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# Some Remarks About Negative Efficiencies in DEA Models

Eliane Gonçalves Gomes[1], João Carlos C. B. Soares de Mello[2],
Lidia Angulo Meza[2], Juliana Quintanilha da Silveira[2],
Luiz Biondi Neto[3] and Urbano Gomes Pinto de Abreu[4]
*[1]Brazilian Agricultural Research Corporation - Embrapa*
*[2]Fluminense Federal University*
*[3]Rio de Janeiro State University*
*[4]Embrapa Pantanal*
*Brazil*

## 1. Introduction

Data Envelopment Analysis appeared in 1978 when the first model, known as CCR was proposed by Charnes et al. (1978). This model calculates the efficiency of productive units, known as DMUs - Decision Making Units, by comparing the use of resources (inputs) and the production (outputs) obtained. This model considers Constant Returns to Scale (CRS), i.e., an increase in resources generates a proportional increment to products. This proportion is constant for all DMUs. An important issue regarding the original CCR model is that all data and variables must be non-negative.

The BCC model proposed by Banker et al. (1984) introduced the hypothesis of variable returns to scale to the DEA models. In this model there is no proportionality between increments in resources and the correspondent increments in products. Among its other characteristics, this model allows the use of negative variables due to some invariability properties in the data translation procedure (Pastor, 1996, Thrall, 1996, Iqbal Ali & Seiford, 1990). In addition, Sharp et al. (2007), Duzakin and Duzakin (2007), Portela et al. (2004) and Sueyoshi (2004) presented examples of DEA models that deal with negative outputs or inputs, including some sophisticated theoretical formulations.

The efficiency achieved by the DEA models was always taken as non negative. In the specific case of classic input orientated models (radials, CCR or BCC), the immediate result of the objective function is the efficiency, measured in the interval [0,1]. This is an immediate consequence of the formulation of the multipliers model, because a set of restrictions establishes that the weights or multipliers are calculated in such a way as to maximize the efficiency of the DMU under analysis. However, a second group of restrictions determines that those multipliers, when used to evaluate other DMUs, cannot generate efficiencies higher than 1. So given that all of the multipliers are not negative, and that the variable data are not negative either, the weighted sum that calculates the efficiency is measured in the interval [0,1].

In the case of the DEA-BCC model, the multipliers model was originally obtained through the dual of its envelope formulation. This model holds an equality restriction such that it finds a convex frontier. The multipliers model in the dual envelope method has a free variable. This means that we cannot guarantee the non negativity of the efficiencies when the multipliers of a DMU are used to evaluate another DMU. When working with the classic DEA models this is not a problem, but negative efficiencies can be generated in typical situations of Cross Evaluation (Sexton et al., 1986, Doyle & Green, 1994) or in the calculation of some non-radial efficiencies (Lins et al., 2004, Quariguasi Frota Neto & Angulo-Meza, 2007).

In this chapter it will be shown that the input orientated DEA BCC model can generate negative efficiencies, which are usually hidden in the model. Using a two-dimensional example (one input and one output), the condition for the possible occurrence of negative efficiencies will be shown. Furthermore, we will show that a small intuitive change in the BCC multipliers model fixes this situation. However, we will show that this modification generates a major change in the dual envelope model, producing alterations to the frontier. We give an interpretation for this new dual model using a non-observed DMU.

Numerical examples will be presented. The first example has two variables, one input and one output. It will be geometrically shown that the modified model causes a change in the efficient frontier. These changes will be interpreted by the introduction of a non-observed DMU. In this first example, the DMU causing negative efficiency in other DMUs has a reduction of its efficiency in the modified model. This reduction does not occur in the second example (two inputs and one output). This happens due to the optimum multipliers set multiplicity.

We will show that the negative efficiencies mentioned in this chapter apply neither to the CCR model nor to the output orientated BCC model. The latter has a restriction in the multipliers model that is of the “greater than” or “equal to” type, ensuring the non negativity of the efficiency measures.

Aside from simple numerical examples, we will also use the proposed theoretical approach to assess the efficiency of cattle breeders in some Brazilian municipalities

## 2. DEA models - general aspects

DEA uses mathematical programming problems to estimate a piecewise linear efficient frontier. DEA can deal with multiple inputs and outputs to calculate the efficiency of the firms, or production units, or DMUs. DEA optimizes each individual observation in order to estimate an efficient frontier (piecewise linear), composed of the units with the best practices within the evaluation sample (Pareto-Koopmans efficient units). These firms are references or benchmarks for the inefficient ones.

There are two classic DEA models. The CCR model (also known as CRS or constant returns to scale), which deals with constant returns to scale (Charnes et al., 1978) and assumes proportionality between inputs and outputs. The BCC model (or VRS), due to Banker et al. (1984), assumes variable returns to scale, i.e. replaces the axiom of proportionality by the axiom of convexity (Lins & Angulo-Meza, 2000). Traditionally, there are two orientations to these models: input oriented, if we want to minimize the resources available, without changing the level of production; oriented to outputs, when the goal is to increase the production, without changing the amount of inputs used. See Cooper et al. (2004) for details. A usual DEA assumption is that resources and products are subject to physical measurement. However, this is not strictly necessary and proxies can be used (Souza, 2006).

The CCR model constructs a non-parametric piecewise linear frontier involving the data. As mentioned before, it assumes constant returns to scale, that is, any change in inputs produces proportional variation in the outputs. This model determines the efficiency by dividing the weighted sum of the outputs (virtual output) by the weighted sum of the inputs (virtual input), generalizing thus the definition of Farrell (1957). The model allows each DMU to choose the weights for each variable (input or output) in the way that is more benevolent. These weights when applied to other DMUs cannot produce a ratio greater than 1. These conditions are formalized in (1), where each DMU $k$, $k$ = 1...$n$, is a production unit that uses $r$ inputs $x_{ik}$, $i$ = 1...$r$, to produce $s$ outputs $y_{jk}$, $j$ = 1...$s$; $x_{i0}$ and $y_{j0}$ are the inputs and outputs of DMU 0; $u_j$ and $v_i$ are the weights calculated by the model for inputs and outputs, respectively.

$$
\begin{aligned}
&\text{Max } \frac{\sum_{j=1}^{s} u_j y_{jo}}{\sum_{i=1}^{r} v_i x_{io}} \\
&\text{subject to} \\
&\frac{\sum_{j=1}^{s} u_j y_{jk}}{\sum_{i=1}^{r} v_i x_{ik}} \le 1, \quad k = 1 \ldots n \\
&v_i\ ,u_j \ge 0, \quad i = 1 \ldots r,\ j = 1 \ldots s
\end{aligned}
\tag{1}
$$

The fractional programming problem (1), which must be solved for each DMU, can be transformed into a Linear Programming Problem (LPP). To do so, we may impose that the denominator of the objective function should be equal to a constant, usually unity. This linear formulation of the CCR model is presented in (2) and is called Multipliers Model with input orientation.

$$
\begin{aligned}
&\text{Max } \sum_{j=1}^{s} u_j y_{j0} \\
&\text{subject to} \\
&\sum_{i=1}^{r} v_i x_{i0} = 1 \\
&\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{r} v_i x_{ik} \le 0,\ k = 1 \ldots n \\
&v_i, u_j \ge 0, \quad i = 1 \ldots r,\ j = 1 \ldots s
\end{aligned}
\tag{2}
$$

There are two equivalent, dual, formulations for DEA. Simply put, we can say that one of the formulations, the Multipliers model, deals with the ratio of weighted sums of products and resources, with the weighting chosen to be more favourable to each DMU, subject to certain conditions. The other formulation, the Envelope model, defines a feasible region of production and works with projections of each DMU in this frontier.

In (3) we present the DEA CCR Envelope model, input oriented, where $\theta_0$ is the efficiency of the DMU under analysis, $\lambda_k$ represents the contribution of DMU $k$ to obtain the target for DMU 0.

$$
\begin{aligned}
&\text{Min } \theta_0 \\
&\text{subject to} \\
&\theta_0 x_{i0} - \sum_{k=1}^{n} x_{ik}\lambda_k \geq 0, \quad i = 1 \ldots r \\
&-y_{j0} + \sum_{k=1}^{n} y_{jk}\lambda_k \geq 0, \quad j = 1 \ldots s \\
&\lambda_k \geq 0, \quad k = 1 \ldots n
\end{aligned}
\tag{3}
$$

The DEA-BCC model was proposed for dealing with situations whereby the proportionality between inputs and outputs is not constant along the efficient frontier, thereby forming a variable returns to scale (VRS) frontier. The BCC model was originally obtained by adding a convexity restriction to the formulation of the CCR envelope model (3). This new restriction is $\sum_{k=1}^{n} \lambda_k = 1$ . The frontier is piecewise linear and takes the different production scales into account: increasing, constant and decreasing returns to scale. The input orientated BCC multipliers model - the dual of the Envelope Model - is presented in (4).

$$
\begin{aligned}
&\text{Max } \sum_{j=1}^{s} u_{j0} y_{j0} + u_* \\
&\text{subject to} \\
&\sum_{i=1}^{r} v_{i0} x_{i0} = 1 \\
&\sum_{j=1}^{s} u_j y_{jk} + u_* - \sum_{i=1}^{r} v_i x_{ik} \leq 0, \; k = 1 \ldots n \\
&v_i, u_j \geq 0, \; u_* \in \Re
\end{aligned}
\tag{4}
$$

In these models, the variable $u_*$ indicates if the observed DMU is found in the area of increasing, constant or decreasing returns to scale. This variable also represents the independent term in the supporting hyper-plane equation and can assume any real value as shown in Fig. 1. As was previously mentioned, this is the dual variable corresponding to the envelope model equality restriction.

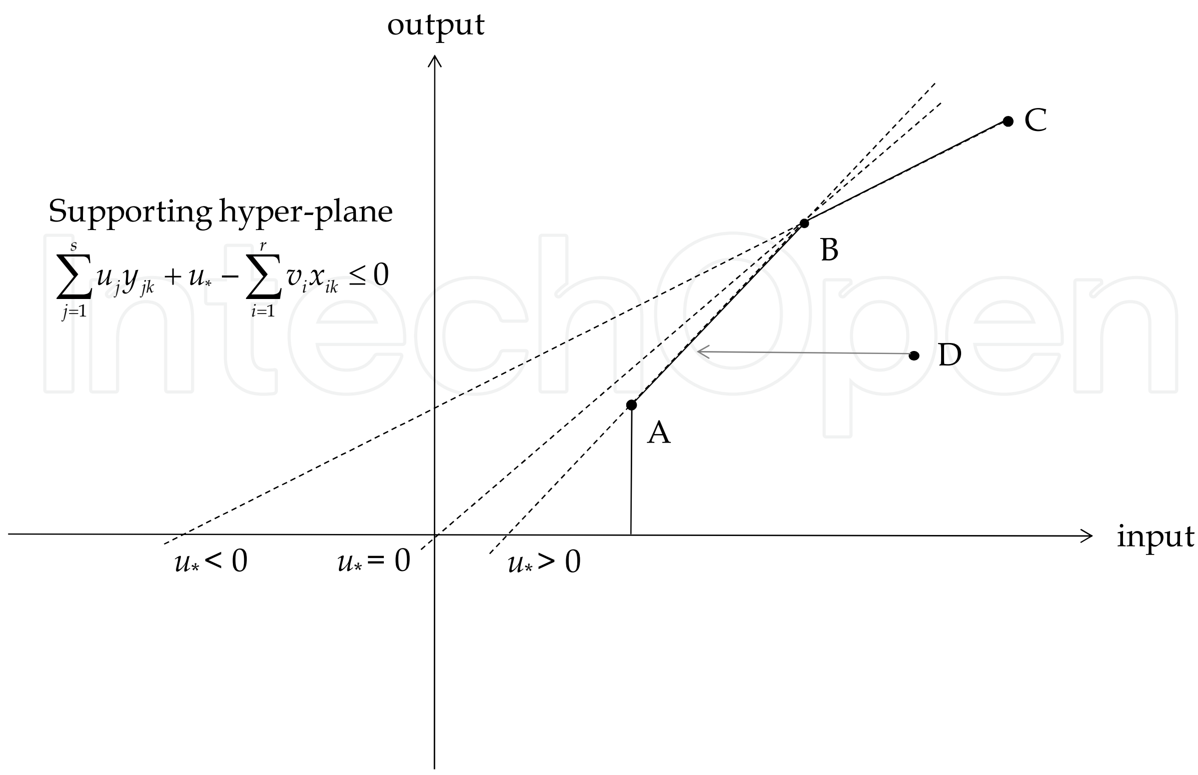



Fig. 1. Supporting hyper-plane to the input orientated BCC model.

Fig. 1 shows the supporting hyper-plane equations for different values of the variable $u_*$. When this term is strictly positive the DMU works in the increasing returns to scale part of the efficient frontier. When it is equivalent to zero it has constant returns, and when is strictly negative it is in the decreasing returns to scale part of the efficient frontier.

In order to model and to interpret correctly the DEA results it is necessary to know their models properties. Two of the most important are (Gomes et al., 2009):

- In any DEA model, the DMU that has the best value of the ratio $(output\ j)/(input\ i)$ will always be efficient. This property requires the existence of a causal relationship between each output and each input. Ignoring this relationship can lead to meaningless results.
- The CCR model, which in its fractional form is a zero degree homogeneous function. Its main property is the proportionality between inputs and outputs at the frontier. As a consequence, an increase (decrease) in the amount of inputs will cause a proportional increase (decrease) in the value of their outputs.

## 3. Negative efficiencies in the DEA BCC model

In model (4), it should be noted that when the multipliers of the DMU 0 are used to evaluate other DMUs, the expression $\sum_{j=1}^{s} u_{j0} y_{j0} + u_{*0}$ may be negative when $u_{*0}$ is sufficiently negative. This means that the efficiency of DMU $j$, when evaluated with the multipliers of the DMU 0, may be negative. This may occur when the DMU 0 is in the decreasing returns to scale portion of the frontier. Negative efficiencies were detected, but not studied, by Soares de Mello et al. (2002).

In the case of just one *output*, $r$ =1, we are able to derive a condition to determine when negative efficiencies will appear in the restrictions for a specific DMU $j$. This happens when $u_{10}y_{1j} + u_{*0} < 0$, that is, $y_{1j} < -u_{*0}/u_{10}$.

Table 1 shows both data and results of a numerical example with one input and one output, illustrating the situation previously described. It is important to highlight that multiple optimum multipliers exist for efficient DMUs (Rosen et al., 1998, Nacif et al., 2009, Soares de Mello et al., 2002) and we here used the first multipliers found by the software SIAD (Angulo-Meza et al., 2005). Note that in Table 1 the $u_{*0}$ is a negative value for DMU A.

| **DMU** | **Input** | **Output** | **Multipliers** | | | **Efficiency** |
|---|---|---|---|---|---|---|
| | | | $v$ | $u$ | $u_*$ | |
| A | 4 | 10 | 0.250 | 0.375 | -2.750 | 1.000 |
| B | 1 | 5 | 1.000 | 0.000 | 1.000 | 1.000 |
| C | 2 | 7 | 0.500 | 0.000 | 0.500 | 0.500 |
| D | 1 | 8 | 1.000 | 0.125 | 0.000 | 1.000 |
| E | 6 | 6 | 0.167 | 0.000 | 0.167 | 0.167 |

Table 1. Numerical example – one input and one output.

Table 2 presents the DMUs' efficiencies calculated using the multipliers of the others, in an approach similar to the Cross Evaluation (Sexton et al., 1986, Doyle & Green, 1994). In this table the values in a column $j$ are the efficiencies of the DMU $j$ when evaluated using the DMU of the respective row.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 1.000 | -3.500 | -0.250 | 1.000 | -0.333 |
| B | 0.250 | 1.000 | 0.500 | 1.000 | 0.167 |
| C | 0.250 | 1.000 | 0.500 | 1.000 | 0.167 |
| D | 0.313 | 0.625 | 0.438 | 1.000 | 0.125 |
| E | 0.250 | 1.000 | 0.500 | 1.000 | 0.167 |

Table 2. Cross efficiencies for the numerical example presented in Table 1.

In Table 2, we can observed that when using the output multiplier of the DMU A $u_A$ = 0.375, and the independent term, $u_{*A}$=-2.750, three negative efficiencies are obtained. This happens for DMUs B, C and E. The outputs of these DMUs are less than 7.333 (result of $-u_{*A}/u_A$). These results have no interpretation in the classic theory of efficiency. Furthermore, these implicit negative efficiencies may be the main reason for the use of Cross Evaluation only with constant returns to scale (CRS) DEA models

## 4. Proposed DEA BCC model with non-negativity constraint

The problem of negative efficiencies can be easily solved by imposing an additional constraint for each DMU, as shown in model (5). This model is called the Modified Input

Orientated DEA BCC Model. It should be mentioned that a similar approach for specific cases was proposed by Wu et al. (2009) and Angulo-Meza et al. (2004), without meaningful interpretation.

$$
\begin{aligned}
&\text{Max } \frac{\sum_{j=1}^{s} u_j y_{jo} + u_*}{\sum_{i=1}^{r} v_i x_{io}} \\
&\text{subject to} \\
&0 \le \frac{\sum_{j=1}^{s} u_j y_{jk} + u_*}{\sum_{i=1}^{r} v_i x_{ik}} \le 1, \quad k = 1 \ldots n \\
&v_i, u_j \ge 0, u_* \in \Re, \ \ i = 1 \ldots r, \ j = 1 \ldots s
\end{aligned}
\tag{5}
$$

The linear form of model (5) is presented in (6). In this model, each restriction $\sum_{j=1}^{s} u_j y_{jk} + u_* \ge 0$ would be called a non-negativity restriction.

$$
\begin{aligned}
&\text{Max } \sum_{j=1}^{s} u_j y_{jo} + u_* \\
&\text{subject to} \\
&\sum_{i=1}^{r} v_i x_{io} = 1 \\
&\sum_{j=1}^{s} u_j y_{jk} + u_* - \sum_{i=1}^{r} v_i x_{ik} \le 0, \quad \forall k \\
&\sum_{j=1}^{s} u_j y_{jk} + u_* \ge 0, \quad \forall k \\
&v_i, u_j \ge 0, u_* \in \Re, \ \ \forall i, j
\end{aligned}
\tag{6}
$$

Table 3 shows the results of model (6) using the data in Table 1. The results show that the Modified Input Oriented BCC Model has changed the efficiency of DMU A. According to this new model, DMU A has become inefficient, because it was the only one that generated negative efficiencies when used to evaluate other DMUs. In fact, the term $-u_{*A}/u_A$ of DMU A is equal to 5.00, being the lowest output value in the set of DMUs.

Once again we used the multipliers obtained with model (6) to calculate the Cross Evaluation Matrix for all DMUs. The results are presented in Table 4. Observe that all of the evaluations carried out by all DMUs generate efficiencies between 0 and 1. It is interesting to note that DMU B, when evaluated by DMU A has an efficiency of 0, i.e. the non-negative restriction is an active restriction. In classic DEA models the null efficiency can only appear when all of the outputs are null.

| DMU | Input | Output | Multipliers | | | Efficiency |
|---|---|---|---|---|---|---|
| | | | $v$ | $u$ | $u_*$ | |
| A | 4 | 10 | 0.250 | 0.083 | -0.417 | 0.417 |
| B | 1 | 5 | 1.000 | 0.000 | 1.000 | 1.000 |
| C | 2 | 7 | 0.500 | 0.000 | 0.500 | 0.500 |
| D | 1 | 8 | 1.000 | 0.000 | 1.000 | 1.000 |
| E | 6 | 6 | 0.167 | 0.000 | 0.167 | 0.167 |

Table 3. Numerical example for the modified DEA-BCC model.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 0.417 | 0.000 | 0.333 | 1.000 | 0.056 |
| B | 0.250 | 1.000 | 0.500 | 1.000 | 0.167 |
| C | 0.250 | 1.000 | 0.500 | 1.000 | 0.167 |
| D | 0.250 | 1.000 | 0.500 | 1.000 | 0.167 |
| E | 0.250 | 1.000 | 0.500 | 1.000 | 0.167 |

Table 4. Cross Efficiencies for the modified DEA BCC model.

## 5. Interpretation of the additional restriction

At this point it is important to discuss the effects on the efficient frontier caused by the new restriction. In Thanassoulis & Allen (1988) it was shown that multipliers restrictions can be replaced by one or more unobserved or artificial DMUs, i.e. DMUs that do not exist in the original data set. As the restrictions of non-negativity, one for each DMU, are in fact multipliers restrictions, they can be replaced by unobserved DMUs. So, the efficient frontier in the model with non-negativity constraints may also depend on inefficient DMUs. Specifically, the inefficient DMUs that may change the frontier are those with, at least, one negative cross efficiency in the classic model.

Analysing the results of the numerical example, we observe that DMU A was the only one whose efficiency was altered by model (6). As a result, the modified model frontier may be obtained including a non-observed DMU. The output of this non-observed DMU is the same output of DMU A. The input of the non-observed DMU is obtained by multiplying DMU A actual input by DMU efficiency in the modified model. For our numerical example the non-observed has an output of 10 and an input of 1.6667.

Fig. 2 shows both DEA BCC and modified DEA BCC frontiers. In this Figure the bold line represents the two models common frontier, the dashed line belongs only to the DEA BCC model and the dotted line represents the frontiers of the modified DEA BCC model. We clearly observe that the new frontier is dislocated from the original DEA BCC frontier.

The geometric representation here above is valid only for the one input and output case. The multidimensional case can be interpreted only using the dual formulation of the modified BCC model. This formulation is presented in (7).

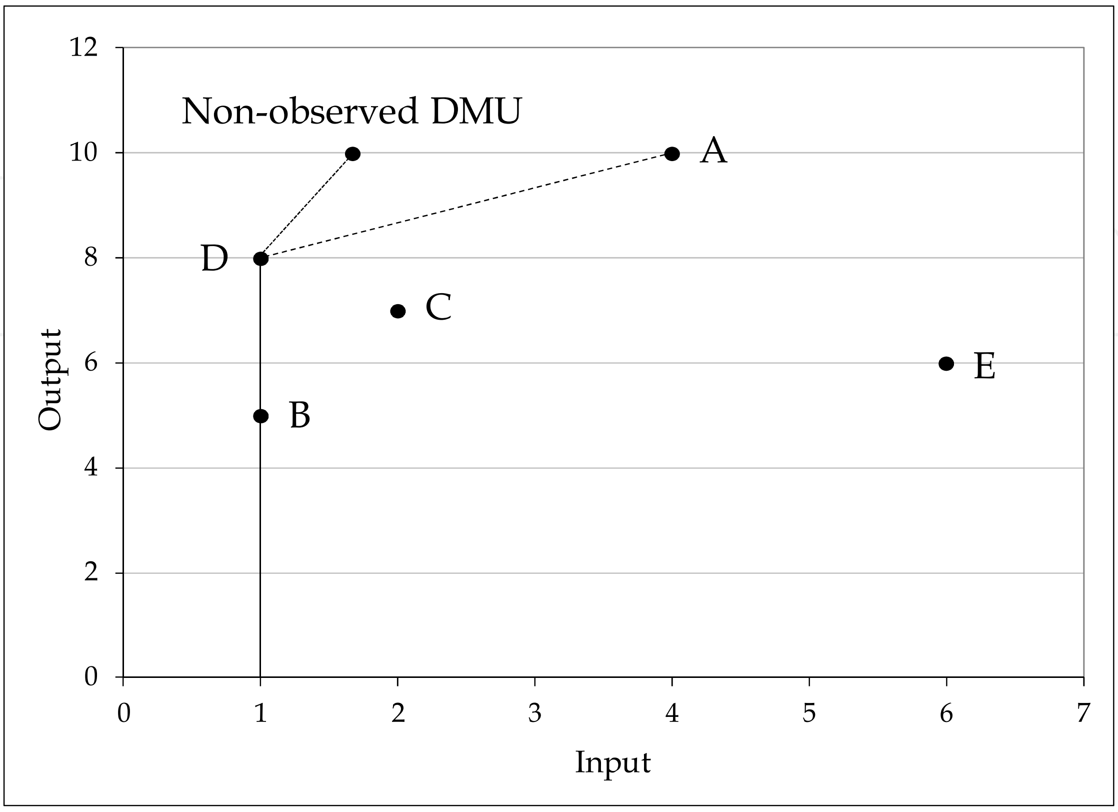



Fig. 2. DEA BCC frontier and the modified DEA BCC frontier.

$$
\begin{aligned}
&\text{Min} \quad \theta_0 \\
&\text{subject to} \\
&\sum_{k=1}^{n} y_{jk}\lambda_k - \sum_{k=1}^{n} y_{jk}\lambda'_k \geq y_{j0}, \ \forall j \\
&\theta_0 x_{i0} \geq \sum_{k=1}^{n} x_{ik}\lambda_k, \ \forall i \\
&\sum_{k=1}^{n} \lambda_k - \sum_{k=1}^{n} \lambda'_k = 1, \ \forall k \\
&\lambda_k, \lambda'_k \geq 0, \ \forall k
\end{aligned}
\tag{7}
$$

The additional restrictions of model (6) - one for each DMU, generate the same number of additional decision variables in the dual model (7). These variables are called $\lambda'$. It should be noted that when the sum of all $\lambda'$ is not null, then the sum of $\lambda$ won't be unitary. Therefore, in this case, there won't be any guarantee of convexity in the modified model.

From the Complementary Slack Theorem, we know that $\lambda'$ times the corresponding restriction slack must be null. Consequently, $\lambda'$ can only be other than zero, if the slack in the corresponding additional restriction of model (6) is null. This happens when the corresponding additional restriction is active. This means that the DMU corresponding to the active restriction would have negative efficiency using DMU *0* multipliers.

In the previous numerical example, only DMU A had its efficiency changed due to the additional restrictions, and this change was caused by the non-negativity restriction relative to DMU B. So, all $\lambda'$ for DMU A are null, except $\lambda'_B$.

## 6. Three-dimensional numerical example

To illustrate the modified BCC model in situations with more than two variables, we will present a new numerical example. Table 5 shows data for the numerical example with seven DMUs, two inputs and one output. The variables' multipliers and efficiencies are depicted in the same table. As previously done, the multipliers shown are the first found by the SIAD software.

| DMU | Input1 | Input2 | Output | Multipliers | | | | Efficiency |
|---|---|---|---|---|---|---|---|---|
| | | | | $v_1$ | $v_2$ | $u$ | $u_*$ | |
| A | 0.489 | 0.637 | 0.607 | 0.300 | 1.340 | 1.627 | 0.012 | 1.000 |
| B | 1.000 | 1.000 | 1.000 | 1.000 | 0.000 | 1.300 | -0.300 | 1.000 |
| C | 0.019 | 0.190 | 0.010 | 30.702 | 2.193 | 0.000 | 1.000 | 1.000 |
| D | 0.032 | 0.008 | 0.005 | 30.702 | 2.193 | 0.000 | 1.000 | 1.000 |
| E | 0.096 | 0.052 | 0.032 | 0.000 | 19.231 | 19.173 | 0.058 | 0.672 |
| F | 0.053 | 0.035 | 0.007 | 17.754 | 1.687 | 15.240 | 0.505 | 0.612 |
| G | 0.898 | 0.164 | 0.115 | 0.000 | 6.098 | 6.079 | 0.018 | 0.717 |

Table 5. Data and results for the three-dimensional numerical example.

As we can observe, the $u_*$ of DMU B is negative. When the multiplier of the sole output ($u$ = 1.30) and the independent term ($u_*$ = -0.300) from DMU B are used to evaluate the other DMUs, negative efficiencies will appear for DMUs that have an output value inferior to 0.231. The DMUs that have output inferior to this value are C, D, E, F and G, whose efficiencies will be negative. The Cross Evaluation Matrix (Table 6) illustrates these comments.

The results of the modified BCC model are depicted in Table 7. Note that the modified model didn't change the efficiency of any DMU. This happened because DMU B, which generated negative efficiencies when evaluating the others, is also efficient in the modified model. This is due to the multiplicity of the optimum multipliers set previously mentioned.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| A | 1.000 | 1.000 | 0.109 | 1.000 | 0.653 | 0.375 | 0.408 |
| B | 1.000 | 1.000 | -15.119 | -9.180 | -2.694 | -5.493 | -0.168 |
| C | 0.061 | 0.030 | 1.000 | 1.000 | 0.327 | 0.587 | 0.036 |
| D | 0.061 | 0.030 | 1.000 | 1.000 | 0.327 | 0.587 | 0.036 |
| E | 0.955 | 1.000 | 0.068 | 1.000 | 0.672 | 0.286 | 0.717 |
| F | 1.000 | 0.810 | 1.000 | 1.000 | 0.554 | 0.612 | 0.139 |
| G | 0.955 | 1.000 | 0.068 | 1.000 | 0.672 | 0.286 | 0.717 |

Table 6. Cross evaluation matrix for the three-dimensional numerical example.

| DMU | Input1 | Input2 | Output | Multipliers | | | | Efficiency |
|---|---|---|---|---|---|---|---|---|
| | | | | $v_1$ | $v_2$ | $u$ | $u_*$ | |
| A | 0.489 | 0.637 | 0.607 | 0.300 | 1.340 | 1.627 | 0.012 | 1.000 |
| B | 1.000 | 1.000 | 1.000 | 0.183 | 0.817 | 0.993 | 0.007 | 1.000 |
| C | 0.019 | 0.190 | 0.010 | 26.989 | 2.564 | 23.168 | 0.768 | 1.000 |
| D | 0.032 | 0.008 | 0.005 | 30.525 | 2.900 | 26.203 | 0.869 | 1.000 |
| E | 0.096 | 0.052 | 0.032 | 0.000 | 19.231 | 19.173 | 0.058 | 0.672 |
| F | 0.053 | 0.035 | 0.007 | 17.754 | 1.687 | 15.240 | 0.505 | 0.612 |
| G | 0.898 | 0.164 | 0.115 | 0.000 | 6.098 | 6.079 | 0.018 | 0.717 |

Table 7. Data and results for the numerical example using modified BCC model.

The Cross Evaluation Matrix for the modified DEA BCC model is shown in Table 8. Note that there are no negative cross efficiencies in this table.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| A | 1.000 | 1.000 | 0.109 | 1.000 | 0.653 | 0.375 | 0.408 |
| B | 1.000 | 1.000 | 0.109 | 1.000 | 0.653 | 0.375 | 0.408 |
| C | 1.000 | 0.810 | 1.000 | 1.000 | 0.554 | 0.612 | 0.139 |
| D | 1.000 | 0.810 | 1.000 | 1.000 | 0.554 | 0.612 | 0.139 |
| E | 0.955 | 1.000 | 0.068 | 1.000 | 0.672 | 0.286 | 0.717 |
| F | 1.000 | 0.810 | 1.000 | 1.000 | 0.554 | 0.612 | 0.139 |
| G | 0.955 | 1.000 | 0.068 | 1.000 | 0.672 | 0.286 | 0.717 |

Table 8. Cross evaluation matrix for the Modified DEA BCC model from the three-dimensional example.

## 7. Case study

We will use the modified DEA BCC model to evaluate the efficiency of some livestock systems.

The central structure in the beef cattle production chain is the biological system of beef production, including the various stages of creation (cow-calf production, stocker production, feedlot beef production) and their combinations, around which the producers are grouped. In Brazil, the cow-calf beef cattle phase occurs predominantly in an extensive continuous grazing, with native and/or cultivated pastures, encompassing: calves (until weaning or even one year old), cows, heifers and bulls. The cow-calf phase is the lower profitability activity and the one of major risk. However, it supports the entire structure of the beef production chain.

This case study, by using the Modified DEA BCC models here proposed, seeks to assess the comparative performance of extensive livestock modal production systems in its cow-calf phase, in some municipalities of Brazil. The objective is to measure the performance of the cattle farmer's decision regarding the composition of the production system, which has a direct impact on the expenditures and on the income generated. A study carried out with the same dataset and based on DEA BCC model with weights restrictions can be found in Gomes et al. (2011).

### 7.1 Data source

Primary data were collected through the panel system, which allows the definition of representative farms, as proposed by Plaxico and Tweeten (1963).

Despite the difficulty of characterizing a single property and a production system that is representative of the locality under study (here the city/municipality), this method looks through the experience of the participating farmers to characterize the property that is the most commonly found in the region. In some cases, the impossibility of determining this typology imposes the specification of more than one representative property or production system.

The panel is a less costly procedure of obtaining information than the census or the sampling of farms. Another advantage is that it provides greater flexibility and versatility in data updating, without affecting their quality. The technique consists in a meeting with a group of one or more researchers, one technician and eight regional farmers, on average (it can range from five to ten). Meetings are scheduled in advance, with the support of rural unions and regional contacts. The subjects and numbers, determined previously in interviews with local technicians, are discussed with the farmers. At the end of that debate, one can say that any characterization of the typical farm in the region has the consent of the farmers. Thus, productivity rates, establishment costs, fixed and variable costs, i.e., all the numbers resulting from the panel, tend to be fairly close to the regional reality.

It is noteworthy that the rates and the costs reported by each participant are not related to their properties, but with a single farm, declared at the beginning of the panel as the one that best represents the scale of operation and the production system of most of the local properties.

This study evaluated 21 beef cattle modal production systems that performed only the cow-calf phase, in seven states of Brazil. The data, derived from the indicators of the project developed by the Centro de Estudos Avançados em Economia Aplicada and the Confederação da Agricultura e Pecuária do Brasil, were collected in municipalities of these seven states: Mato Grosso do Sul - MS (eight), Goiás - GO (four), Rio Grande do Sul - RS (one), Minas Gerais - MG (four), Tocantins - TO (two), São Paulo - SP (one) and Bahia - BA (one). Panels with the farmers, with the support of the local rural technical assistance, were performed to collect the data, according to the methodology described in Centro de Estudos Avançados em Economia Aplicada (2010).

### 7.2 Modelling

#### 7.2.1 DMUs

The objective of the DEA model proposed here is to measure the performance of the farmer's decision regarding the composition of the rearing production system. Thus, the DMUs are the 21 modal systems, identified from the panel discussions in 21 cities in seven Brazilian states.

### 7.2.2 Variables

The technicians and researchers mentioned in item 7.1, analysed the variables set and immediately identified those relevant to our study. They selected “number of bulls” as the input variable, since this variable represents a significant portion of all total expenditures of the ranchers that produce calves, being directly linked to the quality of animals that will be sold in these systems. This is also the only category that is purchased from other herd, especially in ranches with herds of genetic selection.

The products of the system that generate the main revenue from the cow-calf systems were chosen as the outputs variables. These are the “number of calves on the herd” and the “number of cull cows”. All calves produced are sold on the property and generate income. Cull cows are those that are sold, as they are no more be part of the herd production system, either by higher age or by reproductive performance lower than desired.

The variables indicated by experts need to be examined by analysts to determine whether they conform to the properties required by the DEA models. In particular, there must be a causal relationship between each input-output pair (Gomes et al., 2009). There is a clear causal relationship between the output “number of calves on the herd” and the input “number of bulls”. The same cannot be said of the relationship between the input and the output “number of cull cows”. Actually, there is no direct causal relationship between these variables; however there is a cost-benefit relationship. In the case the rancher has a great number of bulls (that represent an expense) he must earn more, either through the sale of calves or cows. Therefore, the “bulls – cull cows” ratio makes sense when using DEA to analyze cost-benefit ratios, and not just pure productive relations. This interpretation of DEA was introduced by Womer et al. (2006) and was used by Kuosmanen & Kortelainen (2007), Kuosmanen et al. (2009). Generalizations of this usage can be seen in Bougnol et al. (2010). Table 9 presents the data. It appears that the herds are of different scales of production.

### 7.2.3 Model

In this chapter we use the DEA BCC model, since there was no evidence of proportionality between inputs and outputs, and the scales of production are known to differ between the modal systems.

We chose the input oriented model, since the objective is to evaluate the performance regarding the farmer’s decision, which is based upon the purchase of bulls, on the most efficient use of the breeding animals kept on the herd, as a strategy to reduce costs.

### 7.3 Results

Table 10 shows the efficiency measurements and the multipliers based on the classic DEA BCC model. The value of $u_*$ for DMU3, DMU5 and DMU8 are negative. When the multipliers of the outputs and the $u_*$ are used to evaluate other DMUs, negative efficiencies will appear. Table 11 and 12 show the results based on the Modified DEA BCC model.

From Table 11 one can see that DMU3 and DMU5 that are efficient in the classic model are not in the modified DEA model. DMU3, Aquidauana was efficient by default, because it has the highest values for the outputs. Efficient by default DMUs may not be really efficient. Its

| DMUs | | | Breeders (input) | Calves (output) | Cull cows (output) |
|---|---|---|---|---|---|
| Municipality | State | Code | | | |
| Alvorada | TO | DMU1 | 12 | 147 | 30 |
| Amanbaí | MS | DMU2 | 15 | 143 | 40 |
| Aquidauana | MS | DMU3 | 92 | 713 | 214 |
| Bonito | MS | DMU4 | 14 | 166 | 75 |
| Brasilândia | MS | DMU5 | 31 | 290 | 178 |
| Camapuã | MS | DMU6 | 9 | 65 | 33 |
| Carlos Chagas | MG | DMU7 | 19 | 297 | 160 |
| Catalão | GO | DMU8 | 8 | 81 | 42 |
| Corumbá | MS | DMU9 | 69 | 455 | 200 |
| Itamarajú | BA | DMU10 | 4 | 44 | 18 |
| Lavras do Sul | RS | DMU11 | 5 | 58 | 30 |
| Montes Claros | MG | DMU12 | 5 | 47 | 28 |
| Niquelândia | GO | DMU13 | 4 | 35 | 18 |
| Paraíso do Tocantins | TO | DMU14 | 12 | 123 | 35 |
| Porangatu | GO | DMU15 | 5 | 46 | 23 |
| Ribas Rio Pardo | MS | DMU16 | 15 | 143 | 70 |
| Rio Verde | GO | DMU17 | 23 | 196 | 82 |
| São Gabriel d'Oeste | MS | DMU18 | 11 | 95 | 40 |
| Tupã | SP | DMU19 | 5 | 46 | 30 |
| Uberaba | MG | DMU20 | 5 | 66 | 36 |
| Uberlândia | MG | DMU21 | 2 | 20 | 10 |

Table 9. DMUs, inputs and outputs.

efficiency may be all due to mathematical distortions. The use of the modified DEA BCC model may help to identify if an efficient by default DMU is really efficient. In our case study, Aquidauana loses its efficiency when using the proposed modified DEA BCC model. We can conclude that its efficiency in the DEA BCC model is due only to a mathematical distortion. DMU5 and DMU9 present a loss of efficiency, and although not efficient by default they are very close to this situation. Further studies are needed to better explain the figures for theses DMUs.

| DMU | Multipliers | | | | Eff |
|---|---|---|---|---|---|
| | $v$ | $u_1$ | $u_2$ | $u_*$ | |
| 1 | 0.083 | 0.005 | 0.000 | 0.064 | 0.816 |
| 2 | 0.067 | 0.004 | 0.000 | 0.052 | 0.637 |
| 3 | 0.011 | 0.002 | 0.000 | -0.360 | 1.000 |
| 4 | 0.071 | 0.004 | 0.000 | 0.055 | 0.783 |
| 5 | 0.032 | 0.000 | 0.022 | -2.828 | 1.000 |
| 6 | 0.111 | 0.007 | 0.000 | 0.086 | 0.529 |
| 7 | 0.053 | 0.003 | 0.000 | 0.000 | 1.000 |
| 8 | 0.125 | 0.008 | 0.000 | 0.097 | 0.718 |
| 9 | 0.014 | 0.000 | 0.025 | -3.922 | 0.990 |
| 10 | 0.250 | 0.015 | 0.000 | 0.193 | 0.868 |

| DMU | Multipliers | | | | Eff |
|---|---|---|---|---|---|
| | $v$ | $u_1$ | $u_2$ | $u_*$ | |
| 11 | 0.200 | 0.012 | 0.000 | 0.155 | 0.866 |
| 12 | 0.200 | 0.000 | 0.023 | 0.173 | 0.808 |
| 13 | 0.250 | 0.015 | 0.000 | 0.193 | 0.730 |
| 14 | 0.083 | 0.005 | 0.000 | 0.064 | 0.693 |
| 15 | 0.200 | 0.012 | 0.000 | 0.155 | 0.719 |
| 16 | 0.067 | 0.004 | 0.000 | 0.052 | 0.637 |
| 17 | 0.043 | 0.003 | 0.000 | 0.034 | 0.557 |
| 18 | 0.091 | 0.006 | 0.000 | 0.070 | 0.600 |
| 19 | 0.200 | 0.000 | 0.023 | 0.173 | 0.853 |
| 20 | 0.200 | 0.000 | 0.023 | 0.173 | 0.989 |
| 21 | 0.500 | 0.031 | 0.000 | 0.386 | 1.000 |

Table 10. Results based on the classic DEA BCC model, input oriented (Eff = Efficiency).

| DMU | Multipliers | | | | Eff |
|---|---|---|---|---|---|
| | $v$ | $u_1$ | $u_2$ | $u_*$ | |
| 1 | 0.083 | 0.005 | 0.000 | 0.064 | 0.816 |
| 2 | 0.067 | 0.004 | 0.000 | 0.052 | 0.637 |
| 3 | 0.011 | 0.001 | 0.000 | -0.015 | 0.517 |
| 4 | 0.071 | 0.004 | 0.000 | 0.055 | 0.783 |
| 5 | 0.032 | 0.000 | 0.004 | -0.041 | 0.686 |
| 6 | 0.111 | 0.007 | 0.000 | 0.086 | 0.529 |
| 7 | 0.053 | 0.003 | 0.000 | 0.041 | 1.000 |
| 8 | 0.125 | 0.008 | 0.000 | 0.097 | 0.718 |
| 9 | 0.014 | 0.001 | 0.000 | -0.020 | 0.432 |
| 10 | 0.250 | 0.015 | 0.000 | 0.193 | 0.868 |

| DMU | Multipliers | | | | Eff |
|---|---|---|---|---|---|
| | $v$ | $u_1$ | $u_2$ | $u_*$ | |
| 11 | 0.200 | 0.012 | 0.000 | 0.155 | 0.866 |
| 12 | 0.200 | 0.000 | 0.023 | 0.173 | 0.808 |
| 13 | 0.250 | 0.015 | 0.000 | 0.193 | 0.730 |
| 14 | 0.083 | 0.005 | 0.000 | 0.064 | 0.693 |
| 15 | 0.200 | 0.012 | 0.000 | 0.155 | 0.719 |
| 16 | 0.067 | 0.004 | 0.000 | 0.515 | 0.637 |
| 17 | 0.043 | 0.003 | 0.000 | 0.336 | 0.557 |
| 18 | 0.091 | 0.006 | 0.000 | 0.070 | 0.600 |
| 19 | 0.200 | 0.000 | 0.023 | 0.173 | 0.853 |
| 20 | 0.200 | 0.000 | 0.023 | 0.173 | 0.989 |
| 21 | 0.500 | 0.000 | 0.057 | 0.433 | 1.000 |

Table 11. Results based on the Modified DEA BCC model, input oriented (Eff = Efficiency).

Other DMUs present similar results in both models. DMU7 and DMU 21 (Carlos Chagas and Uberlândia) are efficient. Uberlândia has the lowest input value and so it is also an efficient by default DMU. As this DMU is located in the decreasing returns to scale region, the modified DEA BCC model does not help to decide if it is really efficient or not.

The production systems developed Carlos Chagas is a medium scale one. The reproductive indexes of the cow matrix are very good, reflecting the good husbandry with breeders' efficient use. That is, the system showed proportionally greater production of calves for sale, with a smaller number of bulls purchased (within the range of each system).

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 2 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 3 | 0.726 | 0.563 | 0.517 | 0.716 | 0.598 | 0.343 | 1.001 | 0.523 | 0.433 | 0.412 | 0.522 | 0.371 | 0.258 | 0.589 | 0.357 | 0.563 | 0.525 | 0.468 | 0.357 | 0.632 | 0.000 |
| 4 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 5 | 0.211 | 0.253 | 0.281 | 0.588 | 0.686 | 0.324 | 1.000 | 0.507 | 0.349 | 0.253 | 0.507 | 0.456 | 0.253 | 0.264 | 0.329 | 0.507 | 0.397 | 0.345 | 0.507 | 0.659 | 0.000 |
| 6 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 7 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 8 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 9 | 0.726 | 0.562 | 0.517 | 0.715 | 0.597 | 0.343 | 1.000 | 0.523 | 0.432 | 0.411 | 0.521 | 0.370 | 0.257 | 0.589 | 0.357 | 0.562 | 0.525 | 0.468 | 0.357 | 0.631 | 0.000 |
| 10 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 11 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 12 | 0.356 | 0.360 | 0.273 | 0.669 | 0.679 | 0.512 | 1.000 | 0.703 | 0.341 | 0.727 | 0.853 | 0.808 | 0.727 | 0.403 | 0.695 | 0.587 | 0.442 | 0.491 | 0.853 | 0.989 | 1.000 |
| 13 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 14 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 15 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 16 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 17 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 18 | 0.816 | 0.637 | 0.484 | 0.783 | 0.599 | 0.529 | 1.000 | 0.718 | 0.416 | 0.868 | 0.866 | 0.731 | 0.730 | 0.693 | 0.719 | 0.637 | 0.557 | 0.600 | 0.719 | 0.965 | 1.000 |
| 19 | 0.356 | 0.360 | 0.273 | 0.669 | 0.679 | 0.512 | 1.000 | 0.703 | 0.341 | 0.727 | 0.853 | 0.808 | 0.727 | 0.403 | 0.695 | 0.587 | 0.442 | 0.491 | 0.853 | 0.989 | 1.000 |
| 20 | 0.356 | 0.360 | 0.273 | 0.669 | 0.679 | 0.512 | 1.000 | 0.703 | 0.341 | 0.727 | 0.853 | 0.808 | 0.727 | 0.403 | 0.695 | 0.587 | 0.442 | 0.491 | 0.853 | 0.989 | 1.000 |
| 21 | 0.356 | 0.360 | 0.273 | 0.669 | 0.679 | 0.512 | 1.000 | 0.703 | 0.341 | 0.727 | 0.853 | 0.808 | 0.727 | 0.403 | 0.695 | 0.587 | 0.442 | 0.491 | 0.853 | 0.989 | 1.000 |

Table 12. Cross evaluation matrix for the Modified DEA BCC model from the case study.

It is interesting to point out that in the study performed by Gomes et al. (2011), based on a DEA BCC model with weights restrictions and with the same dataset, Carlos Chagas and Uberlândia were the production systems that were more referenced as benchmarks (76% of non-zero contributions in the formation of the targets of the inefficient DMUs). The authors state that these modal systems can serve as a reference for the others, when assessing the performance of the cattle farmer's decision in relation to the production criteria. However, it is important to stress again that the DMU Uberlândia is efficient by default, and there must be some caution when indicating it as a benchmark.

One other advantage of the modified DEA BCC model is to allow the use of Cross Evaluation for the variable returns to scale situation as shown in Table 12. This matrix leads to the Cross Evaluation ranking shown in Table 13.

In Table 13 we can see that DMU3, Aquidauana, which was efficient with the original BCC model, has now one of the lowest efficiency measures when using the modified DEA BCC model and the cross evaluation technique. We can conclude that this DMU is a maverick, i.e. it is a false positive.

| DMU | Municipality | Average Cross Efficiency |
|---|---|---|
| DMU7 | Carlos Chagas | 1.0000 |
| DMU20 | Uberaba | 0.9230 |
| DMU21 | Uberlândia | 0.8571 |
| DMU11 | Lavras do Sul | 0.8139 |
| DMU10 | Itamarajú | 0.7685 |
| DMU4 | Bonito | 0.7455 |
| DMU19 | Tupã | 0.7001 |
| DMU12 | Montes Claros | 0.6985 |
| DMU1 | Alvorada | 0.6910 |
| DMU8 | Catalão | 0.6865 |
| DMU13 | Niquelândia | 0.6617 |
| DMU15 | Porangatu | 0.6614 |
| DMU5 | Brasilândia | 0.6182 |
| DMU16 | Ribas Rio Pardo | 0.6138 |
| DMU14 | Paraíso do Tocantins | 0.6076 |
| DMU2 | Amanbaí | 0.5586 |
| DMU18 | São Gabriel d'Oeste | 0.5547 |
| DMU17 | Rio Verde | 0.5241 |
| DMU6 | Camapuã | 0.4983 |
| DMU3 | Aquidauana | 0.4373 |
| DMU9 | Corumbá | 0.4000 |

Table 13. Final ranking using the average cross evaluation index with the modified BCC model.

## 8. Concluding remarks

In this paper it was shown that the input orientated DEA BCC model can generate negative efficiencies that are usually hidden in the model. With the help of an example of one input and one output, the condition for the possible occurrence of negative efficiencies was shown. Furthermore it was shown that a small intuitive change in the BCC multipliers model avoids that situation.

The inclusion of a new set of restrictions in the BCC multipliers model generates an important modification in the dual of the envelope model. The new dual model, with a new set of variables, may change the efficient frontier.

Two numerical examples were presented in this article. The first example has two variables, one input and one output. It was geometrically shown that the modified model causes a change in the efficient frontier. These changes were interpreted by the introduction of a non-observed DMU. In this first example, the DMU causing negative efficiency in other DMUs has a reduction of its efficiency in the modified model. This reduction did not occur in the second example (two inputs and one output). This happened due to the optimum multipliers set multiplicity.

A real case study was carried out with the Modified DEA BCC model here proposed, regarding the evaluation of 21 Brazilian beef cattle modal production systems that performed only the cow-calf phase. The DMUs that could cause negative efficiencies were identified, and the new efficiencies were calculated. We could observe that most of the evaluated systems work with increasing returns to scale and lose efficiency. That is, they could produce more and adjusted the scale if they invest in balancing the number of bulls in the herds. The production systems in the municipalities of Carlos Chagas and Uberlândia are examples of this balance between investment in higher production of bulls and calves for sale, within ranges consistent with the income of ranchers.

It is very important to observe that the negative efficiencies mentioned in this paper apply neither to the CCR model nor to the output orientated BCC model. The latter possesses a restriction in the multipliers model that is of the “greater than” or “equal to” type, guaranteeing the non negativity of the efficiency measures.

Finally, another situation susceptible to the appearance of negative efficiencies in DEA context is when extending the MCDEA model (Li & Reeves, 1999) to the variable returns to scale assumption, especially if the MCDEA-TRIMAP efficiency is used (Soares de Mello et al., 2009). Future studies should verify the conditions in which the phenomenon of negative efficiencies occurs in the MCDEA model, and how to avoid them.

## 9. References

Angulo-Meza, L.; Biondi Neto, L.; Soares de Mello, J. C. C. B. & Gomes, E. G. (2005). ISYDS - Integrated System for Decision Support (SIAD Sistema Integrado de Apoio a Decisão): A Software Package for Data Envelopment Analysis Model. *Pesquisa Operacional,* Vol. 25, No. 3, pp. 493-503

Angulo-Meza, L.; Soares de Mello, J. C. C. B.; Gomes, E. G. & Biondi Neto, L. (2004). Eficiências negativas em modelos DEA-BCC: como surgem e como evitá-las. *VII Simpósio de Pesquisa Operacional e Logística da Marinha - SPOLM*, Niterói.

Banker, R. D.; Charnes, A. & Cooper, W. W. (1984). Some models for estimating technical scale inefficiencies in data envelopment analysis. *Management Science,* Vol. 30, No. 9, pp. 1078-1092, 0025-1909

Bougnol, M. L.; Dulá, J. H.; Estellita Lins, M. P. & Moreira da Silva, A. C. (2010). Enhancing standard performance practices with DEA. *Omega,* Vol. 38, No. 1-2, pp. 33-45

Centro de Estudos Avançados em Economia Aplicada (2010). *Metodologia do índice de preços dos insumos utilizados na produção pecuária brasileira*. Available at: <http://www.cepea.esalq.usp.br/boi/metodologiacna.pdf >. Accessed: 24 March 2010.

Charnes, A.; Cooper, W. W. & Rhodes, E. (1978). Measuring the efficiency of decision-making units. *European Journal of Operational Research,* Vol. 2, pp. 429-444, 0377-2217.

Cooper, W. W.; Seiford, L. M. & Zhu, J. (2004). *Handbook on Data Envelopment Analysis,* Kluwer Academic Publishers, Boston.

Doyle, J. & Green, R. H. (1994). Efficiency and cross-efficiency in DEA derivations, meanings and uses. *Journal of the Operational Research Society,* Vol. 45, pp. 567-578

Duzakin, E. & Duzakin, H. (2007). Measuring the performance of manufacturing firms with super slacks based model of data envelopment analysis: An application of 500 major industrial enterprises in Turkey. *European Journal of Operational Research,* Vol. 182, No. 3, pp. 1412-1432

Farrell, M. J. (1957). The Measurement of Productive Efficiency. *Journal of Royal Statistical Society Series A,* Vol. 120, No. 3, pp. 253-281

Gomes, E. G.; Abreu, U. G. P. d.; Soares de Mello, J. C. C. B.; Carvalho, T. B. & Zen, S. (2011). DEA performance evaluation of livestock systems in Brazil *International Conference on Data Envelopment Analysis and Its Applications to Management.*, Lima.

Gomes, E. G.; Soares de Mello, J. C. C. B.; Souza, G. d. S.; Angulo-Meza, L. & Mangabeira, J. A. d. C. (2009). Efficiency and sustainability assessment for a group of farmers in the Brazilian Amazon. *Annals of Operations Research,* Vol., pp.

Iqbal Ali, A. & Seiford, L. M. (1990). Translation invariance in data envelopment analysis. *Operations Research Letters,* Vol. 9, No. 6, pp. 403-405

Kuosmanen, T.; Bijsterbosch, N. & Dellink, R. (2009). Environmental cost-benefit analysis of alternative timing strategies in greenhouse gas abatement: A data envelopment analysis approach. *Ecological Economics,* Vol. 68, No. 6, pp. 1633-1642

Kuosmanen, T. & Kortelainen, M. (2007). Valuing environmental factors in cost-benefit analysis using data envelopment analysis. *Ecological Economics,* Vol. 62, No. 1, pp. 56-65

Li, X.-B. & Reeves, G. R. (1999). A multiple criteria approach to data envelopment analysis. *European Journal of Operational Research,* Vol. 115, No. 3, pp. 507-517

Lins, M. P. E. & Angulo-Meza, L. (2000). *Análise Envoltória de Dados e perspectivas de integração no ambiente de Apoio à Decisão,* Editora da COPPE/UFRJ, Rio de Janeiro.

Lins, M. P. E.; Angulo-Meza, L. & Moreira da Silva, A. C. (2004). A multi-objective approach to determine alternative targets in data envelopment analysis. *Journal of the Operational Research Society,* Vol. 55, No. 10, pp. 1090–1101

Nacif, F. B.; Soares de Mello, J. C. C. B. & Angulo-Meza, L. (2009). Choosing weights in optimal solutions for DEA-BCC models by means of a n-dimensional smooth frontier. *Pesquisa Operacional,* Vol. 29, No. 3, pp. 623-642

Pastor, J. T. (1996). Translation invariance in data envelopment analysis: A generalization. *Annals of Operations Research,* Vol. 66, pp. 93-102

Plaxico, J. S. & Tweeten, L. G. (1963). Representative farms for policy and projection research. *Journal of Farm Economics,* Vol. 45, pp. 1458-1465

Portela, M. C. A. S.; Thanassoulis, E. & Simpson, G. (2004). Negative data in DEA: A directional distance approach applied to bank branches. *Journal of the Operational Research Society,* Vol. 55, No. 10, pp. 1111-1121

Quariguasi Frota Neto, J. & Angulo-Meza, L. (2007). Alternative targets for data envelopment analysis through multi-objective linear programming: Rio de Janeiro Odontological Public Health System Case Study. *Journal of the Operational Research Society,* Vol. 58, pp. 865–873

Rosen; Schaffnit, C. & Paradi, J. C. (1998). Marginal rates and two dimensional level curves in DEA. *Journal of Productivity Analysis,* Vol. 9, No. 3, pp. 205-232

Sexton, T. R.; Silkman, R. H. & Logan, A. J. (1986). Data Envelopment Analysis: Critique and extensions. IN Silkman, H. (Ed.) *Measuring efficiency: An assessment of data envelopment analysis.* Jossey-Bass Editor, San Francisco.

Sharp, J. A.; Meng, W. & Liu, W. (2007). A modified slacks-based measure model for data envelopment analysis with 'natural' negative outputs and inputs. *Journal of the Operational Research Society,* Vol. 58, No. 12, pp. 1672-1677

Soares de Mello, J. C. C. B.; Climaco, J. C. N. & Angulo-Meza, L. (2009). Efficiency evaluation of a small number of DMUs: an approach based on Li and Reeves's model. . *Pesquisa Operacional,* Vol. 29, Issue 1, pp. 97-110.

Soares de Mello, J. C. C. B.; Lins, M. P. E. & Gomes, E. G. (2002). Construction of a smoothed DEA frontier. *Pesquisa Operacional,* Vol. 28, No. 2, pp. 183-201

Souza, G. d. S. (2006). Significância de Efeitos Técnicos na Eficiencia de Produção da Pesquisa Agropecuária Brasileira. *Revista Brasileira de Economia - RBE,* Vol. 60, No. 1, pp. 94-117

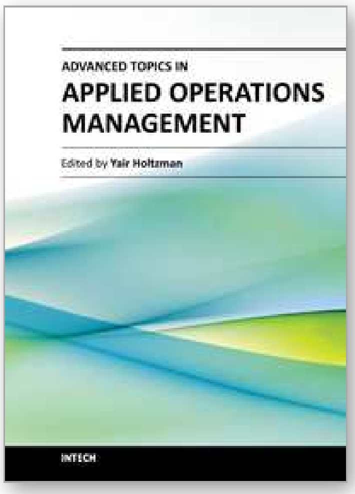
Sueyoshi, T. (2004). Mixed integer programming approach of extended DEA-discriminant analysis. *European Journal of Operational Research,* Vol. 152, No. 1, pp. 45-55

Thanassoulis, E. & Allen, R. (1988). Simulating Weight restrictions in Data Envelopment Analysis by means of unobserved DMUs. *Management Science,* Vol. 44, pp. 586-594.

Thrall, R. M. (1996). The lack of invariance of optimal dual solutions under translation. *Annals of Operations Research,* Vol. 66, pp. 103-108

Womer, N. K.; Bougnol, M. L.; Dulá, J. H. & Retzlaff-Roberts, D. (2006). Benefit-cost analysis using data envelopment analysis. *Annals of Operations Research,* Vol. 145, No. 1, pp. 229-250

Wu, J.; Liang, L. & Chen, Y. (2009). DEA game cross-efficiency approach to Olympic rankings. *Omega,* Vol. 37, No. 4, pp. 909-918

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The chapters in Advanced Topics in Applied Operations Management creatively demonstrate a valuable connection among operations strategy, operations management, operations research, and various departments, systems, and practices throughout an organization. The authors show how mathematical tools and process improvements can be applied effectively in unique measures to other functions. The book provides examples that illustrate the challenges confronting firms competing in today's demanding environment bridging the gap between theory and practice by analyzing real situations.



**InTech Europe**

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
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Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821