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Time-Varying Discrete-Time Wavelet Transforms

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1. Introduction

Discrete-time wavelet transform (DWT) is found to be better than other transforms in the time-varying system analysis, e.g. for time-varying parametric modelling [16], time-varying systems identification [17], time-varying parameter estimation [18] and time domain signal analysis [19]. In the literature the common method to analyze the time-varying system using discrete-time wavelet transform is to model the time-varying system with a time-invariant system firstly, because a general analysis of time-varying discrete-time wavelet transform (TV-DWT) is still missing. To analyze the time-varying system directly using the time-varying discrete-time wavelet transform, we need the theory for the time-varying discrete-time wavelet transform.

The theory of time-invariant discrete-time wavelet transform (DWT) are quite complete [1,2,3]. For time-varying discrete-time wavelet transform, in literature there are some papers related with this topic by studying the changes of two different filter banks [10,11,12]. In [10] the authors analyzed the time-varying wavelet transform through changing the two-band filter banks used in the tree-structured implementation of DWTs with an simple example. In [11] the time-varying wavelet packets built with time-varying cosine-modulated filter banks were investigated. Similar with [10], in [12] the authors studied time-varying wavelet packets more theoretically with changing the two orthogonal two-band filter banks used in tree-structure of DWTs. Generally, in the existed theory of time-varying discrete-time wavelet transform it lacks a basic definition and description of the time-varying discrete-time wavelet transform. A basic analysis of time-varying discrete-time wavelet transform is also missing. The author has studied TV-DWT since some years and has published a series of papers about this topic. In this Chapter we summarize the author's main research results.

In our method the time-varying discrete-time wavelet transform is studied using a time-varying octave-band filter bank with tree structure. With this implementation the analysis of the time-varying discrete-time wavelet transform is equal to the analysis of the time-varying discrete-time octave-band filter bank. Then, the time-varying filter bank theory can be used in TV-DWT analysis. In this chapter we provide some theorems for the time-varying discrete-time wavelet transform with proofs.

2. Formulation of time-varying discrete-time wavelet transforms

From the point of view of digital signal processing, the time-varying discrete-time wavelet transform can be implemented by a time-varying octave-band filter bank with tree structure. Fig. 1 shows the most general time-varying discrete-time wavelet transform implemented with a time-varying octave-band filter bank, where the lowpass and highpass filter $H_l(z, m)$, $H_u(z, m)$, the stage number of the split-merge $J(m)$, all are varying with time index m . In other words, both the frequency characteristic and the time-frequency tiling of the discrete-time wavelet transform are varying with time. Fig. 2 shows the time-varying nonuniform filter bank implementation. With this implementation the analysis of the time-varying discrete-time wavelet transform is equal to the analysis of the time-varying discrete-time octave-band filter bank.

Note that we define the time-varying discrete-time wavelet transform varying with index m which is equivalent to the output index at the last stage of octave-band filter banks. The time indices of the other output are related to m by

$$m_j = 2^{J(m)-i-j} \cdot m, \quad 0 \leq j \leq J(m) - 2. \quad (1)$$

In the literature there are some papers related with this topic by studying changes between two time-invariant filter banks [10,11,12]. In particular, in [10] the authors have discussed the transition behavior during the change between two time-invariant discrete-time wavelet transforms. Different from the existed publications, in this chapter we analyze the general time-varying discrete-time wavelet transform in detail based on the octave-band filter bank and the nonuniform filter bank implementation.

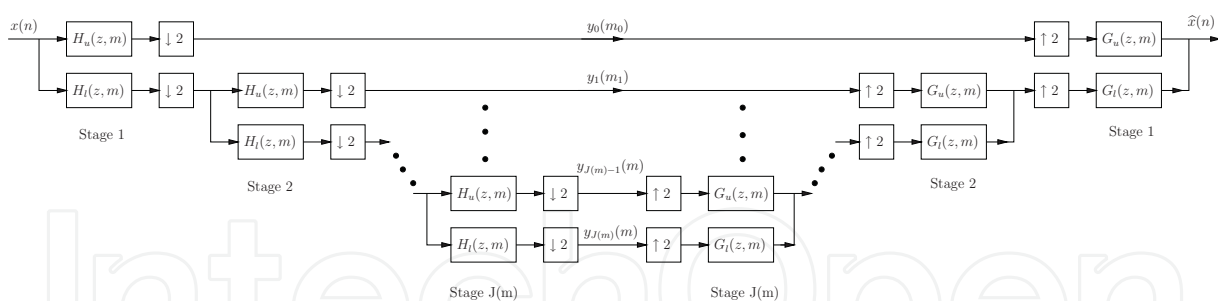


Fig. 1. Time-varying discrete-time wavelet transform implemented with time-varying octave-band filter banks.

3. Implementation with time-varying octave-band filter Banks

To make the analysis simple, in the following analysis we suppose that the stage number J does not change with time and is a constant. Then we get a J -stage time-varying octave-band filter bank. Just as depicted in Fig. 1, a J -stage octave-band time-varying filter bank consists of J stages of two-channel time-varying filter bank. In the analysis side, the input signal $x(n)$ is first split by the two-channel time-varying filter bank at the first stage, then the lowpass output is split again by the same two-band time-varying filter bank at the second stage.

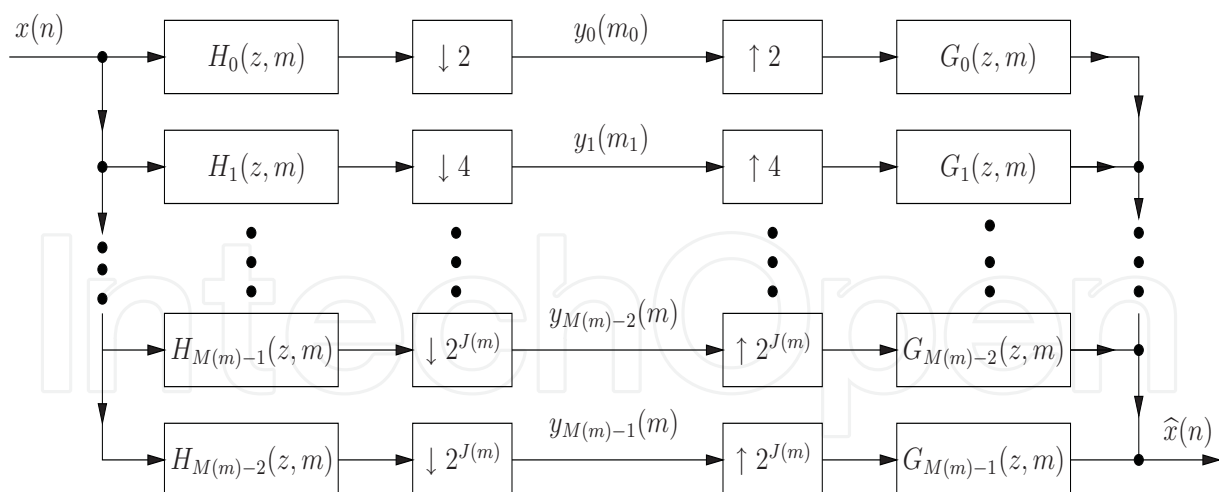


Fig. 2. Time-varying nonuniform filter bank implementation, where $M(m) = 2^{J(m)}$.

The process is ongoing until J -stage. In the synthesis side, the signal is merged to generate the reconstructed signal $\hat{x}(n)$. From the theorem of time-invariant discrete-time wavelet transform [2], we know that if the individual two-channel filter bank, or each split-merge pair is perfectly reconstructed, the octave-band filter bank is as well. Such statement is also valid for the time-varying octave-band filter bank. Therefore, we have following theorem.

Theorem 1: A time-varying discrete-time wavelet transform implemented with a time-varying octave-band filter bank is a biorthogonal time-varying transform if each two-channel time-varying filter bank is perfectly reconstructed.

We cannot use the method used in the time-invariant case to prove the above theorem because the system is time-varying. To prove theorem 1, we define analysis and synthesis matrices of the j -stage two-channel time-varying filter bank shown in Fig. 3 as

$$\mathbf{T}_{ma}^{(j)} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \mathbf{h}_0(m_j) & \mathbf{h}_1(m_j) & \cdots & \mathbf{h}_{N(j)-1}(m_j) & \mathbf{h}_{N(j)}(m_j) & \mathbf{h}_{N(j)+1}(m_j) & \cdots \\ \cdots & 0 & \mathbf{h}_0(m_j) & \cdots & \mathbf{h}_{N(j)-2}(m_j) & \mathbf{h}_{N(j)-1}(m_j) & \mathbf{h}_{N(j)}(m_j) & \cdots \\ \cdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots \\ \cdots & 0 & 0 & \cdots & \mathbf{h}_0(m_j) & \mathbf{h}_1(m_j) & \mathbf{h}_2(m_j) & \cdots \\ \cdots & 0 & 0 & \cdots & 0 & \mathbf{h}_0(m_j+1) & \mathbf{h}_1(m_j+1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \quad (2)$$

$$\mathbf{T}_{ms}^{(j)} = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \mathbf{g}_0(m_j) & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \mathbf{g}_1(m_j) & \mathbf{g}_0(m_j) & \cdots & \mathbf{0} & \mathbf{0} & \cdots \\ \cdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ \cdots & \mathbf{g}_{N(j)-1}(m_j) & \mathbf{g}_{N(j)-2}(m_j) & \cdots & \mathbf{g}_0(m_j) & \mathbf{0} & \cdots \\ \cdots & \mathbf{g}_{N(j)}(m_j) & \mathbf{g}_{N(j)-1}(m_j) & \cdots & \mathbf{g}_1(m_j) & \mathbf{g}_0(m_j+1) & \cdots \\ \cdots & \mathbf{g}_{N(j)+1}(m_j) & \mathbf{g}_{N(j)}(m_j) & \cdots & \mathbf{g}_2(m_j) & \mathbf{g}_1(m_j+1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}, \quad (3)$$

where

$$N(j) = 2^{J-j}, \quad j = 1, 2, \dots, J, \quad (4)$$

and

$$\mathbf{h}_i(m_j) = \begin{bmatrix} h_u(L-2i-1, m_j) & h_u(L-2i-2, m_j) \\ h_l(L-2i-1, m_j) & h_l(L-2i-2, m_j) \end{bmatrix}, \quad (5)$$

$$\mathbf{g}_i(m_j) = \begin{bmatrix} g_u(2i, m_j) & g_l(2i, m_j) \\ g_u(2i+1, m_j) & g_l(2i+1, m_j) \end{bmatrix}, \quad (6)$$

where L is the filter length.

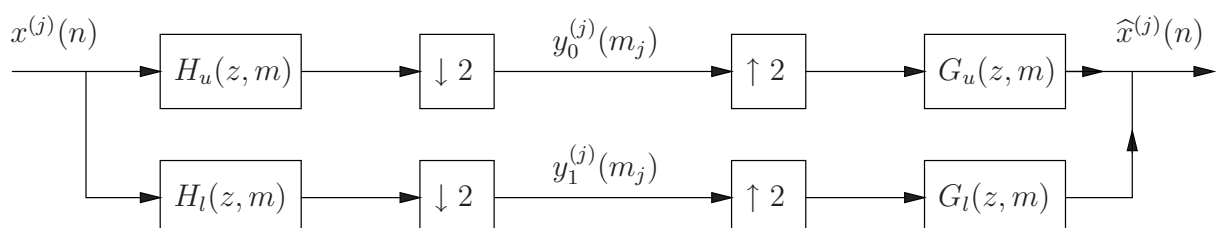


Fig. 3. The j -th stage two-channel time-varying filter bank.

Furthermore, we define two special matrices Λ_0 and Λ_1

$$\Lambda_0 = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & 1 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (7)$$

$$\Lambda_1 = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & 0 & 1 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}, \quad (8)$$

to extract the lowpass and highpass output like

$$\mathbf{y}_0^{(j)} = \Lambda_0 \mathbf{y}^{(j)}, \quad (9)$$

$$\mathbf{y}_1^{(j)} = \Lambda_1 \mathbf{y}^{(j)}, \quad (10)$$

where

$$\mathbf{y}^{(j)} = [\cdots y_0^{(j)}(-1) y_1^{(j)}(-1) y_0^{(j)}(0) y_1^{(j)}(0) y_0^{(j)}(1) y_1^{(j)}(1) \cdots]^T, \quad (11)$$

$$\mathbf{y}_0^{(j)} = [\cdots y_0^{(j)}(-1) y_0^{(j)}(0) y_0^{(j)}(1) \cdots]^T, \quad (12)$$

$$\mathbf{y}_1^{(j)} = [\cdots y_1^{(j)}(-1) y_1^{(j)}(0) y_1^{(j)}(1) \cdots]^T. \quad (13)$$

Based on the above matrix definitions we can describe the filter bank at the j -th stage showed in Fig. 3 as

$$\hat{\mathbf{x}}^{(j)} = \mathbf{T}_{ms}^{(j)} \mathbf{T}_{ma}^{(j)} \mathbf{x}^{(j)}. \quad (14)$$

After adding the $(j+1)$ -th stage with a biorthogonal time-varying two-channel filter bank shown in Fig. 4, we have

$$\begin{aligned} \hat{\mathbf{x}}^{(j)} &= \mathbf{T}_{ms}^{(j)} \mathbf{y}^{(j)} \\ &= \mathbf{T}_{ms}^{(j)} \left\{ \Lambda_0^T \mathbf{y}_0^{(j)} + \Lambda_1^T \mathbf{y}_1^{(j)} \right\} \\ &= \mathbf{T}_{ms}^{(j)} \left\{ \Lambda_0^T \Lambda_0 \mathbf{T}_{ma}^{(j)} \mathbf{x}^{(j)} + \Lambda_1^T \mathbf{T}_{ms}^{(j+1)} \mathbf{T}_{ma}^{(j+1)} \Lambda_1 \mathbf{T}_{ma}^{(j)} \mathbf{x}^{(j)} \right\}. \end{aligned} \quad (15)$$

Because we suppose that the added two-channel filter bank is biorthogonal, we have

$$\mathbf{T}_{ms}^{(j+1)} \mathbf{T}_{ma}^{(j+1)} = \mathbf{I}, \quad (16)$$

$$\Lambda_0^T \Lambda_0 + \Lambda_1^T \Lambda_1 = \mathbf{I}. \quad (17)$$

Then, we can rewrite (15) as

$$\begin{aligned} \hat{\mathbf{x}}^{(j)} &= \mathbf{T}_{ms}^{(j)} \left\{ \Lambda_0^T \Lambda_0 + \Lambda_1^T \Lambda_1 \right\} \mathbf{T}_{ma}^{(j)} \mathbf{x}^{(j)} \\ &= \mathbf{T}_{ms}^{(j)} \mathbf{T}_{ma}^{(j)} \mathbf{x}^{(j)} \\ &= \mathbf{x}^{(j)} \end{aligned} \quad (18)$$

which means that the time-varying octave-band filter bank is still perfectly reconstructed after adding next stage of time-varying biorthogonal two-channel filter bank. In other words, theorem 1 is correct.

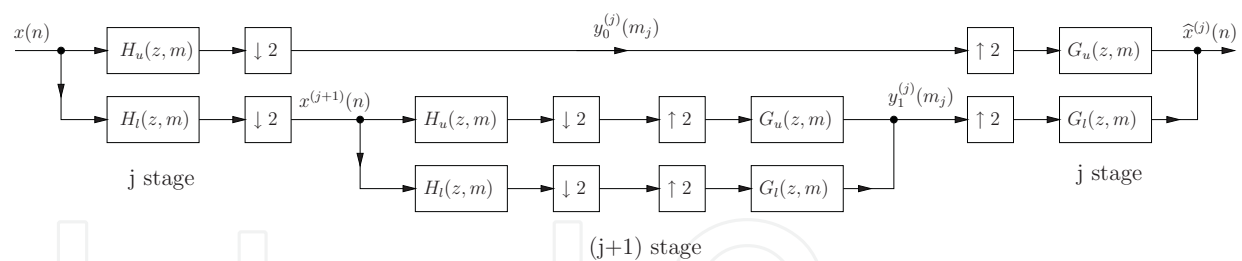


Fig. 4. Adding the $(j + 1)$ -th stage.

4. Implementation with time-varying nonuniform filter banks

Fig. 2 shows another implementation of a time-varying wavelet transform with $(J(m) + 1)$ -channel time-varying nonuniform filter bank. To make the analysis easy we suppose that $J(m)$ does not change with time and is equal to constant J . For analysis of the $(J + 1)$ -channel time-varying nonuniform filter bank we first reconstruct the nonuniform filter bank to a time-varying uniform filter bank through adding following filters between $H_i(z, m)$ and $H_{i+1}(z, m)$ ($0 \leq j \leq J - 1$)

$$H_{i,k}(z, m) = z^{-k \cdot 2^{i+1}} H_i(z, m), \quad 1 \leq k \leq 2^{J-i-1} - 1. \quad (19)$$

After adding additional filters in the nonuniform filter bank in Fig. 2 the filter bank becomes M -channel time-varying uniform filter filter bank. The number of channel M is calculated by

$$\begin{aligned} M &= \sum_{i=0}^{J-2} (2^{J-i-1} - 1) + (J + 1) \\ &= 2^{J-1} \sum_{i=0}^{J-2} 2^{-i} + 2 \\ &= 2^J (1 - 2 \cdot 2^{-J}) + 2 \\ &= 2^J. \end{aligned} \quad (20)$$

For the time-varying system in Fig. 5 we have following theorem.

Theorem 2: A time-varying discrete-time wavelet transform implemented with a time-varying nonuniform filter bank is biorthogonal if each two-channel time-varying filter bank in its tree-structured implementation is perfectly reconstructed.

To prove theorem 2, we need to describe the filter $H_i(z, m)$ in Fig. 5 based on the tree structure in Fig. 1. In the time-invariant discrete-time wavelet transform the description of such filters can be simply got using the convolution role in the transform-domain. However, in the time-varying case, we cannot describe $H_i(z, m)$ as product of functions in the previous stages, like $H_0(z, m)H_1(z^2, m)$, because the system is time-varying and the convolution role does not exist. Referencing to definitions of $\mathbf{T}_{ma}^{(j)}$ and $\mathbf{T}_{ms}^{(j)}$ in (2) and (3), we find that the analysis output

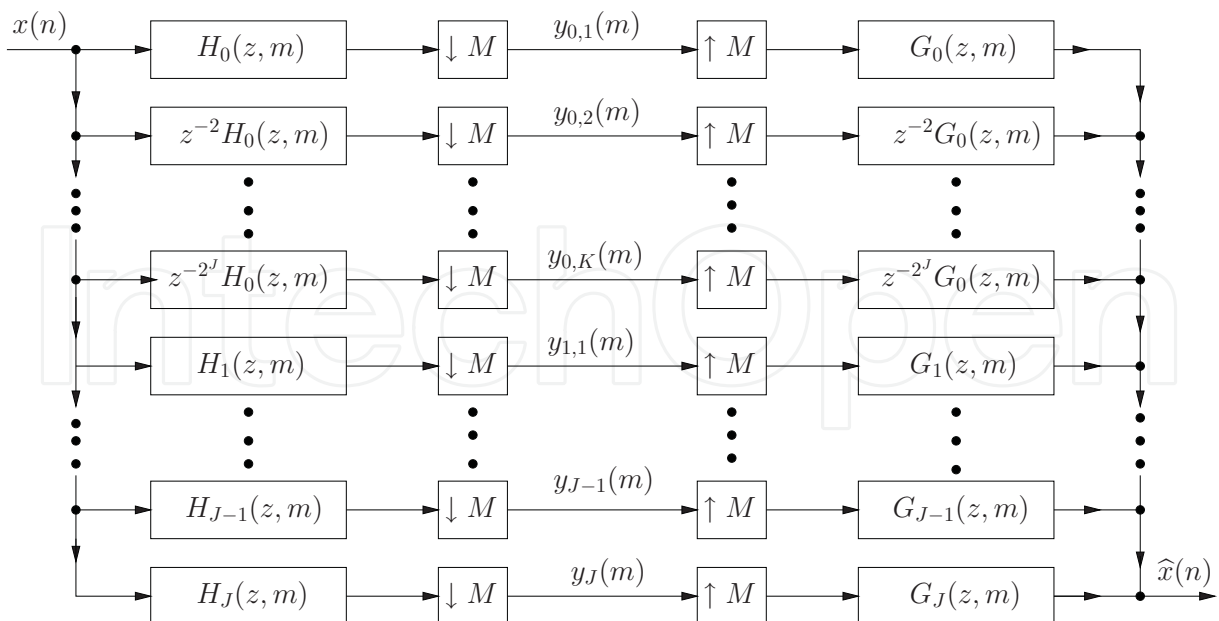


Fig. 5. The equivalent M -channel time-varying uniform filter bank.

$\mathbf{y}_0(m_0)$ can be expressed as

$$\mathbf{y}_0 = \Lambda_0 \mathbf{T}_{ma}^{(1)} \mathbf{x}, \quad (21)$$

where

$$\mathbf{y}_0 = [\cdots \mathbf{y}_0(-1) \mathbf{y}_0(0) \mathbf{y}_0(1) \cdots]^T, \quad (22)$$

$$\mathbf{y}_0(m) = [y_{0,1}(m) y_{0,2}(m) \cdots y_{0,K}(m)]^T, \quad (23)$$

and $K = 2^{2^J-1} - 1$. In general, we have

$$\mathbf{y}_{j-1} = \underbrace{\Lambda_0 \mathbf{T}_{ma}^{(j)} \Lambda_1 \mathbf{T}_{ma}^{(j-1)} \cdots \Lambda_1 \mathbf{T}_{ma}^{(1)}}_{\mathbf{H}_{j-1}} \mathbf{x}, \quad (24)$$

where $1 \leq j \leq J-1$, and

$$\mathbf{y}_{J-1} = \Lambda_0 \mathbf{T}_{ma}^{(J)} \Lambda_1 \mathbf{T}_{ma}^{(J-1)} \cdots \Lambda_1 \mathbf{T}_{ma}^{(1)} \mathbf{x} = \mathbf{H}_{J-1} \mathbf{x}, \quad (25)$$

$$\mathbf{y}_J = \Lambda_1 \mathbf{T}_{ma}^{(J)} \Lambda_1 \mathbf{T}_{ma}^{(J-1)} \cdots \Lambda_1 \mathbf{T}_{ma}^{(1)} \mathbf{x} = \mathbf{H}_J \mathbf{x}, \quad (26)$$

where

$$\mathbf{y}_{j-1} = [\cdots \mathbf{y}_{j-1}(-1) \mathbf{y}_{j-1}(0) \mathbf{y}_{j-1}(1) \cdots]^T, \quad (27)$$

$$\mathbf{y}_{j-1}(m) = [y_{j-1,1}(m) y_{j-1,2}(m) \cdots y_{j-1,K}(m)]^T, \quad (28)$$

for $K = 2^{J-j} - 1$ and $1 \leq j \leq J - 2$, and

$$\mathbf{y}_{J-1} = [\cdots y_{J-1}(-1) y_{J-1}(0) y_{J-1}(1) \cdots]^T, \quad (29)$$

$$\mathbf{y}_J = [\cdots y_J(-1) y_J(0) y_J(1) \cdots]^T. \quad (30)$$

At synthesis side, we have similar definitions as

$$\hat{\mathbf{x}}_{j-1} = \underbrace{\mathbf{T}_{ms}^{(1)} \Lambda_1^T \mathbf{T}_{ms}^{(2)} \cdots \Lambda_1^T \mathbf{T}_{ms}^{(j)} \Lambda_0^T}_{\mathbf{G}_{j-1}} \mathbf{y}_{j-1}, \quad (31)$$

$$\hat{\mathbf{x}}_{J-1} = \mathbf{T}_{ms}^{(1)} \Lambda_1^T \mathbf{T}_{ms}^{(2)} \cdots \Lambda_1^T \mathbf{T}_{ms}^{(J)} \Lambda_0^T \mathbf{y}_{J-1} = \mathbf{G}_{J-1} \mathbf{y}_{J-1}, \quad (32)$$

$$\hat{\mathbf{x}}_J = \mathbf{T}_{ms}^{(1)} \Lambda_1^T \mathbf{T}_{ms}^{(2)} \cdots \Lambda_1^T \mathbf{T}_{ms}^{(J)} \Lambda_1^T \mathbf{y}_J = \mathbf{G}_J \mathbf{y}_J, \quad (33)$$

Now, based on the definition in (23), we can build the analysis output vector for the time-varying filter bank in Fig. 5 as

$$\mathbf{y} = [\cdots \mathbf{y}_0(-1) \cdots \mathbf{y}_J(-1) \mathbf{y}_0(0) \cdots \mathbf{y}_J(0) \mathbf{y}_0(1) \cdots]^T. \quad (34)$$

Suppose that \mathbf{T}_{ma} and \mathbf{T}_{ms} are the analysis and synthesis matrices for the time-varying filter bank in Fig. 5. Referencing (34), \mathbf{T}_{ma} is constructed by interleaving the rows from $\mathbf{T}_{ma}^{(1)}$ to $\mathbf{T}_{ma}^{(J)}$ with same time index m , \mathbf{T}_{ms} is built with similar way, but interleaving the columns. Then, the production $\mathbf{T}_{ma}^{(1)} \mathbf{T}_{ma}^{(J)}$ can be expressed by

$$\mathbf{T}_{ms} \mathbf{T}_{ma} = \sum_{j=0}^J \mathbf{G}_j \mathbf{H}_j. \quad (35)$$

Substituting \mathbf{H}_i and \mathbf{G}_i defined in (24)-(26) and (31)-(33) into (35), and using properties in (16) and (17), we get

$$\mathbf{T}_{ms} \mathbf{T}_{ma} = \mathbf{I}, \quad (36)$$

which means that the time-varying nonuniform filter bank in Fig. 2 is perfectly reconstructed.

Finally, we give another property related with filter coefficients of the time-varying filter bank in Fig. 2. Suppose that $h_i(n, m)$ and $g_i(n, m)$ represent the analysis and synthesis filter coefficients in Fig. 2, then we have following equation

$$\langle g_i(n - kM, m + r), h_j(n - lM, m + s) \rangle = \delta(k - l) \delta(i - j) \delta(r - s), \quad (37)$$

where $M = 2^J$. The proof of equation (37) can be simply got by using the PR condition in (36).

5. Conclusion

In the theory of discrete-time signal expansion, the wavelet transform is very important. In this chapter, we defined the general discrete time-varying dyadic wavelet transform and analyzed its properties in detail. Some theorems describing properties of time-varying discrete-time wavelet transforms were presented. The conditions for a biorthogonal time-varying discrete-time wavelet transform were given. The theory and algorithms presented in this chapter can be used in design of time-varying discrete-time signal expansion in practice.

6. Acknowledgments

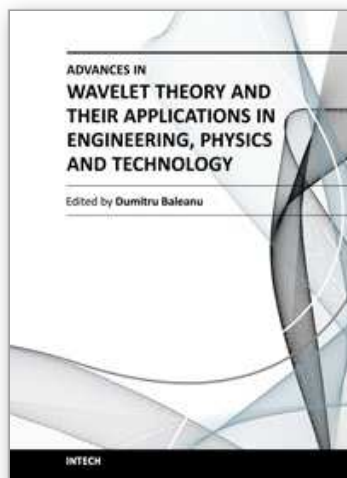
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The use of the wavelet transform to analyze the behaviour of the complex systems from various fields started to be widely recognized and applied successfully during the last few decades. In this book some advances in wavelet theory and their applications in engineering, physics and technology are presented. The applications were carefully selected and grouped in five main sections - Signal Processing, Electrical Systems, Fault Diagnosis and Monitoring, Image Processing and Applications in Engineering. One of the key features of this book is that the wavelet concepts have been described from a point of view that is familiar to researchers from various branches of science and engineering. The content of the book is accessible to a large number of readers.

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