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Fuzzy Modelling Stochastic Processes Describing Brownian Motions

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1. Introduction

Wiener process, as a special mathematical model of Brownian motions, has been investigated and modelling in many probabilistic examples. In the topic literature it is easy to find many procedures of numeric probabilistic simulations of the Wiener process. Fuzzy modelling does not give us more accurate models than probabilistic modelling. Fuzzy knowledge-based modelling allows to determine linguistic description of non-precise relationships between variables and to derive the reasoning procedure from non-crisp facts. More over, using the notions of probabilities of fuzzy events, it is possible to determine a frequency of a conclusion as well as its expected value.

Wiener process and a random walk are very often used for modelling phenomena in physics, engineering and economy. In the area of robot control theory these processes can represent some time-varying parameters of the environments where the object of control operates. Fuzzy models of these processes can constitute a part of a fuzzy model of a tested complex system.

In paragraph 2. of this chapter, the mathematical descriptions of Brownian motions has been reminded, according to the theory of probability and stochastic processes. Some basics of fuzzy modelling has been presented in paragraph 3., to show the method of creating the knowledge base and rules of reasoning. Attention is focused on identification techniques for building empirical probabilities of fuzzy events from input-output data. Exemplary calculations of knowledge bases for real stochastic processes, as well as, some remarks on future works have been presented in paragraphs 4 and 5.

2. Mathematical models of Brownian motion

The Brownian motion it is well known in physics, a random movement of a particle suspended in a liquid or a gas. The name of the movement is given after the botanist Robert Brown (1827), who was studying the movement of pollen grains suspended in water. There are many similar phenomena, where the time evolutions of the object depend on stochastic, microscopic contacts (collisions) with elements of the surrounded system. In mathematics, many models describing Brownian motion are well known and applied, e.g. the random walk stochastic process, Wiener stochastic process, Langevin stochastic differential equation, general diffusion equations and others.

Observations of the microscopic particle behavior show, that at any time step, the particle is changing its position in the space, according to collisions with liquid particles. Crashes of

particles are frequent and irregular. It is usually assumed by mathematicians, that the displacements of the particle $Z_1, Z_2, \dots, Z_n, \dots$ at particular time steps, are independent, identically distributed random variables. The stochastic process $\{Z_i, i = 1, 2, \dots, n, \dots\}$ is named *random walk*.

In macroscopic scale, if the time between two observations of the particle, $t - \tau$, is larger than the time between successive crashes, then the increment of the particle positions, $X_t - X_\tau$, is a sum of many small displacements, $X_t - X_\tau = \sum_{i=1, \dots, k} Z_i$. Since the increments

$X_t - X_\tau$ constitute sums of independent, identically distributed random variables, they are normal distributed random variables.

In mathematics, scalar stochastic process $\{X_t, 0 \leq t < \infty\}$, is the *Wiener process* if and only if

- increments $X_t - X_\tau$, $0 \leq \tau < t < \infty$ are homogeneous (stationary) and independent for disjoint time intervals,
- the initial condition, $P(X_0 = 0) = 1$, is fulfilled,
- trajectories of the process $\{X_t, 0 \leq t < \infty\}$ are continuous (almost surely),
- random variables X_t , are normal distributed, with the probability density function

$$f(t, x) = \frac{1}{\sqrt{2\pi t\sigma^2}} \exp\left(-\frac{x^2}{2t\sigma^2}\right). \quad (1)$$

Wiener process is also known as the *Brownian motion process* (Fisz, 1967; van Kampen, 1990; Kushner, 1983; Sobczyk, 1991).

The increments, $X_t - X_\tau$, $0 \leq \tau < t < \infty$, are normal distributed random variables with the expected value and variance as follows:

$$E(X_t - X_\tau) = 0, \quad D^2(X_t - X_\tau) = (t - \tau)\sigma^2. \quad (2)$$

Random variables X_{t_1}, \dots, X_{t_n} , where

$$X_{t_n} = X_{t_1} + (X_{t_2} - X_{t_1}) + \dots + (X_{t_n} - X_{t_{n-1}}), \quad (3)$$

are also normal distributed with parameters:

$$E(X_{t_k}) = 0, \quad D^2(X_{t_k}) = t_k \sigma^2, \quad k=1, 2, \dots, n; \quad (4)$$

and with a non-zero covariance matrix.

If $\sigma^2 = 1$ then $\{X_t, 0 \leq t < \infty\}$ is the *standard Wiener process*.

Probability, that a particle occurs in some interval $[a, b]$, at the moment t , is given by the relationship

$$\Pr\{X_t \in [a, b]\} = \int_a^b f(t, x) dx = \frac{1}{\sqrt{2\pi t\sigma^2}} \int_a^b \exp\left(-\frac{x^2}{2t\sigma^2}\right) dx. \quad (5)$$

For any $t_1 \leq t_2$ probability density function of the random vector variable (X_{t_1}, X_{t_2}) can be obtained as follows:

$$f(t_1, x_1; t_2, x_2) = f(t_2, x_2 / t_1, X_{t_1} = x_1) f(t_1, x_1) \quad (6)$$

where

$$f(t_2, x_2 / t_1, X_{t_1} = x_1) = \frac{1}{\sqrt{2\pi\sigma(t_2 - t_1)}} \exp\left(-\frac{(x_1 - x_2)^2}{2(t_2 - t_1)\sigma^2}\right) \quad (7)$$

is a conditional probability density function, and $f(t_1, x_1)$ is given by formula (1) with parameters: $E(X_{t_1}) = 0$, $D^2(X_{t_1}) = t_1\sigma^2$.

For any $t_1 \leq t_2 \leq \dots \leq t_n$ probability density function of the multidimensional random vector $(X_{t_1}, \dots, X_{t_n})$ can be obtained, taking into account Markov features of the process and using (3), as follows (Sobczyk, 1991):

$$f(t_n, x_n, \dots, t_1, x_1) = \prod_{i=1, \dots, n} \frac{1}{\sqrt{2\pi\sigma(t_i - t_{i-1})}} \exp\left(-\sum_{i=1, \dots, n} \frac{(x_i - x_{i-1})^2}{2(t_i - t_{i-1})\sigma^2}\right). \quad (8)$$

Stochastic vector process $\{[X_1(t), \dots, X_n(t)], 0 \leq t < \infty\}$ is called the *nD stochastic Wiener process* if its every component, $\{X_i(t), 0 \leq t < \infty\}$, $i=1, \dots, n$, is the scalar stochastic Wiener process and particular scalar stochastic processes are independent.

As an example of the 3D stochastic Wiener process we can show three coordinates of the Brownian particle trajectory.

The Wiener process is also the special diffusion stochastic process, fulfilling the Fokker-Planck diffusion equation

$$\frac{\partial f(x, t)}{\partial t} = \gamma \frac{\partial^2 f(x, t)}{\partial x^2}, \quad \lim_{\Delta t \rightarrow 0, \Delta x \rightarrow 0} \frac{(\Delta x)^2}{\Delta t} = \text{const} = 2\gamma \quad (9)$$

where the solution is given by the normal probability density function, and the diffusion coefficient is equal to $\gamma = \sigma^2 / 2$ (van Kampen, 1990).

In macroscopic scale, in physics and in industrial practice, the probability value $f(x)dx$ that scalar variable X assumes its value from the interval $[x, x+dx]$ is equivalent to the quotient n/N (concentration of particles), where n defines the power of subset of particles, whose feature X determines the value over the interval $[x, x+dx]$, and N is the population size. This idea is consistent with Einstein's experiments who considered collective motion of Brownian particles. He assumed that the density (concentration) of Brownian particles $\rho(x, t)$ at point x at time t , met the following diffusion equation:

$$\frac{\partial \rho(x, t)}{\partial t} = \gamma \frac{\partial^2 \rho(x, t)}{\partial x^2} \quad (10)$$

where γ is a diffusion coefficient. The solution has the known exponential form. From the analytical form of the solution the second central moment of the displacement is expressed as

$$E(X_t^2) = 2\gamma t \quad (11)$$

Diffusion coefficient, γ , has been expressed by Einstein as a function of macro- and microscopic parameters of the fluid and particles, respectively. Einstein confirmed statistical character of the diffusion law (cited by van Kampen, 1990).

3. Fuzzy knowledge representation of the 'short memory' stochastic process

3.1 Stochastic process with fuzzy states

Let $X(t)$ be a 'short memory' stochastic process, the family of time-dependent random variables, where $X \in \chi \subset R$, $t \in T \subset R$ and B is the Borel σ -field of events. Let p be a probability, the normalized measure over the space (χ, B) .

Moreover, assume that according to human experts' suggestions, in the universe of process values, the linguistic random variable has been determined with the set of linguistic values, $L(X) = \{LX_i\}$, $i=1,2,\dots,I$ e.g. $L(X) = \{\text{low}, \text{middle}, \text{high}\}$, according to Zadeh's definition of the linguistic variable (Zadeh, 1975). The meanings of the linguistic values are represented by fuzzy sets A_i , $i=1,2,\dots,I$ determined on χ by their membership functions, $\mu_{A_i}(x): \chi \rightarrow [0,1]$, which are Borel measurable functions, fulfilling the condition

$$\sum_{i=1}^I \mu_{A_i}(x) = 1, \quad \forall x \in \chi. \quad (12)$$

According to above assumptions, the probability distribution of linguistic values of the process $X(t)$ can be determined as follows

$$P(X_t) = \{P(A_i), i=1,2,\dots,I\}, \quad (13)$$

based on Zadeh's definitions of the probability of fuzzy events (Zadeh, 1968)

$$P(A) = \int_{\chi \subseteq R^n} \mu_A(x) dp. \quad (14)$$

The following conditions must be fulfilled

$$0 \leq P(A_i) \leq 1, i=1,2,\dots,I; \quad \sum_{i=1}^I P(A_i) = 1. \quad (15)$$

Let now $t = t_1$, $t = t_2$, $t_2 > t_1$ be fixed, so the stochastic process at that moments is represented by two random variables $(X(t_1), X(t_2))$. Assume, that (χ^2, B, p) is a probability space, where $\chi^2 \subseteq R^2$, B is the Borel σ -field of events and p is a probability, the normalized measure over (χ^2, B) . The assumptions mean that the probability distribution $p(x_{t_1}, x_{t_2})$ over the realizations (X_{t_1}, X_{t_2}) exists.

Let also two linguistic random variables (linguistic random vector) (X_{t_1}, X_{t_2}) be generated in χ^2 , taking simultaneous linguistic values $LX_i \times LX_j$, $i, j=1,2,\dots,I$; corresponding collection of fuzzy events $\{A_i \times A_j\}_{i,j=1,\dots,I}$ is determined on χ^2 by membership functions

$\mu_{A_i \times A_j}(x_{t_1}, x_{t_2})$, $i, j=1, 2, \dots, I$. Membership functions for joint fuzzy events $A_i \times A_j$ should fulfill

$$\sum_i \sum_j \mu_{A_i \times A_j}(x_{t_1}, x_{t_2}) = 1, \quad \forall (x_{t_1}, x_{t_2}) \in \mathcal{X}^2. \quad (16)$$

The joint 2D probability distribution of linguistic values (fuzzy states) of the stochastic process $X(t)$ is determined by the joint probability distribution of the linguistic random vector (X_{t_1}, X_{t_2})

$$P(X_{t_1}, X_{t_2}) = \{P(A_i \times A_j)\}_{i,j=1,2,\dots,I} \quad (17)$$

calculated according to

$$P(A_i \times A_j) = \int_{(x_{t_1}, x_{t_2}) \in \mathcal{X}^2} \mu_{A_i \times A_j}(x_{t_1}, x_{t_2}) dp \quad (18)$$

and fulfilling

$$0 \leq P(A_i \times A_j) \leq 1, \quad \forall i, j = 1, \dots, I \quad \text{and} \quad \sum_{i=1}^I \sum_{j=1}^I P(A_i \times A_j) = 1 \quad (19)$$

(Walaszek-Babiszewska, 2008, 2011). From the joint probability distribution (17), the conditional probability distribution of the fuzzy transition

$$P[(X_{t_2} = A_j) / (X_{t_1} = A_i)], \quad j=1, 2, \dots, I; \quad i=const \quad (20)$$

can be determined according to

$$\begin{aligned} & \{ P[(X_{t_2} / (X_{t_1} = A_i))] \}_{j=1,\dots,I} = \\ & = \frac{\{ P(X_{t_2} = A_j, X_{t_1} = A_i) \}_{j=1,\dots,I}}{P(X_{t_1} = A_i)}. \end{aligned} \quad (21)$$

The following relationships should be fulfilled for the conditional distributions of fuzzy states (probability of the transitions)

$$\sum_{j=1,\dots,I} P[(X_{t_2} = A_j) / (X_{t_1} = A_i)] = 1; \quad i = const. \quad (22)$$

3.2 Rule based fuzzy model

The proposed model of the stochastic process, formulated into fuzzy categories, for two moments t_1, t_2 , $t_2 > t_1$, is a collection of file rules, in the following form (Walaszek-Babiszewska, 2008, 2011):

$$\forall A_i \in L(X), \quad i=1,\dots,I$$

$$\begin{array}{c}
 R^{(i)}: w_i[If(X_{t_1} \text{ is } A_i)] Then(X_{t_2} \text{ is } A_1)w_{1/i} \\
 \text{-----} \\
 \text{Also}(X_{t_2} \text{ is } A_j)w_{j/i} \\
 \text{-----} \\
 \text{Also}(X_{t_2} \text{ is } A_j)w_{j/i}
 \end{array} \quad (23)$$

or as a collection of the elementary rules in the form

$$\forall A_i \in L(X), \forall A_j \in L(X), i, j=1, 2, \dots, I$$

$$R^{(i,j)}: w_{ij}[If(X_{t_1} \text{ is } A_i) Then(X_{t_2} \text{ is } A_j)] \quad (24)$$

where the weights w_i , $w_{j/i}$, w_{ij} represent probabilities of fuzzy states, determined by (13) – (15), (20) – (22) and (17) – (19), respectively. The weights stand for the frequency of the occurrence of fuzzy events in particular parts of rules and show the probabilistic structure of the linguistic values of the linguistic random vector (X_{t_1}, X_{t_2}) . The weights do not change logic values of the conditional sentences.

3.3 Reasoning procedures

Considering reasoning procedure, we assume that some non-crisp (vague) observed value of the stochastic process at moment t_1 is known and equal to $X_{t_1} = A'$, or some crisp value $X_{t_1} = x_{t_1}^*$ of the stochastic process at moment t_1 is given. Then, the level of activation of the elementary rule (24) is determined according to one of the following formulas

$$\tau_i = \max_x \min[\mu_{A_i}(x), \mu_{A'}(x)], \quad (25)$$

$$\tau_i = \mu_{A_i}(x_{t_1}^*), \quad i=1, \dots, I, \quad (26)$$

repectively (Yager & Filev, 1994; Hellendoorn & Driankov, 1997). The conclusion according to the generalized Mamdani-Assilian's type interpretation of fuzzy models has the following form

$$\mu_{A_{j/i}}(x_{t_2}) = T(\tau_i, \mu_{A_j}(x_{t_2})), \quad j=1, \dots, I; \quad i=\text{const}; \quad (27)$$

thus the conclusion derived based on logic type interpretation of fuzzy models is as follows

$$\mu_{A_{j/i}}(x_{t_2}) = I(\tau_i, \mu_{A_j}(x_{t_2})), \quad j=1, \dots, I; \quad i=\text{const}, \quad (28)$$

where T denotes a t -norm and I means the implication operator. Aggregation of the conclusions from particular rules is usually computed by using any s -norm operator (Yager & Filev, 1994; Hellendoorn & Driankov, 1997).

Weights of rules, representing the probability of a fuzzy event in antecedent (w_i), as well as, the conditional probability of a fuzzy event at the consequence part ($w_{j/i}$), can be used to determine probabilistic characteristics of the conclusion. It is worth to note, that fuzzy

conclusions (27) and (28) represent some functions, $\varphi[L(X)]$, of linguistic values of the linguistic random variable X_{t_2} . The fuzzy expected value of the following prediction,

$$E\{(X_{t_2} \text{ is } \varphi(A_j)) / [X_{t_1} \text{ is } A']\} = \bar{A},$$

(29)

computed as the aggregated outputs of all active i -th file rules, can be determined by the following formula (Walaszek-Babiszewska, 2011)

$$\mu_{\bar{A}}(x_{t_2}) = \sum_i w_i \mu_{A_i'}(x_{t_2}) = \sum_i w_i \sum_j w_{j/i} \mu_{A'_{j/i}}(x_{t_2}).$$

(30)

where membership functions, $\mu_{A'_{j/i}}$, of the conclusions from elementary rules are given by (27) or (28), depending on the type of input data and the interpretation of a fuzzy model. Also, it is possible to determine probability of the fuzzy conclusion, taking into account a marginal probability distribution $P(X_{t_2})$ of the output linguistic random variable.

4. Creating fuzzy models of stochastic processes - Exemplary calculations

4.1 Fuzzy model of the stochastic time-discrete increments

First example show the fuzzy representation of the simplest form of the considered above stochastic processes, the one-dimensional time-discrete stochastic process of the increments, $\Delta X_t = X_t - X_{t-1}$. The increments, at given t , are normal distributed random variables, so it is useful to use the standard normal probability distribution function, over the domain of the process values, $\Delta X \in [-3, 3] = \chi \subset R$ (Table 1). Linguistic random variable, Y_t , with the

$x \in [a, b]$	$p(x)$	Fuzzy sets (events)					Probability of fuzzy events $P(Y)$		
		$\mu_{NH}(x)$	$\mu_{NL}(x)$	$\mu_Z(x)$	$\mu_{PL}(x)$	$\mu_{PH}(x)$			
[-3, -2.5)	0.00486	1	0				$P(NH)=$ 0.014		
[-2.5, -2)	0.01654	0.5	0.5						
[-2, -1.5)	0.044057	0	1				$P(NL)=$ 0.220		
[-1.5, -1)	0.091848		1	0					
[-1, -0.5)	0.149882		0.5	0.5					
[-0.5, 0)	0.191463		0	1				$P(Z)=$ 0.532	
[0, 0.5)	0.191463			1	0				
[0.5, 1)	0.149882			0.5	0.5				
[1, 1.5)	0.091848			0	1		$P(PL)=$ 0.220		
[1.5, 2)	0.044057				1	0			
[2, 2.5)	0.01654				0.5	0.5	$P(PH)=$ 0.014		
[2.5, 3]	0.00486				0	1			

Table 1. Probability function of random variable X_t , fuzzy sets representing linguistic values $L\{Y\}$ of the linguistic random variable Y_t and probability distribution $P(Y)$

name 'Increment at moment t ' has been assumed, with the set of its linguistic values: $L(Y) = \{\text{negative high NH, negative low NL, zero Z, positive low PL, positive high PH}\}$. The linguistic values are represented by respective fuzzy sets. The probability distribution of the linguistic random variable, $P(Y)$, calculated according to (13) - (15), has been presented in Table 1.

Also, the second linguistic random variable, Y_{t-1} , with the name 'Increment at moment $t-1$ ' has been determined with the same set of linguistic values $L(Y)$. Increments of the tested process are independent random variables, so conditional probabilities (probabilities of transitions) fulfill the relationship: $P(Y_t / Y_{t-1}) = P(Y_t)$.

The fuzzy knowledge base for the short memory stochastic process consists of the following, five file rules (25 elementary rules) with respective probabilities (according to Table 1.):

$$R^1: 0.014(\text{If } Y_{t-1} \text{ is NH}) \text{ Then } (Y_t \text{ is NH}) 0.014$$

$$\text{Also } (Y_t \text{ is NL}) 0.220$$

$$\text{Also } (Y_t \text{ is Z}) 0.532$$

$$\text{Also } (Y_t \text{ is PL}) 0.220$$

$$\text{Also } (Y_t \text{ is PH}) 0.014;$$

$$R^2: 0.220(\text{If } Y_{t-1} \text{ is NL}) \text{ Then } (Y_t \text{ is NH}) 0.014$$

$$\text{-----} \tag{31}$$

$$R^5: 0.014(\text{If } Y_{t-1} \text{ is PH}) \text{ Then } (Y_t \text{ is NH}) 0.014$$

$$\text{-----}$$

$$\text{Also } (Y_t \text{ is PH}) 0.014.$$

In the created rule base of the stochastic process, the same probability distributions for random variables, ΔX_t and ΔX_{t-1} , have been assumed. It is result of the simplification, under the assumption of a constant time interval $\Delta t = 1$.

4.2 Exemplary fuzzy models constructed based on realizations of stochastic processes

4.2.1 Fuzzy model constructed based on data of a floating particle

In the object literature the problem of the fulfilling the Wiener process assumptions by empirical data is often raised, e.g. the expected values of empirical increments are non-zero or increments do not fulfill the criterion of probabilistic independence. These facts have been also observed based on data representing increments of one coordinate, $\Delta x_t = x_t - x_{t-1}$, $\Delta x \in [-3.3, 3.3] = \chi \subset R$, describing the behavior of the particle floating in some liquid. It was assumed, that data stand for the realization of a certain stochastic process $Y(t)$. Also, the linguistic random variable Y_t has been determined, with the name 'Increment at moment t '.

The set of the linguistic values, $L(Y)=\{negative\ high\ NH, \ negative\ low\ NL, \ positive\ low\ PL, \ positive\ high\ PH\}$ has been assumed. In domain χ of the process values, the linguistic values are represented by respective fuzzy sets. Also, second linguistic random variable, Y_{t-1} , with the name ‘Increment at moment $t-1$ ’ has been determined with the same set of linguistic values $L(Y)$. For the tested process, the criterion of independent increments is not fulfilled, thus, the conditional probabilities (probabilities of transitions) $P(Y_t / Y_{t-1})$ should be found. The empirical joint probability of two linguistic random variables, $P(Y_t, Y_{t-1})$, has been calculated according to (16) - (19), based on the joint probability of numeric values of pairs, $p(\Delta x_{t-1}, \Delta x_t)$, as well as, the assumed fuzzy events, representing the linguistic values $L\{Y_t\} = \{NH, NL, PH, PL\}$ (Table 2). Marginal probability $P(Y_{t-1})$ is presented at the last row of the table. It is not a symmetrical distribution, the highest value of the probability, 0.39251, it is a probability that increments take the linguistic value ‘Positive Low’. Conditional probabilities $P(Y_t / Y_{t-1})$, calculated according to (20) - (22) and presented in Table 3, may be treated as the transitions probabilities from fuzzy states $\{NH, NL, PL, PH\}$ at moment $t-1$ to the particular fuzzy states at moment t .

$P(Y_t, Y_{t-1})$				
$L\{Y_t\}$	$L\{Y_{t-1}\}$			
	NH	NL	PL	PH
PH	0.0178	0.05355	0.10702	0.03572
PL	0.0060	0.06555	0.19024	0.09532
NL	0.0953	0.05955	0.02975	0.06555
NH	0.04765	0.03570	0.0655	0.0298
$P(Y_{t-1})$	0.16675	0.21435	0.39251	0.22639

Table 2. Joint probability distribution, $P(Y_t, Y_{t-1})$, of linguistic random variables representing empirical set of increments at moments t and $t-1$

$P(Y_t / Y_{t-1})$				
$L\{Y_t\}$	$L\{Y_{t-1}\}$			
	NH	NL	PL	PH
PH	0.10675	0.24983	0.27266	0.15778
PL	0.03600	0.30581	0.48466	0.42104
NL	0.57150	0.27781	0.07580	0.28955
NH	0.28575	0.16655	0.16688	0.13163
$\sum P(Y_t / Y_{t-1})$	1.00000	1.00000	1.00000	1.00000

Table 3. Conditional probability distributions $P(Y_t / Y_{t-1})$ of linguistic random variables

Also (Y_t is NL) 0.57150

Also (Y_t is PL) 0.036

Also (Y_t is PH) 0.10675;

(32)

Also (Y_t is PH) 0.15778.

4.2.2 Fuzzy model of the stochastic increments observed in some technological situation

In a certain technological situation some parameter of a non-homogeneous grain material was measured at discrete moments (Figure 1). It is assumed that observed values $X(n)$, $n=1, \dots, 400$ represent realization of a certain stochastic process whose variance is high and changes are very quick. For human experts, engineers of the technological process, it is very

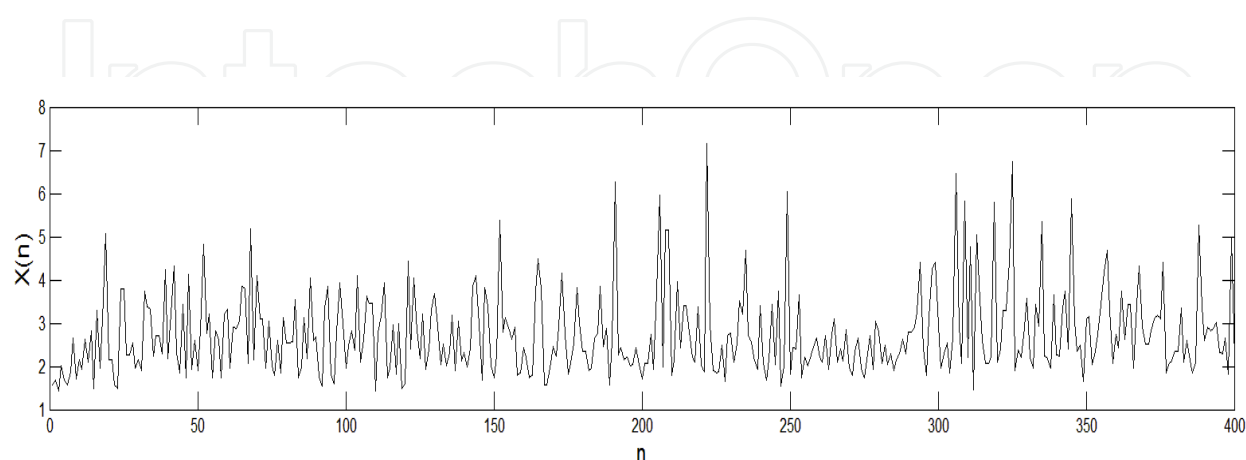


Fig. 1. Realization of the stochastic process $X(n)$

important to recognize a probabilistic character of the changes, especially the changes determined in linguistic categories, like: *Positive Big*, *Positive Small*, *Zero*, *Negative Small*, *Negative Big*. To determine characteristic of the process with fuzzy states, first, we have calculated the increments

$$DX(n)=X(n)-X(n-1) \quad (33)$$

and a joint probability distribution $p(DX(n),DX(n-1))$ of non-fuzzy values of the process. The range of the increments values, a real number interval $[-4.8, 4.8]$, has been divided into 14 disjoint intervals and the frequency of the occurrence of measurements in particular intervals has been determined. The disjoint intervals have been used for the description of membership functions of particular linguistic values of the set $L\{DX(n)\}=\{NB,NS,Z,PS,PB\}$.

The empirical joint probability distribution of the linguistic random variables, $P(L\{DX(n-1)\}, L\{DX(n)\})$, has been calculated and presented in Table 4. In the last row, the marginal probability values of one linguistic random variable, are presented. It is almost symmetrical distribution, with the highest value of the probability, 0.6114, for the linguistic value of increments equal to 'Zero'. Conditional probability distributions for particular linguistic values of the variable $DX(n)$ have been also calculated and they represent weights of particular consequent parts of the rule-base fuzzy model (34). The model of the knowledge base consists of the following five file rules with weights:

R1: 0.6114 IF ($DX(n-1)$ IS Z) THEN ($DX(n)$ IS Z) 0.6450

ALSO ($DX(n)$ IS NS) 0.1945

ALSO ($DX(n)$ IS PS) 0.1462

ALSO ($DX(n)$ IS PB) 0.0113

ALSO ($DX(n)$ IS NB) 0.0030

R2: 0.2123 IF ($DX(n-1)$ IS NS) THEN ($DX(n)$ IS Z) 0.6739

ALSO ($DX(n)$ IS PS) 0.2032

ALSO ($DX(n)$ IS NS) 0.1114

ALSO ($DX(n)$ IS PB) 0.0111

ALSO ($DX(n)$ IS NB) 0.0004

R3: 0.1474 IF ($DX(n-1)$ IS PS) THEN ($DX(n)$ IS NS) 0.4258 (34)

ALSO ($DX(n)$ IS Z) 0.4181

ALSO ($DX(n)$ IS NB) 0.0791

ALSO ($DX(n)$ IS PS) 0.0714

ALSO ($DX(n)$ IS PB) 0.0056

R4: 0.0184 IF ($DX(n-1)$ IS NB) THEN ($DX(n)$ IS Z) 0.6044

ALSO ($DX(n)$ IS PS) 0.2363

ALSO ($DX(n)$ IS NS) 0.1263

ALSO ($DX(n)$ IS PB) 0.0330

R5: 0.0105 IF ($DX(n-1)$ IS PB) THEN ($DX(n)$ IS NB) 0.4762

ALSO ($DX(n)$ IS NS) 0.4286

ALSO ($DX(n)$ IS Z) 0.0952.

	$P(L\{DX(n-1)\}, L\{DX(n)\})$				
	$L\{DX(n-1)\}$				
$L\{DX(n)\}$	NB	NS	Z	PS	PB
PB	0.0051	0.0046	0.0010	0.0	0.0
PS	0.0115	0.0620	0.0609	0.0104	0.0008
Z	0.0017	0.1192	0.3953	0.0895	0.0068
NS	0.0001	0.0235	0.1430	0.0431	0.0023
NB	0	0.0030	0.0112	0.0044	0.0006
$P(L\{DX(n-1)\})$	0.0184	0.2123	0.6114	0.1474	0.0105

Table 4. Joint empirical probability distribution of two linguistic random variables representing increments

To determine the predicted value $DX(n)=b^*$, for given value (crisp or fuzzy) $DX(n-1)=a^*$, the reasoning procedure, described in 3.3 is used, e.g. for $DX(n-1)=1.55$, predicted value is approximated as equal to $DX(n)=0.30538$. This value depends on many parameters of the fuzzy model and the reasoning procedure. It is very useful to create the computing system with many options of changing the reasoning parameters. In Fig. 2 the predicted, mean values of the increments has been underlined by thick line.

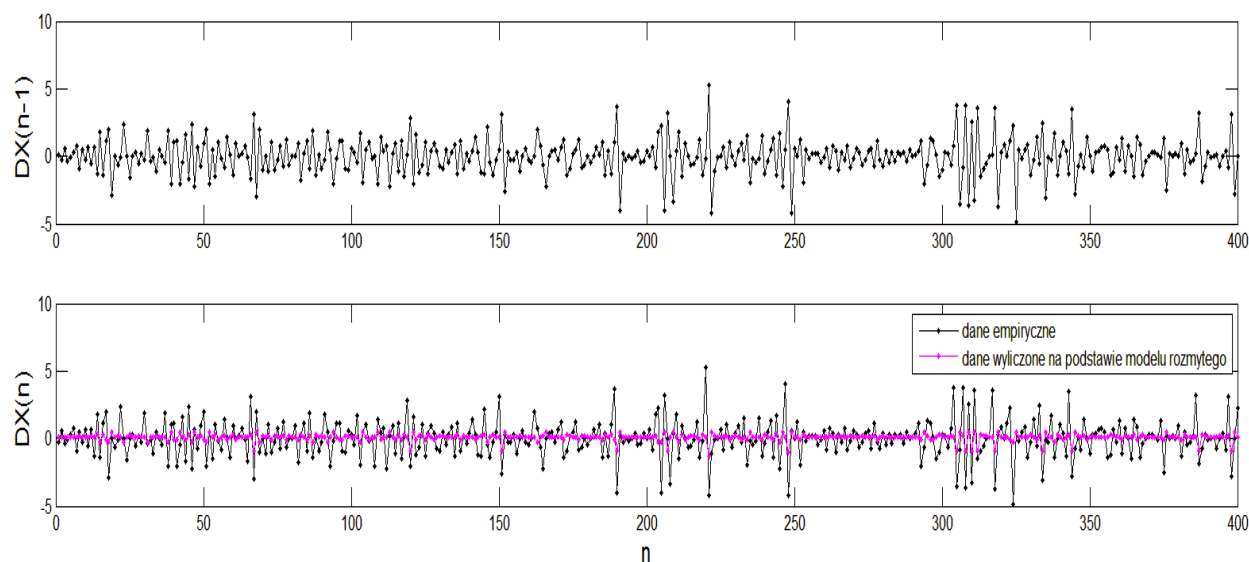


Fig. 2. Realization of the stochastic processes of increments $DX(n)$, $DX(n-1)$ and the predicted mean value

5. Conclusion and future works

In this chapter the new approach to fuzzy modelling has been presented. Knowledge base in the form of weighted fuzzy rules represents in the same time the probability distribution of the fuzzy events occurring in the statements. Considered examples show the creating a few simple models of stochastic increments processes. In the future, in modelling the Wiener process, the time dependent probability of the increments should be taken into account.

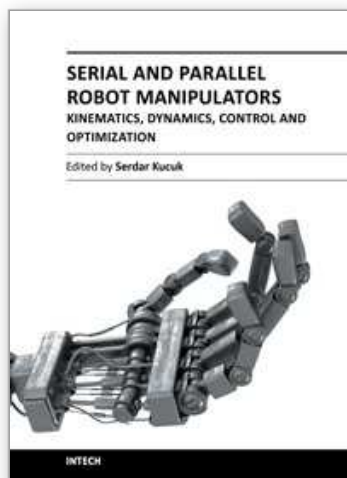
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