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Ampère's Law Proved Not to Be Compatible with Grassmann's Force Law

Jan Olof Jonson
Stockholm University
Sweden

1. Introduction

Efforts have frequently been made to create links between different approaches within electromagnetism. Names like Ampère, Coulomb, Lorentz, Grassmann, Maxwell et.al. are all linked to efforts to create a comprehensive understanding of electromagnetism. In this paper the very focus is on breaking the alleged links between Ampère's law and the Grassmann-Lorentz force. Thanks to extensive mathematical efforts it appears to be possible to disprove earlier assumed links. That will tend to lead the further investigation of the subject effectively forwards

2. The fallacious derivation of Lorentz's law made by Grassmann

2.1 Finding the conceptual roots of the Lorentz force

Graneau discusses the assumption that the Ampère and the Lorentz forces are mathematically equal, claiming that this is not true [1]. He further makes the statement that the magnetic component of the Lorentz force was first proposed by Grassmann [2],[3]. This author shows that when Grassmann makes the derivation beginning with Ampére's law, he commits faults, which finally results in a term that is similar to the Lorentz force.

2.2 Grassmann on the electromagnetic force between currents

Grassmann himself discusses the conceptual problems that arise when studying Ampère's law [2],[3]. He says that the complicated form of Ampère's law arouses suspicion. [4] Among others he complains over the fact that the formula in no way resembles that for gravitational attraction, indicating thereby the lack of analogy between the two kinds of forces

$$(2\cos\varepsilon - r\cos\alpha.\cos\beta).ab\,/\,r^2 \tag{1}$$

It ought to be mentioned that in the time of Grassmann and Ampère the electron has not yet been discovered and hence, the concept of what constituted a current must have been rather vague. It is therefore understandable that neither of them were able to apply Coulomb's law on the problem a law that by form fulfills the requirement to resemble the gravitational force. This author has been successful in doing so, beginning in his first paper on the subject in 1997 [5].

Applying Ampère's law on a configuration [6] consisting of two current carrying conductors, with the elements a and b respectively, an angle ε between the currents, α and β are the angles formed by the elements a and b respectively with the line drawn between the two mid-points, the current from the attracting element being i and its length ds, l being the perpendicular from the midpoints of the attracted element on the circuit element b on to the line of the attracting one, or

$$l = r\sin\alpha \tag{2}$$

gives the force between the two elements

$$-(ids.b / r^2)\cos\beta(2\cos\varepsilon - 3\cos\alpha.\cos\beta) \tag{3}$$

which develops into

$$ds = -d(I\cot\alpha) = l.d\alpha / \sin^2\alpha = r^2.d\alpha / l \tag{4}$$

Where

$$\varepsilon = \alpha - \beta \tag{5}$$

$$\text{and } d\beta = d\alpha \tag{6}$$

This leads the development of eq. (4) into:

$$-(ib / l)(\cos^2\beta.\cos\alpha.d\alpha - 2\sin\alpha.\sin\beta.\cos\beta.d\beta) \tag{7}$$

After he has arbitrarily chosen to put $d\beta$ at the end of the second term, due to the statement (6) above.

He thereafter integrates over the whole attracting line, thereby getting *nothing from the second term*. This is not mathematics. If there were originally undertaken an incremental step $d\alpha$ attached to both terms, the integration along α can of course not be avoided by stating that $d\beta = d\alpha$ and choose to change $d\alpha$ into $d\beta$ on the second term.

He continues the treatment of the two currents by deriving the force perpendicular to the attracted element b due to the attracting one

$$-(ib_l / l)(\cos\varepsilon.\cos\beta + \cos\alpha.\sin^2\beta) \tag{8}$$

This time, when performing the integral, he does it not with respect to α this time; instead he does it with respect to β. He further claims the values of α and β now to be those of the 'initial point' of the line. This indicates that he has not treated the necessary rest of the closed circuit.

$$(ib_l / l)(1 + \cos\varepsilon.\cos\beta + \cos\alpha.\sin^2\beta) \tag{9}$$

Now it has appeared a new term '1' at the very beginning. At infinity he states that α becomes 180°, and β becomes 180°- ε

Hence, the expression (9) may after recalling that

$$\varepsilon = \alpha - \beta \tag{10}$$

be rewritten

$$(ib_l / l)(\cot(½\alpha) + \sin\beta.\cos\beta) \tag{11}$$

Thereafter he neglects the second term of eq. (11), while leaving no indication of the reason for doing so and attains after using eq. (2)

$$(ib_l / r)(\cot(½\alpha) \tag{12}$$

In the following a passage is cited from his paper in order to make it easier to understand his way of thinking [8]

"From this expression he now at once attains the mutual interaction of two current elements, as he prefers to express it. It happens, since he is regarding the attracting current element $i.ds$ as the combination of the two lines through which the current is passing, "these possessing the direction and intensity (i) of this element, and one of them having its current flowing in the same direction as that in the element, and the other in the opposite direction, while the first of them has its starting point in the initial point of the element , and the second has its starting point in the end of the element. We then obtain $(ab_l / r^2).\sin\alpha$ as the effect exerted by a current element a on another b, distant r from it"

This term proportional to $\sin\alpha$ is apparently that which has impelled people to name him as the first to define "Biôt-Savart's law".

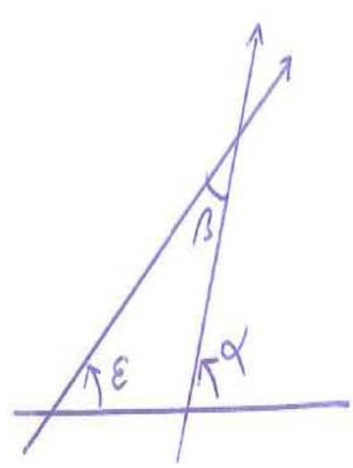

Fig. 1.

2.3 Some comments on the method of Grassmann

If following the derivation Grassmann has made, at several occasions there are clear cases of confusion concerning what he is doing and why. The mathematical steps recall more an 'ad-hoc' way of using mathematical terminology than a real logical way of working. Hence, his results seem to be of no value with respect to factual electrodynamics.

It seems to be totally irrelevant to use the theoretical treatment by Grassmann in order to construct any links between Ampère's law and Biôt-Savart's law and related Lorentz force. They must be kept apart from each other.

3. Analysis of the results by Assis and Bueno in comparing Lorentz's force law and Ampère's law

Assis and Bueno have written a paper [9] in which he claims Ampère's law to be consistent with Grassmann's force law. They derive expressions for the force between the support and the bridge within a set of Ampère's bridge, using both laws. Conceding that the laws are not equal at every point of the circuit, they claim that the result for a whole closed circuit is equal. One special point of observation that they make is that Biôt-Savart's law and the related Lorentz force do not obey Newton's third law, whereas Ampère's law does [10] The set of Ampére's bridge is described by detail in two consecutive figures, with respect to the definitions of the integration domains [11]. The width of the conductor is small relative to the lengths of the branches. The bridge is described by two variables, the laminar thickness is being ignored. The shape is rectangular, the branches being of length l_3 (along the x direction) and l_2 (along the y direction). The branches along the y axis are cut off at the distance l_1 the segments thus attained have been numerated from one to six counterclockwise, beginning with the part of the support being situated along the x axis, where presumably the supporting battery is practically applied. Thus, the cuts appear between segment 2-3 and 5-6 respectively. In this paper it is especially being focused on the force that appears from segment 5 onto segment 4, since the Lorentz force from segment 6 to segment 5 (and similarly between segment 2 and 3) is zero, whereas Ampère's law is not. Interestingly, the Lorentz force from segment 5 onto segment 4 differs from zero, contrary to the force in the opposite direction. The authors claim that this net force **within** this branch (the bridge) is able to account for the force that otherwise Ampère's law produces from segment 6 to 5. Normally, according to Newtons's third law, there cannot be any net force, if there is no acceleration, since during conditions of balance, no net force is active. Biôt-Svart's law and the Lorentz force law implies a single-directed force, that if it were to be real would immediately blow up the circuit, which does not happen. The force that Ampère's law produces between the two segments corresponds to an internal **tension** that is made visible when cutting of the bridge, thereby creating two equal forces, but of opposite direction, that exactly cancel.

To conclude, this discussion seems to verify that the Lorentz force is by nature unphysical, whereas Ampère's law is physical. Furtheron, the derivation that will follow in the next chapter, gains momentum to this conclusion, since it will appear that the derivation by Assis and Bueno that constitutes a fundamental basis for their claim, has been fallaciously performed.

3.1 Basic formulas for the derivation

Assis and Bueno defines the Grassmann force as follows:

$$d^2\vec{F}_{ji}{}^{G} = I_i d\vec{l}_i \times d\vec{B}_j^{B-S} = I_i dl_i \times \left(\frac{\mu_0}{4\pi} I_j \frac{d\vec{l}_j \times \hat{r}}{r^2}\right) = -\frac{\mu_0}{4\pi}\frac{I_i I_j}{r^2}\left[(d\vec{l}_i \bullet d\vec{l}_j)\hat{r} - (d\vec{l}_i \bullet \hat{r})d\vec{l}_j\right] \tag{13}$$

Since the integration involves four variables, a change of the name of the left hand variable will be done according to: $d^4\vec{F}_{ji}{}^G$

It must be mentioned that the Assis and Bueno mention the law as "Grassmann's", without showing the reference. The reference that this author has found is that given by Peter Graneau [13], [14], [15]. It would be preferred to be used the name "Lorentz' force law (based on Biôt-Savart's law)". For the reader's convenience, also Ampère's law is given here:

$$d^2\vec{F}_{ji}{}^A = -\frac{\mu_0}{4\pi} I_i I_j \frac{\hat{r}}{r^2}(2(d\vec{l}_i \bullet d\vec{l}_j) - 3(\hat{r} \bullet d\vec{l}_i)(\hat{r} \bullet d\hat{l}_j)) \tag{14}$$

3.2 Result of the calculations with respect to segment 5-4

The integrations can usually be performed straightforwardly, thereby using the normal rules for integrations. At some points, however, it appears necessary to make approximations, when a term is extremely small as compared to the others, for example terms $\propto w$ in the numerator. That has been dome at *every* actual occasion in this work. This is the precondition for attaining a closed expression at the end.

3.3 The result according to Assis and Bueno

Assis and Bueno have been using two ways to approximate the integrals that have to be performed. In the first case they assume the circuit to be divided into rectangles (21), in the second case they let a diagonal line at the corners define the border between two segments (22). In the first case they claim that the result of the calculation of Grassmann's force on segment 4 due to segment 5 is:

$$(F^G{}_{54})_y \cong \frac{\mu_0}{4\pi}(\ln\frac{l_2 - l_1}{w} - \ln\frac{(l_2 - l_1) + ((l_2 - l_1)^2 + l_3{}^2)^{1/2}}{l_3} + \ln 2 - \frac{3}{2}\ln(1+\sqrt{2}) + \frac{\sqrt{2}}{2} + \frac{1}{2}) \tag{15}$$

In the second case their result is

$$(F^G{}_{54})_y \cong \frac{\mu_0}{2\pi}(\ln\frac{l_2}{w} - \ln\frac{l_2 + (l_2{}^2 + l_3{}^2)^{1/2}}{l_3} + \frac{(l_2{}^2 + l_3{}^2)^{1/2}}{l_2} + \ln 2 + \frac{1}{2}) \tag{16}$$

a result that they identify as equal to that which Ampère's law gives rise to.

3.4 The result according to the analysis of this author, first approach

However, the intention with this paper is to judge the claims by Assis and Bueno. In order to attain that goal, the Grassmann force they have been using will be used in this paper in an independent derivation, by this author.

The first step in the calculation procedure is to give the problem a strict formulation in the shape of an integral, thereby identifying as well the variables of the integrations as the boarders. Applying Eq.(13) above to the segments 5-4 will give rise to the following integral equation:

$$(F^{G}{}_{54})_y \cong \frac{\mu_0 I^2}{4\pi w^2}\int_0^{w} dx_5 \int_0^{l_3} dx_4 \int_{l_1}^{l_2-w} dy_5 \int_{l_2-w}^{l_2} dy_4 \frac{x_4 - x_5}{((x_4 - x_5)^2 + (y_4 - y_5)^2)^{3/2}} \tag{17}$$

3.5 The first step: Integration with respect to x_4

Integrating first with respect to x_4, gives the result:

$$(F^{G}{}_{54})_y = \frac{\mu_0 I^2}{4\pi w^2}\int_{l_1}^{l_2-w} dy_5 \int_{l_2-w}^{l_2} dy_4 \int_0^{w} dx_5 \left(-\frac{1}{((l_3 - x_5)^2 + (y_4 - y_5)^2)^{1/2}} + \frac{1}{(x_5^2 + (y_4 - y_5)^2)^{1/2}}\right) \tag{18}$$

3.6 The second step: Integration with respect to y_5

The subsequent integration with respect to y_5 gives rise to the following expression:

$$\begin{aligned}(F^{G}{}_{54})_y &= \frac{\mu_0 I^2}{4\pi w^2}\int_0^{w} dx_5 \int_{l_2-w}^{l_2} dy_4 (\ln(y_4 - l_2 + w + \sqrt{(y_4 - l_2 + w)^2 + (l_3 - x_5)^2})\\ &-\ln(y_4 - l_1 + \sqrt{(y_4 - l_1)^2 + (l_3 - x_5)^2}) - \ln(y_4 - l_2 + w + \sqrt{(y_4 - l_2 + w)^2 + x_5^2}\\ &+\ln(y_4 - l_1 + \sqrt{(y_4 - l_1)^2} + x_5^2)\end{aligned} \tag{19}$$

For convenience, the four terms may be named (19a), (19b), (19c) and (19d) in consecutive order.

3.7 The third step: Integration with respect to y_4 and x_5

3.7.1 The first term of Eq.(19) above treated, (19a)

In order to solve the integration, some integration formulas must be used:

$$\int \ln(z + (z^2 \pm 1)^{\frac{1}{2}})dz = z\ln(z + (z^2 \pm 1)^{\frac{1}{2}}) - (z^2 \pm 1) \tag{20}$$

$$\int (x^2 + a^2)^{\frac{1}{2}} dx = \frac{x}{2}(x^2 + a^2)^{\frac{1}{2}} \pm \frac{a^2}{2}\ln(x + (x^2 \pm a^2)^{\frac{1}{2}}) \tag{21}$$

$$\int \ln z dz = z\ln z - z \tag{22}$$

In order to solve Eq. (19a), the substitution

$$\frac{y_4 - l_2 + w}{l_3 - x_5} = z \tag{23}$$

will favorably be used. This makes eq. (19a) transform into:

$$(F^G{}_{54})_y = \frac{\mu_0 I^2}{4\pi w^2}\int_0^w dx_5 (l_3 - x_5)\int_{z=0}^{\frac{w}{l_3 - x_5}} (-dz)(\ln(l_3 - x_5) + \ln(z + \sqrt{z^2+1})) \tag{24}$$

This expression contains two terms: The first term becomes after integration:

$$(F^G{}_{54})_{y,1} = \frac{\mu_0 I^2}{4\pi w}\ln l_3 \tag{25}$$

The second term

$$(F^G{}_{54})_{y,2} = \frac{\mu_0 I^2}{4\pi w^2}\int_0^w dx_5 (l_3 - x_5)\int_{z=0}^{\frac{w}{l_3 - x_5}} (-dz)\ln(z + \sqrt{z^2+1}) \tag{26}$$

Solving with respect to z, thereby using formula [10] gives:

$$(F^G{}_{54})_{y,2} = \frac{\mu_0 I^2}{4\pi w^2}\int_0^w dx_5 (l_3 - x_5)(-1)(z\ln(z + \sqrt{z^2+1}) - \sqrt{z^2+1}) \tag{27}$$

Straightforward integration accordingly gives, provided also the approximation

$$w << l_3 - x_5 \tag{28}$$

is used in the final stage, both terms of the integral equal zero in the limit. The second term of those also requires the usage of an integration formula, Eq. [20], before the null result can be achieved. Hence,

$$(F^G{}_{54})^y \langle (19a) \rangle = \frac{\mu_0 I^2}{4\pi}(\ln l_3) \tag{29}$$

3.8 The second term of Eq.(19) above treated, (19b)

In order to solve Eq. (19b), it is at first reasonable to use the substitution

$$t = y_4 - l_1 \tag{30}$$

which gives rise to

$$(F^G{}_{54})^y \langle (19b) \rangle = \frac{\mu_0 I^2}{4\pi}(-\int_{x_5=0}^{w} dx_5 \int_{t=l_2-w-l_1}^{l_2-l_1} dt\ln(t + \sqrt{(t^2 + (l_3 - x_5)^2}\,)) \tag{31}$$

Preparing to perform the substitution

$$z = \frac{t}{l_3 - x_5} \tag{32}$$

an intermediate step will be to write

$$(F^G{}_{54})^y \left\langle (19b) \right\rangle = \frac{\mu_0 I^2}{4\pi}\{-\int_{x_5=0}^{w} dx_5 \int_{t=l_2-w-l_1}^{l_2-l_1} dt \ln((l_3-x_5)(\frac{t}{l_3-x_5}+\sqrt{(\frac{t}{l_3-x_5})^2+1}))\} \tag{33}$$

Here it appears to be evident that this expression might simply be divided into two, according to the logarithm product law (Eq. (61) later in the text):

$$(F^G{}_{54})^y \left\langle (19b) \right\rangle = \frac{\mu_0 I^2}{4\pi}\{-\int_{x_5=0}^{w} dx_5 (l_3-x_5)\cdot \ln(l_3-x_5) \int_{z=\frac{l_2-l_1-w}{l_3-x_5}}^{\frac{l_2-l_1}{l_3-x_5}} dz -$$

$$-\int_{x_5=0}^{w} dx_5 (l_3-x_5) \int_{z=\frac{l_2-l_1-w}{l_3-x_5}}^{\frac{l_2-l_1}{l_3-x_5}} dz \ln(z+\sqrt{z^2+1})\} \tag{34}$$

These terms will hereafter be denoted $(F^G{}_{54})^y \left\langle (19b) \right\rangle_1$ and $(F^G{}_{54})^y \left\langle (19b) \right\rangle_2$ respectively.

Solving straightforwardly, and using series expansions of the ln function, thereby neglecting terms of higher order of w, gives for the first term of the expression (34) above:

$$(F^G{}_{54})^y \left\langle (19b) \right\rangle_1 = \frac{\mu_0 I^2}{4\pi}\{-\ln l_3\} \tag{35}$$

The second part will demand some more computational work, as will appear below:

Evaluating now the integral with respect to z, leads directly to the following, rather complicated expression:

$$\begin{aligned}(F^G{}_{54})^y \left\langle (19b) \right\rangle_2 &= \frac{\mu_0 I^2}{4\pi w^2}(-\int_{x_5=0}^{w} dx_5((l_2-l_1)\ln(\frac{l_2-l_1}{l_3-x_5}+\sqrt{(\frac{l_2-l_1}{l_3-x_5})^2+1})- \\ &-(l_2-l_1-w)\ln(\frac{l_2-l_1-w}{l_3-x_5}+\sqrt{(\frac{l_2-l_1-w}{l_3-x_5})^2+1}))+ \\ &+\frac{\mu_0 I^2}{4\pi w^2}(-\int_{x_5=0}^{w} dx_5 (l_3-x_5)(\sqrt{(\frac{l_2-l_1-w}{l_3-x_5})^2+1}-\sqrt{(\frac{l_2-l_1}{l_3-x_5})^2+1}))\end{aligned} \tag{36}$$

The three terms on the right hand expression will be called (36a), (36b) and (36c) respectively.

Now, instead of evaluating the expressions straightforwardly, it appears to be favorable to find suitable combinations of terms that would be able to make the solution simpler.

3.9 The sum of the first two terms of Eq. (36)

In order to solve the integral (36) it will be favorable to make the following variable substitution:

$$g = \frac{l_2 - l_1}{l_3 - x_5} \tag{37}$$

Applying this on the first term above (36a) gives:

$$\frac{\mu_0 I^2}{4\pi w^2}\Big(-\int_{g=\frac{l_2-l_1}{l_3}}^{\frac{l_2-l_1}{l_3-w}} dg (l_2 - l_1)^2 \frac{1}{g^2} \ln(g + \sqrt{g^2+1})\Big) \tag{38}$$

Applying a similar variable substitution on the second term above (36b) gives:

$$\frac{\mu_0 I^2}{4\pi w^2}\int_{g=\frac{l_2-l_1-w}{l_3}}^{\frac{l_2-l_1-w}{l_3-w}} dg (l_2 - l_1 - w)^2 \frac{1}{g^2} \ln(g + \sqrt{g^2+1}) \tag{39}$$

The treatment of the third term (36c) will be postponed, until the first two have been developed and simplified. Doing so, leads to the result with respect to the first term (36a)

$$\begin{aligned}
&\frac{\mu_0 I^2}{4\pi w^2}\{(l_2 - l_1)\{(l_3 - w)\ln(\frac{1}{l_3}(1+\frac{w}{l_3})(l_2 - l_1 + \sqrt{(l_2-l_1)^2 + (l_3 - w)^2})) - \\
&-l_3 \ln(\frac{1}{l_3}(l_2 - l_1 + \sqrt{(l_2 - l_1)^2 + l_3^2})) + (l_2 - l_1)\ln(\frac{l_3 - w}{l_2 - l_1}(1 + \sqrt{(\frac{l_2 - l_1}{l_3 - w})^2 + 1})) - \\
&-(l_2 - l_1)\ln(\frac{l_3}{l_2 - l_1}(1 + \sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1}))\}\}
\end{aligned} \tag{40}$$

and for (39):

$$\begin{aligned}
&\frac{\mu_0 I^2}{4\pi w^2}\{-(l_2 - l_1 - w)\{(l_3 - w)\ln(\frac{1}{l_3}(1+\frac{w}{l_3})(l_2 - l_1 - w + \sqrt{(l_2-l_1-w)^2 + (l_3 - w)^2})) + \\
&+l_3 \ln(\frac{1}{l_3}(l_2 - l_1 - w + \sqrt{(l_2 - l_1 - w)^2 + l_3^2})) - (l_2 - l_1 - w)\ln(\frac{l_3 - w}{l_2 - l_1 - w}(1 + \sqrt{(\frac{l_2 - l_1 - w}{l_3 - w})^2 + 1})) + \\
&+(l_2 - l_1 - w)\ln(\frac{l_3}{l_2 - l_1 - w}(1 + \sqrt{(\frac{l_2 - l_1 - w}{l_3})^2 + 1}))\}\}
\end{aligned} \tag{41}$$

In the next step the two results above, (40) and (41) will be added. The result is:

$$\begin{aligned}
&\frac{\mu_0 I^2}{4\pi w^2}\{(l_2-l_1)(l_3-w)(\frac{w}{l_3}+\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+(\frac{l_3-w}{l_3})^2}))-\\
&-(l_2-l_1-w)(l_3-w)(\frac{w}{l_3}+\ln(\frac{l_2-l_1-w}{l_3}+\sqrt{(\frac{l_2-l_1-w}{l_3})^2+(\frac{l_3-w}{l_3})^2}))-\\
&-(l_2-l_1)l_3\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})+\\
&+(l_2-l_1-w)l_3\ln(\frac{l_2-l_1-w}{l_3}+\sqrt{(\frac{l_2-l_1-w}{l_3})^2+1})+\\
&+(l_2-l_1)^2\ln(\frac{l_3-w}{l_2-l_1}(1+\sqrt{(\frac{l_2-l_1}{l_3-w})^2+1}))-\\
&-(l_2-l_1-w)^2\ln(\frac{l_3-w}{l_2-l_1-w}(1+\sqrt{(\frac{l_2-l_1-w}{l_3-w})^2+1}))-\\
&-(l_2-l_1)^2\ln(\frac{l_3}{l_2-l_1}(1+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}))+\\
&+(l_2-l_1-w)^2\ln(\frac{l_3}{l_2-l_1-w}(1+\sqrt{(\frac{l_2-l_1-w}{l_3})^2+1}))\}
\end{aligned} \tag{42}$$

It has been used the series expansion of $\ln(1+x)\cong x$ for $0\le x\le 1$ [22]

Since the 'x' is very small, only the first term in the expansion has been taken into account.

The terms within expression (42) will for practical reasons be denoted (42:1) until (42:8).

The next step will be to simplify the expression (42).This must be done in a deliberate way in order to succeed. A practical method is to separate out the dominant terms first, thereafter put the terms of first order in w thereafter those of second order, neglecting the terms of even higher order. The reason for this is that the numerator in the very first term of the expression contains w^2 in the numerator.

In the following this integral formula will again be usable:

$$\int \ln(z+(z^2\pm 1)^{\frac{1}{2}})dz = z\ln(z+(z^2\pm 1)^{\frac{1}{2}})-(z^2\pm 1)^{\frac{1}{2}} \tag{43}$$

Following this procedure gives for the terms of expression (29) respectively:

$$\begin{aligned}
&\frac{\mu_0 I^2}{4\pi w^2}\{(l_2-l_1)(l_3-w)(\{\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}))-\\
&-\frac{w}{l_3}\cdot\frac{1}{\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}+\frac{w}{l_3}\}
\end{aligned} \tag{42:1}$$

$$\frac{\mu_0 I^2}{4\pi w^2}\{(-(l_2-l_1-w)(l_3-w)(\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})-$$
$$-\frac{w}{{l_3}^2}\cdot\frac{l_2-l_1+l_3(1+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})}{\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}+\frac{w}{l_3})\} \tag{42:2}$$

$$\frac{\mu_0 I^2}{4\pi w^2}\{-(l_2-l_1)l_3\cdot\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})\} \tag{42:3}$$

$$\frac{\mu_0 I^2}{4\pi w^2}\{(l_2-l_1-w)l_3\cdot\{\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})-$$
$$-\frac{w}{{l_3}^2}\cdot\frac{l_2-l_1+l_3\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}{\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}\}\} \tag{42:4}$$

$$\frac{\mu_0 I^2}{4\pi w^2}\{(l_2-l_1)^2\cdot\{\ln(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})-$$
$$+$$
$$-\frac{w}{(l_2-l_1)^2}\cdot\frac{l_3+(l_2-l_1)\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}{\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}\}\} \tag{42:5}$$

$$\frac{\mu_0 I^2}{4\pi w^2}\{-(l_2-l_1-w)^2\cdot\{\ln(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})-$$
$$-\frac{w}{(l_2-l_1)^3}\cdot\frac{l_3(l_2-l_1-l_3)+(l_2-l_1-l_3)(l_2-l_1)\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}{\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}\}\} \tag{42:6}$$

$$\frac{\mu_0 I^2}{4\pi w^2}\{-(l_2-l_1)^2\cdot\ln(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})\} \tag{42:7}$$

$$\frac{\mu_0 I^2}{4\pi w^2}\{(l_2 - l_1 - w)^2 \cdot \{\ln(\frac{l_3}{l_2 - l_1} + \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}) +$$

$$+\frac{w}{(l_2 - l_1)^3} \cdot \frac{l_3^2 + l_3(l_2 - l_1)\sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}}{\frac{l_3}{l_2 - l_1} + \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}} \cdot \frac{1}{\sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}}\}\} \quad (42:8)$$

The sum of these eight expressions is equal to expression (42). Now, it will be easier to perform the addition of the terms of Eq. (42), by adding the eight separate terms above to each other. An important key to success has been to approximate every ln and root expression with a series expansion for every case when a 'small' term is appearing besides the big ones. By using this method it has been possible to restrict the complicated ln and root expression to the 'standard'ones, without 'small' terms. In doing so it is possible to gather similar terms from the different expression, even when there have been 'small' terms added inside the ln and root expressions.

The result is:

$$\frac{\mu_0 I^2}{4\pi}\{-\ln l_3 - \ln(\frac{l_2 - l_1}{l_3} + \sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1})\} + 1 -$$

$$-\frac{(l_2 - l_1)^2 + l_3^2 + (l_2 - l_1)\sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1}}{l_3^2} \cdot \frac{1}{\frac{l_2 - l_1}{l_3} + \sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1}} \cdot \frac{1}{\sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1}} +$$

$$+\frac{1}{(\frac{l_3}{l_2 - l_1} + \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}) \cdot \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}} \cdot \{\frac{1}{2} - \frac{1}{4} \cdot \frac{l_3^2}{l_3^2 + (l_2 - l_1)^2} +$$

$$+\frac{1}{2} \cdot \frac{(l_1 - l_2 + l_3)^2 (l_2 - l_1 + 2l_3)}{(l_2 - l_1)^3} +$$

$$+\frac{3}{2} \cdot \frac{l_3^2 (l_2 - l_1)}{(l_2 - l_1)^3} - \frac{1}{4} \cdot \frac{l_3^4 (l_2 - l_1)}{(l_2 - l_1)^5} + \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}\} + \quad (44)$$

$$+\frac{1}{(\frac{l_3}{l_2 - l_1} + \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1})^2 \cdot ((\frac{l_3}{l_2 - l_1})^2 + 1)} \cdot \{\frac{1}{(l_2 - l_1)^4}((2l_3^2 + (l_2 - l_1)^2)(l_2 - l_1)^2 +$$

$$\frac{1}{2} \cdot (l_1 - l_2 + l_3)^2 (l_3^2 + (l_2 - l_1)^2) -$$

$$-\frac{1}{2} \cdot l_3^2 (l_3^2 + (l_2 - l_1)^2) - \frac{1}{2} \cdot l_3^4) - \frac{l_3}{l_2 - l_1} \cdot \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1} - \frac{l_3^3}{(l_2 - l_1)^3} \cdot \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}\}$$

Comment: The four last rows above (in Eq.(44)) arise due to the "other terms" of Eq. (42.5) to (42.8). To be

3.10 The third term of Eq.(36)

The third term of Eq. (36) was denoted (36c) is repeated here for convenience.

$$(F^G{}_{54})^y \langle(19b)\rangle_3 = \frac{\mu_0 I^2}{4\pi w^2}(-\int_{x_5=0}^{w} dx_5(l_3-x_5)(\sqrt{(\frac{l_2-l_1-w}{l_3-x_5})^2+1}-\sqrt{(\frac{l_2-l_1}{l_3-x_5})^2+1})) \tag{36c}$$

It can be simplified easily by recognizing that the integrand may be written approximately

$$\frac{\mu_0 I^2}{4\pi w^2}\sqrt{(l_2-l_1)^2+l_3{}^2}\cdot(\frac{w(l_2-l_1)}{(l_2-l_1)^2+l_3{}^2}) \tag{45}$$

after having neglected terms of higher order than one in w, since the integration implies a multiplication with w.

The result is

$$\frac{\mu_0 I^2}{4\pi}(\frac{l_2-l_1}{\sqrt{(l_2-l_1)^2+l_3{}^2}}) \tag{46}$$

3.11 The sum of all terms due to the second term of Eq.(19) (i.e. 19b)

Now all the partial results due to Eq.(19b) have been attained and the task remains only to sum them together. In order to do so, Eq. (35), (44) and (46) must be added. The result is accordingly:

$$\begin{aligned}
&\frac{\mu_0 I^2}{4\pi}\{-\ln l_3-\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})+1-\\
&-\frac{(l_2-l_1)^2+l_3{}^2+(l_2-l_1)l_3\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}{l_3{}^2}\cdot\frac{1}{\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}+\\
&+\frac{1}{(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}\cdot(\frac{1}{2}-\frac{1}{4}\cdot\frac{l_3{}^2}{l_3{}^2+(l_2-l_1)^2}+\\
&+\frac{1}{2}\cdot\frac{(l_1-l_2+l_3)^2(l_2-l_1+2l_3)}{(l_2-l_1)^3}+\frac{3}{2}\cdot\frac{l_3{}^2(l_2-l_1)}{(l_2-l_1)^3}-\frac{1}{4}\cdot\frac{l_3{}^4(l_2-l_1)}{(l_2-l_1)^5}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})+\\
&+\frac{1}{(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})^2\cdot((\frac{l_3}{l_2-l_1})^2+1)}\cdot(\frac{1}{(l_2-l_1)^4}((2l_3{}^2+(l_2-l_1)^2)(l_2-l_1)^2+\\
&+\frac{1}{2}\cdot(l_1-l_2+l_3)^2(l_3{}^2+(l_2-l_1)^2)-\frac{1}{2}\cdot l_3{}^2(l_3{}^2+(l_2-l_1)^2)-\frac{1}{2}\cdot l_3{}^4)-\frac{l_3}{l_2-l_1}\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}-\\
&-\frac{l_3{}^3}{(l_2-l_1)^3}\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}+\frac{l_2-l_1}{\sqrt{(l_2-l_1)^2+l_3{}^2}}))\}
\end{aligned} \tag{47}$$

3.12 The third term of Eq.(19) above treated, (19c)

Straightforward integration leads again to a result, requiring first a long chain of partial integrations:

In order to solve the third term, Eq. (19c), it is at first reasonable to use the substitution

$$\frac{y_4 - l_2 + w}{x_5} = z \tag{48}$$

This makes it possible to write the integral (19c):

$$\frac{\mu_0 I^2}{4\pi w^2}\{\int_{x_5=0}^{w} dx_5 x_5 \int_{z=0}^{\frac{w}{x_5}} dz \ln(x_5(z+\sqrt{z^2+1}))\} \tag{49}$$

This expression may be dissolved into two terms:

$$\frac{\mu_0 I^2}{4\pi w^2}\{\int_{x_5=0}^{w} dx_5 x_5 \int_{z=0}^{\frac{w}{x_5}} dz \ln(x_5 + \ln(z+\sqrt{z^2+1}))\} \qquad \text{(49a) and (49b)}$$

The first term simply becomes

$$\frac{\mu_0 I^2}{4\pi}(-\ln w + 1) \tag{50}$$

The second term can be rewritten:

$$\frac{\mu_0 I^2}{4\pi w^2}(\int_{x_5=0}^{w} dx_5 w(\ln(w+\sqrt{w^2+x_5^2}) - \ln x_5) - \int_{x_5=0}^{w} dx_5\sqrt{w^2+x_5^2} + \int_{x_5=0}^{w} dx_5 x_5) \tag{51}$$

These three terms will now be denoted (51a), (51b) and (51c) respectively. In order to solve these integrals, it appears practical to make the variable substitution

$$x_5 = \frac{w}{u} \tag{52}$$

Having done this, Eq.(65) may be rewritten:

$$\frac{\mu_0 I^2}{4\pi w^2}\{\int_{u\to+\infty}^{0} du(-\frac{1}{u^2}(\ln(u+\sqrt{u^2+1})) + \int_{u\to+\infty}^{0} du\frac{1}{u^3}\sqrt{u^2+1} - \int_{u\to 0+\infty}^{0} du\frac{1}{u^3}\} \tag{53}$$

Well, straightforwardly performing variable substitution, and, using the integral formula:

$$\int\frac{dx}{x(x^2+a^2)^{\frac{1}{2}}} = -\frac{1}{a}\ln\left|\frac{a+(x^2+a^2)^{\frac{1}{2}}}{x}\right| \tag{54}$$

finally leads to the result: Term (53a) becomes

$$\frac{\mu_0 I^2}{4\pi}(2\ln(1+\sqrt{2})) \tag{55}$$

The same procedure is repeated with the next term, which leads to:

Term (53b) becomes

$$\frac{\mu_0 I^2}{4\pi}(-\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\ln(1+\sqrt{2})) \tag{56}$$

Finally, term (53c) becomes

$$\frac{\mu_0 I^2}{4\pi}(\frac{1}{2}) \tag{57}$$

The total result of term (6c) then will be attained, if adding (50), (55), (56) and (57) to each other.

The result is:

$$(F^{G}{}_{54})_y\left\langle(19c)\right\rangle=\frac{\mu_0 I^2}{4\pi}(-\ln w+\frac{3}{2}-\frac{\sqrt{2}}{2}+\frac{3}{2}\ln(1+\sqrt{2})) \tag{58}$$

3.13 The fourth term of Eq.(19) above treated, (19d)

Straightforward integration leads again to a result. Many partial integration to be done, but in a rather straightforward way. In order to solve the fourth term, Eq. (19c), it is at first reasonable to use the substitution

$$\frac{y_4-l_1}{x_5}=z \tag{59}$$

This makes it possible to write the integral (19d) :

$$\frac{\mu_0 I^2}{4\pi w^2}\{\int_{x_5=0}^{w}dx_5 x_5\int_{z=\frac{l_2-l_1-w}{x_5}}^{\frac{l_2-l_1}{x_5}}dz\ln(x_5 z+\sqrt{(x_5 z)^2+x_5^2})\} \tag{60}$$

This expression may be dissolved into two terms, using the logarithmic product formula:

$$\ln\{A\cdot B\}=\ln A+\ln B \tag{61}$$

The first term simply becomes

$$\frac{\mu_0 I^2}{4\pi}(-\ln w+1) \tag{62}$$

The second term can be rewritten:

$$\frac{\mu_0 I^2}{4\pi w^2}\{\int_{x_5=0}^{w} dx_5((l_2-l_1)(\ln(\frac{l_2-l_1}{x_5}+\sqrt{(\frac{l_2-l_1}{x_5})^2+1})-\sqrt{(l_2-l_1)^2+x_5^2}- \\ -(l_2-l_1-w)\ln(\frac{l_2-l_1-w}{x_5}+\sqrt{(\frac{l_2-l_1-w}{x_5})+1})+\sqrt{(l_2-l_1-w)^2+x_5^2}\} \quad (63)$$

In order to perform the evaluation successfully, the integral formulas [20] and [25] will be needed. Further series expansions have to be done with respect to root and logarithmic expressions that will arise during the work, namely [28], [29] and [30]. It is the hope that these advices will lead the reader successfully to the result, which is

$$(F^G{}_{54})_y\left\langle(19d)\right\rangle=\frac{\mu_0 I^2}{4\pi}\{-\ln w+\frac{1}{2}+\ln(l_2-l_1)+\ln 2\} \quad (64)$$

3.14 The sum of all contributions

In order to attain an expression for the total sum of the integrations above, it would be favorable to gather them in consecutive order below:

$$(F^G{}_{54})^y\left\langle(19a)\right\rangle=\frac{\mu_0 I^2}{4\pi}(\ln l_3) \quad (29)$$

$$(F^G{}_{54})^y\left\langle(19b)\right\rangle=\frac{\mu_0 I^2}{4\pi}\{-\ln l_3+\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})+1- \\ -\frac{(l_2-l_1)^2+l_3{}^2+(l_2-l_1)l_3\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}{l_3{}^2}\cdot\frac{1}{\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}+ \\ +\frac{1}{(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}\cdot(\frac{1}{2}-\frac{1}{4}\cdot\frac{l_3{}^2}{l_3{}^2+(l_2-l_1)^2}+ \\ +\frac{1}{2}\cdot\frac{(l_1-l_2+l_3)^2(l_2-l_1+2l_3)}{(l_2-l_1)^3}+\frac{3}{2}\cdot\frac{l_3{}^2(l_2-l_1)}{(l_2-l_1)^3}-\frac{1}{4}\cdot\frac{l_3{}^4(l_2-l_1)}{(l_2-l_1)^5}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})+ \\ +\frac{1}{(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})^2\cdot((\frac{l_3}{l_2-l_1})^2+1)}\cdot(\frac{1}{(l_2-l_1)^4}((2l_3{}^2+(l_2-l_1)^2)(l_2-l_1)^2+ \\ +\frac{1}{2}\cdot(l_1-l_2+l_3)^2(l_3{}^2+(l_2-l_1)^2)-\frac{1}{2}\cdot l_3{}^2(l_3{}^2+(l_2-l_1)^2)-\frac{1}{2}\cdot l_3{}^4)- \\ -\frac{l_3}{l_2-l_1}\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}-\frac{l_3{}^3}{(l_2-l_1)^3}\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}+\frac{l_2-l_1}{\sqrt{(l_2-l_1)^2+l_3{}^2}}))\} \quad (47)$$

$$(F^G{}_{54})_y\left\langle(19c)\right\rangle=\frac{\mu_0 I^2}{4\pi}(-\ln w+\frac{3}{2}-\frac{\sqrt{2}}{2}+\frac{3}{2}\ln(1+\sqrt{2})) \quad (58)$$

$$(F^{G}{}_{54})_y \left\langle (19d) \right\rangle = \frac{\mu_0 I^2}{4\pi}\{-\ln w + \frac{1}{2} + \ln(l_2 - l_1) + \ln 2\} \tag{64}$$

The sum of all these four terms (omitted here) (65)

3.14.1 The sum of all contributions, simplified expression

After having performed the summation, some simplification occurs and gives rise to the following result:

$$\begin{aligned}
(F^{G}{}_{54})^y \left\langle (19) \right\rangle &= \frac{\mu_0 I^2}{4\pi}\{-\ln(\frac{l_2 - l_1}{l_3} + \sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1}) + 1 - \\
&- \frac{(l_2 - l_1)^2 + l_3{}^2 + (l_2 - l_1)\sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1}}{l_3{}^2} \cdot \frac{1}{\frac{l_2 - l_1}{l_3} + \sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1}} \cdot \frac{1}{\sqrt{(\frac{l_2 - l_1}{l_3})^2 + 1}} + \\
&+ \frac{1}{(\frac{l_3}{l_2 - l_1} + \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}) \cdot \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}} \cdot \{\frac{1}{2} - \frac{1}{4} \cdot \frac{l_3{}^2}{l_3{}^2 + (l_2 - l_1)^2} + \\
&+ \frac{1}{2} \cdot \frac{(l_1 - l_2 + l_3)^2 (l_2 - l_1 + 2l_3)}{(l_2 - l_1)^3} + \frac{3}{2} \cdot \frac{l_3{}^2 (l_2 - l_1)}{(l_2 - l_1)^3} - \frac{1}{4} \cdot \frac{l_3{}^4 (l_2 - l_1)}{(l_2 - l_1)^5} + \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1}\} + \\
&+ \frac{1}{(\frac{l_3}{l_2 - l_1} + \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1})^2 \cdot ((\frac{l_3}{l_2 - l_1})^2 + 1)} \cdot (\frac{1}{(l_2 - l_1)^4}((2l_3{}^2 + (l_2 - l_1)^2)(l_2 - l_1)^2 + \\
&+ \frac{1}{2} \cdot (l_1 - l_2 + l_3)^2 (l_3{}^2 + (l_2 - l_1)^2) - \frac{1}{2} \cdot l_3{}^2 (l_3{}^2 + (l_2 - l_1)^2) - \frac{1}{2} \cdot l_3{}^4) - \frac{l_3}{l_2 - l_1} \cdot \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1} \\
&- \frac{l_3{}^3}{(l_2 - l_1)^3} \cdot \sqrt{(\frac{l_3}{l_2 - l_1})^2 + 1} + \frac{l_2 - l_1}{\sqrt{(l_2 - l_1)^2 + l_3{}^2}}) - 2\ln w + 2 - \frac{\sqrt{2}}{2} + \\
&\frac{3}{2}\ln(1 + \sqrt{2}) + \ln(l_2 - l_1) + \ln 2\}
\end{aligned} \tag{66}$$

4. The result according to the analysis of this author, second approach

The first step in the calculation procedure is to give the problem a strict formulation in the shape of an integral, thereby identifying as well the variables of the integrations as the boarders. Applying Eq.(13) above to the segments 5-4 will give rise to the following integral equation, now in this case with a diagonal line at the corner, defining the border between two segments. The change appears at the upper border in the integral over y_5: $l_2 - x_5$ instead of $l_2 - w$ of Eq. (4) above.

$$(F^{G}{}_{54})_y \cong \frac{\mu_0 I^2}{4\pi w^2} \int_0^w dx_5 \int_0^{l_3} dx_4 \int_{l_1}^{l_2 - x_5} dy_5 \int_{l_2 - w}^{l_2} dy_4 \frac{x_4 - x_5}{((x_4 - x_5)^2 + (y_4 - y_5)^2)^{3/2}} \tag{67}$$

4.1 The first step: Integration with respect to x_4

Integrating first with respect to x_4, gives the result:

$$(F^G{}_{54})_y = \frac{\mu_0 I^2}{4\pi w^2} \int_{l_1}^{l_2 - x_5} dy_5 \int_{l_2 - w}^{l_2} dy_4 \int_0^w dx_5 \Big(-\frac{1}{((l_3 - x_5)^2 + (y_4 - y_5)^2)^{1/2}} + \frac{1}{(x_5^2 + (y_4 - y_5)^2)^{1/2}}\Big) \quad (68)$$

This corresponds to the procedure in section 4.5 and the integration result here (Eq.(68)) is equal to that (Eq.(18)). The border line at the corner has namely not yet been involved.

4.2 The second step: Integration with respect to y_5

The subsequent integration with respect to y_5 gives rise to the following expression:

$$\begin{aligned}(F^G{}_{54})_y &= \frac{\mu_0 I^2}{4\pi w^2} \int_0^w dx_5 \int_{l_2 - w}^{l_2} dy_4 (\ln(y_4 - l_2 + x_5 + \sqrt{(y_4 - l_2 + x_5)^2 + (l_3 - x_5)^2}) \\ &-\ln(y_4 - l_1 + \sqrt{(y_4 - l_1)^2 + (l_3 - x_5)^2}) - \ln(y_4 - l_2 + x_5 + \sqrt{(y_4 - l_2 + x_5)^2 + x_5^2} \\ &+\ln(y_4 - l_1 + \sqrt{(y_4 - l_1)^2} + x_5^2)\end{aligned} \quad (69)$$

Here, at this level, the border line has begun to affect the integration result, now when integrating with respect to y_5.

The second and the fourth term of Eq.(69) are different than the corresponding terms of Eq.(19). For convenience, it might appear suitable to denote the terms of Eq.(69) (69a) etc until (69d). The terms which must be further integrated here are accordingly (69a) and (69c). Eq.(69b) and (69d) are equal to Eq.(19b) and (19d) respectively.

4.3 The third step: Integration with respect to y_4 and x_5

4.3.1 The first term of Eq.(69) above treated, (69a)

In order to solve Eq. (69a), the substitution

$$\frac{y_4 - l_2 + x_5}{l_3 - x_5} = z \quad (70)$$

will favorably be used. This makes eq. (69a) to transform into:

$$(F^G{}_{54})_y = \frac{\mu_0 I^2}{4\pi w^2} \int_0^w dx_5 (l_3 - x_5) \int_{z = \frac{x_5 - w}{l_3 - x_5}}^{\frac{x_5}{l_3 - x_5}{}_2} (-dz)(\ln(z(l_3 - x_5) + \sqrt{z^2 (l_3 - x_5)^2 + 1})) \quad (71)$$

This expression must be separated into several separate terms in order to be solved.

The first term will be:

$$\frac{\mu_0 I^2}{4\pi w^2}\int_0^w dx_5 \ln(l_3 - x_5)\cdot w \tag{72}$$

Integration and, finally, series expansion of the ln function gives the result

$$\frac{\mu_0 I^2}{4\pi w}\{\ln l_3\} \tag{73}$$

The second term will accordingly be:

$$\frac{\mu_0 I^2}{4\pi w^2}\int_0^w dx_5 (l_3 - x_5)\int_{z=\frac{x_5-w}{l_3-x_5}}^{\frac{x_5}{l_3-x_5}} (-dz)\ln(z+\sqrt{z^2+1}) \tag{74}$$

The second terms appears to give a zero result, hence the total result may be written

$$(F^G{}_{54})^y \langle (69a)\rangle = \frac{\mu_0 I^2}{4\pi}\{\ln l_3\} \tag{75}$$

4.4 The second term of Eq.(69) above treated, (69b)

As discussed above, the integration of the second term of Eq.(69) must be equal to the result of the integration of Eq.(19b) in section. 3.12, i.e. Eq.(47). Hence,

$$\begin{aligned}
&(F^G{}_{54})^y \langle (69b)\rangle = \frac{\mu_0 I^2}{4\pi}\{-\ln l_3 - \ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})\} + 1 - \\
&-\frac{(l_2-l_1)^2 + l_3{}^2 + (l_2-l_1)\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}{l_3{}^2}\cdot\frac{1}{\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_2-l_1}{l_3})^2+1}} + \\
&+\frac{1}{(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}\cdot\{\frac{1}{2}-\frac{1}{4}\cdot\frac{l_3{}^2}{l_3{}^2+(l_2-l_1)^2} + \\
&+\frac{1}{2}\cdot\frac{(l_1-l_2+l_3)^2(l_2-l_1+2l_3)}{(l_2-l_1)^3}+\frac{3}{2}\cdot\frac{l_3{}^2(l_2-l_1)}{(l_2-l_1)^3}-\frac{1}{4}\cdot\frac{l_3{}^4(l_2-l_1)}{(l_2-l_1)^5}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1}\} + \\
&+\frac{1}{(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})^2\cdot((\frac{l_3}{l_2-l_1})^2+1)}\cdot\{\frac{1}{(l_2-l_1)^4}((2l_3{}^2+(l_2-l_1)^2)(l_2-l_1)^2 + \\
&\frac{1}{2}\cdot(l_1-l_2+l_3)^2(l_3^2+(l_2-l_1)^2)-\frac{1}{2}\cdot l_3^2(l_3^2+(l_2-l_1)^2)-\frac{1}{2}\cdot l_3{}^4) - \\
&\frac{l_3}{l_2-l_1}\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}-\frac{l_3{}^3}{(l_2-l_1)^3}\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}+\frac{l_2-l_1}{\sqrt{(l_2-l_1)^2+l_3{}^2}})\}
\end{aligned} \tag{76}$$

4.5 The third term of Eq.(69) above treated, (69c)

Due to the rather complicated expressions that will follow, the integral that has to be solved will be repeated here for convenience (from section 5.3.)**:**

$$(F^{G}{}_{54})_{y}\left\langle (69c)\right\rangle = \frac{\mu_0 I^2}{4\pi w^2}\int_0^w dx_5\{-\ln(y_4 - l_2 + x_5 + \sqrt{(y_4 - l_2 + x_5)^2 + x_5^2}\} \tag{69c}$$

In order to solve Eq. (69c), the variable substitution

$$\frac{y_4 - l_2 + x_5}{x_5} = z \tag{77}$$

may appear suitable

In doing so, the following two terms will result:

$$\frac{\mu_0 I^2}{4\pi w^2}\{\int_{x_5=0}^{w} dx_5 x_5 \int_{z=\frac{x_5-w}{x_5}}^{1} dz(\ln x_5 + \ln(z+\sqrt{z^2+1}))\} \tag{78}$$

The first term gives

$$\frac{\mu_0 I^2}{4\pi w^2}\{-\ln w + 1\} \tag{79}$$

Using the integral equation [19] the second term develops to

$$\frac{\mu_0 I^2}{4\pi w^2}\int_0^w dx_5 \cdot x_5\{-\ln(1+\sqrt{2}) + \sqrt{2} + \frac{x_5 - w}{x_5}\ln(\frac{x_5 - w}{x_5} + \sqrt{(\frac{x_5 - w}{x_5})^2 + 1}) - \sqrt{(\frac{x_5 - w}{x_5}) + 1}\} \tag{80}$$

This integral apparently consists of three separate terms, which may be treated separately from each other.

The first term is only a constant term, which has to be integrated with x_5 .over the variable x_5 . Hence, the result is easily written

$$\frac{\mu_0 I^2}{4\pi w^2}\{-\frac{\ln(1+\sqrt{2})}{2} + \frac{\sqrt{2}}{2}\} \tag{81}$$

The second term of Eq. (80) causes a real difficulty when trying to integrate the expression, since no approximations in using seres expansions are allowed. That is so, since the terms are all of the same order. Instead, it is possible to estimate the limits, between which the result must lay. The method is given by the expression

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a) \text{ [23]} \tag{82}$$

By analyzing the asymptotic behavior of the integrand, it may thus be stated that the result has to be within the interval

$$(0, \frac{\mu_0 I^2}{4\pi w} \int_{x_5=0}^{w} dx_5 \{\ln x_5 + \ln(-w + \sqrt{w^2 + x_5{}^2}\}) \tag{83}$$

which may be simplified to

$$(0, \frac{\mu_0 I^2}{4\pi} (-(1 + \ln(1+\sqrt{2})) \ \} \tag{84}$$

The last term of Eq.(80) implies a straightforward integration of a square root expression with the variable squared. A suitable table solution may be used [24]

The result may be written

$$\frac{\mu_0 I^2}{4\pi} \{\frac{1}{2} + \frac{1}{4\sqrt{2}} \ln(\sqrt{2}+1) - \ln(\sqrt{2}-1\} \tag{85}$$

Now it is possible to write the total result of the integral of Eq.(69c):

$$\begin{aligned}(F^G{}_{54})^y \langle (69c) \rangle = \frac{\mu_0 I^2}{4\pi} \{-\ln w + \frac{3}{2} + \frac{\sqrt{2}}{2} + (\frac{1}{4\sqrt{2}} - \frac{1}{2}) \ln(\sqrt{2}+1) + \\ + \frac{1}{4\sqrt{2}} \ln(\sqrt{2}-1) + (0, -(1 + \ln(1+\sqrt{2})\}\end{aligned} \tag{86}$$

4.6 The fourth term of Eq.(69) above treated, (69d)

As discussed above (in section 4.3), the integration of the fourth term of Eq (69) must be equal to the result of the integration of (19d), i.e. Eq.(64) in section 3.14. Hence,

$$(F^G{}_{54})_y \langle (69d) \rangle = \frac{\mu_0 I^2}{4\pi} \{-\ln w + \frac{1}{2} + \ln(l_2 - l_1) + \ln 2\} \tag{87}$$

4.7 The sum of all contributions

$$(F^G{}_{54})^y \langle (69a) \rangle = \frac{\mu_0 I^2}{4\pi} \{\ln l_3\} \tag{75}$$

$$(F^{G}{}_{54})^{y}\left\langle(69b)\right\rangle=\frac{\mu_0 I^2}{4\pi}\{-\ln l_3-\ln(\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1})+1-$$
$$-\frac{(l_2-l_1)^2+l_3{}^2+(l_2-l_1)l_3\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}{l_3{}^2}\cdot\frac{1}{\frac{l_2-l_1}{l_3}+\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}\cdot\frac{1}{\sqrt{(\frac{l_2-l_1}{l_3})^2+1}}+$$
$$+\frac{1}{(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}}\cdot(\frac{1}{2}-\frac{1}{4}\cdot\frac{l_3{}^2}{l_3{}^2+(l_2-l_1)^2}+$$
$$+\frac{1}{2}\cdot\frac{(l_1-l_2+l_3)^2(l_2-l_1+2l_3)}{(l_2-l_1)^3}+\frac{3}{2}\cdot\frac{l_3{}^2(l_2-l_1)}{(l_2-l_1)^3}-\frac{1}{4}\cdot\frac{l_3{}^4(l_2-l_1)}{(l_2-l_1)^5}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})+$$
$$+\frac{1}{(\frac{l_3}{l_2-l_1}+\sqrt{(\frac{l_3}{l_2-l_1})^2+1})^2\cdot((\frac{l_3}{l_2-l_1})^2+1)}\cdot(\frac{1}{(l_2-l_1)^4}((2l_3{}^2+(l_2-l_1)^2)(l_2-l_1)^2+$$
$$\frac{1}{2}\cdot(l_1-l_2+l_3)^2(l_3{}^2+(l_2-l_1)^2)-\frac{1}{2}\cdot l_3{}^2(l_3{}^2+(l_2-l_1)^2)-\frac{1}{2}\cdot l_3{}^4)-\frac{l_3}{l_2-l_1}\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}-$$
$$\frac{l_3{}^3}{(l_2-l_1)^3}\cdot\sqrt{(\frac{l_3}{l_2-l_1})^2+1}+\frac{l_2-l_1}{\sqrt{(l_2-l_1)^2+l_3{}^2}}))\}\tag{76}$$

$$(F^{G}{}_{54})^{y}\left\langle(69c)\right\rangle=\frac{\mu_0 I^2}{4\pi}\{-\ln w+\frac{3}{2}+\frac{\sqrt{2}}{2}+(\frac{1}{4\sqrt{2}}-\frac{1}{2})\ln(\sqrt{2}+1)+\frac{1}{4\sqrt{2}}\ln(\sqrt{2}-1)+\left(0,-(1+\ln(1+\sqrt{2})\right)\}\tag{86}$$

$$(F^{G}{}_{54})_{y}\left\langle(69d)\right\rangle=\frac{\mu_0 I^2}{4\pi}\{-\ln w+\frac{1}{2}+\ln(l_2-l_1)+\ln 2\}\tag{87}$$

The sum of all these four terms (omitted here) (88)

4.8 Numerical check with the Assis/Bueno result necessary

4.8.1 The first approach

By reasons of comparison the result by Assis and Bueno (Eq. (3) above) is repeated here. Obviously, they have not attained the same result as this author (66). Some features are similar, but the differences are also apparent. Therefore it is adequate to claim that the laws are not equal.

$$(F^{G}{}_{54})_{y}\cong\frac{\mu_0}{4\pi}(\ln\frac{l_2-l_1}{w}-\ln\frac{(l_2-l_1)+((l_2-l_1)^2+l_3{}^2)^{1/2}}{l_3}+\ln 2-\frac{3}{2}\ln(1+\sqrt{2})+\frac{\sqrt{2}}{2}+\frac{1}{2})\tag{3}$$

A numerical check for one case shows also that not even an integral gives rise to equal result. It was chosen a wire width $w = 3mm$, $l_1 = 0.7m$, $l_2 = 1.2m$ and, finally, $l_3 = 0.7m$

The result thus attained is

$$(F^G{}_{54})_y \cong \frac{\mu_0}{4\pi} \cdot 10.8 \tag{89}$$

Using, however, the calculation by this author, based on Eq. (76) above, the result will be:

$$(F^G{}_{54})_y \cong \frac{\mu_0}{4\pi} \cdot 12.6 \tag{90}$$

4.8.2 The second approach

Since it was not possible to attain a solution on closed form, a numerical check for one case can be sufficient, provided the result is not in accordance with the formula. It was chosen a wire width $w = 3mm$, $l_1 = 0.7m$, $l_2 = 1.2m$ and, finally, $l_3 = 0.7m$

The result thus attained is

$$(F^G{}_{54})_y \cong \frac{\mu_0}{4\pi} (8.4, 10.3) \tag{91}$$

whereas the formula used by Assis and Bueno gives

$$(F^G{}_{54})_y \cong \frac{\mu_0}{4\pi} \cdot 12.8 \tag{92}$$

Hence, Assis and Bueno are wrong in their claim that the Grassmann force gives the same result as Ampère's law [1]. To conclude, it seemed to be rather wise to reject the claim of coincidence between these two laws, as they did not coincide before integrating. To be stated again,

5. Discussion and conclusions

From the rigorous analysis that has been performed above it is evident that Grassmann's law is not equal to Ampère's law. It is also evident that the very roots of the idea of their equality by Grassmann is false, too. However, it does not exist any need to regard them as equal. It was only one idea among many invented by science in its search for a better understanding of physics. In this respect it constitutes a progress for physics that it has been possible to reject one of the speculative ideas that must inevitably arise in the search for the truth. More seriously, it is of course grave for the proponents of the Lorentz force (identified as the Grassmann force), if it cannot be justified by referring to preceding established ideas but must stand as a mere ad-hoc invention.

6. List of variables

6.1 Variables specified in chapter 3

a, b infinitesimal elements of respective current

r the distance between any two points of two respective electric currents

l the perpendicular from the midpoints of the attracted element on the circuit element b on to the line of the attracting one, or

b_l cosine component of the b element on the perpendicular

ε the angle between two electric currents

α , β the angles formed by the elements a and b respectively with the line drawn between the two mid-points

6.2 Variables specified from chapter 4 onwards

$d^2\vec{F}_{ji}^{\ G}$ the electromagnetic force between two infinitesimal elements of two electric currents (Assis and Bueno: Grassmann's force)

$d^4\vec{F}_{ji}^{\ G}$ the same force as $d^2\vec{F}_{ji}^{\ G}$ above, but with a more adequate notation, indicating the four variables being differentiated

$(F^G{}_{ji})_y$ the y component of the Grassmann force due to two segments

$(F^G{}_{ji})_y \langle ... \rangle$ the y component of the Grassmann force due to two segments, for a specific term

$d^2\vec{F}_{ji}^{\ A}$ the force between two electric currents according to Ampère's law

$\hat{r}$ unit vector along the line connecting the two conductors

I_i , I_j the currents of respective conductor

$d\vec{l}_i$, $d\vec{l}_j$ length elements of respective conductor

l_1 the position of the cut-off point at the y branch

l_2 length of Ampère's bridge along the y axis

l_3 length of Ampère's bridge along the x axis

w the width of branches of Ampère's bridge

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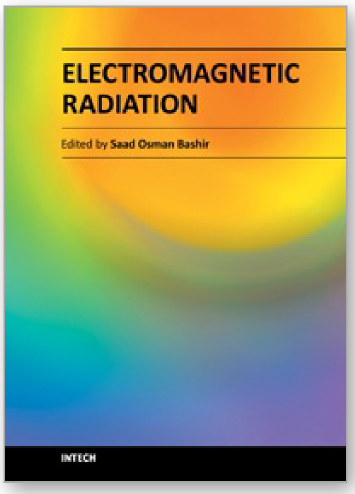

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The application of electromagnetic radiation in modern life is one of the most developing technologies. In this timely book, the authors comprehensively treat two integrated aspects of electromagnetic radiation, theory and application. It covers a wide scope of practical topics, including medical treatment, telecommunication systems, and radiation effects. The book sections have clear presentation, some state of the art examples, which makes this book an indispensable reference book for electromagnetic radiation applications.

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InTech Europe
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Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
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www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
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