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Singularity Analysis, Constraint Wrenches and Optimal Design of Parallel Manipulators

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1. Introduction

In recent years, numerous researchers have investigated parallel manipulators and many studies have been done on the kinematics or dynamics analysis. Parallel manipulators has been only mentioned in several books, as in (Merlet, 2006; Ceccarelli, 2004; Kong, & Gosselin, 2007; Glazunov, et al., 1991). Reference (Gosselin, & Angeles, 1990) has established singularity criteria based on Jacobian matrices when describing the various types of singularity. Then, in (Glazunov, et al. 1990) proposed other singularity criteria for consideration of these problems the screw theory based on the approach of the screw calculus, as in (Dimentberg, 1965). Those criteria are determined by the constraints imposed by the kinematic chains, as in (Angeles, 2004; Kraynev, & Glazunov, 1991), taking into account some problems the Plücker coordinates of constraint wrenches can be applied in (Glazunov, 2006; Glazunov, et al. 1999, 2007, 2009; Thanh, et al. 2009, 2010a).
Dynamical decoupling allows increasing the accuracy for the parallel manipulators presented as in (Glazunov, & Kraynev, 2006; Glazunov, & Thanh, 2008). It is necessary to develop optimal structure have combined (Thanh, et al. 2008), as well as algorithms and multi-criteria optimization (Statnikov, 1999; Thanh, et al. 2010b) obtaining the Pareto set. It is very important to taking into account possible singularity configurations, to find out how they influence the characteristics of constraints restricting working space (Bonev, et al. 2003; Huang, 2004; Arakelian, et al. 2007).
The trend towards highly rapid manipulators due to the demand for greater working volume, dexterity, and stiffness has motivated research and development of new types of parallel manipulator (Merlet, 1991). This paper is focused the constraints and criteria existing in known parallel manipulators in form of a parallel manipulator with linear actuators located on the base.

2. Kinematic of parallel manipulator

In this section, let us consider a 6-DOF parallel manipulator with actuators situated on the base. The mechanical architecture of the considered robot is illustrated in Fig. 1.

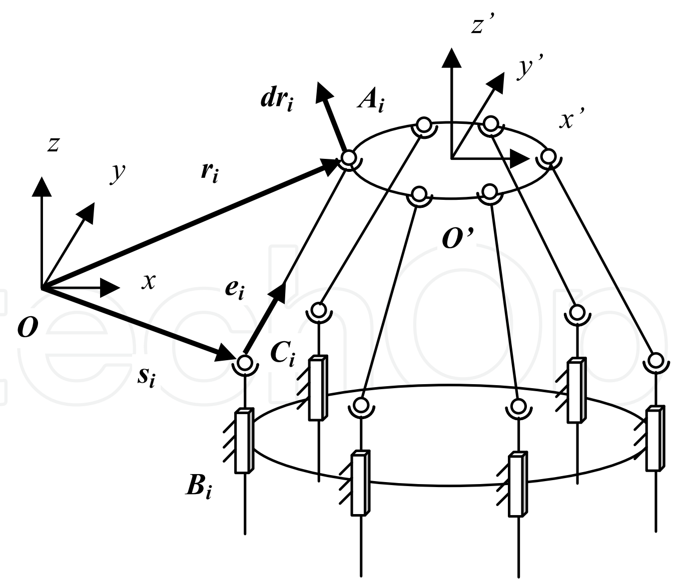


Fig. 1. Parallel manipulator with linear actuators located on the base

The parallel manipulator as seen in Fig. 1 is composed of a mobile platform connected to a fixed base via six kinematic sub-chains (legs) comprising of one prismatic, one universal and one spherical pair (PUS pairs). Parameters of design of the platform and the base form an irregular hexagon positioned in the *(x-y)* plane. A_i, B_i $(i=1,\ldots,6)$ are coordinates of the points of the mobile platform (the output link) and of the base respectively. The points $A_1A_3A_5$ and $A_2A_4A_6$ make form equilateral triangles, the angle ψ_p determines their location and R_p is the radius of the circumscribed circle (Fig. 2, a). Similarly, the angle ψ_b and the radius R_b determines the location of the equilateral triangles $B_1B_3B_5$ and $B_2B_4B_6$ located on the base. Let the distance between the centers of the universal and spherical pairs A_i and C_i of the *i-th* leg be l_i. In addition, the generalized coordinates, which are equal to the distance between the points B_i and C_i are designated θ_i. The radius-vectors of the points A_i and C_i are $r_i(x_{Ai}, y_{Ai}, z_{Ai})$ and $s_i(x_{Ci}, y_{Ci}, z_{Ci})$ respectively $(i=1,\ldots,6)$. We could note that the coordinates of the points B_i and C_i are $x_{Ci}=x_{Bi}$, $y_{Ci}=y_{Bi}$, $z_{Ci}=\theta_i$.

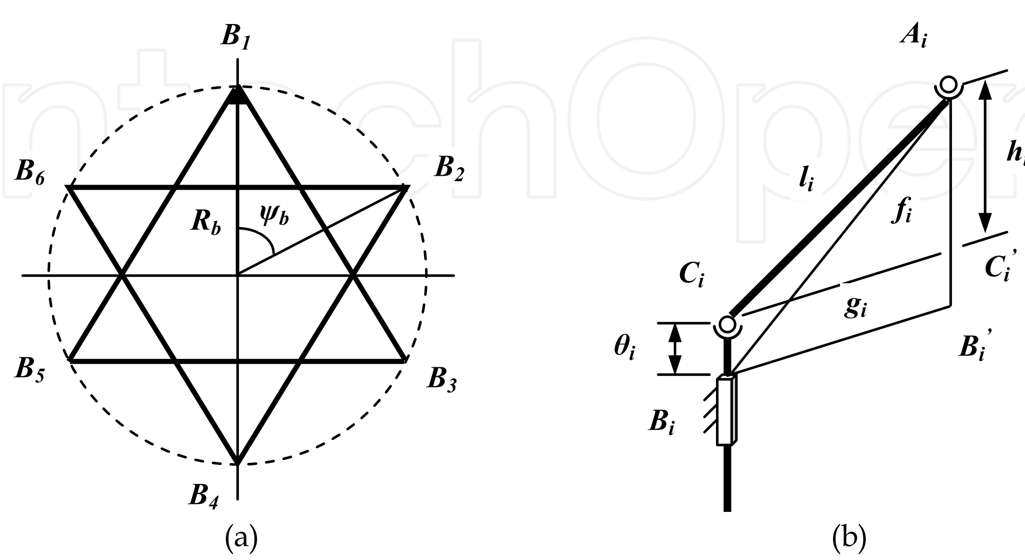


Fig. 2. Parametrical and geometrical design of parallel manipulator

With this approach, the linear actuators can be firmly fixed on the base to reduce high acceleration movements because the power is not used to move heavy actuators but lightweight links. However, the obstacle is a smaller working space in comparison with a Stewart platform, due to the movement of the linear actuators. Moreover, forces acting on the actuator have a perpendicular component, whereas forces exerted upon Stewart actuators have a longitudinal component.
Let us consider an inverse kinematic problem of position of parallel manipulators, which has characteristic relation between the numbers of chains. The manipulator with six kinematic chains offers convenience in optimization of working space in terms of decreased rigidity and load-bearing capacity.
Likewise, the generalized coordinates of the *i-th* segment (the length) which are equal to the distance between the points A_i and B_i, can be expressed as:

$$f_i = \sqrt{\left(x_{Ai} - x_{Bi}\right)^2 + \left(y_{Ai} - y_{Bi}\right)^2 + \left(z_{Ai} - z_{Bi}\right)^2}\ ,\ i = 1,...,6 \tag{1}$$

By geometrical method, the distance between the points C_i and C'_i (Fig. 2, b):

$$g_i = \sqrt{(f_i)^2 - (z_{Ai})^2}\ ,\ i = 1,...,6 \tag{2}$$

when the length of the link l_i is known, the distance between the points A_i and C'_i (Fig. 2, b):

$$h_i = \sqrt{\left(l_i\right)^2 - \left(g_i\right)^2} \tag{3}$$

We could obtain the generalized coordinate θ_i (Fig. 2, c) as follows:

$$\theta_i = z_{Ai} - h_i \tag{4}$$

It is the solution of the inverse kinematic problem. The inverse kinematics for parallel manipulator can be formulated to determine the required actuator heights for a given pose of the mobile platform with respect to the base. The pose consists of both position and orientation in the Cartesian system. Actuators are considered to act linearly in the vertical direction, parallel to the *z*-axis, in order to simplify the mathematics, although that needs not be the case.

3. Multi-criteria optimization

Influence of singularities on parameters of the working space of the parallel manipulator is a significant factor worth investigating. In these singularity configurations, the system is out of control and that greatly affects its functionality. It is necessary to determine the extent of the lack of control to see how that affects the parameters of the working space. These singularity configurations also affect the optimization results.
The constraint wrenches of zero pitch acting to the output link from the legs are located along the unit screws: $E_i = e_i + \chi e^o_i$, $(i=1,\ldots,6)$ where e_i is the unit vector directed along the axis of the line C_iA_i of the corresponding leg, χ is the Clifford factor, $\chi^2=0$ (for a vector, $e_i\, e^o_i=0$). E_i consists of the unit vector e_i and its moment $e^o_i = s_i \times e_i$ corresponding with $e^o_{xi} = s_{Cyi}e_{Czi} - s_{Czi}e_{Cyi}$; $e^o_{yi} = s_{Czi}e_{Cxi} - s_{Cxi}e_{Czi}$; $e^o_{zi} = s_{Cxi}e_{Cyi} - s_{Cyi}e_{Cxi}$ and can be expressed by Plücker coordinates $E_i = (x_i, y_i, z_i, x^o_i, y^o_i, z^o_i)$. These coordinates make form the 6×6 matrix (E):

$$(E) = \begin{pmatrix} x_1 & y_1 & z_1 & x_1^o & y_1^o & z_1^o \\ x_2 & y_2 & z_2 & x_2^o & y_2^o & z_2^o \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ x_6 & y_6 & z_6 & x_6^o & y_6^o & z_6^o \end{pmatrix} \quad (5)$$

Optimization of parameters of the parallel manipulator with linear actuators located on the base is considered. Let us take in account three criteria: working volume, dexterity and stiffness of parallel manipulator. The first criterion N_p is the quantity of the reachable points of the centre of the mobile platform. The second criterion N_c is the average quantity of orientations of the mobile platform in each reachable point. The third criterion D_e is the average module of the determinant $|\det(E)|$ in each configuration. Determinant $|\det(E)|$ constructed from coordinate axes of the drive kinematic couples is used as a third criterion of optimization. Since the value of this criterion is related to one of the important characteristics of the manipulator - its stiffness or load capacity. If determinant are more qualifiers, then the manipulator away from the singularity configuration and the stiffness of the above.

Let us consider optimization of the parameters of the manipulator for different values of the criterion of proximity to singular configurations, as well as the influence of this criterion in the optimization results. We set up four coefficients *H1, H2, H3* and *H4* expressed four parameters of optimization. The coefficient *H1* characterizes the length $l = l_i$ of the links A_iC_i $(i=1,\ldots,6)$ (in Fig. 2, b). The coefficient *H2* characterizes the angle ψ_p (Fig. 2, a) determining the location of the triangles $A_1A_3A_5$ and $A_2A_4A_6$ of the mobile platform. The coefficient *H3* characterizes the angle ψ_b determining the location of the triangles $B_1B_3B_5$ and $B_2B_4B_6$ on the base. Moreover, the coefficient *H4* characterizes the relation between the radius R_p and the radius R_b of the circumscribe circles of the platform and of the base respectively.

The algorithm of determination of the Pareto-optimal solutions can be presented as follows:

Step 1. Establish the limits of the parameters of optimization.

$H_{1min} \le H_1 \le H_{1max}$, $H_{2min} \le H_2 \le H_{2max}$, $H_{3min} \le H_3 \le H_{3max}$, $H_{4min} \le H_4 \le H_{4max}$. The number of steps of scanning in the space of parameters is n_p. The limits of the scanned Cartesian coordinates of the centre of the moving platform and the limits of the scanned orientation angles of this platform in interval are $x_{min} \le x \le x_{max}$, $y_{min} \le y \le y_{max}$, $z_{min} \le z \le z_{max}$, $\alpha_{min} \le \alpha \le \alpha_{max}$, $\beta_{min} \le \beta \le \beta_{max}$, $\gamma_{min} \le \gamma \le \gamma_{max}$. As well as the number n_c of steps of scanning in the space of these coordinates and the limitation of changing of the generalized coordinates $\theta_{imin} \le \theta_i \le \theta_{i\,max}$ $(i=1,\ldots,6)$. The limit of the determinant $|\det(E)| \ge \varepsilon$. At this step assume $i=0$, by this the parameters are $H_{10} = H_{1min}$, $H_{20} = H_{2min}$, $H_{30} = H_{3min}$, $H_{40} = H_{4min}$, $N_p = N_c = D_e = 0$.

Step 2. Determine the values of the criteria for all the values of the parameters.

2.1. Determine the parameters $H_{1i},\ldots, H_{40}$, assume j=0, by this $x_0 = x_{min}$, $y_0 = y_{min}$, $z_0 = z_{min}$, $\alpha_0 = \alpha_{min}$, $\beta_0 = \beta_{min}$, $\gamma_0 = \gamma_{min}$.

2.2. Determine θ_i (i=1,…,6) and $|\det(E)|$; if all the $\theta_{i\,min} \le \theta_i \le \theta_{i\,max}$ (i=1,…,6) and $|\det(E)| \ge \varepsilon$ then $N_c = N_c + 1$, $D_e = D_e + |\det(E)|$; if $x_j \ne x_{j-1}$ or $y_j \ne y_{j-1}$ or $z_j \ne z_{j-1}$ then $N_p = N_p + 1$.

2.3. $j = j+1$, if $j \le n_c$ then go back to 2.2.
2.4. Determine the criteria $N_{pi} = N_p$, $D_{ei} = D_e / N_c$, $N_{ci} = N_c / N_p$; assume $N_p = N_c = D_e = 0$.
2.5. $i = i+1$, if $i \le n_p$ then go back to 2.1.

Step 3. Determine the Pareto-optimal solutions (matrix *(Par)*).

3.1. Assume i= 1, (by this $N_{pi} = N_{p1}$, $D_{ei} = D_{e1}$, $N_{ci} = N_{c1}$), k= 0.
3.2. Determine N_{pi}, D_{ei}, N_{ci}; assume $j = 1$, the criteria of optimal solution $K1=1$, $K2=0$.
3.3. Determine N_{pj}, D_{ej}, N_{cj}; if $N_{pi} > N_{pj}$ or $D_{ei} > D_{ej}$ or $N_{ci} > N_{cj}$ then K2=1; if $N_{pi} = N_{pj}$ and $D_{ei} = D_{ej}$ and $N_{ci} = N_{cj}$ then K2=1.
3.4. If $K2 \neq 1$ then $K1=0$; $K2=0$; $j=j+1$; if $j \le n_p$ then go back to 3.3.
3.5. If $K1 = 1$ then $k = k + 1$, $Par_{1k} = H_{1i}$, …, $Par_{7k} = N_{ci}$, $i=i+1$; if $i \le n_p$ then go back to 3.2.

Singularity of the manipulator is determined by closeness to zero of determinant of matrix *(E)* of Plücker coordinates of unit wrenches. Let us fix certain value $\varepsilon>0$ as a criterion of singularity (the manipulator is in singular position if $|\det(E)| \le \varepsilon$). If $\varepsilon=0$ then the construction of the working space of the manipulator shows that the same results we can get without singularity constraint. Giving various values of the criterion of the singularity, we can get interval of the determinant of matrix (*E*).
Further, analysis influences of the criterion of singularity ε, $|\det(E)| \le \varepsilon$ on the value of the working volume. With the value of the criterion of singularity is equal to $\varepsilon=0.01$ there exist *81* available solutions, but only *8* of them are Pareto-optimal. By the condition of the criterion of singularity is equal to $\varepsilon=0.01$ and the condition $\varepsilon=0$, there Pareto-optimal set consists of *6* and *29* solutions correspondingly. Therefore, the value of ε influences on the results of optimization.
Value of the criterion that determines the proximity to singular configurations is equal to zero, we can assume that the constraints associated with the singularity in general, are not imposed in the analysis of each specific configuration. However, the criterion for determining the load capacity occurs. As a result, the number of Pareto-optimal variants varies very much. Here, *29* variants satisfy the conditions of Pareto set.
Limiting possible module of a determinant of matrix (*E*) to singularity configurations changes the Plücker coordinates of the wrenches transmitted on the output link. Methodology for analyzing the singularities on optimization appearing in the parallel manipulator and their impact in the working space is proposed. The practical significance from the fact is the results obtained in this work increase the effectiveness of design automation.

4. Twist inside singularity

The approach based on matrix *(E)* consisting of the Plücker coordinates of the constraint wrenches allows determining the twists of the platform inside singularity (Glazunov, 2006). Let us consider the increases of the Plücker coordinates of the unit screws E_i after an infinitesimal displacement $\$ = (d\varphi, dr) = (dr_x, dr_x, dr_x, d\varphi_x, d\varphi_y, d\varphi_z)^T$ of the platform corresponding to displacement $dr_i = (dx_{Ai}, dy_{Ai}, dz_{Ai})^T$ of the point A_i of the manipulator presented on the Fig. 1.

$$\begin{aligned} dx_{Ai} &= dr_x + d\varphi_y z_{Ai} - d\varphi_z y_{Ai}, \\ dy_{Ai} &= dr_y + d\varphi_z x_{Ai} - d\varphi_x z_{Ai}, \\ dz_{Ai} &= dr_z + d\varphi_x y_{Ai} - d\varphi_y x_{Ai} \end{aligned} \tag{6}$$

The generalized coordinate after mentioned infinitesimal displacement (dx_i, dy_i, dz_i) is:

$$\theta_i + d\theta_i = z_{Ai} + dz_{Ai} - \sqrt{l_i^2 - (x_{Ai} + dx_{Ai} - x_{Ci})^2 - (y_{Ai} + dy_{Ai} - y_{Ci})^2} \tag{7}$$

After transformations the increase of the generalized coordinate is:

$$d\theta_i = \frac{\left[(x_{Ai} - x_{Ci})dx_{Ai} + (y_{Ai} - y_{Ci})dy_{Ai} + (z_{Ai} - z_{Ci})dz_{Ai}\right]}{(z_{Ai} - z_{Ci})} \tag{8}$$

The unit screw E_i can be rewritten as E_i+dE_i or as e_i+de_i and $e^o{}_i$ + $de^o{}_i$.
Using (6), (7), and (8) the coordinates of the de_i and $de^o{}_i$ can be expressed as:

$$\begin{gathered}
dx_i = \frac{\partial x_i}{\partial \varphi_x}d\varphi_x + \frac{\partial x_i}{\partial \varphi_y}d\varphi_y + \frac{\partial x_i}{\partial \varphi_z}d\varphi_z + \frac{\partial x_i}{\partial r_x}dr_x + \frac{\partial x_i}{\partial r_y}dr_y + \frac{\partial x_i}{\partial r_z}dr_z \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
dz_i^o = \frac{\partial z_i^o}{\partial \varphi_x}d\varphi_x + \frac{\partial z_i^o}{\partial \varphi_y}d\varphi_y + \frac{\partial z_i^o}{\partial \varphi_z}d\varphi_z + \frac{\partial z_i^o}{\partial r_x}dr_x + \frac{\partial z_i^o}{\partial r_y}dr_y + \frac{\partial z_i^o}{\partial r_z}dr_z
\end{gathered} \tag{9}$$

where

$$\frac{\partial x_i}{\partial \varphi_x} = 0,\ \frac{\partial x_i}{\partial \varphi_y} = \frac{z_{Ai}}{l_i},\ \frac{\partial x_i}{\partial \varphi_z} = -\frac{y_{Ai}}{l_i},\ \frac{\partial x_i}{\partial r_x} = \frac{1}{l_i},\ \frac{\partial x_i}{\partial r_y} = 0,\ \frac{\partial x_i}{\partial r_z} = 0,$$

$$\frac{\partial x_i^o}{\partial \varphi_x} = \frac{\left\{(y_{Ai} - y_{Ci})\left[\dfrac{(y_{Ai} - 2y_{Ci})z_{Ai}}{(z_{Ai} - z_{Ci})} - y_{Ai}\right] + z_{Ci}z_{Ai}\right\}}{l_i},$$

$$\frac{\partial x_i^o}{\partial \varphi_y} = \frac{\left[\dfrac{(x_{Ai} - x_{Ci})(2y_{Ci} - y_{Ai})z_{Ai}}{(z_{Ai} - z_{Ci})} - x_{Ai}(y_{Ai} - y_{Ci})\right]}{l_i},$$

$$\frac{\partial x_i^o}{\partial \varphi_z} = \frac{\left\{\dfrac{\left[(y_{Ai} - y_{Ci})x_{Ai} - (x_{Ai} - x_{Ci})y_{Ai}\right](2y_{Ci} - y_{Ai})}{(z_{Ai} - z_{Ci})} - x_{Ai}z_{Ci}\right\}}{l_i},$$

$$\frac{\partial x_i^o}{\partial r_x} = \frac{(x_{Ai} - x_{Ci})(2y_{Ci} - y_{Ai})}{(z_{Ai} - z_{Ci})l_i},\ \frac{\partial x_i^o}{\partial r_y} = \frac{\dfrac{(y_{Ai} - y_{Ci})(2y_{Ci} - y_{Ai})}{(z_{Ai} - z_{Ci})} - z_{Ci}}{l_i},$$

$$\frac{\partial x_i^o}{\partial r_z} = \frac{(y_{Ci} - y_{Ai})}{l_i},\ i = 1,\dots,6$$

Other partial derivatives also can be obtained from Eqs. (6), (7), and (8).
By means of the properties of linear decomposition of determinants d[det(E)] can be obtained as the sum of *36* determinants (Glazunov, 2006). From this, d[det(E)] can be presented as:

$$
\begin{aligned}
d\left[\det(E)\right] = \frac{\partial\left[\det(E)\right]}{\partial\varphi_x d\varphi_x} + \frac{\partial\left[\det(E)\right]}{\partial\varphi_y d\varphi_y} + \frac{\partial\left[\det(E)\right]}{\partial\varphi_z d\varphi_z} + \\
+ \frac{\partial\left[\det(E)\right]}{\partial r_x dr_x} + \frac{\partial\left[\det(E)\right]}{\partial r_y dr_y} + \frac{\partial\left[\det(E)\right]}{\partial r_z dr_z}
\end{aligned}
\tag{10}
$$

Using (10) the criterion of the singularity locus can be presented as d[det(E)]=0. This condition imposes only one constraint. Therefore, there exist five twists of motions of the platform inside singularity.
For example, let us obtain five inside singularity of manipulator. Set up the coordinates of the vectors be r_i are r_1(-1, 0, 4), r_2(-0.5, 1, 4), r_3(0.5, 1, 4), r_4(1, 0, 4), r_5(0.5, -1, 4), r_6(-0.5, -1, 4); s_i are s_1(-1.5, 0, 0.866), s_2(-1, 1.5, 0.707), s_3(1, 1.5, 0.707), s_4(1.5, 0, 0.866), s_5(1, -1.5, 0.707), s_6(-1, -1.5, 0.707). From here, we can see the generalized coordinates as (0.136, 0.305, 0.305, 0.136, 0.305, 0.305). Matrix (E) is determined as:

$$
\begin{pmatrix}
0.158 & 0 & 0.988 & 0 & 1.618 & 0 \\
0.148 & -0.148 & 0.978 & 1.572 & 1.083 & -0.074 \\
-0.148 & -0.148 & 0.978 & 1.572 & -1.083 & 0.074 \\
-0.158 & 0 & 0.988 & 0 & -1.618 & 0 \\
-0.148 & 0.148 & 0.978 & -1.572 & -1.083 & -0.074 \\
0.148 & 0.148 & 0.978 & -1.572 & 1.083 & 0.074
\end{pmatrix}
$$

The determinant consisting of the Plücker coordinates of the unit screws is det(E) = 0. Their partial derivatives are:

$$
\frac{\partial\left[\det(E)\right]}{\partial r_x} = -0.02,\ \frac{\partial\left[\det(E)\right]}{\partial r_y} = 0,\ \frac{\partial\left[\det(E)\right]}{\partial r_z} = 0,
$$

$$
\frac{\partial\left[\det(E)\right]}{\partial\varphi_x} = 0,\ \frac{\partial\left[\det(E)\right]}{\partial\varphi_y} = 0.002,\ \frac{\partial\left[\det(E)\right]}{\partial\varphi_z} = 0
$$

Using the approach presented above, we find five independent twists inside singularity: $\$_1(1, 0, 0, 0, 0, 0)$, $\$_2(0, 0, 1, 0, 0, 0)$, $\$_3(0, 0, 0, 0, 1, 0)$, $\$_4(0, 0, 0, 0, 0, 1)$, $\$_5(0, 4.433, 0, 1, 0, 0)$. The twist-gradient is calculated to be $\$^*(-0.02, 0, 0, 0, 0.002, 0)$. This twist-gradient is practically important as it offers the highest speed.

5. Dynamical decoupling

In this section, let we consider the reduction of the dynamical coupling of the motors of the parallel manipulator with linear actuators located on the base. The basic idea is to represent the kinetic energy as the quadratic polynomial including only the squares of the generalized velocities (Glazunov, & Kraynev, 2006). The kinetic energy can be expressed by means of the matrix *(E)*.
Let m be the mass and J_x, J_y, J_z be the inertia moments of the platform. Assuming that the mass of the platform is much more than the masses of the legs and using the Eqs. (6)-(8), the kinetic energy T can be expressed as follows (Dimentberg, 1965; Kraynev, & Glazunov, 1991):

$$T = \frac{m}{2}\left[\left(\sum_{i=1}^{6} p_i^o \dot{\theta}_i G_{ii}\right)^2 + \left(\sum_{i=1}^{6} q_i^o \dot{\theta}_i G_{ii}\right)^2 + \left(\sum_{i=1}^{6} r_i^o \dot{\theta}_i G_{ii}\right)^2\right] +$$
$$+ \frac{J_x}{2}\left(\sum_{i=1}^{6} p_i \dot{\theta}_i G_{ii}\right)^2 + \frac{J_y}{2}\left(\sum_{i=1}^{6} q_i \dot{\theta}_i G_{ii}\right)^2 + \frac{J_z}{2}\left(\sum_{i=1}^{6} r_i \dot{\theta}_i G_{ii}\right)^2 \tag{11}$$

where $\dot{\theta}_i$ are the generalized velocities, p_i, q_i, r_i, p^o_i, q^o_i, r^o_i are the components of the matrix $(E)^{-1}$, G_{ii} are the components of the diagonal matrix *(G)* *(i=1,…,6)*.
The Lagrange equation of motion for a parallel manipulator can be written as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_i}\right) - \frac{\partial T}{\partial \theta_i} = Q_i \tag{12}$$

where Q_i are the generalized forces *(i=1,…,6)*.
The dynamical coupling can be determined using the Eq. (11). The expression of each generalized force comprises all other generalized velocities and accelerations. In order to reduce the dynamical coupling we represent the kinetic energy as follows:

$$T = \frac{m}{2}\left[\begin{array}{l} \sum_{i=1}^{6}\left(p_i^o \dot{\theta}_i G_{ii}\right)^2 + 2\sum_{i=1,j=1,i\neq j}^{6} p_i^o p_j^o \dot{\theta}_i G_{ii} \dot{\theta}_j G_{jj} + \\ +\ldots+ \sum_{i=1}^{6}\left(r_i^o \dot{\theta}_i G_{ii}\right)^2 + 2\sum_{i=1,j=1,i\neq j}^{6} r_i^o r_j^o \dot{\theta}_i G_{ii} \dot{\theta}_j G_{jj} \end{array}\right] +$$
$$+ \frac{J_x}{2}\left[\sum_{i=1}^{6}\left(p_i \dot{\theta}_i G_{ii}\right)^2 + 2\sum_{i=1,j=1,i\neq j}^{6} p_i p_j \dot{\theta}_i G_{ii} \dot{\theta}_j G_{jj}\right] + \tag{13}$$
$$+\ldots+ \frac{J_z}{2}\left[\sum_{i=1}^{6}\left(r_i \dot{\theta}_i G_{ii}\right)^2 + 2\sum_{i=1,j=1,i\neq j}^{6} r_i r_j \dot{\theta}_i G_{ii} \dot{\theta}_j G_{jj}\right]$$

According to the Eq. (13) dynamical decoupling can be satisfied if the columns of the following matrix *(D)* are orthogonal:

$$(D) = \begin{pmatrix} p_1^o G_{11}\sqrt{m} & . & . & . & . & p_6^o G_{66}\sqrt{m} \\ q_1^o G_{11}\sqrt{m} & . & . & . & . & q_6^o G_{66}\sqrt{m} \\ r_1^o G_{11}\sqrt{m} & . & . & . & . & r_6^o G_{66}\sqrt{m} \\ p_1 G_{11}\sqrt{J_x} & . & . & . & . & p_6 G_{66}\sqrt{J_x} \\ q_1 G_{11}\sqrt{J_y} & . & . & . & . & q_6 G_{66}\sqrt{J_y} \\ r_1 G_{11}\sqrt{J_z} & . & . & . & . & r_6 G_{66}\sqrt{J_z} \end{pmatrix}$$

$$(D)=\begin{pmatrix} \sqrt{m} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{m} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{J_x} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{J_y} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{J_z} \end{pmatrix}(E)^{-1}(G)$$

$$(D)=(M)(E)^{-1}(G) \tag{14}$$

Let the axes of the links A_iC_i be parallel to the axes of the links B_iC_i and the matrix *(G)* is unit matrix. Then in order to satisfy the condition of orthogonal columns of the matrix *(D)* the rows of the inverse matrix $(D)^{-1} = (E)(M)^{-1}$ are to be orthogonal. From this, the following matrix *(E)* is proposed:

$$(E)=\begin{pmatrix} 0 & 0 & 1 & 0 & -\sqrt{J_y/m} & 0 \\ 0 & 0 & -1 & 0 & -\sqrt{J_y/m} & 0 \\ 1 & 0 & 0 & 0 & 0 & -\sqrt{J_z/m} \\ -1 & 0 & 0 & 0 & 0 & -\sqrt{J_z/m} \\ 0 & 1 & 0 & -\sqrt{J_x/m} & 0 & 0 \\ 0 & -1 & 0 & -\sqrt{J_x/m} & 0 & 0 \end{pmatrix} \tag{15}$$

The determinant of the matrix *(E)* (15) can be written as:

$$\det(E) = 8\sqrt{J_x J_y J_z / m^3} \tag{16}$$

The matrix (15) corresponds to the Fig. 3. Here the center of the mass of the platform coincides with the center of the coordinate system xyz and the axes of the links of the legs

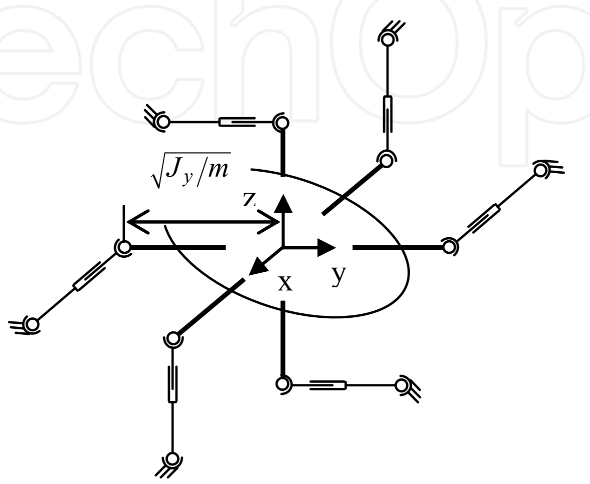


Fig. 3. Parallel manipulator with dynamical decoupling

are parallel to the main central inertia axes of the platform. The proposed approach can be applicable for manipulators characterized by small displacements and high speeds. Moreover, this architecture causes partial kinematic decoupling because if the generalized coordinates corresponding to the opposite legs are equivalent then the moving platform keeps constant orientation.

6. Pressure angles

The parallel manipulators have singularity configurations in which there is an uncontrolled mobility because some of the wrenches acting on the output link are linearly dependent. The local criterion of singular configurations is the singular matrix of the screw coordinates of the wrenches, such as:

$$\det(E) = \varepsilon^* \tag{17}$$

where ε^* is the preassigned minimal determinant value. The pressure angle of the linear dependent sub-chain is equal to $\pi/2$, as a reciprocal twist to five-member group of screws has a perpendicular moment at about any points of the axis. All stalled actuators but one the manipulator has *DOF=1* and its output link can move along some twist $\Omega = \omega + \chi\omega^0$ ($\chi^2 = 0$) reciprocal to five-member group of the wrenches corresponding to stalled actuators. We can find this twist from the reciprocity condition:

$$mom(\Omega, R_i) = 0,\ i = 1,\ldots,5 \tag{18}$$

In general the six-member group of the unit wrenches of zero parameter $R_i(r_i, r_i^o)$ $(i=1,\ldots,6)$ is acting on the output link of the such manipulators, determinant composed of the screw coordinates of these wrenches as given in (5).
The velocity of any point A_i $(i=1,\ldots,6)$ of the mobile platform can be found as a twist moment relative to this point:

$$V_{A_i} = \omega^0 + \omega \times r_{A_i},\ i = 1,\ldots,6 \tag{19}$$

where r_{Ai} is radius-vectors of the points A_i.
The pressure angle α_i for the stalled actuator *i-th* of the parallel manipulators (Fig.1) can be determined as:

$$\alpha_i = \arccos\left(\frac{V_{A_i}.F_i}{\left|V_{A_i}\right|.\left|F_i\right|}\right),\ i = 1,\ldots,6 \tag{20}$$

where F_i is the force vector on the actuator axis. For normal functions of the manipulator it is necessary that working space be limited by positions:

$$\alpha_i \le \alpha_{KP},\ i = 1,\ldots,6 \tag{21}$$

where α_{KP} is maximum pressure angle is defined by friction coefficient.
The manipulator control system must be provided by algorithm testing the nearness to singular configurations based on the analysis of singular matrix (5) or on the pressure angle determination.

7. Manipulator for external conditions

In Fig. 4, (a, b) the six-DOFs parallel mechanisms and their sub-chain has a parallel connection of links and actuators are shown in (Glazunov, et al. 1999) which were invented by (Kraynev, & Glazunov, 1991). Such mechanisms may be utilized to manipulate the corrosive medium at all actuators that are located out of the working space. Existence of several sub-chains and many closed loops determine the essential complication of the mathematical description of these mechanisms. Screw calculus using screw groups is universal and effective for parallel mechanisms analysis. Here, *1* describes the fixed base, *2* describes the output link and *3* describes the actuators. Addition, A_i expresses the spherical joint center situated on the fixed base; B_j expresses the center of the spherical joints combined with translational; C_j expresses the output link spherical joint centers; l_i, d_j expresses the generalized coordinates and s_j, f_j expresses the link lengths *(i=1,…,6; j=1,…, 3)*.

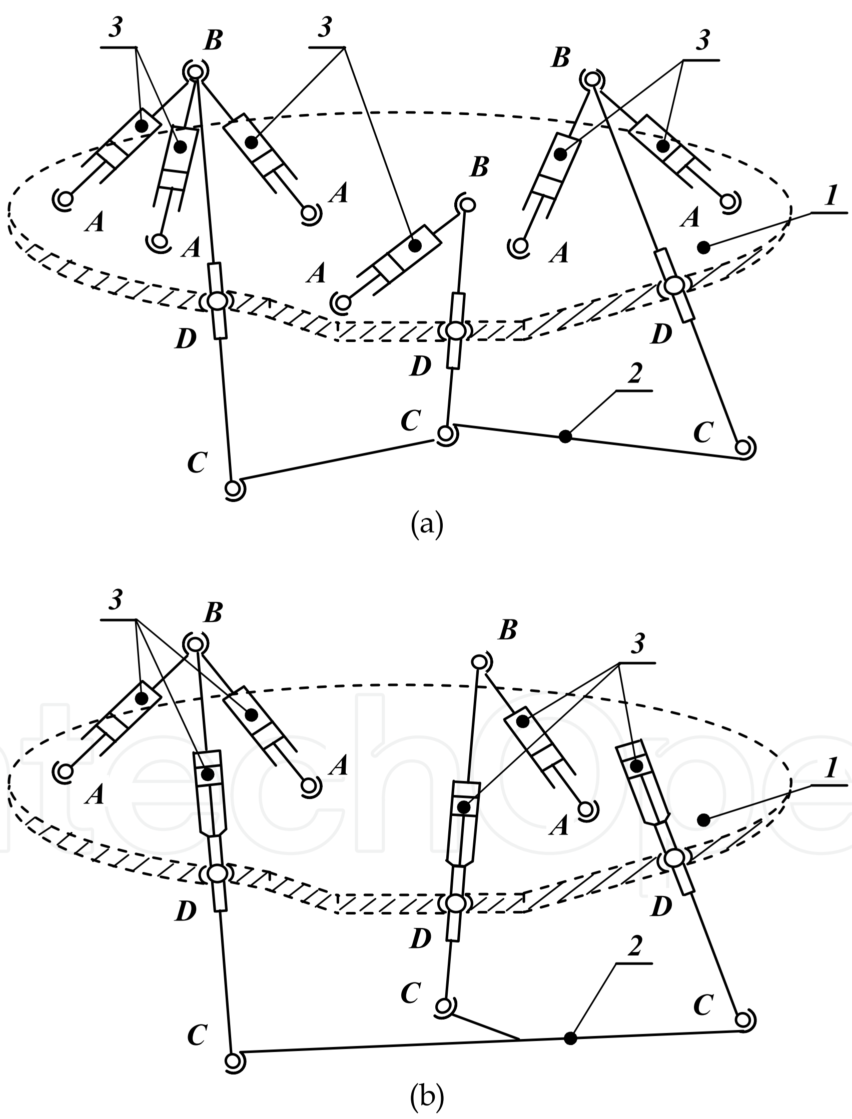


Fig. 4. The six-DOFs parallel mechanisms

In general, the wrench axis corresponding to *i-th* stalled actuator is located in the plane ($A_iB_jC_j$), passes through center joint C_j and is directed perpendicular to its possible displacement. With the mechanisms as in (Fig. 3, a) the components of the wrench R_i can be find as:

$$r_i = \frac{1}{p_i\left\{a_i + \left[\frac{s_j}{f_j} - \left(\frac{1}{f_j} + \frac{1}{s_j}\right)\frac{\left(a_i.b_j\right)}{d_j}\right]b_j\right\}}\,,\,; r_i^0 = \rho_{C_j} \times r_i \tag{22}$$

where p_i - vector defining the wrench axis, a_i - vector from point A_i to point C_j, b_i - vector from point D_j to point C_j and ρ_{C_j} - radius vector point C_j.

Besides, with the mechanism as in (Fig. 4, b) the wrench axis *(j=1, …,3)* coincides with actuator axis. The wrench of the *i-th* stalled actuator *(i=, …,3)* is given as

$$r_i = \frac{1}{l_i}\left\{a_i - \frac{\left(a_i.b_j\right)b_j}{d_j^2}\right\}\,,\,;\; r_i^0 = \rho_{C_j} \times r_i \tag{23}$$

The present approach may be applied for different types of mechanisms such as sub-chains with varied actuator connection using spherical pairs.

8. Conclusion

Thus in this paper various criteria of design and singularity analysis of parallel manipulators are presented. The constraint wrenches imposed to the platform by kinematic chains is proposed to rely on the screw theory by determinants of matrix consisting of the Plücker coordinates of the unit screws. Criteria for design and singularity analysis of parallel manipulators with linear actuators located on the base are presented. The kinematic criterion of singularity corresponds to linear dependence of wrenches supporting the mobile platform; the static criterion corresponds to the limitation of pressure angles. The dynamical decoupling allows increasing the accuracy, parametrical optimization allows designing the mechanisms with optimal working volume, dexterity and stiffness, determination of the twists inside singularity allows finding the differential conditions of singular loci. Furthermore, the use of screw groups in order to determination of the singular zones of the multi-DOFs parallel mechanisms that make form of continuous areas and manipulators for external conditions are expressed.

9. References

Angeles, J. (2004). The Qualitative Synthesis of Parallel Mechanisms, *In Journal of Mechanical Design*, 126: 617-624.

Arakelian, V.; Briot, S. & Glazunov, V. (2007). Improvement of functional performance of spatial parallel mechanisms using mechanisms of variable structure, *In Proceedings of the 12th World Congress in Mechanism and Machine Science*, Besancon, France, 1: 159-164.

Bonev, I.; Zlatanov, D. & Gosselin, C. (2003). Singularity analysis of 3-DOF planar parallel mechanisms via screw theory, *In Transactions of the ASME, Journal of Mechanical Design*, 125: 573-581.

Ceccarelli, M. (2004). Fundamentals of Mechanics of Robotic Manipulations, *Kluwer Academic Publishers*.

Dimentberg, F. (1965). The Screw calculus and its Applications in Mechanics, *Nauka*, (English translation: AD680993, Clearinghouse for Federal Technical and Scientific Information, Virginia).

Glazunov, V. & Kraynev, A. (2006). Design and Singularity Criteria of Parallel mechanisms, *In ROMANSY 16, Robot Design, Dynamics, and Control, Proceedings of 16 CISM-IFToMM Symposium*, Springer Wien New York, 15-22.

Glazunov, V. & Thanh, N.M. (2008). Determination of the parameters and the Twists inside Singularity of the parallel Manipulators with Actuators Situated on the Base, *ROMANSY 17, Robot Design, Dynamics, and Control. In Proceedings of the Seventeenth CISM-IFToMM Symposium*, Tokyo, Japan, 467-474.

Glazunov, V. (2006). Twists of Movements of Parallel Mechanisms inside Their Singularities, *In Mechanism and Machine Theory*, 41: 1185-1195.

Glazunov, V.; Gruntovich, R.; Lastochkin, A. & Thanh, N.M. (2007). Representations of constraints imposed by kinematic chains of parallel mechanisms, *In Proceedings of 12th World Congress in Mechanism and Machine Science*, Besancon, France, 1: 380-385.

Glazunov, V.; Hue, N.N. & Thanh, N.M. (2009). Singular configuration analysis of the parallel mechanisms, *In Journal of Machinery and engineering education*, ISSN 1815-1051, No. 4, 11-16.

Glazunov, V.; Koliskor, A. & Kraynev, A. (1991). Spatial Parallel Structure Mechanisms, *Nauka*.

Glazunov, V.; Koliskor, A.; Kraynev, A. & B. Model, (1990). Classification Principles and Analysis Methods for Parallel-Structure Spatial Mechanisms, *In Journal of Machinery Manufacture and Reliability*, Allerton Press Inc., 1: 30-37.

Glazunov, V.; Kraynev, A.; Rashoyan, G. & Trifonova, A. (1999). Singular Zones of Parallel Structure Mechanisms, *In X World Congress on TMM*, Oulu, Finland, 2710-2715.

Gosselin, C. & Angeles, J. (1990). Singularity Analysis of Closed Loop Kinematic Chains, *In IEEE Trans. on Robotics and Automation*, 6(3): 281-290.

Huang, Z. (2004). The kinematics and type synthesis of lower-mobility parallel robot manipulators, *Proceedings of the XI World Congress in Mechanism and Machine Science*, Tianjin, China, 65–76.

Kong, X. & Gosselin, C. (2007). Type Synthesis of Parallel Mechanisms, *Springer-Verlag Berlin Heidelberg*, 272p.

Kraynev, A. & Glazunov, V. (1991). Parallel Structure Mechanisms in Robotics, *In MERO'91, Sympos. Nation. de Robotic Industr.*, Bucuresti, Romania, 1: 104-111.

Merlet, J.-P. (1991). Articulated device, for use in particular in robotics, *United States Patent 5,053,687*.

Merlet, J.-P. (2006). Parallel Robots, *Kluwer Academic Publishers*, 394p.

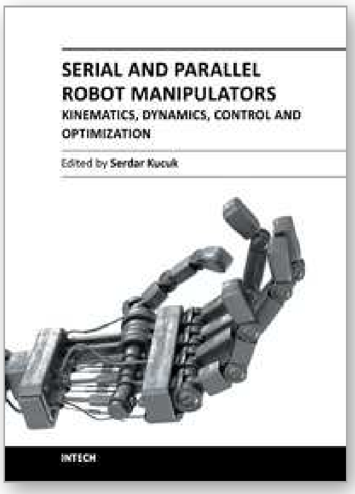
Statnikov, R.B. (1999). Multicriteria Design. Optimization and Identification, Dordrecht, Boston, London: *Kluwer Academic Publishers*, 206p.

Thanh, N.M.; Glazunov, V. & Vinh, L.N. (2010). Determination of Constraint Wrenches and Design of Parallel Mechanisms, *In CCE 2010 Proceedings, Tuxtla Gutiérrez, Mexico, 2010, International Conference on Electrical Engineering, Computing Science and Automatic Control, IEEE 2010*, 46-53.

Thanh, N.M.; Glazunov, V.; Tuan, T.C. & Vinh, N.X. (2010). Multi-criteria optimization of the parallel mechanism with actuators located outside working space, *In ICARCV 2010 Proceedings, Singapore, International Conference on Control, Automation, Robotics and Vision, IEEE 2010*, 1772-1778.

Thanh, N.M.; Glazunov, V.; Vinh, L.N. & Mau, N.C. (2008). Parametrical optimization of the parallel mechanisms while taking into account singularities, *In ICARCV 2008 Proceedings, Hanoi, Vietnam, International Conference on Control, Automation, Robotics and Vision, IEEE 2008*, 1872-1877.

Thanh, N.M.; Quoc, L.H. & Glazunov, V. (2009). Constraints analysis, determination twists inside singularity and parametrical optimization of the parallel mechanisms by means the theory of screws, *In CCE 2009 Proceedings, Toluca, Mexico, International Conference on Electrical Engineering, Computing Science and Automatic Control, IEEE 2009*, 89-95.



Serial and Parallel Robot Manipulators - Kinematics, Dynamics, Control and Optimization

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The robotics is an important part of modern engineering and is related to a group of branches such as electric & electronics, computer, mathematics and mechanism design. The interest in robotics has been steadily increasing during the last decades. This concern has directly impacted the development of the novel theoretical research areas and products. This new book provides information about fundamental topics of serial and parallel manipulators such as kinematics & dynamics modeling, optimization, control algorithms and design strategies. I would like to thank all authors who have contributed the book chapters with their valuable novel ideas and current developments.