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Inverse Dynamics of RRR Fully Planar Parallel Manipulator Using DH Method

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1. Introduction

Parallel manipulators are mechanisms where all the links are connected to the ground and the moving platform at the same time. They possess high rigidity, load capacity, precision, structural stiffness, velocity and acceleration since the end-effector is linked to the movable plate in several points (Kang et al., 2001; Kang & Mills, 2001; Merlet, J. P. 2000; Tsai, L., 1999; Uchiyama, M., 1994). Parallel manipulators can be classified into two fundamental categories, namely spatial and planar manipulators. The first category composes of the spatial parallel manipulators that can translate and rotate in the three dimensional space. Gough-Stewart platform, one of the most popular spatial manipulator, is extensively preferred in flight simulators. The planar parallel manipulators which composes of second category, translate along the x and y-axes, and rotate around the z-axis, only. Although planar parallel manipulators are increasingly being used in industry for micro-or nano-positioning applications, (Hubbard et al., 2001), the kinematics, especially dynamics analysis of planar parallel manipulators is more difficult than their serial counterparts. Therefore selection of an efficient kinematic modeling convention is very important for simplifying the complexity of the dynamics problems in planar parallel manipulators. In this chapter, the inverse dynamics problem of three-**D**egrees **O**f **F**reedom (**DOF**) RRR **F**ully **P**lanar **P**arallel **M**anipulator (**FPPM**) is derived using DH method (Denavit & Hartenberg, 1955) which is based on 4x4 homogenous transformation matrices. The easy physical interpretation of the rigid body structures of the robotic manipulators is the main benefit of DH method. The inverse dynamics of 3-DOF RRR FPPM is derived using the virtual work principle (Zhang, & Song, 1993; Wu et al., 2010; Wu et al., 2011). In the first step, the inverse kinematics model and Jacobian matrix of 3-DOF RRR FPPM are derived by using DH method. To obtain the inverse dynamics, the partial linear velocity and partial angular velocity matrices are computed in the second step. A pivotal point is selected in order to determine the partial linear velocity matrices. The inertial force and moment of each moving part are obtained in the next step. As a last step, the inverse dynamic equation of 3-DOF RRR FPPM in explicit form is derived. To demonstrate the active joints torques, a butterfly shape Cartesian trajectory is used as a desired end-effector's trajectory.

2. Inverse kinematics and dynamics model of the 3-DOF RRR FPPM

In this section, geometric description, inverse kinematics, Jacobian matrix & Jacobian inversion and inverse dynamics model of the 3-DOF RRR FPPM in explicit form are obtained by applying DH method.

2.1 Geometric descriptions of 3-DOF RRR FPPM

The 3-DOF RRR FPPM shown in Figure 1 has a moving platform linked to the ground by three independent kinematics chains including one active joint each. The symbols θ_i and α_i illustrate the active and passive revolute joints, respectively where i=1, 2 and 3. The link lengths and the orientation of the moving platform are denoted by l_j and ϕ, respectively, j=1, 2, ··· ,6. The points B_1, B_2, B_3 and M_1, M_2, M_3 define the geometry of the base and the moving (Figure 2) platform, respectively. The {XYZ} and {xyz} coordinate systems are attached to the base and the moving platform of the manipulator, respectively. O and M_1 are the origins of the base and moving platforms, respectively. $P(X_B, Y_B)$ and ϕ illustrate the position of the end-effector in terms of the base coordinate system {XYZ} and orientation of the moving platform, respectively.

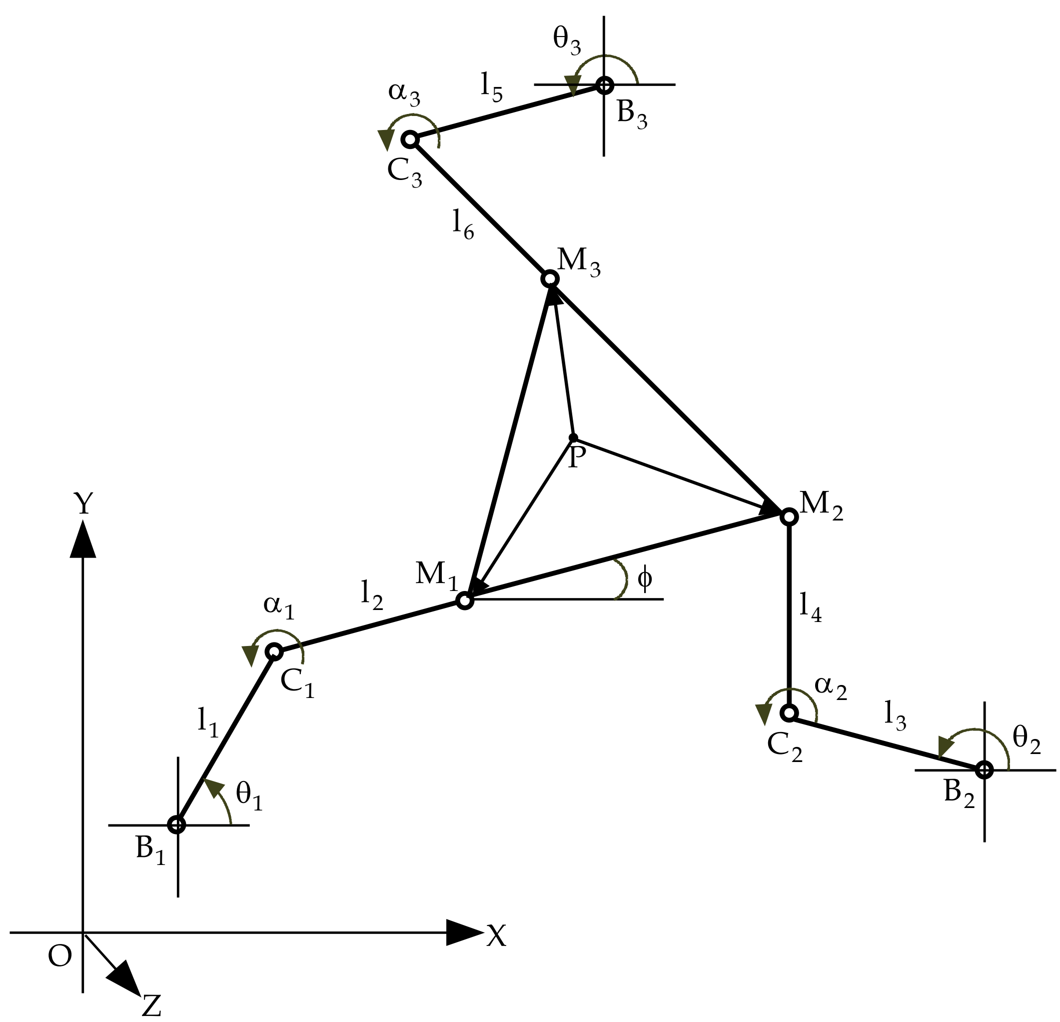


Fig. 1. The 3-DOF RRR FPPM

The lines M_1P, M_2P and M_3P are regarded as n_1, n_2 and n_3, respectively. The γ_1, γ_2 and γ_3 illustrate the angles $B\widehat{P}M_1$, $M_2\widehat{P}B$, and $B\widehat{P}M_3$, respectively. Since two lines AB and M_1M_2 are parallel, the angles $P\widehat{M}_1M_2$ and $P\widehat{M}_2M_1$ are equal to the angles $A\widehat{P}M_1$ and $M_2\widehat{P}B$, respectively. $P(x_m, y_m)$ denotes the position of end-effector in terms of {xyz} coordinate systems.

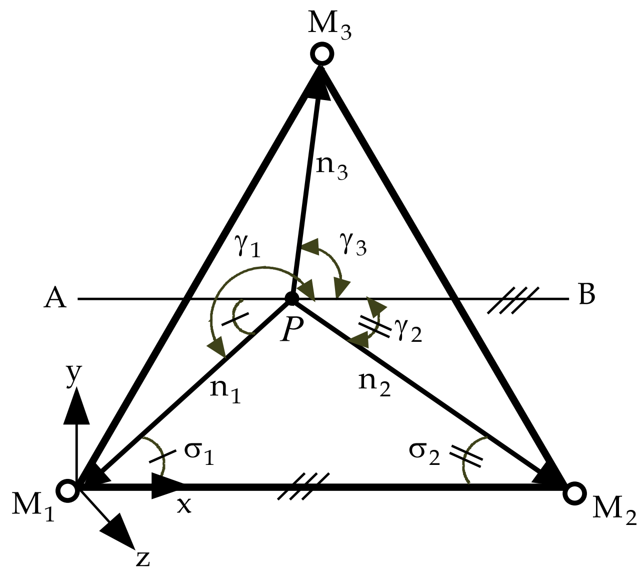


Fig. 2. The moving platform

2.2 Inverse kinematics

The inverse kinematic equations of 3-DOF RRR FPPM are derived using the DH (Denavit & Hartenberg, 1955) method which is based on 4x4 homogenous transformation matrices. The easy physical interpretation of the rigid body structures of the robotic manipulators is the main benefit of DH method which uses a set of parameters ($\alpha_{i-1}, a_{i-1}, \theta_i$ and d_i) to describe the spatial transformation between two consecutive links. To find the inverse kinematics problem, the following equation can be written using the geometric identities on Figure 1.

$$OB_i + B_iM_i = OP + PM_i \tag{1}$$

where i=1, 2 and 3. If the equation 1 is adapted to the manipulator in Figure 1, the ${}^{O_i}_{M_i}T^1$ and ${}^{O_i}_{M_i}T^2$ transformation matrices can be determined as

$$
\begin{aligned}
{}^{O_i}_{M_i}T^1 &= \begin{bmatrix} 1 & 0 & 0 & o_{x_i} \\ 0 & 1 & 0 & o_{y_i} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha_i & -\sin\alpha_i & 0 & l_{2i-1} \\ \sin\alpha_i & \cos\alpha_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_{2i} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos(\theta_i+\alpha_i) & -\sin(\theta_i+\alpha_i) & 0 & o_{x_i} + l_{2i}\cos(\theta_i+\alpha_i) + l_{2i-1}\cos\theta_i \\ \sin(\theta_i+\alpha_i) & \cos(\theta_i+\alpha_i) & 0 & o_{y_i} + l_{2i}\sin(\theta_i+\alpha_i) + l_{2i-1}\sin\theta_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{2}
$$

$$
{}^{O_i}_{M_i}T^2 = \begin{bmatrix} 1 & 0 & 0 & P_{X_B} \\ 0 & 1 & 0 & P_{Y_B} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\gamma_i+\phi) & -\sin(\gamma_i+\phi) & 0 & 0 \\ \sin(\gamma_i+\phi) & \cos(\gamma_i+\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & n_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$= \begin{bmatrix} \cos(\gamma_i + \phi) & -\sin(\gamma_i + \phi) & 0 & P_{X_B} + n_i\cos\gamma_i\cos\phi - n_i\sin\gamma_i\sin\phi \\ \sin(\gamma_i + \phi) & \cos(\gamma_i + \phi) & 0 & P_{Y_B} + n_i\cos\gamma_i\sin\phi + n_i\sin\gamma_i\cos\phi \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

where (P_{X_B}, P_{Y_B}) corresponds the position of the end-effector in terms of the base {XYZ} coordinate systems, $\gamma_1 = \pi + \sigma_1$ and $\gamma_2 = -\sigma_2$. Since the position vectors of ${}^{O_i}_{M_i}T^1$ and ${}^{O_i}_{M_i}T^2$ matrices are equal, the following equation can be obtained easily.

$$\begin{bmatrix} l_{2i}\cos(\theta_i + \alpha_i) \\ l_{2i}\sin(\theta_i + \alpha_i) \end{bmatrix} = \begin{bmatrix} P_{X_B} + b_{x_i}\cos\phi - b_{y_i}\sin\phi - o_{x_i} - l_{2i-1}\cos\theta_i \\ P_{Y_B} + b_{x_i}\sin\phi + b_{y_i}\cos\phi - o_{y_i} - l_{2i-1}\sin\theta_i \end{bmatrix} \tag{4}$$

where $b_{x_i} = n_i\cos\gamma_i$ and $b_{y_i} = n_i\sin\gamma_i$. Summing the squares of the both sides in equation 4, we obtain, after simplification,

$$\begin{aligned} & l_{2i-1}^2 - 2P_{Y_B}o_{y_i} - 2P_{X_B}o_{x_i} + b_{x_i}^2 + b_{y_i}^2 + o_{x_i}^2 + o_{y_i}^2 + P_{X_B}^2 + P_{Y_B}^2 \\ & +2l_{2i-1}b_{y_i}[\sin(\phi - \theta_i) - \cos(\phi - \theta_i)] + 2\cos\phi\left(P_{X_B}b_{x_i} + P_{Y_B}b_{y_i} - b_{x_i}o_{x_i} - b_{y_i}o_{y_i}\right) \\ & +2\sin\phi\left(P_{Y_B}b_{x_i} - P_{X_B}b_{y_i} - b_{x_i}o_{y_i} + b_{y_i}o_{x_i}\right) + 2l_{2i-1}\cos\theta_i(o_{x_i} - P_{X_B}) \\ & +2l_{2i-1}\sin\theta_i\left(o_{y_i} - P_{Y_B}\right) - l_{2i}^2 = 0 \end{aligned} \tag{5}$$

To compute the inverse kinematics, the equation 5 can be rewritten as follows

$$A_i\sin\theta_i + B_i\cos\theta_i = C_i \tag{6}$$

where

$$A_i = 2l_{2i-1}\left(o_{y_i} - b_{x_i}\sin\phi - b_{y_i}\cos\phi - P_{Y_B}\right)$$

$$B_i = 2l_{2i-1}\left(o_{x_i} + b_{y_i}\sin\phi - b_{x_i}\cos\phi - P_{X_B}\right)$$

$$\begin{aligned} C_i = -\big[& l_{2i-1}^2 - 2P_{Y_B}o_{y_i} - 2P_{X_B}o_{x_i} + b_{x_i}^2 + b_{y_i}^2 + o_{x_i}^2 + o_{y_i}^2 + P_{X_B}^2 + P_{Y_B}^2 - l_{2i}^2 \\ & +2\cos\phi\left(P_{X_B}b_{x_i} + P_{Y_B}b_{y_i} - b_{x_i}o_{x_i} - b_{y_i}o_{y_i}\right) + 2\sin\phi\left(P_{Y_B}b_{x_i} - P_{X_B}b_{y_i} - b_{x_i}o_{y_i} + b_{y_i}o_{x_i}\right)\big] \end{aligned}$$

The inverse kinematics solution for equation 6 is

$$\theta_i = \mathrm{Atan2}(A_i, B_i) \mp \mathrm{Atan2}\left(\sqrt{A_i^2 + B_i^2 - C_i^2}, C_i\right) \tag{7a}$$

Once the active joint variables are determined, the passive joint variables can be computed by using equation 4 as follows.

$$\alpha_i = \mathrm{Atan2}(D_i, E_i) \mp \mathrm{Atan2}\left(\sqrt{D_i^2 + E_i^2 - G_i^2}, G_i\right) \tag{7b}$$

where

$$D_i = -\sin\theta_i,\ \ E_i = \cos\theta_i$$

and

$$G_i = \left(P_{X_B} + b_{x_i}\cos\phi - b_{y_i}\sin\phi - o_{x_i} - l_{2i-1}\cos\theta_i\right)/l_{2i}$$

Since the equation 7 produce two possible solutions for each kinematic chain according to the selection of plus '+' or mines '–' signs, there are eight possible inverse kinematics solutions for 3-DOF RRR FPPM.

2.3 Jacobian matrix and Jacobian inversion

Differentiating the equation 5 with respect to the time, one can obtain the Jacobian matrices.

$$B\dot{\mathbf{q}} = A\dot{\boldsymbol{\chi}}$$

$$\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix} \tag{8}$$

where

$$a_i = -2\left(P_{X_B} - o_{x_i} + b_{x_i}\cos\phi - l_{2i-1}\cos\theta_i - b_{y_i}\sin\phi\right)$$

$$b_i = -2\left(P_{Y_B} - o_{y_i} + b_{y_i}\cos\phi - l_{2i-1}\sin\theta_i + b_{x_i}\sin\phi\right)$$

$$c_i = -2\left[l_{2i-1}b_{y_i}\cos(\phi - \theta_i) + l_{2i-1}b_{x_i}\sin(\phi - \theta_i) + \cos\phi\left(P_{Y_B}b_{x_i} - P_{X_B}b_{y_i} - b_{x_i}o_{y_i} + b_{y_i}o_{x_i}\right) + \sin\phi\left(b_{x_i}o_{x_i} + b_{y_i}o_{y_i} - P_{X_B}b_{x_i} - P_{Y_B}b_{y_i}\right)\right]$$

$$d_i = 2\left[l_{2i-1}\cos\theta_i\left(o_{y_i} - P_{Y_B}\right) + l_{2i-1}\sin\theta_i\left(P_{X_B} - o_{x_i}\right) - l_{2i-1}b_{y_i}\cos(\phi - \theta_i) - l_{2i-1}b_{x_i}\sin(\phi - \theta_i)\right]$$

The A and B terms in equation 8 denote two separate Jacobian matrices. Thus the overall Jacobian matrix can be obtained as

$$J = B^{-1}A = \begin{bmatrix} \frac{a_1}{d_1} & \frac{b_1}{d_1} & \frac{c_1}{d_1} \\ \frac{a_2}{d_2} & \frac{b_2}{d_2} & \frac{c_2}{d_2} \\ \frac{a_3}{d_3} & \frac{b_3}{d_3} & \frac{c_3}{d_3} \end{bmatrix} \tag{9}$$

The manipulator Jacobian is used for mapping the velocities from the joint space to the Cartesian space

$$\dot{\theta} = J\dot{\chi} \tag{10}$$

where $\dot{\chi} = [\dot{P}_{X_B} \quad \dot{P}_{Y_B} \quad \dot{\phi}]^T$ and $\dot{\theta} = [\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3]^T$ are the vectors of velocity in the Cartesian and joint spaces, respectively.

To compute the inverse dynamics of the manipulator, the acceleration of the end-effector is used as the input signal. Therefore, the relationship between the joint and Cartesian accelerations can be extracted by differentiation of equation 10 with respect to the time.

$$\ddot{\theta} = J\ddot{\chi} + \dot{J}\dot{\chi} \tag{11}$$

where $\ddot{\chi} = [\ddot{P}_{X_B} \quad \ddot{P}_{Y_B} \quad \ddot{\phi}]^T$ and $\ddot{\theta} = [\ddot{\theta}_1 \quad \ddot{\theta}_2 \quad \ddot{\theta}_3]^T$ are the vectors of acceleration in the Cartesian and joint spaces, respectively. In equation 11, the other quantities are assumed to be known from the velocity inversion and the only matrix that has not been defined yet is the time derivative of the Jacobian matrix. Differentiation of equation 9 yields to

$$\dot{J} = \begin{bmatrix} K_1 & L_1 & R_1 \\ K_2 & L_2 & R_2 \\ K_3 & L_3 & R_3 \end{bmatrix} \tag{12}$$

K_i, L_i and R_i in equation 12 can be written as follows.

$$K_i = \frac{\dot{a}_i d_i - a_i \dot{d}_i}{d_i^2} \tag{13}$$

$$L_i = \frac{\dot{b}_i d_i - b_i \dot{d}_i}{d_i^2} \tag{14}$$

$$R_i = \frac{\dot{c}_i d_i - c_i \dot{d}_i}{d_i^2} \tag{15}$$

where

$$\dot{a}_i = -2\left(\dot{P}_{X_B} - \dot{\phi} b_{x_i} \sin\phi + \dot{\theta}_i l_{2i-1} \sin\theta_i - \dot{\phi} b_{y_i} \cos\phi\right)$$

$$\dot{b}_i = -2\left(\dot{P}_{Y_B} - \dot{\phi} b_{y_i} \sin\phi - \dot{\theta}_i l_{2i-1} \cos\theta_i + \dot{\phi} b_{x_i} \cos\phi\right)$$

$$\begin{aligned} c_i = -2\big[&-l_{2i-1} b_{y_i}\left(\dot{\phi} - \dot{\theta}_i\right)\sin(\phi - \theta_i) + \left(\dot{\phi} - \dot{\theta}_i\right) l_{2i-1} b_{x_i} \cos(\phi - \theta_i) \\ &-\dot{\phi} \sin\phi\left(P_{Y_B} b_{x_i} - P_{X_B} b_{y_i} - b_{x_i} o_{y_i} + b_{y_i} o_{x_i}\right) + \cos\phi\left(\dot{P}_{Y_B} b_{x_i} - \dot{P}_{X_B} b_{y_i}\right) \\ &+\dot{\phi} \cos\phi\left(b_{x_i} o_{x_i} + b_{y_i} o_{y_i} - P_{X_B} b_{x_i} - P_{Y_B} b_{y_i}\right) - \sin\phi\left(\dot{P}_{X_B} b_{x_i} + \dot{P}_{Y_B} b_{y_i}\right)\big] \end{aligned}$$

$$\begin{aligned} \dot{d}_i = 2\big[&-l_{2i-1} \dot{\theta}_i \sin\theta_i\left(o_{y_i} - P_{Y_B}\right) - l_{2i-1} \cos\theta_i \dot{P}_{Y_B} + l_{2i-1} \dot{\theta}_i \cos\theta_i\left(P_{X_B} - o_{x_i}\right) + l_{2i-1} \sin\theta_i \dot{P}_{X_B} \\ &+ l_{2i-1} b_{y_i}\left(\dot{\phi} - \dot{\theta}_i\right)\sin(\phi - \theta_i) - l_{2i-1} b_{x_i}\left(\dot{\phi} - \dot{\theta}_i\right)\cos(\phi - \theta_i)\big] \end{aligned}$$

2.4 Inverse dynamics model

The virtual work principle is used to obtain the inverse dynamics model of 3-DOF RRR FPPM. Firstly, the partial linear velocity and partial angular velocity matrices are computed by using homogenous transformation matrices derived in Section 2.2. To find the partial linear velocity matrix, B_{2i-1}, C_{2i-1} and M_3 points are selected as pivotal points of links l_{2i-1}, l_{2i} and moving platform, respectively in the second step. The inertial force and moment of each moving part are determined in the next step. As a last step, the inverse dynamic equations of 3-DOF RRR FPPM in explicit form are derived.

2.4.1 The partial linear velocity and partial angular velocity matrices

Considering the manipulator Jacobian matrix in equation 10, the joint velocities for the link l_{2i-1} can be expressed in terms of Cartesian velocities as follows.

$$\dot{\theta}_i = \begin{bmatrix} \frac{a_i}{d_i} & \frac{b_i}{d_i} & \frac{c_i}{d_i} \end{bmatrix} \begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix}, \quad i = 1, 2 \text{ and } 3. \tag{16}$$

The partial angular velocity matrix of the link l_{2i-1} can be derived from the equation 16 as

$$\boldsymbol{\omega_{2i-1}} = \begin{bmatrix} \frac{a_i}{d_i} & \frac{b_i}{d_i} & \frac{c_i}{d_i} \end{bmatrix}, \quad i = 1,2 \text{ and } 3. \tag{17}$$

Since the linear velocity on point B_i is zero, the partial linear velocity matrix of the point B_i is given by

$$\boldsymbol{\upsilon_{2i-1}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad i = 1,2 \text{ and } 3. \tag{18}$$

To find the partial angular velocity matrix of the link l_{2i}, the equation 19 can be written easily using the equality of the position vectors of ${}^{O_i}_{M_i}T^1$ and ${}^{O_i}_{M_i}T^2$ matrices.

$$\begin{bmatrix} o_{x_i} + l_{2i}\cos(\theta_i + \alpha_i) + l_{2i-1}\cos\theta_i \\ o_{y_i} + l_{2i}\sin(\theta_i + \alpha_i) + l_{2i-1}\sin\theta_i \end{bmatrix} = \begin{bmatrix} P_{X_B} + b_{x_i}\cos\phi - b_{y_i}\sin\phi \\ P_{Y_B} + b_{x_i}\sin\phi + b_{y_i}\cos\phi \end{bmatrix} \tag{19}$$

The equation 19 can be rearranged as in equation 20 since the link l_{2i} moves with $\delta_i = \theta_i + \alpha_i$ angular velocity.

$$\begin{bmatrix} o_{x_i} + l_{2i}\cos\delta_i + l_{2i-1}\cos\theta_i \\ o_{y_i} + l_{2i}\sin\delta_i + l_{2i-1}\sin\theta_i \end{bmatrix} = \begin{bmatrix} P_{X_B} + b_{x_i}\cos\phi - b_{y_i}\sin\phi \\ P_{Y_B} + b_{x_i}\sin\phi + b_{y_i}\cos\phi \end{bmatrix} \tag{20}$$

Taking the time derivative of equation 20 yields the following equation.

$$\begin{bmatrix} -l_{2i}\dot{\delta}_i\sin\delta_i - l_{2i-1}\dot{\theta}_i\sin\theta_i \\ l_{2i}\dot{\delta}_i\cos\delta_i + l_{2i-1}\dot{\theta}_i\cos\theta_i \end{bmatrix} = \begin{bmatrix} \dot{P}_{X_B} - \dot{\phi}b_{x_i}\sin\phi - \dot{\phi}b_{y_i}\cos\phi \\ \dot{P}_{Y_B} + \dot{\phi}b_{x_i}\cos\phi - \dot{\phi}b_{y_i}\sin\phi \end{bmatrix} \tag{21}$$

Equation 21 can also be stated as follows.

$$\begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \end{bmatrix} l_{2i}\dot{\delta}_i + \begin{bmatrix} -l_{2i-1}\sin\theta_i \\ l_{2i-1}\cos\theta_i \end{bmatrix} \dot{\theta}_i = \begin{bmatrix} 1 & 0 & -b_{x_i}\sin\phi - b_{y_i}\cos\phi \\ 0 & 1 & b_{x_i}\cos\phi - b_{y_i}\sin\phi \end{bmatrix} \begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix} \tag{22}$$

If $\dot{\theta}$ in equation 16 is substituted in equation 22, the following equation will be obtained.

$$\begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \end{bmatrix} l_{2i}\dot{\delta}_i = \left(\begin{bmatrix} 1 & 0 & -b_{x_i}\sin\phi - b_{y_i}\cos\phi \\ 0 & 1 & b_{x_i}\cos\phi - b_{y_i}\sin\phi \end{bmatrix} - \begin{bmatrix} -l_{2i-1}\sin\theta_i \\ l_{2i-1}\cos\theta_i \end{bmatrix} \begin{bmatrix} \frac{a_i}{d_i} & \frac{b_i}{d_i} & \frac{c_i}{d_i} \end{bmatrix} \right) \begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix} \tag{23}$$

If the both sides of equation 23 premultiplied by $[-\sin\delta_i \quad \cos\delta_i]$, angular velocity of link l_{2i} is obtained as.

$$\dot{\delta}_i = \begin{bmatrix} -\frac{\sin\delta_i}{l_{2i}} & \frac{\cos\delta_i}{l_{2i}} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & -b_{x_i}\sin\phi - b_{y_i}\cos\phi \\ 0 & 1 & b_{x_i}\cos\phi - b_{y_i}\sin\phi \end{bmatrix} - \begin{bmatrix} -l_{2i-1}\sin\theta_i \\ l_{2i-1}\cos\theta_i \end{bmatrix} \begin{bmatrix} \frac{a_i}{d_i} & \frac{b_i}{d_i} & \frac{c_i}{d_i} \end{bmatrix} \right) \begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix} \tag{24}$$

Finally the angular velocity matrix of l_{2i} is derived from the equation 24 as follows.

$$\omega_{2i} = \begin{bmatrix} -\frac{\sin\delta_i}{l_{2i}} & \frac{\cos\delta_i}{l_{2i}} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & -b_{x_i}\sin\phi - b_{y_i}\cos\phi \\ 0 & 1 & b_{x_i}\cos\phi - b_{y_i}\sin\phi \end{bmatrix} - \begin{bmatrix} -l_{2i-1}\sin\theta_i \\ l_{2i-1}\cos\theta_i \end{bmatrix} \begin{bmatrix} \frac{a_i}{d_i} & \frac{b_i}{d_i} & \frac{c_i}{d_i} \end{bmatrix} \right) \tag{25}$$

The angular acceleration of the link l_{2i} is found by taking the time derivative of equation 21.

$$\begin{aligned} &\begin{bmatrix} -l_{2i}(\ddot{\delta}_i\sin\delta_i + \dot{\delta}_i^2\cos\delta_i) - l_{2i-1}(\ddot{\theta}_i\sin\theta_i + \dot{\theta}_i^2\cos\theta_i) \\ l_{2i}(\ddot{\delta}_i\cos\delta_i - \dot{\delta}_i^2\sin\delta_i) + l_{2i-1}(\ddot{\theta}_i\cos\theta_i - \dot{\theta}_i^2\sin\theta_i) \end{bmatrix} \\ &= \begin{bmatrix} \ddot{P}_{X_B} - (\ddot{\phi}b_{x_i}\sin\phi + \dot{\phi}^2 b_{x_i}\cos\phi) - (\ddot{\phi}b_{y_i}\cos\phi - \dot{\phi}^2 b_{y_i}\sin\phi) \\ \ddot{P}_{Y_B} + (\ddot{\phi}b_{x_i}\cos\phi - \dot{\phi}^2 b_{x_i}\sin\phi) - (\ddot{\phi}b_{y_i}\sin\phi + \dot{\phi}^2 b_{y_i}\cos\phi) \end{bmatrix} \end{aligned} \tag{26}$$

Equation 26 can be expressed as

$$\begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \end{bmatrix} l_{2i}\ddot{\delta}_i = \begin{bmatrix} s_{i1} \\ s_{i2} \end{bmatrix} \tag{27}$$

where

$$s_{i1} = \ddot{P}_{X_B} - (\ddot{\phi}b_{x_i}\sin\phi + \dot{\phi}^2 b_{x_i}\cos\phi) - (\ddot{\phi}b_{y_i}\cos\phi - \dot{\phi}^2 b_{y_i}\sin\phi) + l_{2i}\dot{\delta}_i^2\cos\delta_i$$

$$+l_{2i-1}(\ddot{\theta}_i\sin\theta_i + \dot{\theta}_i^2\cos\theta_i)$$

$$s_{i2} = \ddot{P}_{Y_B} + (\ddot{\phi}b_{x_i}\cos\phi - \dot{\phi}^2 b_{x_i}\sin\phi) - (\ddot{\phi}b_{y_i}\sin\phi + \dot{\phi}^2 b_{y_i}\cos\phi) + l_{2i}\dot{\delta}_i^2\sin\delta_i$$

$$-l_{2i-1}(\ddot{\theta}_i\cos\theta_i - \dot{\theta}_i^2\sin\theta_i)$$

If the both sides of equation 27 premultiplied by $[-\sin\delta_i \quad \cos\delta_i]$, angular acceleration of link l_{2i} is obtained as.

$$\ddot{\delta}_i = \begin{bmatrix} -\frac{\sin\delta_i}{l_{2i}} & \frac{\cos\delta_i}{l_{2i}} \end{bmatrix} \begin{bmatrix} s_{i1} \\ s_{i2} \end{bmatrix} \tag{28}$$

where i=1,2 and 3. To find the partial linear velocity matrix of the point C_i, the position vector of ${}^{O_i}_{C_i}T^1$ is obtained in the first step.

$$\begin{aligned} {}^{O_i}_{C_i}T^1 &= \begin{bmatrix} 1 & 0 & 0 & o_{x_i} \\ 0 & 1 & 0 & o_{y_i} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_{2i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & o_{x_i} + l_{2i-1}\cos\theta_i \\ \sin\theta_i & \cos\theta_i & 0 & o_{y_i} + l_{2i-1}\sin\theta_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{29}$$

The position vector of ${}^{O_i}_{C_i}T^1$ is obtained from the fourth column of the equation 29 as

$${}^{O_i}_{C_i}T^1_{P(x,y)} = \begin{bmatrix} o_{x_i} + l_{2i-1}\cos\theta_i \\ o_{y_i} + l_{2i-1}\sin\theta_i \end{bmatrix} \tag{30}$$

Taking the time derivative of equation 30 produces the linear velocity of the point C_i.

$$\upsilon_{C_i} = \frac{d}{dt}\left({}^{O_i}_{C_i}T^1_{P(x,y)}\right) = \begin{bmatrix} -l_{2i-1}\sin\theta_i \\ l_{2i-1}\cos\theta_i \end{bmatrix} \dot{\theta}_i \tag{31}$$

If $\dot{\theta}$ in equation 16 is substituted in equation 31, the linear velocity of the point C_i will be expressed in terms of Cartesian velocities.

$$\upsilon_{C_i} = \begin{bmatrix} -l_{2i-1}\sin\theta_i \\ l_{2i-1}\cos\theta_i \end{bmatrix} \begin{bmatrix} \frac{a_i}{d_i} & \frac{b_i}{d_i} & \frac{c_i}{d_i} \end{bmatrix} \begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix}$$

$$= \frac{l_{2i-1}}{d_i} \begin{bmatrix} -a_i\sin\theta_i & -b_i\sin\theta_i & -c_i\sin\theta_i \\ a_i\cos\theta_i & b_i\cos\theta_i & c_i\cos\theta_i \end{bmatrix} \begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix} \tag{32}$$

Finally the partial linear velocity matrix of l_{2i} is derived from the equation 32 as

$$\boldsymbol{\upsilon}_{2i} = \frac{l_{2i-1}}{d_i} \begin{bmatrix} -a_i\sin\theta_i & -b_i\sin\theta_i & -c_i\sin\theta_i \\ a_i\cos\theta_i & b_i\cos\theta_i & c_i\cos\theta_i \end{bmatrix} \tag{33}$$

The angular velocity of the moving platform is given by

$$a_{mp} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix} \tag{34}$$

The partial angular velocity matrix of the moving platform is

$$\boldsymbol{\omega_{mp}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \tag{35}$$

The linear velocity ($l_{v_{mp}}$) of the moving platform is equal to right hand side of the equation 22. Since point M_3 is selected as pivotal point of the moving platform, the b_{x_i} is equal to b_{x_3}.

$$l_{v_{mp}} = \begin{bmatrix} 1 & 0 & -b_{x_3}\sin\phi - b_{y_3}\cos\phi \\ 0 & 1 & b_{x_3}\cos\phi - b_{y_3}\sin\phi \end{bmatrix} \begin{bmatrix} \dot{P}_{X_B} \\ \dot{P}_{Y_B} \\ \dot{\phi} \end{bmatrix} \tag{36}$$

The partial linear velocity matrix of the moving platform is derived from the equation 36 as

$$\boldsymbol{v_{mp}} = \begin{bmatrix} 1 & 0 & -b_{x_3}\sin\phi - b_{y_3}\cos\phi \\ 0 & 1 & b_{x_3}\cos\phi - b_{y_3}\sin\phi \end{bmatrix} \tag{37}$$

2.4.2 The inertia forces and moments of the mobile parts of the manipulator

The Newton-Euler formulation is applied for deriving the inertia forces and moments of links and mobile platform about their mass centers. The m_{2i-1}, m_{2i} and m_{mp} denote the masses of links l_{2i-1} , l_{2i} and moving platform, respectively where i=1,2 and 3. The c_{2i-1} c_{2i} and c_{mp} are the mass centers of the links l_{2i-1}, l_{2i} and moving platform, respectively. Figure 3 denotes dynamics model of 3-DOF RRR FPPM.

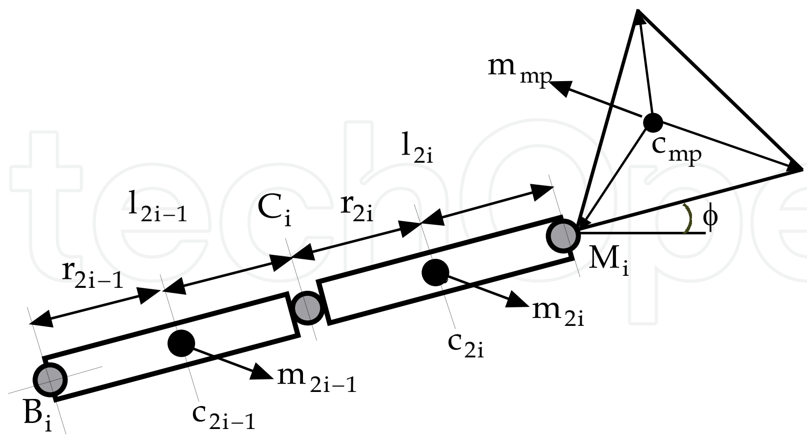


Fig. 3. The dynamics model of 3-DOF RRR FPPM

To find the inertia force of the mass m_{2i-1}, one should determine the acceleration of the link l_{2i-1} about its mass center first. The position vector of the link l_{2i-1} has already been obtained in equation 30. To find the position vector of the center of the link l_{2i-1}, the length r_{2i-1} is used instead of l_{2i-1} in equation 30 as follows

$$ {}^{O_i}_{C_i}T^{1}_{Pc_{2i-1}} = \begin{bmatrix} o_{x_i} + r_{2i-1}\cos\theta_i \\ o_{y_i} + r_{2i-1}\sin\theta_i \end{bmatrix} \tag{38} $$

The second derivative of the equation 30 with respect to the time yields the acceleration of the link l_{2i-1} about its mass center.

$$ a_{c_{2i-1}} = \frac{d}{dt}\left(\frac{d}{dt}\begin{bmatrix} o_{x_i} + r_{2i-1}\cos\theta_i \\ o_{y_i} + r_{2i-1}\sin\theta_i \end{bmatrix}\right) = r_{2i-1}\begin{bmatrix} -\ddot{\theta}_i\sin\theta_i - \dot{\theta}_i^2\cos\theta_i \\ \ddot{\theta}_i\cos\theta_i - \dot{\theta}_i^2\sin\theta_i \end{bmatrix} \tag{39} $$

The inertia force of the mass m_{2i-1} can be found as

$$ \boldsymbol{F_{2i-1}} = -m_{2i-1}\left(a_{c_{2i-1}} - g\right) $$

$$ = m_{2i-1}r_{2i-1}\begin{bmatrix} \ddot{\theta}_i\sin\theta_i + \dot{\theta}_i^2\cos\theta_i \\ -\ddot{\theta}_i\cos\theta_i + \dot{\theta}_i^2\sin\theta_i \end{bmatrix} \tag{40} $$

where g is the acceleration of the gravity and $\mathbf{g} = [0 \quad 0]^T$ since the manipulator operates in the horizontal plane. The moment of the link l_{2i-1} about pivotal point B_i is

$$ \boldsymbol{M_{2i-1}} = -\left[\ddot{\theta}_i I_{2i-1} + m_{2i-1}\left(\frac{d}{d\theta_i}\, {}^{O_i}_{C_i}T^{1}_{Pc_{2i-1}}\right)^T a_{B_i}\right] $$

$$ = \ddot{\theta}_i I_{2i-1} \tag{41} $$

where I_{2i-1}, ${}^{O_i}_{C_i}T^1_{Pc_{2i-1}}$ and a_{B_i}, denote the moment of inertia of the link l_{2i-1}, the position vector of the center of the link l_{2i-1} and the acceleration of the point B_i, respectively. It is noted that $a_{B_i} = 0$.
The acceleration of the link l_{2i} about its mass center is obtained first to find the inertia force of the mass m_{2i}. The position vector of the link l_{2i} has already been given in the left side of the equation 20 in terms of δ_i and θ_i angles. To find the position vector of the center of the link l_{2i} $\left({}^{O_i}_{M_i}T^1_{Pc_{2i}}\right)$, the length r_{2i} is used instead of l_{2i} in left side of the equation 20.

$$ {}^{O_i}_{M_i}T^1_{Pc_{2i}} = \begin{bmatrix} o_{x_i} + r_{2i}\cos\delta_i + l_{2i-1}\cos\theta_i \\ o_{y_i} + r_{2i}\sin\delta_i + l_{2i-1}\sin\theta_i \end{bmatrix} \tag{42} $$

The second derivative of the equation 42 with respect to the time produces the acceleration of the link l_{2i} about its mass center.

$$ \begin{aligned} a_{c_{2i}} &= \frac{d}{dt}\left(\frac{d}{dt}\begin{bmatrix} o_{x_i} + r_{2i}\cos\delta_i + l_{2i-1}\cos\theta_i \\ o_{y_i} + r_{2i}\sin\delta_i + l_{2i-1}\sin\theta_i \end{bmatrix}\right) \\ &= \begin{bmatrix} -r_{2i}\left(\ddot{\delta}_i\sin\delta_i + \dot{\delta}_i^2\cos\delta_i\right) - l_{2i-1}\left(\ddot{\theta}_i\sin\theta_i + \dot{\theta}_i^2\cos\theta_i\right) \\ r_{2i}\left(\ddot{\delta}_i\cos\delta_i - \dot{\delta}_i^2\sin\delta_i\right) + l_{2i-1}\left(\ddot{\theta}_i\cos\theta_i - \dot{\theta}_i^2\sin\theta_i\right) \end{bmatrix} \end{aligned} \tag{43} $$

The inertia force of the mass m_{2i} can be found as

$$ \begin{aligned} \boldsymbol{F_{2i}} &= -m_{2i}\left(a_{c_{2i}} - g\right) \\ &= -m_{2i}\begin{bmatrix} -r_{2i}\left(\ddot{\delta}_i\sin\delta_i + \dot{\delta}_i^2\cos\delta_i\right) - l_{2i-1}\left(\ddot{\theta}_i\sin\theta_i + \dot{\theta}_i^2\cos\theta_i\right) \\ r_{2i}\left(\ddot{\delta}_i\cos\delta_i - \dot{\delta}_i^2\sin\delta_i\right) + l_{2i-1}\left(\ddot{\theta}_i\cos\theta_i - \dot{\theta}_i^2\sin\theta_i\right) \end{bmatrix} \end{aligned} \tag{44} $$

where $\mathbf{g} = [0 \quad 0]^T$. The moment of the link l_{2i} about pivotal point C_i is

$$ \begin{aligned} \boldsymbol{M_{2i}} &= -\left[\ddot{\delta}_i I_{2i} + m_{2i}\left(\frac{d}{d\delta_i}\,{}^{O_i}_{M_i}T^1_{Pc_{2i}}\right)^T a_{C_i}\right] \\ &= -\left(\ddot{\delta}_i I_{2i} + m_{2i}r_{2i}l_{2i-1}\left[\sin\delta_i\left(\ddot{\theta}_i\sin\theta_i + \dot{\theta}_i^2\cos\theta_i\right) \quad \cos\delta_i\left(\ddot{\theta}_i\cos\theta_i - \dot{\theta}_i^2\sin\theta_i\right)\right]\right) \end{aligned} \tag{45} $$

where I_{2i}, ${}^{O_i}_{M_i}T^1_{Pc_{2i}}$ and a_{C_i}, denote the moment of inertia of the link l_{2i}, the position vector of the center of the link l_{2i} in terms of the base coordinate system {XYZ} and the acceleration of the point C_i, respectively. The terms $\frac{d}{d\delta_i}\,{}^{O_i}_{M_i}T^1_{Pc_{2i}}$ and a_{C_i} are computed as

$$ \frac{d}{d\delta_i}\,{}^{O_i}_{M_i}T^1_{Pc_{2i}} = \frac{d}{d\delta_i}\begin{bmatrix} o_{x_i} + r_{2i}\cos\delta_i + l_{2i-1}\cos\theta_i \\ o_{y_i} + r_{2i}\sin\delta_i + l_{2i-1}\sin\theta_i \end{bmatrix} = r_{2i}\begin{bmatrix} -\sin\delta_i \\ \cos\delta_i \end{bmatrix} \tag{46} $$

$$ a_{C_i} = \frac{d}{dt}\left(\frac{d}{dt}\begin{bmatrix} o_{x_i} + l_{2i-1}\cos\theta_i \\ o_{y_i} + l_{2i-1}\sin\theta_i \end{bmatrix}\right) = -l_{2i-1}\begin{bmatrix} \ddot{\theta}_i\sin\theta_i + \dot{\theta}_i^2\cos\theta_i \\ -\ddot{\theta}_i\cos\theta_i + \dot{\theta}_i^2\sin\theta_i \end{bmatrix} \tag{47} $$

The acceleration of the moving platform about its mass center is obtained in order to find the inertia force of the mass m_{mp}. The position vector of the moving platform has already been given in the right side of the equation 20.

$$ {}^{O_i}_{M_i}T^2 = \begin{bmatrix} P_{X_B} + b_{x_i}\cos\phi - b_{y_i}\sin\phi \\ P_{Y_B} + b_{x_i}\sin\phi + b_{y_i}\cos\phi \end{bmatrix} \tag{48} $$

The second derivative of the equation 48 with respect to the time produces the acceleration of the moving platform about its mass center (c_{mp}).

$$ a_{c_{mp}} = \frac{d}{dt}\left(\frac{d}{dt}\begin{bmatrix} P_{X_B} + b_{x_i}\cos\phi - b_{y_i}\sin\phi \\ P_{Y_B} + b_{x_i}\sin\phi + b_{y_i}\cos\phi \end{bmatrix}\right) $$

$$ = \begin{bmatrix} \ddot{P}_{X_B} - \ddot{\phi}\left(b_{x_3}\sin\phi + b_{y_3}\cos\phi\right) - \dot{\phi}^2\left(b_{x_3}\cos\phi - b_{y_3}\sin\phi\right) \\ \ddot{P}_{Y_B} + \ddot{\phi}\left(b_{x_3}\cos\phi - b_{y_3}\sin\phi\right) - \dot{\phi}^2\left(b_{x_3}\sin\phi + b_{y_3}\cos\phi\right) \end{bmatrix} \tag{49} $$

The inertia force of the mass m_{mp} can be found as

$$ \boldsymbol{F_{mp}} = -m_{mp}\left(a_{c_{mp}} - g\right) $$

$$ = -m_{mp}\begin{bmatrix} \ddot{P}_{X_B} - \ddot{\phi}\left(b_{x_3}\sin\phi + b_{y_3}\cos\phi\right) - \dot{\phi}^2\left(b_{x_3}\cos\phi - b_{y_3}\sin\phi\right) \\ \ddot{P}_{Y_B} + \ddot{\phi}\left(b_{x_3}\cos\phi - b_{y_3}\sin\phi\right) - \dot{\phi}^2\left(b_{x_3}\sin\phi + b_{y_3}\cos\phi\right) \end{bmatrix} \tag{50} $$

where $\mathbf{g} = [0 \quad 0]^T$. The moment of the moving platform about pivotal point M_3 is

$$ \boldsymbol{M_{mp}} = -\left[\ddot{\phi} I_{mp} + m_{mp}\left(\frac{d}{d\phi}\,{}^{O_i}_{M_3}T^2_{P(x,y)}\right)^T a_{c_{mp}}\right] $$

$$ = -\left(\ddot{\phi} I_{mp} + m_{mp}\left[\ddot{P}_{X_B}\left(-b_{x_3}\sin\phi - b_{y_3}\cos\phi\right) + \ddot{P}_{Y_B}\left(b_{x_3}\cos\phi - b_{y_3}\sin\phi\right)\right]\right) \tag{51} $$

where I_{mp}, ${}^{O_i}_{M_3}T^2_{P(x,y)}$ and $a_{c_{mp}}$, denote the moment of inertia of the moving platform, the position vector of the moving platform in terms of {XYZ} coordinate system and the acceleration of the point c_{mp}, respectively. The terms $\frac{d}{d\phi}\,{}^{O_i}_{M_3}T^2_{P(x,y)}$ and $a_{c_{mp}}$ are computed as

$$ \frac{d}{d\phi}\,{}^{O_i}_{M_3}T^2_{P(x,y)} = \frac{d}{d\phi}\begin{bmatrix} P_{X_B} + b_{x_3}\cos\phi - b_{y_3}\sin\phi \\ P_{Y_B} + b_{x_3}\sin\phi + b_{y_3}\cos\phi \end{bmatrix} = \begin{bmatrix} -b_{x_3}\sin\phi - b_{y_3}\cos\phi \\ b_{x_3}\cos\phi - b_{y_3}\sin\phi \end{bmatrix} \tag{52} $$

$$ a_{c_{mp}} = \begin{bmatrix} \ddot{P}_{X_B} \\ \ddot{P}_{Y_B} \end{bmatrix} \tag{53} $$

The inverse dynamics of the 3-DOF RRR FPPM based on the virtual work principle is given by

$$ J^T\tau + F = 0 \tag{54} $$

where

$$ F = \sum_{i=1}^{3}\left(\left[\boldsymbol{v}_{2i-1}^T \quad \boldsymbol{\omega}_{2i-1}^T\right]\begin{bmatrix} \boldsymbol{F}_{2i-1} \\ \boldsymbol{M}_{2i-1} \end{bmatrix}\right) + \sum_{i=1}^{3}\left(\left[\boldsymbol{v}_{2i}^T \quad \boldsymbol{\omega}_{2i}^T\right]\begin{bmatrix} \boldsymbol{F}_{2i} \\ \boldsymbol{M}_{2i} \end{bmatrix}\right) + \left[\boldsymbol{v}_{mp}^T \quad \boldsymbol{\omega}_{mp}^T\right]\begin{bmatrix} \boldsymbol{F}_{mp} \\ \boldsymbol{M}_{mp} \end{bmatrix} \tag{55} $$

The driving torques ($\tau_1 \quad \tau_2 \quad \tau_3$) of the 3-DOF RRR FPPM are obtained from equation 54 as

$$ \tau = -(J^T)^{-1}F \tag{56} $$

where $\tau = [\tau_1 \quad \tau_2 \quad \tau_3]^T$.

3. Case study

In this section to demonstrate the active joints torques, a butterfly shape Cartesian trajectory with constant orientation ($\phi = 30^o$) is used as a desired end-effector's trajectory. The time dependent Cartesian trajectory is

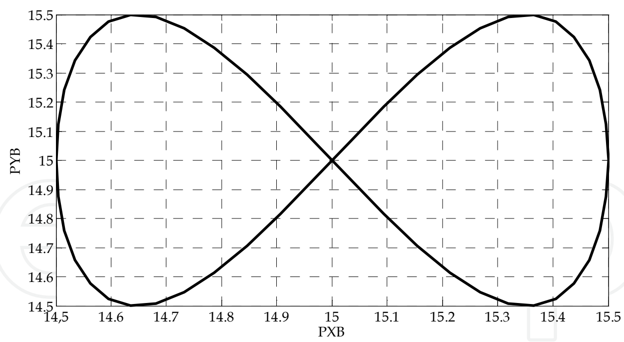
$$P_{X_B} = P_{X_0} + a_m \cos(\omega_c \pi t) \quad 0 \le t \le 5 \text{ seconds} \tag{57}$$

$$P_{Y_B} = P_{Y_0} + a_m \sin(\omega_s \pi t) \quad 0 \le t \le 5 \text{ seconds} \tag{58}$$

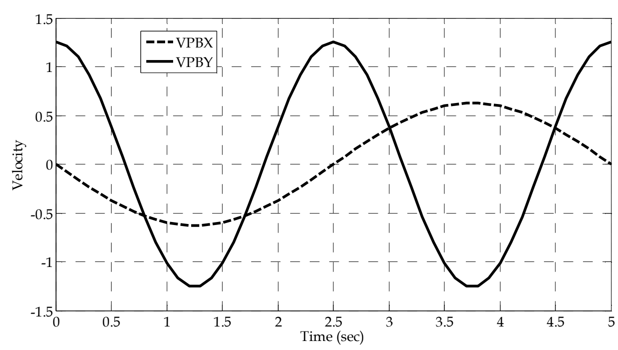
A safe Cartesian trajectory is planned such that the manipulator operates a trajectory without any singularity in 5 seconds. The parameters of the trajectory given by 57 and 58 are as follows: $P_{X_{B0}} = P_{Y_{B0}} = 15$, $a_m = 0.5$, $\omega_c = 0.4$ and $\omega_s = 0.8$. The Cartesian trajectory based on the data given above is given by on Figure 4a (position), 4b (velocity) and 4c (acceleration). On Figure 4, the symbols VPBX, VPBY, APBX and APBY illustrate the velocity and acceleration of the moving platform along the X and Y-axes. The first inverse kinematics solution is used for kinematics and dynamics operations. The moving platform is an equilateral triangle with side length of 10. The position of end-effector in terms of {xyz} coordinate systems is $P(x_m, y_m)=(5, 2.8868)$ that is the center of the equilateral triangle moving platform. The kinematics and dynamics parameters for 3-DOF RRR FPPM are illustrated in Table 1. Figure 5 illustrates the driving torques ($\tau_1 \quad \tau_2 \quad \tau_3$) of the 3-DOF RRR FPPM based on the given data in Table 1.

Link lengths		Base coordinates		Masses		Inertias	
l_1	10	o_{x_1}	0	m_1	10	I_1	10
l_2	10	o_{y_1}	0	m_2	10	I_2	10
l_3	10	o_{x_2}	20	m_3	10	I_3	10
l_4	10	o_{y_2}	0	m_4	10	I_4	10
l_5	10	o_{x_3}	10	m_5	10	I_5	10
l_6	10	o_{y_3}	32	m_6, m_{mp}	10	I_6, I_{mp}	10

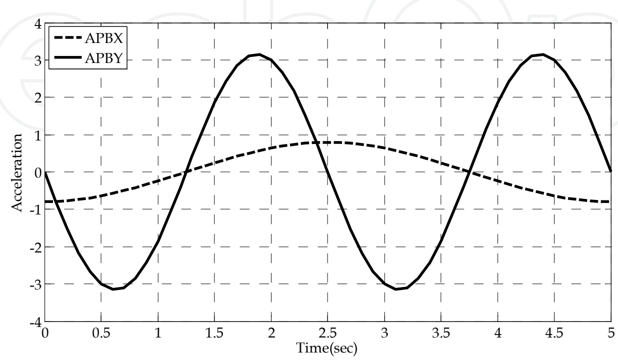
Table 1. The kinematics and dynamics parameters for 3-DOF RRR FPPM



(a)



(b)



(c)

Fig. 4. a) Position, b) velocity and c) acceleration of the moving platform

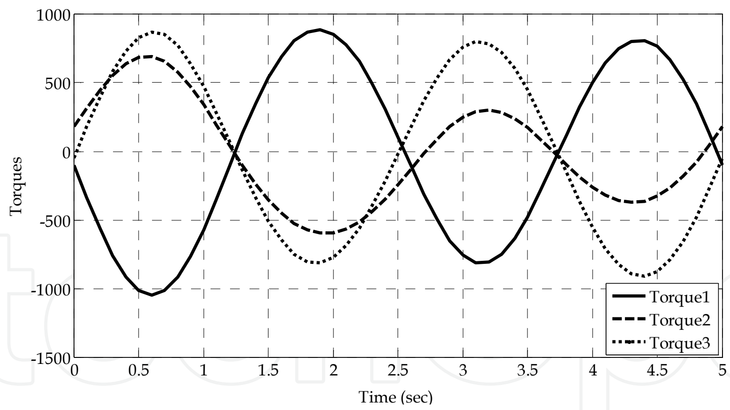


Fig. 5. The driving torques ($\tau_1 \quad \tau_2 \quad \tau_3$) of the 3-DOF RRR FPPM

4. Conclusion

In this chapter, the inverse dynamics problem of 3-DOF RRR FPPM is derived using virtual work principle. Firstly, the inverse kinematics model and Jacobian matrix of 3-DOF RRR FPPM are determined using DH method. Secondly, the partial linear velocity and partial angular velocity matrices are computed. Pivotal points are selected in order to determine the partial linear velocity matrices. Thirdly, the inertial force and moment of each moving part are obtained. Consequently, the inverse dynamic equations of 3-DOF RRR FPPM in explicit form are derived. A butterfly shape Cartesian trajectory is used as a desired end-effector's trajectory to demonstrate the active joints torques.

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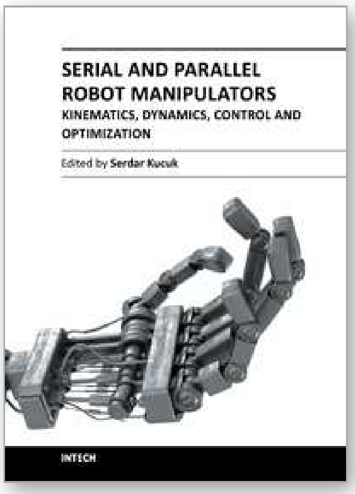

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The robotics is an important part of modern engineering and is related to a group of branches such as electric & electronics, computer, mathematics and mechanism design. The interest in robotics has been steadily increasing during the last decades. This concern has directly impacted the development of the novel theoretical research areas and products. This new book provides information about fundamental topics of serial and parallel manipulators such as kinematics & dynamics modeling, optimization, control algorithms and design strategies. I would like to thank all authors who have contributed the book chapters with their valuable novel ideas and current developments.

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