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Ranking Indices for Fuzzy Numbers

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1. Introduction

Fuzzy set theory has been studied extensively over the past 30 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes (see for example Zadeh (1965)). Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. In an effort to gain a better understanding of the use of fuzzy set theory in production management research and to provide a basis for future research, a literature review of fuzzy set theory in production management has been conducted. While similar survey efforts have been undertaken for other topical areas, there is a need in production management for the same. Over the years there have been successful applications and implementations of fuzzy set theory in production management. Fuzzy set theory is being recognized as an important problem modeling and solution technique.

Kaufmann and Gupta (1988) report that over 7,000 research papers, reports, monographs, and books on fuzzy set theory and applications have been published since 1965.

As evidenced by the large number of citations found, fuzzy set theory is an established and growing research discipline. The use of fuzzy set theory as a methodology for modeling and analyzing decision systems is of particular interest to researchers in production management due to fuzzy set theory's ability to quantitatively and qualitatively model problems which involve vagueness and imprecision. Karwowski and Evans (1986) identify the potential applications of fuzzy set theory to the following areas of production management: new product development, facilities location and layout, production scheduling and control, inventory management, quality and cost benefit analysis. Karwowski and Evans identify three key reasons why fuzzy set theory is relevant to production management research. First, imprecision and vagueness are inherent to the decision maker's mental model of the problem under study. Thus, the decision maker's experience and judgment may be used to complement established theories to foster a better understanding of the problem. Second, in the production management environment, the information required to formulate a model's objective, decision variables, constraints and parameters may be vague or not precisely measurable. Third, imprecision and vagueness as a result of personal bias and subjective opinion may further dampen the quality and quantity of available information. Hence, fuzzy set theory can be used to bridge modeling gaps in descriptive and prescriptive decision models in production management research.

2. Fuzzy ranking and neural network

Ordering fuzzy subsets is an important event in dealing with fuzzy decision problems in many areas. This issue has been of concern for many researchers over the years. Also, in the last several years, there has been a large and energetic upswing in neuroengineering research aimed at synthesizing fuzzy logic with computational neural networks. The two technologies often complement each other: neural networks supply the brute force necessary to accommodate and interpret large amounts of sensor data and fuzzy logic provides a structural framework that utilizes and exploits these low-level results. As a neural network is well known for its ability to represent functions, and the basis of every fuzzy model is the membership function, so the natural application of neural networks in fuzzy models has emerged to provide good approximations to the membership functions that are essential to the success of the fuzzy approach. Many researchers evaluate and analyze the performance of available methods of ranking fuzzy subsets on a set of selected examples that cover possible situations we might encounter as defining fuzzy subsets at each node of a neural network. Along with prosperity of computer and internet technology, more and more pepole used e-learning system to lecture and study. Therefore, how to evaluate the students' proficiency by arranging is the topic that deverses our attention. This chapter focus on fuzzy ranking approaches to evaluate fuzzy numbersas a tool in neural network.

3. Ranking fuzzy numbers

In many applications, ranking of fuzzy numbers is an important component of the decision process. Since fuzzy numbers do not form a natural linear order, like real numbers, a key issue in operationalzing fuzzy set theory is how to compare fuzzy numbers. Various approaches have been developed for ranking fuzzy numbers. In the existing research, the commonly used technique is to construct proper maps to transform fuzzy numbers into real numbers so called defuzzification. These real numbers are then compared. Herein, in approaches (; Abbasbandy & Asady, 2006; Abbasbandy & Hajjari, 2009, 2011; Asady, 2010; S. J. Chen & S. M. Chen, 2003, 2007, 2009; Deng & Liu, 2005; Deng et al., 2006; Hajjari, 2011a; Hajjari, 2011b; Z.-X. Wang et al. 2009) a fuzzy number is mapped to a real number based on the area measurement. In approaches (L. H. Chen & Lu, 2001, 2002; Liu & Han, 2005), $\alpha -$cut set and decision-maker's preference are used to construct ranking function. On the other hand, another commonly used technique is the centroid-based fuzzy number ranking approach (Cheng, 1998; Chu, & Tsao, 2002; Y.J. Wang et al. 2008). It should be noted that with the development of intelligent technologies, some adaptive and parameterized defuzzification methods that can include human knowledge have been proposed. Halgamuge et al. (Halgamuge et al. 1996) used neural networks for defuzzification. Song and Leland (Song & Leland, 1996) proposed an adaptive learning defuzzification technique. Yager (1996) proposed knowledge based on defuzzification process, which becomes more intelligent. Similar to methods of Filve and Yager (Filev & Yager, 1991), Jiang and Li (Jiang & Li, 1996) also proposed a parameterized defuzzification method with Gaussian based distribution transformation and polynomial transformation, but in fact, no method gives a right effective defuzzification output. The computational results of these methods are often conflict.

We often face difficultly in selecting appropriate defuzzification, which is mainly based on intuition and there is no explicit decision making for these parameters. For more comparison details on most of these methods, in this chapter we review some of ranking methods.

4. Basic notations and definitions

4.1 Definition

First, In general, a generalized fuzzy number A is membership $\mu_A(x)$ can be defined as (Dubios & Prade, 1978)

$$\mu_A(x) = \begin{cases} L_A(x) & a \le x \le b \\ \omega & b \le x \le c \\ R_A(x) & c \le x \le d \\ 0 & otherwise, \end{cases} \tag{1}$$

where $0 \le \omega \le 1$ is a constant, and $L_A : [a,b] \to [0,\omega]$, $R_A : [c,d] \to [0,\omega]$ are two strictly monotonical and continuous mapping from R to closed interval $[0,\omega]$. If $\omega = 1$, then A is a normal fuzzy number; otherwise, it is a trapezoidal fuzzy number and is usually denoted by $A = (a,b,c,d,\omega)$ or $A = (a,b,c,d)$ if $\omega = 1$.

In particular, when $b = c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number denoted by $A = (a,b,d,\omega)$ or $A = (a,b,d)$ if $\omega = 1$. Therefore, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers.

Since L_A and R_A are both strictly monotonical and continuous functions, their inverse functions exist and should be continuous and strictly monotonical. Let $L_A^{-1} : [a,b] \to [0,\omega]$ and $R_A^{-1} : [a,b] \to [0,\omega]$ be the inverse functions of $L_A(x)$ and $R_A(x)$, respectively. Then $L^{-1}{}_A(r)$ and $R^{-1}{}_A(r)$ should be integrable on the close interval $[0,\omega]$. In other words, both $\int_0^{\omega} L_A^{-1}(r)dr$ and $\int_0^{\omega} R_A^{-1}(r)dr$ should exist. In the case of trapezoidal fuzzy number, the inverse functions $L^{-1}{}_A(r)$ and $R^{-1}{}_A(r)$ can be analytically expressed as

$$\cdot L_A^{-1}(r) = a + (b-a)r / \omega \quad 0 \le \omega \le 1$$

$$R_A^{-1}(r) = d - (d-c)r / \omega \quad 0 \le \omega \le 1$$

The set of all elements that have a nonzero degree of membership in A, it is called the support of A, i.e.

$$S(A) = \{x \in X \mid \mu_A(x) \succ 0\} \tag{2}$$

The set of elements having the largest degree of membership in A, it is called the core of A, i.e.

$$C(A) = \left\{ x \in X \mid \mu_A(x) = \sup_{x \in X} L_A(x) \right\} \tag{3}$$

In the following, we will always assume that A is continuous and bounded support $S(A)$. The strong support of A should be $S(A) = [a,d]$.

4.2 Definition

The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalent represented in (Zadeh, 1965; Ma et al., 1999; Dubois & Prade, 1980) as follows.

For arbitrary $A = \left(L_A^{-1}(r), R_A^{-1}(r)\right)$ and we define addition $B = \left(L_B^{-1}(r), R_B^{-1}(r)\right)$ $(A+B)$ and multiplication by scalar $k \succ 0$ as

$$\begin{aligned}
&(\underline{A+B})(r) = \underline{A}(r) + \underline{B}(r) \\
&\left(\overline{A+B}(r)\right) = \overline{A}(r) + \overline{B}(r) \\
&(\underline{kA})(r) = k\underline{A}(r), \left(\overline{kA}\right)(r) = k\overline{A}(r).
\end{aligned}$$

To emphasis, the collection of all fuzzy numbers with addition and multiplication as defined by (8) is denoted by E, which is a convex cone. The image (opposite) of $A = (a,b,c,d)$ is $-A = (-d,-c,-b,-a)$ (Zadeh, L.A, 1965; Dubois, D. and H. Prade, 1980).

4.3 Definition

A function $f:[0,1] \to [0,1]$ is a reducing function if is s increasing and $f(0) = 0$ and $f(1) = 1$. We say that s is a regular function if $f(r)dr = 1/2$.

4.4 Definition

If A is a fuzzy number with r-cut representation, $\left(L_A^{-1}(r), R_A^{-1}(r)\right)$ and s is a reducing function, then the value of A (with respect to s); it is defined by

$$Val(A) = \int_0^1 f(r)[L_A^{-1}(r) + R_A^{-1}(r)]dr \tag{4}$$

4.5 Definition

If A is a fuzzy number with r-cut representation $\left(L_A^{-1}(r), R_A^{-1}(r)\right)$, and s is a reducing function then the ambiguity of A (with respect to s) is defined by

$$Amb(A) = \int_0^1 f(r)[R_A^{-1}(r) - L_A^{-1}(r)]dr \tag{5}$$

Let also recall that the expected interval $EI(A)$ of a fuzzy number A is given by

$$EI(A)=\left[\int_0^1 L_A^{-1}(r)dr,\int_0^1 R_A^{-1}(r)dr\right]. \quad (6)$$

Another parameter is utilized for representing the typical value of the fuzzy number is the middle of the expected interval of a fuzzy number and it is called the expected value of a fuzzy number A i.e. number A is given by (Bodjanova, 2005)

$$EV(A)=\frac{1}{2}\left[\int_0^1 L_A^{-1}(r)dr+\int_0^1 R_A^{-1}(r)dr\right]. \quad (7)$$

4.6 Definition

The first of maxima (FOM) is the smallest element of $core(A)$. i.e.

$$FOM=\min core(A). \quad (8)$$

4.7 Definition

The last of maxima (LOM) is the greatest element of $core(A)$. i.e.

$$LOM=\max core(A). \quad (9)$$

4.8 Definition

For arbitrary fuzzy numbers $A=\left(L_A^{-1}(r),R_A^{-1}(r)\right)$ and $B=\left(L_B^{-1}(r),R_B^{-1}(r)\right)$ the equality

$$D(A,B)=\left[\int_0^1\left(L_A^{-1}(r)-L_B^{-1}(r)\right)^2 dr+\int_0^1\left(R_A^{-1}(r)-R_B^{-1}(r)\right)^2 dr\right]^{1/2} \quad (10)$$

is the distance between A and B. The function $D(A,B)$ is a metric in E and (E,D) is a complete metric space.

The ordering indices are organized into three categories by Wang and Kerre (Wang & Kerre, 2001) as follows:

- **Defuzzification method**: Each index is associated with a mapping from the set of fuzzy quantities to the real line. In this case, fuzzy quantities are compared according to the corresponding real numbers.
- **Reference set method**: in this case, a fuzzy set as a reference set is set up and all the fuzzy quantities to be ranked are compared with the reference set.
- **Fuzzy relation method**: In this case, a fuzzy relation is constructed to make pair wise comparisons between the fuzzy quantities involved.

Let M be an ordering method on E. The statement two elements A_1 and A_2 in E satisfy that A_1 has a higher ranking than A_2 when M is applied will be written as $A_1 \succ A_1$ by M. $A_1 \approx A_1$ and $A_1 \geq A_1$ are similarly interpreted. The following reasonable properties for the ordering approaches are introduced by Wang and Kerre (Wang & Kerre 2001).

1. For an arbitrary finite subset Γ of E and $A_1 \in \Gamma$, $A_1 \geq A_1$.
2. For an arbitrary finite subset Γ of E and $(A_1, A_2) \in \Gamma^2, A_1 \geq A_2$ and $A_2 \geq A_1$, we should have $A_1 \approx A_2$.
3. For an arbitrary finite subset Γ of E and $(A_1, A_2, A_3) \in \Gamma^3, A_1 \geq A_2$ and $A_2 \geq A_3$, we should have $A_1 \geq A_3$.
4. For an arbitrary finite subset Γ of E and $(A_1, A_2) \in \Gamma^2$, inf{supp(A_1)}> sup{supp(A_2), we should have $A_1 \geq A_2$.
5. For an arbitrary finite subset Γ of E and $(A_1, A_2) \in \Gamma^2$, inf{supp(A_1)}> sup{supp(A_2)}, we should have $A_1 \succ A_2$.
6. Let $A_1, A_2, A_1 + A_3$ and $A_2 + A_3$ be elements of E. If $A_1 \geq A_2$, then $A_1 + A_3 \geq A_2 + A_3$.

5. Ranking indices

a. Methods of centroid point

In order to determine the centroid points (x_0, y_0) of a fuzzy number A, Cheng (Cheng, 1998) provided a formula then Wang et al. (Y. M. Wang et al., 2006) found from the point of view of analytical geometry and showed the corrected centroid points as follows:

$$x_0 = \frac{\int_a^b xL_A(x)dx + \int_b^c xdx + \int_c^d xR_A(x)dx}{\int_a^b L_A(x)dx + \int_b^c dx + \int_c^d R_A(x)dx}$$

$$y_0 = \frac{\left[\int_0^\omega yR_A^{-1}(y)dy - \int_0^\omega yL_A^{-1}(y)dy\right]}{\int_0^\omega R_A^{-1}(y)dy - \int_0^\omega L_A^{-1}(y)dy}. \tag{11}$$

For non-normal trapezoidal fuzzy number $A = (a, b, c, d, \omega)$ formulas (11) lead to following results respectively.

$$x_0 = \frac{1}{3}\left[a + b + c + d - \frac{dc - ab}{(d + c) - (a + b)}\right]$$

$$y_0 = \frac{\omega}{3}\left[1 + \frac{c - d}{(d + c) - (a + b)}\right]. \tag{12}$$

Since non-normal triangular fuzzy numbers are, special cases of normal trapezoidal fuzzy numbers with $b = c$, formulas (12) can be simplified as

$$x_0 = \frac{1}{3}\left[a + b + d\right]$$

$$y_0 = \frac{\omega}{3}. \tag{13}$$

In this case, normal triangular fuzzy numbers could be compared or ranked directly in terms of their centroid coordinates on horizontal axis.

Cheng (Cheng, 1998) formulated his idea as follows:

$$R(A) = \sqrt{x_0(A)^2 + y_0(A)^2}. \tag{14}$$

To overcome the drawback of Cheng's distance Chu and Tsao's (Chu & Tsao, 2002) computed the area between the centroid and original points to rank fuzzy numbers as:

$$S(A) = x_0(A).y_0(A). \tag{15}$$

Then Wang and Lee (Y. J. Wang, 2008) ranked the fuzzy numbers based on their x_0's values if they are different. In the case that they are equal, they further compare their y_0's values to form their ranks.

Further, for two fuzzy numbers A and B if $y_0(A) \geq y_0(B)$ based on $x_0(A) = x_0(B)$, then $A \geq B$.

By shortcoming of the mentioned methods finally, Abbasbandy and Hajjari (Abbasbandy & Hajjari 2010) improved Cheng's distance centroid as follows:

$$IR(A) = \gamma(A)\sqrt{x_0(A)^2 + y_0(A)^2} \tag{16}$$

Where

$$\gamma(A) = \begin{cases} 1 & \int_0^1 \left(L_A^{-1}(x) + R_A^{-1}(x)\right) dx \succ 0, \\ 0 & \int_0^1 \left(L_A^{-1}(x) + R_A^{-1}(x)\right) dx = 0, \\ -1 & \int_0^1 \left(L_A^{-1}(x) + R_A^{-1}(x)\right) dx \prec 0. \end{cases} \tag{17}$$

However, there are some problems on the centroid point methods. In next section, we will present a new index for ranking fuzzy numbers. The proposed index will be constructed by fuzzy distance and centroid point.

b. Method of D-distance (Ma et al. 2000)

Let all of fuzzy numbers are positive or negative. Without less of generality, assume that all of them are positive. The membership function of $a \in R$ is $\mu_a(x) = 1$, if $x = a$ and $\mu_a(x) = 0$ if $x \neq a$. Hence if $a = 0$ we have the following

$$\mu_0(0) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0. \end{cases}$$

Since $\mu_0(x) \in E$, left fuzziness and right fuzziness are 0, so for each $\mu_A \in E$

$$D(A,\mu_0)=\left[\int_0^1\left(L_A^{-1}(r)^2+R_A^{-1}(r)_2\right)dr\right]^{1/2} \tag{18}$$

Thus, we have the following definition

5.1 Definition

For A and $B\in E$, define the ranking of A and B by saying

$$\begin{aligned} A\succ B \quad & iff \quad d(A,\mu_0)\succ d(B,\mu_0),\\ A\prec B \quad & iff \quad d(A,\mu_0)\prec d(B,\mu_0),\\ A\approx B \quad & iff \quad d(A,\mu_0)=d(B,\mu_0). \end{aligned}$$

5.2 Property

Suppose A and $B\in E$, are arbitrary, therefore

If $A=B$ then $A\approx B$.

If $B\subseteq A$ and $L_A^{-1}(r)^2+R_A^{-1}(r)^2\succ L_B^{-1}(r)^2+R_B^{-1}(r)^2$ for all $r\in[0,1]$ then $B\prec A$.

5.3 Remark

The distance triangular fuzzy number $A=(x_0,\sigma,\beta)$ of μ_0 is defined as following:

$$d(A,\mu_0)=\left[2x_0^2+\sigma^2/3+\beta^2/3+x_0(\beta-\sigma)\right]^{1/2} \tag{19}$$

The distance trapezoidal fuzzy number $A=(x_0,y_0,\sigma,\beta)$ of μ_0 is defined as following

$$d(A,\mu_0)=\left[2x_0^2+\sigma^2/3+\beta^2/3-x_0\sigma+y_0\beta)\right]^{1/2} \tag{20}$$

If $A\approx B$ it is not necessary that $A=B$.

If $A\neq B$ and $(L_A^{-1}(r)^2+R_A^{-1}(r)^2)^{1/2}=(L_B^{-1}(r)^2+R_B^{-1}(r)^2)^{1/2}$ then $A\approx B$.

c. Method of sign distance (Abbasbandy & Asady 2006)

5.4 Definition

For arbitrary fuzzy numbers $A=\left(L_A^{-1}(r),R_A^{-1}(r)\right)$ and $B=\left(L_B^{-1}(r),R_B^{-1}(r)\right)$ the function

$$D(A,A_0)=\left(\int_0^1(\left|L_A^{-1}(x)\right|^p+\left|R_A^{-1}(x)\right|^p)dx\right)^{\frac{1}{p}}. \tag{21}$$

is the distance between A and B.

5.5 Definition

Let $\gamma(A): E \longrightarrow \{-1,1\}$ be a function that is defined as follows:

$$\gamma(A) = sign\int_0^1 [L_A^{-1}(r) + R_A^{-1}(r)]dr$$

Where

$$\gamma(A) = \begin{cases} 1 & \int_0^1 (L_A^{-1}(r) + R_A^{-1}(r)) \succ 0 \\ -1 & \int (L_A^{-1}(r) + R_A^{-1}(r)) \prec 0 \end{cases}$$

5.6 Remark

1. If supp $(A) \geq 0$ or inf $L_A^{-1}(r) \geq 0$ then $\gamma(A) = 1$.
2. If supp $(A) \prec 0$ or sup $R_A^{-1}(r) \prec 0$ then $\gamma(A) = -1$.

5.7 Definition

For $A \in E$, $d_p(A, \mu_0) = \gamma(A) D(A, \mu_0)$ is called sign distance.

5.8 Definition

For A and $B \in E$ define the ranking order of A and B by d_p on E. i.e.

$$\begin{aligned} A_i \succ A_j \quad & iff \quad d_p(A_i, \mu_0) \succ d_p(A_j, \mu_0) \\ A_i \prec A_j \quad & iff \quad d_p(A_i, \mu_0) \prec d_p(A_j, \mu_0) \\ A_i \approx A_j \quad & iff \quad d_p(A_i, \mu_0) = d_p(A_j, \mu_0) \end{aligned}$$

5.9 Remark

1. The function d_p, sign distance has the Wang and Kerre's properties .
2. The function d_p, sign distance for $p = 1$ has the following properties

if

inf {supp (A), supp (B), supp $(A + C)$, supp $(B + C)$ } ≥ 0

or

sup {supp (A), supp (B), supp $(A + C)$, supp $(B + C)$ } ≤ 0

3. Suppose A and $B \in E$ are arbitrary then

a. If $A = B$ then $A \approx B$,

b. If $B \subseteq A$ and $\gamma(A)\left(\left|L_A^{-1}(r)\right|^p + \left|R_A^{-1}(r)^p\right|\right) \succ \gamma(B)\left(\left|L_B^{-1}(r)\right|^p + \left|R_B^{-1}(r)\right|^p\right)$, that for all $r \in [0,1]$ then $B \prec A.$,

4. If $A \approx B$ it is not necessary that $A = B$. Since if $A \neq B$ and $\gamma(A)\left(\left|L_A^{-1}(r)\right|^p + \left|R_A^{-1}(r)^p\right|\right) = \gamma(B)\left(\left|L_B^{-1}(r)\right|^p + \left|R_B^{-1}(r)\right|^p\right)$ that for all $r \in [0,1]$ then $B \approx A.$,
5. If $A \leq B$ then $-A \geq -B$.

Therefore, we can simply rank the fuzzy numbers by the defuzzification of $d_p(A, \mu_0)$. By Remark 3.12 part (5) we can logically infer ranking order of the image of the fuzzy numbers.

d. Method of H-distance

5.10 Definition

A continuous function $s:[0,1] \longrightarrow [0,1]$ with the following properties is a source function $s(0) = 0,\ s(1) = 1,\ s(r)$ is increasing, and $\int_0^1 s(r)dr = \frac{1}{2}$.

In fact, a reducing has the reflection of weighting the influence of the different r-cuts and diminishes the contribution of the lower r-levels. This is reasonable since these levels arise from values of membership function for which there is a considerable amount of uncertainty. For example, we can use $s(r) = r$.

5.11 Definition

For A and $B \in E$ we define H-distance of A and B by

$$D_H^*(A,B) = \frac{1}{2}\left\{\left|Val(A) - Val(B)\right| + \left|Amb(A) - Amb(B)\right| + d_H\left(\left|A\right|^1 + \left|B\right|^1\right)\right\} \tag{22}$$

where d_H is the Housdorf metric between intervals and $[.]^1$ is the 1-cut representation of a fuzzy number.

e. Method of source distance (Ma et al., 2000)

5.12 Definition

For A and $B \in E$ we define source distance of A and B by

$$D_s(A,B) = \frac{1}{2}\left\{\left|Val_s(A) - Val_s(B)\right| + \left|Amb_s(A) - Amb_s(B)\right| + \max\{|\,t_A - t_B\,|, |\,m_A - m_B\,|\}\right\},$$

where $[m_A, t_A]$ and $[m_B, t_B]$ are the cores of fuzzy numbers A and B respectively.

5.13 Property

The source distance metric D_s is a metric on E_{TR} and a pseudo-metric on E.

f. Method of magnitude (Abbasbandy & Hajjari 2009)

For an arbitrary trapezoidal fuzzy number $A=(x_0,y_0,\sigma,\beta)$ with parametric form $A=\left(L_A^{-1}(r),R_A^{-1}(r)\right)$, we define the magnitude of the trapezoidal fuzzy number A as

$$Mag(A)=\frac{1}{2}\left(\int_0^1 (L_A^{-1}(r)+R_A^{-1}(r)+x_0+y_0)f(r)dr\right).\quad f(r)=r \tag{23}$$

where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0)=0, f(1)=1$ and $\int_0^1 f(r)dr=\frac{1}{2}.$ for example, we can use. The resulting scalar value is used to rank the fuzzy numbers. The larger $Mag(A)$, the larger fuzzy number. Therefore for any two fuzzy number A and $B\in E$. We defined the ranking of A and B by the $Mag(A)$ on E as follows

$Mag(A)\succ Mag(B)$ if and only if $A\succ B$.

$Mag(A)\prec Mag(B)$ if and only if $A\prec B$.

$Mag(A)=Mag(B)$ if and only if $A\approx B$.

Then we formulate the order $\geq$ and $\leq$ as $A\geq B$ if and only if $A\succ B$ or $A\approx B$, $A\leq B$ if and only if $A\prec B$ or $A\approx B$. In other words, this method is placed in the first class of Kerre's categories(X. Wang & Kerre 2001).

g. Method of promoter operator (Hajjari & Abbasbandy 2011)

Let $A=(a,b,c,d)$ be a non-normal trapezoidal fuzzy number with $r-$cut representation $A=\left(L_A^{-1}(r),R_A^{-1}(r)\right)$. . Consequently, we have

$$Mag(A)=\frac{(3\omega^2+2)(b+c)}{12\omega}+\frac{(3\omega-2)(a+d)}{12\omega}. \tag{24}$$

It is clear that for normal trapezoidal fuzzy numbers the formula (24) reduces to

$$Mag(A)=\frac{5}{12}(b+c)+\frac{1}{12}(a+d). \tag{25}$$

In the following, we use an example to illustrate the ranking process of the proposed method.

Moreover, for normal fuzzy numbers we have

$$Mag(A)=\frac{1}{2}\left[\int_0^1 (L_A^{-1}(r)dr+R_A^{-1}(r)+L_A^{-1}(1)+R_A^{-1}(1))f(r)dr\right]. \tag{26}$$

h. Methods of deviation degree

Ranking L-R fuzzy numbers based on deviation degree (Z.X. Wang et al., 2009)

5.14 Definition

For any groups of fuzzy numbers $A_1, A_2, ..., A_n$ in E with support sets $S(A_i)$, $i = 1, . . ., n.$ Let $S = \bigcap_{i=1}^{n} S(A_i)$ and $x_{\min} = \inf S$ and $x_{\max} = \sup S$. Then minimal and maximal reference sets $A_{\min}$ and $A_{\max}$ are defined as

$$\mu_{A_{\min}}(x) = \begin{cases} \dfrac{x_{\max} - x}{x_{\max} - x_{\min}}, & if \quad x \in S \\ 0, & otherwise, \end{cases} \tag{27}$$

$$\mu_{A_{\max}}(x) = \begin{cases} \dfrac{x - x_{\min}}{x_{\max} - x_{\min}}, & if \quad x \in S \\ 0, & otherwise. \end{cases} \tag{28}$$

5.15 Definition

For any groups of fuzzy numbers $A_1, A_2, ..., A_n$ in E, let $A_{\min}$ and $A_{\max}$ be minimal and maximal reference sets of these fuzzy numbers, respectively. Then left and right deviation degree of A_i, $i = 1, . . ., n,$ are defined as follows:

$$d_i^L = \int_{x_{\min}}^{t_i} \left(\mu_{A_{\min}}(x) - L_A^{-1}(x) \right) dx$$

$$d_i^R = \int_{u_i}^{x_{\max}} \left(\mu_{A_{\max}}(x) - R_A^{-1}(x) \right) dx \tag{29}$$

where t_i and u_i, $i = 1, 2, ..., n$ are the abscissas of the crossover points of L_{A_i} and $\mu_{A_{\min}}$, and R_{A_i} and $\mu_{A_{\max}}$, respectively.

5.16 Definition

For any groups of fuzzy numbers $A_i = (a_i, b_i, c_i, d_i, \omega)$ in E, its expectation value of centroid is defined as follows:

$$M_i = \frac{\int_{a_i}^{d_i} x \mu_{A_i}(x) dx}{\int_{a_i}^{d_i} \mu_A(x) dx} \tag{30}$$

$$\lambda_i = \frac{M_i - M_{\min}}{M_{\max} - M_{\min}} \tag{31}$$

where $M_{\max} = \max\{M_1, M_2, ..., M_n\}$ and $M_{min} = min\{M_1, M_2, ..., M_n\}$

Based on mentioned formulae, the ranking index value of fuzzy numbers $A_i,\ i = 1, \ . \ . \ ., \ n,$ is given by

$$d_i = \begin{cases} \dfrac{d_i^L \lambda_i}{1 + d_i^R (1 - \lambda_i)}, & M_{\max} \neq M_{\min},\ i = 1, 2, ..., n, \\ \dfrac{d_i^L}{1 + d_i^R} & M_{\max} = M_{\min},\ i = 1, 2, ..., n. \end{cases} \tag{32}$$

Now, by using (15), for any two fuzzy numbers A_i and Aj the ranking order is based on the following rules.

1. $A_i \succ A_j$ if and only if $d_i \succ d_j$,
2. $A_i \prec A_j$ if and only if $d_i \prec d_j$,
3. $A_i \approx A_j$ if and only if $d_i = d_j$.

- The revised method of ranking L-R fizzy number based on deviation degree (Asady, 2010)

Asady (Asady, 2010) revised Wang et al. (Z.X. Wang et al. 2009) method and suggested $D(.)$ operator for ranking of fuzzy numbers as follows:

Consider two fuzzy numbers A and B the ranking order is based on the following situations:

1. If $D(A) \prec D(B)$, then $A \prec B$.
2. If $D(A) \succ D(B)$, then $A \succ B$.
3. If $D(A) = D(B)$, then

if $\gamma_A \neq \gamma_B$, $D^*(A) \prec D^*(B)$ then $A \prec B$,

if $\gamma_A \neq \gamma_B$, $D^*(A) \succ D^*(B)$ then $A \succ B$,

else $A \approx B$.

where

$$D(A) = \frac{D_A^L}{1 + D_A^R} \tag{33}$$

$$D^*(A) = \frac{D_A^L \gamma}{1 + D_A^R \gamma} \tag{34}$$

where

$$D_A^L = \int_0^1 \left(R_A^{-1}(x) + L_A^{-1}(x) - 2x_{\min} \right) dx \tag{35}$$

$$D_A^R = \int_0^1 \left(2x_{\max} - R_A^{-1}(x) - L_A^{-1}(x) \right) dx \tag{36}$$

Ranking fuzzy numbers based on the left and the right sides of fuzzy numbers (Nejad & Mashinchi, 2011)

Recently Nejad and Mashinchi (Nejad & Mashinchi, 2011) pointed out the drawback of Wang et al. (Z.X. Wang et al. , 2009) hen they presented a novel ranking method as follows.

5.17 Definition

Let $A_i = (a_i, b_i, c_i, d_i, \omega)$, $i = 1,2,...,n$, are fuzzy numbers in E, $a_{\min} = \min\{a_1, a_2, ..., a_n\}$ and $d_{\max} = \max\{d_1, d_2, ..., d_n\}$. The areas s_i^L and s_i^R of the left and right sides of the fuzzy number A_i are defined as

$$s_i^L = \int_0^{\omega} \left(L_A^{-1}(r) - a_{\min} \right) dr \tag{37}$$

$$s_i^R = \int_0^{\omega} \left(d_{\max} - R_A^{-1}(r) \right) dr. \tag{38}$$

Based on above definitions, the proposed ranking index is

$$s_i = \frac{s_i^L \lambda_i}{1 + s_i^R (1 - \lambda_i)}, \quad i = 1,2,...,n. \tag{39}$$

Then the ranking order follows next rules.

1. $A_i \succ A_j$ if and only if $s_i \succ s_j$,
2. $A_i \prec A_j$ if and only if $s_i \prec s_j$,
3. $A_i \approx A_j$ if and only if $s_i = s_j$.

To obtain the reasonable they added two triangular fuzzy numbers A_0 and A_{n+1}, where

$$\begin{aligned} &A_0 = (a_0, b_0, d_0), \\ &a_0 = 2b_0 - d_0, \; b_0 = \min\{a_i, \; i = 1,2,...,n\}, \\ &d_0 = (d + b_0)/2, \; d = \min\{d_i, \; i = 1,2,...,n\} \end{aligned} \tag{40}$$

and

$$\begin{aligned} &A_{n+1} = (a_{n+1}, b_{n+1}, d_{n+1}), \\ &a_{n+1} = (b_{n+1} + a)/2, \quad b_{n+1} = \max\{d_i, \quad i = 1,2,...,n\}, \\ &d_{n+1} = 2b_{n+1} - a_{n+1}), \quad a = \max\{a_i, \quad i = 1,2,...,n\}. \end{aligned} \tag{41}$$

Then they ranked fuzzy numbers $A_1, A_2, ..., A_n$ based on the ranking area values $s_1, s_2, ..., s_n$.

Nevertheless, the new ranking method has drawback.

In the next section, we discuss on those methods that based on deviation degree by a number numerical counter examples.

6. Discussion and counter examples

6.1 Example

Let two fuzzy numbers *A= (3,6,9)* and *B= (5,6,7)* from (Z.-X. Wang et al., 2009) as shown in Fig. 1.

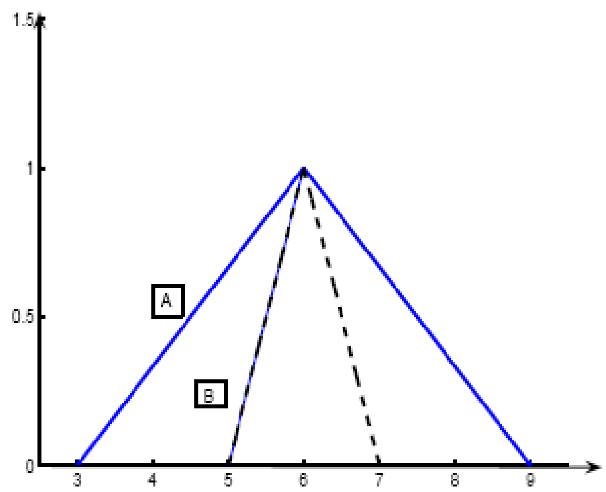


Fig. 1. Fuzzy numbers *A=(3,6,9)* and *B=(5,6,7)*

Through the approaches in this paper, the ranking index can be obtained as *Mag (A)=Mag(B)=12* and *EV(A) = EV(B) = 6*. Then the ranking order of fuzzy numbers is $A \approx B$. Because fuzzy numbers *A* and *B* have the same mode and symmetric spread, most of existing approaches have the identical results. For instance, by Abbasbandy and Asady's approach (Abbasbandy & Asady, 2006), different ranking orders are obtained when different index values *p* are taken. When *p = 1* and *p = 2* the ranking order is the same, i.e., $A \approx B$ Nevertheless, the same results produced when distance index, *CV* index of Cheng's approach and Chu and Tsao's area are respectively used, i.e., $x_A = x_B = 6$ and $y_A = y_B = \frac{1}{3}$ then from Cheng's distance and Chau and Tsao's area we get that *R(A) = R(B) = 2.2608*, *S(A) = S(B) = 1.4142* respectively.

From the obtained results we have $A \approx B$, for two triangular fuzzy numbers *A=(3, 6, 9)* and *B=(5, 6, 7)*. Now we review the ranking approaches by promoter operator. Since *A* and *B* have the same ranking order and the same centroid points we then compute their ambiguities. Hence, from (Deng et al., 2006) it will be obtained $amb(A) = 1$ and $amb(B) = \frac{1}{3}$.

Consequently, by using promoter operator we have

$$P(A)=\left(Mag(A),\frac{1}{1+amb(A)}\right)=(12,\frac{1}{2}),\quad P(B)=\left(Mag(B),\frac{1}{1+amb(B)}\right)=(12,\frac{3}{4})$$

$$P(A)=\left(EV(A),\frac{1}{1+amb(A)}\right)=(6,\frac{1}{2}),\quad P(B)=\left(EV(B),\frac{1}{1+amb(B)}\right)=(6,\frac{3}{4})$$

$$P(A)=\left(R(A),\frac{1}{1+amb(A)}\right)=(2.2608,\frac{1}{2}),\quad P(B)=\left(R(B),\frac{1}{1+amb(B)}\right)=(2.2608,\frac{3}{4})$$

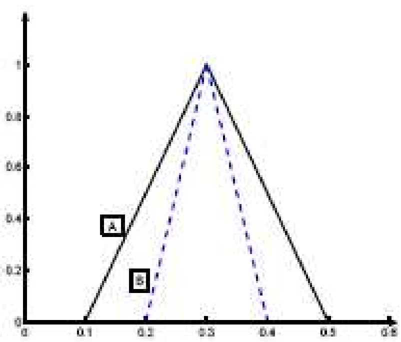
$$P(A)=\left(S(A),\frac{1}{1+amb(A)}\right)=(12,\frac{1}{2}),\quad P(B)=\left(S(B),\frac{1}{1+amb(B)}\right)=(12,\frac{3}{4}).$$

The ranking order is $A \prec B$ Through the proposed approach by Wang et al., the ranking index values can be obtained as d_1 = 0.1429 and d_2 = 0.1567. Then the ranking order of fuzzy numbers is also $A \prec B$.

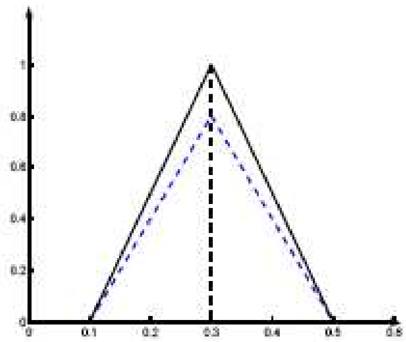
In the following, we use the data sets shown in Chen and Chen (S. J. Chen et al. 2009) to compare the ranking results of the proposed approaches with Cheng method (Cheng, 1998), Chu and Tsao's method (Chu & Tsao 2002) and Chen and Chen (S. J. Chen et al. 2009). The comparing of ranking results for different methods will be explained in the following.

For the fuzzy numbers *A* and *B* shown in Set 1 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu's method (Chu, T. and Tsao, C., 2002), Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) and *Mag*- method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order $A \prec B$**.**

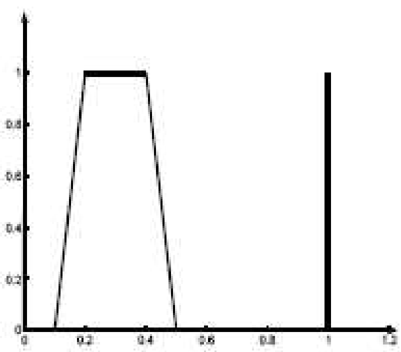
4. For the fuzzy numbers *A* and *B* shown in Set 2 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu 's method (Chu, T. and Tsao, C., 2002) and *Mag*- method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order $A \approx B$, which is unreasonable. Whereas by applying the promoter operator the ranking order is the same as Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009), i.e. $A \prec B$.
5. For the fuzzy numbers *A* and *B* shown in Set 3 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002) and *Mag*- method (Abbasbandy, S. and Hajjari, T., 2009) get an inaccurate ranking order $A \approx B$ whereas by applying the promoter operator the ranking order is the same as Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) i.e. $A \prec B$.
6. For the fuzzy numbers *A* and *B* shown in Set 4 of Fig. 4, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002), Chen and Chen's method (Chen, S. J. and Chen, S. M., 2007; Chen, S.-M. and Chen, J.-H., 2009) and *Mag*- method (Abbasbandy, S. and Hajjari, T., 2009) get the same ranking order: $A \prec B$**.**
7. For the fuzzy numbers *A* and *B* shown in Set 5 of Fig. 2, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu, T. and Tsao, C., 2002) cannot calculate the crisp-value fuzzy number, whereas Chen and Chen's method (S. J. Chen, 2009) and $Mag-$ *Mag*- method (Abbasbandy & Hajjari, 2009) get the same ranking order: $A \prec B$.



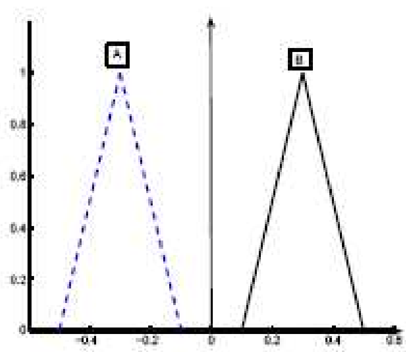
Set 3
A=(0.1, 0.3, 0.3, 0.5; 1)
B=(0.2, 0.3, 0.3, 0.4; 1)



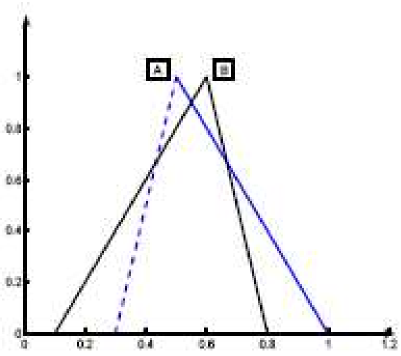
Set 4
A=(0.1, 0.3, 0.3, 0.5; 0.8)
B=(0.1, 0.3, 0.3, 0.5; 1)



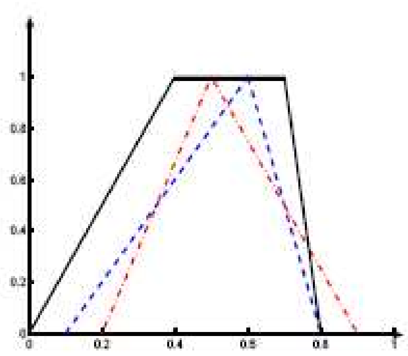
Set 5
A=(0.1, 0.2, 0.4, 0.5; 1)
B=(1.0, 1.0, 1.0, 1.0; 1)



Set 6
A=(-0.5, -0.3, -0.3, -0.1; 1)
B=(0.1, 0.3, 0.3, 0.5; 1)



Set 7
A=(0.3, 0.5, 0.5, 1.0; 1)
B=(0.1, 0.6, 0.6, 0.8; 1)



Set 8
A=(0.0, 0.4, 0.6, 0.8; 1)
B=(0.2, 0.5, 0.5, 0.9; 1)
C=(0.1, 0.6, 0.7, 0.8; 1)

Fig. 2.

Methods	F.n	set 1	set 2	set 3	set 4	set 5	set 6	set 7	set 8
Cheng's	A	.583	.583	.583	.461	.424	-.583	.767	.68
(1998)	B	.707	.583	.583	.583	$*$	.583	.724	.726
	C	$*$	$*$	$*$	$*$	$*$	$*$	$*$	.746
Results		$A \prec B$	$A \sim B$	$A \sim B$	$A \prec B$	$*$	$A \prec B$	$B \prec A$	$A \prec B \prec C$
P.O Results		$*$	$A \prec B$	$A \prec B$	$*$	$*$	$*$	$*$	$*$
Chau's	A	.15	.15	.15	.12	.15	-.15	.287	.228
(2002)	B	.25	.15	.15	.15	$*$	.15	.262	.262
	C	$*$	$*$	$*$	$*$	$*$	$*$	$*$	.278
Results		$A \prec B$	$A \sim B$	$A \sim B$	$A \prec B$	$*$	$A \prec B$	$B \prec A$	$A \prec B \prec C$
P.O Results		$*$	$A \prec B$	$A \prec B$	$*$	$*$	$*$	$*$	$*$
Chen's	A	.446	.424	.446	.357	.42	.446	.413	.372
(2007)	B	.489	.446	.473	.446	.860	.747	.401	.416
	C	$*$	$*$	$*$	$*$	$*$	$*$	$*$	.398
Results		$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$B \prec A$	$A \prec C \prec B$
P.O Results		$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
Chen's	A	.258	.254	.258	.254	.206	-.258	.443	.335
(2009)	B	.430	.258	.278	.258	1	.258	.404	.408
	C	$*$	$*$	$*$	$*$	$*$	$*$	$*$	.420
Results		$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$B \prec A$	$A \prec B \prec C$
P.O Results		$*$	$*$	$*$	$*$	$*$	$*$	$*$	$*$
Mag	A	.3	.3	.3	.27	.3	-.3	.525	.483
(2009)	B	.5	.3	.3	.3	1	.3	.575	.508
	C	$*$	$*$	$*$	$*$	$*$	$*$	$*$	.617
Results		$A \prec B$	$A \sim B$	$A \sim B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B$	$A \prec B \prec C$
P.O Results		$*$	$A \prec B$	$A \prec B$	$*$	$*$	$*$	$*$	$*$

Table 1.

8. For the fuzzy numbers A and B shown in Set 6 of Fig. 2, Cheng's method (Cheng, C. H., 1998), Chu and Tsao's method (Chu & Tsao, 2002), Chen and Chen's method (S. J. Chen & S. M. Chen, 2007; S.-M. Chen & J.-H. Chen, 2009) and *Mag*- method (Abbasbandy & Hajjari, 2009) get the same ranking order: $A \prec B$.
9. For the fuzzy numbers A and B shown in Set 7 of Fig. 2, Cheng's method (Cheng, 1998), Chu and Tsao's method (Chu & Tsao, 2002), Chen and Chen's method (S. J. Chen et al. 2009) get the same ranking order: $B \prec A$, whereas the ranking order by *Mag*- method (Abbasbandy & Hajjari, 2009) is $A \prec B$. By comparing the ranking result of *Mag*-method with other methods with respect to Set 7 of Fig. 2, we can see that *Mag*- method considers the fact that defuzzified value of a fuzzy number is more important than the spread of a fuzzy number.
10. For the fuzzy numbers A and B shown in Set 8 of Fig. 2, Cheng's method (Cheng, 1998), Chu and Tsao's method (Chu & Tsao, 2002), Chen and Chen's method (S. J. Chen et al., 2009) and *Mag*- method (Abbasbandy & Hajjari, 2009) get the same ranking order: $A \prec B \prec C$, whereas the ranking order by Chen and Chen's method is $A \prec C \prec B$. By comparing the ranking result of mentioned method with other methods with respect to Set 8 of Fig. 4, we can see that Chen's method considers the fact that the spread of a fuzzy number is more important than defuzzified value of a fuzzy number.

The idea of ranking fuzzy numbers by deviation degree is useful, but a significant approaches should be reserved the important properties such that

- $A \le B \;\Leftrightarrow\; -B \le -A$
- $A \le B \;\Leftrightarrow\; A + C \le B + C$
- $A \le B \;\wedge\; B \le C \;\Rightarrow\; A \le C$

Now we give some numerical example to show the drawback of the aforementioned methods.

6.2 Example

Given two triangular fuzzy number $A = (0.2, 0.5, 0.8)$ and $B = (0.4, 0.5, 0.6)$ (Nejad & Mashinchi, 2011), which are indicated in Fig. 3.

The ranking order by Nejad and Mashinchi is $A \prec B$. The images of two numbers A and B are *A=(-0.8, -0.5, -0.2), B=(-0.6, -0.5, -0.4)* respectively, then the ranking order is $-B \prec -A$.

On the other hand, ranking order for A and B and their images by Wang et al.'s method and Asady's revised are $A \approx B$, $-A \approx -B$ respectively.

This example could be indicated that all methods are reasonable. However, we will show that functions of all three methods are not the same in different conditions.

6.3 Example

Consider the three triangular fuzzy numbers A=(1, 2, 6), B = (2.5, 2.75,3) and C = (2, 3, 4), which are taken from Asady's revised (Asady, 2010) (See Fig. 4).

Utilizing Nejad and Mashinchi's method the ranking order is $A \prec B \prec C$ and the ranking order of their images will be obtained $-C \prec -A \prec -B$, which is illogical.

By using Wang et al.'s method the ranking order is $B \prec A \prec C$ and for their images is $-A \approx -C \prec -B$, which is unreasonable too.

From point of revised deviation degree method (Asady, 2010) the ranking orders are $B \prec A \prec C$, $-C \prec -A \prec -B$, respectively.

From this example, it seems the revised method can rank correctly.

In the next example, we will indicate that none of the methods based on deviation degree can rank correctly in all situations.

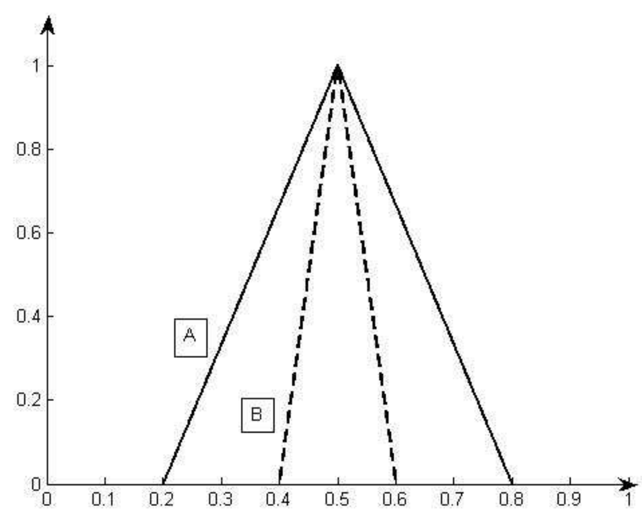


Fig. 3.

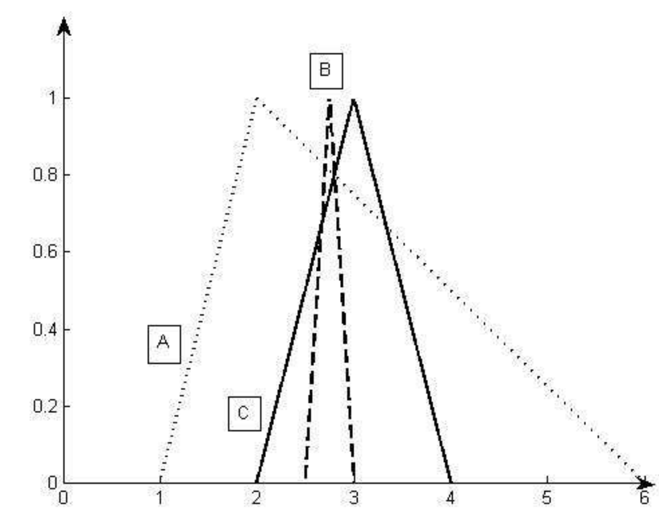


Fig. 4.

6.4 Example

Let the triangular fuzzy number $A=(1,2,3)$ and the fuzzy number $B=(1,2,4)$ with the membership function (See Fig. 5)

$$\mu_B(x)=\begin{cases}\left[1-(x-2)^2\right]^{1/2} & 1\le x\le 2,\\ \left[1-\frac{1}{4}(x-2)^2\right]^{1/2} & 2\le x\le 4,\\ 0 & otherwise.\end{cases}$$

Using Asady's method the ranking order is obtained $A \prec B.$ However, the ranking order of their images is $-A \prec -B,$ which is unreasonable.

From mentioned examples, we can theorize that ranking fuzzy numbers based on deviation degree cannot rank fuzzy numbers in all situations.

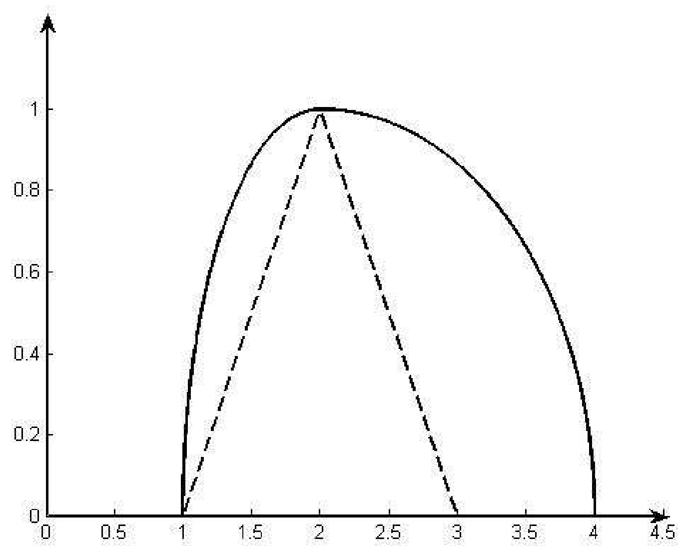


Fig. 5.

7. Conclusion

With the increasing development of fuzzy set theory in various scientific fields and the need to compare fuzzy numbers in different areas. Therefore, Ranking of fuzzy numbers plays a very important role in linguistic decision making, neural network and some other fuzzy application systems . Several strategies have been proposed for ranking of fuzzy numbers. Each of these techniques has been shown to produce non-intuitive results in certain case. In this chapter, we reviewed some recent ranking methods, which will be useful for the researchers who are interested in this area.

8. Acknowledgment

This work was partly supported by Islamic Azad University, FiroozKooh Branch.

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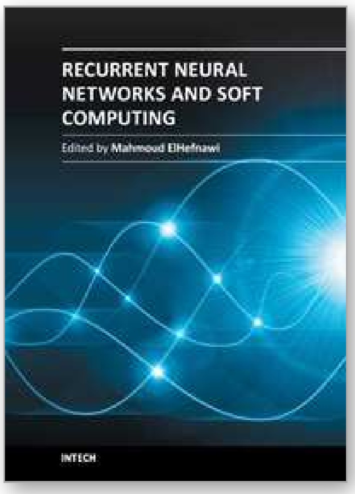

Recurrent Neural Networks and Soft Computing

Edited by Dr. Mahmoud ElHefnawi

ISBN 978-953-51-0409-4
Hard cover, 290 pages
Publisher InTech
Published online 30, March, 2012
Published in print edition March, 2012