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Industrial Robot Control System Parametric Design on the Base of Methods for Uncertain Systems Robustness

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1. Introduction

Industrial robots often operate in conditions of their parameters substantial variation that causes variation of their control systems characteristic equations coefficients values, thus generating the equations families. Analysis of the dynamic systems characteristic polynomial families stability, the stable polynomials and polynomial families synthesis represent complicated and important task (Polyak, 2002, a). Within the parametric approach to the problem the series of the effective methods for analysis have been developed (Bhattaharyya et al., 1995; Polyak, 2002, a). In this way, V. L. Kharitonov (Kharitonov, 1978) proved that for the interval uncertain polynomials family asymptotic stability verification it is necessary and enough to verify only four polynomials of the family with the definite constant coefficients. In the works of Y. Z. Tsypkin and B. T. Polyak the frequency approach to the polynomially described systems robustness was offered (Polyak & Tsypkin, 1990; Polyak & Scherbakov, 2002; Tsypkin & Polyak, 1990; Tsypkin, 1995). This approach comprises the robust stability criteria for linear continuous systems, the methods for calculating the maximal disturbance swing for the nominal stable system on the base of the Tsypkin – Polyak hodograph. These results were generalized to the linear discrete systems (Tsypkin & Polyak, 1990). The robust stability criterion for the relay control systems with the interval linear part was obtained (Tsypkin, 1995). The super-stable linear systems were considered (Polyak & Scherbakov, 2002).

The problem for calculating the polynomial instability radius on the base of the frequency approach is investigated (Kraev & Fursov, 2004). The technique for composing the stability domain in the space of a single parameter or two parameters of the system with the *D*-decomposition approach application is developed (Gryazina & Polyak. 2006).

The method for definition of the nominal polynomial coefficients deviations limit values, ensuring the hurwitz stability, has been offered (Barmish, 1984). The task here is reduced to the single-parameter optimization problem. The similar tasks are solved by A. Bartlett (Bartlett et al., 1987) and C. Soh (Soh et

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al., 1985). Conditions for the generalized stability of polynomials with the linearly dependent coefficients (polytopes) have been obtained (Bartlett et al., 1987; Rantzer, 1992).

One of the most important stages, while calculating dynamic systems with uncertain parameters, is ensuring robust quality. The control process qualitative characteristics are defined by the characteristic equations roots location in the complex plane (the plane of system fundamental frequencies). In this connection, three main groups of tasks being solved can be distinguished: determining the assured roots location domain (region) for the given system, finding conditions of whether roots get into the given region or not (determination of the Λ-stability conditions) and locating roots in the given domain (ensuring Λ-stability).

The frequency stability criteria for the linear systems families and also the method for finding the largest disturbance range of their characteristic equations coefficients, which guarantees the system asymptotic stability, are considered by B. T. Polyak and Y. Z. Tsypkin (Polyak & Tsypkin, 1990). The assured domain of the interval polynomial roots location is found in (Soh et al., 1985). The root locus theory is used in (Gaivoronsky, 2006) for this task solution. Conditions (Vicino, 1989; Shaw & Jayasuriya, 1993) for the interval polynomial roots getting into the given domain of some convex shape are defined.

The parametric approach to robustness, based on the root locus theory (Rimsky, 1972; Rimsky & Taborovetz, 1978; Nesenchuk, 2002; Nesenchuk, 2005), is considered in this chapter in application to the industrial anthropomorphous robot control system parametric design. The developed techniques allow to set up the values of the parameter variation intervals limits for the cases when the stability verification showed, that the given system was unstable, and to ensure the system robust quality by locating the characteristic equations family roots within the given quality domain.

2. Industrial robot and its control system description

Most industrial robots are used for transportation of various items (parts), e. g. for installing parts and machine tools in the cutting machines adjustments, for moving parts and units, etc. During the robot operation due to some internal or external reasons its parameters vary, causing variation of the system characteristic equation coefficients. This variation can be rather substantial. In such conditions the system is considered, as the uncertain system.

2.1 General description of the anthropomorphous industrial robot

The industrial robot considered here is used for operation as an integrated part of the flexible industrial modules including those for stamping, mechanical as-

sembly, welding, machine cutting, casting production, etc. The industrial robot is shown in fig. 1. It comprises manipulator 1 of anthropomorphous structure, control block 2 including periphery equipment and connecting cables 3. Manipulator has six units (1–8 in fig. 1) and correspondingly is of six degrees of freedom (see fig. 1): column 4 turn, shoulder 5 swing, arm 6 swing, hand 7 swing, turn and rotation. The arm is connected with the joining element 8. Controlling robots of such a type, belonging to the third generation, is based on the hierarchical principle and features the distributed data processing. It is based on application of special control processors for autonomous control by every degree of freedom (lower executive control level) and central processor coordinating their operation (higher tactical control level).

2.2 Industrial robot manipulator unit control system, its structure and mathematical model

Executive control of every manipulator unit is usually executed in coordinates of this unit (Nof, 1989) and is of the positional type. It is the closed-loop servo-control system not depending on the other control levels. Although real unit control is executed by a digital device (microprocessor, controller) in a discrete way, the effect of digitization is usually neglected, as the digitization frequency is high enough to consider the unit and the controller as the analog (continuous) systems. As for the structure, the unit control loops are almost similar and differ only in the parameter values. Therefore, any unit of the industrial robot can be considered for investigating the dynamic properties.

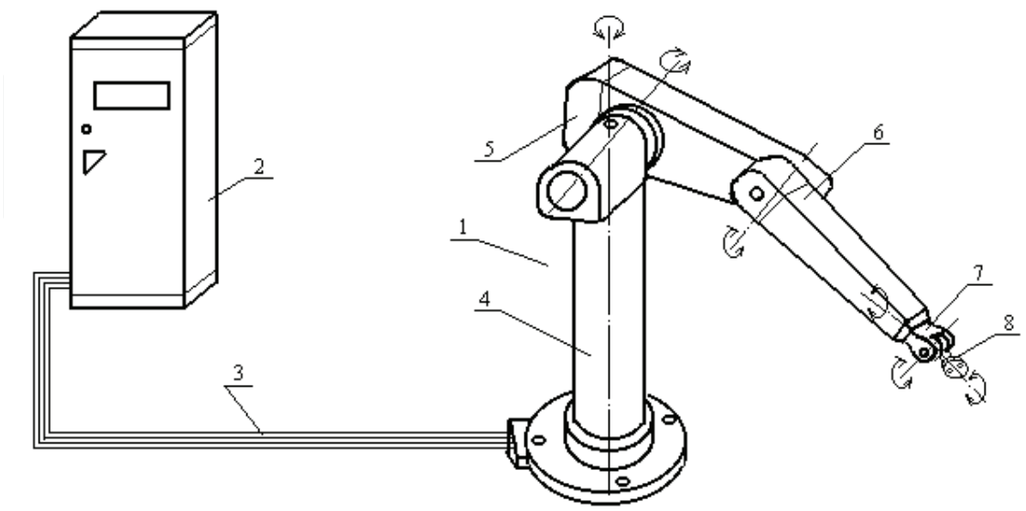


Figure 1. Anthropomorphous industrial robot

The structure of the manipulator unit subordinate control is shown in fig. 2. The simplified version of the structure is presented in fig 3.
In fig. 2 the plant is represented by elements 1–4 (a DC motor); 5 is the sensor transforming the analog speed signal into the speed code (photo-pulse sensor), 6 is the element combining the speed PI regulator, code-pulse width transformer and capacity amplifier, 7 is the transformer of analog position signal into the position code (photo-pulse sensor), 8 is the proportional regulator of the manipulator shoulder position, 9 is the transfer mechanism (reducer). In fig. 3 the transfer function

$$W_p^{'}(s) = W_p(s)s$$

where $W_p(s)$ is the plant transfer function.

Substitute corresponding parameters and express the plant transfer function as follows:

$$W_p(s) = \frac{\varphi}{U_g} = \frac{1}{(j_m + j_l)\frac{L_A}{C_M}s^3 + (j_m + j_l)\frac{R_A}{C_M}s^2 + C_e s}, \quad (1)$$

where U_g is the input voltage, φ is the object shaft angle of rotation.

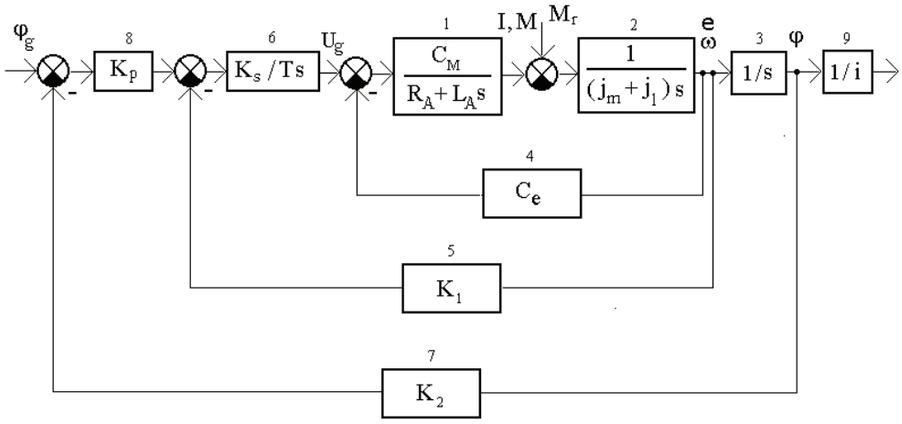


Figure 2. Control system for the industrial robot manipulator shoulder unit

On the basis of (1) write the manipulator unit control system characteristic equation

$$s^4 + \frac{R_A}{L_A}s^3 + \frac{C_e C_M}{j_m L_A}s^2 + \frac{C_M K_1 K_s}{j_m L_A T}s + \frac{C_M K_2 K_p K_s}{j_m L_A T} = 0$$

or as

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0, \quad (2)$$

where

$$a_0 = 1;\ a_1 = R_A \big/ L_A;\ a_2 = \frac{C_e C_M}{(j_m + j_{l'})L_A};\ a_3 = \frac{C_M K_1 K_s}{(j_m + j_l)L_A T};\ a_4 = \frac{C_M K_2 K_p K_s}{(j_m + j_l)L_A T};$$

- R_A is the motor anchor resistance;
- L_A is the anchor inductance;
- j_l is the load inertia moment;
- j_m is the anchor inertia moment;
- C_e is the electric-mechanical ratio of the motor;
- C_M is the constructive constant of the motor;
- T is the time constant of the PI regulator;
- K_1 and K_2 are photo-electric sensor coefficients;
- K_s and K_p are gains of regulators by speed and position correspondingly.

Suppose the robot unit has the following nominal parameters:

- $R_A = 0{,}63\ \Omega$;
- $L_A = 0{,}0014$ henry;
- $j_l = 2{,}04 \cdot 10^{-5}\ kg/m^2$
- $j_m = 40{,}8 \cdot 10^{-5}\ kg/m^2$;
- $C_e = 0{,}16 \frac{V \cdot s}{rad}$;
- $C_M = C_e$;
- $T = 0{,}23\ s$;
- $K_1 = 66{,}7,\ K_2 = 250$;
- $K_s = 0{,}078,\ K_p = 2{,}5$.

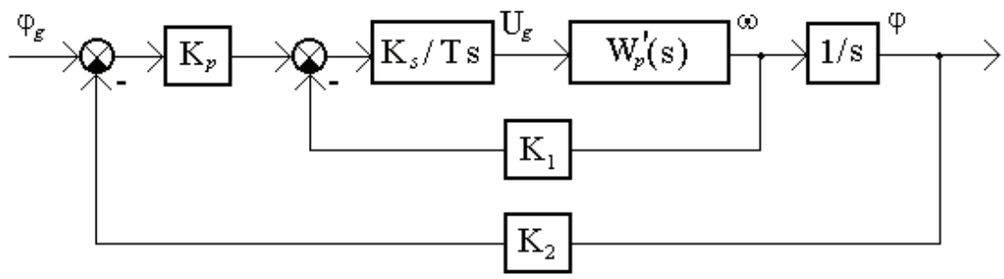


Figure 3. Structure of the position control system loop for the manipulator shoulder unit

After substitution of the nominal values into (2) rewrite the unit characteristic equation as

$$s^4 + 0{,}5 \cdot 10^3 s^3 + 0{,}427 \cdot 10^5 s^2 + 0{,}6 \cdot 10^7 s + 0{,}56 \cdot 10^8 = 0 \quad (3)$$

The coefficients of (3) are the nominal ones and while robot operation they often vary within the enough wide intervals. For this reason when calculating the robot control system it is necessary to consider the parameters uncertainty and ensure the control system robustness.

3. The techniques for robust stability of systems with parametric uncertainty

The method is described for synthesis of the interval dynamic system (IDS) stable characteristic polynomials family from the given unstable one, based on the system model in the form of the free root locus portrait. This method allows to set up the given interval polynomial for ensuring its stability in cases, when it was found, that this polynomial was unstable. The distance, measured along the root locus portrait trajectories, is defined as the setting up criterion, in particular, the new polynomial can be selected as the nearest to the given one with consideration of the system quality requirements. The synthesis is carried on by calculating new boundaries of the polynomial constant term variation interval (stability interval), that allows to ensure stability without the system root locus portrait configuration modification

3.1 The task description

While investigating uncertain control systems for getting more complete representation of the processes, which occur in them, it seems substantial to discover correlation between algebraic, frequency and root locus methods of in-

vestigation. Such correlation exists and can be applied for finding dependence between the system characteristic equation coefficients values (parameters) and its dynamic properties to determine how and what coefficients should be changed for ensuring stability. One of the ways for establishing the above mentioned correlation can be investigation of the systems root locus portraits and Kharitonov's polynomials root loci (Kharitonov, 1978).
Consider the IDS, described by the family of characteristic polynomials

$$P(s) = \sum_{j=0}^{n} a_j s^{n-j} = 0, \tag{4}$$

where $a_j \in [\underline{a}_j, \overline{a}_j]$, $\underline{a}_0 > 0$, $j = 0, \ldots, n$; $\underline{a}_j$ and $\overline{a}_j$ are correspondingly the lower and upper boundaries of the closed interval of uncertainty, $[\underline{a}_j, \overline{a}_j]$; $s = \sigma + i\omega$.
The coefficients of polynomial (4) are in fact the uncertain parameters.
The task consists in synthesis of the stable interval family of polynomials (4) on the basis of the initial (given) unstable one, i. e., when the initial system stability verification by application of Kharitonov's polynomials gave the negative result. Calculation of new parameter variation intervals boundaries is made on the base of the initial boundaries in correspondence with the required dynamic characteristics of the system. The new boundaries values definition criteria can be different, in particular they can be selected the nearest to the given ones. In this case the distance, measured along the system roots trajectories, is accepted to be the criterion of such proximity.

3.2 The interval system model in the form of the root locus portrait

Introduce the series of definitions.

Definition 1. Name the root locus of the dynamic system characteristic equation (polynomial), *as the dynamic system root locus*.

Definition 2. Name the family (the set) of the interval dynamic system root loci, *as the root locus portrait of the interval dynamic system*.

Definition 3. The algebraic equation coefficient or the parameter of the dynamic system, described by this equation, being varied in a definite way for generating the root locus, when it is assumed, that all the rest coefficients (parameters) are constant, name as the algebraic equation root locus free parameter or simply the root locus parameter.

Definition 4. The root locus, which parameter is the coefficient a_k, name as the algebraic equation *root locus relative to the coefficient* a_k.

Definition 5. The root locus relative to the dynamic system characteristic equation constant term name as the *free root locus of the dynamic system*.

Definition 6. The points, where the root locus branches begin and the root locus parameter is equal to zero, name as the *root locus initial points.*

Remark 1. One of the free root locus initial points is always located at the origin of the roots complex plane.
The above remark correctness follows from the form of equation (4).

Remark 2. The free root locus positive real branch portion, adjacent to the initial point, located at the origin, is directed along the negative real half-branch σ of the complex plane to the left half-plane.

Remark 2 is correct due to the root loci properties (Uderman, 1972) and because real roots of equations with positive coefficients are always negative (see fig. 4).

The peculiarity of the free root loci, which distinguishes them from another types of root loci, consists in the fact, that all their branches strive to infinity, approaching to the corresponding asymptotes.
For carrying on investigation apply the Teodorchik – Evans free root loci (TEFRL) (Rimsky, 1972), i. e. the term "root locus" within this section will mean the TEFRL, which parameter is the system characteristic equation constant term.
To generate the IDS root locus portrait apply the family of the mapping functions

$$s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-2} s^2 + a_{n-1} s = u(\sigma,\omega) + iv(\sigma,\omega) = -a_n, \quad (5)$$

where $u(\sigma,\omega)$ and $v(\sigma,\omega)$ are harmonic functions of two independent variables σ and ω; a_n is the root locus parameter; $s = \sigma + i\omega$. Analytical and graphical root loci are formed using mapping function (5). The root locus equation is as follows:

$$iv(\sigma,\omega) = 0 \quad (6)$$

and the parameter equation (Rimsky, 1972) as follows:

$$u(\sigma,\omega) = -a_n. \quad (7)$$

The fragmentary root locus portrait for the IDS of the forth order, which is made up of four Kharitonov's polynomials free root loci, is shown in fig. 4. The Kharitonov's polynomials h_1, h_2, h_3 and h_4 in this figure are represented by points (roots), marked with circles, triangles, squares and painted over squares correspondingly. There are the following designations: $\sigma_{h\,i}$, i = 1, 2, 3, 4, – the cross centers of asymptotes for the root loci of every polynomial h_i, t_l, l = 1, 2, 3, – cross points of the root loci branches with the system asymptotic stability boundary, axis $i\omega$. The root loci initial points, which represent zeroes of mapping function (5), are depicted by X-s. Because in fig. 4 all roots of the Kharitonov's polynomials are completely located in the left half-plane, the given interval system is asymptotically stable (Kharitonov, 1978).

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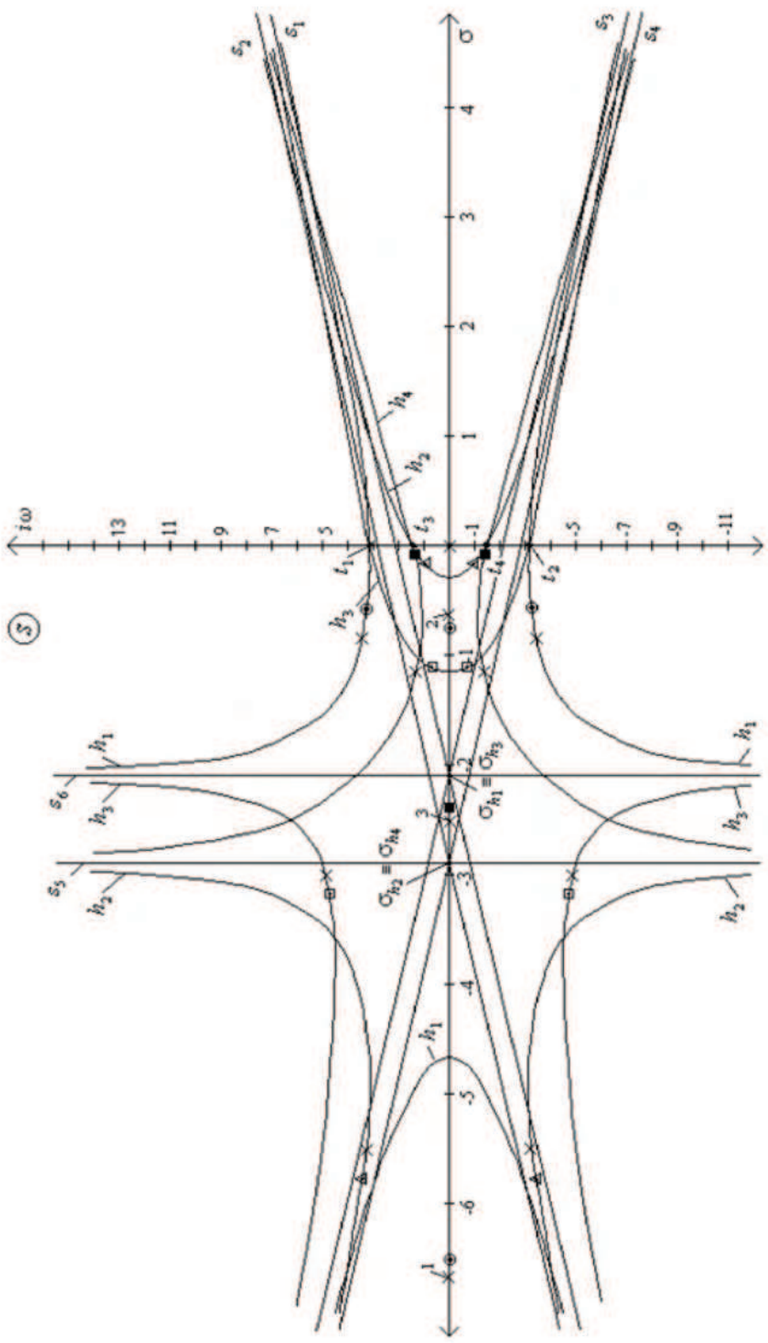


Figure 4. Root loci of the Kharitonov's polynomials for the system of class [4;0]

3.3 Investigation of the characteristic polynomial family root loci branches behavior at the asymptotic stability boundary of the system

The branches of the IDS root locus portrait, when crossing the stability boundary, generate on it the region (set) of cross points. Name this region, as *the cross region* and designate it as R_ω. According to the theory of the complex variable (Lavrentyev & Shabat, 1987) and due to the complex mapping function (5) continuity property, this region is the many-sheeted one and is composed of the separate sheets with every sheet (continuous subregion), formed by the separate branch while it moves in the complex plane following the parameters variation. The cross region portion, generated by only positive branches of the system root locus portrait, name as *the positive cross region* and designate it as R_ω^+.

$$R_\omega^+ \subset R_\omega. \tag{8}$$

Define also the subregion r_ω^+ (either continuous or discrete one) of the cross region R_ω^+ (8) generated by the root loci branches of any arbitrary *subfamily f* of the interval system polynomials family (4), and name it as *the* (*positive*) *cross subregion*, thus,

$$r_\omega^+ \subset R_\omega^+. \tag{9}$$

Introduce the following sets:

$$W_r^+ = \left\{\omega_{r\,i}^+\right\} \tag{10}$$

$$A_r^+ = \left\{a_{r\,i}^+\right\} \tag{11}$$

where W_r^+ is the set (family) of the cross subregion r_ω^+ (9) points coordinates $\omega_{r\,i}^+$; A_r^+ is the set (family) of values $a_{r\,i}^+$ of the root locus parameter a_n at the set W_r^+ points.

Define the minimal positive value $a_{r\,\min}^+$ of the root locus parameter within the cross subregion r_ω^+:

$$a_{r\,\min}^+ = \inf A_r^+. \tag{12}$$

Peculiarities of the IDS root loci initial points location make it possible to draw a conclusion about existence of its characteristic equation coefficients variation intervals, ensuring asymptotic stability of the given system.

Statement. If the initial points of the IDS characteristic polynomials arbitrary subfamily *f* free root loci, excluding points always situated at the origin, are located in the left complex half-plane *s,* there exists the interval *d* of the root loci parameter a_n values, ensuring asymptotic stability of the subfamily *f*.

$$d = (0, a^+_{r\,\min}), \tag{13}$$

Proof. The subfamily *f* free root loci generate at the system stability boundary the cross subregion $r_\omega{}^+$ (9) of cross points, which is formed by the set (10) of the cross points coordinates and corresponding set (11) of the parameters values. If the initial points are located, as it is defined by the statement, on every *i*-th branch of every polynomial root loci there exist an interval $r_i = (\sigma_{l_i}, 0)$ of roots values (starting from the branch initial point with coordinate σ_{l_i} until the point, where it crosses the stability boundary, axis $i\omega$ of the complex plane), which is completely located in the left half-plane. Therefore, there exists also the appropriate maximum possible common interval d_m (which is common for all the branches) of the root loci parameter a_n values (beginning from zero up to the definite maximum possible value $a_n = a^+_{r\,m}$), corresponding to the values of roots within some interval $r_k = (\sigma_{l_k}, 0)$, which ensures the system stability. Name this interval d_m the *dominating interval* and define it as $d_m = (0, a^+_{r\,m})$. Designate the roots σ_i coordinates values interval, located on every positive *i*-th branch of the family and corresponding to the dominating interval, as $r_d = (\sigma_{l_i}, \sigma_{r_i})$. It is evident, that $a^+_{r\,m}$ will be maximum possible at the stability boundary, i. e. at $\sigma_{r_i} = 0$. Then, $\forall \sigma_{r_i} [a^+_{r\,m} = a^+_{r\,\min} \rightarrow \sigma_{r_i} \le 0]$, i. e. the dominating one is the interval $d_m = (0, a^+_{r\,\min})$, which represents itself the interval *d* (13). Hence, the statement is correct.

Definition 7. The interval of polynomial (4) root loci parameter values name *the polynomial stability interval by this parameter* or simply *the polynomial stability interval*, if the polynomial asymptotic stability property holds within this interval.

In case, if some initial points are located at the stability boundary (excluding the point, which is always located at the origin), and on the assumption, that all the rest points are located in the left half-plane, the additional analysis is required for finding the stability interval existence. For this purpose it is necessary to define the root loci branches direction at their outcome from the initial

points, located at the stability boundary, i. e. just to determine what half-plane they are directed to: left one or right one. Obviously, such stability interval exists in the following cases:

a) all the root loci branches with initial points, located at the stability boundary, are directed from these points to the left half-plane;
b) all positive root loci branches with initial points, located at the stability boundary, are directed from these points to the left half-plane.

To determine the above indicated branches direction at the initial points, it is enough to define the root locus sensitivity vector (Nesenchuk, 2005) direction at them.

As a result of the IDS root locus portraits analysis several general regularities have been discovered, being inherent in Kharitonov's polynomials free root loci: paired convergence of the root loci branches at the complex plane imaginary axis (points t_1, t_2, t_3, t_4 in fig. 4); paired convergence of the corresponding asymptotes at the real axis of the complex plane (points σ_{h1}, σ_{h2}, σ_{h3}, σ_{h4} in fig. 4); the tendency for the system robust properties variation while varying its characteristic polynomial coefficients values. It gives the possibility to fix the fact of existence of the system characteristic equation coefficients variation intervals, ensuring its robust stability and also to determine how the coefficients values should be changed for the system dynamic characteristics correction, if it is unstable.

The IDS root locus portraits investigation, which has been carried out, confirms that they can be successfully applied for the in-depth studying robust properties of these systems.

3.4 Parametric synthesis of stable uncertain systems

The conditions for existence of the polynomials (4) family coefficients stability intervals were formulated in the previous section. Here we define what these intervals values should be. For this purpose consider the polynomials (4) subfamily *f*, consisting of the system Kharitonov's polynomials, and develop the procedure for synthesis of the stable Kharitonov's polynomials on the base of the unstable ones, which depends on the root loci initial points location in relation to the asymptotic stability boundary. For the synthesis procedure development apply the Kharitonov's polynomials free root loci. Consider the case, when initial points are located in the left half-plane. In this case the algorithm of synthesis can be divided into the following stages.

Stage 1. Obtaining the Teodorchik – Evans free root loci equation (6) for each one of the IDS four Kharitonov's polynomials.
As the Kharitonov's polynomials represent the subfamily of the IDS polynomials family, they generate the above described cross subregion r_ω^+ (9) on the stability boundary, which is formed by the set (10) of the cross points coordinates.

Stage 2. Calculating coordinates $\omega_{r\,i}^+$ of the set (10) by solution of the TEFRL equations, obtained in stage 1, relative to ω in condition, that σ = 0. In this way the set W_r^+ (10) is formed.
For every obtained value of $\omega_{r\,i}^+$ from W_r^+ the corresponding value of the variable coefficient a_n is calculated by formula (7), thus, forming the set A_r^+ (11).

Stage 3. Definition of the stability interval by the coefficient a_n.
For this purpose, using (12), define the minimal one, $a_{r\,\min}^+$, of the parameter values at points of the set A_r^+. Thus obtain the interval d (13) of the parameter a_n variation, which ensures stability of the Kharitonov's polynomials and, therefore, the system in whole.

Before describing the next stage of synthesis formulate the following theorem.

Theorem. For robust stability of the polynomial family (4) it is necessary and enough to ensure the upper limit of the constant term a_n variation interval to satisfy the inequality

$$\bar{a}_n < a_{r\,\min}^+, \tag{14}$$

if the family is stable at $a_n = 0$.

Proof. Let the coefficient a_n to be the polynomial (4) root locus parameter. Under the theorem condition family of (4) is stable at $a_n = 0$, i.e. the root loci initial points are located in the left half-plane. Therefore, in view of statement 1 the theorem is valid.

Stage 4. Comparing the obtained stability interval (13) with the given interval $a_n \in [\underline{a}_n, \bar{a}_n]$ of the parameter a_n variation in correspondence with inequality (14).

In case, if condition (14) is not satisfied, the upper limit $\bar{a}_n$ of the parameter variation interval is set up in correspondence with this inequality.
When the power n of the polynomial is less or equal than 3, n ≤ 3, the above given theorem is applied without any conditions, i. e. it is not required to sat-

isfy condition of the Kharitonov's polynomials roots real parts negativity at $a_n = 0$, because in this case the coefficients positivity always guarantees negativity of the roots real parts.
The above described algorithm allows to carry on the parametric synthesis of the stable interval system without modification of its root locus portrait configuration, by simple procedure of setting up the characteristic polynomial constant term variation interval limits.
The *numerical example* , demonstrating the results obtained, is given below
Consider the interval system, described by the initial characteristic polynomial

$$s^4 + 10s^3 + 35s^2 + 50s + 24 = 0, \tag{15}$$

where the real coefficients are: $a_0 = 1$; $8{,}4 \le a_1 \le 11{,}6$; $24 \le a_2 \le 48$; $26{,}5 \le a_3 \le 83{,}1$; $8{,}99 \le a_4 \le 50{,}3$.

Let the coefficient a_4 to be the root locus parameter. Then, define the mapping function:

$$-a_4 = a_0\sigma^4 + 4a_0\sigma^3 i\omega - 6a_0\sigma^2\omega^2 - 4a_0\sigma i\omega^3 + a_0\omega^4 + a_1\sigma^3 + 3a_1\sigma^2 i\omega - \\ - 3a_1\delta\sigma^2 - a_1 i\omega^3 + a_2\sigma^2 + 2a_2\delta i\sigma - a_2\omega^2 + a_3\sigma + a_3 i\omega.$$

Write correspondingly the TEFRL and the parameter equations::

$$\omega(4a_0\sigma^3 - 4a_0\sigma\omega^2 + 3a_1\sigma^2 - a_1\omega^2 + 2a_2\sigma + a_3) = 0;$$
$$a_0\sigma^4 - 6a_0\sigma^2\omega^2 + a_0\omega^4 + a_1\sigma^3 - 3a_1\sigma\omega^2 + a_2\sigma^2 - a_2\sigma = -a_4.$$

Define the Kharitonov's polynomials for the interval system with the initial characteristic polynomial (15):

$$h_1(s) = s^4 + 8{,}4s^3 + 24s^2 + 83{,}1s + 50{,}3;$$
$$h_1(s) = s^4 + 11{,}6s^3 + 48s^2 + 26{,}5s + 8{,}99;$$
$$h_1(s) = s^4 + 8{,}4s^3 + 48s^2 + 83{,}1s + 8{,}99;$$
$$h_1(s) = s^4 + 11{,}6s^3 + 24s^2 + 26{,}5s + 50{,}3.$$

The root loci of these polynomials are represented in fig. 4, described above.
Number of asymptotes n_a (in fig. 4 they are indicated as s_1, s_2, …, s_6) is constant for every one of Kharitonov's polynomials and is equal to

$$n_a = n - m = 4 - 0 = 4,$$

where m is the number of poles for function (5).

The centers of asymptotes are located on the axis σ and have coordinates: σ_{h1} = 2,10; σ_{h2} = 2,90; σ_{h3} = 2,10; σ_{h4} = 2,90 (see fig. 4). The asymptotes centers coordinates coincide in pairs: for the pair $h_1(s)$ and $h_3(s)$, and also for the pair $h_2(s)$ and $h_4(s)$.
The inclination angles of asymptotes for the given root loci are correspondingly the following:

$$\varphi_1 = 0^0; \quad \varphi_3 = 135^0;$$
$$\varphi_2 = 45^0; \quad \varphi_4 = 180^0.$$

According to fig. 4, every pair of the root loci strives to the same asymptotes, i.e. the pairs are formed by those root loci, which asymptotes centers coincide, as it was indicated above.
For definition of equation (15) coefficients intervals, ensuring the system stability, stability condition (14) is applied. Thus, the following values $a_{r\,i}^{+}$ of the set A_r^+ have been defined:

$a_{r\,1}^{+}$ = 139,67 for the polynomial h_1;
$a_{r\,2}^{+}$ = 116,33 for the polynomial h_2;
$a_{r\,3}^{+}$ = 377,75 for the polynomial h_3;
$a_{r\,4}^{+}$ = 54,89 for the polynomial h_4.

The minimal value is

$$a_{r\,\min}^{+} = a_{r\,4}^{+} = 54{,}89.$$

Because $\bar{a}_4 < 54{,}89$, in correspondence with (14) the given interval system is asymptotically stable.

4. The method for ensuring uncertain systems quality

In this section the task is solved for locating the uncertain system roots within the trapezoidal domain. The method allows to locate roots of the uncertain system characteristic equations family within the given quality domain, thus ensuring the required system quality (generalized stability). The task is solved by inscribing the system circular root locus field into the given quality domain. The trapezoidal domain, bounded by the arbitrary algebraic curve, is considered. Peculiarity of the method consists in the root locus fields application.

The systems with parametric uncertainty are considered, described by the family of characteristic polynomials

$$p(s) = s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n \tag{16}$$

where a_1, ..., a_n are coefficients, which depend linearly of some uncertain parameter k, and can be either real or complex ones.
For selection of the uncertain parameter k, transform equation (16) and rewrite it in the following form:

$$\phi(s) + k\psi(s) = 0 \tag{17}$$

where $\phi(s)$ and $\psi(s)$ are some polynomials of the complex variable s; k is the system uncertain parameter.

Based on (17), derive the expression for k in the form

$$k = f(s) = -\frac{\varphi(s)}{\psi(s)} = u(\sigma,\omega) + iv(\sigma,\omega) \tag{18}$$

where $u(\sigma,\omega)$, $v(\sigma,\omega)$ are harmonic functions of two independent real variables σ and ω.

Consider some provisions about the root locus fields.

Definition 8. *The root locus field of the control system* is the field with the complex potential

$$\varphi(s) = u(\sigma,\omega) + i\nu(\sigma,\omega),$$

that is defined in every point of the extended free parameter complex plane by setting the *root locus image* existence over the whole plane (Rimsky & Taborovetz, 1978).
Then, set the root locus image by the real function $h = h(u,\nu,t)$, where t is the constant value for every image. Name t, as *the image parameter*. Suppose the image is defined over the whole free parameter plane by setting the corresponding boundaries of the parameter t. Thus, using mapping function (18), define in the general form the scalar root locus field function

$$f^* = f^*(\sigma,\omega) \tag{19}$$

and the root locus field level lines equation

$$f^*(\sigma,\omega) = L, \tag{20}$$

where $L = const = t_j$, t_j is the parameter of the j-th image, $-\infty \leq t_j \leq +\infty$, $j = 1, 2, 3, \ldots$

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4.1 The task formulation

Define the quality domain Q (fig. 5) in the left complex half-plane of the system fundamental frequencies (roots plane), bounding the equation (16) roots location by the lines L_η' and L_η'' of the equal degree of stability (stability margin) and the lines $L_{+\beta}$ and $L_{-\beta}$ of constant damping, that is equivalent to setting permissible limits for the following system quality indicators: degree of the system stability η and oscillation β. In fig. 5 the quality domain Q has the shape of a trapezoid.

The task consists in locating the characteristic equation (16) roots within the domain Q, i. e. in determination of such a domain D of the uncertain parameter k values, which ensure location of this equation roots (e. g., p_1, p_2, p_3, p_4 in fig. 5) within the given domain Q, when the system qualitative characteristics do not get beyond the preset limits for η and β, ensuring thus the system Q-stability and fulfillment of the condition. bounded by the lines of equal degree of stability and constant damping

$$k \in D \rightarrow s_i \in Q, \tag{21}$$

where $i = 1, 2, 3, \ldots, n$.

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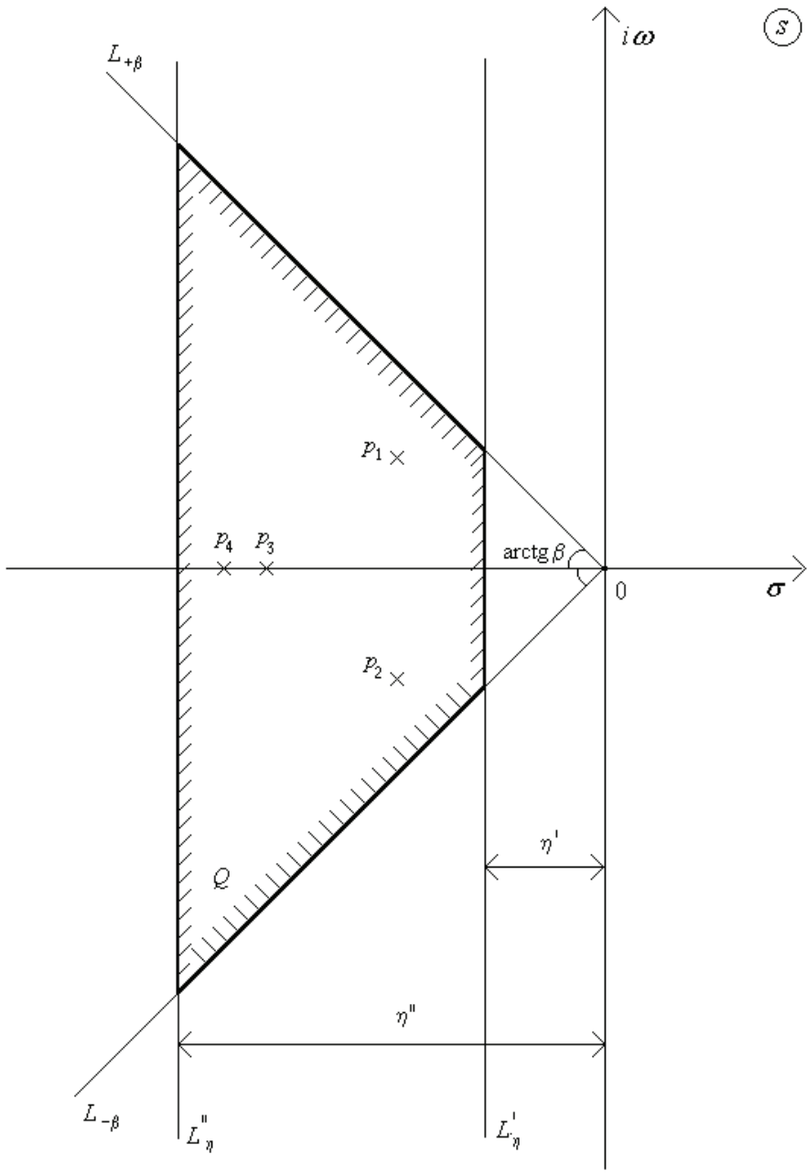


Figure 5. The domain Q of the desired characteristic equation roots location, bounded by the lines of equal degree of stability and constant damping.

For solving the task, apply the root locus fields of the circular image (circular root locus fields – CRLF) (Rimsky, 1972; Nesenchuk, 2005).
The field function (19) and the level lines equation (20) for the CRLF in the general form:

$$f^* = f^*(\sigma, \omega, a, b) \tag{22}$$

$$f^*(\sigma, \omega, a, b) = \rho^2. \quad (23)$$

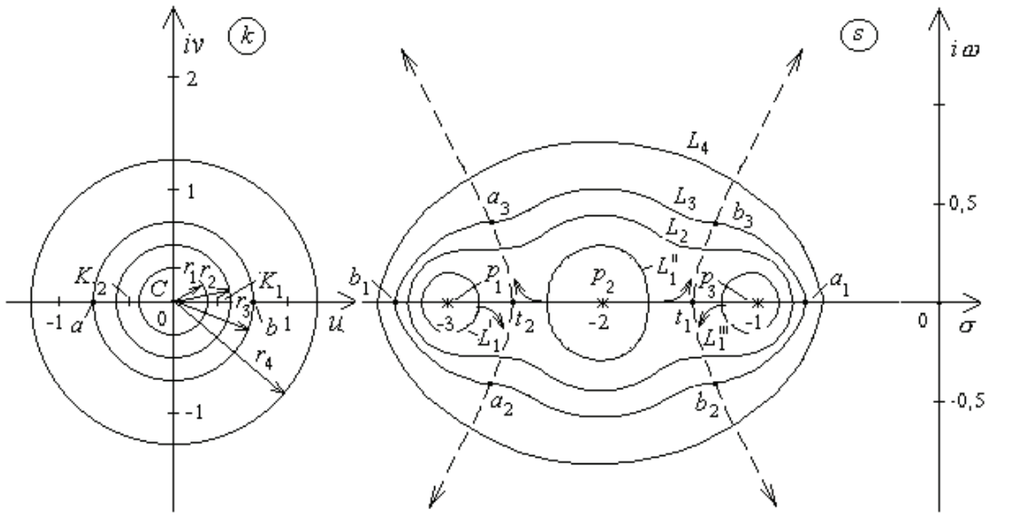
where a and b are the image center coordinates by axes u and υ correspondingly, a = const and b = const; ρ is the circular image radius.
The circular root locus fields for the systems of class [3;0] are represented in fig. 6 and 7.
The CRLF location in the complex plane to the great extent is defined by the given circular image center location, which is mapped onto the complex plane by the field localization centers (see definition 2.4 in (Nesenchuk, 2005)).
Localization centers of the field, described by the level lines L_1 (L_1', L_1'', L_1'''), L_2, L_3, L_4, are located in the points C_1, C_2, C_3 (fig. 6, b). The level lines bound the corresponding domains (W_1, W_2, W_3, W_4 in fig. 6, b) in the plane s. Every such many-sheeted domain W represents the mapping of the root locus level line disk-image of the certain radius.

4.2 Locating roots in the given domain

The given task is solved by inscribing the level line of the CRLF, previously oriented in a special way in the complex plane, into the given quality domain of the system. This level line image in the free parameter plane k, that represents some circle of the radius r, will be the boundary of the required domain D (the required disk). Then, in case, if the circular image center is located in the origin, the following condition should be satisfied: $|k| \le r$.

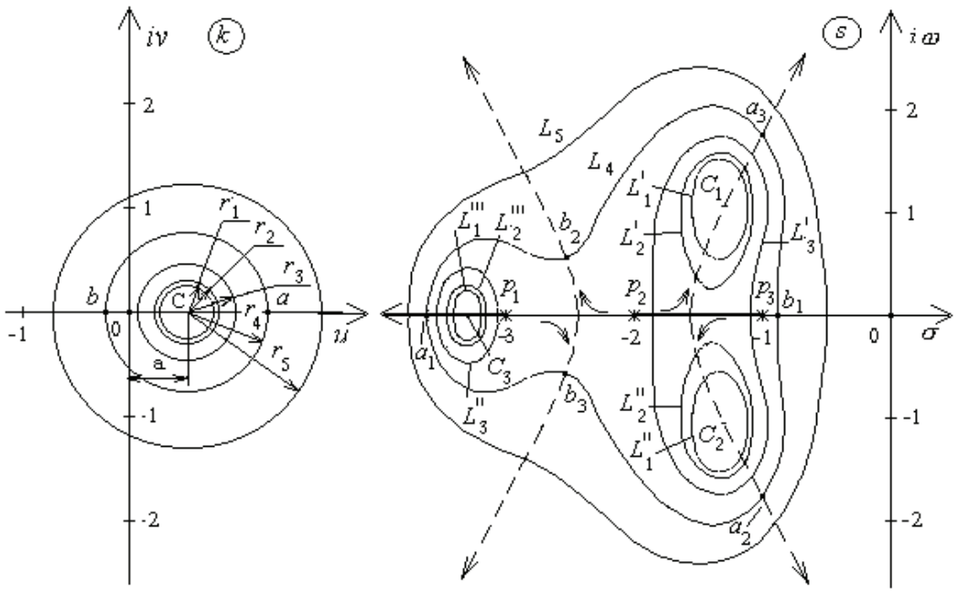
The field orientation
For realization of the above indicated task solution algorithm, at first it is necessary to set orientation (location) of the scalar CRLF in relation to the system quality domain in such a way to ensure the possibility of the field level lines inscription into this domain. Assume the circular image center is located on the positive real axis u, including the origin. The desired location of the circular field is attained, when all its localization centers (i. e. the points, which represent mappings of the circular image center onto the complex plane s) are located inside the quality domain. The enough condition for ensuring such orientation of the field localization centers is location of function (18) zeroes within this domain.
As it was initially assumed, that the circular image center was located on the real axis, the localization centers can be set in two ways:

- in zeroes of function (18), i. e. in poles of the open-loop system transfer function;
- on the branches of the investigated control system Teodorchik – Evans root locus (TERL).



(a) (b)

Figure 6. Circular root locus field when setting the image center in the origin of the variable parameter plane *k*



(a) (b)

Figure 7. Circular root locus field when shifting the image center in relation to the origin of the variable parameter plane *k*

In the first case the circular image center will be located in point *C*, where $k = 0$ (fig. 6, *a*). In the second case the field localization centers should be located on the TERL positive branches segments being completely located within the given quality domain. Coordinates $u = a$ and $v = b$ (fig. 7, *a*) of the corresponding image center are determined from formula (18).

The level lines inscription

After setting the field localization centers it is possible to start inscription of its level lines into the given quality domain. The inscription procedure consists in finding such a level line, which completely belongs to the given quality domain and which represents itself the mapping of the circular image with the maximal possible radius, that evidently will guarantee the required *Q*-stability of the family (16).

Conditionally divide the task into two subtasks of the level line inscription into the domain, bounded only by:

- the vertical lines of equal degree of stability;
- the inclined lines of constant damping.

Consider the first subtask. For its solution find the extreme points of contact of the CRLF level line and the lines L_η', L_η'' of equal degree of stability (fig. 5). Apply the formula for the gradient of the root locus field:

$$gradf^* = \frac{\partial f^*}{\partial \sigma}\vec{i} + \frac{\partial f^*}{\partial \omega}\vec{j}, \tag{24}$$

where $f^*(\sigma,\omega)$ is the field function; $\vec{i}, \vec{j}$ are projections of the identity vector, directed along the normal to the field level line, onto the axes σ and ω correspondingly.

Because in the desired points of contact the gradient (24) projections onto the axis $i\omega$ are equal to zero, determine these points coordinates by composing two systems of equations:

$$\left.\begin{array}{l} \dfrac{\partial f^*(\sigma,\omega)}{\partial \omega} = 0 \\ \sigma = \sigma_\eta' \end{array}\right\}; \tag{25}$$

$$\left.\begin{array}{l} \dfrac{\partial f^*(\sigma,\omega)}{\partial \omega} = 0 \\ \sigma = \sigma_\eta'' \end{array}\right\}, \tag{26}$$

where the first equation of every system represents projection of the gradient onto the axis ω; σ_η' and σ_η'' are coordinates of cross points of the axis σ and the lines L_η' and L_η' correspondingly. From the first system of equations the coordinate ω of the extreme contact point of the line L_η', bounding the quality domain from the right side, and the CRLF level line is determined. The second system allows to determine the coordinate ω of the extreme contact point (e. g., point t_3 in fig. 8) of the line L_η'', bounding the domain Q on the left side, and the CRLF level line.

Turn to the second subtask consideration. For its solution it is necessary to find the extreme contact point (points) of the CRLF level line and the line $L_{+\beta}$ or $L_{-\beta}$ (fig. 5) of constant damping. The only one line, $L_{+\beta}$ or $L_{-\beta}$, is chosen because when the image center is set on the axis u of the free parameter plane, the CRLF is symmetric in relation to the axis $i\omega$. The line $L_{+\beta}$ will be considered as a tangent to the CRLF level line.

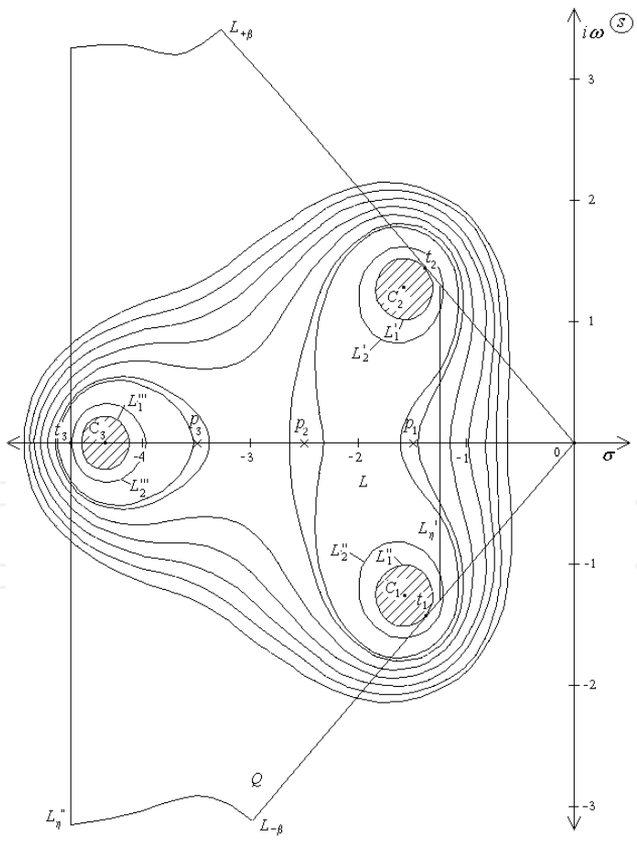


Figure 8. The domain of roots location, inscribed into the given quality domain

Write the equation of a tangent to the scalar CRLF level line (a tangent to the curve) in the general form:

$$\frac{\partial f^*(\sigma,\omega)}{\partial\sigma}(\Delta-\sigma)+\frac{\partial f^*(\sigma,\omega)}{\partial\omega}(\Omega-\omega)=0, \tag{27}$$

where Δ, Ω are current coordinates of a point on the tangent; σ, ω are the point of contact coordinates.
As in this case the tangent to the level line passes through the origin, set coordinates Δ and Ω to zero and rewrite (27) in the following form:

$$\frac{\partial f^*(\sigma,\omega)}{\partial\sigma}(-\sigma)+\frac{\partial f^*(\sigma,\omega)}{\partial\omega}(-\omega)=0\,. \tag{28}$$

On the other hand, the equation of the level line $L_{+\beta}$ is

$$\omega = \mu\sigma,$$

where μ is the real constant, $\mu = \mathrm{tg}\,\beta$ (fig. 5), β is the angle between the constant damping line and the axis ω. By composing on the basis of the last two equations the system

$$\left.\begin{aligned}\frac{\partial f^*(\sigma,\omega)}{\partial\delta}(-\delta)+\frac{\partial f^*(\sigma,\omega)}{\partial\omega}(-\omega)=0\\ \omega=\mu\sigma\end{aligned}\right\} \tag{29}$$

and solving (29), obtain coordinates σ and ω of the desired contact point.
It is necessary to note, that when solving both the first and the second subtasks, several points of contact to every quality domain boundary can be found. It is explained by the fact, that contact points are determined for both global and every local field level line. In this case the level line corresponding to the circular image of the minimal radius is always chosen. Thus, from three points t_1, t_2 and t_3 (fig. 8), found by the above described method, the point t_1 located on the level line L_1, corresponding to the circular image of the minimal radius, is chosen. This line represents itself the boundary of the desired domain D of the uncertain parameter k values, ensuring the required system operational quality indicators.
Consider *the numerical example*. The system quality domain Q (see fig. 5) is bounded by the lines of equal degree of stability, described by equations

$$\sigma = -1.2,\ \sigma = -4.7,$$

and the lines of constant damping with equations

$\sigma = \omega$, $\sigma = -\omega$.

Set the characteristic equation, describing the dynamic system of class [3;0] and select polynomials $\phi(s)$ and $\psi(s)$ (see (17)):

$$\phi(s) = s^3 + 7{,}5s^2 + 17{,}8s + 13{,}1; \tag{30}$$

$$\psi(s) = 1. \tag{31}$$

Suppose, that the polynomial constant term a_n is the uncertain parameter. It is required to determine the domain of the perturbed coefficient a_n values, belonging to the given quality domain Q.
Evidently, the poles $p_1 = -1.5$, $p_2 = -2.5$ and $p_3 = -3.5$ (in fig. 8 are marked by X-s) of the open loop transfer function are located inside the quality domain Q.
Define the circular root locus field by setting the root locus image existence region over the whole plane of the free parameter a_n. For this purpose set the circular field location by defining circular image center in the point C with coordinates $a = 5$, $b = 0$ (fig. 7, *a*) in the free parameter plane a_n. Then, its localization centers are located in points C_1, C_2 and C_3 on the branches of the system Teodorchik – Evans root locus, as shown in fig. 7, b.
Calculations were carried on with application of the computer program for ensuring the required quality of control systems with parametric uncertainty, developed for the above described method realization. Polynomials (30), (31) and the domain Q boundaries equations were entered as the input data. The following results have been obtained.
The circular image root locus equation for the given system:

$$\omega^6 + 3\sigma^2\omega^4 + 15\sigma\omega^4 + 20{,}8\omega^4 + 3\omega^2\sigma^4 + 30\omega^2\sigma^3 + 113\omega^2\sigma^2 + 158\omega^2\sigma +$$
$$+ 4{,}32\omega^2 + \sigma^6 + 15\sigma^5 + 91{,}8\sigma + 303\sigma^3 + 587\sigma^2 + 643\sigma + 304 = 0.$$

The CRLF function, applied for the system investigation:

$$f^*(\sigma, \omega) = \omega^6 + 3\omega^4\sigma^2 + 15\omega^4\sigma + 20{,}7\omega^4 + 3\omega^2 + 30\omega^2\sigma^3 + 113\omega^2\sigma^2 +$$
$$+ 158\omega^2\sigma + 45{,}3\omega^2 + \sigma^6 + 15{,}2\sigma^5 + 91{,}9\sigma^4 + 303\sigma^3 + 588\sigma^2 + 644\sigma + 328.$$

For determination of the CRLF level line, inscribed into the quality domain, the following systems of equations (25), (26) and (29) were solved:

$$\left.\begin{array}{r} 6\omega^4+12\omega^2\sigma^2+60\omega^2\sigma+82{,}8\omega^2+6\sigma^4+60\sigma^3+226\sigma^2+316\sigma+90{,}6=0 \\ \sigma=-1{,}28 \end{array}\right\};$$

$$\left.\begin{array}{r} 6\omega^4+12\omega^2\sigma^2+60\omega^2\sigma+82{,}8\omega^2+6\sigma^4+60\sigma^3+226\sigma^2+316\sigma+90{,}6=0 \\ \sigma=-4{,}68 \end{array}\right\};$$

$$\left.\begin{array}{r} (6\omega^4\sigma+15\omega^4+12\omega^2\sigma^3+90\omega^2\sigma^2+226\omega^2\sigma+158\omega^2+6\sigma^5+75\sigma^4+ \\ +368\sigma^3+909\sigma^2+1180\sigma+644)(-\sigma)+(6\omega^4+12\omega^2\sigma^2+60\omega^2\sigma+ \\ +82{,}8\omega^2+6\sigma^4+60\sigma^3+226\sigma^2+316\sigma+90{,}6)(-\omega)=0 \\ \omega=\sigma \end{array}\right\}.$$

The first equation of the first and the second system represents the CRLF gradient value in the contact points of the field level line and the lines, bounding the quality domain from the left and right (the lines of equal degree of stability), the second equation represents the equation of the lines of equal degree of stability. The first equation of the third system represents the equation of a tangent to the CRLF level line, which passes through the origin. As a result of these equations three points of contact of the CRLF level lines and the lines L_η', L_η'' and $L_{+\beta}$, bounding the quality domain, are defined. In fig. 8 these points are t_1, t_2 for contact of level lines L_1'', L_1' correspondingly and the constant damping line $L_{+\beta}$ and point t_3 for contact of the level line L_2'' and the line L_η'' of equal degree of stability. It has been found, that the point t_2 belongs to the level line, inscribed into the domain Q, and the lines L_2', L_2'', which correspond to the contact point t_3, and the level line L_2''' get beyond this domain (the lines L_2', L_2'' and L_2''' represent mappings of a single circular image). Thus, three simply connected closed regions (in fig. 8 they are cross-hatched) are formed, bounded correspondingly by three level lines L_1', L_1'' and L_1'', representing three sheets of the three-sheeted domain, defined by mapping of the image disc onto the plane s using three branches of the three-valued mapping function. This three-sheeted domain represents the domain of the characteristic equation roots, satisfying the required quality. The image of this domain boundary onto the plane a_n is the circle of radius $r = 2$, bounding the desired closed domain D of the free parameter a_n values, which comply with the given conditions of the system stability.

The developed method for parametric synthesis of the dynamic systems, meeting the robust quality requirements, is based on the circular root locus fields application. It allows to select some regions of the system characteristic equation roots location, belonging to the given quality domain, which defines the required quality indicators values (degree of stability and oscillation), and also to define the corresponding regions of the variable parameters values, ensuring the status when the system quality characteristics do not get beyond the boundaries set. The main advantage of the method is, that it allows to determine the system parameters values, which ensure the required quality indica-

tors for cases when the given system does not comply with the requirements, i.e. to carry on the dynamic systems parametric synthesis. The method can be generalized to the tasks of roots location within the domains of other shapes.

5. Parametric design of the industrial robot robust control systemon the base of the method for interval control systems synthesis

Operation of the industrial robot (see section 2) in conditions of uncertainty is considered, when its parameters are subject to substantial variation. The above described technique is applied for solving the task of the anthropomorphous robot manipulator units control system parametric synthesis. It allows to find analytically the manipulator parameters values variation ranges, which will ensure maintaining the system stability property and the required operational quality within their limits, i. e. to ensure the system robustness.

5.1 Control system model for the case of operation in conditions of uncertainty

Robots loads change with variation of the weights of the items they carry, that causes variation of the load inertia moment j_l, which is linearly included into the characteristic equation coefficients (see (1) and (2)), generating their variation intervals. Currently during the design procedure robots parameters values in the cases of substantial parameters variation are obtained by the technique of tests and mistakes. Conduct parametric synthesis of the manipulator shoulder control system in conditions of its parameters uncertainty using the analytical method described in 3.

Let coefficients of the characteristic equation (3) for the manipulator shoulder unit vary within the following limits:

$$a_0 = 1;\ 0{,}4 \cdot 10^3 \le a_1 \le 0{,}5 \cdot 10^3;\ 0{,}373 \cdot 10^3 \le a_2 \le 0{,}427 \cdot 10^3;\ 0{,}52 \cdot 10^7 \le a_3 \le 0{,}6 \cdot 10^7;$$
$$0{,}488 \cdot 10^9 \le a_4 \le 0{,}56 \cdot 10^9.$$

Suppose any of the coefficients, e. g. a_4, varies continuously along the real axis in the plane of system fundamental frequencies. Taking into account expression (1), the complex mapping function (5), that determines root loci of the interval system relative to a_4, is defined as

$$f(s) = -\frac{\phi(s)}{\psi(s)} = -(s^4 + a_1 s^3 + a_2 s^2 + a_3 s). \tag{32}$$

The control system characteristic equation is

$$\phi(s) + a_4\psi(s) = s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0. \quad (33)$$

The limit values of equation (33) coefficients variation intervals are entered to the input of the package ANALRL for computer-aided investigation of control systems with variable parameters. During the package functioning the Kharitonov's polynomials of the system characteristic equation are formed:

$$h_1(s) = s^4 + 0{,}4 \cdot 10^3 s^3 + 0{,}373 \cdot 10^5 s^2 + 0{,}6 \cdot 10^7 s + 0{,}56 \cdot 10^9 = 0,$$
$$h_2(s) = s^4 + 0{,}5 \cdot 10^3 s^3 + 0{,}427 \cdot 10^5 s^2 + 0{,}52 \cdot 10^7 s + 0{,}488 \cdot 10^9 = 0,$$
$$h_3(s) = s^4 + 0{,}4 \cdot 10^3 s^3 + 0{,}427 \cdot 10^5 s^2 + 0{,}6 \cdot 10^7 s + 0{,}488 \cdot 10^9 = 0,$$
$$h_4(s) = s^4 + 0{,}5 \cdot 10^3 s^3 + 0{,}373 \cdot 10^5 s^2 + 0{,}52 \cdot 10^7 s + 0{,}56 \cdot 10^9 = 0.$$

These four equations form the basis for generation of the mathematical model for the robot interval control system in the form of the Kharotiniv's polynomials root loci.

Considering presence of the load inertia moment j_l substantial variations, it is required to find the coefficients variation intervals, ensuring stability of the characteristic equations family.

5.2 Procedure of the control system parametric synthesis

For the task solution apply the method, described in section 3, which allows to calculate the characteristic equations family coefficients intervals, ensuring the system robust stability.

First, zeroes of functions (32) (the poles of the open loop transfer function) are calculated for the above Kharitonov's polynomials and, if they are located in the left-half plane of the plane s (see statement given in subsection 3.3) or on the stability bound $i\omega$, the root loci of Kharitonov's polynomials are generated on the basis of these functions.

As for our example one of zeroes of function (32) is located on the stability bound (in the point $s = 0$), the direction of the corresponding root locus is verified. It is stated that the positive branch is directed from this zero to the left half plane that, according to the above given statement (see 3.3), means the existence of positive stability intervals of the system investigated.

After constructing the parameter functions according to the technique suggested above (see section 3.4), the variable coefficient values from the set A_r^+ (11)) in the cross points of the Kharitonov's polynomials root loci branches with the system stability bound $i\omega$ are determined. For the given case the following values have been obtained:

$a_{r\,1}^{+}$=0,334·10^9 (polynomial h_1);
$a_{r\,2}^{+}$=0,336·10^9 (polynomial h_2);
$a_{r\,3}^{+}$=0,414·10^9 (polynomial h_3);
$a_{r\,4}^{+}$= 0,280·10^9 (polynomial h_4).

According to the corresponding task algorithm and the obtained values, the minimal positive value $a_{r\,\min}^{+}$ = 0.280·10^9 is determined. The interval (13) of a_4 values variation is calculated that ensures system asymptotic stability: d = (0; 0.280·10^9)). On the basis of the theorem, formulated in 3.4, the following stability condition of the interval system is formed:

$0 < a_4 < 0.280 \cdot 10^9$.

As the root locus parameter varies within the limits $\overline{a}_4$= 0.56·10^9, and $\underline{a}_4$=0.488·10^9, the upper one doesn't comply with the stability condition. For ensuring stability of the considered interval control system the upper limit should be set to $\overline{a}_4$= 0.280·10^9. The limits of the acceptable interval of the coefficient a_4 variation are the following:

$\overline{a}_4 = 0.280 \cdot 10^9$, $\underline{a}_4 = 0$.

From the above described calculations it is evident that the developed method can be applied not only for investigating the interval system stability, but also for calculating intervals of its variable parameters in case the initial system is not stable. It is worth to pinpoint that the method allows to ensure the system asymptotic stability by setting up only one coefficient of its characteristic equation.

6. Conclusion

Industrial robots represent devices, which usually operate in conditions of substantial uncertainty. Therefore, in this chapter the problem of uncertain control systems stability and quality is considered in application to the industrial robot analysis and synthesis tasks solution. The task for synthesis of the interval control systems stable polynomials has been solved. For its solution the investigation of the system root locus portrait behavior at the asymptotic stability boundary has been carried out. On this basis the system robust stability condition was formulated. The method has been developed for setting up the interval polynomial for ensuring its stability in cases, when the stability verification showed, that the initial polynomial was unstable. If the system order $n > 3$, this method is applicable when the Kharitonov's polynomials free root loci initial points are located in the left complex half-plane, because in this case the root locus portrait configuration ensures existence of the stability interval on every branch of the root loci. When $n \leq 3$, the method is applied without any conditions (limitations). The algorithm considered allows to carry on parametric synthesis of the stable interval system without its root locus portrait modification by setting up the limit values of the characteristic polynomial coefficients variation intervals. Thus, the stability interval for the initially unstable polynomial is defined. The obtained stable polynomial can be selected to be the nearest to the initial (given) one in the sense of the distance, measured along its root trajectories with consideration of the appropriate system quality requirements.

The root locus method has also been developed for ensuring the required quality (Q-stability) of the uncertain control system. It is based on inscription of the circular root locus field level line into the given quality domain.

Currently during the industrial robots design procedure in the cases of substantial parameters variation the robots control systems parametric synthesis is often conducted by the method of tests and mistakes. The techniques, considered here, allow to carry on the analysis and parametric synthesis of the robot control system, operating in conditions of parametric uncertainty, using analytical procedures.

7. References

Barmish, B. (1984). Invariance of the strict hurwitz property for polynomials with perturbed coefficients. *IEEE Trans. Automat. Control*, Vol. 29, No. 10, (October 1984) 935–936, ISSN 0018-9286.

Bartlett, A.; Hollot, C. & Lin, H. (1987). Root Location of an entire polytope of polynomials in suffices to check the edges. *Math. Contr.*, Vol. 1, No. 1, (1987) 61–71, ISSN 0932-4194.

Bhattacharyya, S.; Chapellat, H. & Keel, L. (1995). *Robust Control: the Parametric Approach*, Prentice Hall, ISBN 013781576X, New York.

Gaivoronsky, S. (2006). Vertex analysis for the interval polynomial roots localization in the given sector [in Russian], *Theses of the Third International Conference on Control Problems*, p. 95, ISBN 5-201-14987-1, Moscow, June 20–22, 2006, Institute of Control Sciences of the Russian Academy of Sciences, Moscow.

Gryazina, E. & Polyak, B. (2006). Stability regions in the parameter space: D-decomposition revisited. *Automatica*, Vol. 42, No. 1, (January 2006) 13–26, ISSN 0005-1098.

Handbook of Industrial Robotics (1989). Edited by Nof, S. John Wiley and Sons, ISBN 04711778, New York.

Kharitonov, V. (1978). About asymptotic stability of the linear differential equations family equilibrium [in Russian]. *Differentsyal'nye Uravneniya*, Vol. XIV, No. 11, (November 1978) 2086–2088, ISSN 0374-0641.

Kraev, A. & Fursov, A. (2004). Estimating the instability radii for polynomials of arbitrary power [in Russian]. *Nonlinear Dynamics and Control*. Issue 4: Collection of Papers / Eds. Emelyanov, S. & Korovin, S. FIZMATLIT, ISBN 5-9221-0557-4, Moscow, pp. 127–134.

Lavrentyev, M. & Shabat, B. (1987). *Methods of the Complex Variable Functions Theory*, Nauka, Moscow.

Nesenchuk, A. (2002). Parametric synthesis of qualitative robust control systems using root locus fields. *Proceedings of the 15th Triennial World Congress of IFAC*, vol. E, pp. 331–335, ISBN 0-08-044220-X, Barcelona, Spain, July 21–26, 2002, Elsevier Science, Oxford.

Nesenchuk, A. (2005). *Analysis and Synthesis of Robust Dynamic Systems on the Basis of the Root Locus Approach* [in Russian], United Institute of Informatics Problems of the Belarusian National Academy of Sciences, ISBN 985-6744-18-0, Minsk.

Polyak, B. & Tsypkin, Y. (1990). Frequency criteria for robust stability and aperiodicity [in Russian]. *Avtomatika i Telemekhanika*, No. 9, (September 1990) 45–54, ISSN 005-2310.

Polyak, B. & Scherbakov, P. (2002), a. *Robust Stability and Control* [in Russian], Nauka, ISBN 5-02-002561-5, Moscow.

Polyak, B. & Scherbakov, P. (2002), b. Superstable linear control systems I, II ([n Russian]. *Avtomatika i Telemekhanika*, No. 8, (2002) 37–53, ISSN 005-2310.

Rantzer, A. (1992). Stability conditions for polytopes of polynomials. *IEEE Trans. Automat. Control*, Vol. 37, No. 1, (January 1992) 79–89, ISSN 0018-9286.

Rimsky, G. (1972). *Basics of the General Root Locus Theory for Control Systems* [in Russian], Nauka i Tekhnika, Minsk.

Rimsky, G & Taborovetz, V (1978). *Computer-Aided Investigation of the Automatic Systems*, Nauka I Tekhnika, Minsk.

Shaw, J. & Jayasuriya, S. (1993). Robust stability of an interval plant with respect to a convex region in the complex plane. *IEEE Trans. Automat. Control*, Vol. 38, No. 2, (February 1993) 284-287, ISSN 0018-9286.

Soh, C.; Berger, C. & Dabke, K. (1985). On the stability properties of polynomials with perturbed coefficients. *IEEE Trans. Automat. Control*, Vol. 30, No. 10, (October 1985) 1033–1036, ISSN 0018-9286.

Soh, Y. (1989). Strict hurwitz property of polynomials under coefficient perturbations. *IEEE Trans. Automat. Control*, Vol. 34, No. 6, (June 1989) 629–632, ISSN 0018-9286.

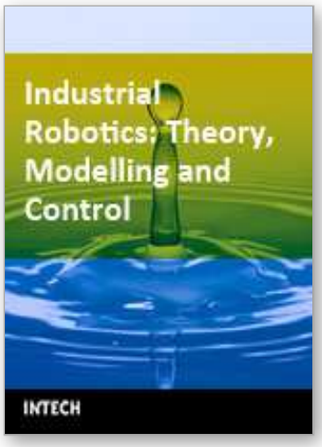
Tsypkin, Y. & Polyak, B. (1990). Frequency criteria of robust modality for the linear discrete systems [in Russian]. *Avtomatika i Teeamekhanika*, No. 4, (April 1990) 3–9, ISSN 005-2310.

Tsypkin, Y. (1995). Robust stability of the relay control systems [in Russian]. *Doklady RAN*, Vol. 340, No. 6, (February 1995) 751–753, ISSN 0869-5652.

Uderman, E. (1972). *The Root Locus Method in the Control Systems Theory* [in Russian], Nauka, Moscow.

Vicino, A. (1989). Robustness of pole location in perturbed systems. *Automatica*, Vol. 25, No. 1, (January 1989) 109-113, ISSN 0005-1098.

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This book covers a wide range of topics relating to advanced industrial robotics, sensors and automation technologies. Although being highly technical and complex in nature, the papers presented in this book represent some of the latest cutting edge technologies and advancements in industrial robotics technology. This book covers topics such as networking, properties of manipulators, forward and inverse robot arm kinematics, motion path-planning, machine vision and many other practical topics too numerous to list here. The authors and editor of this book wish to inspire people, especially young ones, to get involved with robotic and mechatronic engineering technology and to develop new and exciting practical applications, perhaps using the ideas and concepts presented herein.

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