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Multimedia Approach in Teaching Mathematics – Examples of Interactive Lessons from Mathematical Analysis and Geometry

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1. Introduction

Research on multimedia approach efficiency was carried out using lessons about the definite integral and isometric transformations (line and point reflection, translation and rotation), since these areas are among the basic ones in the fields of mathematical analysis and geometry. These topics are also important because of their presence in the mathematics programmes in the great majority of high schools and faculties, both directly and through their multipurpose character. Consequently, proper approach in presenting these topics is one of the most important segments of teaching mathematics on all educational levels.

In teaching mathematics, it is remarkably important to avoid so-called 'knowledge/ information adoption' as the only way of work. Students often solve problems mechanically, by following the algorithm steps without real awareness of their actual meaning. For example, in case of the definite integral, one of the common problems is that students calculate its value by following steps of the algorithm, without real understanding of its definition, the meaning of upper and lower limit or relation between the definite integral and the size of an area or a volume etc. In case of isometric transformations, it was noticed that students learned what are line and point reflection, translation and rotation, but the problem appeared when they have been faced to the realistic task, where they were supposed to use the adopted knowledge. One of the most important aims of the modern approach in teaching mathematics is to combine information adoption with so-called 'knowledge transfer', i.e. learning through defining the facts and explaining their interrelations.

Mathematics teachers show great interest in visualisation of the mathematical terms and emphasize that visualised lectures are of the great help in abstract thinking in mathematics (Bishop, 1989; Tall, 1986) believes that it is of the major importance to connect the existing pictures that students have on certain terms in order to develop them further and to enable students to accept the further knowledge. Therefore, in teaching mathematics it is necessary to combine the picture method and the definition method in order to improve the existing knowledge and to enlarge it with the new facts, which is one of the points of the cognitive theory of multimedia learning (Mayer, 2001, 2005).

Nowadays, use of different kinds of multimedia is largely included in the education because it allows the wider spectrum of possibilities in teaching and learning. Visualisation is very useful in the process of explaining mathematical ideas, abstract terms, theorems, problems etc.

Experience in work with students showed that they are highly interested in modern methods in learning which include all kinds of multimedia, such as educational software, internet, etc.

Modern methods in multimedia approach to learning include the whole range of different possibilities applicable in mathematics lectures for different levels of education and with different levels of interactivity (Deliyiannis et al. 2008a, 2008b; D. Đ. Herceg, 2009; Milovanovic, 2005, 2009; Milovanovic et al., 2011; Takači, et al., 2003, 2004, 2006, 2008). The authors usually work on suggestions on using different kinds of software in education, especially in the field of mathematics: geometry, algebra, numerical analyses etc. (Deliyiannis et al., 2008a, 2008b; D. Đ. Herceg, 2009; Milovanovic, 2005, 2009; Milovanovic et al., 2011), as well as the definite integrals and isometric transformations.

All the above-mentioned resulted in an idea of making applicative software which would be helpful in modern and more interesting approach to the field of teaching mathematics and rising the students' knowledge from the scope of definite integrals and isometric transformation to a higher level. The aim of our research was to recognize the importance of multimedia in the teaching process as well as to examine the students' reaction to this way of learning and teaching. Therefore, we have developed experimental software with multimedia lessons about the definite integral and isometric transformations and tested them in class in order to see how they would affect teaching process and results.

2. Multimedia presentation of given problems from the fields of integrals and isometric transformations

We would like to emphasize the importance of using computers, i.e. multimedia software in teaching and learning in the both areas, because visual presentation offers much more possibilities. Beside that, using multimedia lessons on isometric transformations enables students to actually see not only the final solution, but also the „movements" that have led to it.

2.1 Multimedia presentation of the area calculation

The problem of area calculation in the filed of integration calculations lead us directly to the definition of the definite integral. The basis for the defining and calculation of the definite integrals was made by Archimedes[1] and his quadrature of the parabola. Because of that, we decided to use modern, multimedia approach in explaining Archimedes' quadrature of the parabola to the students. In order to use PC as a teaching aid, we were led by suggestions of Tall (Tall, 1991), who emphasized the importance of PC in the teaching because of its great possibilities in the scope of visual presentations.

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[1] Archimedes of Syracuse (Greek: Ἀρχιμήδης; c. 287 BC – c. 212 BC) was a Greek mathematician, physicist, engineer, inventor, and astronomer.

The quadrature of the parabola problem is formulated as follows: *For any given parabola $y = x^2$ and rectangle with nodes $A(0,0)$, $B(a, 0)$, $C(a, a^2)$ and $D(0, a^2)$, $(a > 0)$ the parabola divides the rectangle in two zones of which the area of one is twice bigger than the area of the other one.*

Given problem is show on Figure 1[2]. The area of the rectangle is:

$$P = a \cdot a^2 = a^3 \tag{1}$$

If we mark the zone under the parabola – limited by sides AB and BC of the rectangle and the arc AC – with S, and the other one with P, our next task is to prove the following equation:

$$S : (P - S) = 1 : 2 \text{ , i.e. } S = \frac{1}{3}P = \frac{1}{3}a^3 \tag{2}$$
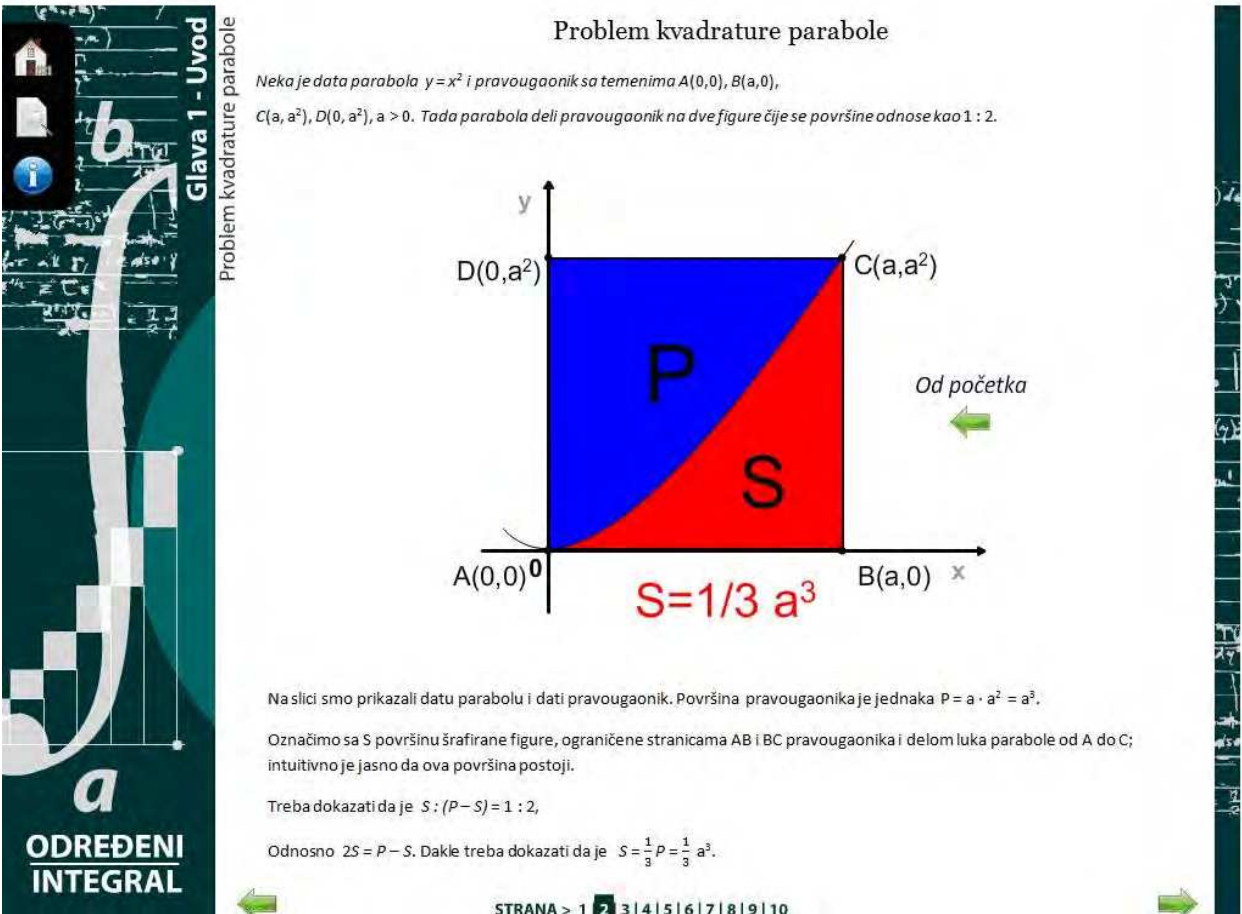
Fig. 1. Illustration of given problem taken from the multimedia lesson about the definite integrals

The next step is to divide the interval $[0, a]$ of the *Ox* axis in *n* equal parts of length *a/n*, where *n* is a natural number, which should be shown to the students by animation (step-by-step), as shown in Figure 2a. Within each of these intervals, we construct two rectangles:

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[2] Figure 1 is taken from the multimedia lesson about the definite integrals.

'circumscribed' one, with upper right vertex on the parabola (the animation of this step is shown on Figure 2b), and 'inscribed' one, with upper left vertex on the parabola (shown on Figure 2d). It is obvious that the first part have no inscribed rectangle. The heights of these rectangles are shown on Figure 2b, and their areas are as follows:

$$\frac{a}{n}\cdot\left(\frac{a}{n}\right)^2,\frac{a}{n}\cdot\left(2\frac{a}{n}\right)^2,...,\frac{a}{n}\cdot\left((n-1)\frac{a}{n}\right)^2,\frac{a}{n}\cdot\left(n\frac{a}{n}\right)^2.$$

(Part of this animation is shown on Figure 2c.). Area of each inscribed rectangle is difference between the area of adjoining circumscribed rectangle and the area of 'added' rectangle (part of this animation is shown on Figures 2d and 2e).

It is obvious that:

$$P_U < S < P_O \tag{3}$$

Therefore: $\frac{a^3}{3}-\frac{a^3}{2n}+\frac{a^3}{6n^2}<S<\frac{a^3}{3}+\frac{a^3}{2n}+\frac{a^3}{6n^2}$, that is: $-\frac{a^3}{2n}+\frac{a^3}{6n^2}<S-\frac{a^3}{3}<\frac{a^3}{2n}+\frac{a^3}{6n^2}$.

These inequalities are correct for any given natural number n.

Since $\lim_{n\to\infty}(-\frac{a^3}{2n}+\frac{a^3}{6n^2})=\lim_{n\to\infty}(\frac{a^3}{2n}+\frac{a^3}{6n^2})=0$, we can conclude that:

$$S=\frac{1}{3}P=\frac{1}{3}a^3 \tag{4}$$

This was the solution of the given problem via numerical method, but we can offer much more by using the multimedia lessons. In animation shown on Figure 3, students can clearly see that with increasing of n, i.e. the number of circumscribed and inscribed rectangles, these areas will get closer and closer, until they, according to our intuition and visual perception, both become equal to the area S.

Led by the similar idea as in previous example, we will try to calculate the area of the curvilinear trapezium (students will see it in animation, as shown in Figure 4a and b, etc.) formed by the graph of the function $y=f(x), x\in[a,b]$, the abscissa's segment $[a,b]$ and the two segments of the lines $x=a$ and $x=b$ making the figure closed (Figure 4a).

If the values $x_0,x_1,...,x_{n-1},x_n$ define points on x axis as follows:

$$a=x_0<x_1<...<x_{n-1}<x_n=b$$

these points divide interval [a, b] into n sub-intervals:

$$[x_0,x_1],[x_1,x_2],...,[x_{n-1},x_n].$$

Therefore, we can name the $(n+1)$-plet $(x_0,x_1,...,x_{n-1},x_n)$ as *division of interval* $[a,b]$. For simplification, we will mark it as $\Pi=(x_0,x_1,...,x_{n-1},x_n)$.

If we choose any of these sub-intervals (Figure 4b), for example (x_{i-1},x_i), and if ξ_i is arbitrary value within that sub-interval, the area of the rectangle whose basis is sub-interval $[x_{i-1},x_i]$ and height is $f(\xi_i)$. can be calculated as:
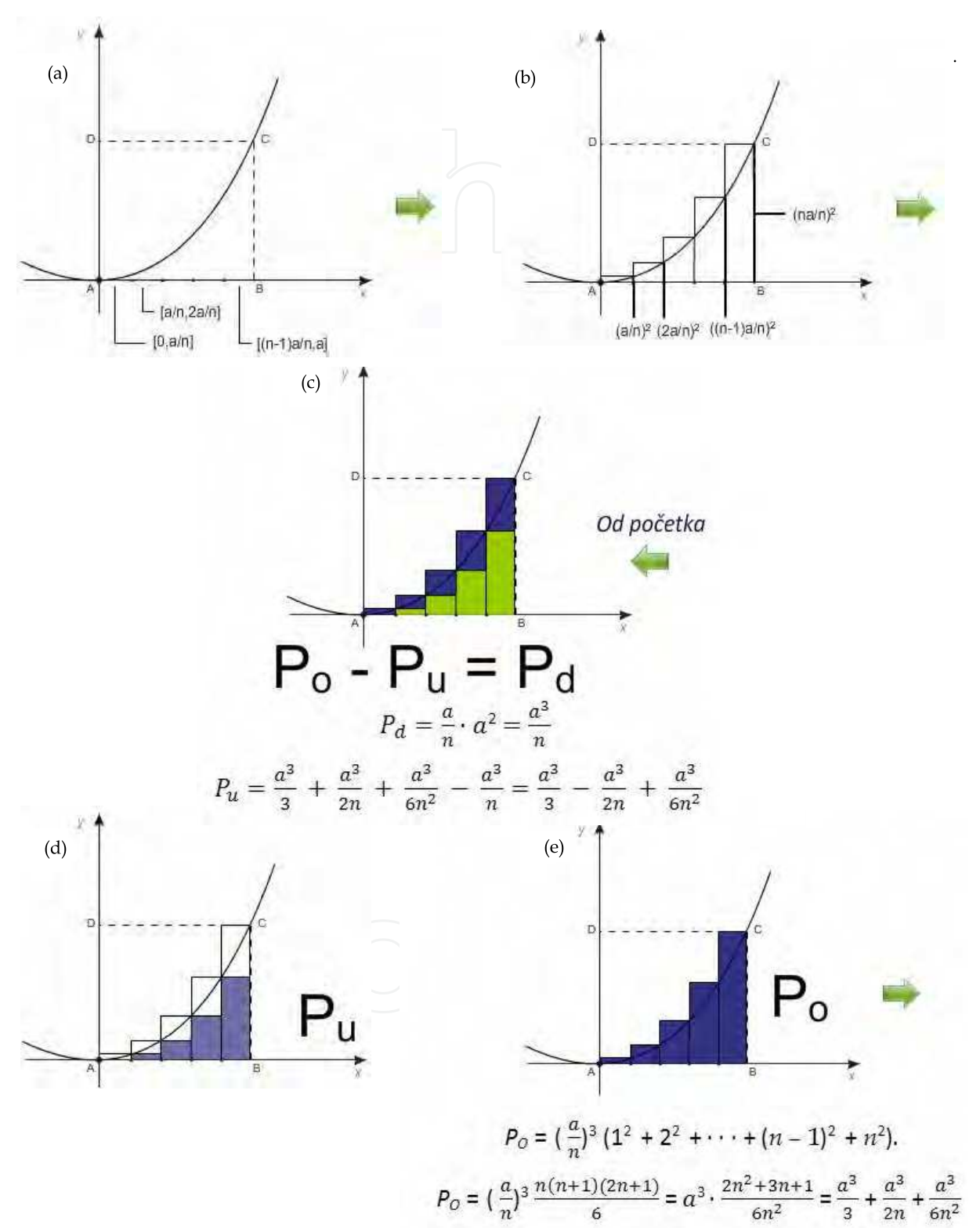
$$P_O - P_u = P_d$$

$$P_d = \frac{a}{n} \cdot a^2 = \frac{a^3}{n}$$

$$P_u = \frac{a^3}{3} + \frac{a^3}{2n} + \frac{a^3}{6n^2} - \frac{a^3}{n} = \frac{a^3}{3} - \frac{a^3}{2n} + \frac{a^3}{6n^2}$$

$$P_O = (\frac{a}{n})^3 (1^2 + 2^2 + \cdots + (n-1)^2 + n^2).$$

$$P_O = (\frac{a}{n})^3 \frac{n(n+1)(2n+1)}{6} = a^3 \cdot \frac{2n^2 + 3n + 1}{6n^2} = \frac{a^3}{3} + \frac{a^3}{2n} + \frac{a^3}{6n^2}$$

Fig. 2. Illustrations of the quadrature of the parabola problem taken from the multimedia lesson about the definite integrals
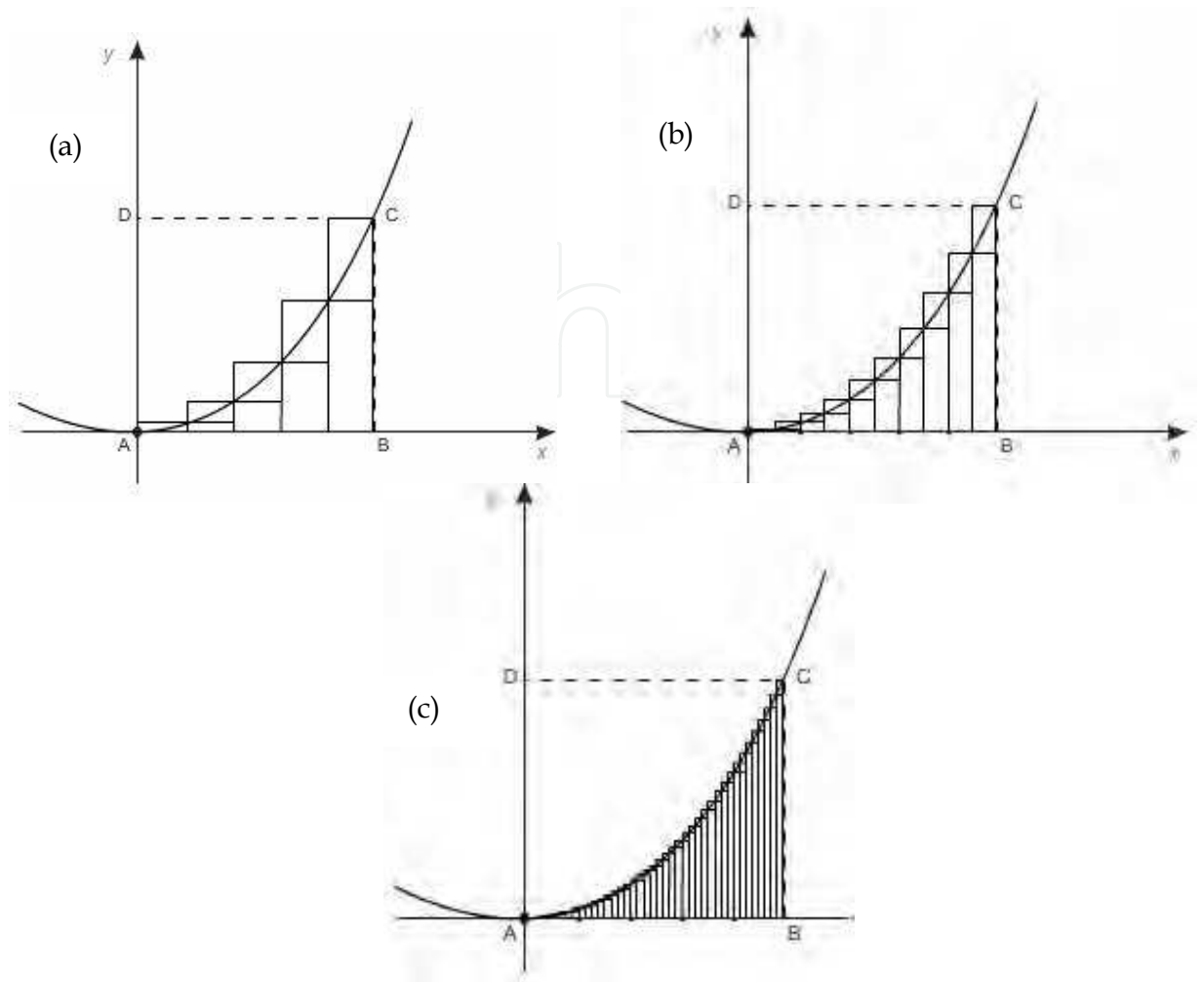
Fig. 3. Illustration of the quadrature of the parabola problem taken from the multimedia lesson (step-by-step)
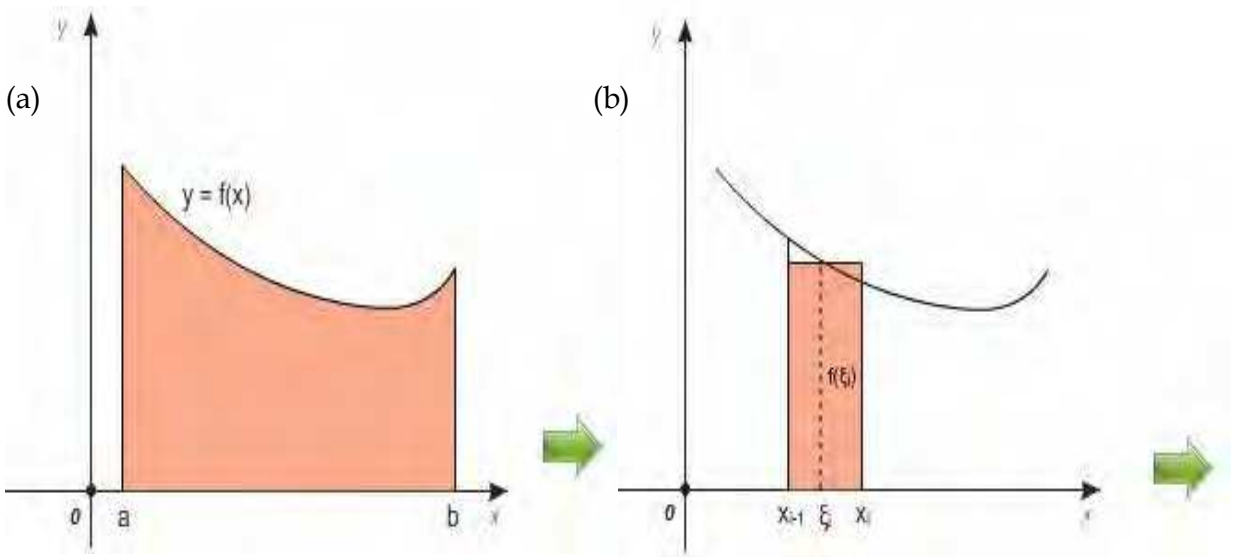
Fig. 4. The first part of solution of the quadrature of the parabola problem taken from the multimedia lesson (step-by-step)

$$P_i = f(\xi_i)(x_i - x_{i-1}) \tag{5}$$

If we do the same with every sub-interval $[x_{i-1}, x_i]$, $i = 1, 2, ..., n$, we will get the series of rectangles – figure S – with total area:

$$P(S) = \sum_{i=1}^{n} P_i = f(\xi_i)(x_i - x_{i-1}) \tag{6}$$

For a given curvilinear trapezium – i.e. for given interval $[a, b]$ and given function $f(x)$ – the shape of figure S depends on division $\Pi = (x_0, x_1, ..., x_{n-1}, x_n)$ and on choice of values $\xi_i \in [x_{i-1}, x_i]$, $i = 1, 2, ..., n$. Let us mark this n-plet of choices as $\xi = (\xi_1, \xi_2 ..., \xi_n)$. If all the sub-intervals $[x_{i-1}, x_i]$, $i = 1, 2, ..., n$, are 'small', shape of figure S will be 'very' similar to the curvilinear trapezium F (which are shown on Figures 5 and 6a).

If we mark value of $\Delta x_i = x_i - x_{i-1}, i - 1, 2, ... n$, than set $\{\Delta x_1, \Delta x_2, ..., \Delta x_n\}$ is finite set of positive numbers, and consequently has the largest element, which we will mark as d:

$$d = d(\Pi) = \max\{\Delta x_1, \Delta x_2, ..., \Delta x_n\}$$

If the value of d is small enough natural number, it means that sub-intervals are smaller and the division Π is 'fine'. If we introduce new breaking points, d gets smaller and smaller so the division gets finer. Consequently – and according to our intuition and multimedia presentation – figure S will get more and more similar to the curvilinear trapezium, so we can conclude that following definition of the area of the curvilinear trapezium F is valid:

Definition 1: Real number S is the area of the curvilinear trapezium F if for every $\varepsilon > 0$, there exists $\delta > 0$, such that for every division Π for which $d(\Pi) < \delta$ and for any chosen set of values $\xi = (\xi_1, \xi_2 ..., \xi_n)$ in correspondent sub-intervals:
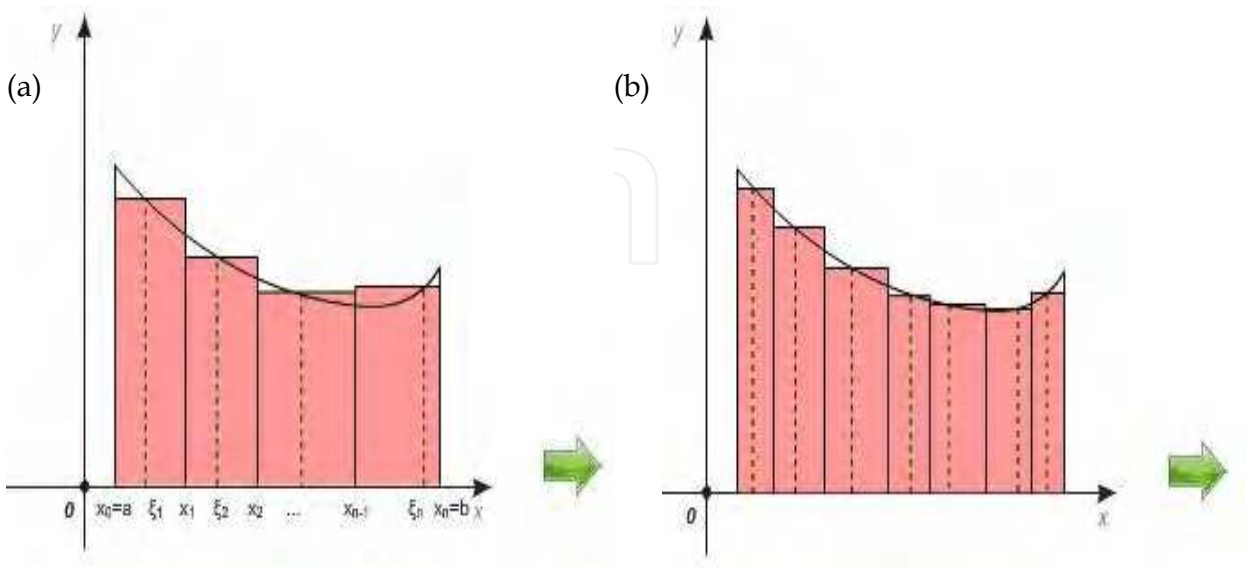

Fig. 5. The second part of solution of the quadrature of the parabola problem taken from the multimedia lesson (step-by-step)
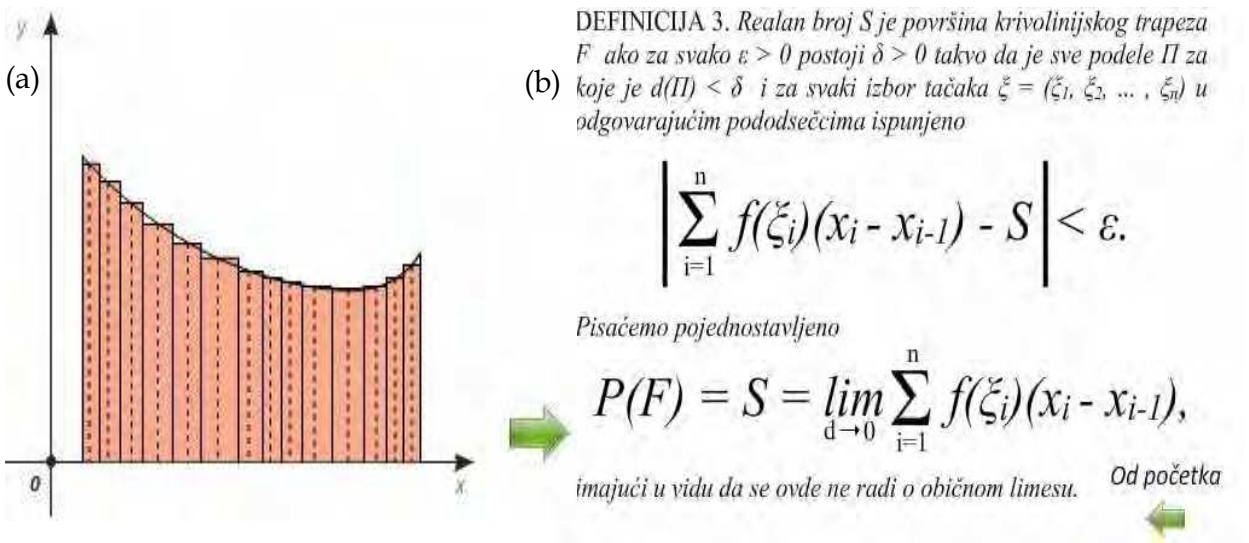
Fig. 6. *Definition 1*: Area of the curvilinear trapezium F – visualization method

$$\left| \sum_{i=1}^{n} f(\xi_i)(x_i - x_{i-1}) - S \right| < \varepsilon \tag{7}$$

Or it is simplified as:

$$P(F) = S = \lim_{d \to 0} \sum_{i=1}^{n} f(\xi_i)(x_i - x_{i-1}), \text{ i.e.} \tag{8}$$

Definition 2: Let the real-valued function f be defined on interval $[a,b]$. The real number I is definite integral of the function f on the interval $[a,b]$ if for every $\varepsilon > 0$, there exists $\delta > 0$, such that for every division $\Pi = (x_0, x_1, ..., x_{n-1}, x_n)$, $a = x_0 < x_1 < ... < x_{n-1} < x_n = b$ for which $d = d(\Pi) = \max\{\Delta x_1, \Delta x_2, ..., \Delta x_n\} < \delta$ and for any chosen set of values $\xi = (\xi_1, \xi_2 ... \xi_n)$, $\xi_i \in [x_{i-1}, x_i]$, $i = 1, 2, ..., n$ (animation, Figure 7 shows further development step-by-step).
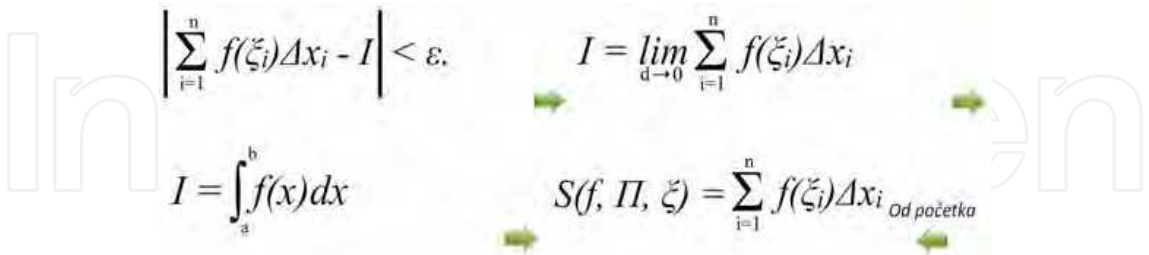
Fig. 7. *Definition 2:* Area of the curvilinear trapezium F – visualization method

With numerous visual presentations, animations, illustrations and examples we can also introduce and explain *integrability*, *integral sum*, *integrand*, *limits of integration*, *Newton-Leibniz formula*, *applications of integrals,* etc.

Example: Determining the area of plane figure.
Task: Determine the area of the figure in the *xOy* plane bounded by the curves *x – y = 0* and *x – y³ = 0*.

Solution: Animation shows the graphs of given curves (Figure 8a) and their intersection points $A(1, 1)$, $O(0, 0)$ and $B(-1, 1)$ (with numerical and graphic presentation). We can see that given figure consists of two identical parts. The next step in the animation is solving the given problem step-by-step. The final result is illustrated on Figure 8b.
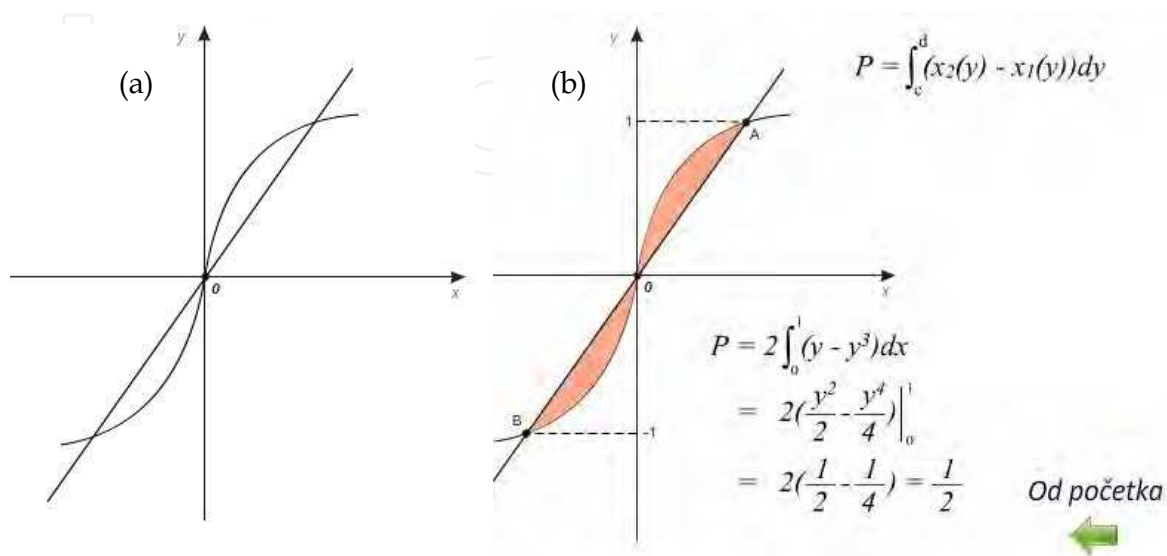
Fig. 8. Animation parts which represents the graphs of the given task and solution in determining the area of a plane figure

In a similar way as shown for determining the area of a given figure, we used multimedia animations to explain application of the determined integrals for calculus of volumes of solids, as well as volumes of solids of revolution obtained by revolving a plane figure around Ox or Oy axis.

Example: Determining the volume of body by revolving.
Task: Determine the volume of a right circular cone with altitude h and base radius r.
Solution: The cone is generated by revolving the right-angled triangle OAB around the Ox-axis (Figure 9a), which can be clearly shown by using animation (Figures 9b, and 9c). Animation parts which represents the given task and the triangle revolution.

Numerical solution of given problem is also shown step-by-step, by using animation.

Slant height of the cone is defined as line:

$$y = x \cdot tg\alpha = \frac{r}{h} \cdot x$$

Therefore, according to the formula for calculus of volume:

$$V = \pi \int_0^h \left(\frac{r}{h} \cdot x\right)^2 dx = \frac{\pi \cdot r^2}{h^2} \cdot \frac{x^3}{3}\Big|_0^h = \frac{\pi \cdot h \cdot r^2}{3}.$$
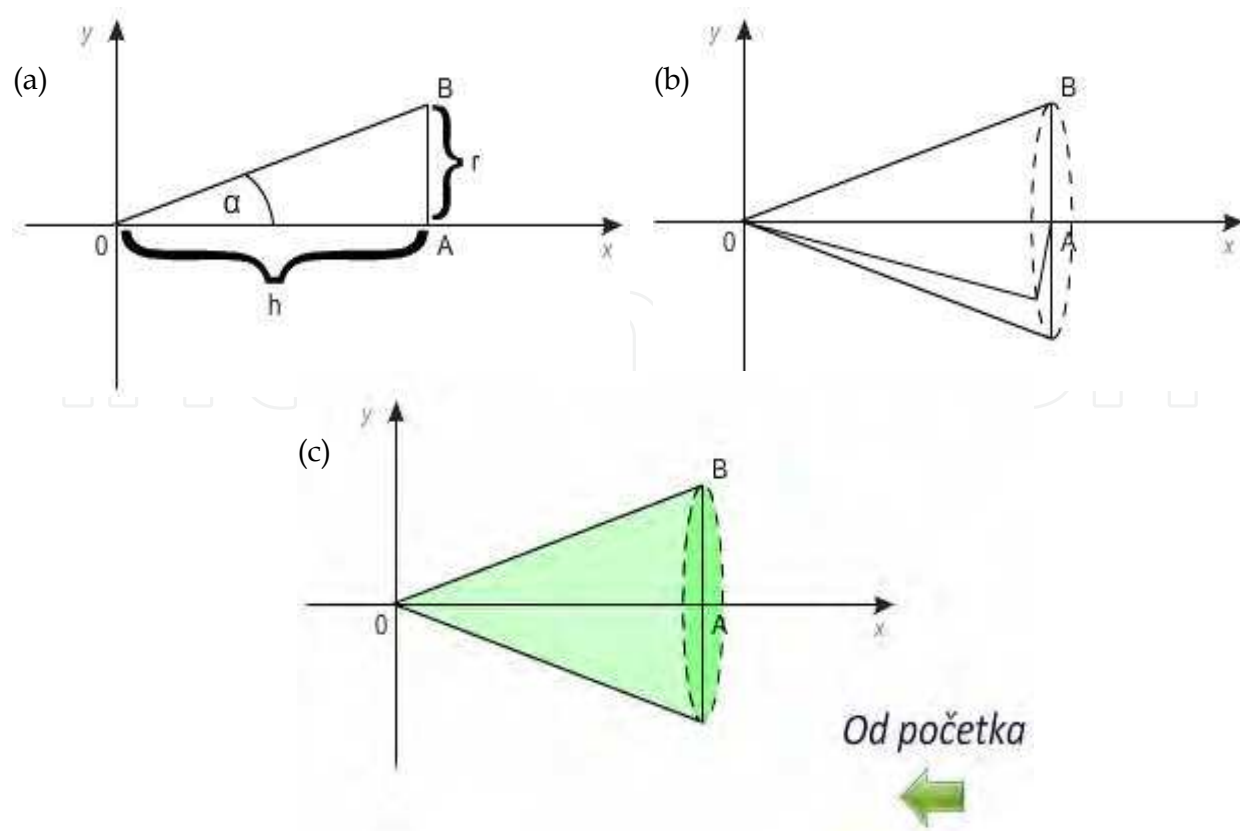
Fig. 9. Animation parts which represents the graphs of the given task and solution in determine the volume of a right circular cone with altitude *h* and base radius *r*

2.2 Assorted examples and problems from multimedia lectures on isometric transformations

Our lectures on isometric transformations (homepage shown on Figure 10), consist of four units: line and point reflection, translation and rotation. Every transformation is presented by the following chapters:

1. Basics
2. Examples
3. Some characteristics
4. Exercises
5. Problems
6. Examples from everyday life

Since the field of isometric transformations is very broad, we have conducted the research in only one area – line reflection. Multimedia lessons in line reflection are presented here by characteristic examples which have enabled us to use different, multimedia approach than in classical lectures.

Great advantage of multimedia lessons is particularly evident in chapter *Basics*, because the definitions of line reflection, axis of symmetry etc. are not only 'given' (written and drawn), but also illustrated by numerous animations which show 'movements', i.e. isometric transformation. We have also paid special attention to enabling students to find out the solutions by themselves.

Fig. 10. Homepage of animation about isometric transformations

Example: Basic idea of this example is to help students to see, comprehend and implement the line reflection in different cases before giving them the exact definition. In the first task (Figure 11a), students were asked to recognize the common characteristic of given figures and to find which two of them do not belong in the group. After that, the solution was

offered (for all the figures except the third and the last one) in which it was shown that there is at least one line along which we can fold the paper and every point from one side would fall on corresponding point on the other side (Figure 11b).
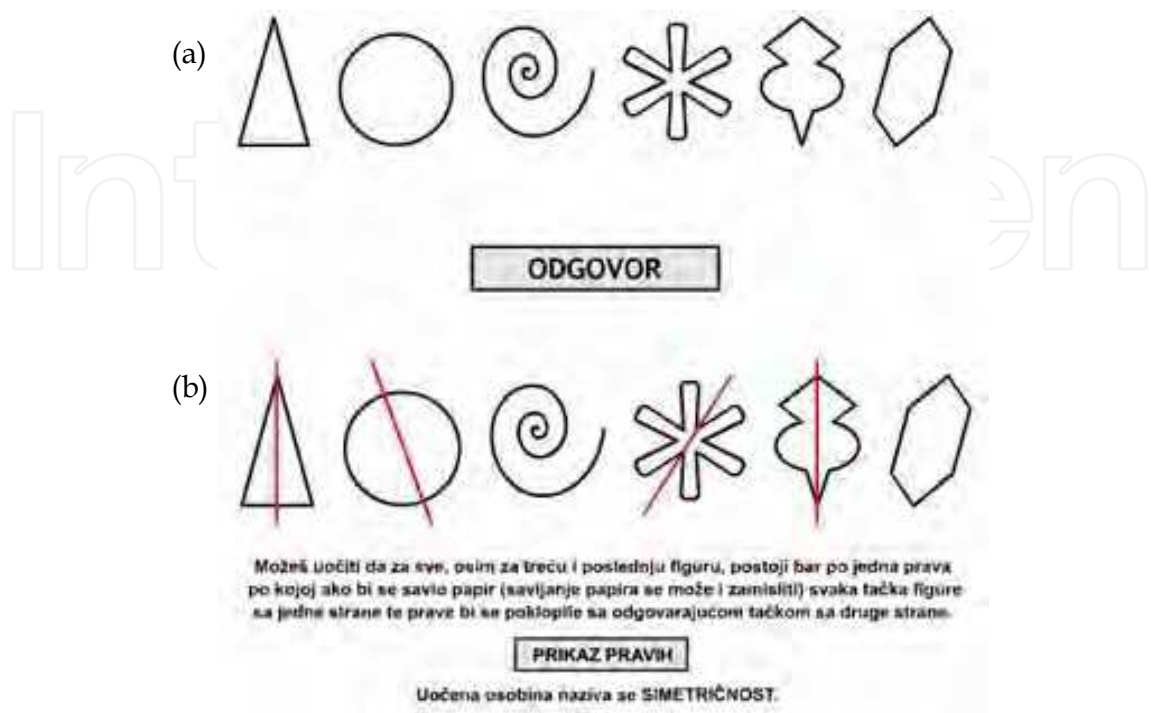
Fig. 11.

In next step, students were asked to look at the figures shown on Figure 12a and to find out if there is an axis of symmetry for any given pair of figures. Afther that, multimedia animation led them to the correct answer (Figure 12b)
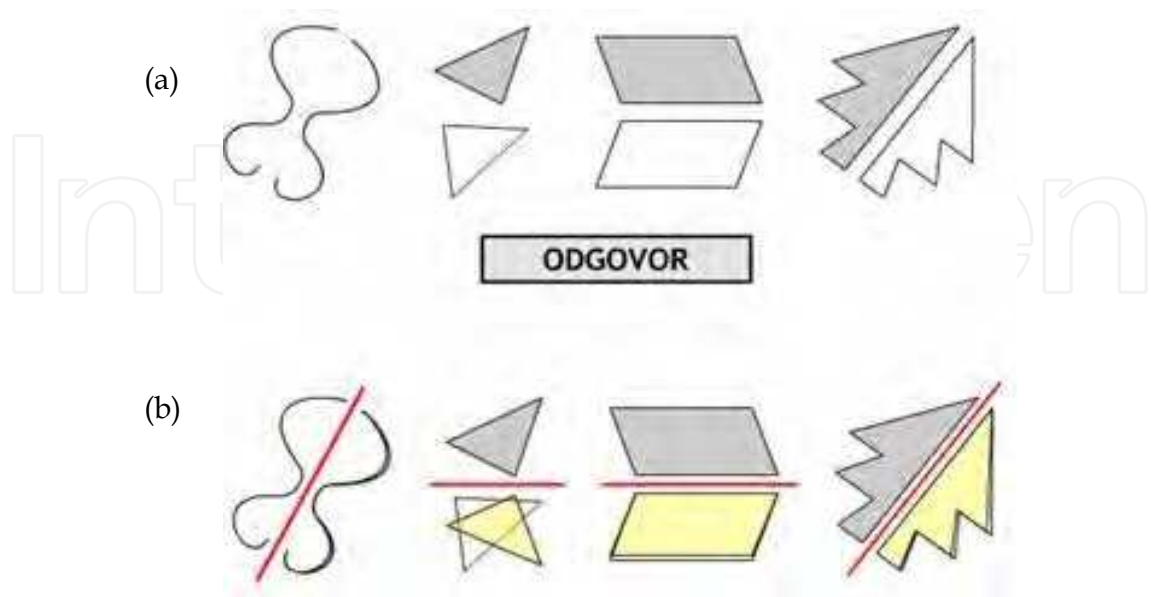

Fig. 12.

In chapter 'Examples', we have used series of animations to ask different questions, such as which of the given figures are symmetrical, how many axis of symmetry they have etc. All the answers were illustrated by complete multimedia presentations of isometric transformations. (Figure 13)
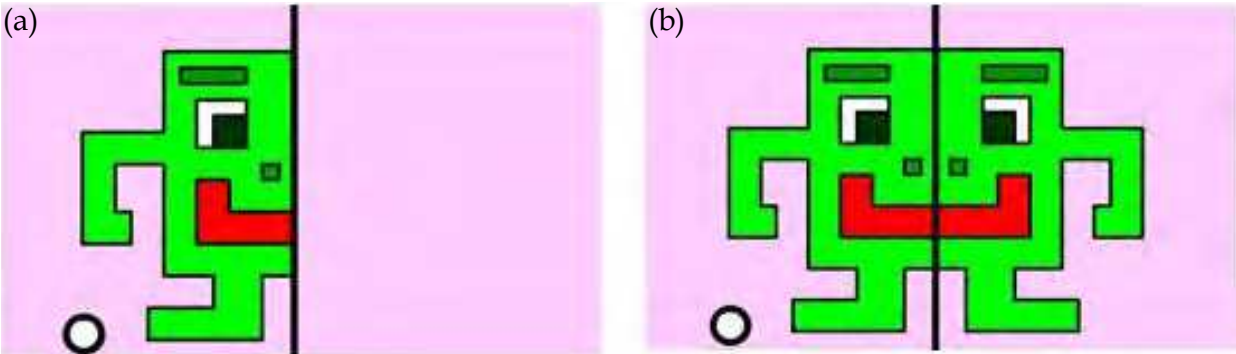

Fig. 13. Example of symmetrical figure (b), which is obtained by animated isometric transformation of picture (a), point by point, by pressing the button (left)

Chapter 'Exercises' offers variety of multimedia Q/A, quizzes and tests with purpose of resuming and exercising adopted knowledge.

One of the examples is presented in Figure 14. Its purpose was to use an interesting example from the everyday life in order to enable students to resume their knowledge.


Fig. 14. Task: How many axes of symmetry have these flags?

Chapter 'Problems' include variety of different interesting tasks, ranged from easier to more complex ones. All tasks are solved, majority with complete solutions and some with instruction how to solve them. The main idea was to enable student to get to the right solution individually, before he or she see it on the screen. Animations do not offer complete solution instantly, but gradually, step-by-step. Some of tasks are typical ones, as in traditional classes, but there are also non-standard tasks taken from the mathematics competitions.

Example: Two billiard balls, A and B, are on the rectangular table, as shown on figure. How should we hit the ball A if we want it to strike all four rails before hitting the ball B?

Let us mark the rectangle (billiard table) as XYUV, and A'=I_{XV}(A), A''=I_{XY}(A'), B'=I_{UV}(B), B''=I_{UY}(B').
(Multimedia presentation shows transformation step by step.)

(a)
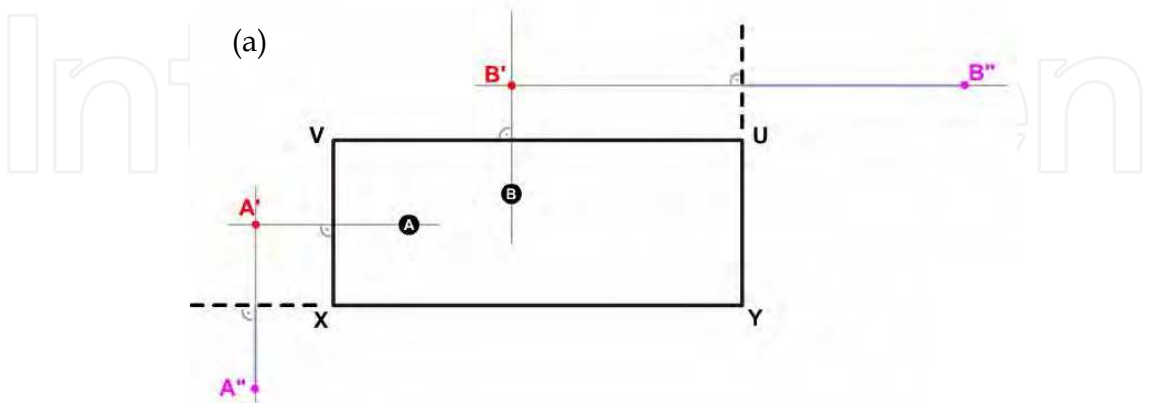
If we mark the intersection of lines A''B'' and XY as M, the intersection of lines A''B'' and UY as N, the intersection of lines A'M and XV as P, and the intersection of lines B'N and UV as Q, it can be noticed that the following angles are equal: APV=A'PV=XPM, PMX=XMA''=NMY, MNY=B''NU=UNQ, and NQU=B'QV=VQB.

(Multimedia presentation shows drawing of every line and their intersections, i.e. above mentioned points)

(b)
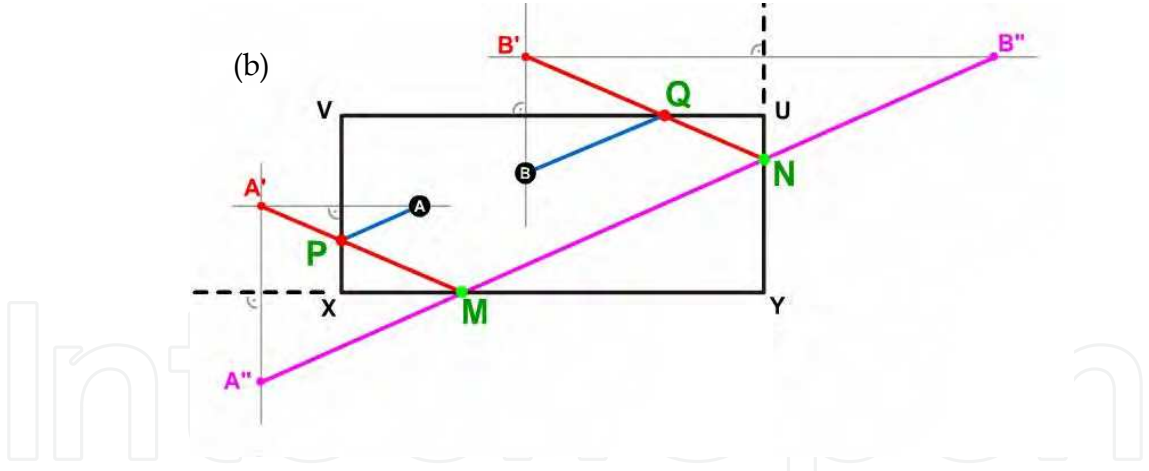
Therefore, ball in point A should be hit in such a way that would send it through points P, M, N and Q, and it will finally hit the ball in point B.
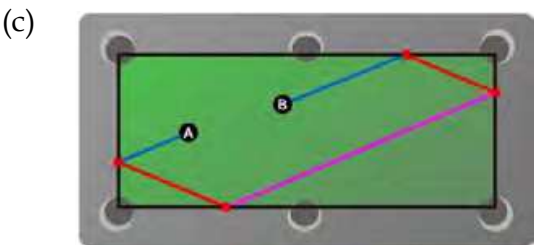

(c)

Fig. 15. Solution of a given task given by the multimedia animation

In teaching mathematics, we are sometimes supposed to explain abstract terms that rarely or hardly can be seen in reality, but students often ask for proof that theory can be seen and implemented in everyday life. With help of multimedia aids, we can show numerous examples of symmetry in architecture, art, nature, psychology, religion, etc.
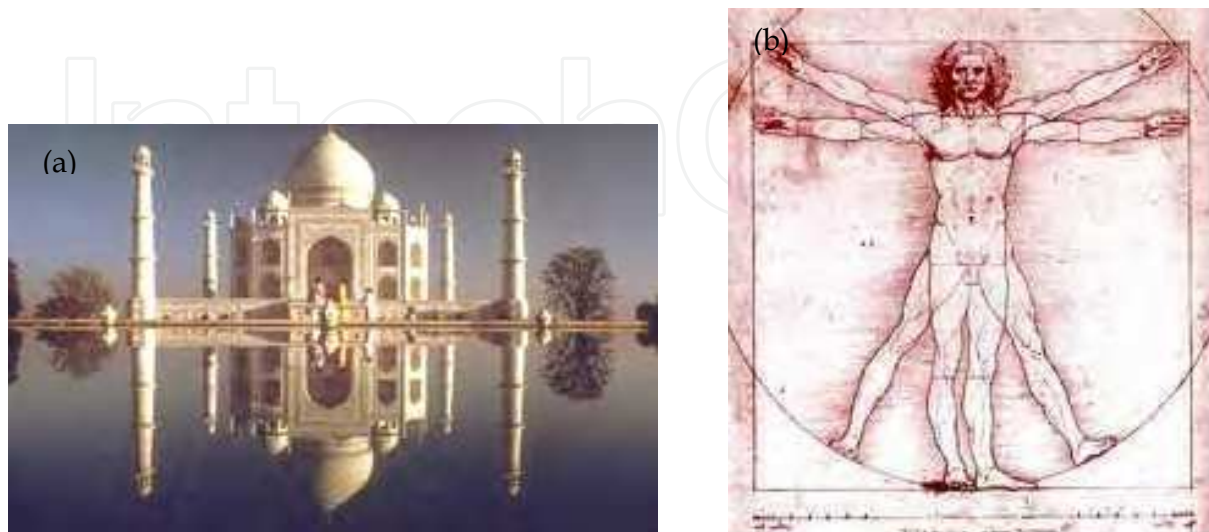

Fig. 16. Examples of symmetry in everyday life: (a) This photograph of the Taj Mahal has two axes of symmetry; Beside the vertical one, there is also a horizontal one, along the water line; (b) Famous Leonardo da Vinci's drawing The Vitruvian Man is also called Canon of Proportions or Proportions of Man. It shows the symmetry of the human body

3. Research methodology

3.1 Aim and questions of the research

Thanks to the experiences of some previous researches and results, some of the questions during this research were as follows:

1. Are there any differences between results of the first group of students, who had traditional lectures (control group – *traditional group*) and the second group, who had multimedia lectures (experimental group – *multimedia group*)? Where were these differences the most obvious?
2. What do students from the experimental group think about multimedia lectures? Do they prefer this or traditional way and why?
3. In students' opinion, where did multimedia learning help them more, in geometry or analysis (based on lessons on the definite integrals and the isometric transformations)?
4. Do students think it is easier to understand and learn the matter individually and during the classes by multimedia lectures?

3.2 Participants of the research

The research was conducted on two groups of 50 students, divided on subgroups of 25, at two faculties: the Faculty of Architecture and the Faculty of Civil Construction Management of the UNION University, Belgrade, Serbia. In both cases, one subgroup, consisting of 25 students,

had traditional lectures while the other one had multimedia lectures. Groups were formed randomly, so the previous knowledge needed for the lectures about limited integrals and isometric was practically the same, which was confirmed by pre-test. The pre-test included tasks from the area of the continuous functions (analysis and graph-drawing), as well as the tasks about the basic figures in analytic geometry (circle, ellipse, parabola etc.). Average score of this pre-test was practically equal in these groups (I: 72.35, II: 71.25 out of 100).

3.3 Methods, techniques and apparatus

Lectures in both groups included exactly the same information on given topics, i.e. axioms, theorems, examples and tasks. It is important to emphasize that the lecturer and the number of classes were the same, too. The main information source for the multimedia group was software created in Macromedia Flash 10.0, which is proven to be very successful and illustrative for creating multimedia applications in mathematics lectures (Bakhoum, 2008). Our multimedia lecturing material was created in accordance with methodical approach, i.e. cognitive theory of multimedia learning (Mayer, 2001, 2005), as well as with principles of multimedia teaching and design based on researches in the field of teaching mathematics (Atkinson, 2005). This material includes a large number of dynamic and graphic presentations of definitions, theorems, characteristics, examples and tests from the area of the definite integrals based on step-by-step method with accent on visualisation. An important quality of making one's own multimedia lectures is the possibility of creating combination of traditional lecture and multimedia support in those areas we have mentioned as the 'weak links' (definite integral definition, area, volume, etc.)

After the lectures were finished, all students had the same tests consisting of tasks on definite integrals and isometric transformations. Besides that, students were interviewed after the classes and transcripts of the most characteristic opinions are also included here. In order to get as objective results as possible, participation in the interviews was voluntary and anonymous, and the interviewer was not a member of the teaching staff at any of the faculties.

Test 1 – Definite integral:

1. Use Archimedes' method to determine the area of plane figure bounded by the Ox-axis, line $x = a$, and part of the curve $y = x^3$ for $0 \le x \le a$.
2. Write the definite integral definition.
3. Determine the definite integral $\int\limits_{0}^{\pi/4} \sin x \cdot \cos^2 x \, dx$.
4. Determine the area of the plane figure F bounded by Oy axis, graph of the function $y = x^2$ and the tangent on this graph in point (1, 1).
5. Determine the volume of the body of revolution obtained by rotating figure F bounded by parabolas $y^2 = 8x$ and $x^2 = y$ in the xOy plane around the Oy axis.

Test 2 – Isometric transformations:

1. Which of the following figures are symmetrical:
   - Ray

- Circle
- Line
- Parallelogram
- Isosceles triangle
- Isosceles trapezium
- Kite

2. Line reflection:
   a. is a plane isometry
   b. is not a plane isometry
   How many axes of symmetry are there in the circle?
   a. 2
   b. 4
   c. Infinite number
   Immovable lines in the line reflection are:
   a. parallel with axis
   b. intersect axis
   c. rectangular with axis
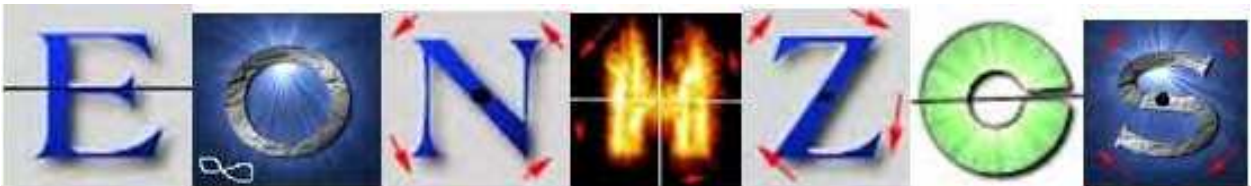3. How many axes of symmetry are there in the following alphabet letters?

Fig. 17.

4. For given sharp angle aOb and point C, find the points A and B, such that A belongs to Oa, B belongs to Ob, and the triangle ABC has the minimal possible circumference.
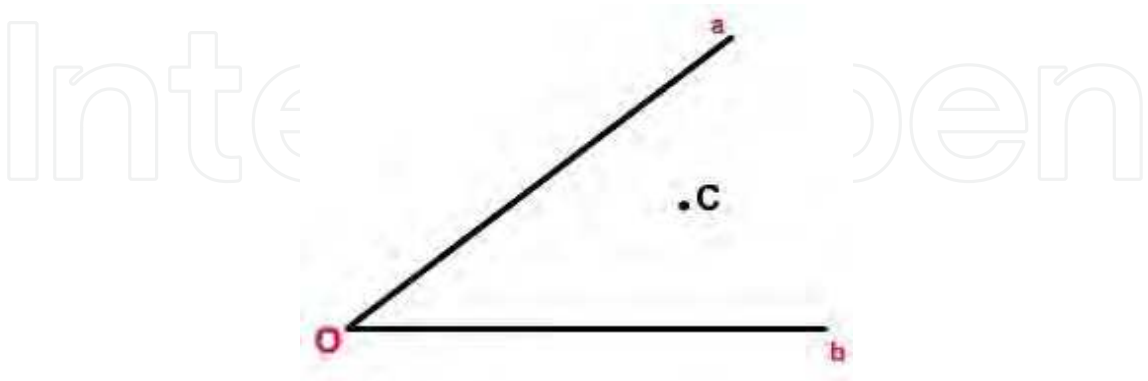
Fig. 18.

5. For given line p and points A and B on the same side of p, find the point P on the line p, so the ray of light which starts in point A hits the point P and passes through the point B. (Use the fact that entering ray is symmetrical with reflected ray.)
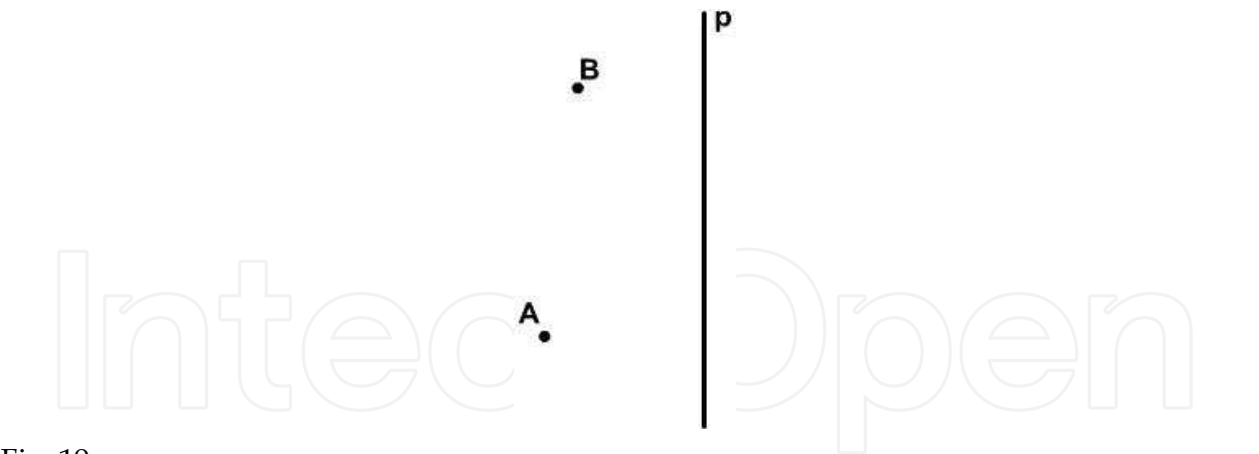
Fig. 19.

Both tests were scored within the interval from 0 to 100 (20 points per task) and the average scores in both tests separately were calculated for the traditional and the multimedia group.

Results were analysed with *Student's t-test for independent samples* using SPSS (version 10.0) software. The result was considered significant if the probability *p* was less than 0.05.

3.4 Results

At the Faculty of Architecture, average score on Test 1 (definite integrals) in the traditional group was 67.75 with standard deviation 14.51, and in the multimedia group, average score was 83.21 with standard deviation 15.01. Average score on Test 2 (line reflection) in the traditional group was 76.04 with standard deviation 15.25, and in the multimedia group, average score was 87.92 with standard deviation 12.5. (Figure 20)
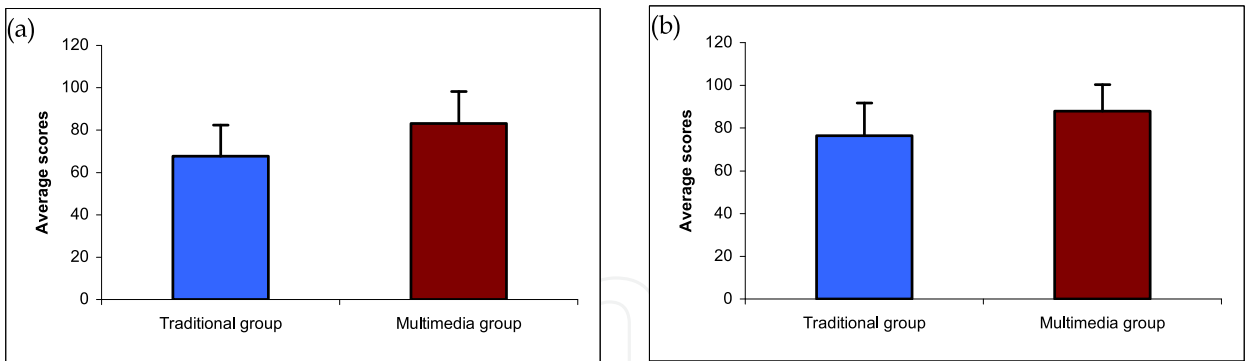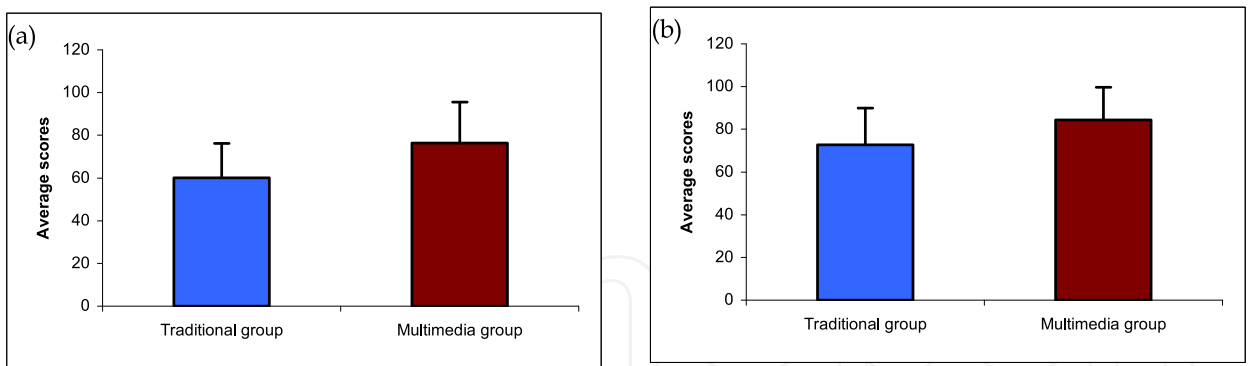

Fig. 20. Total average test scores for (a) Test 1 (definite integral) and (b) Test 2 (line reflection) for traditional and multimedia groups at the faculty of Architecture

At the Faculty of Civil Construction Management, average score on Test 1 (definite integrals) in the traditional group was 60.04 with standard deviation 16.2, and in the multimedia group, average score was 76.37 with standard deviation 19.13. Average score on Test 2 (line reflection) in the traditional group was 72.21 with standard deviation 17.32, and in the multimedia group, average score was 84.37 with standard deviation 15.27. (Figure 21)

In all groups, statistical comparison with t-test for two independent samples showed that multimedia groups had remarkably higher score in comparison with the traditional groups, with statistical significance p < 0.05.

Fig. 21. Total average test scores for (a) Test 1 (definite integral) and (b) Test 2 (line reflection) for traditional and multimedia groups at the Faculty of Civil Construction Management

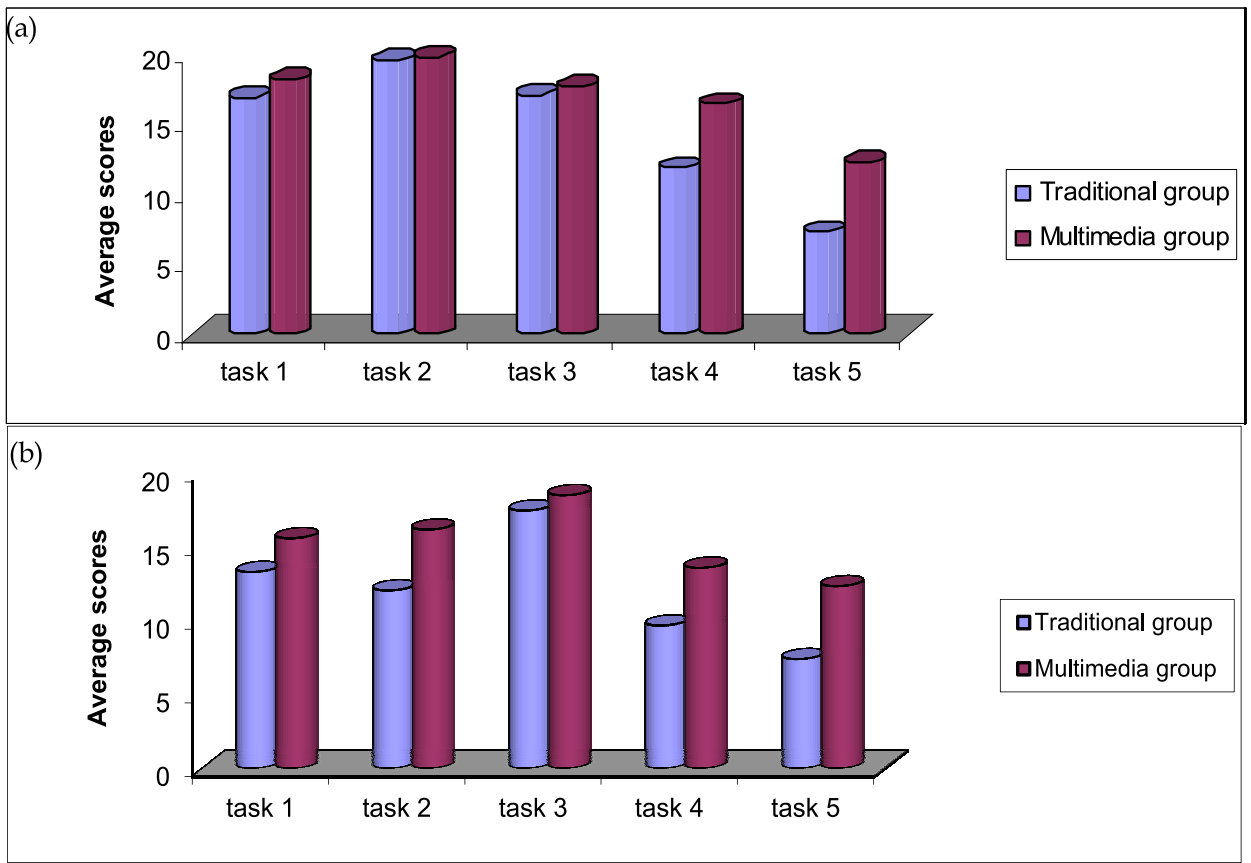Test scores by tasks for all groups are given in Figures 22 and 23.

Fig. 22. Average test scores by single tasks for (a) Test 1 (definite integral) and (b) Test 2 (line reflection) for traditional and multimedia groups at the faculty of Architecture

When asked whether they prefer classical or multimedia way of learning, 12% (3 students) answered classical and 82% (22 students) answered multimedia at the Faculty of Architecture, while at the Faculty of Civil Construction Management 20% (5 students) answered classical and 80% (20 students) answered multimedia, explaining it with the following reasons:
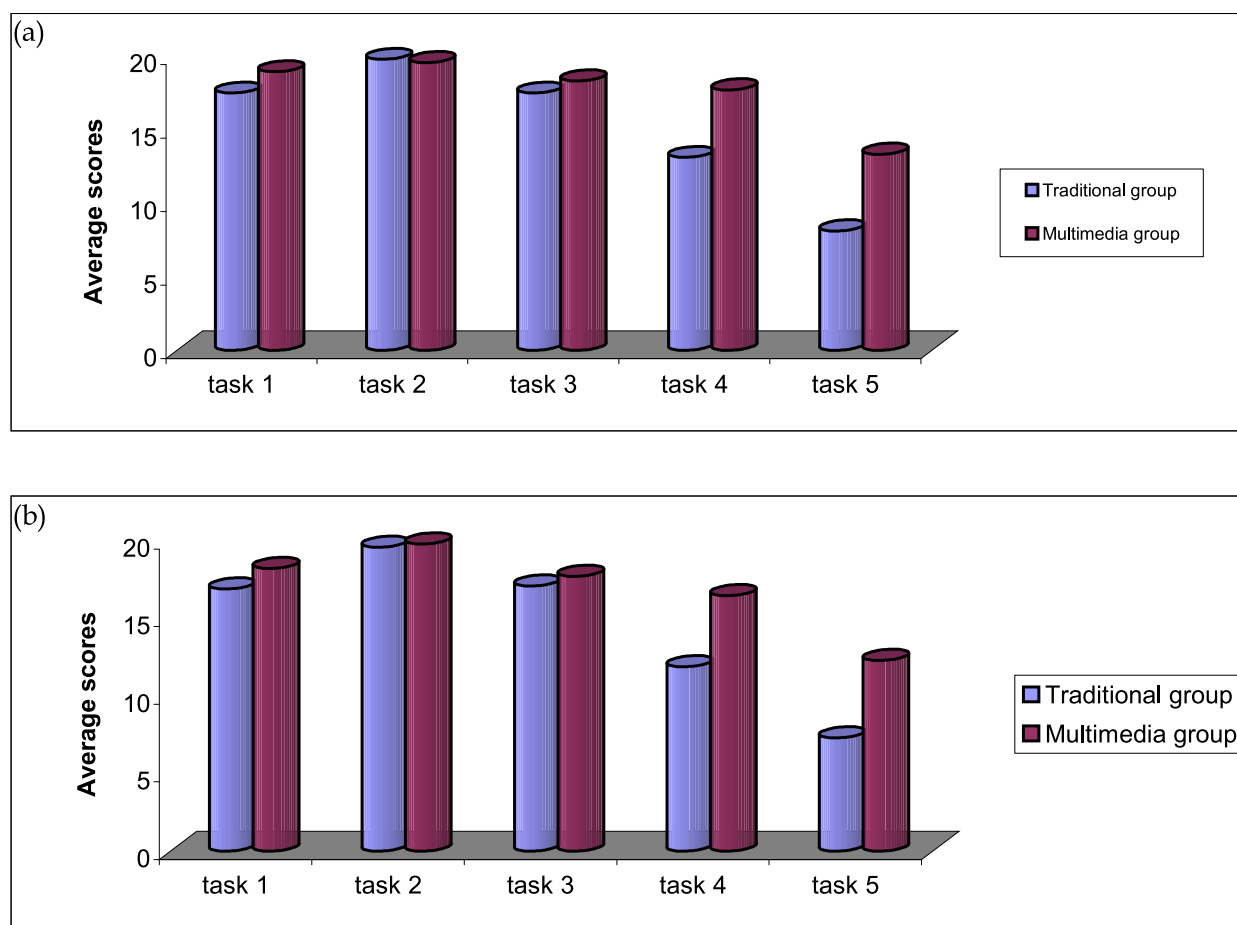
Fig. 23. Average test scores by single tasks for (a) Test 1 (definite integral) and (b) Test 2 (line reflection) for traditional and multimedia groups at the Faculty of Civil Construction Management

- 'It is much easier to see and understand some things, and much easier to comprehend with the help of step-by-step animation.'
- 'Much more interesting and easier to follow, in opposite to traditional monotonous lectures with formulas and static graphs.'
- 'More interesting and easier to see, understand and remember.'
- 'I understand it much better this way and I would like to have similar lectures in other subjects, too.'
- 'This enables me to learn faster and easier and to understand mathematical problems which demand visualisation.'
- 'Quite interesting, although classical lectures can be interesting – depending on teacher.'

Students have also commented in which area (analyses or geometry) multimedia lessons were more helpful:

- 'Multimedia lessons certainly make learning easier, especially in the fields which are more abstract and which are better understood with help of pictures and animations, such was the one about the integrals.'

- 'I have comprehended the integrals much better now than in the high school, while in case of symmetry it was much easier to understand and solve more difficult problems with help of multimedia.'
- 'Line reflection is not very difficult to understand and learn, but it was much easier and even funny through the multimedia lessons. I have always thought that integrals are horrible, but now I understand them and know how to calculate area or volume by drawing a figure.'

When asked whether it was easier for them to learn, understand and solve problems after having lectures and individual work with multimedia approach, students answered the question as shown in Figure 24:
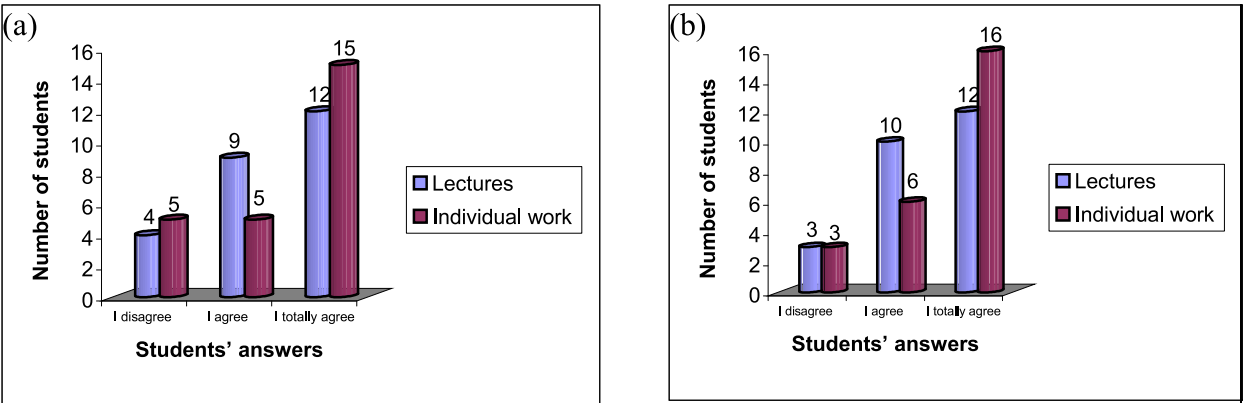

Fig. 24. Students' answers to the question: Should PC be used in lecturing and learning mathematics? (a) Architecture; (b) Civil Construction Management

4. Discussion and conclusions

During past few years, multimedia learning has become very important and interesting topic in the field of teaching methodology. Researches conducted by Mayer (Mayer, 2001, 2005) and Atkinson (Atkinson, 2005) resulted in establishing the basic principles of multimedia learning and design, which were confirmed in our research, too. Multimedia lessons about the definite integrals and the isometric transformations, created in accordance with these principles, proved to be successful. According to the students' reactions, highly understandable animations from multimedia lessons are the best proof that a picture is worth a thousand words. Their remark, and consequently one of this research's conclusions, was that there should be much more of this kind of lessons in education, made – of course – in accordance with certain rules and created in the right way. Many research works in different scientific fields, including mathematics, have proven that multimedia makes learning process much easier.

The tests of adopted knowledge conducted during this research showed that students from multimedia group had much higher average scores in comparison with students from traditional group, which also correspondents with results of some other authors (D. Đ. Herceg, 2009; Wishart, 2000). At the Faculty of Architecture, average score on Test 1 (definite integrals) was 15.49 points greater in multimedia group than in traditional group, while the

average score on Test 2 (line reflection) was 11.52 points greater in multimedia group. At the Faculty of Civil Construction Management, average score was 15.97 points greater on Test 1 and 12.6 points greater in multimedia group.

Research on learning the definite integrals with software packages Mathematica and GeoGebra (D. Đ. Herceg, 2009) has shown that students who had used PC in learning process had higher scores on tests. Although this research was conducted with different multimedia teaching tools for the same subject – the definite integral as one of the most important areas in mathematical analyses – our results only proved the universality of multimedia in the process of teaching mathematics.

According to Figures 22a and 23a, which show average scores in single tasks from the field of definite integral, we can conclude that students from multimedia group were remarkably more successful in problems which demand visual comprehension (tasks 1, 2, 4 and 5), while the average score in the third task was practically the same on the both groups. Additionally, according to Figures 22b and 23b, which show average scores in single tasks from the line reflection, it can be seen that the students from multimedia groups at both faculties were remarkably more successful in solving the tasks that demanded visual comprehension, as well as in using the adopted knowledge for more complicated problems (tasks 4 and 5), while the average scores for other tasks were not significantly different.

Wishart's (Wishart, 2000) research included analyses of comments on how much multimedia approach affects teaching and learning processes. Teachers emphasized that multimedia lectures have made their work easier and have proved to be motivating for students, while students said that multimedia lessons, in comparison with traditional methods, have offered better visual idea about the topic. As shown in Figure 24, a great number of them insisted that multimedia tools enabled easier understanding, learning and implementation of knowledge. Students' remark, and consequently one of this research's conclusions, was that there should be more multimedia lessons, i.e. that multimedia is an important aspect of teaching and learning process.

One of this research's conclusions can be put in the way one student did it during the survey (by answering the question: What is multimedia learning): 'Multimedia learning is use of multimedia as an addition to the traditional way of learning. Multimedia enables us to have better understanding of many mathematical problems and to experiment with them.'

Since the experimental lessons on definite integral and isometric transformations have proven to be very successful, we have decided to continue our work and to develop similar multimedia lessons for other areas of mathematics (where applicable) and to make them available for other researchers and teachers when the whole package is ready for publishing.

5. Guidelines for further researches

During our research, several new questions appeared that should be solved in the future: (a) In which scientific fields does the multimedia approach give the best results? (b) How much success of the multimedia approach depends on an individual student's ability and how much on a teacher's skills? (c) How can we improve the understanding of lectures by using the multimedia approach, because our aim is learning and understanding, not the multimedia *per se*.

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Interactive Multimedia

Edited by Dr Ioannis Deliyannis

Interactive multimedia is clearly a field of fundamental research, social, educational and economical importance, as it combines multiple disciplines for the development of multimedia systems that are capable to sense the environment and dynamically process, edit, adjust or generate new content. For this purpose, ideas, theories, methodologies and inventions are combined in order to form novel applications and systems. This book presents novel scientific research, proven methodologies and interdisciplinary case studies that exhibit advances under Interfaces and Interaction, Interactive Multimedia Learning, Teaching and Competence Diagnosis Systems, Interactive TV, Film and Multimedia Production and Video Processing. The chapters selected for this volume offer new perspectives in terms of strategies, tested practices and solutions that, beyond describing the state-of-the-art, may be utilised as a solid basis for the development of new interactive systems and applications.

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