17

Risk, Return and Market Condition: From a Three-Beta to a Functional-Beta Capital Asset Pricing Model

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1. Introduction

In this chapter, we are concerned with how a time-varying beta is linked to market condition in capital asset pricing model (CAPM). This is a question that is related to the recent interest in these large and unexpected swings in asset values, revived after the publication of Taleb's (2007) book, "The Black Swan: The Impact of the Highly Improbable", to explore the merits of beta in the presence of large market fluctuations (c.f., Estrada and Vargas 2011).

It is well known that capital asset pricing model due to (Sharpe 1964) and (Lintner 1965) conveys important information that individual securities are priced so that their expected return will compensate investors for their expected risk. Symbolically, CAPM can be expressed in a general non-expected form as

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i, \tag{1}$$

where R_i is the return on security i, R_m is the return on the market portfolio and β_i is the measure of security i's non-diversifiable risk relative to that of the market portfolio. Here the return on individual security R_i can be decomposed into the specific return, including expected specific return α_i and random specific return ε_i, and the systematic return, $\beta_i R_m$, owing to the common market return R_m. In this model, the quantity β_i is of particular importance, which is an alternative measure of the risk that an investor has to bear owing to the systematic market movement.

In the traditional CAPM, β_i is assumed to be constant. This assumption has been widely documented to be untrue in the literature. Blume (1971) was among the first to consider the time-varying beta market model, which showed that the estimated beta tended to regress toward the mean; see also (Blume 1975). Earlier studies that attempted to apply random coefficient model to beta include, among others, (Sunder 1980) and (Simonds, LaMotte and McWhorter 1986) who suggested a random-walk coefficient model, and (Ohlson and Rosenberg 1982) and (Collins, Ledolter and Rayburn 1987) who proposed an ARMA(1,1) model for the beta coefficient. More recent literature has widely recognized that the systematic risk of asset changing over time may be due to both the microeconomic factors in the level of the firm and the macroeconomic factors; see (Fabozzi and Francis 1978; Bos and Newbold

1984). Considerable empirical evidences have suggested that beta stability assumption is invalid. The literature is abundant, see, for example, (Kim 1993), (Bos and Ferson 1992, 1995), (Wells 1994), (Bos, Ferson, Martikainen and Perttunen 1995), (Brooks, Faff and Lee 1992) and (Cheng 1997).

The time-varying beta models have also been investigate by using Australian stock market data sets. Brooks, Faff and Lee (1992), and (Faff, Lee and Fry 1992) were among the first to investigate the time-varying beta models. Faff, Lee and Fry (1992) employed a locally best invariant test to study the hypothesis of stationary beta, with evident finding of nonstarionarity across all of their analysis. The random coefficient model was further suggested by (Brooks, Faff and Lee 1994) as the preferred model to best describe the systematic risk of both individual shares and portfolios. However, (Pope and Warrington 1996) reported that random coefficient model was appropriate only for a bit more than 10% companies in their studies. Faff, Lee and Fry (1992) investigated the links between beta's nonstationarity and the three firm characteristics: riskiness, size and industrial sector, without finding the strong pattern between firm size or industry sector and nonstationarity. Faff and Brooks (1998) modelled industrial betas by different regimes based on market returns and volatility of the risk-free interest rate, their univariate and multivariate tests providing mixed evidence concerning the applicability of a time-varying beta model which incorporates these variables. Groenewold and Fraser (1999) argued that the industrial sectors could be divided into two groups: one of them has volatile and non-stationary betas and the other group has relatively constant and generally stationary beta. Other recent studies include (Gangemi, Brooks and Faff 2001), (Josev, Brooks and Faff 2001), and others. An interesting study recently made by (Yao and Gao 2004) investigated the problem of choosing a best possible time-varying beta for each individual industrial index using the state-space framework, including the random walk models, random coefficient models and mean reverting models, which were examined in detail by using the Kalman filter approach.

When testing the validity of asset pricing models, many studies account for market movements, defined as up and down markets. For example, (Kim and Zumwalt 1979) used the average monthly market return, the average risk-free rate and zero as three threshold levels; when the realized market return is above (below) the threshold level the market is said to be in the up (down) market state. Crombez and Vennet (2000) conducted an extensive investigation into the risk-return relationship in the tails of the market return distribution; they defined up and down markets with two thresholds: zero and the risk-free rate. Further, to define three regimes for market movements, that is substantially upward moving, neutral and substantial bear, different threshold points were used, such as: the average positive (negative) market return, the average positive (negative) market return plus (less) half the standard deviation of positive (negative) market returns, and the average positive (negative) market return plus (less) three-quarters of the standard deviation of positive (negative) market returns. The conditional beta risk-return relation has been found to be stronger if the classification of up and down markets is more pronounced.

Galagedera and Faff (2005) has recently argued as in the finance literature and media that high volatility leads to high returns. High volatility in equity prices in many situations has been related to negative shocks to the real economy. On one hand, the volatility of macro-economic variables may partially explain the equity market price variation. On the other hand, the volatility in equity market prices may also be entrenched more in financial market disturbances. In particular, when the market volatility becomes extreme, it could have

an impact on financial markets. Some securities are more susceptible to market volatility than others. Two interesting questions that arise in this setting were posed by (Galagedera and Faff 2005): (i) Does the beta risk-return relationship depend on the various market volatility regimes? (ii) Are the betas corresponding to these volatility regimes priced? There have been empirical evidences raising concern about the ability of a single beta to explain cross-sectional variation of security and portfolio returns. Security or portfolio systematic risk is known to vary considerably over time, as documented in the literature in the above. It is further well known that the volatility of financial time series, particularly in high frequency data, changes over time.

In their pioneering work of three-beta CAPM, (Galagedera and Faff 2005) made an assumption that the market conditions can play an important part in explaining a changing beta and could be divided into three states specified as “low", “neutral" or “high" market volatility. First, they fit a volatility model for daily market returns and obtain the estimates for conditional variance. Then, based on the magnitude of these estimates, (Galagedera and Faff 2005) classify daily market volatility σ^2_{Mt} into one of three market volatility regimes, using appropriately defined indicator functions:

$$I_{Lt} = \begin{cases} 1 & if \quad \sigma^2_{Mt} < \sigma^2_L \\ 0 & if \quad otherwise \end{cases} \tag{2}$$

$$I_{Nt} = \begin{cases} 1 & if \quad \sigma^2_L < \sigma^2_{Mt} < \sigma^2_H \\ 0 & if \quad otherwise \end{cases} \tag{3}$$

$$I_{Ht} = \begin{cases} 1 & if \quad \sigma^2_H < \sigma^2_{Mt} \\ 0 & if \quad otherwise. \end{cases} \tag{4}$$

Here σ^2_L and σ^2_H are constants; I_{Lt} represents the low market condition, I_{Nt} represents the neutral market condition, I_{Ht} represents high market condition. By investigating empirically on the single factor CAPM $R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$, to estimate the betas in the low, neutral and high volatility markets, Galagedera and Faff extended the market model given in (1), in the form:

$$R_{it} = \alpha_i + \beta_{iL}(I_{Lt}R_{Mt}) + \beta_{iN}(I_{Nt}R_{Mt}) + \beta_{iH}(I_{Ht}R_{Mt}) + \varepsilon_i, \tag{5}$$

where $\beta_{iL}, \beta_{iN}, \beta_{iH}$ are three constants defined as the systematic risks corresponding to the low, neutral and high market volatility regimes, respectively. This model is a richer specification than the traditional single factor CAPM. It is a three-state regime-switching model with the percentiles of market volatility used as threshold parameters. This model raises an interesting question: How is the beta risk linked to the market condition measured by the market volatility? *Our main objective in this chapter is to investigate whether and how the securities' responses to the market vary depending on the changing market volatility.*

We are making a careful investigation into this question in the following sections. In Section 2.1, we shall extend the three-beta model (5) to a more general functional-beta CAPM framework. Nonparametric estimation of the beta functional will be introduced in Section 2.2. We will use the data sets from the Australian stock market to empirically examine the evidences of functional-beta structure in CAPM. An introduction into the data will be provided in Section 3, where an estimation of the unobserved market volatility will be established. Section 4 will carefully examine the linkage of the beta to the market volatility by using both nonparametric and parametric approaches. Based on the findings from the

nonparametric estimation, we can suggest reasonable regime switching thresholds, by which a regime-switching threshold CAPM will be proposed and investigated. We will conclude in Section 5.

2. Methodology: From a Three-Beta CAPM to a Functional-Beta CAPM

In this section, we will first propose a Functional-Beta CAPM that is a generalization of the Three-Beta CAPM suggested by (Galagedera and Faff 2005) in Subsection 2.1, and then introduce a nonparametric method to estimate the unknown functional beta in the Functional-Beta CAPM in Subsection 2.2.

2.1 Model

Following the idea of (Galagedera and Faff 2005), we consider a new more general structural framework to incorporate market movements into asset pricing models by including the changes in the conditional market volatility. We achieve this by noting that the model (5) can be expressed as

$$R_{it} = \alpha_i + (\beta_{iL} I_{Lt} + \beta_{iN} I_{Nt} + \beta_{iH} I_{Ht}) R_{Mt} + \varepsilon_{it} \equiv \alpha_i + \beta_{it} R_{Mt} + \varepsilon_{it}, \tag{6}$$

which is a time-varying beta model, with

$$\beta_{it} = \beta_{iL} I_{Lt} + \beta_{iN} I_{Nt} + \beta_{iH} I_{Ht}. \tag{7}$$

We note that the volatility of market returns is partitioned into three regimes in (2)–(4), which are the functions of the size of the conditional market volatility, say, σ^2_{Mt}. Therefore β_{it} is a simple functional of the market volatility σ^2_{Mt}, that is

$$\beta_{it} = \begin{cases} \beta_{iL} & if \quad \sigma^2_{Mt} < \sigma^2_L, \\ \beta_{iN} & if \quad \sigma^2_L \le \sigma^2_{Mt} < \sigma^2_H, \\ \beta_{iH} & if \quad \sigma^2_{Mt} \ge \sigma^2_H. \end{cases} \tag{8}$$

So the three-beta CAPM proposed by (Galagedera and Faff 2005) is a simple functional beta model.

In this chapter, we will extend the model (5) that was suggested by (Galagedera and Faff 2005) and propose a general functional-beta model as follows:

$$R_{it} = \alpha_i + \beta_i(\sigma^2_{Mt}) R_{Mt} + \varepsilon_{it}, \tag{9}$$

where as before, R_{it} is the return of financial asset i at time t, R_{Mt} is the market return at time t, σ^2_{Mt} is the market volatility at time t, α_i is the conditional expected specific return, ε_{it} is random specific return, and β_i is the coefficient of the contribution due to the market factor, changing with the market volatility. Here $\beta_i(\cdot)$ is particularly important, which is the systematic risk functional, in capital asset pricing modelling. We may also treat α_i as varying with σ^2_{Mt} although its value is usually rather small to be often assumed as a constant. We call (9) the functional-beta CAPM. For our objective, we need to estimate the unknown α_i and $\beta_i(\cdot)$ in (9).

2.2 Estimation of functional-beta CAPM: nonparametric method

Given the historical observations (R_{it}, R_{Mt}), $t = 1, 2, \cdots, T$, we are concerned with how to estimate the unknown functional beta. First of all, we need some way to estimate the unobservable market volatility σ^2_{Mt}. Using the market returns R_{Mt}, $t = 1, 2, \cdots, T$, we can try to estimate σ^2_{Mt} in various ways. A simple way is to apply the econometric models of ARCH of (Engle 1982) or GARCH of (Bollersleve 1986), as done in (Galagedera and Faff 2005). More involved stochastic volatility models can also be applied (c.f., Gao, 2007, page 169). Alternatively, we can use realized market volatility as an estimate of σ^2_{Mt}; see (Allen *et al*. 2008) for a comprehensive review on realized volatility. In the following we assume the market volatility σ^2_{Mt} has been estimated, denoted by $\hat{\sigma}^2_{Mt}$, $t = 1, 2, \cdots, T$.

We will estimate the unknown functional $\beta(v)$ at the market volatility $\sigma^2_{Mt} = v$ by least squares local linear modelling technique (c.f. Fan and Gijbels, 1996). The basic idea of least squares local linear modelling technique with $\beta(\cdot)$ can be described as follows. When σ^2_{Mt} is equal or close to v, then $\beta(\sigma^2_{Mt})$ can be expressed or approximated by

$$\beta(v) + \beta'(v)(\sigma^2_{Mt} - v) \equiv \beta_0 + \beta_1(\sigma^2_{Mt} - v) \tag{10}$$

Locally at v, the model can then be approximately expressed as:

$$R_{it} \approx \alpha + (\beta_0 + \beta_1(\sigma^2_{Mt} - v))R_{Mt} + \varepsilon_t, \tag{11}$$

where though we can also assume α depending on σ^2_{Mt} in model (9) and apply local linear idea to $\alpha(\cdot)$, the estimation of $\alpha(\cdot)$ is of less interest in capital asset pricing modelling, which is very close to zero, therefore in (11) α is treated as a local constant to reduce the number of unknown local parameters.

Therefore, replacing σ^2_{Mt} by $\hat{\sigma}^2_{Mt}$, the least squares local linear estimate of α and $\beta(\cdot)$ in (9) can be made by setting $\widehat{\alpha(v)} = \widehat{\alpha}$ and $\widehat{\beta(v)} = \widehat{\beta_0}$, where $(\widehat{\alpha}, \widehat{\beta_0}, \widehat{\beta_1})$ minimizes:

$$L(\alpha, \beta_0, \beta_1) = \sum_{t=1}^{T} (R_{it} - [\alpha + (\beta_0 + \beta_1(\hat{\sigma}^2_{Mt} - v)R_{Mt})])^2 K(\frac{\hat{\sigma}^2_{Mt} - v}{h}), \tag{12}$$

where $h = h_T \to 0$ is a bandwidth that controls the length of the local neighborhood of v in which the observations locally used fall, *K(x)* is a kernel function, which may, for example, take

$$K(x) = \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}. \tag{13}$$

Therefore, we have three unknown local parameters α, β_0 , β_1 . Applying partial differentiation, we get the expression of the estimators of the three unknown local parameters at v as follows:

$$\begin{pmatrix} \widehat{\alpha} \\ \widehat{\beta_0} \\ \widehat{\beta_1} \end{pmatrix} = A_T^{-1} B_T, \tag{14}$$

where

$$A_T = \sum_{t=1}^{T} \begin{pmatrix} 1 & R_{Mt} & R_{Mt}(\hat{\sigma}^2_{Mt} - v) \\ R_{Mt} & R^2_{Mt} & R^2_{Mt}(\hat{\sigma}^2_{Mt} - v) \\ (\hat{\sigma}^2_{Mt} - v)R_{Mt} & (\hat{\sigma}^2_{Mt} - v)R^2_{Mt} & (\hat{\sigma}^2_{Mt} - v)^2 R^2_{Mt} \end{pmatrix} K\left(\frac{\hat{\sigma}^2_{Mt} - v}{h}\right),$$

$$B_T = \begin{pmatrix} \sum_{t=1}^{T} R_{it} K(\frac{\hat{\sigma}^2_{Mt}-v}{h}) \\ \sum_{t=1}^{T} R_{Mt} R_{it} K(\frac{\hat{\sigma}^2_{Mt}-v}{h}) \\ \sum_{t=1}^{T} (\hat{\sigma}^2_{Mt}-v) R_{Mt} R_{it} K(\frac{\hat{\sigma}^2_{Mt}-v}{h}) \end{pmatrix}.$$

It is well-known (c.f., Fan and Gijbels, 1996) that the bandwidth h plays an important role in the process of estimation. Therefore, how to choose the bandwidth becomes an important step in the estimation. A popular method is the cross-validation (CV) selection of bandwidth (cf., Stone 1974). We here apply a leave-one-out CV, defined below, which is relatively computationally less intensive in comparison with other more involved CV principles:

$$CV(h) = \sum_{s=1}^{T} \{R_{is} - \hat{\alpha}_{-s}(\hat{\sigma}^2_{Ms}) - \hat{\beta}_{-s}(\hat{\sigma}^2_{Ms}) R_{Ms}\}^2, \tag{15}$$

where $(\hat{\alpha}_{-s}(\cdot), \hat{\beta}_{-s}(\cdot))$ are the estimators of $(\alpha(\cdot), \beta(\cdot))$ obtained by minimizing (12) with the term $t = s$ removed from the sum of (12). We select the h_{opt} that minimizes $CV(\cdot)$ over $h \in [h_L, h_U]$, where $0 < h_L < h_U$ are appropriately given. To simplify the computation, a partition of $[h_L, h_U]$ into q points $h_1, h_2, \cdots, h_q$ is applied. We adapt an empirical rule for selecting a bandwidth by (Fan *et al*. 2003) to determining the bandwidth h; see also (Lu *et al*. 2009): Up to first order asymptotics, the optimal bandwidth is $h_{opt} = \{c_2/(4Tc_1)\}^{1/5}$, minimising $CV(h) = c_0 + c_1 h^4 + \frac{c_2}{Th} + o_P(h^4 + T^{-1}h^{-1})$. In practice, the coefficients c_0, c_1 and c_2 will be estimated from $CV(h_k)$, $k = 1, 2, \cdots, q$, via least squares regression,

$$\min_{c_0, c_1, c_2} \sum_{k=1}^{q} \left\{CV_k - c_0 - c_1 h_k^4 - c_2/(Th_k)\right\}^2, \tag{16}$$

where $CV_k = CV(h_k)$ obtained from (15). We let $h_{opt} = \{\hat{c}_2/(4T\hat{c}_1)\}^{1/5}$ when both $\hat{c}_1$ and $\hat{c}_2$, the estimators of c_1 and c_2, are positive. In the likely event that one of them is nonpositive, we let $h_{opt} = \arg\min_{1 \le k \le q} CV(h_k)$. This bandwidth selection procedure is computationally efficient as q is moderately small, i.e. we only need to compute q CV-values.

Before we can apply the estimation method introduced in this section to examine the empirical evidence of the functional-beta CAPM, we need to introduce the data sets that we will use, in next section.

3. Data sets from Australian stock market

We will use a set of the stocks data collected from Australian stock market to explore the evidence of functional-beta CAPM in this chapter. The reason why we use the Australian data is because we believe an Australian dataset is ideal for this task. Firstly, the Australian evidence regarding the CAPM is well studied by (Ball, Brown and Officer 1976); (Faff 1991); (Wood 1991); (Faff 1992); (Brailsford and Faff 1997); and (Faff and Lau 1997) as well as (Yao and Gao 2004). Ball, Brown and Officer (1976) may be the first published test of the CAPM using Australian data. They employed the basic univariate testing methodology and found evidence supporting their model. Therefore using the Australian data set will help us to better understand the varying-coefficient nature of CAPM. Secondly, it can be seen that a relatively few, very large companies dominate the Australian market. For example, around 40 per cent of market capitalization and trading value is produced by just 10 stocks, whereas a similar number of the largest US stocks constitute only about 15 per cent of the total US

market. Moreover, there are typically prolonged periods in which many smaller Australian companies do not trade. Therefore the market risk may be a significant factor that impacts the risk of the individual stock or index measured by the beta in CAPM. Thirdly, despite the above argument, the Australian equity market belongs to the dominant group of developed markets. For instance, as at the end of 1996 it ranked tenth among all markets in terms of market capitalization at about $US312,000 million. Interestingly, this is not greatly dis-similar from the size of the Canadian market which ranked sixth (Faff, Brooks, Fan 2004). Therefore the Australian data may be of some typical properties that the other markets may share.

According to ASX Indices (including All Ordinaries Index, ASX 200 GICS Sectors Index), we take sample size 986, from August 2nd 2004 to August 8th 2008, for an illustration. The sectors indexes include ASX 200 GICS Energy, ASX 200 GICS Materials, ASX 200 GICS Health Care, ASX 200 GICS Financials, ASX 200 GICS Finance-v-property trusts. Moreover, we also take a group of individual stock data which is ANZ bank group limited as survey sample of individual stock analysis. An introduction to individual and market return series is presented in Subsection 3.1; estimation of the market volatility that we need in estimating functional-beta CAPM is detailed in Subsection 3.2.

3.1 Individual and market return series

At first we review the market return of Australia Index from August 2nd 2004 to August 8th 2008. The time series plot of the 5 sector daily indices and the ANZ stock daily closing price are depicted in Figure 1 together with their return series in Figure 2, where the daily return data denoted as R_t (for individual sector index or for individual security), can be expressed as:

$$R_t = (\log P_t - \log P_{t-1}) \times 100, \tag{17}$$

where P_t represents the closing price of individual sector index in day t. The daily market return data, R_{Mt} , can be expressed as:

$$R_{Mt} = (\log P_{Mt} - \log P_{M,t-1}) \times 100, \tag{18}$$

and P_{Mt} represents the closing price of all ordinaries index in day t, both of which are plotted in Figure 3.

In addition to market return data, R_{Mt}, we also need the daily market volatility, σ^2_{Mt}, which is unobservable directly. We discuss how to estimate σ^2_{Mt} based on the market return data, R_{Mt}, in the next subsection.

3.2 Market volatility

The market volatility that can not be observed directly needs to be estimated by using the market return series R_{Mt}. A popular method to estimate the market volatility in the literature is by using the family of GARCH models proposed by (Engle 1982) and (Bollersleve 1986); in particular, we produce the market volatility by the most popular GARCH(1,1) model applied to the return series of All Ordinaries Index. In the GARCH(1,1) model

$$\begin{cases} R_{Mt} = a_0 + a_1 R_{M,t-1} + \epsilon_t \\ \epsilon_t = e_t \sigma^2_{Mt} \\ \sigma^2_{Mt} = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 \sigma^2_{Mt}, \end{cases} \tag{19}$$

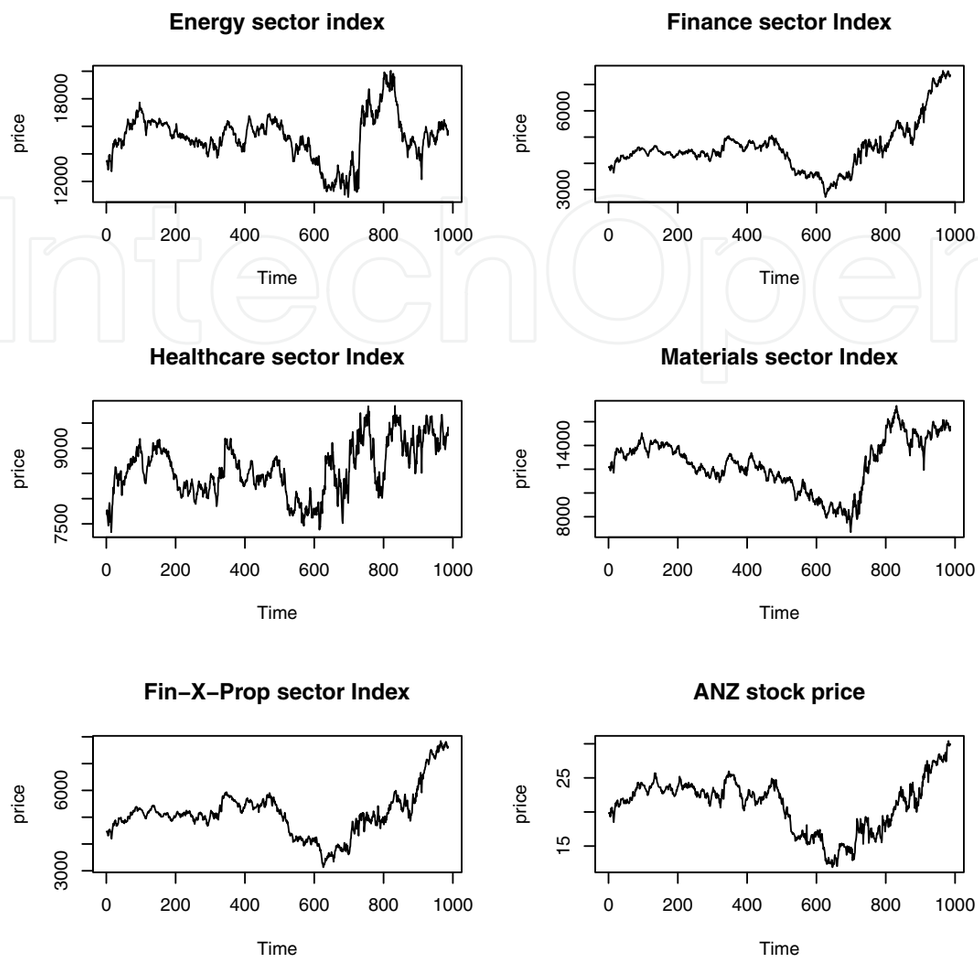


Fig. 1. The 5 ASX 200 GICS Sectors Indexes and ANZ daily closing price in Australia from August 2nd 2004 to August 8th 2008. Sample size =986.

we use R fGarch package to calculate the parameters under e_t satisfying $Ee_t = 0$ and $\text{var}(e_t) = 1$. In order to examine the impacts of distribution for e_t on the estimation of the volatility, we tried different distributions for e_t in the fGarch package, including normal, t and generalized error distribution and their skewed versions, and found that the non-skewed distributions are more acceptable according to their AIC values (Akaike, 1974), with the results listed in Table 1: Quite obviously it follows from the p-value in Table 1 that a_0 and a_1 in the AR part are close to zero while the GARCH parameters are all away from zero at the significance level of 5%. Also by the AIC values, these three GARCH models are quite close to each other and well fitted to the market return data set, with the GARCH-GED (i.e., with e_t of generalized error distribution) preferred. The estimated volatility series under GARCH-GED is plotted Figure 4, where the kernel density estimators of the estimated volatility series under different GARCH models are also displayed, confirming that the estimated volatility under different models are very close. In the following we will use the estimated volatility from the GARCH-GED model as the volatility series in the estimation of the functional-beta CAPM. The summary statistics

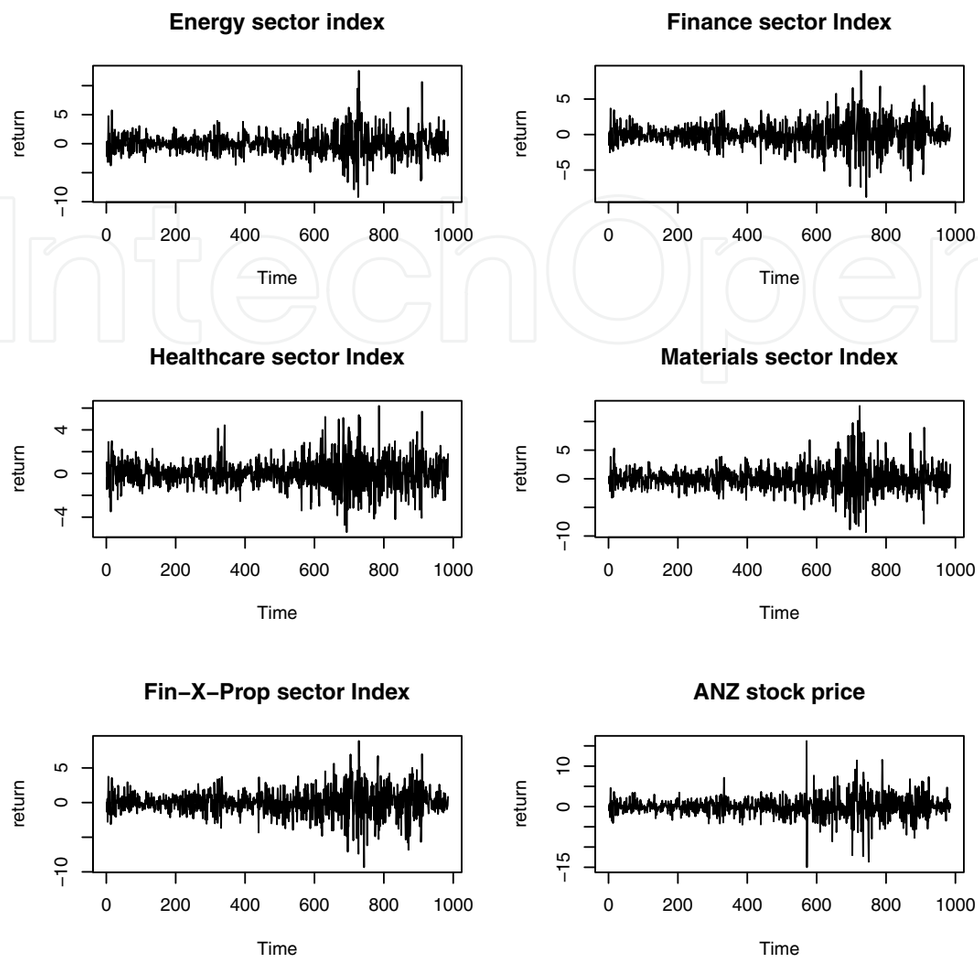


Fig. 2. The return series of the 5 ASX 200 GICS Sectors Indexes and ANZ daily closing price in Australia from August 2nd 2004 to August 8th 2008. Sample size =985.

	a_0	a_1	α_0	α_1	β_1	shape	AIC
garch-normal	-0.019310	-0.022673	0.033037	0.110890	0.871949		3.248890
(p-value)	(0.5731)	(0.4995)	(0.0146)	(4.53e-08)	(2e-16)		
garch-t	-0.032541	-0.015214	0.031242	0.114034	0.873597	10.00000	3.248322
(p-value)	(0.3363)	(0.6457)	(0.0360)	(1.14e-06)	(2e-16)	(7.50e-05)	
garch-ged	-0.031626	-0.020063	0.031619	0.111393	0.872692	1.682406	3.244669
(p-value)	(0.3536)	(0.5460)	(0.0267)	(3.94e-07)	(2e-16)	(2e-16)	

Table 1. Estimated GARCH models under different innovations for the return of all ordinaries index

on the market return and volatility are provided in Table 2, from which we can see that the market is quite volatile with large extreme values.

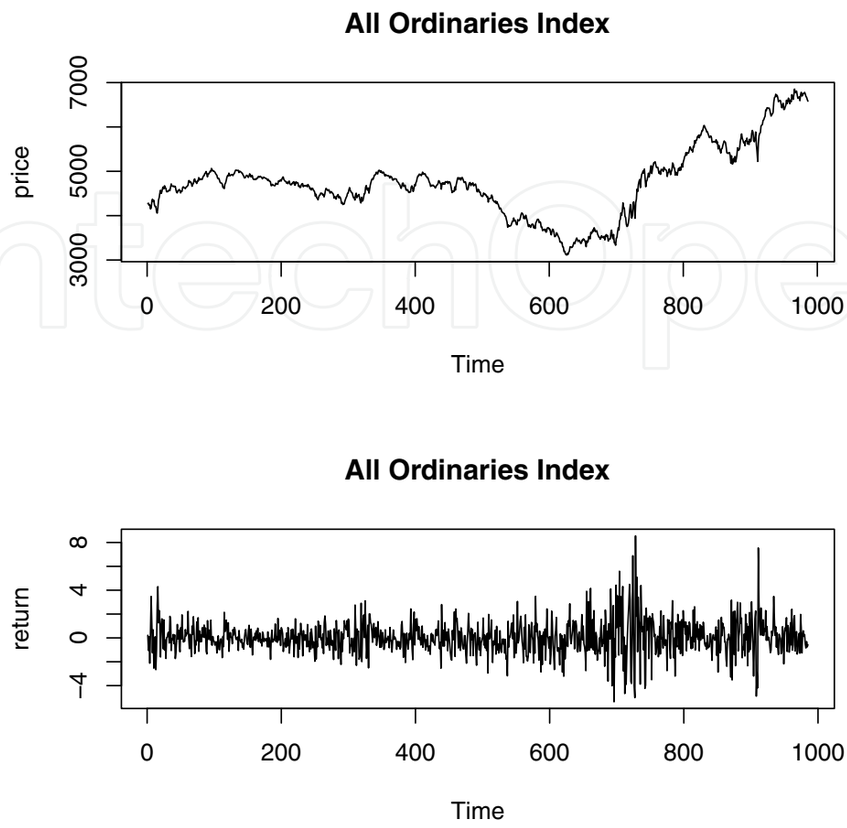


Fig. 3. All Ordinaries Index (sample size =986) and its return series (sample size=985) in Australia from August 2nd 2004 to August 8th 2008.

	mean	standard deviation	skewness	kurtosis	median	min	max
Market return	0.04388	2.0214	0.4596	6.5235	-0.0099	-5.3601	8.5536
Market volatility	1.3089	0.3169	2.1493	8.7173	1.1729	0.6023	4.3905

Table 2. Some statistics data for the Australia index

In the next section, we shall explore the functional form of the beta risk associated with the market volatility in CAPM with the stocks data sets from Australian stock market introduced in this section.

4. Functional-beta CAPM: Empirical evidences

To carefully examine the evidences of functional-beta model (9), we are using the methodology introduced in Section 2, under the semiparametric beta structure that is estimated by local linear fitting introduced in Section 2.2. The advantage of this semiparametric method lies in that the data will determine the relationship of the beta coefficient associated with the market volatility, without pre-specifying a parametric structure to avoid model mis-specification. Based on the findings from the semiparametric method,

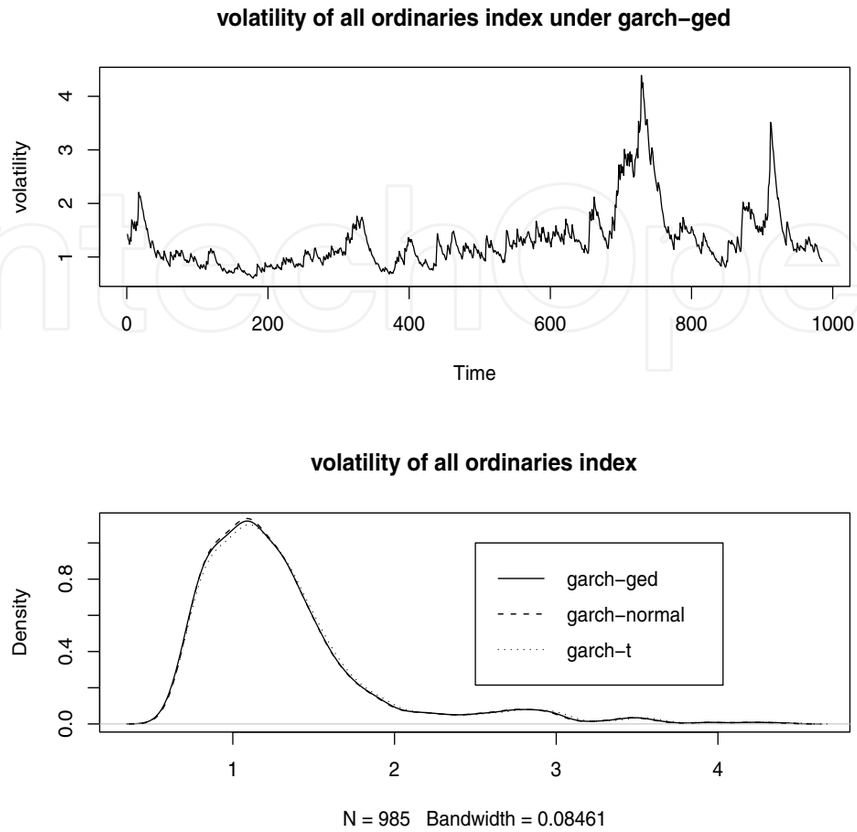


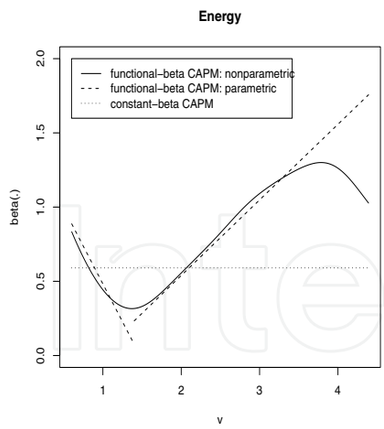
Fig. 4. The estimated volatility of All Ordinaries Index and its kernel probability density estimates under different GARCH models

we shall clearly see whether the beta coefficient associated with the market volatility is of three- or multi-beta structure or not. Differently from the three- or multi-beta CAPM in (Galagedera and Faff 2005), our new findings will indicate that the functional beta may probably be parameterized as threshold (regime-switching) stepwise linear functionals of the market volatility, rather than three or more simple constant beta's.

We will present our empirical evidence based nonparametric estimation method in Subsection 4.1 and further investigation into the parametric evidence in Subsection 4.2.

4.1 Nonparametric evidence

Referred to Section 2.2, when we apply local linear fitting method to estimate the unknown beta function in model (9), we need to use the real data to choose the ideal bandwidth for each Sector index. The values of CV are calculated against 40 points of the bandwidth h for eight groups of data (with bandwidth ranging from 0.2 to 0.6 with partition interval of length 0.01). Hence using the CV calculation procedure given in Section 2.2, we can have the chosen bandwidths as follows in Table 3:



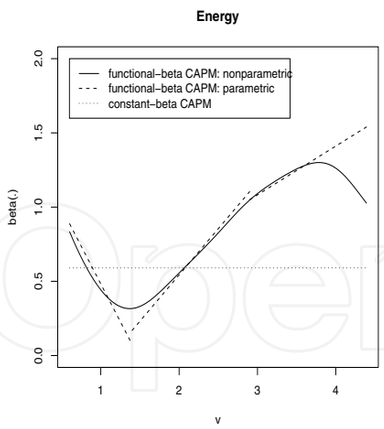
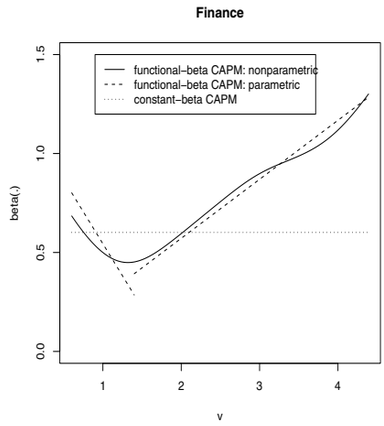


Fig. 5. For Energy sector index:



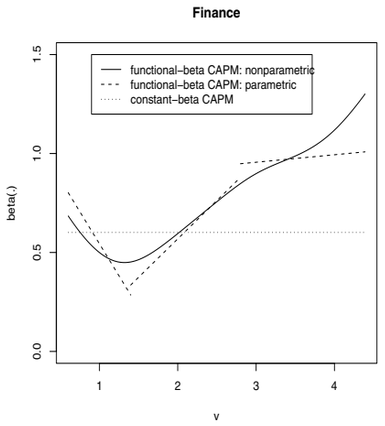
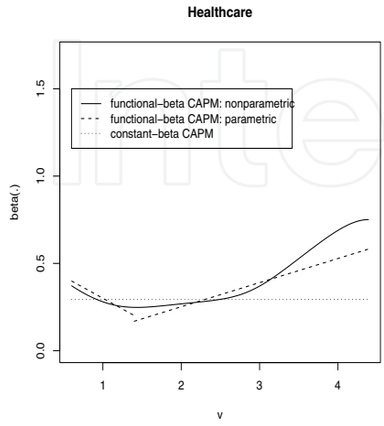


Fig. 6. For Finance sector index:



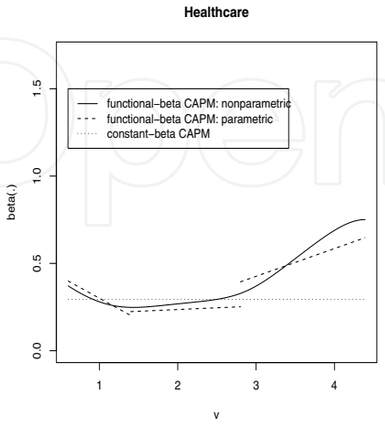


Fig. 7. For Health care sector index:

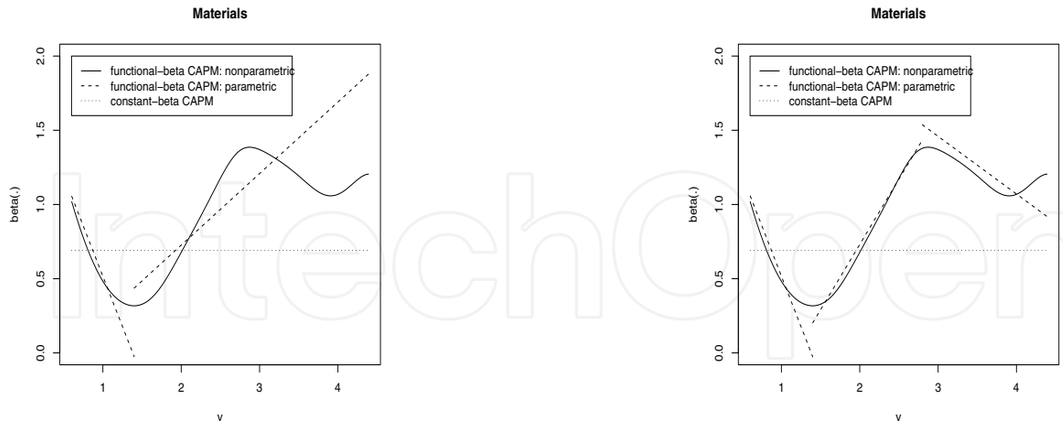


Fig. 8. For Materials sector index:

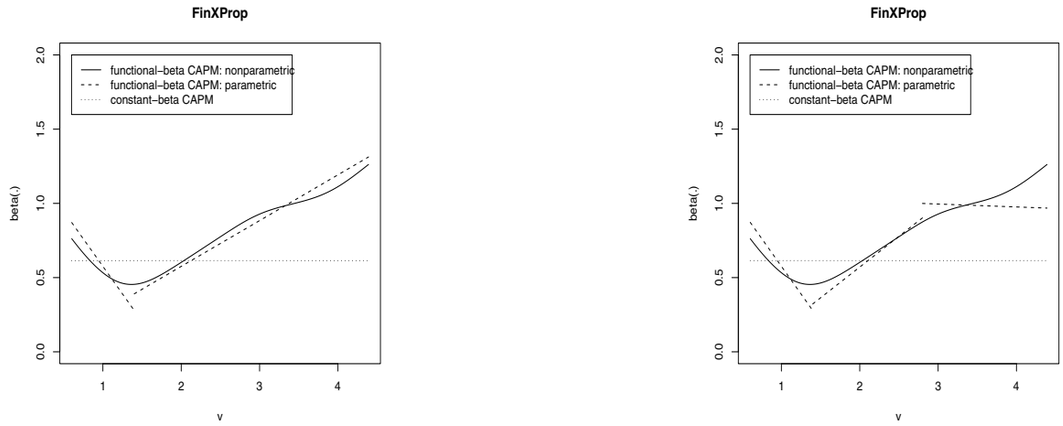


Fig. 9. For Financial-v-Properties Trusts sector index:

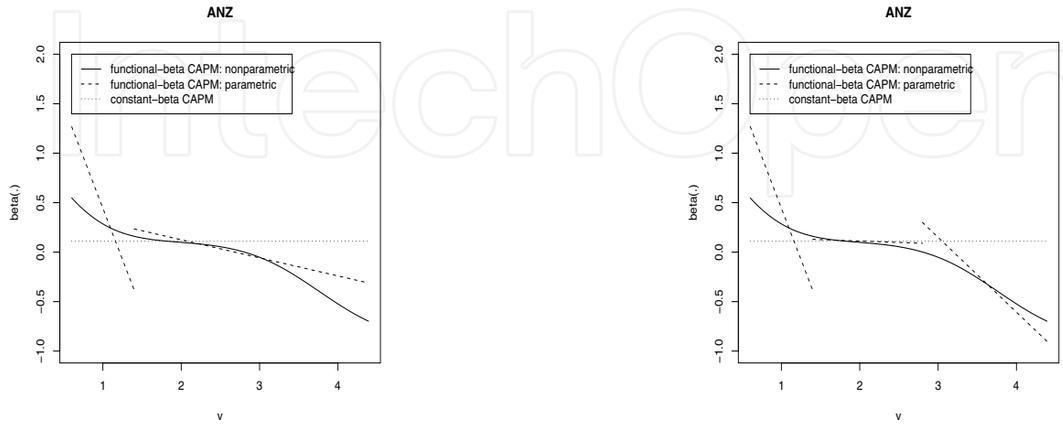


Fig. 10. For ANZ bank group limited:

	Energy	Finance	Healthcare	Materials	Fin-V-Prop	ANZ
Bandwidth	0.45	0.56	0.58	0.38	0.53	0.78

Table 3. Bandwidth Selection

Based on the chosen bandwidth in Table 3, the results of nonparametric estimation of beta functional can be plotted in graphs. For each of eight groups of data (mentioned in Section 3), we can have a curve of beta function plotted in the solid line in Figures 5–10, respectively. As most of the beta functions except for ANZ are positive, it means that the market return has positive effects on all individual sector return. Moreover, the time changing of the beta factor is obvious; it also shows that the individual returns are influenced by the market returns under conditions of market volatility at different levels. We will examine the findings of regime-switching phenomena from the nonparametric estimation more carefully by considering different parametric beta structures in CAPM, which are studied in the next subsection.

4.2 Parametric analysis

In this part we focus on further parametric investigation according to the previous work of nonparametric outcomes. How to specify parametric structures for functional beat? In a recent pioneering work of three-beta CAPM, (Galagedera and Faff 2005) made an assumption that the market conditions can play an important part in explaining a changing beta and could be divided into three states specified as -"low", "neutral" and "high". The nonparametric outcomes in Section 4.1 provide us with some possible ways of parametrization of the beta functional.

To capture the findings of regime-switching phenomena in functional-beta CAPM, we need to suitably specify the switching regimes of market condition in a parametric analysis of functional-beta. The difficult choice of the specific switching regimes of market condition can be suggested in accordance with the nonparametric outcomes of the functional beta, by which we can select reasonable changing points (thresholds) that are needed in parametric estimation. The problem of how many thresholds we should choose in the functional-beta model will be solved by Akaike's information criterion (AIC). This way, we shall have a general flexible functional-beta model which fits the financial data more adaptively.

With reference to the results of non-parametric estimates in Figures 5–10, the market volatility changing regime points σ_L^2 and σ_H^2 are quite amazingly very close to 1.4 and 2.8, respectively, for all the individual sector indexes and the ANZ stock, if we would apply a three-regime (two-threshold) CAPM as done in the three-beta CAPM of (Galagedera and Faff 2005). We therefore run the comparisons of the following five types of parametric models to examine which one appears more flexible and better fitted to the real data. (i) The first one is the traditional CAPM with a constant beta as coefficient. (ii) The second one is similar to the first one but it has a linear functional beta. (iii) In the third one, we divide the market volatility into two regimes and the beta functional can be parameterized as a two stepwise linear function. From the nonparametric estimates in Figures 5–10, it looks reasonable to set $\sigma_L^2 = 1.4$ as the threshold. (iv) In the fourth one, we divide the market volatility into three regimes and the beta functional can be parameterized as a three stepwise function, where we take $\sigma_L^2 = 1.4$ and $\sigma_H^2 = 2.8$. (v)The fifth model is the three-beta CAPM of (Galagedera and Faff 2005), with $\sigma_L^2 = 1.4$ and $\sigma_H^2 = 2.8$. Specifically,

(i) Traditional CAPM: $R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \qquad \beta_i = \beta_{i0}$
(ii)Linear-beta CAPM: $R_{it} = \alpha_i + \beta_{it} R_{mt} + \varepsilon_{it} \qquad \beta_{it} = \beta_{i0} + \beta_{i1}\sigma^2_{Mt}$
(iii) Two-regime (one-threshold) CAPM: $R_{it} = \alpha_i + \beta_{it,L}(I_{Lt}R_{Mt}) + \beta_{it,H}(I_{Ht}R_{Mt}) + \varepsilon_{it}$

$$\begin{cases} \beta_{it,L} = \beta_{i0} + \beta_{i1}\sigma^2_{Mt}, & \sigma^2_{Mt} \le \sigma^2_L \\ \beta_{it,H} = \beta_{i2} + \beta_{i3}\sigma^2_{Mt}, & \sigma^2_{Mt} > \sigma^2_L. \end{cases}$$

(iv) Three-regime (two-threshold) CAPM:
$R_{it} = \alpha_i + \beta_{it,L}(I_{Lt}R_{Mt}) + \beta_{it,N}(I_{Nt}R_{Mt}) + \beta_{it,H}(I_{Ht}R_{Mt}) + \varepsilon_{it}$

$$\begin{cases} \beta_{it,L} = \beta_{i0} + \beta_{i1}\sigma^2_{Mt}, & \sigma^2_{Mt} \le \sigma^2_L \\ \beta_{it,N} = \beta_{i2} + \beta_{i3}\sigma^2_{Mt}, & \sigma^2_L < \sigma^2_{Mt} \le \sigma^2_H \\ \beta_{it,H} = \beta_{i4} + \beta_{i5}\sigma^2_{Mt}, & \sigma^2_{Mt} > \sigma^2_H. \end{cases}$$

(v)Three-beta CAPM: $R_{it} = \alpha_i + \beta_{it,L}(I_{Lt}R_{Mt}) + \beta_{it,N}(I_{Nt}R_{Mt}) + \beta_{it,H}(I_{Ht}R_{Mt}) + \varepsilon_{it}$

$$\begin{cases} \beta_{it,L} = \beta_{i0}, & \sigma^2_{Mt} \le \sigma^2_L \\ \beta_{it,N} = \beta_{i2}, & \sigma^2_L < \sigma^2_{Mt} \le \sigma^2_H \\ \beta_{it,H} = \beta_{i4}, & \sigma^2_{Mt} > \sigma^2_H. \end{cases}$$

Here all the $\alpha_i, \beta_{i0}, \beta_{i1}, \beta_{i2}, \beta_{i3}, \beta_{i4}, \beta_{i5}$ are constants to be estimated by linear regression method.

One important problem in practice is the model selection, that is, which model is the best suitable for a real data set among Models (i)–(v). In order to verify which type of CAPM best suits each group of data respectively, Akaike's information criterion, AIC, is to be applied in this part by minimizing the value of $AIC(m)$, where m is the number of unknown parameters in the model. Note that all 5 models (i)–(v) can be expressed in a linear model in the form $R_i = (R_{i1}, \cdots, R_{iT})' = X\mathbf{b} + (\varepsilon_{i1}, \cdots, \varepsilon_{iT})'$ by suitably defining a $T \times m$ matrix X, where m is the number of parameters in each of the five CAPMs (See Table 4). Then we can define

$$AIC(m) = T \log \widehat{\sigma}^2 + T(2m)$$
$$\widehat{R}_i = HR_i \qquad H = X(X'X)^{-1}X' \tag{20}$$

$$\widehat{\sigma}^2 = \frac{1}{n}\sum_{t=1}^{n}(R_{it} - \widehat{R_{it}})'(R_{it} - \widehat{R_{it}}) = \frac{1}{n}R_i'(I-H)'(I-H)R_i. \tag{21}$$

The results in Table 4 show that except for the health care sector index selecting Model (v), all the other data sets select either Model (iii), Model (iv), which means the beta functional could be divided into two or three regimes. Models (i) and (ii) may be too simple to describe the relationship between market return and individual return.

The two-stepwise beta functional in Model (iii) estimated by using the common changing point σ^2_L for each data set is plotted in dashed line in the left panel of Figures 5–10, respectively, and the three-stepwise beta functional in Model (iv) in dashed line in the right panel of Figures 5–10, respectively. Obviously, due to the sparseness of highly extreme market volatility σ^2_{Mt} (c.f., Figure 4), the results of nonparametric estimation are poor and unreliable at

Model	(i): m=2	(ii): m=3	(iii): m=5	(iv): m=7	(v): m=4	chosen CAPM
Energy	3869.947	3799.963	3778.340	3779.698	3805.315	(iii)
Finance	3677.854	3650.706	3643.563	3645.087	3655.554	(iii)
HealthCare	3321.021	3318.320	3318.096	3320.025	3316.039	(v)
Materials	4137.633	4066.287	4044.999	4026.751	4077.107	(iv)
F-v-P Trusts	3778.767	3755.012	3746.462	3747.036	3758.679	(iii)
ANZ bank	4577.802	4571.992	4558.787	4558.482	4578.409	(iv)

Table 4. $AIC(m)$ and The Type of CAPM Chosen

extreme market volatility, while the parametric results of two-stepwise or three-stepwise beta functionals provide reasonable outcomes in Figures 5–10. Under moderate market volatility, both nonparametric and parametric outcomes are pretty consistent.

In (Galagedera and Faff 2005), the functional beta is assumed as three constants over three regimes, which is a special case of Model (iv) with $\beta_{i1} = \beta_{i3} = \beta_{i5} = 0$. To examine their work, here we test the significance of $\beta_{i1},\ \beta_{i3},\ \beta_{i5}$ by applying T-statistics:

$$H_0 : \beta_{i1} = \beta_{i3} = \beta_{i5} = 0 \tag{22}$$

Applying linear regression method we get $\widehat{\mathbf{b}}$, and residuals $\widehat{r}$:

$$\hat{\mathbf{b}} = (X'X)^{-1}X'R_i,\ where\ \mathbf{b} = (\alpha_i, \beta_{i0}, \beta_{i1}, \beta_{i2}, \beta_{i3}, \beta_{i4}, \beta_{i5})', \tag{23}$$

and let δ stand for the standard deviation of $\widehat{r} = R_i - X\widehat{\mathbf{b}}$. Then the T-statistics value of each estimated parameter is $T_j = \widehat{b}_j / \{(XX^T)^{-1}_{jj}\delta^2\}^{1/2}$, where $\widehat{b}_j$ represents the jth element of $\widehat{\mathbf{b}}$. In a standard normal distribution, only 5% of the values fall outside the range plus-or-minus 2. Hence, as a rough rule of thumb, a t-statistic larger than 2 in absolute value would be significant at the significance level of 5%. The outcomes of the T-statistics with p-values for Models (iii), (iv) and (v) are listed in Tables 5–10, respectively, which indicate that the (Galagedera and Faff 2005)' three-beta model is basically rejected except for the health care sector index.

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}	AIC
Threshold CAPM (2 step)								3778.340
Estimate	-0.0222	1.5001	-1.0176	-0.4794	0.5093			
T statistic	-0.4199	4.2287	-3.2583	-2.9629	7.9788			
p-value	0.6746	2.6e-05	0.0012	0.0031	4.1e-15			
Threshold CAPM (3 step)								3779.698
Estimate	-0.0192	1.5024	-1.0196	-0.5792	0.5527	0.5988	0.1933	
T statistic	-0.3625	4.2365	-3.2654	-2.2719	4.5061	0.8689	0.9408	
p-value	0.7171	2.5e-05	0.0011	0.0233	7.4e-06	0.3851	0.3470	
Three-beta CAPM (3 step)								3805.315
Estimate	-0.0336	0.3597		0.5410		1.2432		
T statistic	-0.6315	6.2018		9.2367		14.035		
p-value	0.5278	8.2e-10		1.5e-19		6.4e-41		

Table 5. Estimate, T-statistic and p-value for $\hat{\alpha}$ and each component of $\hat{\beta}$ in Models (iii), (iv) and (v) for Energy Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}	AIC
Threshold CAPM (2 step)								3643.563
Estimate	0.0332	1.1911	-0.6470	-0.0248	0.2982			
T statistic	0.6728	3.5952	-2.2182	-0.1642	5.0018			
p-value	0.5013	0.0003	0.0268	0.8696	6.7e-07			
Threshold CAPM (3 step)								3645.087
Estimate	0.0355	1.1928	-0.6484	-0.2079	0.3883	0.8428	0.0377	
T statistic	0.7187	3.6012	-2.2237	-0.8730	3.3893	1.3094	0.1965	
p-value	0.4725	0.0003	0.0264	0.3829	0.0007	0.1907	0.8442	
Three-beta CAPM (3 step)								3655.554
Estimate	0.0255	0.4660		0.5790		0.9690		
T statistic	0.5174	8.6687		10.667		11.805		
p-value	0.6050	1.8e-17		3.3e-25		3.6e-30		

Table 6. Estimate, T-statistic and p-value for $\hat{\alpha}$ and each component of $\hat{\beta}$ in Models (iii), (iv) and (v) for Finance Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}	AIC
Threshold CAPM (2 step)								3318.096
Estimate	0.0080	0.5473	-0.2468	-0.0246	0.1382			
T statistic	0.1909	1.9487	-0.9983	-0.1924	2.7342			
p-value	0.8486	0.0516	0.3183	0.8475	0.0064			
Threshold CAPM (3 step)								3320.025
Estimate	0.0082	0.5475	-0.2469	0.1961	0.0198	-0.0505	0.1589	
T statistic	0.1969	1.9494	-0.9990	0.9716	0.2043	-0.0926	0.9764	
p-value	0.8440	0.0515	0.3181	0.3315	0.8381	0.9263	0.3291	
Three-beta CAPM (3 step)								3316.039
Estimate	0.0054	0.2707		0.2365		0.4784		
T statistic	0.1308	5.9824		5.1772		6.9238		
p-value	0.8960	3.1e-09		2.7e-07		7.9e-12		

Table 7. Estimate, T-statistic and p-value for $\hat{\alpha}$ and each component of $\hat{\beta}$ in Models (iii), (iv) and (v) for Health care Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}	AIC
Threshold CAPM (2 step)								4044.999
Estimate	-0.0213	1.8714	-1.3549	-0.2381	0.4821			
T statistic	-0.3518	4.6074	-3.7889	-1.2850	6.5962			
p-value	0.7251	4.6e-06	0.0002	0.1991	6.9e-11			
Threshold CAPM (3 step)								4026.751
Estimate	-0.0140	1.8769	-1.3594	-1.0294	0.8800	2.6250	-0.3886	
T statistic	-0.2341	4.6686	-3.8409	-3.5619	6.3288	3.3600	-1.6685	
p-value	0.8150	3.5e-06	0.0001	0.0004	3.8e-10	0.0008	0.0955	
Three-beta CAPM (3 step)								4077.107
Estimate	-0.0392	0.3529		0.7542		1.3335		
T statistic	-0.6415	5.3006		11.218		13.115		
p-value	0.5214	1.4e-07		1.5e-27		2.5e-36		

Table 8. Estimate, T-statistic and p-value for $\hat{\alpha}$ and each component of $\hat{\beta}$ in Models (iii), (iv) and (v) for Materials Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}	AIC
Threshold CAPM (2 step)								3746.462
Estimate	0.0238	1.3180	-0.7423	-0.0411	0.3083			
T statistic	0.4574	3.7760	-2.4154	-0.2579	4.9091			
p-value	0.6475	0.0002	0.0159	0.7966	1.1e-06			
Threshold CAPM (3 step)								3747.036
Estimate	0.0267	1.3202	-0.7441	-0.2572	0.4141	1.0547	-0.0197	
T statistic	0.5133	3.7850	-2.4231	-1.0259	3.4323	1.5560	-0.0975	
p-value	0.6078	0.0002	0.0156	0.3052	0.0006	0.1200	0.9224	
Three-beta CAPM (3 step)								3758.679
Estimate	0.0146	0.4861		0.5820		0.9900		
T statistic	0.2780	8.5826		10.176		11.445		
p-value	0.7797	3.6e-17		3.4e-23		1.5e-28		

Table 9. Estimate, T-statistic and p-value for $\hat{\alpha}$ and each component of $\hat{\beta}$ in Models (iii), (iv) and (v) for Financial-x-trusts Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}	AIC
Threshold CAPM (2 step)								4558.787
Estimate	0.0851	2.5144	-2.0707	0.4902	-0.1826			
T statistic	1.0836	4.7693	-4.4614	2.03844	-1.9251			
p-value	0.2788	2.1e-06	9.1e-06	0.0418	0.0545			
Threshold CAPM (3 step)								4558.482
Estimate	0.0903	2.5183	-2.0740	0.1687	-0.0282	2.4172	-0.7560	
T statistic	1.1504	4.7822	-4.4736	0.4455	-0.1547	2.3621	-2.4779	
p-value	0.2503	2.0e-06	8.6e-06	0.6560	0.8771	0.0184	0.0134	
Three-beta CAPM (3 step)								4578.409
Estimate	0.0453	0.1929		0.1149		-0.0954		
T statistic	0.5749	2.2468		1.3247		-0.7278		
p-value	0.5655	0.0249		0.1856		0.4669		

Table 10. Estimate, T-statistic and p-value for $\hat{\alpha}$ and each component of $\hat{\beta}$ in Models (iii), (iv) and (v) for ANZ bank group limited

5. Closure and future work

In this chapter, we have suggested a functional-beta single-index CAPM, extending the work of three-beta CAPM (Galagedera and Faff, 2004) that takes into account the condition of market volatility. Differently from the three-beta CAPM, we allow systematic risk β_i changing functionally with the market volatility σ_M^2, which is more flexible and adaptive to the changing structure of financial systems.

By using a set of stocks data sets collected from Australian stock market, empirical evidences of the functional-beta CAPM in Australia have been carefully examined under both nonparametric and parametric model structures. Differently from the three- or multi-beta (constant) CAPM in (Galagedera and Faff 2005), our new findings have convincingly demonstrated that the functional beta can be reasonably parameterized as threshold (regime-switching) linear functionals of market volatility with two or three regimes of market condition, taking as a special case the three-beta model of (Galagedera and Faff 2005) which was mostly rejected except for the health care sector index. In the condition of extreme market volatility, a parametric threshold functional-beta CAPM is found useful, which is of potential interest in exploring the Black Swan effect of the merits of beta in the presence of large market fluctuations.

The CAPM provides a usable measure of risk that helps investors determine what return they deserve for putting their money at risk. Our new model is no doubt helpful to better understand the relationship between risk and return under different market conditions. It can be potentially applied widely, for example, it may be useful both for market investors and financial risk managers in their investment/management decision-making, such as portfolio selection.

In addition, as done in (Galagedera and Faff 2005), it is interesting to investigate how the functional beta systematic risk is priced in the real financial assets.

We shall leave the above questions for future work.

6. Acknowledgements

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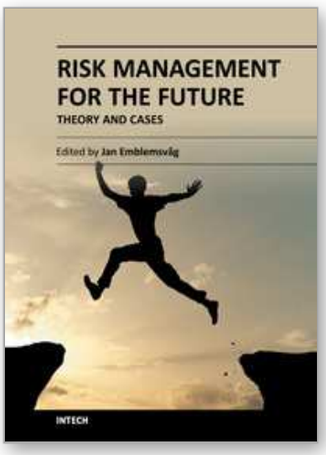

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A large part of academic literature, business literature as well as practices in real life are resting on the assumption that uncertainty and risk does not exist. We all know that this is not true, yet, a whole variety of methods, tools and practices are not attuned to the fact that the future is uncertain and that risks are all around us. However, despite risk management entering the agenda some decades ago, it has introduced risks on its own as illustrated by the financial crisis. Here is a book that goes beyond risk management as it is today and tries to discuss what needs to be improved further. The book also offers some cases.

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