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Modeling of Woven Fabrics Geometry and Properties

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1. Introduction

There are many ways of making fabrics from textile fibers. The most common and most complex category comprises fabrics made from interlaced yarns. These are the traditional methods of manufacturing textiles. The great scope lies in choosing fibers with particular properties, arranging fibers in the yarn in several ways and organizing in multiple ways, interlaced yarn within the fabric. This gives textile designer great freedom and variation for controlling and modifying the fabric. The most common form of interlacing is weaving, where two sets of threads cross and interweave with one another. The yarns are held in place due to the inter-yarn friction. Another form of interlacing where the thread in one set interlocks with the loops of neighboring thread by looping is called knitting. The interloping of yarns results in positive binding. Knitted fabrics are widely used in apparel, home furnishing and technical textiles. Lace, Crochet and different types of Net are other forms of interlaced yarn structures. Braiding is another way of thread interlacing for fabric formation. Braided fabric is formed by diagonal interlacing of yarns. Braided structures are mainly used for industrial composite materials.

Other forms of fabric manufacture use fibers or filaments laid down, without interlacing, in a web and bonded together mechanically or by using adhesive. The former are needle punched nonwovens and the later spun bonded. The resulting fabric after bonding normally produces a flexible and porous structure. These find use mostly in industrial and disposable applications. All these fabrics are broadly used in three major applications such as apparel, home furnishing and industrial.

The traditional methods of weaving and hand weaving will remain supreme for high cost fabrics with a rich design content. The woven structures provide a combination of strength with flexibility. The flexibility at small strains is achieved by yarn crimp due to freedom of yarn movement, whereas at high strains the threads take the load together giving high strength. A woven fabric is produced by interlacing two sets of yarns, the warp and the weft which are at right angles to each other in the plane of the cloth (Newton, 1993). The warp is along the length and the weft along the width of the fabric. Individual warp and weft yarns are called ends and picks. The interlacement of ends and picks with each other produces a coherent and stable structure. The repeating unit of interlacement is called the weave (Robinson & Marks, 1973). The structure and properties of a woven fabric are dependent upon the constructional parameters as thread density, yarn fineness, crimp, weave etc.

The present chapter establishes some interesting mathematical relationships between these constructional parameters so as to enable the fabric designer and researcher to have a clear understanding of the engineering aspects of woven fabrics. This is an attempt to transform from an experience based designing into an engineered approach to model woven fabric constructions.

1.1 Elements of fabric structure

Plain weave has the simplest repeating unit of interlacement. It also has the maximum possible frequency of interlacements. Plain weave fabrics are firm and resist yarn slippage. Figure 1 shows plain weave in plan view and in cross-section along warp and weft. The weave representation is shown by a grid in which vertical lines represent warp and horizontal lines represent weft. Each square represents the crossing of an end and a pick. A mark in a square indicates that the end is over the pick at the corresponding place in the fabric that is warp up. A blank square indicates that the pick is over the end that is weft up. One repeat of the weave is indicated by filled squares and the rest by crosses. The plain weave repeats on two ends and two picks.

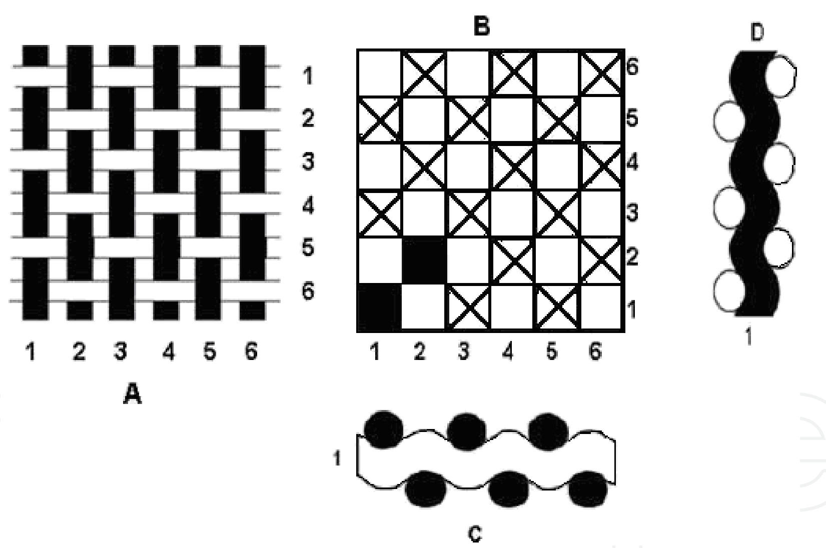

Fig. 1. Plan (A), Weave representation (B) Cross-sectional view along warp (D) Cross-sectional view along weft (C) for plain weave

1.2 Regular and irregular weaves

1.2.1 Regular weaves

Regular weaves (Grosicki, 1988) give a uniform and specific appearance to the fabric. The properties of the fabric for such weaves can be easily predicted. Examples of some of the common regular weaves are given in figure 2.

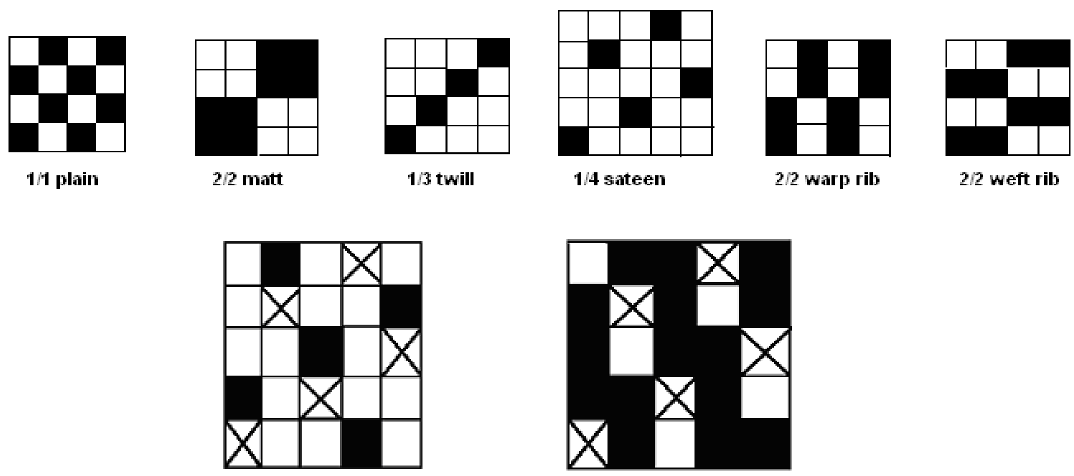

Fig. 2. Regular weaves

1.2.2 Irregular weaves

Irregular weaves are commonly employed when the effect of interlacement is masked by the coloured yarn in the fabric. Such weaves are common in furnishing fabric. In such structures the prediction of mechanical properties is difficult. Examples of some of the common irregular weaves are given in figure 3.

Fig. 3. Irregular weaves

1.3 Mathematical representation of different weaves

The firmness of a woven fabric depends on the density of threads and frequency of interlacements in a repeat. Fabrics made from different weaves cannot be compared easily with regard to their physical and mechanical properties unless the weave effect is normalized. The concept of average float has been in use since long, particularly for calculating maximum threads per cm. It is defined as the average ends per intersection in a unit repeat. Recently this ratio termed as weave factor (Seyam, 2002; Weiner, 1971) has been used to estimate tightness factor in fabric.

1.3.1 Weave factor

It is a number that accounts for the number of interlacements of warp and weft in a given repeat. It is also equal to average float and is expressed as:

$$M = \frac{E}{I} \tag{1}$$

Where E is number of threads per repeat, I is number of intersections per repeat of the cross-thread.

The weave interlacing patterns of warp and weft yarns may be different. In such cases, weave factors are calculated separately with suffix1 and 2 for warp and weft respectively.

Therefore, $M_1 = \frac{E_1}{I_2}$; E_1 and I_2 can be found by observing individual pick in a repeat

and $M_2 = \frac{E_2}{I_1}$; E_2 and I_1 can be found by observing individual warp end in a repeat.

1.3.2 Calculation of weave factor

1.3.2.1 Regular weaves

Plain weave is represented as $\frac{1}{1}$; for this weave, E_1 the number of ends per repeat is equal to 1+1=2 and I_2 the number of intersections per repeat of weft yarn =1+ number of changes from up to down (vice versa) =1+1=2.

Table 1 gives the value of warp and weft weave factors for some typical weaves.

Weave	E_1	I_2	E_2	I_1	M_1	M_2
1/1 Plain	2	2	2	2	1	1
2/1 Twill	3	2	3	2	1.5	1.5
2/2 Warp Rib	2	2	4	2	1	2
2/2 Weft Rib	4	2	2	2	2	1

Table 1. Weave factor for standard weaves

E_1 and E_2 are the threads in warp and weft direction
I_2 and I_1 are intersections for weft and warp threads

1.3.2.2 Irregular weaves

In some weaves the number of intersections of each thread in the weave repeat is not equal. In such cases the weave factor is obtained as under:

$$M = \frac{\sum E}{\sum I} \tag{2}$$

Using equation 2 the weave factors of a ten-end irregular huckaback weave shown in figure 4 is calculated below.

Weave factor, $M = \frac{10+10+10+10+10+10+10+10+10+10}{10+6+10+6+10+6+10+6+10+6} = \frac{100}{84} = 1.19$

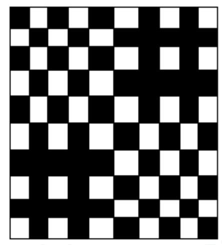

Fig. 4. Ten-end Huck-a-back weave

2. Geometrical model of woven structures

The properties of the fabric depend on the fabric structure. The formal structure of a woven fabric is defined by weave, thread density, crimp and yarn count. The interrelation between fabric parameters can be obtained by considering a geometrical model of the fabric. The model is not merely an exercise in mathematics. It is not only useful in determining the entire structure of a fabric from a few values given in technological terms but it also establishes a base for calculating various changes in fabric geometry when the fabric is subjected to known extensions in a given direction or known compressions or complete swelling in aqueous medium. It has been found useful for weaving of maximum sett structures and also in the analysis and interpretation of structure-property relationship of woven fabrics. Mathematical deductions obtained from simple geometrical form and physical characteristics of yarn combined together help in understanding various phenomena in fabrics.

2.1 Basic relationship between geometrical parameters

The geometrical model is mainly concerned with the shape taken up by the yarn in the warp or weft cross-section of the fabric. It helps to quantitatively describe the geometrical parameters. The basic model (Pierce, 1937) is shown in figure 5. It represents a unit cell interlacement in which the yarns are considered inextensible and flexible. The yarns have circular cross-section and consist of straight and curved segments. The main advantages in considering this simple geometry are:

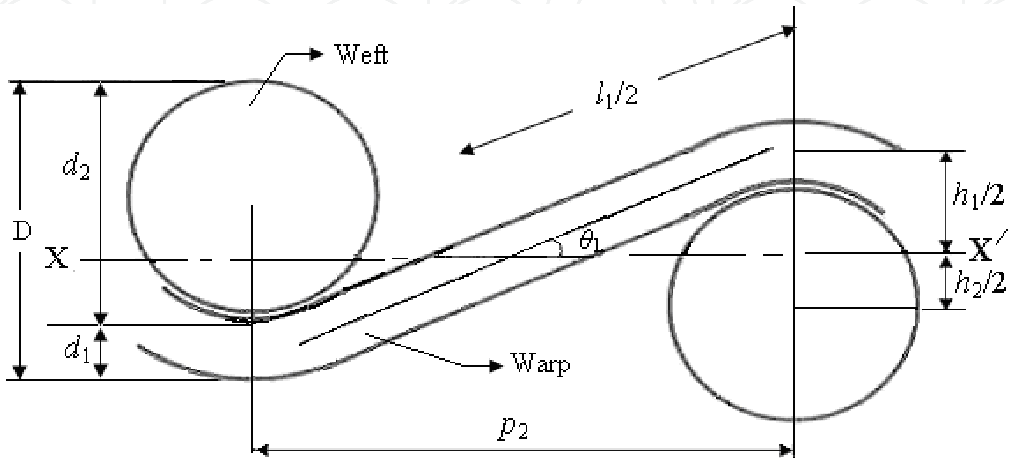

Fig. 5. Peirce's model of plain weave

1. Helps to establish relationship between various geometrical parameters
2. Able to calculate the resistance of the cloth to mechanical deformation such as initial extension, bending and shear in terms of the resistance to deformation of individual fibers.
3. Provide information on the relative resistance of the cloth to the passage of air, water or light.
4. Guide to the maximum density of yarn packing possible in the cloth.

From the two-dimensional unit cell of a plain woven fabric, geometrical parameters such as thread-spacing, weave angle, crimp and fabric thickness are related by deriving a set of equations. The symbols used to denote these parameters are listed below.

d - diameter of thread
p - thread spacing
h - maximum displacement of thread axis normal to the plane of cloth (crimp height)
θ - angle of thread axis to the plane of cloth (weave angle in radians)
l - length of thread axis between the planes through the axes of consecutive cross- threads (modular length)
c - crimp (fractional)
$D = d_1 + d_2$

Suffix 1 and 2 to the above parameters represent warp and weft threads respectively.

In the above figure projection of yarn axis parallel and normal to the cloth plane gives the following equations:

$$c_1 = \frac{l_1}{p_2} - 1 \tag{3}$$

$$p_2 = (l_1 - D\theta_1)\cos\theta_1 + D\sin\theta_1 \tag{4}$$

$$h_1 = (l_1 - D\theta_1)\sin\theta_1 + D(1 - \cos\theta_1) \tag{5}$$

Three similar equations are obtained for the weft direction by interchanging suffix from 1 to 2 or vice-versa as under:

$$c_2 = \frac{l_2}{p_1} - 1 \tag{6}$$

$$p_1 = (l_2 - D\theta_2)\cos\theta_2 + D\sin\theta_2 \tag{7}$$

$$h_2 = (l_2 - D\theta_2)\sin\theta_2 + D(1 - \cos\theta_2) \tag{8}$$

Also, $$d_1 + d_2 = h_1 + h_2 = D \tag{9}$$

In all there are seven equations connecting eleven variables. If any four variables are known then the equations can be solved and the remaining variables can be determined. Unfortunately, these equations are difficult to solve. Researchers have tried to solve these equations using various mathematical means to find new relationships and also some simplified useful equations.

2.1.1 Relation between weave composition and structural parameters

When the interlacement pattern is modified by changing the float length, the structure of the fabric changes dramatically. It has a profound effect on the geometry of the yarn interlacement and related properties in the woven fabric. The maximum weavability limit is predicted by extending the Peirce's geometrical model for non-plain weaves by soft computing. This information is helpful to the weavers in avoiding attempts to weave impossible constructions thus saving time and money. It also helps to anticipate difficulty of weaving and take necessary steps in warp preparations. The relationship between the cover factors in warp and weft direction is demonstrated for circular and racetrack cross-section for plain, twill, basket and satin weave in later part of this chapter. Non plain weave fabric affords further flexibility for increasing fabric mass and fabric cover. As such they enlarge scope of the fabric designer and researcher. Figure 6 shows the relationship between warp and weft thread spacing for different weaves for a given yarn.

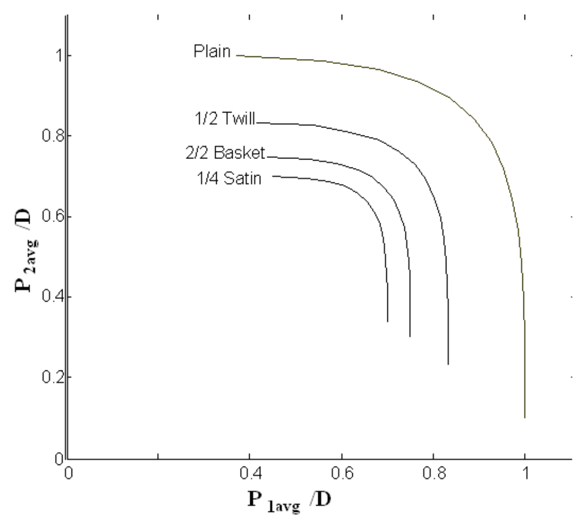

Fig. 6. Relation between average thread spacing in warp and weft for different weaves (Circular cross-section, yarn tex=30, Ø=0.6, ρ=1.52)

2.2 Some derivatives

2.2.1 Relation between *p*, *h*, *θ* and *D*

From equations 4 and 5 we get:

$$\left(l_1 - D\theta_1\right) = \frac{p_2 - D\sin\theta_1}{\cos\theta_1} = \frac{h_1 - D(1-\cos\theta_1)}{\sin\theta_1}$$

$$\text{or } D(\sec\theta_1 - 1) - p_2\tan\theta_1 + h_1 = 0$$

Substituting $x_1 = \frac{\tan\theta_1}{2}$

$$\text{we get, } x_1^2\left(D - \frac{h_1}{2}\right) - p_2 x_1 + \frac{h_1}{2} = 0$$

For real fabrics $x_1 = \frac{tan\theta_1}{2} = p_2 - \frac{\sqrt{p_2^2 - 2h_1(D - \frac{h_1}{2})}}{2D - h_1} = \frac{p_2 - \sqrt{p_2^2 - h_2^2 - D^2}}{D + h_2}$

Using value of x_1, one can calculate θ, l and c and also other parameters.

Similarly, using equation 7 and 8, and by eliminating l and substituting x_1 as above, we will arrive at a more complex equation as:

$$\frac{c_1}{2} + x_1\frac{D}{p_2} - x_1^2(1 + \frac{c_1}{2}) = \frac{D}{p_2}\left(1 - x_1^2\right)\tan^{-1}x_1$$

It is difficult to solve this equation algebraically for x_1. However one can substitute value of x_1 obtained earlier to solve this equation just for an academic interest.

These seven equations have been solved by soft computing in order to establish several useful relationships. However, at this stage, one can generalize the relationship as:

$$h_1 = f\,(p_2, c_1)$$

This function f can be obtained by plotting p and h for different values of c.

2.2.2 Functional relationship between *p, h, c*

Trigonometric expansion of equations 4 and 5 gives:

$$p_2 = l_1 - \frac{l_1\theta_1^2}{2} + \frac{D\theta_1^3}{3} + \frac{l_1\theta_1^4}{24} + ----$$

$$h_1 = l_1\theta_1 - \frac{D\theta_1^2}{2} - \frac{l_1\theta_1^3}{6} + \frac{D\theta_1^4}{8} + ----$$

When θ is small, higher power of θ can be neglected which gives:

$$h_1 = l_1\theta_1,\ p_2 = l_1,\ c_2 = \frac{\theta_1^2}{2},\ h_1 = p_2\sqrt{2c_1}$$

and these equations reduce to:

$$\theta_1 = \left(2c_1\right)^{\frac{1}{2}} \tag{10}$$

$$\theta_2 = \left(2c_2\right)^{\frac{1}{2}} \tag{11}$$

$$h_1 = \frac{4}{3}p_2\sqrt{c_1} \tag{12}$$

$$h_2 = \frac{4}{3} p_1 \sqrt{c_2} \tag{13}$$

These four equations are not new equations in this exercise. They are derived from the previous seven original equations. However they give simple and direct relationships between four fabric parameters *h, p, c* and *θ*.

2.2.3 Jammed structures

A woven fabric in which warp and weft yarns do not have mobility within the structure as they are in intimate contact with each other are called jammed structures. In such a structure the warp and weft yarns will have minimum thread spacing. These are closely woven fabrics and find applications in wind-proof, water-proof and bullet-proof requirements.

During jamming the straight portion of the intersecting yarn in figure 5 will vanish so that in equation 4 and 5, $l_1 - D\theta_1 = 0$

$$\frac{l_1}{D} = \theta_1$$

Equations 4 and 5 will reduce to

$$h_1 = D(1 - \cos\theta_1)$$

$$p_2 = D\sin\theta_1$$

Similarly, for jamming in the weft direction $l_2 - D\theta_2 = 0$, equations 7 and 8 will reduce to the above equations with suffix interchanged from 1 to 2 and vice-versa.

For a fabric being jammed in both directions we have:

$$D = h_1 + h_2 = D(1 - \cos\theta_1) + D(1 - \cos\theta_2)$$

$$\text{or } \cos\theta_1 + \cos\theta_2 = 1 \tag{14}$$

$$\sqrt{1 - \left(\frac{p_1}{D}\right)^2} + \sqrt{1 - \left(\frac{p_2}{D}\right)^2} = 1 \tag{15}$$

This is an equation relating warp and weft spacing of a most closely woven fabric.

2.2.4 Cross threads pulled straight

If the weft yarn is pulled straight $h_2 = 0$ and $h_1 = D$,

Equation 5 will give $D = (l_1 - D\theta_1)\sin\theta_1 + D(1 - \cos\theta_1)$

$$\cos\theta_1 = \left(\frac{l_1}{D} - \theta_1\right)\sin\theta_1$$

$$\text{or } \theta_1 + \cot\theta_1 = \frac{l_1}{D} \tag{16}$$

This equation gives maximum value of θ_1 for a given value of l_1/D

The above equation will be valid for warp yarn being straight by interchange of suffix from 1 to 2.

However, the weft thread can be restricted in being pulled straight by the jamming of warp threads.

In such a case,

$$l_1 - D\theta_1 = 0$$

$$\text{or } \theta_1 = \frac{l_1}{D}$$

Equation 5 will become

$$h_2 = D - h_1 = D - D(1-\cos\theta_1) = D\cos\frac{l_1}{D} \tag{17}$$

If the weft thread is pulled straight and warp is just jammed

$$\text{Then } \frac{l_1}{D} = \theta_1 = \frac{\pi}{2} \tag{18}$$

These are useful conditions for special fabric structure.

2.2.5 Non circular cross-section

So far, it is assumed that yarn cross-section is circular and yarn is incompressible. However, the actual cross-section of yarn in fabric is far from circular due to the system of forces acting between the warp and weft yarns after weaving and the yarn can never be incompressible. This inter-yarn pressure results in considerable yarn flattening normal to the plane of the cloth even in a highly twisted yarn. Therefore many researchers have tried to correct Peirce's original relationship by assuming various shapes for the cross-section of yarn. Two important cross-sectional shapes such as elliptical and race-track are discussed below.

2.2.5.1 Elliptical cross-section

Peirce's elliptical yarn cross-section is shown in figure 7; the flattening factor is defined as

$$e = \sqrt{\frac{b}{a}}$$

Where b = minor axis of ellipse, a = major axis of ellipse

The area of ellipse is $(\pi/4)ab$. If d is assumed as the diameter of the equivalent circular cross–section yarn, then

$$d = \sqrt{ab}$$

$$h_1 + h_2 = d_1 + d_2 = b_1 + b_2$$

$$b_1 + b_1 = h_1 + h_2 = \frac{4}{3}\left[p_1\sqrt{c_2} + p_2\sqrt{c_1}\right] \tag{19}$$

Yarn diameter is given by its specific volume, v and yarn count as under:

$d_{\text{mils}} = 34.14\frac{\sqrt{v}}{\sqrt{N}}$, N is the English count.

$d_{\text{cm}} = \frac{\sqrt{\text{Tex}}}{280.2\sqrt{\varphi\rho_{\text{f}}}} = \frac{\sqrt{\text{Tex}}}{280}$, assuming, φ = 0.65, ρ_f = 1.52 for cotton fiber

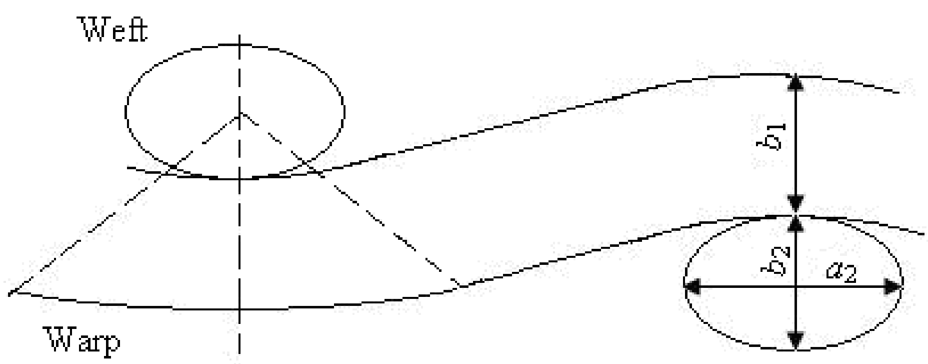

Fig. 7. Elliptical cross-section

This can be used to relate yarn diameter and crimp height by simply substituting in equation 19 to obtain:

$$h_1 + h_2 = d_1 + d_2 = D = 34.14\left(\sqrt{\frac{v_1}{N_1}} + \sqrt{\frac{v_2}{N_2}}\right) \tag{20}$$

$$h_1 + h_2 = d_1 + d_2 = \frac{1}{280.2}\left(\sqrt{\frac{T_1}{\varphi_1\rho_{\text{f1}}}} + \sqrt{\frac{T_2}{\varphi_2\rho_{\text{f2}}}}\right) = \frac{1}{280}\left(\sqrt{T_1} + \sqrt{T_2}\right)$$

assuming, φ = 0.65, ρ_f = 1.52 for cotton fiber

These are useful equation to be used subsequently in the crimp interchange derivation.

2.2.5.2 Race track cross-section

In race track model (Kemp, 1958; Love, 1954) given in figure 8, a and b are maximum and minimum diameters of the cross-section. The fabric parameters with superscript refer to the zone AB, which is analogous to the circular thread geometry; the parameters without superscript refer to the race track geometry, a repeat of this is between CD. Then the basic equations will be modified as under:

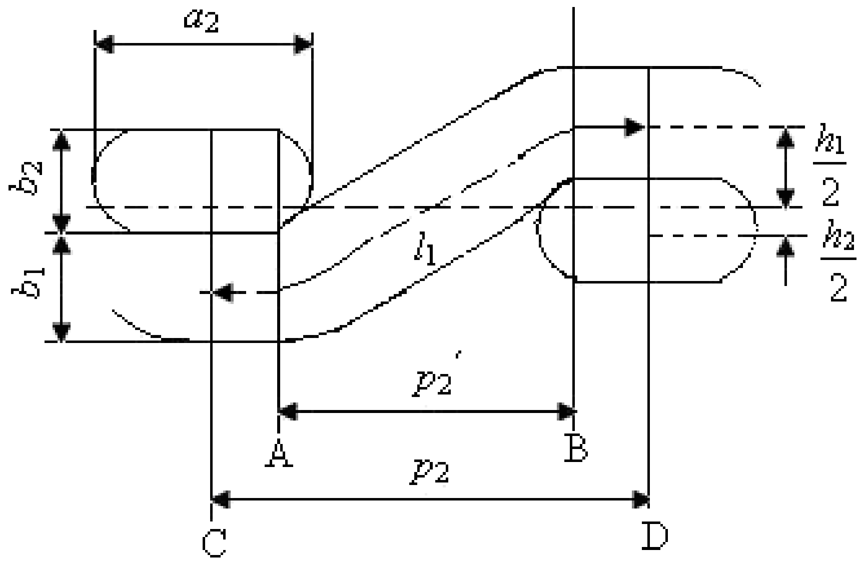

Fig. 8. Race track cross-section

$$p_2' = p_2 - (a_2 - b_2) \tag{21}$$

$$l_1' = l_1 - (a_2 - b_2) \tag{22}$$

$$c_1' = \frac{l_1' - p_2'}{p_2'} = \frac{c_1 p_2}{p_2 - (a_2 - b_2)} \tag{23}$$

Similarly,

$$c_2' = \frac{c_2 p_1}{p_1 - (a_1 - b_1)} \tag{24}$$

$$h_1 = \frac{4}{3} p_2' \sqrt{c_1'} \tag{25}$$

$$h_2 = \frac{4}{3} p_1' \sqrt{c_2'} \tag{26}$$

$h_1 + h_2 = B = b_1 + b_2$

And also if both warp and weft threads are jammed, the relationship becomes

$$\sqrt{B^2 - \left(p_1'\right)^2} + \sqrt{B^2 - \left(p_2'\right)^2} = B \tag{27}$$

2.3 Prediction of fabric properties

Using the fabric parameters discussed in the previous section it is possible to calculate the *Fabric thickness, Fabric cover, Fabric mass and Fabric specific volume.*

2.3.1 Fabric thickness

Fabric thickness for a circular yarn cross-section is given by

h_1+d_1 *or* h_2+d_2, whichever is greater.

When the two threads project equally, then $h_1+d_1 = h_2+d_2$
In this case the fabric gives *minimum thickness* $=1/2(h_1+d_1+h_2+d_2)=D$; $h_1=D-d_1$

Such a fabric produces a smooth surface and ensures uniform abrasive wear.

In a fabric with coarse and fine threads in the two directions and by stretching the fine thread straight, maximum crimp is obtained for the coarse thread. In this case the fabric gives *maximum thickness* as under;

$$\text{Maximum Thickness} = D + d_{\text{coarse}}, \text{ since } h_{\text{coarse}} = D$$

When yarn cross- section is flattened, the fabric thickness can be expressed as
h_1+b_1 or h_2+b_2, whichever is greater

2.3.2 Fabric cover

In fabric, cover is considered as fraction of the total fabric area covered by the component yarns. For a circular cross-section cover factor is given as:

$$\frac{d}{p} = \frac{E\sqrt{T}}{280.2\sqrt{\varphi\rho_{\text{f}}}} = \frac{K}{28.02\sqrt{\varphi\rho_{\text{f}}}}$$

$$K = E\sqrt{T} \times 10^{-1} \; K \text{ is cover factor}$$

T is yarn tex, E is threads per cm = $1/p$ suffix 1 and 2 will give warp and weft cover factors.

for $\frac{d}{p} = 1$, cover factor is maximum and given by,

$$K_{\max} = 28.02\sqrt{\varphi\rho_{\text{f}}}$$

Fractional fabric cover is given by:

$$\frac{d_1}{p_1} + \frac{d_2}{p_2} - \frac{d_1 d_2}{p_1 p_2} = \frac{1}{28.02}\left[K_1 + K_2 - \frac{K_1 K_2}{28.02}\right]$$

Multiplying by 28.02 and taking 28.02 ≈ 28 we get fabric cover factor as under:

$$\text{Fabric cover factor} = \mathrm{K}_1 + \mathrm{K}_2 - \left(\mathrm{K}_1\mathrm{K}_2 / 28\right) \tag{28}$$

For race track cross-section the equation will be

$$\frac{a}{p}=\frac{d}{ep\sqrt{1+\frac{4}{\pi}\left(\frac{1}{e}-1\right)}\times 28.02\sqrt{\varphi\rho_{\mathrm{f}}}} \quad \text{here } e=b/a$$

$$=\frac{K}{e\sqrt{1+\frac{4}{\pi}\left(\frac{1}{e}-1\right)}\times 28.02\sqrt{\varphi\rho_{\mathrm{f}}}}$$

For elliptical cross-section the equation will be;

$$\frac{a}{p}=\frac{d}{ep}=\frac{E\sqrt{T}}{280.2\ e\sqrt{\varphi_{\rho\mathrm{f}}}}=\frac{K}{28.02\,e\sqrt{\varphi_{\rho\mathrm{f}}}} \tag{29}$$

$$\text{Here } e=\sqrt{\frac{b}{a}} \text{ and } d=\sqrt{ab}$$

2.3.3 Fabric mass (Areal density)

$$\text{gsm} = [T_1E_1(1+c_1)+T_2E_2(1+c_2)]\times 10^{-1} \tag{30}$$

$$\text{gsm} =\sqrt{T_1}\ [(1+c_1)\ K_1 + (1+c_2)\ K_2\beta] \tag{31}$$

E_1, E_2 are ends and picks per cm.
T_1, T_2 are warp and weft yarn tex
Here K_1 and K_2 are the warp and weft cover factors, c is the fractional crimp and $d_2/d_1= \beta$.

In practice the comparison between different fabrics is usually made in terms of gsm. The fabric engineer tries to optimize the fabric parameters for a given gsm. The relationship between the important fabric parameters such as cloth cover and areal density is warranted.

2.3.4 Fabric specific volume

The apparent specific volume of fabric, υ_F is calculated by using the following formula:

$$\upsilon_F=\frac{\text{fabric thickness (cm)}}{\text{fabric mass (g/cm}^2)} \tag{32}$$

Fabric mass (g/cm^2) = 10^{-4} x gsm

Fabric packing factor, $\Phi = \upsilon_f / \upsilon_F$ (33)

Here υ_f, υ_F are respectively fiber and fabric specific volume.

A knowledge of fiber specific volume helps in calculating the packing of fibers in the fabric. Such studies are useful in evaluating the fabric properties such as warmth, permeability to air or liquid.

2.4 Maximum cover and its importance

Maximum cover in a jammed fabric is only possible by keeping the two consecutive yarns (say warp) in two planes so that their projections are touching each other and the cross thread (weft) interlaces between them. In this case the weft will be almost straight and maximum bending will be done by the warp.

$$d_1/p_1 = 1 \text{ will give } K_1 = K_{max}$$

and the spacing between the weft yarn, $p_2 = D\sin\theta_1 = D$ (for $\theta = 90^0$) , $p_2 = d_1 + d_2$

$$\frac{d_2}{p_2} = \frac{d_2}{d_1 + d_2} = \frac{2}{3} \quad \text{for } d_2 = 2d_1$$

This will give $K_2 = 2/3\ K_{max}$,

If $d_1 = d_2$ then $d_1 = d_2/p_2 = 0.5$ and $K_2 = 0.5\ K_{max}$

This is the logic for getting maximum cover in any fabric.

The principles are as under:

1. Use fine yarn in the direction where maximum cover is desired and keep them in two planes so that their projections touch each other and use coarse yarn in the cross direction.
2. As in (1) instead of coarse yarn insert two fine yarns in the same shed.

Both options will give maximum cover in warp and weft but first option will give more thickness than the second case.

The cover factor indicates the area covered by the projection of the thread. The ooziness of yarn, flattening in finishing and regularity further improves the cover of cloth. It also gives a basis of comparison of hardness, crimp, permeability, transparency. Higher cover factor can be obtained by the lateral compression of the threads. It is possible to get very high values only in one direction where threads have higher crimp. Fabrics differing in yarn counts and average yarn spacing can be compared based on the fabric cover. The degree of flattening for race track and elliptical cross-section can be estimated from fabric thickness measurements to evaluate b and a from microscopic measurement of the fabric surface.

The classical example in this case is that of a poplin cloth in which for warp threads

$p_1 = d_1$ and for $d_1 = d_2 = D/2$ and for jamming in both directions

$$p_1 = D\sin\theta_2$$

$$d = D/2 = D\sin\theta_2$$

$$\theta_2 = 30^o = 0.5236$$

$$\theta_1 = 82^0 18' 1.4364 (\text{using } \cos\theta_1 + \cos\theta_2 = 1)$$

$$p_2 = D\sin\theta_1 = 0.991D \approx 2p_1$$

$$l_1 = D\theta_1 = 1.14364$$

$$l_2 = D\theta_2 = 0.5236$$

$$c_1' = 0.45,\ c_2' = 0.0472$$

This is a specification of good quality poplin which has maximum cover and ends per cm is twice that of picks per cm.

3. Application of geometrical model

3.1 Computation of fabric parameters

The basic equations derived from the geometrical model are not easy to handle. Research workers (Nirwan & Sachdev, 2001; Weiner, 1971) obtained solutions in the form of graphs and tables. These are quite difficult to use in practice. It is possible to predict fabric parameters and their effect on the fabric properties by soft computing (Newton, 1995). This information is helpful in taking a decision regarding specific buyers need. A simplified algorithm is used to solve these equations and obtain relationships between useful fabric parameters such as thread spacing and crimp, fabric cover and crimp, warp and weft cover. Such relationships help in guiding the directions for moderating fabric parameters.

Peirce's geometrical relationships can be written as

$$\frac{p_2}{D} = (K_1 - \theta_1)\cos\theta_1 + \sin\theta_1 \tag{34}$$

$$\frac{h_1}{D} = (K_1 - \theta_1)\sin\theta_1 + (1 - \cos\theta_1) \tag{35}$$

Where $K_1 = l_1/D$ and two similar equations for the weft direction will be obtained by interchanging the suffix 1 with 2 and vice versa. The solution of p_2/D and h_1/D is obtained for different values of θ_1 (weave angle) ranging from 0.1– $\pi/2$ radians. Such a relationship is shown in figure 9.

It is a very useful relationship between fabric parameters for engineering desired fabric constructions. One can see its utility for the following three cases

1. Jammed structures
2. Non-jammed fabrics
3. Special case in which cross-threads are straight

3.1.1 Jammed structures

Figure 9 shows non linear relationship between the two fabric parameters p and h on the extreme left. In fact, this curve is for jamming in the warp direction. It can be seen that the jamming curve shows different values of p_2/D for increasing h_1/D, that is warp crimp. The theoretical range for p_2/D and h_1/D varies from 0-1. Interestingly this curve is a part of circle and its equation is:

$$\left(\frac{p_2}{D}\right)^2 + \left(\frac{h_1}{D} - 1\right)^2 = 1 \tag{36}$$

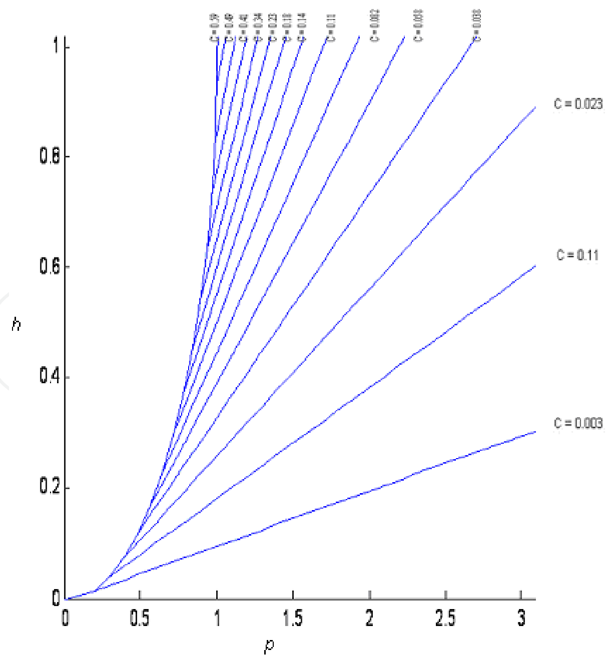

Fig. 9. Relation between thread spacing and crimp height

with centre at (0, 1) and radius equal to 1.

For jamming in the warp direction of the fabric the parameters p_2/D and corresponding h_1/D can be obtained either from this figure or from the above equation.

The relationship between the fabric parameters over the whole domain of structure being jammed in both directions can be obtained by using an algorithm involving equations from the previous section.

Another useful relationship between the crimps in the two directions is shown in figure 10. It indicates inverse non-linear relationship between c_1 and c_2. The intercepts on the X and Y axis gives maximum crimp values with zero crimp in the cross-direction. This is a fabric configuration in which cross-threads are straight and all the bending is being done by the intersecting threads.

Figure 11 shows the relation between h_1/p_2 and h_2/p_1. The figure shows inverse linearity between them except at the two extremes. This behavior is in fact a relationship between the square root of crimp in the two directions of the fabric.

Other practical relations are obtained between the warp and weft cover factor and between cloth cover factor and fabric mass (gsm).

Figure 12 gives the relation between warp and weft cover factor for different ratio of weft to warp yarn diameters (β). The relation between the cover factors in the two directions is sensitive only in a narrow range for all values of β. The relation between the cover factors in the two directions are inter- dependent for jammed structures. Maximum threads in the warp or weft direction depend on yarn count and weave. Maximum threads in one direction of the fabric will give unique maximum threads in the cross-direction. The change in the value of β causes a distinct shift in the curve. A comparatively coarse yarn in one direction with respect to the other direction helps in increasing the cover factor. For β = 0.5, the warp yarn is coarser than the weft, this increases the warp cover factor and decrease the weft cover factor. This is due to the coarse yarn bending less than the fine yarn. Similar effect can be noticed for β =2, in which the weft yarn is coarser than the warp yarn. These results are similar to earlier work reported by Newton (Newton, 1991 & 1995; Seyam, 2003).

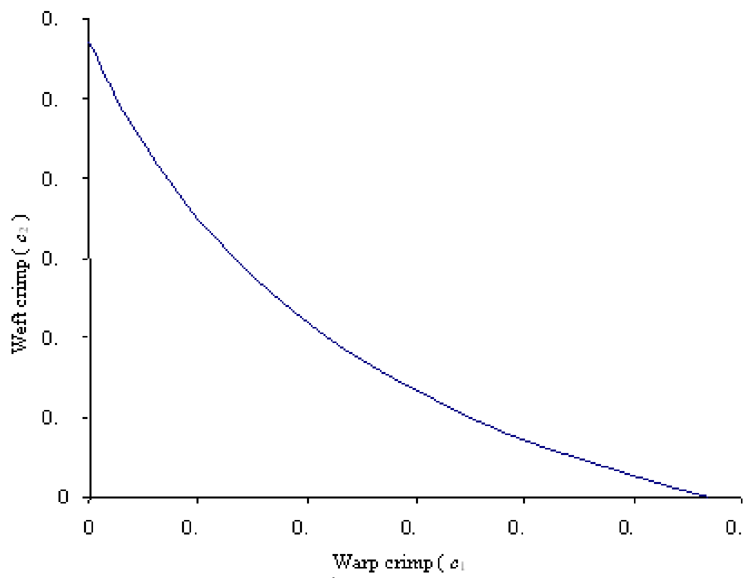

Fig. 10. Relation between warp and weft crimp for jammed fabric

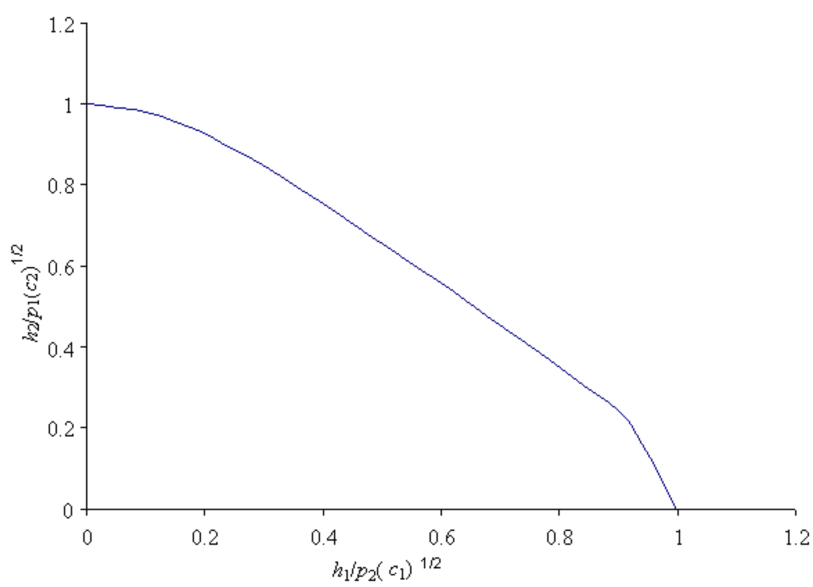

Fig. 11. Relation between warp and weft crimp in jammed fabric

The relation between fabric mass, (gsm) with the cloth cover (K_1+K_2) is positively linear (Singhal & Choudhury, 2008). The trend may appear to be self explanatory. Practically an increase in fabric mass and cloth cover factor for jammed fabrics can be achieved in several ways such as with zero crimp in the warp direction and maximum crimp in the weft direction; zero crimp in the weft direction and maximum crimp in the warp direction; equal or dissimilar crimp in both directions. This explanation can be understood by referring to the non-linear part of the curve in figure 11.

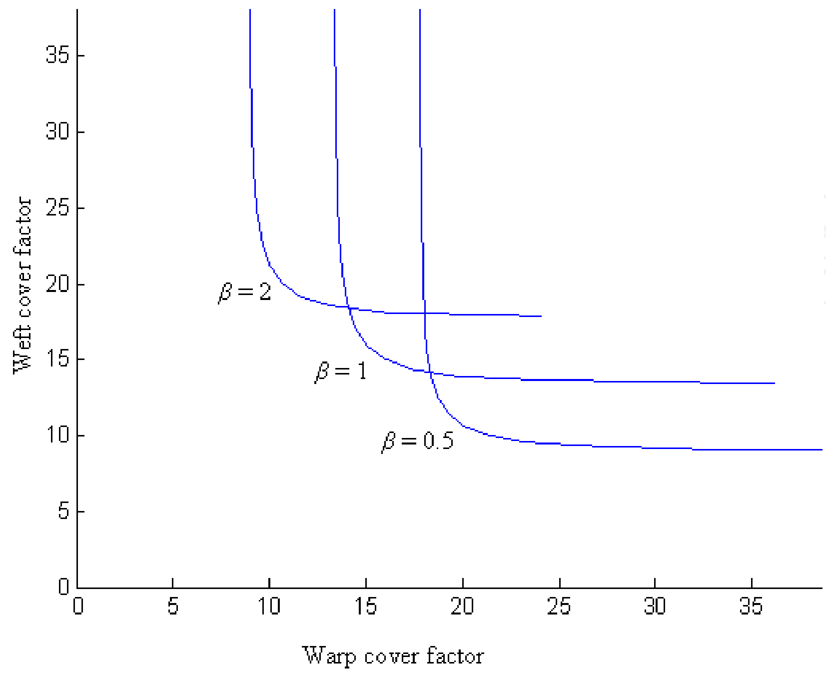

Fig. 12. Relation between warp and weft cover factor for different β in jammed fabric

3.1.2 Non-jammed structure

It can be seen that the relation between p_2/D and corresponding h_1/D is linear for different values of crimp. This relationship is useful for engineering non-jammed structures for a range of values of crimp. The fabric parameters can be calculated from the above non-jammed linear relation between p_2/D and h_1/D for any desired value of warp crimp. Then h_2/D can be obtained from $(1–h_1/D)$ and for this value of h_2/D one can obtain the corresponding value of p_1/D for the desired values of weft crimp. Thus all fabric parameters can be obtained for desired value of p_2/D, picks per cm, warp and weft yarn tex, warp and weft crimp. One can choose any other four parameters to get all fabric parameters.

3.1.3 Straight cross threads

The intersection of horizontal line corresponding to h_1/D=1 gives all possible structures ranging from relatively open to jammed configurations. In this case h_2= 0, h_1= D; This gives interesting structures which have stretch in one direction only, enabling maximum fabric thickness and also being able to use brittle yarns. The fabric designer gets the options to choose from the several possible fabric constructions. These options include jamming and other non jammed constructions. Using the above logic it is also possible to get fabric parameters for:

1. fabric jammed in both directions.
2. fabric with maximum crimp in one direction and cross-threads being straight.
3. fabric which is neither jammed nor has zero crimp in the cross-threads.

3.2 Weavability limit

The maximum number of ends and picks per unit length that can be woven with a given yarn and weave defines weavability limit (Hearle et al., 1969). This information is helpful to the weavers in avoiding attempts to weave impossible constructions thus saving time and money. It also helps to anticipate difficulty of weaving and take necessary preparations. (Dickson, 1954) demonstrated the usefulness of theoretical weavability limit and found agreement with the loom performance. Most of the work in this area was done using empirical relationships. The geometrical model is very useful in predicting this limit for a given warp, weft diameter (tex) and any weave. Maximum weavability limit is calculated in the model by using jamming conditions for plain and non-plain weaves for circular and race track cross-sections.

3.2.1 Yarn diameter

Two important geometrical parameters are needed for calculating weavability for a general case. These are yarn diameter and weave factor.

Yarn diameter in terms of linear density in tex for a general case is given as:

$$d = \frac{\sqrt{T}}{280.2\sqrt{\varphi_{\rho_f}}} \tag{37}$$

Where d = yarn diameter (cm), T = yarn linear density (tex, i.e. g/km),

ρ_f = fiber density(g/cm^3), ρ_y = yarn density(g/cm^3), Φ is yarn packing factor.

This equation for the yarn diameter is applicable for any yarn type and fiber type. The packing factor depends on fiber variables such as fiber crimp, length, tex and cross-section shape.

Table 2 and 3 give the fiber density and yarn packing factor for different fiber and yarn type respectively.

Acetate	1.32
Cotton	1.52
Lycra	1.20
Nylon 6	1.14
Nylon 66	1.13-1.14
Polyester	1.38
Polypropylene	0.91
Rayon	1.52
Wool	1.32

Table 2. Fiber density, g/cm^3

Ring-spun	0.60
Open-end-spun	0.55
Worsted	0.60
Woolen	0.55
Continuous-filament	0.65

Table 3. Yarn packing factor

For blended yarns, average fiber density is given by the following

$$\frac{1}{\rho} = \sum_{1}^{n} \frac{p_i}{p_{ft}} \tag{38}$$

where $\overline{\rho}$ = average fiber density ,

p_i = weight fraction of the ith component,

p_{ft} = fiber density of the ith component and

n = number of components of the blend

3.2.2 Effect of variation in beta (d_2/d_1) on the relation between warp and weft cover factor for jammed fabrics

An increase in the value of beta from 0.5-2 increases the range of warp cover factors but raises the level for the weft cover factor. This means with an increase in beta higher weft cover factors are achievable and vice-versa. However it may be noted that for cotton fibers having higher fiber density the sensitivity range between the warp and weft cover factor is relatively large compared to polypropylene fiber as shown in figure 13a and 13b. This shows a very important role played by fiber density in deciding warp and weft cover factors for the jammed fabrics.

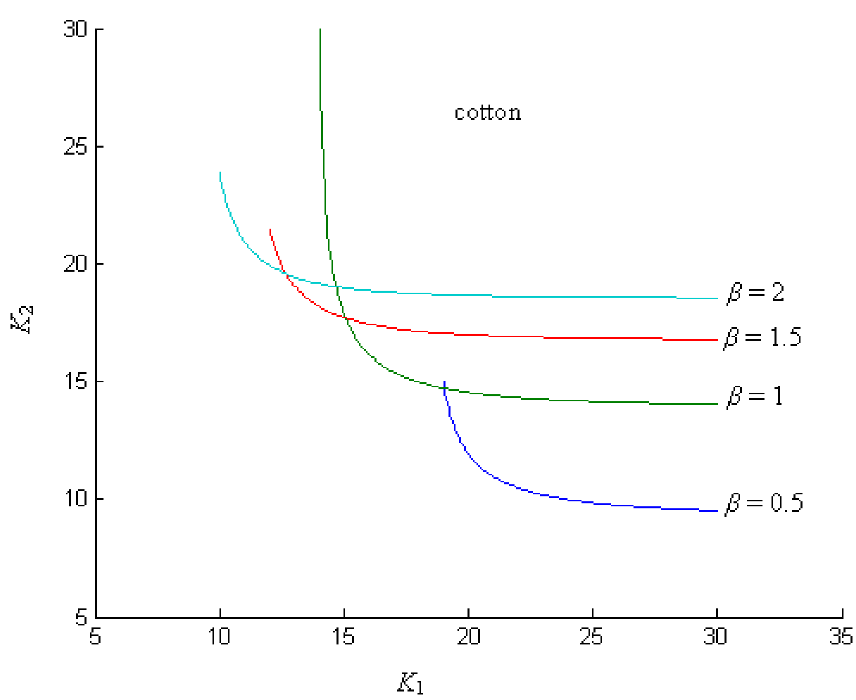

Fig. 13a. Effect of β on the relation between warp and weft cover factor

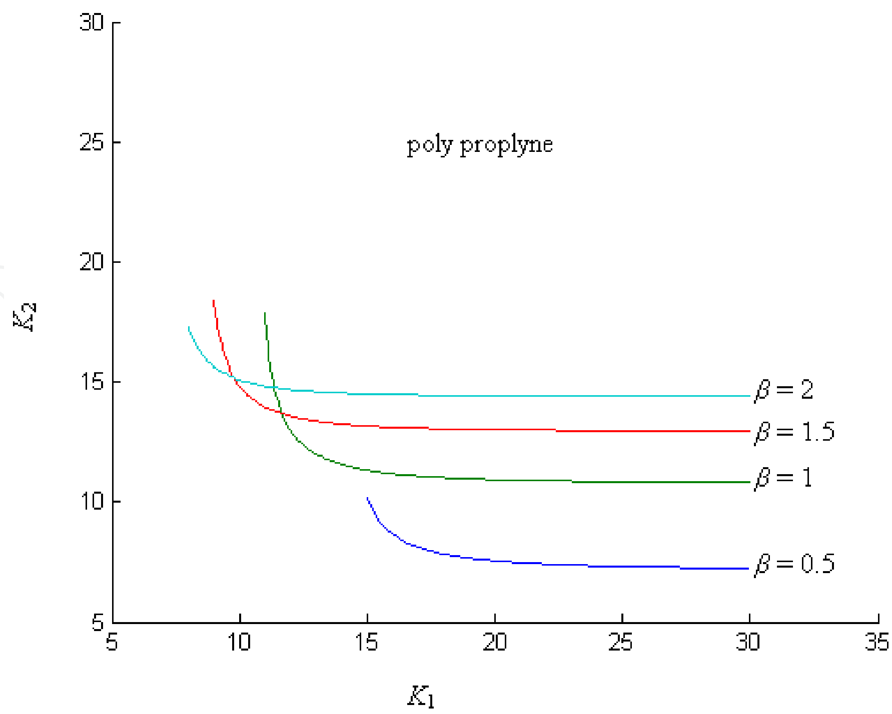

Fig. 13b. Effect of β on the relation between warp and weft cover factor

3.2.3 Equation for jammed structure for circular cross-section in terms of weave factor

Weave factor is useful in translating the effect of weave on the fabric properties. For circular cross-section the general equation for jammed cloth is desired.

Thread spacing P_{t1} for a non-plain weave per repeat is shown in figure 14 and is given as:

$$P_{t1} = I_2 p_1 + (E_1 - I_2) d_1 \tag{39}$$

Average thread spacing $\overline{P_1} = \dfrac{I_2 p_1 + (E_1 - I_2) d_1}{E_1}$

That means, $\dfrac{E_1 \overline{P_1}}{I_2} = p_1 + \left(\dfrac{E_1}{I_2} - 1 \right) d_1$

$$M_1 \overline{P_1} = p_1 + (M_1 - 1) d_1$$

$$\frac{p_1}{D} = M_1 \frac{\overline{P_1}}{D} - (M_1 - 1) \frac{d_1}{D}$$

$$\frac{p_1}{D} = M_1 \frac{\overline{P_1}}{D} - \frac{(M_1 - 1)}{1 + \beta} \tag{40}$$

where $\beta = d_2/d_1$

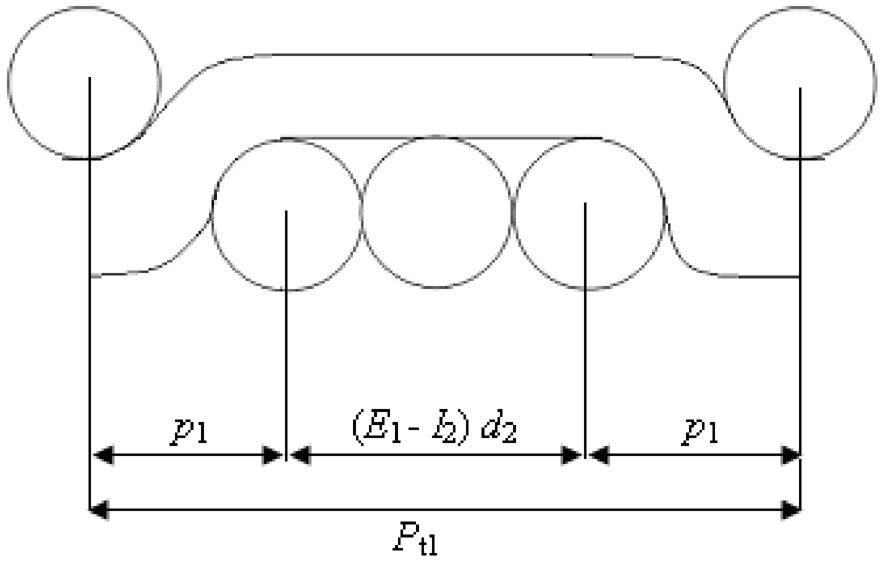

Fig. 14. Jammed structure for 1/3 weave (circular cross-section along warp)

Similarly, interchanging suffix 1, 2 we get

$$\frac{p_2}{D} = M_2 \frac{\overline{P_2}}{D} - (M_2 - 1)\frac{d_2}{D}$$

$$\frac{p_2}{D} = M_2 \frac{\overline{P_2}}{D} - (M_2 - 1)\frac{\beta}{1+\beta} \tag{41}$$

For a jammed fabric the following equation is valid:

$$\sqrt{1 - \left(\frac{p_1}{D}\right)^2} + \sqrt{1 - \left(\frac{p_2}{D}\right)^2} = 1$$

$$\sqrt{1 - \left(M_1 \frac{\overline{P_1}}{D} - \frac{(M_1 - 1)}{1+\beta}\right)^2} + \sqrt{1 - \left(M_2 \frac{\overline{P_2}}{D} - \frac{(M_2 - 1)\beta}{1+\beta}\right)^2} = 1$$

This equation can easily be transformed in terms of warp and weft cover factor (K_1 and K_2)

$$\sqrt{1 - \left[\left(\frac{28.02\sqrt{\varphi\rho_f}\, M_1}{K_1} - (M_1 - 1)\right)\frac{1}{1+\beta}\right]^2} + \sqrt{1 - \left[\left(\frac{28.02\sqrt{\varphi\rho_f}\, M_2}{K_2} - (M_2 - 1)\right)\frac{\beta}{1+\beta}\right]^2} = 1 \tag{42}$$

3.2.4 Relation between fabric parameters for circular cross-section for different weaves

The effect of weave in the jammed structures is examined using the above equations for plain, twill, basket and satin weave. *M*, the weave factor value (average float length) for these weaves are 1, 1.5, 2 and 2.5 respectively for all the discussion which follows.

The relation between p_{1avg}/D and p_{2avg}/D is established and it is seen that with the increase in float length, the sensitivity of the curve decreases in general. Also the range of p_1/D and p_2/D values gets reduced. This means a weave with longer float length decreases the flexibility for making structures.

Figure 15 shows the relationship between the warp and weft cover factor for circular cross-section. It is interesting to note that the behavior is similar for different weaves. However with the increase in float, the curve shifts towards higher values of weft cover factor. It should be borne in mind that the behavior shown in this figure is for virtual fabrics. In real fabrics jammed structure is unlikely to retain circular cross-section.

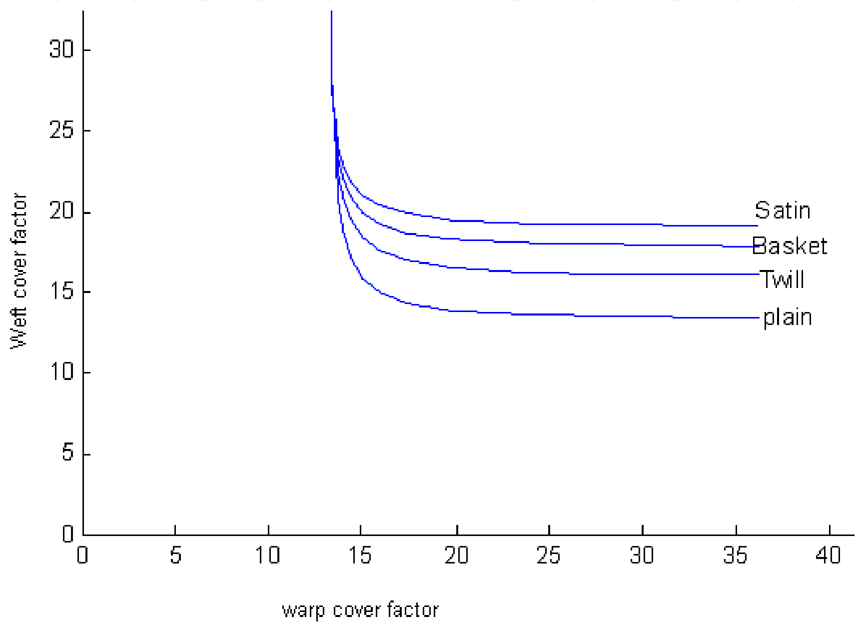

Fig. 15. Relation between warp and weft cover factor for jammed fabric (circular cross-section)

3.2.5 Equation for jammed structure for a race track cross-section in terms of weave factor

In jammed fabrics, the yarn cross-section cannot remain circular. The cross-section will change. It is easy to modify the geometry for circular cross-section by considering race track cross-section. Figure 16 shows the configuration of jammed structure for 1/3 weave for race track cross-section along weft direction of the fabric.

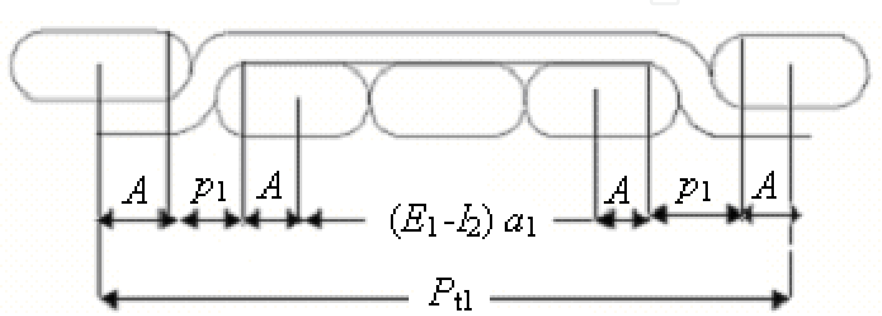

Fig. 16. Jammed structure for 1/3 weave (race track cross-section along warp)

Here, $$A = \left(\frac{a_1 - b_1}{2}\right)$$

Thread spacing P_{t1} for a non-plain weave per repeat is given as:

$$P_{t1} = I_2 p_1 + (E_1 - I_2)a_1 + 4\left(\frac{a_1 - b_1}{2}\right) \tag{43}$$

Simillarly,

$$P_{t2} = I_1 p_2 + (E_2 - I_1)a_2 + 4\left(\frac{a_2 - b_2}{2}\right) \tag{44}$$

Where, p_1 and p_2 are horizontal spacing between the semi-circular threads in the intersection zone. Here, a and b are the major and minor diameters of race track cross-section.

The average thread spacing

$$\overline{P_1} = \frac{p_1}{M_1} + \left(1 - \frac{1}{M_1}\right)a_1 + \frac{4}{M_1 I_2}\left(\frac{a_1 - b_1}{2}\right) \tag{45}$$

Similarly,

$$\overline{P_2} = \frac{p_2}{M_2} + \left(1 - \frac{1}{M_2}\right)a_2 + \frac{4}{M_2 I_1}\left(\frac{a_2 - b_2}{2}\right) \tag{46}$$

As such analysis of circular thread geometry can be applied for the intersection zone of the race track cross-section.

$$L_1 = 4\left(\frac{a_2 - b_2}{2}\right) + (E_2 - I_1)a_2 + I_1 \times l_1 \tag{47}$$

Total warp crimp in the fabric is given by:

$$C_1 = \frac{L_1}{P_{t2}} - 1$$

p_1 and p_2 can be calculated from the jamming considerations of the circular thread geometry using:

$$\sqrt{1 - \left(\frac{p_1}{B}\right)^2} + \sqrt{1 - \left(\frac{p_2}{B}\right)^2} = 1$$

It should be remembered that p/B corresponds to the semi-circular region of the race track cross-section and is similar to p/D for circular cross-section. As such the values of p/D ratio can be used for p/B

$$\sqrt{1-\left(M_1\frac{\overline{P_1}}{B}-\frac{4}{BI_2}\left(\frac{a_1-b_1}{2}\right)-(M_1-1)\frac{a_1}{B}\right)^2}+\sqrt{1-\left(M_2\frac{\overline{P_2}}{B}-\frac{4}{BI_1}\left(\frac{a_2-b_2}{2}\right)-(M_2-1)\frac{a_2}{B}\right)^2}=1$$

This equation can be simplified to the following usable forms.

$$\sqrt{1-\left(M_1\frac{\overline{P_1}}{B}-\frac{2(1-e)}{e(1+\beta)I_2}-\frac{(M_1-1)}{e(1+\beta)}\right)^2}+\sqrt{1-\left(M_2\frac{\overline{P_2}}{B}-\frac{2(1-e)\beta}{e(1+\beta)I_1}-\frac{(M_2-1)}{e(1+\beta)}\right)^2}=1$$

It is assumed that $e_1 = e_2 = e$

where $e = b/a$

The above equation can easily be transformed in terms of warp and weft cover factor as under:

$$\sqrt{1-\left(\frac{28.02\sqrt{\varphi\rho_f}M_1}{(1+\beta)K_1}\sqrt{1+\frac{4}{\pi}\left(\frac{1}{e}-1\right)}-\frac{2(1-e)}{e(1+\beta)I}-\frac{(M_1-1)}{e(1+\beta)}\right)^2}$$

$$+\sqrt{1-\left(\frac{28.02\sqrt{\varphi\rho_f}M_2\beta}{(1+\beta)K_2}\sqrt{1+\frac{4}{\pi}\left(\frac{1}{e}-1\right)}-\frac{2(1-e)\beta}{e(1+\beta)I}-\frac{(M_2-1)\beta}{e(1+\beta)}\right)^2}=1 \qquad (48)$$

3.2.6 Relationship between fabric parameters in race track cross-section

The relationship between fabric parameters such as p_2 and p_1, p_1 and c_2 for the race track cross-section in jammed condition is discussed below.

The parameters are similar to that for the circular cross-section but it shifts towards higher values of thread spacing.

Figure 17 shows the relationship between warp and weft cover factors for different weaves. As discussed above in real fabrics the weaves show distinct differences between them unlike in circular cross-section. Increase in float length decreases the scope of cover factors.

From these equations crimp and fabric cover can be evaluated using the above two equations along with:

$$d^2 = b^2\left[\frac{\pi}{4}+\left(\frac{1}{e}-1\right)\right] \text{ and } \frac{b_2}{b_1}=\beta \qquad (49)$$

3.3 Square cloth

A truly square fabric has equal diameter, spacing and crimp.

$$p_1 = p_2,\ c_1 = c_2,\ d_1 = d_2,\ h_1 = h_2 = D/2,\ \theta_1 = \theta_2$$

From the basic equations of the geometrical model from the previous section we have:

$$\tan\theta / 2 = \frac{2}{3}\left[\frac{p}{D} - \sqrt{\left(\frac{p}{D}\right)^2 - 0.75}\right] \tag{50}$$

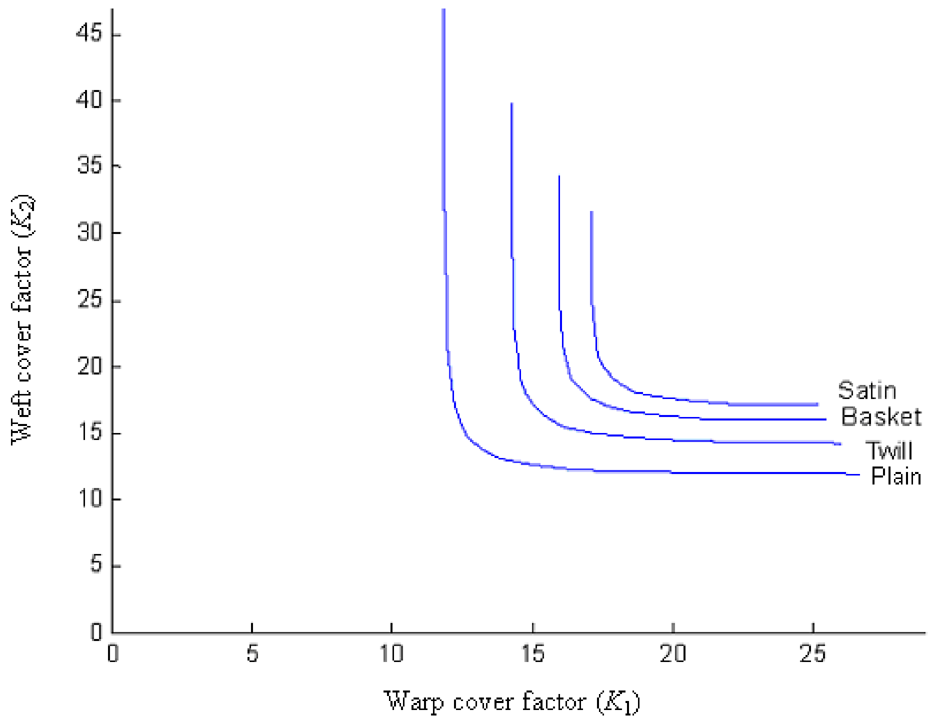

Fig. 17. Relation between warp and weft cover factor for jammed fabric (race track cross-section)

This is valid for all values of (p/D) 2 ≥ 0.75 or $p/D \geq 0.866$

$$p/d \geq 1.732;\ d/p \leq 0.5773$$

$$\text{Also } D = 2d = h_1 + h_2 = 2\times(4/3)p\sqrt{c}$$

$$\sqrt{c} = \frac{3}{4}\frac{d}{p} = \frac{0.75}{p} \times \frac{\sqrt{Tex}}{280.2\sqrt{\varphi\rho_f}}$$

$$c = \left(\frac{0.02677K}{\sqrt{\varphi\rho_f}}\right)^2 \tag{51}$$

Crimp in % can be calculated from, $\%c = \left(\frac{K}{3.57}\right)^2$

For jammed square cloth

$$\cos\theta_1 + \cos\theta_2 = 1 \text{ will give}$$

$$\cos\theta = ½ \text{ and } \theta = 60^0$$

$$p = 2d\sin\theta,\ 1 = D\theta = 2d\frac{\theta}{3}\quad \therefore \frac{1}{d} = 2\frac{\theta}{3}$$

$$\frac{p}{d} = \sqrt{3} = 1.732$$

$$=> \frac{p}{d} = 0.5773$$

$$\text{crimp} = \frac{1/d}{p/d} - 1 = 0.2092 = 20.9\%$$

Therefore complete cover is not possible with square cloth.

4. Crimp in the fabric

The crimp in fabric is the most important parameter which influences several fabric properties such as extensibility, thickness, compressibility and handle. It also decides quantity of yarn required to weave a fabric during manufacturing. Therefore control of crimp is vital for geometrical analysis of fabric structure.

4.1 Crimp interchange equation

Normally crimp interchange equation is used to predict the change in crimp in the fabric when it is extended in any direction by keeping the ratio of modular length to the sum of thread diameter (l_1/D and l_2/D)constant. An attempt is made by soft computing to exploit the crimp interchange equation in a different way instead of keeping the usual three invariants l_1, l_2 and D and the relationship between warp crimp (C_1) and weft crimp (C_2) is determined by varying l_1/D and l_2/D. Such a strategy enables bias of crimp in a preferred direction. This is a new concept and entirely a different use of crimp interchange equation.

Following equation gives a useful relationship between the two directions of the fabric.

$$D = h_1 + h_2 = h_1' + h_2'$$

Superscript represents changes in the fabric parameter after modification

$$D = h_1' + h_2' = \frac{4}{3}\left[p_2'\sqrt{c_1'} + p_1'\sqrt{c_2'}\right]$$

$$D = \frac{4}{3}\left[\frac{l_1\sqrt{c_1'}}{1+c_1'} + \frac{l_2\sqrt{c_2'}}{1+c_2'}\right]$$

$$\frac{l_1}{D}\frac{\sqrt{c_1'}}{(1+c_1')} + \frac{l_2}{D}\frac{\sqrt{c_2'}}{(1+c_2')} = \frac{3}{4} \tag{52}$$

The above equation is called crimp interchange equation. It gives the relationship between the warp and weft crimp for the new configuration after the application of stretch in warp/weft direction. It may be noted that the parameters l_1, l_2 and D are invariant; they have the same value in the original fabric and in the new configuration. This basically means it is assumed that the geometry in deformed fabric is same as in undeformed fabric.

In the crimp interchange equation one of the parameter $c_1^{/}$ or $c_2^{/}$ is determined based on the requirement of modification and the other parameter is calculated.

The most general manner of solving crimp interchange problems is getting relation between $\frac{\sqrt{C_1}}{1+C_1}$ and $\frac{\sqrt{C_2}}{1+C_2}$ for constant l_1/D and l_2/D.

4.2 Crimp balance equation

Textile yarns are not flexible as assumed in Peirce's geometrical model. They offer resistance to bending. The elastica model demonstrates the existence of inter yarn force at the crossover points during fabric formation. The crimp balance equation is an offshoot of this analysis. It shows the importance of bending rigidity of warp and weft yarns in influencing the ratio of crimp in both warp and weft directions.

The analysis using the rigid thread thread model [R] gives the value of inter yarn force

$$V = 16\,M \sin\theta / p^2$$

The balance of inter yarn force in two direction gives

$$V_1 = V_2$$

$$M_1 \sin\theta_1 / \, p_2^2 = M_2 \sin\theta_2 \, p_1^2$$

Since $\sin\theta ; \sqrt{C}$

$$\frac{\sqrt{C_1}}{\sqrt{C_2}} = \frac{M_2}{M_1}\left(\frac{p_2}{p_1}\right)^2 \tag{53}$$

$$\frac{C_1}{C_2} = \left(\frac{1+C_2}{1+C_1}\right)^4 \left(\frac{M_2}{M_1}\right)^2 \left(\frac{l_1 / D}{l_2 / D}\right)^4 \tag{54}$$

The solution for C_1 and C_2 for this equation is obtained in terms of M_2/M_1 and l_1/D and L_2/D using special algorithm in MATLAB.

4.3 Interaction of crimp interchange and crimp balance equations

The interaction of crimp interchange and crimp balance equations for given values of l_1/D , l_2/D and M_2/M_1 (ratio of bending moment of warp and weft) gives desired C_1 and C_2. It is impossible to solve these equations mathematically however soft computing facilitates solutions using iterations. It is the aim of this paper to facilitate fabric engineer in determining the fabric parameters for a given value of warp and weft crimp. This approach gives another alternative to engineer fabrics. The important variables of crimp balance equations are M_2/M_1, l1/D and l_2/D. For a given crimp interchange equation in terms of l1/D and l_2/D, the crimp balance equation gives intersections. The scales are also calibrated in terms of crimp.

Figures 18, 19 and 20 show the interaction of crimp interchange and crimp balance equation corresponding to $l_1/D = l_2/D$, l1/D> l_2/D and $l_1/D < l_2/D$ respectively. It is interesting to note that in all these curves with the increase in M_2/M_1, warp crimp increases and weft decreases. Another interesting result can be seen from these figures when l1/D not equal to l_2/D. $l_1/D > l_2/D$ or $l_1/D < l_2/D$ causes a reduction in a range and shift towards lower values for both C1 and C2

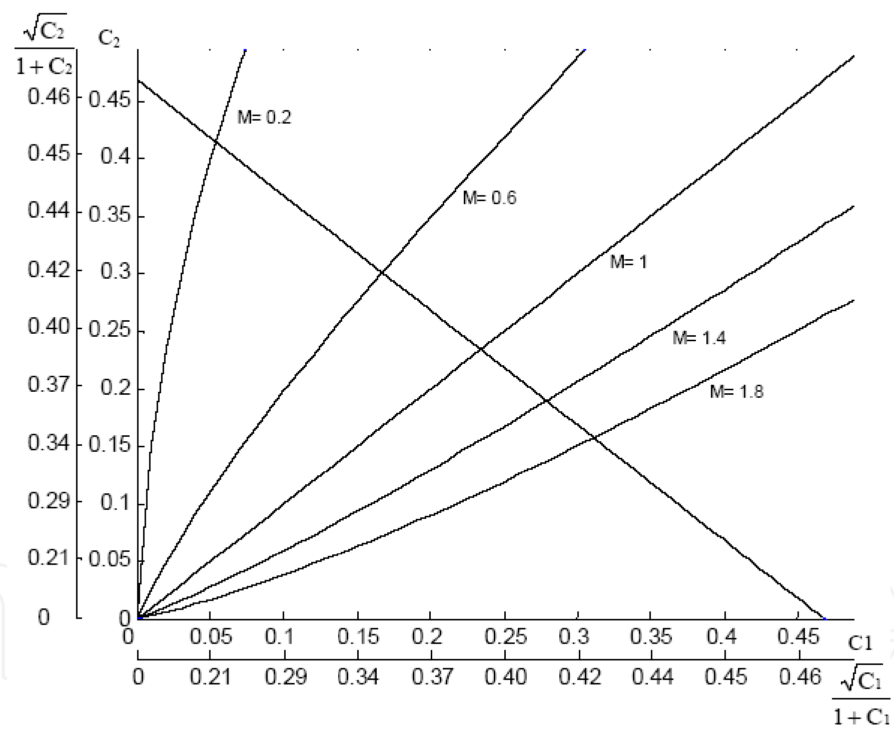

Fig. 18. Interaction of crimp interchange and crimp balance equations ($l_1/D = l_2/D$)

These three curves show very interesting ways in which the values of crimp in warp and weft can be varied in a wide range. Therefore the three parameters M_2/M_1, l_1/D and l_2/D can influence the crimp in warp and weft in a wide range and this is what gives maneuverability to the fabric designer.

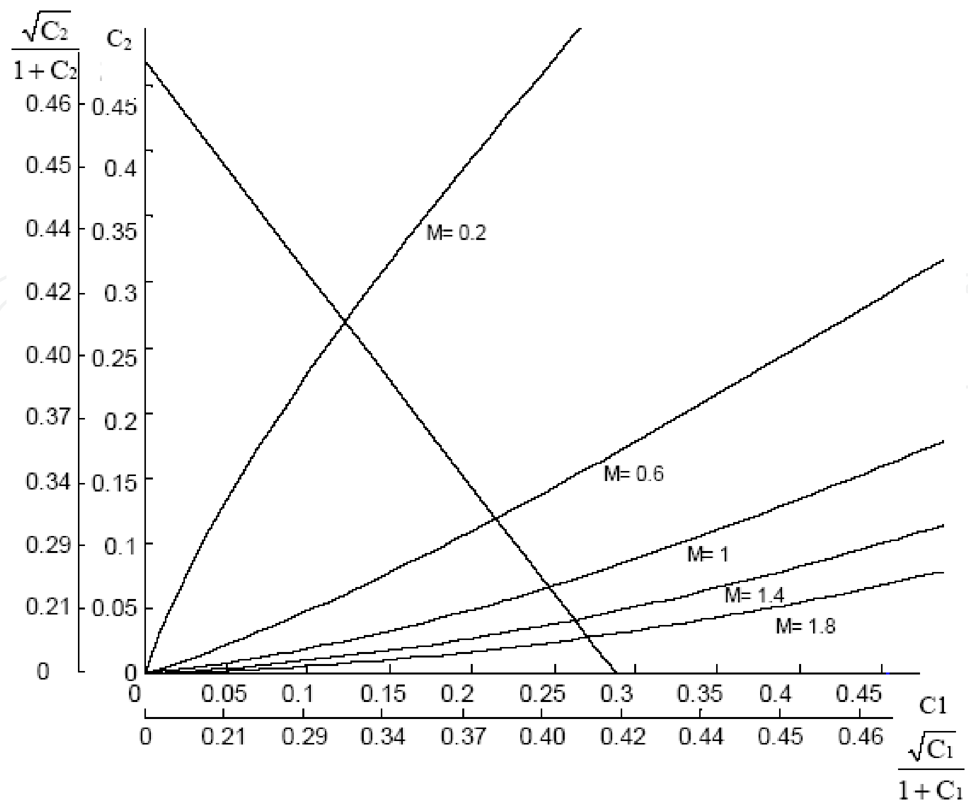

Fig. 19. Interaction of crimp interchange and crimp balance equations ($l_1/D > l_2/D$)

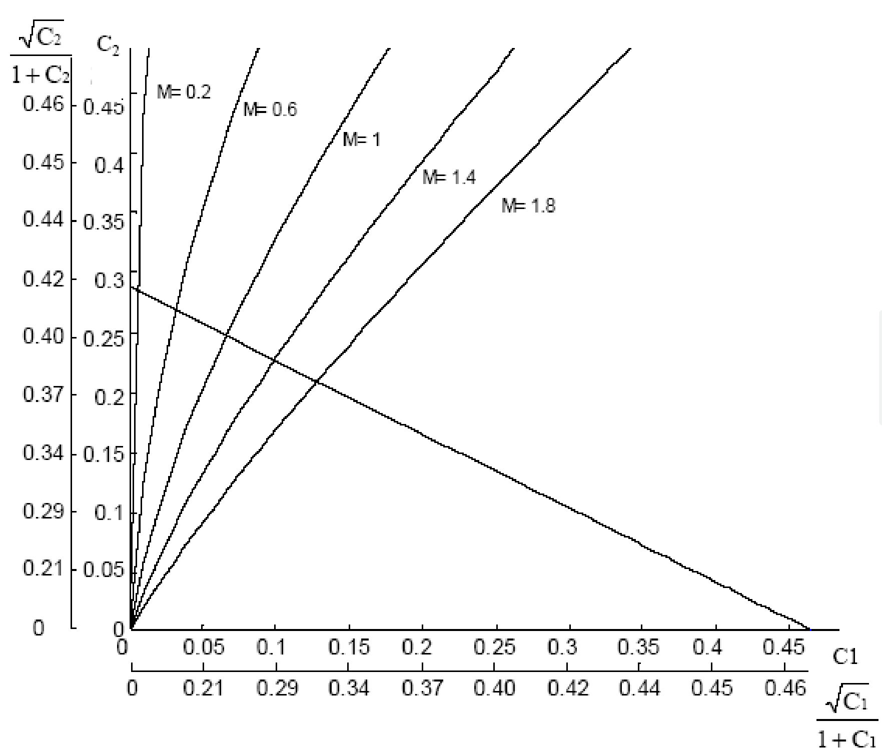

Fig. 20. Interaction of crimp interchange and crimp balance equations ($l_1/D < l_2/D$)

5. Conclusion

An attempt has been made to optimize engineering attributes of plain and non-plain weave fabrics as per requirement. Soft computing is used to solve fabric geometrical model equations and relationships between useful fabric parameters such as thread spacing and crimp, fabric cover and crimp, warp and weft cover are obtained. Such relationships help in guiding the direction for moderating fabric parameters. The full potential of Peirce fabric geometrical model for plain weave has been exploited by soft computing and the same is extended for non-plain constructions. The inter-relationships between different fabric parameters for jammed structures, non jammed structures and special case in which cross threads are straight are obtained using suitable computing techniques. It is hoped that the fabric designer will be benefited by the flexibility to choose fabric parameters for achieving any end use with desired fabric properties. This information is helpful to the weavers in avoiding attempts to weave impossible constructions thus saving time and money. It also helps to anticipate difficulty of weaving and take necessary steps in warp preparations. The relationship between the cover factors in warp and weft direction is demonstrated for circular and racetrack cross-section for plain, twill, basket and satin weave. Non plain weave fabric affords further flexibility for increasing fabric mass and fabric cover. As such they enlarge scope of the fabric designer.

Soft computing can successfully provide a platform to manoeuvre crimp in warp and weft over a wide range with only three fabric parameters; yarn tex, modular length of warp and modular length of weft yarn. This has enabled solutions by interaction of crimp interchange and crimp balance equations. This exercise offers several solutions for fabric engineering by varying the above three parameters.

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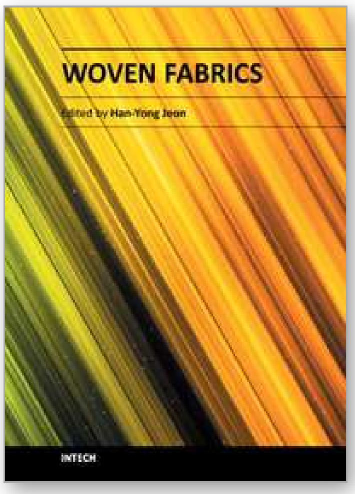

Woven Fabrics

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"Woven Fabrics" is a unique book which covers topics from traditional to advanced fabrics widely used in IT, NT, BT, ET, ST industry fields. In general, woven fabrics are known as the traditional textile fabrics for apparel manufacturing and are used widely in various fabric compositions as intermediate goods that affect human activities. The relative importance of woven fabrics as traditional textile materials is extremely large and currently application fields of woven fabrics as technical textiles are rapidly expanded by utilizing its geometric features and advantages. For example, the book covers analytical approaches to fabric design, micro and nano technology needed to make woven fabrics, as well as the concept for industrial application.

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