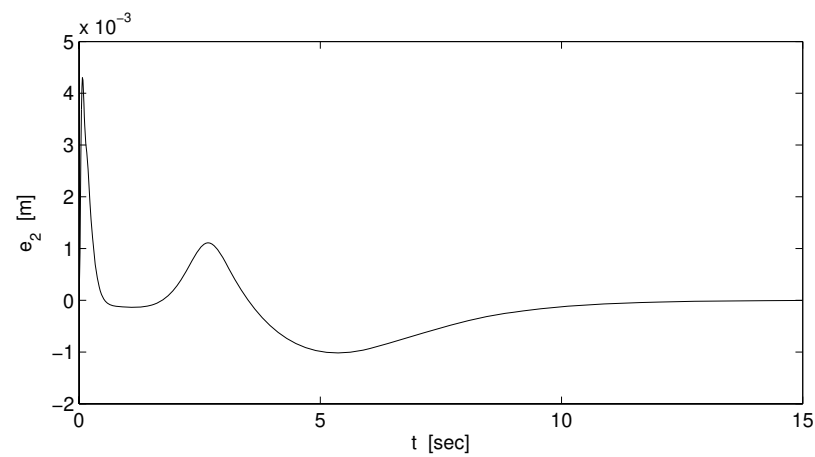


Figure 3. Path following error e_2 vs. time

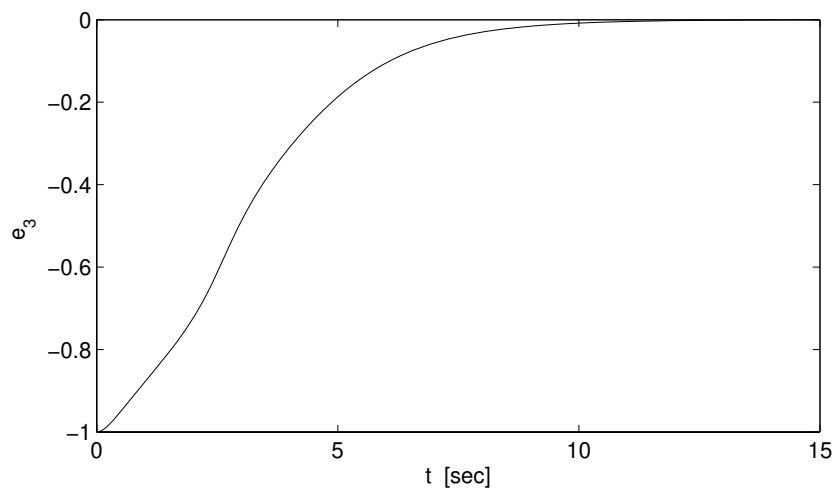


Figure 4. Path following error e_3 vs. time

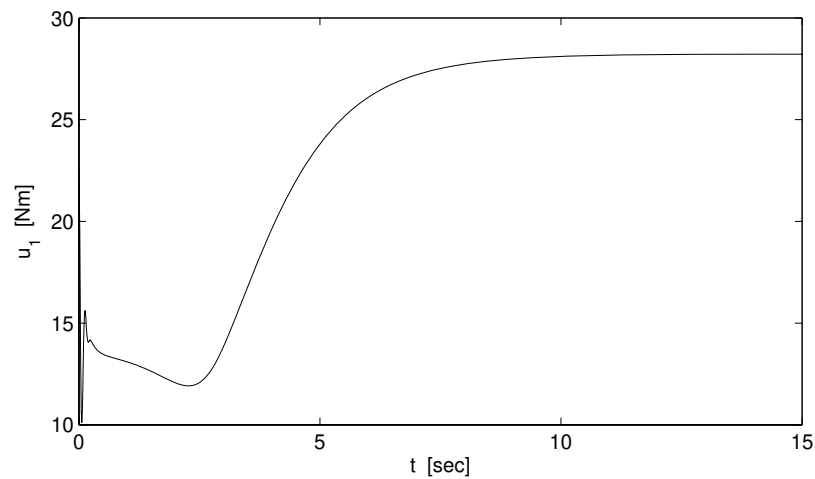


Figure 5. Input torque u_1 vs. time

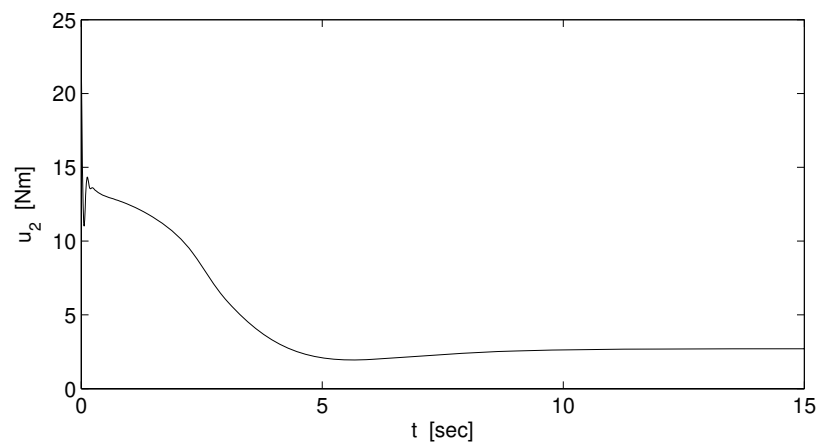


Figure 6. Fig. 6 Input torque u_2 vs. time

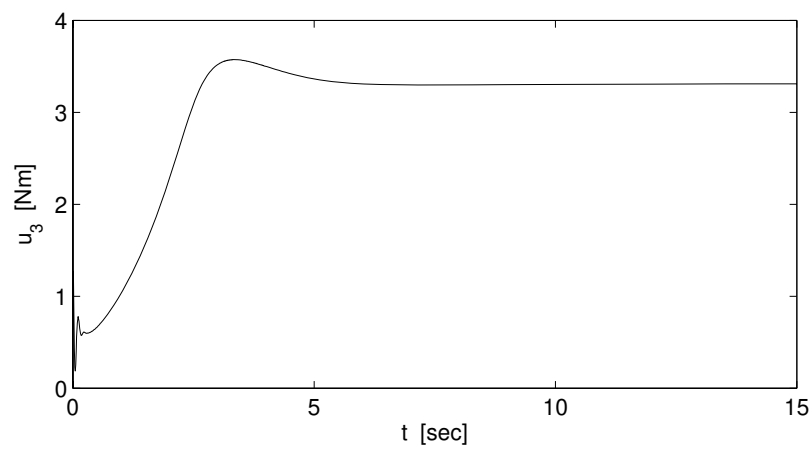


Figure 7. Input torque u_3 vs. time

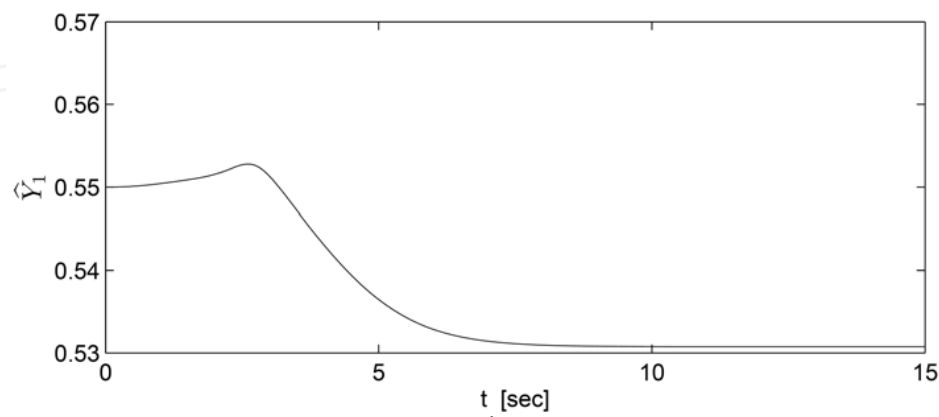


Figure 8. Time course of adaptive estimate $\hat{\gamma}_1$

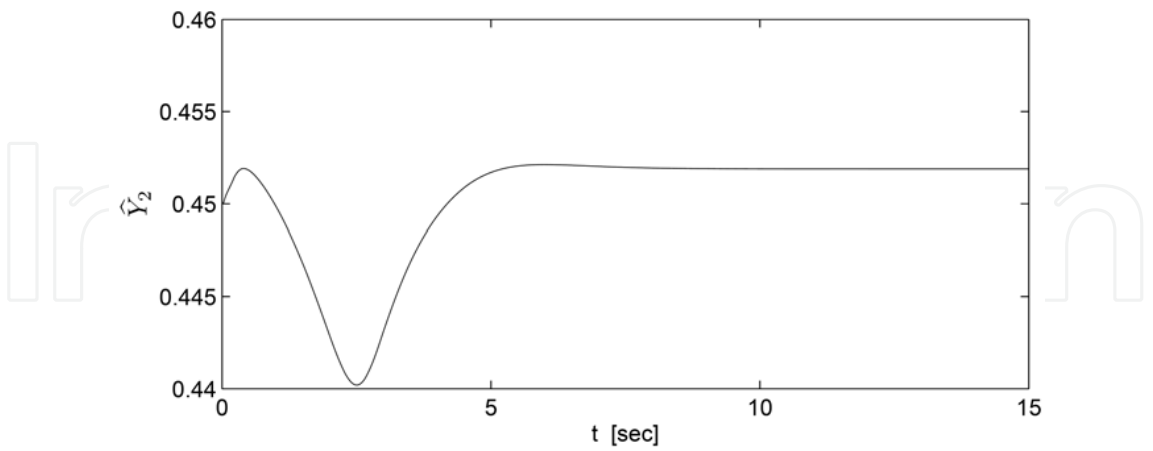


Figure 9. Time course of adaptive estimate \hat{Y}_2

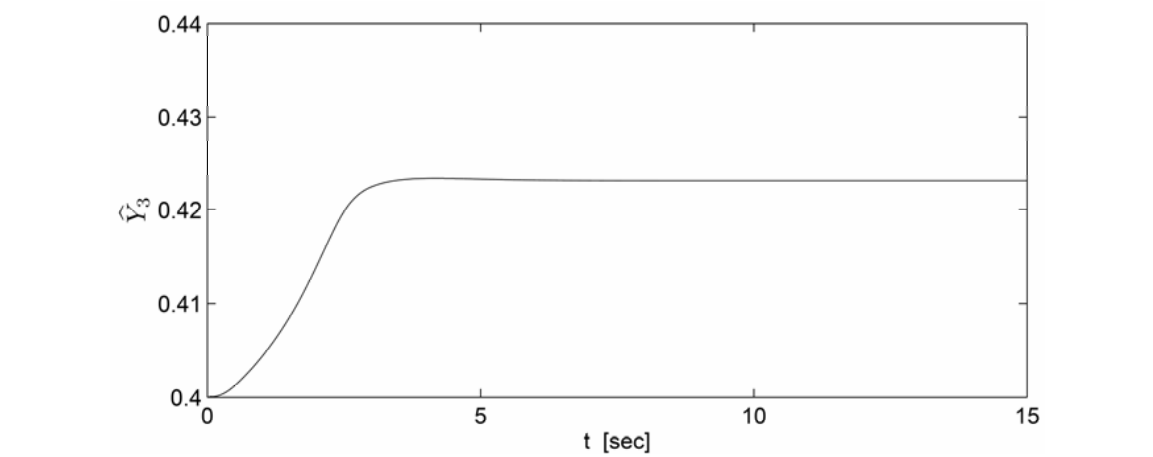


Figure 10. Time course of adaptive estimate \hat{Y}_3

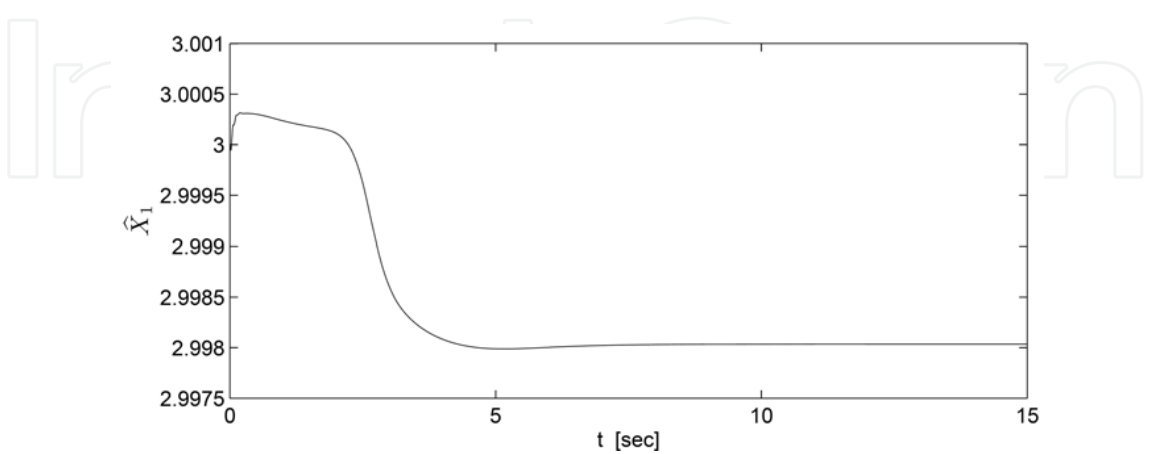


Figure. 11. Time course of adaptive estimate \hat{X}_1

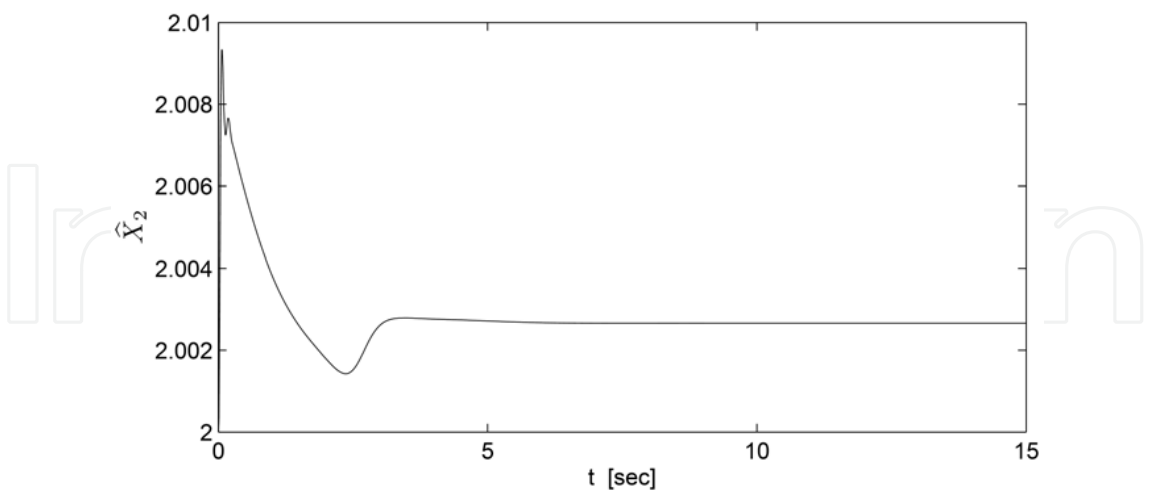


Figure 12. Time course of adaptive estimate \hat{X}_2

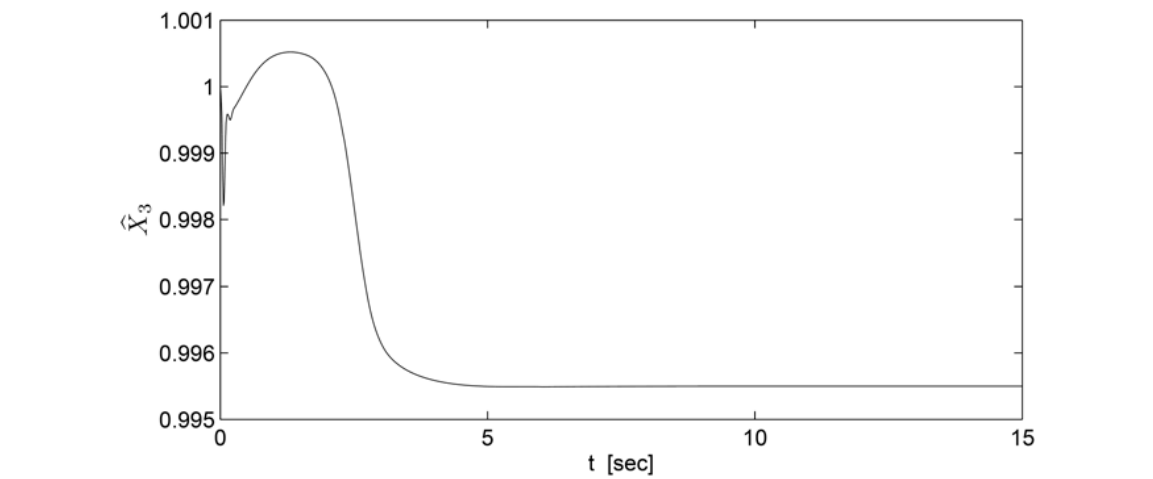


Figure 13. Time course of adaptive estimate \hat{X}_3

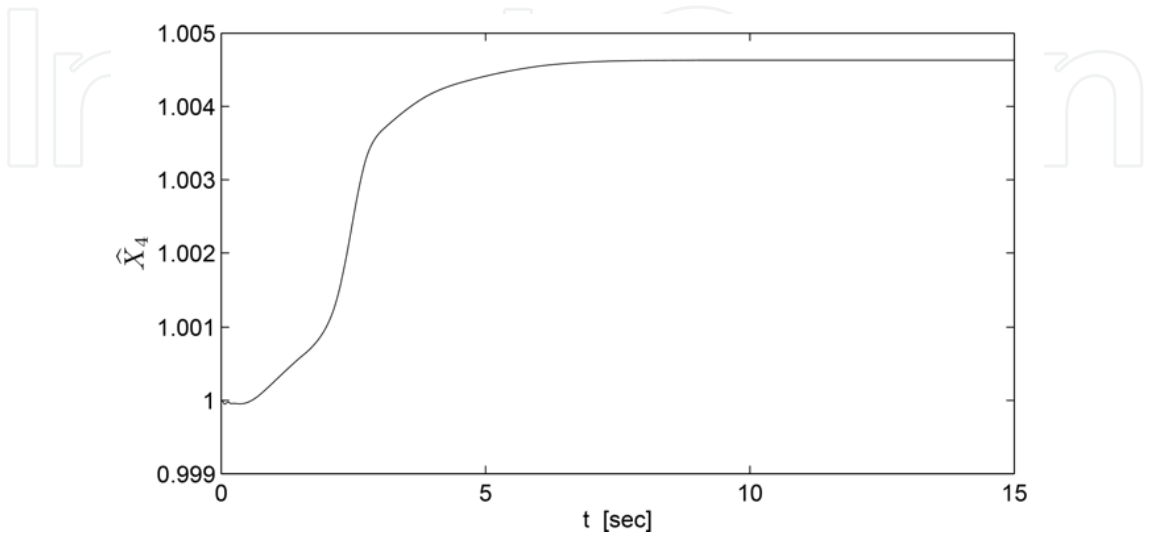


Figure 14. Time course of adaptive estimate \hat{X}_4

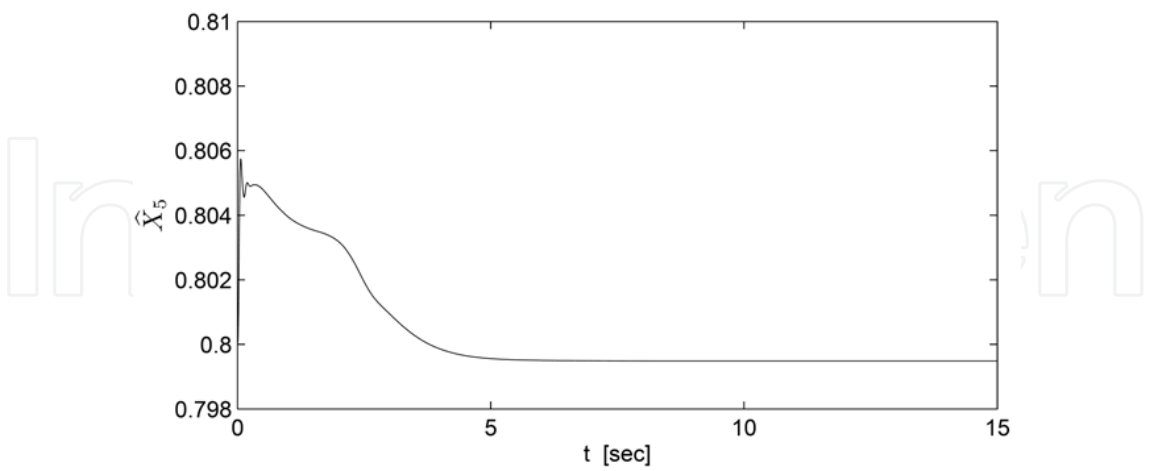


Figure 15. Time course of adaptive estimate \hat{X}_5

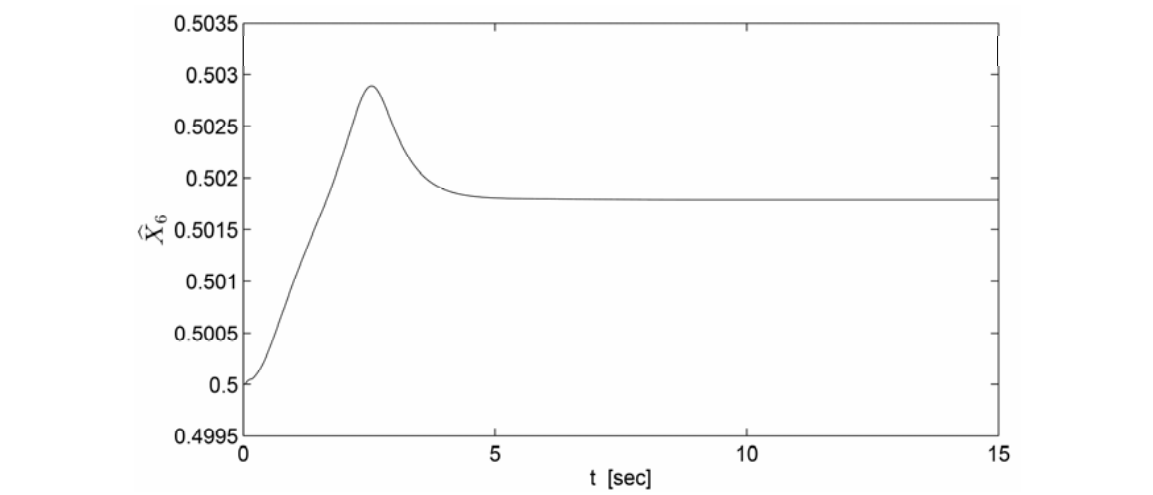


Figure 16. Time course of adaptive estimate \hat{X}_6

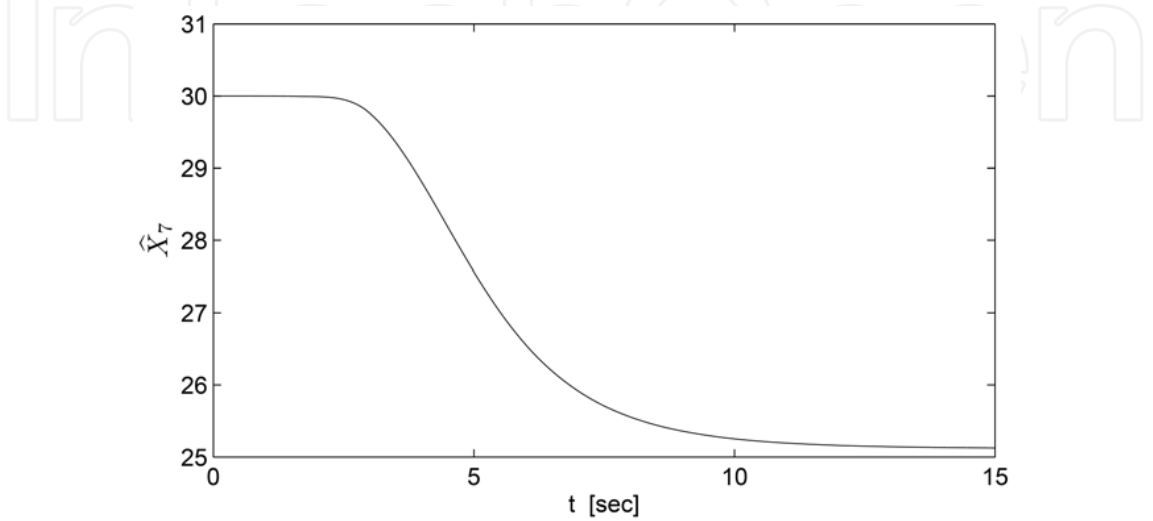


Figure 17. Time course of adaptive estimate \hat{X}_7

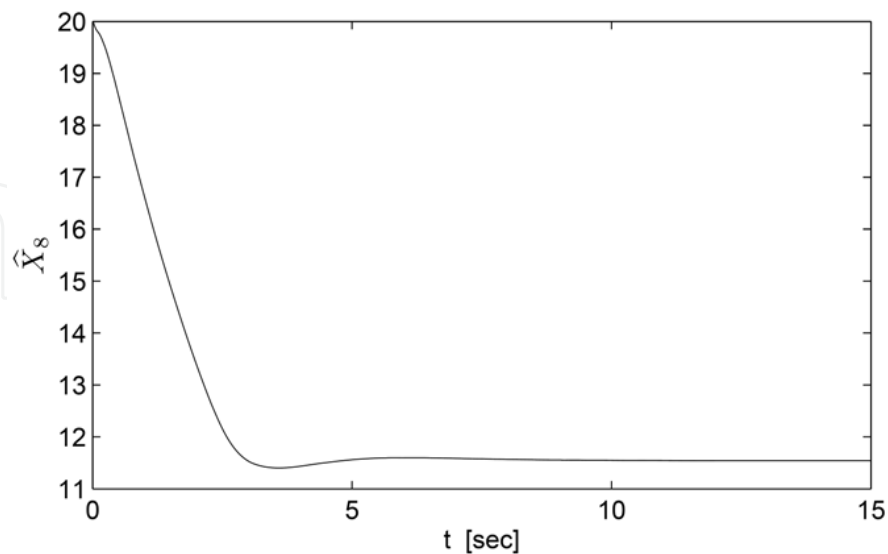


Figure 18. Time course of adaptive estimate \hat{X}_8

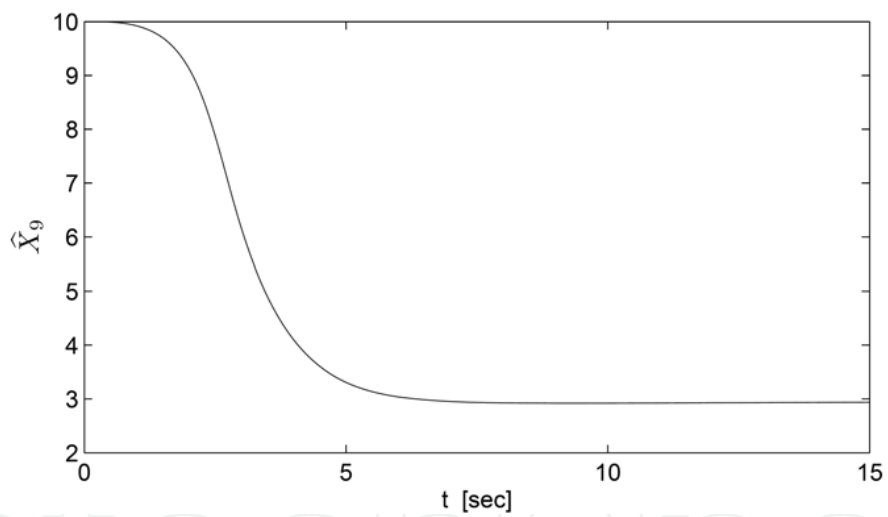


Figure 19. Time course of adaptive estimate \hat{X}_9

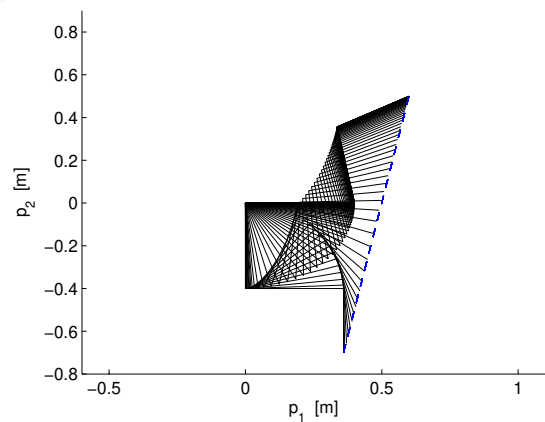


Figure 20. Manipulator motion along the geometric path

As might be expected, the path following errors from Figs 2-3 are much smaller than those obtained from the conservative dependence (26). Moreover, as one can observe from Figs 2-7, the time dependent damping function k_s decreases (eliminates) errors and torques oscillations at the very beginning of time histories. Furthermore, as seen from Figs 8-19, estimations \hat{X} , \hat{Y} do not converge to their real (nominal) values.

5. Conclusion

This study has presented an adaptive robot controller for the path following by the end-effector. The control generation scheme has been derived using the Lyapunov stability theory. An advantage of the proposed control law (18) is that it requires, in fact no information regarding the parameters of the robot dynamic equations. The control strategy (18) is shown to be asymptotically stable (by fulfilment of practically reasonable assumptions). The proposed robot controller has been applied to a planar redundant manipulator of three revolute kinematic pairs operating in a two dimensional task space. Numerical simulations have shown that the results obtained are in accordance with the theoretical analysis. The novelty of the strategy proposed lies in its relative simplicity in design, program code and real-time implementation. The approach presented here will also be in future directly applicable to cooperating kinematically redundant manipulators.

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This book covers a wide range of topics relating to advanced industrial robotics, sensors and automation technologies. Although being highly technical and complex in nature, the papers presented in this book represent some of the latest cutting edge technologies and advancements in industrial robotics technology. This book covers topics such as networking, properties of manipulators, forward and inverse robot arm kinematics, motion path-planning, machine vision and many other practical topics too numerous to list here. The authors and editor of this book wish to inspire people, especially young ones, to get involved with robotic and mechatronic engineering technology and to develop new and exciting practical applications, perhaps using the ideas and concepts presented herein.

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