We are IntechOpen,
the world's leading publisher of
Open Access books
Built by scientists, for scientists

6,900
Open access books available

185,000
International authors and editors

200M
Downloads

154
Countries delivered to

Our authors are among the

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

BOOK CITATION INDEX
CLARIVATE ANALYTICS
INDEXED

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Performance Parameters Measurement and Stochastic Processes

Miroslav Svítek
Czech Technical University in Prague
Faculty of Transportation Sciences, Prague
Czech Republic

1. Introduction

In this chapter, the emphasis is on the analysis of performance parameters, assigned to different system components, functions or processes, together with the methodology of their statistical assessment based on limited measured samples, fulfilling the condition of normal distribution. The practical examples of GNSS (Global Navigation Satellite Systems) applications assessment and certification are presented.

2. Definition of performance parameters

The methodology for the definition and measurement of following individual system parameters is being developed within the frame of complex system's assessment. The basic performance parameters can be defined as follows:

- **Accuracy** is the degree of conformance between system true parameters and its measured values that can be defined as the probability

$$P\left(\left|p_i - p_{m,i}\right| \le \varepsilon_1\right) \ge \gamma_1 \tag{1}$$

  that the difference between the required system parameter p_i and the measured parameter $p_{m,i}$ will not exceed the value ε_1 on probability level γ_1 where this definition is applicable for all N system parameters $p_1, p_2, ..., p_N$.

- **Reliability** is the ability to perform a required function (process) under given conditions for a given time interval that can be defined as the probability

$$P\left(\left|\vec{v}_t - \vec{v}_{m,t}\right| \le \varepsilon_2\right) \ge \gamma_2, t \in \langle 0, T \rangle \tag{2}$$

  that the difference between required system functions (processes) represented by parameters $\vec{v}_t$ and the vector of measured parameters $\vec{v}_{m,t}$ will not exceed the value ε_2 on probability level γ_2 in each time interval *t* from the interval $\langle 0, T \rangle$.

- **Availability** is the ability to perform required functions (processes) at the initialization (triggering) of the intended operation that can be defined as the probability

$$P\left(\left|q_i - q_{m,i}\right| \le \varepsilon_3\right) \ge \gamma_3 \tag{3}$$

that the difference between the required rate[1] of successful performing of the function i (process i) q_i and the measured $q_{m,i}$ will not exceed the value ε_3 at the probability level γ_3.

- **Continuity** is the ability to perform required functions (processes) without non-scheduled interruption during the intended operation that can be defined as the probability

$$P\left(\left|r_i - r_{m,i}\right| \leq \varepsilon_4\right) \geq \gamma_4 \qquad (4)$$

that the difference between the required rate of successful performing of the function i (process i) without interruption r_i and the measured $r_{m,i}$ will not exceed the value ε_4 at the probability level γ_4.

- **Integrity** is the ability to provide timely and valid alerts to the user, when a system must not be used for the intended operation, that can be defined as the probability

$$P\left(\left|S_i - S_{m,i}\right| \leq \varepsilon_5\right) \geq \gamma_5 \qquad (5)$$

that the difference between the required rate of successful performing of the alert limit (AL) i not later than predefined time to alert (TTA) S_i and the measured $S_{m,i}$ will not exceed the value ε_5 on the probability level γ_5.

- **Safety** can also be covered among the performance parameters, but the risk analysis and the risk classification must be done beforehand with a knowledge of the system environment and potential risk, and then the safety can be defined as the probability

$$P\left(\left|W_i - W_{m,i}\right| \leq \varepsilon_6\right) \geq \gamma_6 \qquad (6)$$

that the difference between the required rate of i risk situations W_i and the measured ones $W_{m,i}$ will not exceed the value ε_6 on the probability level γ_6.

A substantial part of the system parameters analysis is represented by a decomposition of system parameters into individual sub-systems of the telematic chain. One part of the analysis is the establishment of requirements on individual functions and information linkage so that the whole telematic chain can comply with the above defined system parameters.

The completed decomposition of system parameters will enable the development of a methodology for a follow-up analysis of telematic chains according to various criteria (optimisation of the information transfer between a mobile unit and a processing centre, maximum use of the existing information and telecommunication infrastructure, etc.).

The following communication performance parameters quantify the quality of telecommunication service [16]:

---

[1] $q_{m,i} = \dfrac{Q_i}{Q}$ where Q_i is the number of successful experiments (successful performing of the function i, successful performing of the process i) and Q is the number of all experiments (both successful and unsuccessful).

- **Availability** – (i) Service Activation Time, (ii) Mean Time to Restore (MTTR), (iii) Mean Time between Failure (MTBF) and (iv) Virtual Connection Availability
- **Delay** - is an accumulative parameter effected by (i) Interfaces Rates, (ii) Frame Size, and (iii) Load / Congestion of all active nodes (switches) in the line
- **Packet/Frames Loss**
- **Security**

Performance indicators described for communications applications must be transformed into telematic performance indicators structure, and vice versa. Such transformation allows for a system synthesis.

Transformation matrix construction is dependent on detailed communication solution and its integration into telematic system. Probability of each phenomena appearance in the context of other processes is not deeply evaluated in the introductory period. Each telematic element is consequently evaluated in several steps, based on a detailed analysis of the particular telematic and communications configuration and its appearance probability in the context of the whole system performance. This approach represents a subsequent iterative process, managed with the goal of reaching the stage where all minor indicators (relations) are eliminated, and the major indicators are identified under the condition that relevant telematic performance indicators are kept within a given tolerance range.

3. Quality of measured performance parameters

In this chapter, unified approach applicable for all above mentioned performance parameters [18] will be introduced.

- **Absolute measuring error (μ_a)** is the difference between a measured value and the real value or the accepted reference

$$\mu_a = x_d - x_s \tag{7}$$

x_d - measured dynamic value

x_s - corresponding real value or accepted reference

- **Relative measuring error (μ_r)** is the absolute measuring error divided by a true value given by

$$\mu_r = \frac{x_d - x_s}{x_s} \tag{8}$$

- **Accuracy (δ) of a measuring system** is the range around the real value in which the actual measured value must lie. The measurement system is said to have accuracy δ if:

$$x_s - \delta \leq x_d \leq x_s + \delta \tag{9}$$

or straightforwardly:

$$-\delta \leq \mu_a \leq +\delta \tag{10}$$

Accuracy is often expressed as a relative value in $\pm\ \delta$ %.

- **Reliability (1-α) of a measuring system** is the minimal probability of a chance that a measuring error μ_a lies within the accuracy interval $[-\delta, \delta]$:

$$(1-\alpha) \leq P(|\mu_a| \leq \delta) \tag{11}$$

where P(.) means the probability value.

- **Error probability (α) of a measuring system** is the probability that a measured value lies further from the actual value than the accuracy:

$$\alpha \geq P(|\mu_a| > \delta) \tag{12}$$

The reliability of measuring system is often controlled by the end-user of the measurement system while error probability is generally assessed by the International Organization for Legal Metrology (OIML).

- **Dependability (β) of an acceptance test** is the probability that - on the basis of the sample - a correct judgment is given on the accuracy and reliability of the tested system:

$$P(\alpha \leq P(-\delta < \mu_a < \delta)) \geq \beta \tag{13}$$

The desired dependability determines the size of the sample; the higher the sample, the higher the dependability of the judgment.

4. Estimation of performance parameters

4.1 Tests of normality

With regard to [12] normal distribution will be expected, because using different kinds of statistics, such as order statistics (distribution independent) for small sample sizes, typical for performance parameters, the result may be fairly imprecise. Testing normality is important in the performance parameters procedure, because in analyses containing a lot of data this data is required to be at least approximately normally distributed. Furthermore, the confidence limits assessment requires the assumption of normality. Several kinds of normality tests are available, such as [1]:

- Pearson test (Chi-Square Goodness-of-Fit Test)
- Kolmogorov-Smirnov test
- Anderson-Darling and Cramer-von Mises test

All the above mentioned tests for normality are based on the *empirical distribution function* (EDF) and are often referred to as *EDF tests*. The empirical distribution function is defined for a set of *n* independent observations $X_1, X_2, ..., X_n$ with a common distribution function F(x). Under the null hypothesis, F(x) is the normal distribution. Denote the observations ordered from the smallest to the largest as $X_{(1)}, X_{(2)}, .., X_{(n)}$. The empirical distribution function, $F_n(x)$, is defined as

$$F_0(x) = 0, \quad x < X_{(1)}$$
$$F_i(x) = \frac{i}{n}, \quad X_{(i)} \leq x < X_{(i+1)}, \quad i = 1, ..., \text{n-1} \tag{14}$$
$$F_n(x) = 1, \quad X_{(n)} \leq x$$

Note that $F_n(x)$ is a step function that takes a step of height *1/n* at each observation. This function estimates the distribution function F(x). At any value x, $F_n(x)$ is the proportion of observations less than or equal to x, while F(x) is the probability of an observation less than or equal to x. *EDF statistics* measure the discrepancy between $F_n(x)$ and F(x).

In the following part the *Pearson test* (Chi-Square Goodness-of-Fit Test) will be introduced as a practical example of EDF tests. The chi-square goodness-of-fit statistic χ_q^2 for a fitted parametric distribution is computed as follows:

$$\chi_q^2 = \sum_{i=1}^{L} \frac{(m_i - n \cdot p_i)^2}{n \cdot p_i} \tag{15}$$

where L is the number of histogram intervals, m_i is the observed percentage in *i*-th histogram interval, *n* is the number of observations, p_i is the probability of *i*-th histogram interval computed by means of theoretical distribution. The degree of freedom for the chi-square test χ^2 is equal to L-r-1, where r is parameters number of theoretical distribution (in case of normal distribution r=2).

4.2 Estimation of measuring system's accuracy, reliability and dependability

Let us assume we have a normally distributed set of *n* measurements of performance parameters $\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n}$ (absolute error between prescribed and measured parameters as defined in (7)).

If the mean value or a standard deviation is not known we can estimate both the mean value $\bar{\mu}_a$ and standard deviation s_a from the measured data as follows:

$$\bar{\mu}_a = \frac{1}{n} \sum_{i=1}^{n} \mu_{a,i}$$

$$s_a = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(\mu_{a,i} - \bar{\mu}_a \right)^2} \tag{16}$$

Let *n* be non-negative integer, α, β are given real numbers $(0 < \alpha, \beta < 1)$ and let $\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n}, \mu_{a,y}$ be *n+1* independent identically distributed random variables.

Tolerance limits $L = L(\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n})$ and $U = U(\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n})$ are defined as values so that the probability is equal to β that the limits include at least a proportion $(1 - \alpha)$ of the population. It means that such limits L and U satisfy:

$$P\left\{ P\left(L < \mu_{a,y} < U \right) \geq 1 - \alpha \right\} = \beta \tag{17}$$

A *confidence interval* covers population parameters with a stated confidence. The *tolerance interval* covers a fixed proportion of the population with a stated confidence. *Confidence limits* are limits within which we expect a given population parameter, such as the mean, to lie. *Statistical tolerance limits* are limits which we expect a stated proportion of the population to lie within.

For the purpose of this chapter we will present only results derived under the following assumptions:

- $\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n}, \mu_{a,y}$ are *n+1* independent normally distributed random variables with the same mean μ_0 and variance σ_0^2 (equivalently $\mu_{a,1}, \mu_{a,2}, ..., \mu_{a,n}, \mu_{a,y}$ is a random sample of size *n+1* from the normal distribution with mean μ_0 and variance σ_0^2).
- The symmetry about the mean or its estimation is required.
- The tolerance limits are restricted to the simple form $\overline{\mu}_a - k \cdot s_a$ and $\overline{\mu}_a + k \cdot s_a$, where *k* is a so called *tolerance factor*, $\overline{\mu}_a$ and s_a are *sample mean and sample standard deviations*, respectively, given by (16).

Under the above given assumptions, the condition (17) can be rewritten as follows:

$$P\left\{\Phi\left(\frac{U-\mu_0}{\sigma_0}\right) - \Phi\left(\frac{L-\mu_0}{\sigma_0}\right) \geq 1 - \alpha\right\} = \beta \qquad (18)$$

where Φ is the distribution function of the normal distribution with mean zero and standard deviation equal to one:

$$\Phi(u) = \frac{1}{\sqrt{2 \cdot \pi}} \int_{-\infty}^{u} e^{-\frac{1}{2} \cdot t^2} dt \qquad (19)$$

The solution of the problem to construct tolerance limits depend on the level of knowledge of the normal distribution, i.e., on the level of knowledge of mean deviation $\overline{\mu}_a$ and standard deviation s_a.

In the following part the *accuracy, reliability and dependability of the measuring system* will be mathematically derived for a known mean value and standard deviation, for a known mean value and unknown standard deviation, and for both an unknown mean value and standard deviation.

Known mean value and standard deviation

We can start with the equation [3]:

$$P\left\{P\left[\mu_0 - z_{(1-\alpha/2)} \cdot \sigma_0 \leq \mu_{a,y} \leq \mu_0 + z_{(1-\alpha/2)} \cdot \sigma_0\right] \geq (1-\alpha)\right\} = 1 \qquad (20)$$

where $\mu_{a,y}$ is the measured value, μ_0, σ_0 are known mean value and standard deviation and $z_{(1-\alpha/2)}$ is a percentile of normal distribution (e.g. for $\alpha = 0.05$ we can find in statistical table $z_{0.975} = 1.96$).

Based on (20) we can decide that measuring system's accuracy $\delta = z_{(1-\alpha/2)} \cdot \sigma_0$ is guaranteed with measuring system's reliability $(1-\alpha)$. Because the mean value and standard deviation are known, the measuring system's dependability is equal to $\beta = 1$.

Known standard deviation and unknown mean value

Now we expect that the mean value is estimated according to (16). Then we can write the equation [3]:

$$P\left\{P\left[\bar{\mu}_a - k \cdot \sigma_0 \leq \mu_{a,y} \leq \bar{\mu}_a + k \cdot \sigma_0\right] \geq (1-\alpha)\right\} = \beta \qquad (21)$$

where σ_0^2 is the known variance and k is computed from the following equation:

$$\Phi\left(\frac{z_{(1+\beta)/2}}{\sqrt{n}} + k\right) - \Phi\left(\frac{z_{(1+\beta)/2}}{\sqrt{n}} - k\right) = 1-\alpha \qquad (22)$$

where the function $\Phi(u)$ was defined in (19) and sample $\bar{\mu}_a$ computed according to (16).

Based on the equation (22) we can say that for the predefined values of measuring system's reliability $(1-\alpha)$ and dependability β and the number of measurements n the accuracy of measuring system will be

$$\delta = \left(z_{1-\frac{\alpha}{2}} + \frac{1}{\sqrt{n}} \cdot z_{\frac{(1+\beta)}{2}}\right) \cdot \sigma_0 \qquad (23)$$

Known mean value and unknown standard deviation

For a known mean value and unknown standard deviation we can write the equation:

$$P\left\{P\left[\mu_0 - \left(z_{(1-\alpha/2)} \cdot \left(\frac{n}{\chi_{(1-\beta)}^2(n)}\right)^{\frac{1}{2}}\right) \cdot s_a \leq \mu_{a,y} \leq \mu_0 + \left(z_{(1-\alpha/2)} \cdot \left(\frac{n}{\chi_{(1-\beta)}^2(n)}\right)^{\frac{1}{2}}\right) \cdot s_a\right] \geq (1-\alpha)\right\} = \beta \quad (24)$$

where s_a is estimated according to (16), $\chi_{(1-\beta)}^2(n)$ means chi-quadrate distribution with n degree of freedom.

Based on the equation (24) we can say that for predefined values of measuring system's reliability $(1-\alpha)$ and dependability β and the number of measurements n the accuracy of measuring system will be:

$$\delta = \left(z_{(1-\alpha/2)} \cdot \left(\frac{n}{\chi_{(1-\beta)}^2(n)}\right)^{\frac{1}{2}}\right) \cdot s_a \qquad (25)$$

Unknown mean value and standard deviation

This variant is the most important in many practical cases, but the solution is theoretically very difficult. However, a lot of approximation forms exist based on which the practical simulation could be feasible.

We start by the task description

$$P\left\{P\left[\bar{\mu}_a - k \cdot s_a \leq \mu_{a,y} \leq \bar{\mu}_a + k \cdot s_a\right] \geq (1-\alpha)\right\} = \beta \tag{26}$$

where the sample mean value $\bar{\mu}_a$ and sample standard deviation s_a are estimated from n samples according to (16).

Howe [4] defines a very simple approximation form for *k*:

$$k \approx \left(\frac{n+1}{n}\right)^{\frac{1}{2}} \cdot z_{\left(1-\frac{\alpha}{2}\right)} \cdot \left(\frac{n-1}{\chi_{(1-\beta)}^2(n-1)}\right)^{\frac{1}{2}} \tag{27}$$

Bowker [5] defines:

$$k \approx z_{\left(1-\frac{\alpha}{2}\right)} \cdot \left[1 + \frac{z_\beta}{\sqrt{2n}} + \frac{5 \cdot z_\beta^2 + 10}{12n}\right] \tag{28}$$

Ghosh [6] defines the next approximation form:

$$k \approx z_{\left(1-\frac{\alpha}{2}\right)} \cdot \left(\frac{n}{\chi_{(1-\beta)}^2(n-1)}\right)^{\frac{1}{2}} \tag{29}$$

If we take the approximation forms for z_x for x>0.5 [2][2]:

$$z_x = u_x - \frac{2.30753 + 0.27061 \cdot u_x}{1 + 0.99229 \cdot u_x + 0.04481 \cdot u_x^2}$$

$$u_x = \left[\ln(1-x)^{-2}\right]^{\frac{1}{2}} \tag{30}$$

and for $\chi_x^2(\gamma)$ [3][3] (the number of degree of freedom is usually $\gamma = n-1$):

$$\chi_x^{'2}(\gamma) = \gamma + z_x \cdot \sqrt{2} \cdot \gamma^{\frac{1}{2}} + \frac{2}{3} \cdot \left(z_x^2 - 1\right) + \frac{1}{9\sqrt{2}}\left(z_x^3 - 7 \cdot z_x\right) \cdot \gamma^{-\frac{1}{2}} - \frac{1}{405}\left(6 \cdot z_x^4 + 14 \cdot z_x^2 - 32\right) \cdot \gamma^{-1} +$$

$$+ \frac{1}{4860\sqrt{2}} \cdot \left(9 \cdot z_x^5 + 256 \cdot z_x^3 - 433 \cdot z_x\right) \cdot \gamma^{-\frac{3}{2}} \tag{31}$$

or a much simpler approximation form from [3]:

$$\chi_x^{'2}(\gamma) = \frac{1}{2} \cdot \left[z_x + \left(2 \cdot \gamma - 1\right)^{\frac{1}{2}}\right]^2 \tag{32}$$

---

[2] The approximation error is not greater than 0.003

[3] For $x \in \langle 0.01, 0.99 \rangle$ and $\gamma \geq 20$ the absolute error of approximation is not greater than 0.001

then the analytical equation for the estimation of measuring system's accuracy δ based on an estimated mean value and standard deviation of n-sample data with the predefined measuring system's reliability $(1-\alpha)$ and measuring system's dependability β can be computed.

4.3 Illustrative example 1 - Simulation result

A very important question can be addressed with regard to accuracy: How the measuring system's accuracy δ depends on the number of measurements for the prescribed measuring system's reliability and measuring system's dependability? We consider the following prescribed values for α, β :

a. $\alpha = 0.3, \beta = 0.5$
b. $\alpha = 0.05, \beta = 0.99$

From (26) the measuring system's accuracy δ is given $\delta = k \cdot s_a$. For finding the parameter k the equations (30), (31) and (32) were used. The Fig. 1 and Fig. 2 show the dependence of the parameter k on the number of measurements n for cases a) and b) respectively.

4.4 Illustrative example 2 - Simulation result

Can the results from example 1 be proved by a simulation, if the number of measured values is n=30?

For both cases a) $\alpha = 0.3, \beta = 0.5$ and b) $\alpha = 0.05, \beta = 0.99$ the values of k were found in Fig. 1 and Fig. 2 respectively:

a.   k=1.0584,
b.   k=2.797.

In MATLAB, a set of 1000 samples of normal distribution with zero mean and standard deviations equal to one was generated. From the first 30 samples the mean value and standard deviation were estimated (16). Then the interval $(\overline{\mu}_a - k \cdot s_a, \overline{\mu}_a + k \cdot s_a)$ was selected in accordance to (26) and the probability of falling into interval was computed from the whole set of 1000 samples - which is a procedure of how to compute measuring system's reliability.

The above mentioned procedure was repeated 5 000 times, and the probability of exceeding the predefined measuring system's reliability limit was computed - thus the test of measuring system's dependability was performed.

The obtained results for measuring system's dependability through simulation will be summarized as follows:

a. $\beta = 0.553$
b. $\beta = 0.9864$

The test was repeated many times and the predefined parameters for measuring system's reliability and dependability were achieved in all experiments.
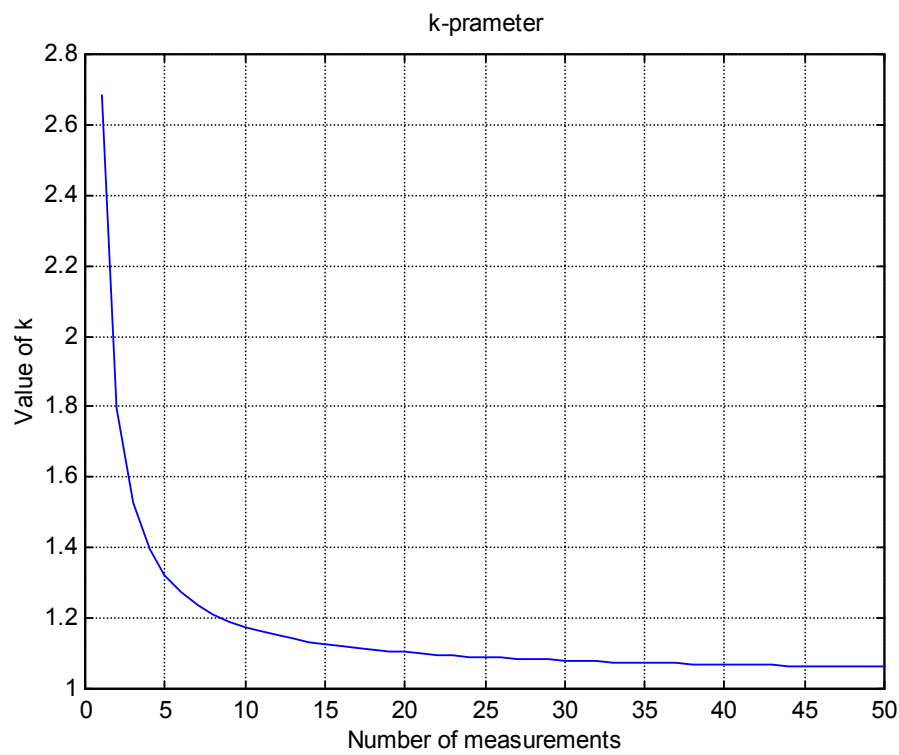
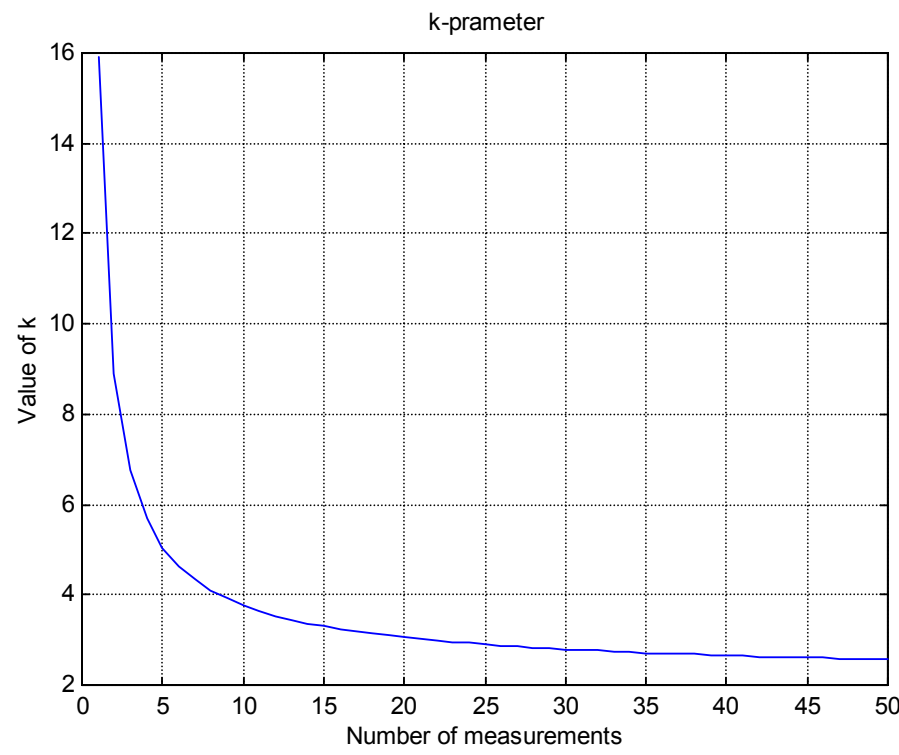Fig. 1. Dependence of parameter k on number of measurements $(\alpha = 0.3, \beta = 0.5)$

Fig. 2. Dependence of parameter k on number of measurements $(\alpha = 0.05, \beta = 0.99)$

5. Assessment of safety performance parameters

We can suppose N sensors data available where the probability of right error detection is marked as P_{RD} and the probability of non-correct error detection as P_{FD}. Because of the enormous safety and economical impact in case of non-correct error alert, the method of filtering "*M from N*" will be presented.

Let us have N sensors and for simplicity let us suppose the same probabilities of correct P_{RD} and non-correct error detection P_{FD} on each sensor. If this assumption is not fulfilled the method can be easily extended to a more general case.

As mentioned above, the hypothesis H_0 represents perfect system behavior (non system error, no sensor error) and hypothesis H_1 as a state with detected error (error of system, or error of sensors).

In the next equation, the probability of error detection on *k* sensors of *N* sensors (*N-k* sensors do not detect errors) is given in case the system does not display any error (conditioned by hypothesis H_0):

$$P[k \mid H_0] = \binom{N}{k} \cdot P_{FD}^k \cdot \left(1 - P_{FD}\right)^{N-k} \tag{33}$$

In the same way, the probability of error detection by *k* of *N* sensors is given in case the system is in an error state (conditioned by hypothesis H_1):

$$P[k \mid H_1] = \binom{N}{k} \cdot P_{RD}^k \cdot \left(1 - P_{RD}\right)^{N-k} \tag{34}$$

The main idea of "M from N" filtering is in selection of value M (threshold) defining the minimum number of sensors that detected error. If M sensors detect error then this error is taken as the real system error and the system starts sending error alert signals. The threshold M should be selected with respect to the following probabilities:

$$\begin{aligned} P_F &= \sum_{k=M}^{N} \binom{N}{k} \cdot P_{RD}^k \cdot \left(1 - P_{RD}\right)^{N-k} \\ P_D &= \sum_{k=M}^{N} \binom{N}{k} \cdot P_{FD}^k \cdot \left(1 - P_{FD}\right)^{N-k} \end{aligned} \tag{35}$$

where P_F, P_D means probability of a *false alert* (an error is detected but the system works without any errors) and the probability of the *right detection* (the system error is correctly detected).

The number of detectors N and the threshold M can be chosen based on sensors parameters P_{RD}, P_{FD} and required probabilities P_F, P_D.

Methods of data fusion and comparison are the main tools for estimation of system performance parameters (accuracy, reliability, integrity, continuity, etc.) and can be used for a derivation of an exact definition of false alert and right detection probabilities.

5.1 Illustrative example - Geo-object detection

In this example the measurement data comparison will be used as a tool for better geo-object detection in, e.g., electronic tolling application.

We can suppose N available position measurements of a geo-object where the probability of the right geo-object detection is marked as P_{RD} and the probability of non-correct (false) geo-object detection as P_{FD}. Let the hypothesis H_0 represent the assumption of a perfect geo-object detection (no detection error reported). The hypothesis H_1 represents a non-correct geo-object detection (error caused, for example, by wrong position accuracy, etc.).

The probability of *k* non-correct geo-object detections of *N* measurements for the final assumption that the geo-object is perfectly detected (conditioned on the hypothesis H_0) can be given:

$$P[k \mid H_0] = \binom{N}{k} \cdot P_{FD}^k \cdot \left(1 - P_{FD}\right)^{N-k} \tag{36}$$

The probability of *k* correct geo-object detections of N measurements for the final assumption of non-correct geo-object detection (conditioned on hypothesis H_1) is given:

$$P[k \mid H_1] = \binom{N}{k} \cdot P_{RD}^k \cdot \left(1 - P_{RD}\right)^{N-k} \tag{37}$$

The main idea of "M matches from N measurements" principle is in the selection of the threshold M with respect to the following probabilities:

$$
\begin{aligned}
P_F &= \sum_{k=M}^{N} \binom{N}{k} \cdot P_{FD}^k \cdot \left(1 - P_{FD}\right)^{N-k} \\
P_D &= \sum_{k=M}^{N} \binom{N}{k} \cdot P_{RD}^k \cdot \left(1 - P_{RD}\right)^{N-k}
\end{aligned}
\tag{38}
$$

where P_F, P_D means the probability of a *false alert of geo-object detection* (the geo-object is detected even though the vehicle did not go through it) and the probability of a *right geo-object detection* (the right geo-object is detected based on the measured data, and the vehicle went through it).

The number of measurements N and the threshold M can be chosen based on the position probabilities P_{FD}, P_{RD} and the required probabilities P_F, P_D. Further discussion will be presented within an illustrative example below.

There are two parallel roads (one under tolling, the other one free of charge) and the distance D between them of 20 meters, as it is shown in Fig 3. The length L is supposed to be 1 kilometer.

In this example, we will try to tune the parameter *M* to increase the probability of the correct toll road detection in order to reach the expected value of more than 99%.

We expect a maximum vehicle speed of 200 km/h or 55 m/s. If the length is 1000 m and the GPS receiver monitors the position every second, we can obtain as many as 18 position measurements per one road. The road can be distinguished by GPS received with probability app. 70% (we can assign the measurement to the right road, if the error is lower than D/2 which is in our case 10 meter - this accuracy is typically achieved by a GPS receiver at a probability level of 70%).
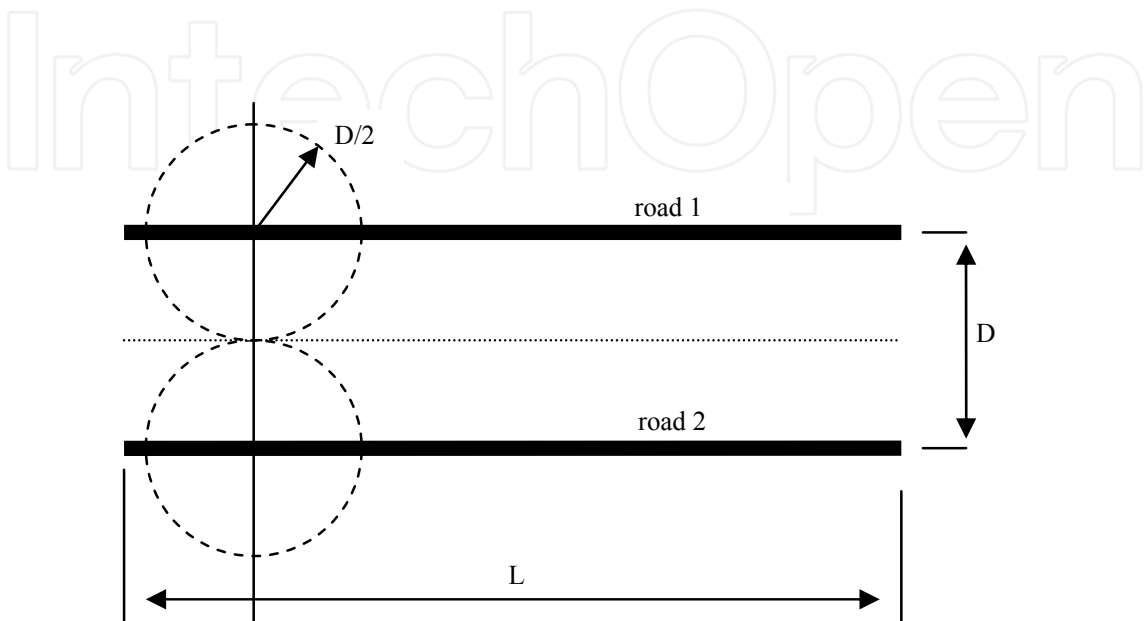

Fig. 3. Two parallel roads and Toll detection being on one of the roads

Based on the above mentioned assumptions, we can summarize the following parameters:

$$P_{RD} = 0.7, \ P_{FD} = 0.3, \ N = 18$$

Using the equations (38), the probabilities P_F, P_D for different parameters M will be as given in Tab.1.

Parameter M	P_D	P_F
6 from 18	0.9997	0.4656
8 from 18	0.9939	0.1407
10 from 18	0.9404	0.0210
12 from 18	0.7217	0.0014

Table 1. Probabilities P_F, P_D and their dependence on parameter M

If the parameter M is 6 or 8, we can achieve the requested probability of the geo-object detection higher than 99%. On the other hand, for M=6 the probability of lost vehicles is higher (the vehicles used the toll road, but the system did not detect them). For M=8 we can achieve a better balance between both probabilities P_F, P_D. If the user needs to minimize the loss of a vehicle and to keep the acceptable detection probability, the variant M=10 could be a good compromise.

6. Performance cluster of telematic applications using GNSS

Transport telematics architecture [22] displays the arrangement of subsystems and functional blocks, including information relationships according to the defined point of view. The task also covers the selection of representative telematics applications ("*cluster*") that shows identical systems requirements.

Among the individual representative applications using GNSS (Global Navigation Satellite Systems) the following may be included:

- Securing the movement of a means of transport in a transport infrastructure (from the point of view of performance parameters within the GNSS, it is a question of securing accuracy, reliability, availability, integrity, etc., at exactly defined points of the transport infrastructure – the application lays high stress both on the locator proper and the information transmission and processing systems; the solution should comply with the "fail-safe" principle; for typical transport telematics applications we may refer to railway interlocking technology, monitoring the transport of dangerous goods, or monitoring the movement of means of transport at the airport.

- Navigation of the means of transport in a transport network (from the point of view of performance parameters, it is a matter of coverage with a signal, time lag in on-line navigation, requirements as for the exactly working maps of an entire geographical area, requirements on the speed of information processing, both within a mobile unit and the processing centre, as well as minimisation of the delay when establishing the position – TTFF - Time to Fix Face); as typical transport telematics applications, the following may be referred to: the navigation of safety and rescue units for a localised accident place or dynamic and/or on-line automobile navigation.

- Monitoring and operating the maintenance of transport networks (from the point of view of performance requirements, it is particularly a matter of an exact transport infrastructure information retrieval, interoperability of individual GIS (Geographical Information Systems) systems of various organisations dealing with maintenance, and achievement of high statistical accuracy in establishing position); as it concerns typical transport telematic applications, the following ones should be mentioned: mapping the river channel by means of a measuring ship, or measuring the carriageway parameters by means of special measuring vehicles.

- Monitoring the movement of persons and goods in a transport infrastructure (from the point of view of performance requirements, it is a matter of transmission and central processing of large amount of information from resources of various accuracy, fast identification of individual sub-sets of the objects of transport, sophisticated information processing in the centre, for instance, the "Floating Car Data"); as typical transport telematic applications, the following can be referred to: the use of taxi cabs, public transport passenger vehicles or other utility vehicles equipped with the GNSS systems for traffic flow modelling, or the use of localised mobile telephones for modelling the mobility of persons.

- Transport infrastructure charging according to its utilisation (from the point of view of performance parameters, it is a matter of reliability, integrity and time lag because the GNSS system is used to calculate the amount of the charge and, furthermore, the application places demands on the "fail-safe" principle in terms of the distance covered – if there is an uncertainty about correct charging of the driver, the distance covered is

not taken account of); as a typical transport telematic application, it is electronic charging of the transport infrastructure according to the vehicle parameters and distance covered.

As a follow-up to the completed analysis and decomposition of performance parameters to individual subsystems, a table can be obtained containing performance requirements of the above mentioned representatives as for the locator proper, telecommunications environment or the information processing centre.

6.1 Illustrative example - Assessment of telecommunications solution of GNSS monitoring system on airport surface

Described system methodology can be demonstrated on the telematic application based on GNSS (Global Navigation Satellite System) developed for the airport area moving objects management with good potential to be integrated into the already operated airport monitoring and management system.

The service central server collects and processes data received from all service vehicles. The obtained information is combined with the data gained from the existing systems. Processed and obtained result is distributed not the only to the airport management, but as well as to each vehicle equipped with active On-Board Unit (OBU) equipped with display of relevant size and quality. Each OBU receives also the managerial data generated by either airport control system or by dispatchers. Principal schema of the subsystem organization is displayed on Figure 4.
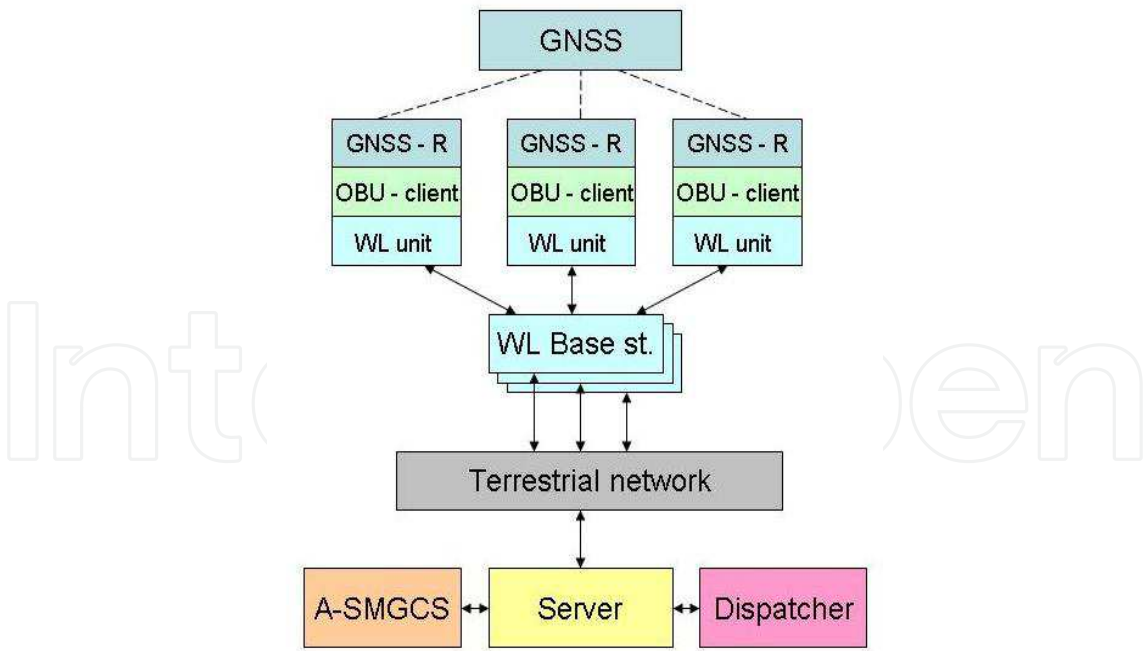

Fig. 4. Telematic service structure (WL – wireless, A-SMGCS - Advanced Surface Movement Guidance & Control System)

An airport area is precisely and transparently regulated area. The telematic sub-system performance indicators are introduced in Table 2.

Perform. Indicator	limit value	probability level	time interval
Accuracy	7.5m	99%	-
Availability	30s	99%	after init.
Reliability	36s	99%	3,600s
Continuity	5s	99%	180s
Integrity	5s	99%	-

Table 2. Required Telematics Performance Indicators in airport application

Using a transformation method described in [14] - [16] the telematic performance indicator "accuracy" was identified as the performance indicator with the dominant impact on the whole system performance. Its dominance is caused by the specific character of studied application. The requested level of accuracy (see Table 2) must be reached for every object moving with speed up to 120km/hour, If 1m GNSS sensor accuracy can be reached (the differential GNSS alternative must be applied) 195ms remain for the delay caused by all devices including the potential error healing in case of any sub-system problem (all on probability level 99% - see table 2).

Mobile WiMax (IEEE Std. 802.16d) was identified as the only possible alternative of the wireless access solution for the critical areas of the airport. All the other available access systems like GSM based products DTMF, HSCSD GPRS and EDGE as well as UMTS were identified as inappropriate. WiFi system operated in the open frequency band does not provide any service quality guarantee. Table 3 displays the obtained dynamical parameters of the WiMax channel (ART – Average Round Trip delay) in two critical stages of the Signal to Noise Ratio (SNR)

Site	Visibility	ART [ms]	SNR [db]
1	LOS	45.6	33
2	LOS	47.1	32
3	NLOS	44.6	-26
4	NLOS	44.8	-27

Table 3. Principle parameters of the WiMax access

Even though WiMax was selected as the core mobile access system for the airport critical areas, the whole airport area coverage with this technology is not economical. Some of alternative access solutions (EDGE/GPRS/UMTS or even WiFi) for Mobile WiMax difficult/irrelevant areas can be applied, if system parameters of these technologies meet these areas system parameters requirements. For such case multi-path solution like CALM, IEEE 802.21 based or by authors announced adaptive multi-path alternative (see e.g. in [20] - [21].) are identified as the appropriate alternatives.

The L2 ring based solution of the terrestrial chain part with the local QoS management (Hirschman) was applied to fulfill time limits of the whole chain.

The applied method nosily disqualified such communications "gurus" like the MPLS backbone terrestrial networking or GPRS/EDGE wireless access and combination of the wireless solution based on IEEE Std. 802.16d with terrestrial L2 switching solution with QoS management tools implemented were applied as core technology, they were tested and reasonable results were obtained. Critical issue, however, represent implementation of the effective decision processes to manage the multi-path solution to be kept in the required time limits – see [13] - [20].

7. Certification and testing of telematic applications

General methodology for telematic certification and testing of performance parameters [21] is figured on Fig. 5.
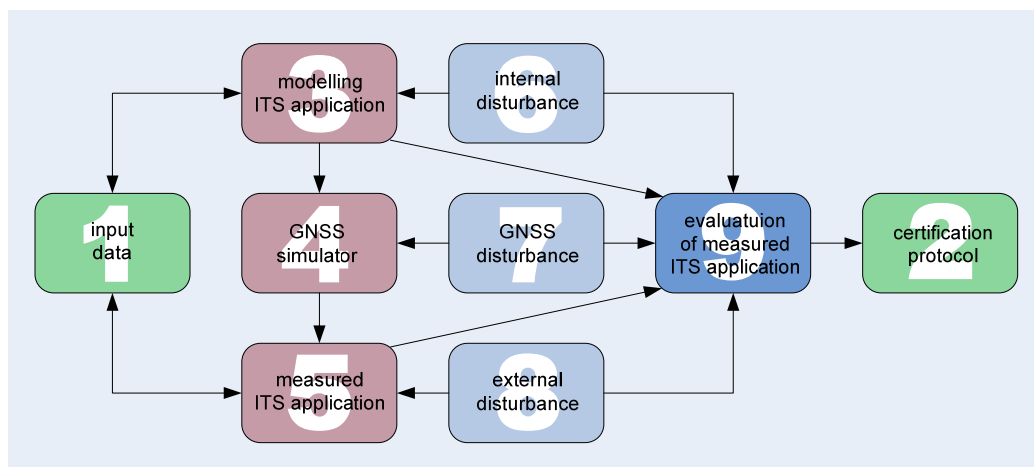
Fig. 5. General certification system for performance evaluation of telematic applications

Telematic application certification process consists of these steps:

1. definition of initial conditions for application (block 1);
2. optimally operating (modelling) telematic application definition (block 3);
3. disturbance statistics definition – internal disturbance of telematic application (block 6, disturbance of whole set of tested vehicles or OBUs), disturbance of GNSS signal (block 7) and external disturbance (block 8);
4. activation of measured (certified) telematic application (block 5);
5. testing of measured telematic application with simulated GNSS signal (block 4) for all defined situations/scenarios (initial conditions – block 1) with goal to cover all suitable situations;
6. real testing of selected (available) scenarios;
7. conformity assessment of output data from appropriate (tested) telematic application and output data from model (optimally operating) application for defined initial conditions and defined disturbances;
8. measurement results processing for certification protocol (protocol of measurement) and final assessment of telematic system parameters guarantee (block 2);
9. performance parameters guarantee must be statistically verified on a sufficient number of measurements to be able to guarantee monitored properties in defined statistical parameters.

7.1 Illustrative example - Pilot certification of telematic applications using GNSS

Pilot tests of GNSS signal reception conditions and their evaluation were carried out during September and November 2009 at e-Ident laboratory in Prague [21], the laboratory for electronic identification systems and communications. The signal was simulated with Spirent GNSS Simulator, type GSS 8000. For pilot testing the GPS positioning system was selected as a reference GNSS system.

The routes were generated by creating NMEA messages (a special file was created for each route), every case is located in Prague area and its nearby neighbourhood. NMEA log files of routes for pilot testing were recorded during September in real test rides in cars, with the GPS unit. This data was processed and adjusted according to requirements for pilot testing in laboratory.

For virtual gate passage tests the hardware equipment of universal telematic mobile unit as OBU (On-Board Unit) was used. Installed DEFT application (Dynavix, EFC, Fleet, Toll) was also developed on testing hardware device.

For testing were chosen 2 following testing sections. Both sections are situated in the west of Prague, where both directions of travel through the section are considered and prepared for testing (Fig.6):

- section of road II/605 around the bridge over the highway R1; to initialize the GNSS unit, previous section of road II/605 (in the direction from the city centre) was used; section transit speed between 50 and 70 kph;
- section of highway R5 near the Metropole shopping centre; to initialize the GNSS unit, previous sections of highways R1 and R5 were used; section transit speed between 80 and 100 kph;

Test sections have been recorded into the log file in GPS unit placed on the windscreen of testing vehicle. This data was then processed into various NMEA files (each represents one testing section and one direction of travel).

The NMEA records were cut and connected into series. Series of measurement consist of 10 experiments, 5 of them in one direction and 5 of them in the opposite direction, so that it can test the entire sequence, i. e. all 10 passages through the defined segment. Before each series testing, there was simulated passing of the initialization section (to get fix of GPS signal and calibrate the equipment).
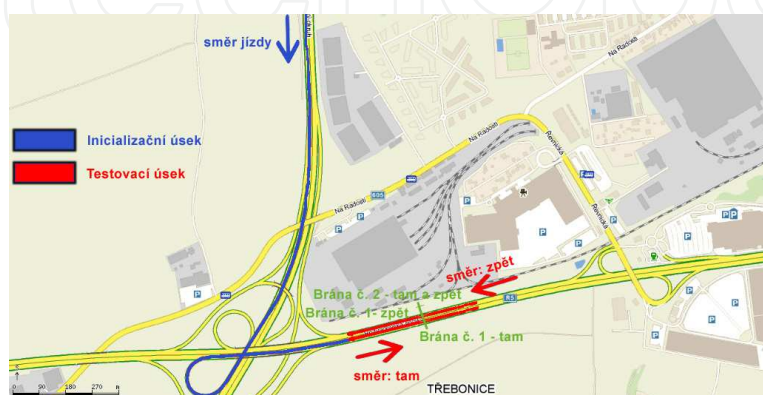

Fig. 6. Initialization (blue) and testing (red) section on highway R5, western part of Prague

Virtual gates locations were defined by geographical coordinates and the direction was determined by the starting azimuth of drive. For testing there was set azimuth angle of ±90° (from the road axis, 45° on each side).

Virtual gate passage was detected with software by the principle of measuring the distance from defined positions of the virtual gate in the outer circle of radius r_1, which takes the value of the distance travelled by a vehicle at time $t_1 = 2$ s, respectively, crossing the border of inside circle of radius r_2, which takes the value of the distance that the vehicle travels at time $t_2 = 1$ s (Fif.7).
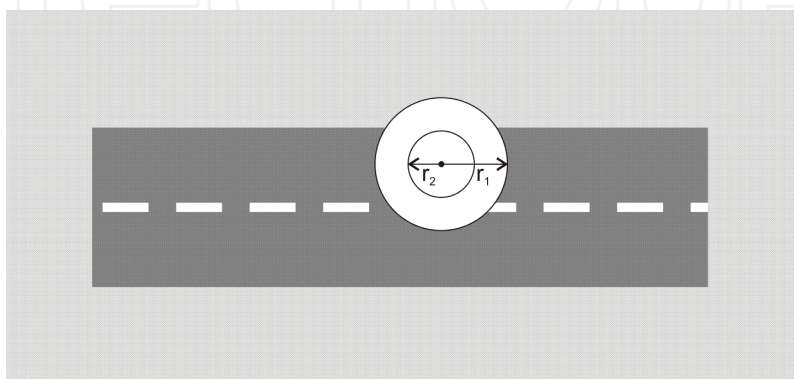
Fig. 7. Radius of detection circles

The geographical coordinates defining the location of the virtual gates were deducted from the publicly available maps on Internet, the azimuth was derived from the test drive data. Number of defined virtual gates was not restricted. For each section have always been defined 2 positions of virtual gate in each direction:

- the first on the passing road;
- the second in order of tens of meters from the passing road for the simulation of parallel road, where it is possible to expect the most common mistakes, so it is a gate on a fictitious road (for parallel road testing does not matter whether the road is fictitious or real).

For the section on road II/605 the location of the second gate (parallel communication) was defined at a distance of approx. 20 m from the axis of the road. In the case of section on highway R5 the virtual gate for parallel communication is located at a distance of approx. 30 m from the axis of one belt of the highway.

The virtual gate passage detection testing (with usage of GNSS simulator) was carried out at a total of 26 series of 10 measurements (Fig.8):

- 15 series for the section of road II/605, i. e. a total of 150 measurements,
- 11 series for the section of highway R5, i. e. a total of 110 measurements.

The tests were carried out at first with the default GNSS signal without any restrictions and with the defined parameters of the route. After that further test scenarios were developed and the simulated signal was influenced in the following way:

- changing in signal power from different satellites;
- turning off selected satellites;

- simulation of built scenes of surroundings – highway, city, suburbs – which contain a typical set of ground clutter, multipath signal transmission and signal shading;
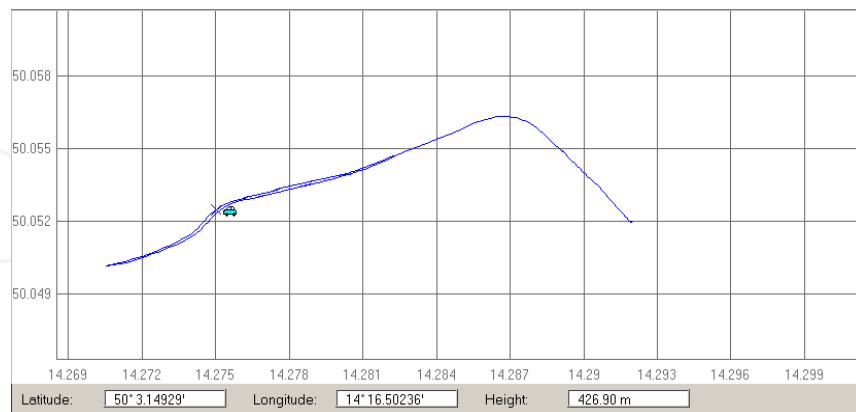- simulation of various predefined atmospheric changes.

Fig. 8. Simulation output of the position on road II/605

The simulated signal was influenced at first for whole measurement series, this approach was later changed to influencing the signal for various tests in the series, always another way, such as switching on and off the satellites at a defined angle above the vehicle, respectively above the ground (road plane). Signal influence has been prepared based on detailed description of possible signal influence. For the tests the spherical characteristics of the receiving antenna were chosen, with an open top of the antenna (i. e. position of the antenna on the roof of the vehicle).

Running of the tests has been recorded in the "log file" stored directly in the hardware unit. Based on this data further processing took place already on PC. The hardware unit recorded the following data of a virtual gate passage:

- virtual gate passage time;
- ID and a description of the gate (defined positions and azimuth).

Each measurement result was then classified as "passed" or "failed" according to the following categorization:

- **passed**, if all the following conditions are true:
  - gate is identified on the running road;
  - gate is not identified in other nearby road;
  - any upstream gate is not identified;
  - more passages through the same gate during one test are not evaluated;
- **failed** in other cases.

Results of carried out testing are listed in Tab.4 [22]. Based on the measurements, it was demonstrated, that the results of the passage identification varies for different parameters of the GNSS signal, environment and other influences on the signal reception. The relatively low percentage of successful running gate identification may be due to high sensitivity software in the OBU. It can be assumed that for usage of OBU for telematic applications the higher success rate for negative detection of upstream or neighbouring gates will be demanded.

Section and direction	Type of test	Passed in %	Failed in %
Road II/605 forward	Passed gate	60.0	40.0
	Upstream gate	100.0	0.0
	Nearby downstream gate	97.3	2.7
	Nearby upstream gate	98.7	1.3
Road II/605 backwards	Passed gate	50.7	49.3
	Upstream gate	98.7	1.3
	Nearby downstream gate	96.0	4.0
	Nearby upstream gate	100.0	0.0
Highway R5 forward	Passed gate	65.5	34.5
	Upstream gate	96.4	3.6
	Nearby downstream gate	100.0	0.0
	Nearby upstream gate	100.0	0.0
Highway R5 backwards	Passed gate	69.1	30.9
	Upstream gate	100.0	0.0
	Nearby downstream gate	100.0	0.0
	Nearby upstream gate	100.0	0.0
On the whole	Passed gate	60.4	39.6
	Upstream gate	98.8	1.2
	Nearby downstream gate	98.1	1.9
	Nearby upstream gate	99.6	0.4

Table 4. Results of pilot testing, depending on the segment, direction and type of test

Within implementation of a sufficiently large count of measurements the resulting values in Tab.4 show the probability of conformity of the tested OBU properties with the desired properties of the measured applications.

Proposed test protocol is divided into a part of recording the individual measurements and a part of evaluation of the application as a whole, including evaluation of system parameters. It is already possible on the basis of pilot testing to summarize the partial results and define partial requirements for selected system parameters of tested applications:

a. **Accuracy** – the required value of 15 meters in the horizontal plane at 95 % level of probability. Thus defined accuracy corresponds to usual accuracy requirements on OBU GPS unit using a standard statistical distribution of signal parameters. The tested OBU and its software configuration were selected based on the required accuracy. Accuracy of the OBU can be further increased.

b. **Reliability** – the specific desired value will be defined on basis of experience in the follow-up testing. Assuming a value of around 90 %, eventually higher, at 95 % probability level. After pilot tests result with 60.4 % success of virtual gate passage detection can be seen that the higher reliability will be required for the approval. To approve the usage of tested OBU it will be necessary to adjust the software in OBU or to use another OBU. Reliability is affected by the security parameter, see below.

c. **Availability** – the specific desired value will again be defined on the basis of experience in further testing. Assuming the OBU unit activation at start-up of travel and the availability value up to 60 seconds. Pilot testing showed in most cases the availability of tens of seconds.

d. **Continuity** – for telematics application this is not a critical system parameter. Depends on ability of the telematic system to assess and calculate the travelled route in case of system failure while driving the vehicle; this parameter does not depend only on the OBU and GNSS signal reception. In case of the usage of virtual sections or more virtual gates on the road sections the requirements on this parameter are significantly decreasing.

e. **Integrity** – this parameter has not been considered for the tested systems, mainly because it represents rather the quality of OBU diagnostics, resp. informing the user within a reasonable time for failure of proper OBU function. This parameter is not so much related to position determining or frequency of the virtual gates and sections passage records.

f. **Safety** – "dangerous conditions" are defined for both tested applications in the field of electronic toll system (EFC). This is a situation where the system assesses charges, which in fact did not occur. This is part of the conditions to test status "failed" – so-called "false alarms" – i. e. the identification of neighbouring gates or division, identification or evaluation of multiple transits the same gate or the same road section. With regard to safety (as one of the important system parameters of tested applications) appropriate testing scenarios were selected by pilot tests – all the scenes are containing unpleasant situation in a similar gate on nearby parallel road (i. e. identically oriented and situated at a distance of tens of meters from the passing road). It should be emphasized that effort to eliminate false alarms is associated with a reduction on the reliability parameter of applications, i. e. reducing the probability of passed gates/sections correct detection. It is therefore necessary to seek to balance, which on the one hand significantly reduces the risk of false positives, on the other hand, provides useful reliability parameter. This balanced condition can be found by testing various OBU units, resp. by testing one OBU at various software settings.

8. References

[1] Test of normality
    [http://www.caspur.it/risorse/softappl/doc/sas_docs/qc/chap1/sect19.htm]

[2] Novovicova J.: *Probability and mathematical statistics*, textbook, Faculty of transportation sciences, CTU Prague, 1999.

[3] Jilek M.: *Statistical confidence intervals*, Teoreticka kniznice inzenyra, SNTL, Prague, 1988 [in Czech].

[4] Howe W.G.:*Two-sided tolerance limits for normal populations*, JASA, 64, 1969.

[5] Bowker A.H.: *Computation of factors for tolerance limits on a normal distribution when sample is large*, AMS 17, 1946.

[6] Ghosh D.T.: A *note on computation of factor for tolerance limits for a normal distribution*, Sankhya B42, 1980.

[7] COST 323, *European Specification on Weigh-In-Motion of Road Vehicles.* Draft 3.0, EUCO-COST/323/x/99, June 1999.

[8] Jacob, B., *Assessment of the Accuracy and Classification of Weigh-In-Motion Systems*, Part 1 - Statistical Background. March 1997.

[9] Lieshout, R.A. van, Zoutenbier, M.H.H., *Weigh-In-Motion of Road Vehicles.* WIM-VID/IMP1, nr 6800.0770, E1655-01, CQM. May 1998.

[10] Lieshout, R.A. van, Zoutenbier, M.H.H., *Weigh-In-Motion of Road Vehicles.* WIM-VID/IMP1, nr 68000.0821- Studying Measurement Accuracy-, E1657-01, CQM. Sept 1999.

[11] Owen, D.B., *Handbook of Statistical Tables.* Reading, Mass, 1962.

[12] Stig Danielson: *Accuracy in WIM systems - An examination of different methods for determining precision*, Report, Linkoping University, Department of Mathematics Statistics

[13] M. Svitek,, T. Zelinka, T., Communications Solutions for ITS Telematic Subsystems, WSEAS Transactions on Business and Economics, Issue 4 (2006), Vol. 3, pp 361 – 367, Athens 2006,   ISSN 1109-9526,

[14] T. Zelinka, M. Svitek, Communication solution for Vehicles Navigation on the Airport territory, Proceedings of the 2007 IEEE Intelligent Vehicle Symposium, Istanbul, Turkey, pp 528–534, IEEE Catalogue number 07TH8947, ISBN 1-4244-1068-1.

[15] T. Zelinka, M. Svitek, Communication Scheme of Airport Over-ground Traffic Navigation System. ISCIT 2007. IEEE Sydney, 2007, pp 329 - 334. IEEE Catalogue No. 07EX1682(C), ISBN 1-4244-977-2, Library of Congress 2007920360.

[16] M. Svitek,, T. Zelinka, Monitoring of Transport Means on Airport Surface. Advances in Transport Systems Telematics, edited by Jerzy Mikulski, Selesian University of Technology, Katowice, pp. 285 – 292, ISBN 978-83-917156-6-6.

[17] T. Zelinka, M. Svitek, Decision processes in telematic multi-path communications access systems. International Journal of Communications, North Atlantic University Network NOUN, Issue 2, Volume 1, 2007, pp.11 – 16.

[18] M. Svitek,, T. Zelinka, Communications multi-path access decision scheme. Neural Network World, ICS AS CR and CTU, FTS, Praha, No. 6.,2008, pp 3 - 14, 2008, ISSN 1210 0552,

[19] T. Zelinka, M. Svitek, Adaptive communications solutions in complex transport telematics systems. Computers and Simulation in Modern Science- II, WSEAS Press, Athens 2009, pp. 234 -241, ISBN 978-960-474-032-1.

[20] T. Zelinka, M. Svitek, M. Srotyr, M. Vosatka: Adaptive Multi-path Telecommunications
      Solutions for ITS, Journal of Systemics, Cybernetics and Informatics Volume 9, No.
      1, pp. 14 – 20, Orlando, 2011, ISSN: 1690-4524.

[21] Svitek, M; Faltus, V; Lokaj, Z: Pilot Testing of Certification Methods Developed for ITS
      Applications Based on GNSS Systems, 13th International IEEE Conference on
      Intelligent Transportation Systems, Madeira, PT, 2010, ISBN 978-1-4244-7658-9,
      ISSN 2153-0009.

[22] M. Svítek: Intelligent Transport Systems - Architecture, Design methodology and
      Practical Implementation , [Key-note lesson], World Scientific and Engineering
      Academy and Society, 5th WSEAS/IASME Int. Conf. on Systems Theory and
      Scientific Computation, Malta, 2005.

Modern Metrology Concerns

Edited by Dr. Luigi Cocco

"What are the recent developments in the field of Metrology?" International leading experts answer this question providing both state of the art presentation and a road map to the future of measurement science. The book is organized in six sections according to the areas of expertise, namely: Introduction; Length, Distance and Surface; Voltage, Current and Frequency; Optics; Time and Relativity; Biology and Medicine. Theoretical basis and applications are explained in accurate and comprehensive manner, providing a valuable reference to researchers and professionals.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Miroslav Svítek (2012). Performance Parameters Measurement and Stochastic Processes, Modern Metrology Concerns, Dr. Luigi Cocco (Ed.), ISBN: 978-953-51-0584-8, InTech, Available from:
http://www.intechopen.com/books/modern-metrology-concerns/system-performance-parameters

INTECH
open science | open minds