We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900
Open access books available

185,000
International authors and editors

200M
Downloads

Our authors are among the

154
Countries delivered to

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



12

Hilbert Transform and Applications

Yi-Wen Liu
National Tsing Hua University
Taiwan

1. Introduction

Hilbert transform finds a companion function $y(t)$ for a real function $x(t)$ so that $z(t) = x(t) + iy(t)$ can be analytically extended from the real line $t \in \mathcal{R}$ to upper half of the complex plane.

In the field of signal processing, Hilbert transform can be computed in a few steps: First, calculate the Fourier transform of the given signal $x(t)$. Second, reject the negative frequencies. Finally, calculate the inverse Fourier transform, and the result will be a complex-valued signal where the real and the imaginary parts form a Hilbert-transform pair.

When $x(t)$ is narrow-banded, $|z(t)|$ can be regarded as a slow-varying envelope of $x(t)$ while the phase derivative $\partial_t[\tan^{-1}(y/x)]$ is an instantaneous frequency. Thus, Hilbert transform can be interpreted as a way to represent a narrow-band signal in terms of amplitude and frequency modulation. The transform is therefore useful for diverse purposes such as latency analysis in neuro-physiological signals (Recio-Spinoso et al., 2011; van Drongelen, 2007), design of bizarre stimuli for psychoacoustic experiments (Smith et al., 2002), speech data compression for communication (Potamianos & Maragos, 1994), regularization of convergence problems in multi-channel acoustic echo cancellation (Liu & Smith, 2002), and signal processing for auditory prostheses (Nie et al., 2006).

The rest of this review chapter is organized as follows: Sec. 2 reviews the mathematical definition of Hilbert transform and various ways to calculate it. Secs. 3 and 4 review applications of Hilbert transform in two major areas: Signal processing and system identification. The chapter concludes with remarks on the historical development of Hilbert transform in Sec. 6.

2. Mathematical foundations of Hilbert transform

The desire to construct the Hilbert transform stemmed from this simple quest: Given a real-valued function $f : \mathcal{R} \to \mathcal{R}$, can we find an imaginary part ig such that $f_c = f + ig$ can be analytically extended? For example, if $f(x) = \cos(x)$, then by inspection we can find $g(x) = \sin(x)$ such that $f_c(x) = f + ig = \exp(ix)$. This function can obviously be extended analytically to the entire complex plane by replacing the real variable x with the complex variable z in the expression; the result is $f_{\text{ext}}(z) = \exp(iz)$ and we have

$$\text{Re}\{f_{\text{ext}}(z)\}|_{z=x} = f(x), \tag{1}$$

which states that real part of the extended function is equal to the original given function $f(x)$ on the real line. The companion function $g(x)$ is called the Hilbert transform of $f(x)$.

This simple example brings forth a few questions. First, is analytic extension always possible for reasonably smooth $f(x)$? If so, is the companion function $g(x)$ unique, in the sense that Eq. (1) would hold? The answer to the second question is a definite 'No', because any $f_c(x) = f(x) + i\{g(x) + g_0\}$ also satisfies Eq. (1), where g_0 is an arbitrary constant. Therefore, the original simple quest needs to be refined as follows.

2.1 Hilbert transform as a boundary-value problem

To establish the uniqueness of the companion function, we first note that any analytic function $f_{\text{ext}}(z) = f_R(z) + if_I(z)$ defined on the complex plane $z = x + iy$ must satisfy Cauchy-Riemann equations,

$$\frac{\partial f_R}{\partial x} = \frac{\partial f_I}{\partial y},$$
$$\frac{\partial f_I}{\partial x} = -\frac{\partial f_R}{\partial y}.$$

Consequently, both f_R and f_I satisfy Laplace's equation,

$$\frac{\partial^2 F_R}{\partial x^2} + \frac{\partial^2 F_R}{\partial y^2} = 0, \tag{2}$$
$$\frac{\partial^2 F_I}{\partial x^2} + \frac{\partial^2 F_I}{\partial y^2} = 0 \tag{3}$$

over the region where $f_{\text{ext}}(z)$ is analytic. Conventionally, by requiring $f_{\text{ext}}(z)$ to be analytic in the upper half-plane, the quest of finding the Hilbert transform for any given function $f(x)$ can be formulated as a boundary value problem (Scott, 1970). By specifying the boundary conditions that

- $f_R(x, 0) = f(x)$, and that
- $f_R(x, y) = 0$ as $x \to \pm\infty$ or $y \to \infty$,

$f_R(x, y)$ can be uniquely determined by solving Laplace's equation (Eq. 2) in the upper half plane. Then, through Cauchy-Riemann equations, $f_I(x, y)$ can be calculated in the entire upper half plane and in particular on its boundary the x-axis (Scott, 1970). Thus $g(x) = F_I(x, 0)$ is the Hilbert transform of the given function $f(x)$.

2.2 Calculation through improper integrals

The above formulation of Hilbert transform as a boundary-value problem is hardly mentioned in recent texts [except as an exercise problem in Oppenheim & Schafer (2010)].[1]: Instead,

[1] The boundary-value-problem formulation is missing for a reason — it does not tell us how to *compute* the Hilbert transform.

Hilbert transform is commonly introduced and defined through an improper integral [e.g., (Hahn, 1996)]:

$$g(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(u) \frac{1}{x-u} du. \tag{4}$$

Here, note that the convolution kernel function $h(x) = 1/\pi x$ is singular at $x = 0$. Therefore, the integral in Eq. 4 is improper in the sense of Cauchy's principal value:

$$g(x) = \lim_{\epsilon \to 0} \left(\int_{-\infty}^{x-\epsilon} + \int_{x+\epsilon}^{\infty} \right) f(u) \cdot h(x-u) du. \tag{5}$$

To be convinced that Eq. 4 indeed produces the Hilbert transform, we need to think about the effects of Hilbert transform in the frequency domain. First, for any frequency k, note that the Hilbert transform of $f_k(x) = \cos(kx)$ is $g_k(x) = \sin(kx)$. So, we can understand Hilbert transform as a *phase shifter* which gives every sinusoidal function -90 degrees of phase shift. Therefore, in the frequency domain, we have

$$G(k) = F(k) \cdot (-i \cdot \mathrm{sgn}(k)), \tag{6}$$

where $G(k)$ and $F(k)$ are the Fourier transform of $g(x)$ and $f(x)$, respectively, and $\mathrm{sgn}(x)$ is the sign function (i.e., $\mathrm{sgn}(k) = 1$ if $k > 0$ and $\mathrm{sgn}(k) = -1$ if $k < 0$.) Therefore, if we think of $H(k) = -i \cdot \mathrm{sgn}(k)$ as the transfer function of a phase-shift kernel $h(x)$, the kernel can be written as the inverse Fourier transform of the transfer function; that is,

$$h(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(k) e^{ikx} dk. \tag{7}$$

Note that $H(k) = -i$ for $k > 0$ and $H(k) = i$ for $k < 0$. Therefore, $H(k)$'s first derivative with respect to k is

$$\frac{\partial H}{\partial k} = -2i\delta(k), \tag{8}$$

where $\delta(k)$ is the Dirac delta function. Since the operator $\partial/\partial k$ in the frequency domain corresponds to multiplication by $-ix$ in the space domain, we can take the inverse Fourier transform on both sides of Eq. 8 and obtain the following,

$$-ix \cdot h(x) = -2i \cdot \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(k) e^{ikx} dk \right) = \frac{-i}{\pi}. \tag{9}$$

Dividing both sides by $-ix$, we conclude that the convolution kernel $h(x) = 1/\pi x$.

2.3 The notion of Hilbert transform "pairs"

The phase-shifter interpretation of Hilbert transform leads to the fact that if $f(x)$'s Hilbert transform is $g(x)$, then $g(x)$'s Hilbert transform is $-f(x)$; in this sense, $f(x)$ and $g(x)$ form a *Hilbert transform pair*.

This symmetric property can be understood as follows. Note that the $H^2(k) = -1$ for all k since $H(k) = \pm i$. This means that if we take the Hilbert transform twice, the result would be the original function with a negative sign.

This makes sense because Hilbert transform introduces a 90-degree phase shift to all simple harmonics. Therefore, Hilbert transform repeated twice introduces a 180-degree phase shift to all simple harmonics, which means multiplication of the original function by -1.

A table of commonly used Hilbert transform pairs can be found in the Appendix of Hahn (1996) for applications in signal processing. A thorough 80-page table of Hilbert transform pairs can be found in the Appendix of King (2009b) and transform pairs are also plotted in a 20-page atlas.

2.4 The convolution kernel $h(x)$ as the Hilbert transform of $\delta(x)$

We now introduce another way to derive the convolution kernel of Hilbert transform. To begin, note that Eq. 5 essentially states that Hilbert transform is a filtering process which is characterized by its impulse response $h(x)$. Therefore, $h(x)$ must be regarded as the Hilbert transform of the impulse function $\delta(x)$. Then, it is of our interest to check that

$$f_c(x) = \delta(x) + ih(x)$$

can be regarded as an analytic function in the sense of Eq. 1. To see it, consider a family of complex analytic functions $f(z) = i/\pi(z + i\eta)$ parametrized by a variable $\eta > 0$. Since the only singularity of $f(z)$ is at $z = -i\eta$, $f(z)$ is analytic in the entire upper half plane. Therefore, the real part and imaginary part of $f(z)$ form a Hilbert transform pair on the real line $x \in \mathcal{R}$. With a little algebra, the real and imaginary parts can be written as

$$f(x) = \frac{i}{\pi(x + i\eta)} = f_R(x) + if_I(x), \tag{10}$$

where

$$f_R(x) = \frac{\eta}{\pi(x^2 + \eta^2)}$$

and

$$f_I(x) = \frac{x}{\pi(x^2 + \eta^2)}$$

form a Hilbert transform pair for any $\eta > 0$.

Now we let η approach zero and observe $f_R(x)$ and $f_I(x)$. Note that $\int_{-\infty}^{\infty} f_R(x)dx = 1$ regardless of the value of η, and that $f_R(0) = 1/\pi\eta$ approaches infinity as $\eta \to 0$. So we can claim that

$$\lim_{\eta \to 0} f_R(x) = \delta(x). \tag{11}$$

Meanwhile, it is trivial that

$$\lim_{\eta \to 0} f_I(x) = 1/\pi x. \tag{12}$$

From the arguments above, we can be convinced that the Hilbert transform of $\delta(x)$ is indeed $1/\pi x$, for $f_c(x) = \delta(x) + i \cdot (1/\pi x)$ is equal to the limit function of $f(x)$ as $\eta \to 0$ (Scott, 1970).

3. Applications in signal processing

Signal processing is nowadays conveniently conducted in the digital domain. The first step of signal digitization involves sampling an analog signal $x(t)$ at a constant rate $f_s = 1/T$ where

T is the sampling period. In this section, we denote the sampled waveform as $x[n] = x(nT)$, using the square brackets $[\cdot]$ to indicate that the signal is sampled in discrete time. So the *discrete-time Fourier transform* (DTFT) is defined as follows:[2]

$$X(j\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}.$$

Note that $X(j\omega)$ is periodic at every 2π in the frequency domain.[3] Next, we show how Hilbert transform can be defined in discrete time.

3.1 The discrete-time Hilbert transform and Hilbert transformers

Recall that the Hilbert transform introduce a 90-degree phase shift to all sinusoidal components. In the discrete-time periodic-frequency domain, the transfer function of Hilbert transform is specified as follows,

$$H(j\omega) = \begin{cases} -j, & 0 < \omega < \pi \\ j, & -\pi < \omega < 0 \end{cases} \tag{13}$$

The convolution kernel for $H(j\omega)$ can be calculated through inverse Fourier transform (Oppenheim & Schafer, 2010):

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(j\omega)e^{j\omega n} d\omega \tag{14}$$

$$= \begin{cases} \frac{2}{\pi} \frac{\sin^2(\pi n)}{n}, & n \neq 0 \\ 0, & n = 0 \end{cases} \tag{15}$$

Note that $h[n]$ has a infinite support from $n = -\infty$ to ∞. In practice, the entire function can not be stored digitally. To circumvent this difficulty, we now discuss two major methods for calculating the discrete-time Hilbert transform.

3.1.1 The MATLAB approach

The universally popular scientific-computing software MATLAB (MathWorks, Natick, Massachusetts, USA) has a `hilbert()` function that "computes the so-called discrete-time analytic signal `X = Xr + i*Xi` such that `Xi` is the Hilbert transform of `Xr`".[4] MATLAB's implementation of the `hilbert()` function takes advantage of the fast Fourier transform (FFT). Essentially, the `hilbert()` function completes the calculation in three steps:

- Do the FFT of `Xr`.
- Set the elements in FFT which correspond to frequency $-\pi < \omega < 0$ to zero.
- Do the inverse FFT.

[2] We now switch to the electrical-engineering convention of using j to refer to $\sqrt{-1}$.
[3] Hereafter, we use capital letters $X, Y, Z, H, \ldots$ to denote spectrums in the frequency domain, and lowercase letters x, y, z, h for signals in the time domain.
[4] This is what MATLAB's help file shows.

That the three steps above can work is a consequence of the fact that, if $x_i[n]$ is the discrete Hilbert transform of $x_r[n]$, the Fourier transform of $x_r[n] + jx_i[n]$ vanishes for all negative frequencies $-\pi < \omega < 0$. This can be verified by inspecting the definition given in Eq. 13.

Remarks: Perhaps because of MATLAB's popularity and easiness to use, many have allegedly mis-regard the returned vector `X=hilbert(Xr)` as the Hilbert transform of `Xr`. One must be aware of these deviations from the conventional definition to avoid unnecessary confusion.

3.1.2 Hilbert transform as a filter design problem

Thanks to FFT, MATLAB's implementation of Hilbert transform is very efficient. However, it should be used with caution — note that the Hilbert transform defined in Eq. 13 has a discontinuity at $\omega = 0$. Consequently (due to Gibb's phenomenon), the convolution kernel $h[n]$ in Eq. 15 has an infinite support in time. As a result, when implemented through FFT, the Hilbert transform kernel wraps around itself and time-domain aliasing comes in (Oppenheim & Schafer, 2010). The time-domain aliasing could be perceived as artifacts in applications such as audio and video signal processing.

To avoid time-domain aliasing, one can formulate discrete Hilbert transform as a filter-design problem. The ideal transfer function is specified by Eq. 13, and there are standard techniques to design finite impulse response (FIR) or infinite impulse response (IIR) filters that appraoch the ideal transfer function. For example, one can truncate the ideal impulse response by multiplying it with a window function $w[n]$ which has a finite support (let's say from $n = -N$ to N). Then the resulting function $w[n]h[n]$ yields an approximate magnitude response $H_w(j\omega)$ that has a smooth transition between negative and positive frequencies as well as ripples in both regions. The height of the ripples can be reduced by selecting the window function wisely, while the transition bandwidth is inversely proportional to the window length. Interested readers can refer to Chapter 7 and 12 of Oppenheim & Schafer (2010).

The FIR or IIR filters designed to approximate Hilbert transform are called *Hilbert transformers*. Next, we discuss applications of Hilbert transformers in communication and in biomedical engineering.

3.2 Sampling of bandpass signals for communication

An important application of Hilbert transformers is in sampling bandpass signals.[5] To explain this, let us assume that a bandpass signal $s(t)$ is has a region of support $f_c \le f \le f_c + \Delta f$ in the frequency domain, where $\Delta f = 0.2 f_c$. Based on Nyquist's theorem, the sampling rate needs to be at least two times the highest frequency, or $2.4 f_c$, to avoid frequency-domain aliasing. However, the bandwidth of this signal is really $\Delta f = 0.2 f_c$, so $f_s = 2.4 f_c$ is in fact an *oversampling*.

To take advantage of the narrow bandwidth, we initially need to sample at $2.4 f_c$ to obtained an oversampled signal $s_r[n] = s(nT)$, where $T = 1/f_s$. Then, we can use a Hilbert transformer to obtain $s_i[n]$ such that the Fourier transform of $s[n] = s_r[n] + js_i[n]$ has no components at negative frequencies $\pi < \omega < 0$. Now, $S(j\omega)$'s region of support is $\omega \in (5\pi/6, \pi)$. Therefore, we can down sample $s[n]$ by a factor of 6 and there would be no frequency-domain aliasing.

[5] This part is adapted from Sec. 12.4.3 of Oppenheim & Schafer (2010)

Transmitting $s_d[n] = s[6n]$ is more efficient than transmitting $s_r[n]$ because the sampling rate is lowered.

To reconstruct $s_r[n]$ from $s_d[n]$, we can do the following:

- Expand $s_d[n]$ by a factor of 6; i.e., construct $s_e[n] = \begin{cases} s_d[n/6], & \text{if } n = 0, \pm 6, \pm 12, ... \\ 0, & \text{otherwise.} \end{cases}$
- Filter $s_e[n]$ with the passband of $(5\pi/6, \pi)$.
- Take the real part, and the result is a reconstructed copy of $s_r[n]$.

In practice, since all Hilbert bandpass filters are not ideal, one needs to consider sampling at slightly higher than $2.4f_c$. That would protect the passband from ripple interference both during downsampling and during signal reconstruction.

3.3 AM-FM decomposition for auditory prostheses

A cochlear implant device consists of up to tens of electrodes which are inserted as an array to the cochlea to stimulate auditory nerves by electrical currents (Zeng et al., 2008). Each channel (electrode) represents an acoustic frequency band; the amount of currents sent to an electrode should faithfully reflect how acoustic energy entering through the ear microphone varies in time in the corresponding frequency band. Typically, acoustic waveforms are processed with a bank of filters and the resulting envelopes control the electrical currents sent to individual electrodes.

In this application scenario, a Hilbert transformer has been found useful for envelope extraction. This can be understood by noting that the Hilbert transform produces a $\sin(\omega t)$ for every $\cos(\omega t)$. For a narrow-band signal $s_R(t)$, we can factor it as a product of slow-varying *envelope* $A(t)$ and fast-varying *fine structure* $f(t)$:

$$s_R(t) = A(t)f(t) = A(t)\cos[\phi(t)]$$

where $d\phi(t)/dt$ can be regarded as an *instantaneous frequency* of the signal. It can be shown that, if $A(t)$ varies sufficiently slowly, the Hilbert transform produces approximately $s_I(t) = A(t)\sin[\phi(t)]$. Then, the envelope $A(t)$ can simply be estimated by taking the root of sum of squares:

$$A(t) \simeq \sqrt{s_R^2(t) + s_I^2(t)}. \tag{16}$$

Clinically, encoding the currents based on $A(t)$ provides clear speech perception for cochlear-implant users (Nie et al., 2006). Moreover, auditory pitch information can be extracted by taking the time-derivation of $\phi(t)$, which can be determined by the relative phase between s_I and s_R:

$$\phi(t) = \tan^{-1}\left(\frac{s_I(t)}{s_R(t)}\right).$$

Pitch is arguably the utmost important feature for music perception as well as understanding tonal languages. Here, we see Hilbert transform can serve as an efficient computation tool to extract it.

4. Applications in system identification

Hilbert transform relates the real part and the imaginary part of the transfer function of any *physically viable* linear time-invariant system. By "physical viability" we mean a system should be stable and causal. Stability requires the systems to produce bounded output if the input is bounded [so called the bounded-input bounded-output criterion, (Oppenheim & Schafer, 2010)]. Causality prohibits the system from producing responses before any stimulus comes in. Denote the impulse response as $h(t)$ and its Laplace transform as $H(s)$. The above conditions requires that

- $h(t) = 0$ for all $t < 0$ (causality)
- All sigularities of $H(s)$ are located in the left half-plane (stability).

The two conditions above ensure that $H(s)$ converges and is analytic in the entire right half-plane, and in particular on the imaginary axis $s = j\omega$. Therefore, the real and imaginary part of $H(j\omega) = H_R(\omega) + jH_I(\omega)$ are inter-dependent in term of the *Kramers-Krönig relations* (King, 2009b):

$$H_I(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{H_R(u)du}{\omega - u} \tag{17}$$

$$H_R(\omega) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{H_I(u)du}{\omega - u} \tag{18}$$

which is basically Hilbert transform in its time-frequency dual form.

The Kramers-Krönig relations govern how physical viable transfer functions can vary in frequency. For instance, the real and imaginary part of an electromagnetic wave propagation function defines the attenuation and the wavenumber per unit length as a function of frequency. The frequency dependence of both functions is referred to as *dispersion*, for in optics it describes how red light travels faster than violet light in water — so we behold the beauty of rainbows after raining. It is now intriguing to realize that the attenuation and velocity of light are two inter-dependent functions of frequency. More physics-inclined readers can refer to King's thorough discussion on dispersion relations in electrodynamics and optics (King, 2009b).

To a certain extent, the concept that the real and the imaginary parts are inter-dependent similarly applies to the magnitude and phase of transfer functions of a physically viable system. Note that any transfer function $H(j\omega)$ can be decomposed logarithmically into magnitude and phase:

$$\log H(j\omega) = \log|H(j\omega)| + j\angle H(j\omega).$$

This shows that the log-magnitude and the phase are real and imaginary parts of the log-spectrum, respectively. It might appear that they must satisfy the Kramers-Krönig relations. Unfortunately, this is a wishful thinking since apparently $\tilde{H}(j\omega) = \exp(-j\omega\tau)H(j\omega)$, where τ is a constant, would have the same magnitude as $H(j\omega)$ but a different phase response.[6]

It turns out that, for any given magnitude response, the uniqueness of phase response can be established if the transfer function satisfies a *minimum-phase* criterion; the criterion requires

[6] in fact, $\tilde{h}(t) = h(t - \tau)$.

that all zeros and poles of the transfer function $H(s)$ to be located in the left-half plane. This criterion ensures that all the singularities of $\log H(s)$ are located in the left-half plane so the real and imaginary parts of $\log H(s)$ become a Hilbert transform pair. Otherwise, any transfer function can be uniquely factorized as a product of a minimum-phase function $M(j\omega)$ and an all-pass function $P(j\omega)$. It is noteworthy that the system whose transfer function is $M(j\omega)$ has the minimal energy delay among all linear time-invariant systems of the same magnitude response. Further readings are recommended in Chapter 5 and 12 of Oppenheim & Schafer (2010).

5. Concluding remarks and historical developments

In this chapter, we first presented Hilbert transform as an analytic extension problem. Hilbert transform uniquely exists due to Cauchy-Riemann equations. We then reviewed several different ways to calculate Hilbert transform. A few important points stand out: First, Hilbert transform can be regarded as a 90-degree phase shifter. Secondly, the real part and imaginary part of a physically viable transfer function must satisfy Kramers-Krönig relations, which is the Hilbert transform applied in time-frequency duality. A good reference on these topics can be found in Oppenheim & Schafer (2010).

The construction of Hilbert transform pairs through Cauchy-Riemann equations in Sec. 2.1 was found in the appendices of an old text on microwave electronics (Scott, 1970). The original formulation was stated in terms of Kramers-Krönig relation, and in this chapter that formulation is adapted so the signal is defined on the real line instead of the frequency axis $j\omega$.

As a matter of fact, the definition of Hilbert transform was not given by David Hilbert himself. The name "Hilbert transform" was first given by the British mathematician G. H. Hardy in honor of Hilbert's pioneering work on integral equations (King, 2009a). Hardy's early work established mathematical rigor of the transform (Hardy, 1932), which we now apply in various areas such as physiology and telecommnication. A review of these contributions from brilliant mathematicians reminds us that the transform really is a heritage from the 20th century. Nevertheless, it is also amusing that nowadays we calculate Hilbert transform with super fast computers, which might never had been envisioned by 20th-century pioneers, including Hilbert, Hardy, Scott, or even Oppenheim.

6. Acknowledgement

This work is supported by Taiwan's National Science Council through research grant No. 100-2221-E-007-011.

7. References

Hahn, S. L. (1996). *Hilbert Transforms in Signal Processing*, Artech House, Inc., Norwood, MA, USA.

Hardy, G. H. (1932). On hilbert transforms, *Quart. J. Math. (Oxford)* 3: 102–112.

King, F. W. (2009a). *Hilbert Transforms, Vol. 1*, Cambridge University Press, Cambridge, UK.

King, F. W. (2009b). *Hilbert Transforms, Vol. 2*, Cambridge University Press, Cambridge, UK.

Liu, Y.-W. & Smith, J. O. (2002). Perceptually similar orthogonal sounds and applications to multichannel acoustic echo canceling, *Proc. Audio Eng. Soc. 22nd Int. Conf.*, Espoo, Finland.

Nie, K., Barco, A. & Zeng, F.-G. (2006). Spectral and temporal cues in cochlear implant speech perception, *Ear Hear.* 27: 208–217.

Oppenheim, A. V. & Schafer, R. W. (2010). *Discrete-Time Signal Processing, 3rd Edition*, Pearson, Boston.

Potamianos, A. & Maragos, P. (1994). A comparison of the energy operator and the hilbert transform approach to signal and speech demodulation, *Signal Processing* 37(1): 95 – 120.

Recio-Spinoso, A., Fan, Y.-H. & Ruggero, M. (2011). Basilar-membrane responses to broadband noise modeled using linear filters with rational transfer functions, *Biomedical Engineering, IEEE Transactions on* 58(5): 1456 –1465.

Scott, A. (1970). *Active and Nonlinear Wave Propagation in Electronics*, Wiley-Interscience, New York.

Smith, Z. M., Delgutte, B. & Oxenham, A. J. (2002). Chimaeric sounds reveal dichotomies in auditory perception, *Nature* 416: 87–90.

van Drongelen, W. (2007). *Signal Processing for Neuroscientists: Introduction to the analysis of physiological signals*, Academic Press, London.

Zeng, F.-G., Rebscher, S., Harrison, W., Sun, X. & Feng, H. (2008). Cochlear implants: System design, integration, and evaluation, *IEEE Rev. Biomed. Eng.* 1: 115–142.

IntechOpen



Fourier Transform Applications

Edited by Dr Salih Salih

ISBN 978-953-51-0518-3
Hard cover, 300 pages
Publisher InTech
Published online 25, April, 2012
Published in print edition April, 2012

The book focuses on Fourier transform applications in electromagnetic field and microwave, medical applications, error control coding, methods for option pricing, and Helbert transform application. It is hoped that this book will provide the background, reference and incentive to encourage further research and results in these fields as well as provide tools for practical applications. It provides an applications-oriented analysis written primarily for electrical engineers, control engineers, signal processing engineers, medical researchers, and the academic researchers. In addition the graduate students will also find it useful as a reference for their research activities.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Yi-Wen Liu (2012). Hilbert Transform and Applications, Fourier Transform Applications, Dr Salih Salih (Ed.), ISBN: 978-953-51-0518-3, InTech, Available from: http://www.intechopen.com/books/fourier-transform-applications/hilbert-transform-and-applications

INTECH
open science | open minds

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2012 The Author(s). Licensee IntechOpen. This is an open access article distributed under the terms of the Creative Commons Attribution 3.0 License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

IntechOpen

IntechOpen