4

Evolutionary Algorithms Based on the Automata Theory for the Multi-Objective Optimization of Combinatorial Problems

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1. Introduction

In this chapter we are going to study metaheuristics based on the Automata Theory for the Multi-objective Optimization of Combinatorial Problems. As well known, Combinatorial Optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution(Yong-Fa & Ming-Yang, 2004). In this field it is possible to find a lot of problems denominated NP-Hard, that is mean that the problem does not have a solution in Polynomial Time. For instance, problems such as Multi-depot vehicle routing problem(Lim & Wang, 2005), delivery and pickup vehicle routing problem with time windows(Wang & Lang, 2008), multi-depot vehicle routing problem with weight-related costs(Fung et al., 2009), Railway Traveling Salesman Problem(Hu & Raidl, 2008), Heterogeneous, Multiple Depot, Multiple Traveling Salesman Problem(Oberlin et al., 2009) and Traveling Salesman with Multi-agent(Wang & Xu, 2009) are categorized as NP-Hard problems.

One of the most classical problems in the Combinatorial Optimization Field is the Traveling Salesman Problem (TSP), it has been analyzed for years(Sauer & Coelho, 2008) either in a Mono or Multi-objective way. It is defined as follows: "Given a set of cities and a departure city, visit each city only once and go back to the departure city with the minimum cost". Basically, that is mean, visiting each city once, to find an optimal tour in a set of cities, an instance of TSP problem can be seen in figure 1. Formally, TSP is defined as follows:

$$min \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} \cdot X_{ij} \tag{1}$$

Subject to:

$$\sum_{j=1}^{n} X_{ij} = 1, \forall i = 1, \ldots, n \tag{2}$$

$$\sum_{j=1}^{n} X_{ij} = 1, \forall j = 1, \ldots, n \tag{3}$$

$$\sum_{i \in \kappa} \sum_{j \in \kappa} X_{ij} \leq |\kappa| - 1, \forall \kappa \subset \{1, \ldots, n\} \tag{4}$$

$$X_{ij} = 0, 1 \forall i, j \tag{5}$$

Where C_{ij} is the cost of the path X_{ij} and κ is any nonempty proper subset of the cities $1, \ldots, m$. (1) is the objective function. The goal is the optimization of the overall cost of the tour. (2), (3) and (5) fulfills the constrain of visiting each city only once. Lastly, Equation (4) set the subsets of solutions, avoiding cycles in the tour.

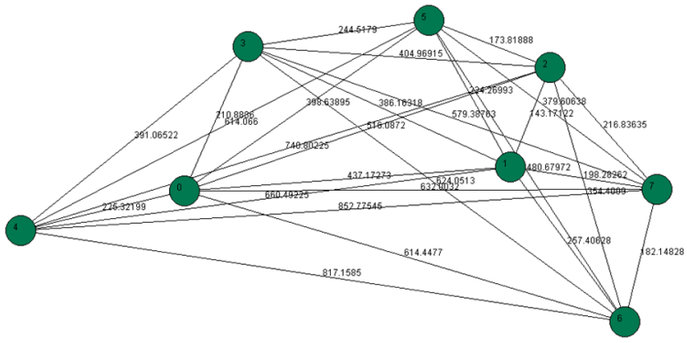

Fig. 1. TSP instance of ten cities

TSP has an important impact on different sciences and fields, for instance in Operations Research and Theoretical Computer Science. Most problems related to those fields, are based in the TSP definition. For instance, problems such as Heterogeneous Machine Scheduling(Kim & Lee, 1998), Hybrid Scheduling and Dual Queue Scheduling(Shah et al., 2009), Project Management(de Pablo, 2009), Scheduling for Multichannel EPONs(McGarry et al., 2008), Single Machine Scheduling(Chunyue et al., 2009), Distributed Scheduling Systems(Yu et al., 1999), Relaxing Scheduling Loop Constraints(Kim & Lipasti, 2003), Distributed Parallel Scheduling(Liu et al., 2003), Scheduling for Grids(Huang et al., 2010), Parallel Scheduling for Dependent Task Graphs(Mingsheng et al., 2003), Dynamic Scheduling on Multiprocessor Architectures(Hamidzadeh & Atif, 1996), Advanced Planning and Scheduling System(Chua et al., 2006), Tasks and Messages in Distributed Real-Time Systems(Manimaran et al., 1997), Production Scheduling(You-xin et al., 2009), Cellular Network for Quality of Service Assurance(Wu & Negi, 2003), Net Based Scheduling(Wei et al., 2007), Spring Scheduling Co-processor(Niehaus et al., 1993), Multiple-resource Periodic Scheduling(Zhu et al., 2003), Real-Time Query Scheduling for Wireless Sensor Networks(Chipara et al., 2007), Multimedia Computing and Real-time Constraints(Chen et al., 2003), Pattern Driven Dynamic Scheduling(Yingzi et al., 2009), Security-assured Grid Job Scheduling(Song et al., 2006), Cost Reduction and Customer Satisfaction(Grobler & Engelbrecht, 2007), MPEG-2 TS Multiplexers in CATV Networks(Jianghong et al., 2000), Contention Awareness(Shanmugapriya et al., 2009) and The Hard Scheduling Optimization(Niño, Ardila, Perez & Donoso, 2010) had been derived from TSP. Although several algorithms have been implemented to solve TSP, there is no one that optimal solves it. For this reason, this chapter discuss novel metaheuristics based on the Automata Theory to solve the Multi-objective Traveling Salesman Problem.

This chapter is structured as follows: Section 2 shows important definitions to understand the Multi-objective Combinatorial Optimization and the Metaheuristic Approximation. Section 3, 4 and 5 discuss Evolutionary Metaheuritics based on the Automata Theory for the Multi-objective Optimization of Combinatorial Problems. Finally, Section 6 and 7 discuss the Experimental Results of each proposed Algorithm using Multi-objective Metrics from the specialized literature.

2. Preliminaries

2.1 Multi-objective optimization

The Multi-objective optimization consists in two or more objectives functions to optimize and a set of constraints. Mathematically, the Multi-objective Optimization model is defined as follows:

$$optimize \quad F(X) = \{f_1(X), f_2(X), \ldots, f_n(X)\} \tag{6}$$

Subject to:

$$H(X) = 0 \tag{7}$$

$$G(X) \leq 0 \tag{8}$$

$$X_l \leq X \leq X_u \tag{9}$$

Where $F(X)$ is the set of objective functions, $H(X)$ and $G(X)$ are the constraints of the problem. Lastly, X_l and X_u are the bounds for the set of variables X.

Unlike to Mono-objective Optimization, Multi-objective Optimization deal with searching a set of Optimal Solutions instead of a Optimal Solution. For instance, table 1 shows three solutions for a particualr Mono-objective Problem. If we suppose that those are related to a maximization problem then the Optimal Solution (found) is the solution 1 otherwise (minimization) will be the solution 2. On the other hand, in table 2 can be seen three solutions for a particular Tri-objective Problem. Thus, if we suppose that all the components of the solutions are related with a minimization problem, solution 2 is a *dominated solution* due to all the components (0.8, 0.9 and 1.0) are the biggest values. On the other hand, solution 0 and 1 are *no-dominated solutions* due to in the first and second component (0.6 and 0.4) solution 0 is bigger than the relative components of the solution 1 but in the third component (0.5) solution 0 is lower than the same component in solution 1. Both examples show the

k	$F(X_k)$
0	10
1	20
2	5

Table 1. Solutions for a particular Mono-objective Problem

difference between Mono-objective and Multi-objective Optimization. While the first deal with finding the Optimal Solution, the last does with finding a set of Optimal Solutions. In Combinatorial Optimization, the set of Optimal Solution is called *Pareto Front*. It contains all the no-dominated solutions for a Multi-objective Problem. Figure 2 shows a Pareto Front for a particular Tri-objective Problem. Lastly, it is probably that some Multi-objective Problems

k	$F(X_k) = \{f_0(X_k), f_1(X_k), f_2(X_k)\}$
0	$\{0.6, 0.4, 0.5\}$
1	$\{0.2, 0.3, 0.8\}$
2	$\{0.8, 0.9, 1.0\}$

Table 2. Solutions for a particular Tri-objective Problem

have an infinite Pareto Front, in those cases is necessary to determinate how many solutions are required, for instance, using a maximum number of solution permitted in the Pareto Front.

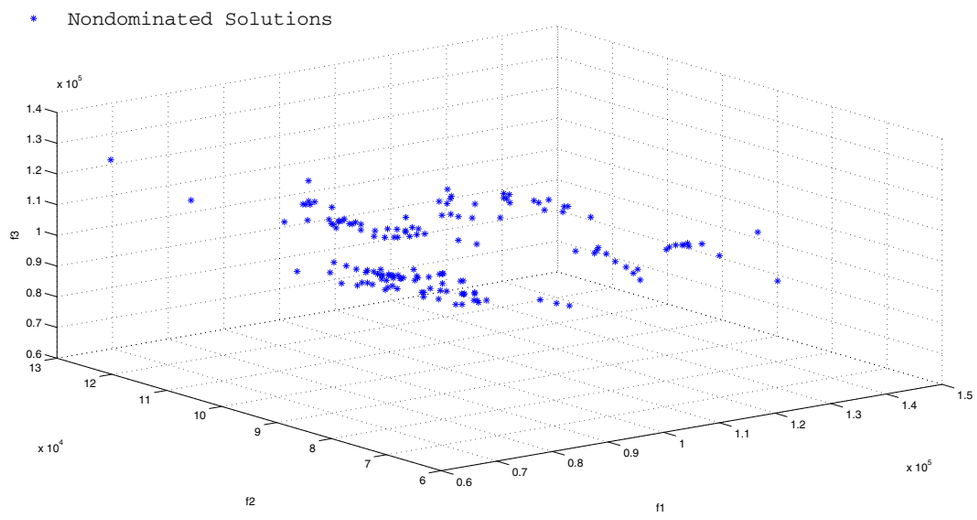

Fig. 2. Pareto Front for a particular Tri-objective Problem

2.2 Tabu search

Tabu Search(Glover & Laguna, 1997) is a basic local search strategy for the Optimization of Combinatorial Problems. It is defined as follows: Given S as the Initial Solutions Set.

Step 1. Selection. Select $x \in S$

Step 2. Perturbation. Perturbs the solution x for the purpose of knowing its Neighborhood ($N(x)$). Perturbing a solution means to modify the solution x in order to obtain a new solution ($x_i^{'}$). The solutions found are called Neighbors, and those represent the Neighborhood. For instance, figure 3 shows three perturbations for a x solutions and the new solutions $x_1^{'}$,$x_2^{'}$ and $x_3^{'}$ found. The perturbation can be done according to the representation of the solutions. Regularly, the representations of the solutions in Combinatorial Problems are based on Discrete Structures such as Vectors, Matrices, Queues and Lists. Lastly, good solutions are added to S.

Setp 3. Check Stop Condition. The stop condition can be delimited using rules such as number of execution without improvement or maximum number of iteration exceeded.

Recently, novels Tabu Search inspired Algorithms have been developed in order to solve Combinatorial Problems such as Permutation Flow Shop Scheduling(Ren et al., 2011), Displacement based on Support Vector(Fei et al., 2011), Examination Timetabling Problem(Malik et al., 2011), Partial Transmit Sequences for PAPR Reduction(Taspinar et al., 2011), Inverse Problems(An et al., 2011), Fuzzy PD Controllers(Talbi & Belarbi, 2011b), Instrusion Detection(Jian-guang et al., 2011), Tel-Home Care Problems(Lee et al., 2011), Ant Colony inspired Problems(Zhang-liang & Yue-guang, 2011), Steelmaking-Continuous Casting Production Scheduling(Zhao et al., 2011), Fuzzy Inference System(Talbi & Belarbi, 2011a) and Coordination of Dispatchable Distributed Generation and Voltage Control Devices(Ausavanop & Chaitusaney, 2011).

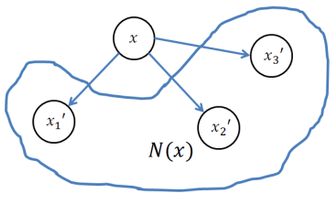

Fig. 3. The Neighborhood of a solution x is known after being perturbed

2.3 Genetic algorithms

Genetic Algorithms are Algorithms based on the Theory of Natural Selection(Wijkman, 1996). Thus, Genetic Algorithms mimics the realBehavior Genetic Algorithms(Fisher, 1930) through three basic steps: Given a set of Initial Solutions S

Step 1. Selection. Select solutions from a population. In pairs, select two solutions $x, y \in S$

Step 2. Crossover. Cross the selected solutions avoiding local optimums.

Step 3. Mutation. Perturbs the new solutions found for increasing the population. The perturbation can be done according to the representation of the solution. In this step, good solutions are added to S

Figure 4 shows the basics steps of a Genetic Algorithm. The most known Genetic

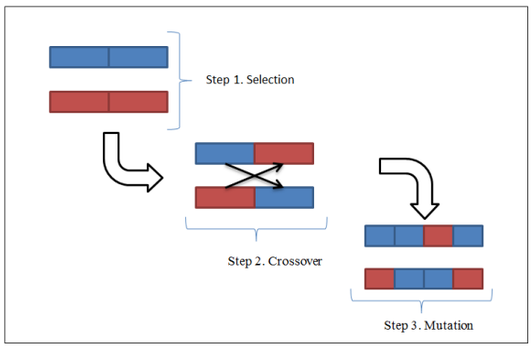

Fig. 4. Basics steps of a Genetic Algorithm

Algorithms from the literature(Dukkipati & Narasimha Murty, 2002) are the Non-Dominated Sorting Genetic Algorithm(Deb et al., 2002) (NSGA-II) and the Strength Pareto Evolutionary Algorithm 2(Zitzler et al., 2001; 2002) (SPEA 2). NSGA-II uses a no-dominated sort for sorting the solutions in different Pareto Sets. Consequently, it demands a lot of time, but it allows a global verification of the solutions for avoiding the Local Optimums. On the other hand, SPEA 2 is an improvement of SPEA. The difference with the first version is that SPEA 2 works using strength for every solution according to the number of solutions that it dominates. Consequently, at the end of the iterations, SPEA 2 has the non dominated solutions stronger avoiding Local Optimums. SPEA 2 and NSGA-II have been implemented to solve a lot of problems in the Multiobjective and Combinatorial Optimization fiel. For instance, problems such as Pattern-recognition based Machine Translation System(Sofianopoulos & Tambouratzis, 2011), Tuning of Fuzzy Logic controllers for a heating(Gacto et al., 2011), Real-coded Quantum Clones(Xiawen & Yu, 2011), Optimization Problems with Correlated Objectives(Ishibuchi et al., 2011), Production Planning(Yu et al., 2011), Optical and Dynamic Networks Designs(Araujo et al., 2011; Wismans et al., 2011), Benchmark multi-objective

optimization(McClymont & Keedwell, 2011) and Vendor-managed Inventory(Azuma et al., 2011) have been solved using SPEA and NSGA-II.

2.4 Simulated Annealing algorithms

Simulated Annealing(Kirkpatrick et al., 1983) is a generic probabilistic metaheuristic based in the Annealing in Metallurgy. Similar to Tabu Search, Simulated Annealing explores the neighborhood of solutions being flexible with no-good solutions. That is mean, accepting bad solutions as well as good solution, but only in the first iterations. The acceptation of a bad solution is based on the Boltzmann Probabilistic Distribution:

$$P(x) = e^{\left(-\left(\frac{E}{T_i}\right)\right)} \tag{10}$$

Where E is the change of the Energy and T_i is the temperature in the moment i. In the first level of the temperature, bad solutions are accepted as well, anyways, when the temperature go down, Simulated Annealing behaves similar to Tabu Search (only accept good solutions).

Recentrly, similar to Genetic Algoritms and Tabu Search, many problems have been solved using Simulated Annealing metaheuristic. For instance, Neuro Fuzzy - SystemsCzabaski (2006), Contrast Functions for BSSGarriz et al. (2005), Cryptanalysis of Transposition CipherSong et al. (2008), Transmitter-Receiver Collaborative-Relay BeamformingZheng et al. (2011) and Two-Dimensional Strip Packing ProblemDereli & Sena Da (2007) have been solved through Simulated Annealing inspired algorithms.

2.5 Deterministic Finite Automata

Formally, a Deterministic Finite Automata is a Quint-tuple defined as follows:

$$A = (Q, \Sigma, \delta, q_0, F) \tag{11}$$

Set of transitions δ. The set of transitions (δ) describes the behavior of the automata. Let $a \in S$ and $q, r \in Q$, then the function is defined as follows:

$$\delta(q, a) = r \tag{12}$$

Example 1. Let $A = (Q, \Sigma, \delta, q_0, F)$, where $Q = \{q_0, q_1, q_2\}$, $S = \{0, 1\}$, $F = \{q_1\}$ and the set of transitions δ defined in table 3, the representation of A using a state diagram can be derived as shown in figure 5. Notice that each state of DFA has transitions with all the elements of Σ.

	0	1
q_0	q_2	q_0
q_1	q_1	q_1
q_2	q_2	q_1

Table 3. Set of transitions for the DFA of example 1

2.6 Metaheuristic Of Deterministic Swapping (MODS)

Metaheuristic Of Deterministic Swapping (MODS) (Niño et al., 2011) is a local search strategy that explores the Feasible Solution Space of a Combinatorial Problem supported in a data structure named Multi Objective Deterministic Finite Automata (MDFA) (Niño, Ardila, Donoso & Jabba, 2010). A MDFA is a Deterministic Finite Automata that allows the

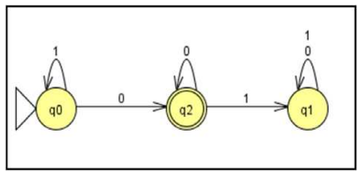

Fig. 5. Automata state diagram for the example 1.

representation of the feasible solution space of a Combinatorial Problem. Formally, a MDFA is defined as follows:

$$M = (Q, \Sigma, \delta, Q_0, F(X)) \tag{13}$$

Where Q represents all the set of states of the automata (feasible solution space), Σ is the input alphabet that is used for δ (transition function) to explore the feasible solution space of a combinatorial problem, Q_0 contains the initial set of states (initial solutions) and $F(X)$ are the objectives to optimize.

Example 1. MDFA for a Scheduling Parallel Machine Problem:

A Company has three machines. It is necessary to schedule three processes in parallel P_1,P_2 and P_3. Each process has a duration of 5, 10 y 50 minutes respectively. If the processes can be executed in any of the machines, how many manners the machines can be assigned to the processes? Given the Bi-objective function in (10), what is the optimal Pareto Front?

$$F(X) = \left\{ f_1(X) = \sum_{i=1}^{3} i \cdot X_i, f_2(X) = \sum_{i=1}^{3} \left(\frac{1}{i}\right) \cdot X_i \right\} \tag{14}$$

First of all, we need to build the MDFA. For doing this, we must define the states of the MDFA setting the structure of the solution for each state. Therefore, if we state that $X_q = (P_k, P_i, P_j)$ represents the solution for the state q: machine 1 executes the process P_k, machine 2 executes the process P_i and machine 3 executes the process P_j then the arrays solution for each state will be $X_q0 = (P_1, P_2, P_3)$, $X_q1 = (P_1, P_3, P_2)$, $X_q2 = (P_2, P_1, P_3)$, $X_q3 = (P_2, P_3, P_1)$, $X_q4 = (P_3, P_1, P_2)$ y $X_q5 = (P_3, P_2, P_1)$. Now, we have six states q_0,q_1,q_2,q_3,q_4 and q_5, those represent the feasible solution space of the Scheduling problem proposed. The set of states for the MDFA of this problem can be seen in figure 6. Once the set of states is defined, the Input Alphabet

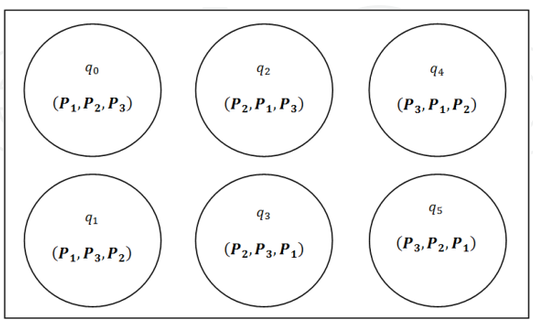

Fig. 6. Set of states for the MDFA of example 2

(Σ) and the Transition Function (δ) be done. It is very important to take into account, first, the bond of both allows to perturb the solutions in all the possible manners, in other words, we can change of state using the combination of Σ and δ. Obviously, doing this, we avoid

unfeasible solutions. Regarding the proposed problem, we propose the set Σ as follows:

$$\Sigma = \{(P_1, P_2), (P_1, P_3), (P_2, P_3)\} \tag{15}$$

Hence, it is elemental that $\delta(q_0, (P_1, P_2)) = q_2$, $\delta(q_0, (P_1, P_3)) = q_5$, ... , $\delta(q_5, (P_2, P_3)) = q_3$. At this part, the transitions has been defined therefore the MDFA can be seen in figure 7.

Finally, the solution of each state is replaced in (10). The results can be seen in table 4 and the Optimal Pareto Front is shown in figure 8.

State	Assignments			Times			$F(X)$	
q_i	M_1	M_2	M_3	M_1	M_2	M_3	$f_1(X)$	$f_2(X)$
q_0	P_1	P_2	P_3	10	50	5	125	36.66
q_1	P_1	P_3	P_2	10	5	50	170	29.16
q_2	P_2	P_1	P_3	50	10	5	85	56.66
q_3	P_2	P_3	P_1	50	5	10	90	55.83
q_4	P_3	P_1	P_2	5	10	50	175	26.66
q_5	P_3	P_2	P_1	5	50	10	135	33.33

Table 4. Values of $F(X)$ for the states of example 2

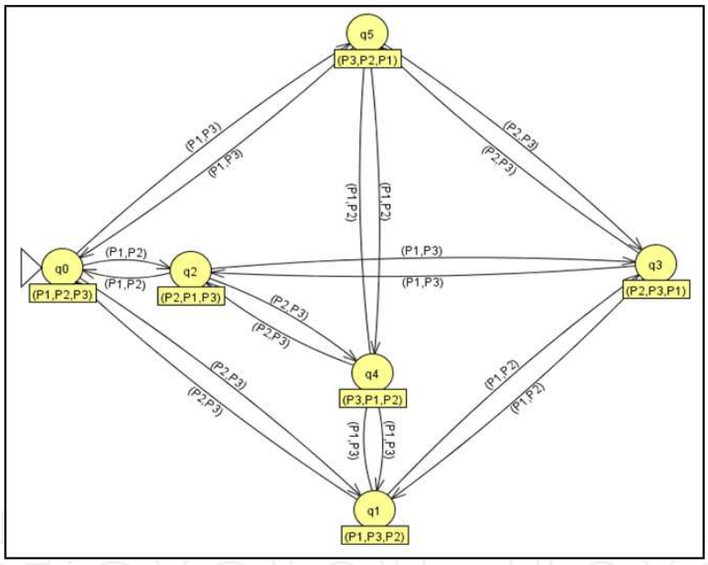

Fig. 7. MDFA for example 2, Parallel execution of processes

As can be seen in figure 7, the feasible solution space for this problem was described using a MDFA. Also, unfeasible solutions are not allowed because of the definition of Σ. Nevertheless, the general problem was not solved, only a particular case of three variables (machines) was done. For this reason, it was easy to draw the entire MDFA. However, problems like this are intractable for a large number of variables, in other words, when the number of variables grow the feasible solution space grows exponentially. In this manner, it is not a good idea to draw the entire feasible solution space and pick the best solutions. Thus, what should we do in order to solve any combinatorial problem, without taking into account its size, using a MDFA? Looking an answer to this question, MODS was proposed.

MODS explores the feasible solution space represented through a MDFA using a search direction given by an elitist set of solutions (Q_*). The elitist solution are states that, when

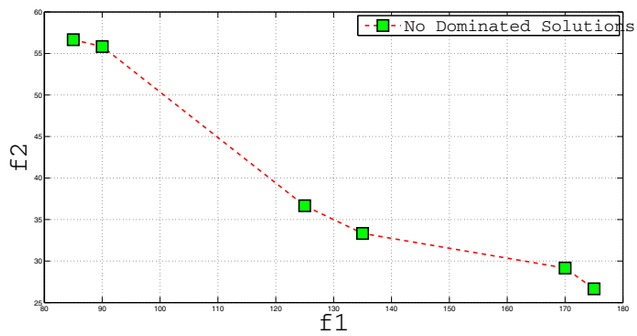

Fig. 8. Pareto Front for the MDFA of example 2, Parallel execution of processes

were visited, their solution dominated at least one solution of an element in Q_ϕ. Q_ϕ contains all the states with non-dominated solutions. Due to this, it can be inferred that the elements of Q_* are contained in Q_ϕ, for this reason is true that:

$$Q_\phi = Q_\phi \cup Q_* \tag{16}$$

Lastly, the template algorithm of MODS is defined as follows:

Step 1. Create the initial set of solutions Q_0 using a heuristic relative to the problem to solve.

Step 2. Set Q_ϕ as Q_0 and Q_* as ϕ.

Step 3. Select a random state $q \in Q_\phi$ or $q \in Q_*$

Step 4. Explore the neighborhood of q using δ and Σ. Add to Q_ϕ the solutions found that are not dominated by elements of Q_f. In addition, add to Q_* those solutions found that dominated at least one element from Q_ϕ.

Step 5. Check stop condition, go to 3.

3. Simulated Annealing Metaheuristic Of Deterministic Swapping (SAMODS)

Simulated Annealing & Metaheuristic Of Deterministic Swapping(Niño, 2012) (SAMODS) is a hybrid local search strategy based on the MODS theory and Simulated Annealing Algorithm for the Multiobjective Optimization of combinatorial problems. Its main propose consists in optimizing a combinatorial problem using a Search Direction and an Angle Improvement. SAMODS is based in the next Automata:

$$M = (Q, Q_0, P(q), F(X), A(n)) \tag{17}$$

Alike MODS, Q_0 is the set of initial solutions, Q is the feasible solution space and $F(X)$ are the functions of the combinatorial problem. $P(q)$ and $A(n)$ are defined as follows:

$P(q)$ is the *Permutation Function*, formally it is defined as follows:

$$P(q) : Q \to Q \tag{18}$$

P receives a solution $q \in Q$ and perturbs it returning a new solution $r_i \in Q$. The perturbation can be done based on the representation of the solutions. An example of some perturbations based on the representation of the solution can be seen in figure 15.

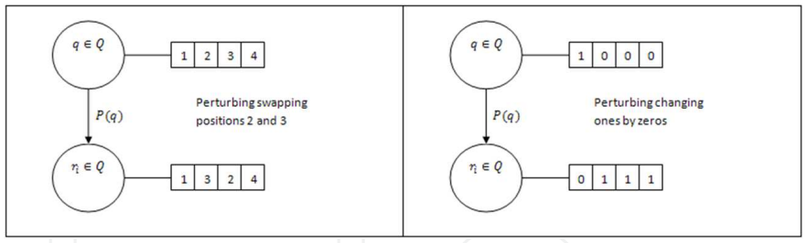

Fig. 9. Different representation and perturbation of solutions.

$A(n)$ is the *Weight Function*. Formally, it is defined as follow:

$$A(n) : \mathbb{N} \rightarrow \Re^n \tag{19}$$

Where n is the number of objectives of the problem.

Function A receives a natural number as parameter and it returns a vector with the weights. The weight values are randomly generated with an uniform distribution. Those represent the weight to assign to each function of the combinatorial problem. The weight values returned by the function fulfill the next constrain:

$$\sum_{i=1}^{n} \alpha_i = 1, 0 \leq \alpha_i \leq 1 \tag{20}$$

Where α_i is the weight assigned to function i. Table 5 shows some vectors randomly generated by $A(n)$.

Input Parameter	*Function*	*Vector of Weights*
2	$A(2)$	$\{0.6, 0.4\}$
3	$A(3)$	$\{0.2, 0.4, 0.4\}$
4	$A(4)$	$\{0.3, 0.8, 0.1, 0.0\}$

Table 5. Some weight vectors generated by A(n)

But, what is the importance of those weights? The weights, in an implicit manner, allow setting the angle direction to the solutions. The angle direction is the course being followed by the solutions for optimizing F(X). Hence, when the weights values are changed, the angle of optimization is changed and a new search direction is obtained. For instance, different search directions for different weight values are shown in figure 16 in a Bi-objective combinatorial problem. Due to this, (6) is rewritten as follows:

$$F(X) = \sum_{i=1}^{n} \alpha_i \cdot f_i(X) \tag{21}$$

Where n is the number of objectives of the problem and α_i is the weight assigned to the function i. The weights fulfills the constrain established in (20).

SAMODS main idea is simple: it takes advantage of the search directions given by MODS and it proposed an angle direction given by the function $A(n)$. Thus, there are two directions; the first helps in the convergence of the Pareto Front and the second helps the solutions to find neighborhoods where $F(X)$ is optimized. Due to this, SAMODS template is defined as follows:

Step 1. Setting sets. Set Q_0 as the set of Initial Solutions. Set Q_ϕ and Q_* as Q_0.

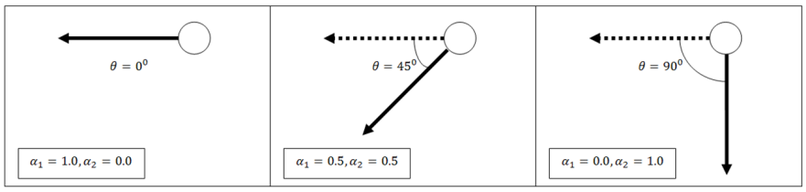

Fig. 10. Different angles given by different weights for a Bi-objective Problem.

Step 2. Settings parameters. Set T as the initial temperature, n as the number of objectives of the problem and ρ as the cooler factor.

Step 3. Setting Angle. If T is equal to 0 then got to 8, else set $T_{i+1} = \rho \times T_i$, randomly select $s \in Q_\phi$, set $W = A(n) = \{w_1, w_2, \cdots, w_n\}$ and go to step 4.

Step 4. Perturbing Solutions. Set $s^{'} = P(s)$, add to Q_ϕ and Q_* according to the next rules:

$$Q_\phi = Q_\phi \cup \left\{ s^{'} \right\} \Leftrightarrow (\nexists r \in Q_\phi)(r \quad dominated \quad to \quad s^{'}) \tag{22}$$

$$Q_* = Q_* \cup \left\{ s^{'} \right\} \Leftrightarrow (\exists r \in Q_*)(s^{'} \quad dominated \quad to \quad r) \tag{23}$$

If Q_ϕ has at least one element that dominated to $s^{'}$ go to step 5, otherwise go to step 7.

Step 5. Guess with dominated solutions. Randomly generated a number $n \in [0, 1]$. Set z as follows:

$$z = e^{(-(\gamma / T_i))} \tag{24}$$

Where T_i is the temperature value in moment i and γ is defined as follows:

$$\gamma = \sum_{i=1}^{n} w_i \cdot f_i(s_X) - \sum_{i=1}^{n} w_i \cdot f_i(s^{'}{}_X) \tag{25}$$

Where s_X is the vector X of solution s, $s^{'}{}_X$ is the vector X of solution $s^{'}$, w_i is the weight assigned to the function i and n is the number of objectives of the problem. If $n < z$ then set s as $s^{'}$ and go to step 4 else go to step 6.

Step 6. Change the search direction. Randomly select a solution $s \in Q_*$ and go to step 4.

Step 7. Removing dominated solutions. Remove the dominated solution for each set (Q_* and Q_ϕ). Go to step 3.

Step 8. Finishing. Q_ϕ has the non-dominated solutions.

As can be seen in figure 11, alike MODS, SAMODS removes the dominated solutions when the new solution found is not dominated. Besides, if the new solution found dominated at least one element from the solution set (Q_ϕ) then it will be added to the elitisms set (Q_*) that works as a search direction for the Pareto Front. As far as here, SAMODS could sounds as a simple local search strategy but not, when a new solution found is dominated, SAMODS tries to improve it using guessing. Guessing is done accepting dominated solution as good solutions. Alike Simulated Annealing inspired algorithms, the dominated solutions are accepted under the Boltzmann Distribution Probability assigning weights to the objectives of the problem. It is probably that perturbing a dominated solution, a non-dominated solution can be found as can be seen in figure 12. Due to this, local optimums are avoided. When the

temperature is low, the bad solutions are avoided because z value is low therefore SAMODS accepts only non-dominated solutions. However, by that time, Q_ϕ will be leaded on by Q_*.

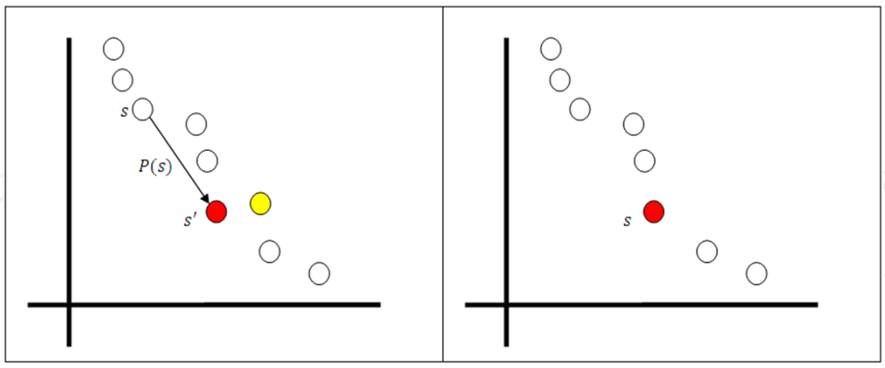

Fig. 11. Behavior of SAMODS when the new solution found is not dominated. Once a new solution found is non-dominated, it is added to the elitism set Q_* and the dominated solutions from Q_ϕ are removed.

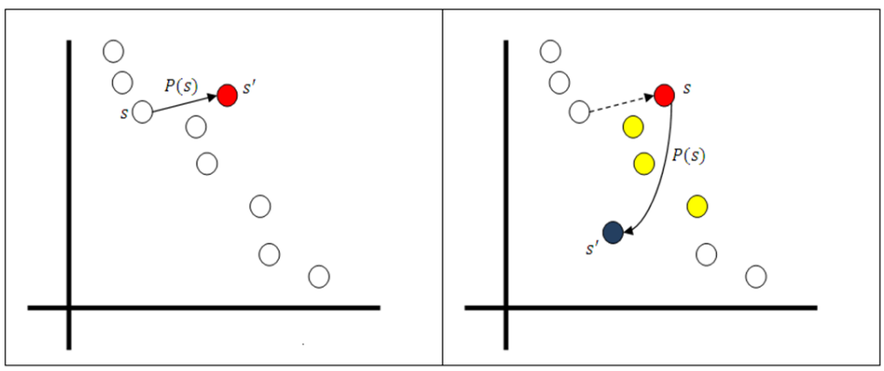

Fig. 12. Behavior of SAMODS when the new solution found is dominated. In this case, guessing gives a new solution non-dominated.

4. Genetic Simulated Annealing Metaheuristic Of Deterministic Swapping (SAGAMODS)

Simulated Annealing, Genetic Algorithm & Metaheuristic Of Deterministic Swapping(Niño, 2012) (SAGAMODS) is a hybrid search strategy based on the Automata Theory, Simulated Annealing and Genetics Algorithms. SAGAMODS is an extension of the SAMODS theory. It comes up as result of the next question: could SAMODS avoid quickly local optimums? Although, SAMODS avoids local optimums guessing, it can take a lot of time accepting dominated solutions for finding non-dominated. Thus, the answer to this question is based on the Evolutionary Theory. SAGAMODS proposes crossover step before SAMODS template is executed. Due to this, SAGAMODS supports to SAMODS for exploring distant regions of the solution space.

Formally, SAGAMODS is based on the next automata:

$$M = (Q, Q_S, C(q, r, k), F(X)) \tag{26}$$

Where Q is the feasible solutions space, Q_S is the initial solutions and $F(X)$ are the objectives of the problem. $C(q,r,k)$ is defined as follows:

Formally, *Cross Function K* is defined as follows:

$$C(q,r,k) : Q \to Q \tag{27}$$

Where $q,r \in Q$ and $k \in N$. q and r are named parents solutions and k is the cross point. The main idea of this function is cross two solutions in the same point and returns a new solution. For instance, two solutions of 4 variables are cross in figure 13. Obviously, the crossover is made regarding the representation of the solutions. Lastly, SAGAMODS template is defined

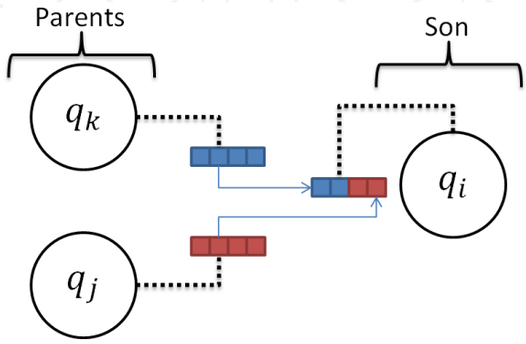

Fig. 13. Crossover between two solutions. Solutions of the states q_k and q_j are crossed in order to get state q_i

as follows:

Step 1. Setting parameters. Set Q_S as the solution set, x as the number of solutions to cross for each iteration.

Step 2. Selection. Set Q_C (crossover set) as selection of x solutions in Q_S, Q_M (mutation set) as ϕ and k as a random value.

Step 3. Crossover. For each $s_i, s_{i+1} \in Q_C / 1 \leq i < |Q_C|$:

$$Q_M = Q_M \cup \{C(s_i, s_{i+1}, k)\} \tag{28}$$

Step 4. Mutation. Set Q_0 as Q_M. Execute SAMODS as a local search strategy.

Step 5. Check stop conditions. Go to 2.

5. Evolutionary Metaheuristic Of Deterministic Swapping (EMODS)

Evolutionary Metaheuristic of Deterministic Swapping (EMODS), is a novel framework that allows the Multiobjective Optimization of Combinatorial Problems. Its framework is based on MODS template therefore its steps are the same: create Initial Solutions, Improve the Solutions (Optional) and Execute the Core Algorithm. Unlike SAMODS and SAGAMODS, EMODS avoids the slowly convergence of Simulated Annealing's method. EMODS explores different regions from the feasible solution space and search for non-dominated solution using Tabu Search.

The Core Algorithm is defined as follows:

Step 1. Set θ as the maximum number of iterations, β as the maximum number of state selected in each iteration, ρ as the maximum number of perturbations by state and Q_ϕ as Q_0

Step 2. Selection. Randomly select a state $q \in Q_\phi$ or $q \in Q_*$

Step 3. Mutation - Tabu Search. Set N as the new solutions found as result of perturbing q. Add to Q_ϕ and Q_* according to the next equations:

$$\left(Q_\phi = Q_\phi \cup \{q\}\right) \Longleftrightarrow \left(\nexists r \in Q_\phi / q \quad is \quad dominated \quad by \quad r\right) \tag{29}$$

$$\left(Q_* = Q_* \cup \{q\}\right) \Longleftrightarrow \left(\exists r \in Q_\phi / r \quad is \quad dominated \quad by \quad q\right) \tag{30}$$

Remove the states with dominated solutions for each set.

Step 4. Crossover. Randomly select states from Q_ϕ and Q_*. Generate a random point of cross.

Step 5. Check stop condition, go to 3.

Step 2 and 3 support the algorithm in removing dominated solutions from the set of solutions Q_ϕ as can be seen in figure 3. However, one of the most important steps in the EMODS algorithm is step 4. There, similar to SAGAMODS, the algorithm applies an Evolutionary Strategy based in the crossover step of Genetic Algorithms for avoiding Local Optimums. Due to the crossover is not always made in the same point (the k-value is randomly generated in each state analyzed) the variety of solutions found are diverse avoiding local optimums. An overview of EMODS behavior for a Tri-objective Combinatorial Optimization problem can be seen in figure 14

6. Experimental analysis

6.1 Experimental settings

The algorithms were tested using well-known instances from the Multi-objective Traveling Salesman Problem taken from TSPLIB(Heidelberg, n.d.). The instances worked are shown in table 6 and the input parameters for the algorithms are shown in table 7. The test of the algorithms was made using a Dual Core Computer with 2 Gb RAM. The optimal solutions were constructed based in the best non-dominated solutions of all algorithms in comparison for each instance worked.

6.2 Performance metrics

There are metrics that allow measuring the quality of a set of optimal solutions and the performance of an Algorithm (Corne & Knowles, 2003). Most of them use two Pareto Fronts. The first one is PF_{true} and it refers to the real optimal solutions of a combinatorial problem. The second is PF_{know} and it represents the optimal solutions found by an algorithm.

Generation of Non-dominated Vectors (GNDV) It measures the number of No Dominates Solutions generated by an algorithm.

$$GNDV = |PF_{know}| \tag{31}$$

A higher value for this metric is desired. *Rate of Generation of No-dominated Vectors (RGNDV)* This metric measures the proportion of the No Dominates Solutions (31) generated by an

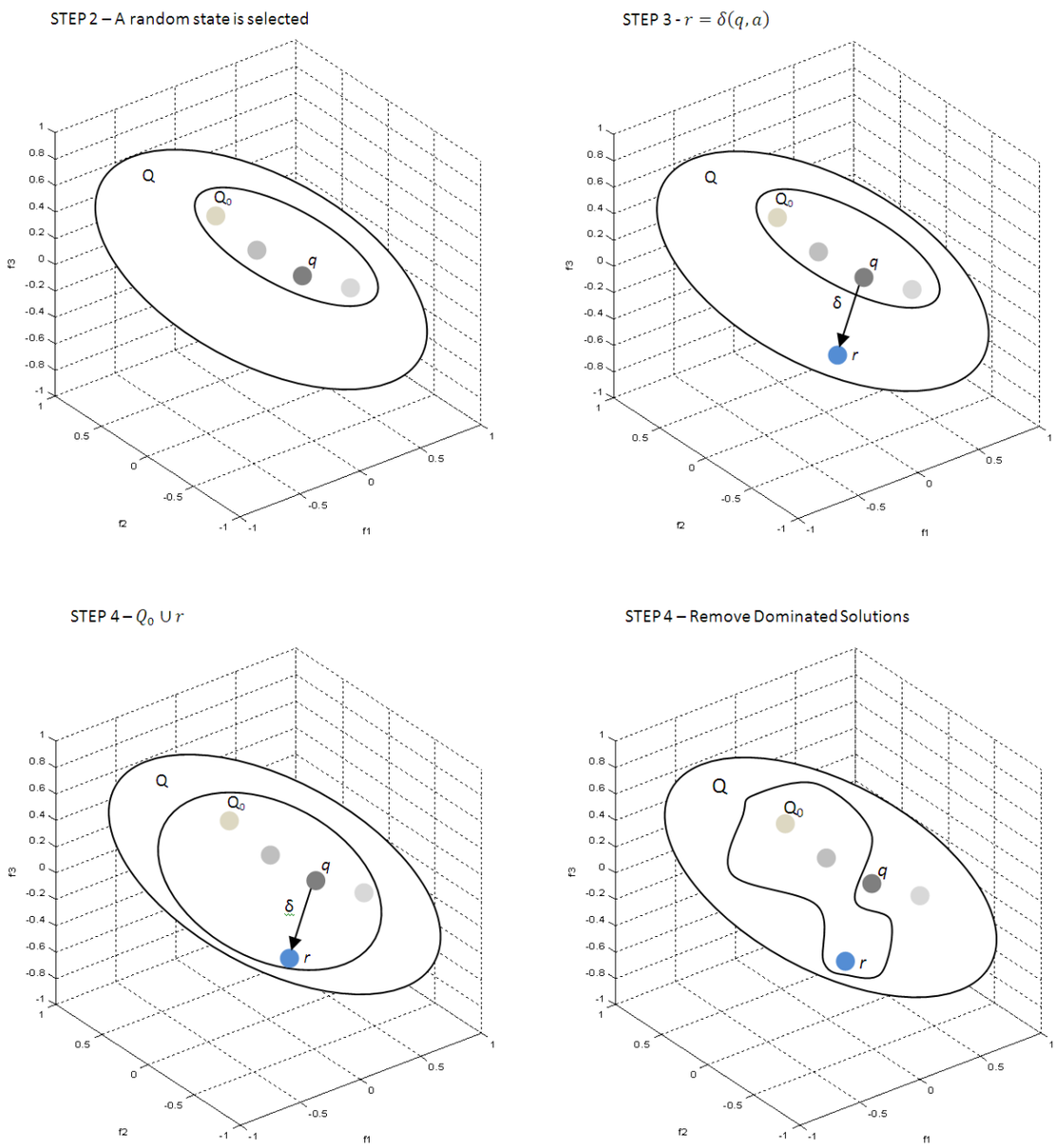

Fig. 14. An overview of EMODS behavior for a Tri-objective Problem.

algorithm and the Real Solutions.

$$RGNDV = \left(\frac{GNDV}{|PF_{true}|}\right) \cdot 100\% \tag{32}$$

A value closer to 100% for this metric is desired. *Real Generation of Non-dominated Vectors (ReGNDV)* This metric measures the number of Real Solutions found by an algorithm.

$$ReGNDV = |\{y | y \in PF_{know} \wedge y \in PF_{true}\}| \tag{33}$$

Combinatorial Problem Instance	Number of Objectives
KROAB100	2
KROAC100	
KROAD100	
KROAE100	
KROBC100	
KROBD100	
KROBE100	
KROCD100	
KROCE100	
KRODE100	
KROABC100	3
KROABD100	
KROABE100	
KROACD100	
KROACE100	
KROADE100	
KROBCD100	
KROBCE100	
KROBDE100	
KROCDE100	
KROABCD100	4
KROABCE100	
KROABDE100	
KROACDE100	
KROBCDE100	
KROABCDE100	5

Table 6. Instances worked for testing the proposed algorithms.

Algorithm	Max. Iterations	Max. Perturbations	Initial Temperature	Cooler Value	Crossover Rate
MODS	100	80	NA	NA	NA
SAMODS	100	80	1000	0.95	NA
SAGAMODS	100	80	1000	0.95	0.6
EMODS	100	80	NA	NA	0.6

Table 7. Parameters setting for each compared algorithm.

A value closer to $|PF_{true}|$ for this metric is desired.

Generational Distance (GD) This metric measures the distance between PF_{know} and PF_{true}. It allows to determinate the error rate in terms of the distance of a set of solutions relative to the real solutions.

$$GD = \left(\frac{1}{|PF_{know}|}\right) \cdot \left(\sum_{i=1}^{|PF_{know}|} d_i\right)^{(1/p)} \tag{34}$$

Where d_i is the smallest Euclidean distance between the solution i of FP_{know} and the solutions of FP_{true}. p is the dimension of the combinatorial problem, it means the number of objective functions. *Inverse Generational Distance (IGD)* This is another distance measurement between FP_{know} and FP_{true}:

$$IGD = \left(\frac{1}{|PF_{true}|}\right) \cdot \left(\sum_{i=1}^{|PF_{know}|} d_i\right) \tag{35}$$

Where d_i is the smallest Euclidean distance between the solution i of PF_{know} and the solutions of PF_{true}. *Spacing (S)* It measures the range variance of neighboring solutions in PF_{know}

$$S = \left(\frac{1}{|PF_{know}| - 1}\right)^2 \cdot \left(\sum_{i=1}^{|PF_{know}|} \left(\overline{d} - d_i\right)^2\right)^{(1/p)} \tag{36}$$

Where d_i is the smallest Euclidean distance between the solution i of PF_{know} and the rest of solutions of PF_{know}. $\overline{d}$ is the mean of all d_i. p is the dimension of the combinatorial problem.

A value closer to 0 for this metric is desired. A value of 0 means that all the solutions are equidistant.

Error Rate (ε) It estimates the error rate respect to the precision of the Real Algorithms Solutions (33) as follows:

$$\varepsilon = \left(\left|\frac{PF_{true}}{ReGNDV}\right|\right) \cdot 100\% \tag{37}$$

A value of 0% in this metric means that the values of the Real Pareto Front are constructed from the values of the Algorithm Pareto Front.

Lastly, notice that every metric by itself does not have sense. It is necessary to support in the other metrics for a real judge about the quality of the solutions. For instance, if a Pareto Front has a higher value in $GNDV$ but a lower value in $ReGNDV$ then the solutions has a poor-quality.

6.3 Experimental results

The tests made with Bi-objectives, Tri-objectives, Quad-objectives and Quin-objectives TSP instances are shown in tables 8, 9, 10 and 11 respectively. The average of the measurement is shown in table 12. Furthermore, a graphical comparison for bi-objectives and tri objectives instances worked is shown in figures 15 and 16 respectively.

6.4 Analysis

It can be concluded, that, in the case of two and three objectives, metrics such as S, IGD, GD and ε determine the best algorithm. In this case, the measurement of the metrics is similar for SAMODS and SAGAMODS. On the other hand, MODS has the most poor-quality measurement for the metrics used and EMODS has the best quality measurement for the same metrics.

Lastly, why are the results of the metrics similar for quint-instances? In this case, all the solutions for each solution set are in the optimal set. The answer to this question is based in the angle improvement. MODS as a local search strategy explore a part of the feasible solution using its search direction (Q_*). However, SAMODS and SAGAMODS, in addition, use a search direction given by the change of the search angle. While SAMODS was looking in a

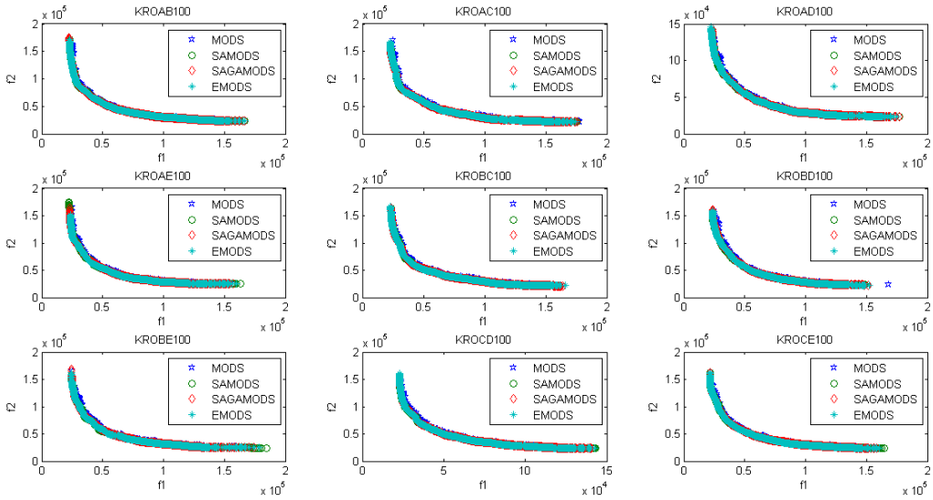

Fig. 15. Graphical comparison between MODS, SAMODS, SAGAMODS and EMODS for Bi-objective TSP instances.

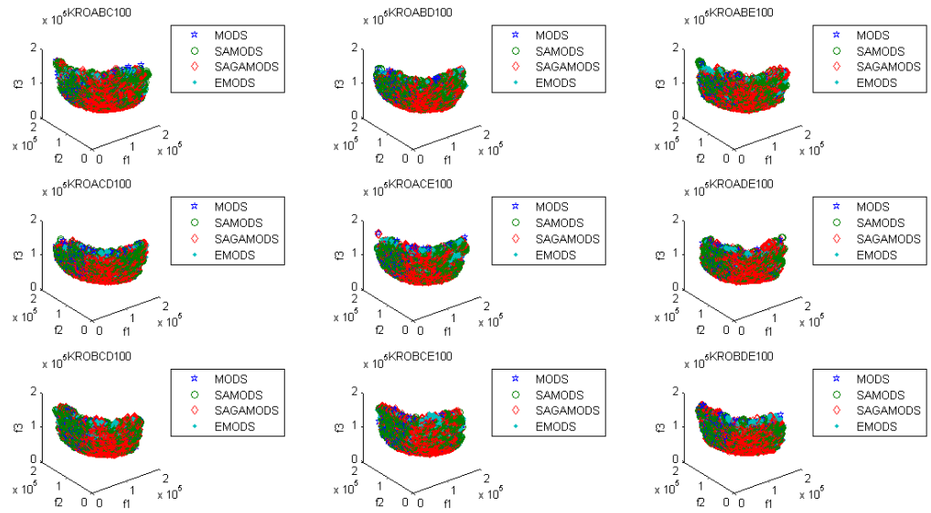

Fig. 16. Graphical comparison between MODS, SAMODS, SAGAMODS and EMODS for Tri-objective TSP instances.

part of the feasible solution space, SAGAMODS was doing the same in other. The same reason applies to EMODS. It can be possible because of the large size of the feasible solution space ($\Re^5$). The possibility of exploring the same part of the solution space for different algorithms is low.

Instance	*Algorithm*	*GNDV*	*RGNDV*	*ReGNDV*	$\left(\frac{ReGNDV}{GNDV}\right)$ %	*S*	*GD*	*IGD*	ε
AB	MODS	289	0.0189	0	0%	0.0193	21.2731	2473.4576	100%
	SAMODS	7787	0.5096	1247	16.01%	0.001	0.2404	229.2593	91.84%
	SAGAMODS	8479	0.5549	2974	35.07%	0.0007	0.1837	158.8229	80.54%
	EMODS	26125	1.7096	11060	42.33%	0.0002	0.0412	75.8814	27.62%
AC	MODS	217	0.0155	0	0%	0.034	28.575	2751.3232	100%
	SAMODS	6885	0.4927	2303	33.45%	0.0008	0.2297	179.0354	83.52%
	SAGAMODS	7023	0.5025	2431	34.61%	0.0008	0.2628	243.7536	82.6%
	EMODS	20990	1.502	9241	44.03%	0.0002	0.0617	119.8825	33.87%
AD	MODS	281	0.0187	0	0%	0.0139	20.4429	2198.883	100%
	SAMODS	6383	0.4253	1464	22.94%	0.0029	0.3188	275.9153	90.24%
	SAGAMODS	6289	0.4191	764	12.15%	0.0016	0.2835	211.7935	94.91%
	EMODS	17195	1.1458	12779	74.32%	0.0005	0.0521	53.5314	14.85%
AE	MODS	283	0.0189	0	0%	0.0533	21.3238	2433.63	100%
	SAMODS	5693	0.3804	1433	25.17%	0.0016	0.4308	402.0483	90.42%
	SAGAMODS	6440	0.4304	1515	23.52%	0.0013	0.3906	422.7859	89.88%
	EMODS	20695	1.383	12016	58.06%	0.0002	0.066	124.7197	19.7%
BC	MODS	298	0.0212	0	0%	0.0158	19.537	2411.8365	100%
	SAMODS	6858	0.488	789	11.5%	0.0024	0.3597	433.0882	94.39%
	SAGAMODS	6919	0.4923	2201	31.81%	0.0015	0.2378	192.5601	84.34%
	EMODS	21902	1.5584	11064	50.52%	0.0003	0.0582	115.5673	21.28%
BD	MODS	241	0.0198	0	0%	0.0239	21.7441	2251.0972	100%
	SAMODS	6844	0.561	2054	30.01%	0.0021	0.2542	248.0971	83.16%
	SAGAMODS	5934	0.4864	1971	33.22%	0.0018	0.2818	229.2093	83.84%
	EMODS	19420	1.5919	8174	42.09%	0.0003	0.0432	57.6434	32.99%
BE	MODS	280	0.0259	0	0%	0.0309	19.0193	2622.5243	100%
	SAMODS	6260	0.5789	952	15.21%	0.001	0.2601	245.1433	91.2%
	SAGAMODS	5802	0.5365	1622	27.96%	0.0025	0.3912	476.4848	85%
	EMODS	17362	1.6055	8240	47.46%	0.0004	0.0631	111.0209	23.8%
CD	MODS	286	0.022	0	0%	0.0184	18.0035	2040.9722	100%
	SAMODS	6171	0.4751	1912	30.98%	0.0007	0.2588	196.3394	85.28%
	SAGAMODS	6301	0.4851	994	15.78%	0.0014	0.2852	248.5855	92.35%
	EMODS	18628	1.434	10084	54.13%	0.0002	0.0426	48.3785	22.37%
CE	MODS	224	0.0187	0	0%	0.0285	23.1312	2245.6542	100%
	SAMODS	5881	0.4919	946	16.09%	0.0017	0.2894	242.2535	92.09%
	SAGAMODS	4613	0.3859	939	20.36%	0.0028	0.481	411.854	92.15%
	EMODS	15211	1.2724	10070	66.2%	0.0003	0.0339	22.2645	15.77%
DE	MODS	228	0.0147	0	0%	0.0477	23.6222	1864.9602	100%
	SAMODS	6110	0.3928	1157	18.94%	0.0022	0.2942	207.7947	92.56%
	SAGAMODS	7745	0.4979	407	5.26%	0.0012	0.2644	269.6204	97.38%
	EMODS	20058	1.2896	13990	69.75%	0.0005	0.0304	23.8829	10.06%

Table 8. Measuring algorithms performance for Bi-objectives instances of Traveling Salesman Problem with Multi-objective optimization metrics.

Instance	*Algorithm*	*GNDV*	*RGNDV*	*ReGNDV*	$\left(\frac{ReGNDV}{GNDV}\right)\%$	*S*	*GD*	*IGD*	ε
ABC	MODS	2115	0.0307	83	3.92%	0.1567	0.2819	3075.4309	99.88%
	SAMODS	12768	0.1853	227	1.78%	0.0722	0.0421	2256.4593	99.67%
	SAGAMODS	12523	0.1818	328	2.62%	0.073	0.0427	2220.5614	99.52%
	EMODS	70474	1.023	68254	96.85%	0.0477	0.001	5.6388	0.93%
ABD	MODS	1951	0.0292	74	3.79%	0.1524	0.305	3153.9212	99.89%
	SAMODS	12094	0.1811	317	2.62%	0.0746	0.0441	2270.3475	99.53%
	SAGAMODS	12132	0.1817	250	2.06%	0.0726	0.0441	2286.7374	99.63%
	EMODS	68001	1.0184	66133	97.25%	0.0471	0.0011	6.3212	0.96%
ABE	MODS	1931	0.0281	63	3.26%	0.1496	0.315	3278.1554	99.91%
	SAMODS	12461	0.1815	373	2.99%	0.0743	0.0438	2371.7641	99.46%
	SAGAMODS	12391	0.1805	370	2.99%	0.0745	0.0436	2304.3277	99.46%
	EMODS	70411	1.0257	67839	96.35%	0.0474	0.0012	8.0639	1.17%
ACD	MODS	2031	0.0305	66	3.25%	0.1425	0.2945	3213.7378	99.9%
	SAMODS	12004	0.1802	241	2.01%	0.0734	0.0444	2277.0343	99.64%
	SAGAMODS	12123	0.182	206	1.7%	0.0735	0.0442	2310.3683	99.69%
	EMODS	67451	1.0127	66090	97.98%	0.0468	0.001	4.5012	0.77%
ACE	MODS	1950	0.0306	57	2.92%	0.1628	0.3024	3215.8357	99.91%
	SAMODS	11382	0.1785	263	2.31%	0.074	0.0461	2271.6542	99.59%
	SAGAMODS	11476	0.18	303	2.64%	0.0734	0.0456	2241.9933	99.52%
	EMODS	64804	1.0162	63145	97.44%	0.048	0.0012	7.3103	0.98%
ADE	MODS	1824	0.0274	67	3.67%	0.1487	0.3289	3248.597	99.9%
	SAMODS	12149	0.1827	179	1.47%	0.0733	0.0442	2336.2798	99.73%
	SAGAMODS	11773	0.1771	258	2.19%	0.0771	0.0457	2346.6414	99.61%
	EMODS	67767	1.0193	65981	97.36%	0.0468	0.0011	5.7824	0.76%
BCD	MODS	2065	0.03	43	2.08%	0.1451	0.2927	3206.9305	99.94%
	SAMODS	13129	0.1908	260	1.98%	0.0712	0.0417	2387.8219	99.62%
	SAGAMODS	12889	0.1873	253	1.96%	0.0786	0.042	2308.1811	99.63%
	EMODS	70035	1.0176	68270	97.48%	0.0452	0.001	5.6235	0.81%
BCE	MODS	2009	0.0286	58	2.89%	0.1505	0.3065	3327.787	99.92%
	SAMODS	12992	0.1852	229	1.76%	0.0701	0.0428	2448.3577	99.67%
	SAGAMODS	12582	0.1794	201	1.6%	0.0736	0.0445	2503.0421	99.71%
	EMODS	71176	1.0147	69654	97.86%	0.0464	0.0011	7.8122	0.7%
BDE	MODS	2039	0.0316	45	2.21%	0.1532	0.2914	3252.7813	99.93%
	SAMODS	12379	0.192	205	1.66%	0.0728	0.0434	2401.8804	99.68%
	SAGAMODS	12427	0.1928	195	1.57%	0.0742	0.0431	2377.0621	99.7%
	EMODS	65509	1.0163	64015	97.72%	0.0476	0.0011	5.6322	0.69%
CDE	MODS	2010	0.0278	83	4.13%	0.1463	0.3022	3094.2824	99.89%
	SAMODS	13084	0.1807	399	3.05%	0.0712	0.0414	2193.6586	99.45%
	SAGAMODS	13009	0.1796	347	2.67%	0.0719	0.0418	2224.4738	99.52%
	EMODS	74063	1.0227	71589	96.66%	0.0453	0.0011	7.2279	1.14%

Table 9. Measuring algorithms performance for Tri-objective instances of TSP with Multi-objective optimization metrics.

Instance	*Algorithm*	*GNDV*	*RGNDV*	*ReGNDV*	$\left(\frac{ReGNDV}{GNDV}\right)$ %	*S*	*GD*	*IGD*	ε
ABCD	MODS	5333	0.0925	3303	61.94%	0.3497	0.0256	6030.5288	94.27%
	SAMODS	28523	0.4947	12178	42.7%	0.231	0.0042	3454.1238	78.88%
	SAGAMODS	36802	0.6382	14967	40.67%	0.2203	0.0031	3092.5232	74.04%
	EMODS	201934	3.502	27214	13.48%	0.1754	0.0005	1991.148	52.8%
ABCE	MODS	5533	0.0973	3439	62.15%	0.3452	0.0244	5861.8605	93.95%
	SAMODS	27684	0.4868	11471	41.44%	0.2331	0.0043	3444.6454	79.83%
	SAGAMODS	35766	0.6289	14552	40.69%	0.2232	0.0032	3118.6397	74.41%
	EMODS	204596	3.5976	27408	13.4%	0.1754	0.0005	1885.4464	51.81%
ABDE	MODS	5259	0.0942	3142	59.75%	0.3487	0.0256	5864.5036	94.37%
	SAMODS	27180	0.4869	11247	41.38%	0.232	0.0043	3398.9429	79.85%
	SAGAMODS	34930	0.6257	14472	41.43%	0.2236	0.0033	2986.5319	74.08%
	EMODS	195756	3.5067	26963	13.77%	0.1775	0.0005	1916.7141	51.7%
ACDE	MODS	5466	0.094	3400	62.2%	0.3405	0.0246	5617.5202	94.15%
	SAMODS	26757	0.4602	11336	42.37%	0.235	0.0044	3394.8396	80.5%
	SAGAMODS	34492	0.5932	14638	42.44%	0.2265	0.0033	2965.4482	74.83%
	EMODS	196800	3.3845	28774	14.62%	0.1764	0.0005	1793.4489	50.52%
BCDE	MODS	5233	0.0879	3082	58.9%	0.3499	0.0259	5677.9988	94.83%
	SAMODS	28054	0.471	11739	41.84%	0.2315	0.0042	3296.196	80.29%
	SAGAMODS	36258	0.6087	15145	41.77%	0.2218	0.0032	2902.8041	74.57%
	EMODS	203017	3.4083	29599	14.58%	0.1752	0.0005	1873.1748	50.31%

Table 10. Measuring algorithms performance for Quad-objectives instances of TSP with Multi-objective optimization metrics.

Instance	*Algorithm*	*GNDV*	*RGNDV*	*ReGNDV*	$\left(\frac{ReGNDV}{GNDV}\right)$ %	*S*	*GD*	*IGD*	ε
ABCDE	MODS	7517	0.0159	7517	100%	0.5728	0.0125	15705.6864	98.41%
	SAMODS	26140	0.0554	26140	100%	0.4101	0.0033	10801.6382	94.46%
	SAGAMODS	26611	0.0564	26611	100%	0.4097	0.0033	10544.8901	94.36%
	EMODS	411822	0.8723	411822	100%	0.3136	0.0001	950.4252	12.77%

Table 11. Measuring algorithms performance for Quint-objectives instances of TSP with Multi-objective optimization metrics.

Objectives	*Algorithm*	*GNDV*	*RGNDV*	*ReGNDV*	$\left(\frac{ReGNDV}{GNDV}\right)$ %	*S*	*GD*	*IGD*	ε
2	MODS	262.7	0.0194	0	0%	0.0286	21.6672	2329.4338	100%
	SAMODS	6487.2	0.4796	1425.7	22.03%	0.0016	0.2936	265.8974	89.47%
	SAGAMODS	6554.5	0.4791	1581.8	23.97%	0.0015	0.3062	286.547	88.3%
	EMODS	19758.6	1.4492	10671.8	54.89%	0.0003	0.0492	75.2773	22.23%
3	MODS	1992.5	0.0295	63.9	3.21%	0.1508	0.302	3206.7459	99.91%
	SAMODS	12444.2	0.1838	269.3	2.16%	0.0727	0.0434	2321.5258	99.6%
	SAGAMODS	12332.5	0.1822	271.1	2.2%	0.0743	0.0437	2312.3389	99.6%
	EMODS	68969.1	1.0187	67097	97.3%	0.0468	0.0011	6.3914	0.89%
4	MODS	5364.8	0.0932	3273.2	60.99%	0.3468	0.0252	5810.4824	94.31%
	SAMODS	27639.6	0.4799	11594.2	41.94%	0.2325	0.0043	3397.7495	79.87%
	SAGAMODS	35649.6	0.619	14754.8	41.4%	0.2231	0.0032	3013.1894	74.39%
	EMODS	200420.6	3.4798	27991.6	13.97%	0.176	0.0005	1891.9864	51.43%
5	MODS	7517	0.0159	7517	100%	0.5728	0.0125	15705.6864	98.41%
	SAMODS	26140	0.0554	26140	100%	0.4101	0.0033	10801.6382	94.46%
	SAGAMODS	26611	0.0564	26611	100%	0.4097	0.0033	10544.8901	94.36%
	EMODS	411822	0.8723	411822	100%	0.3136	0.0001	950.4252	12.77%

Table 12. Measuring algorithms performance for Multi-objectives instances of TSP with Multi-objective optimization metrics.

7. Conclusion

SAMODS, SAGAMODS and EMODS are algorithms based on the Automata Theory for the Multi-objective Optimization of Combinatorial Problems. All of them are derived from the MODS metaheuristic, which is inspired in the Theory of Deterministic Finite Swapping. SAMODS is a Simulated Annealing inspired Algorithm. It uses a search direction in order to optimize a set of solution (Pareto Front) through a linear combination of the objective functions. On the other hand, SAGAMODS, in addition to the advantages of SAMODS, is an Evolutionary inspired Algorithm. It implements a crossover step for exploring far regions of a solution space. Due to this, SAGAMODS tries to avoid local optimums owing to it takes a general look of the solution space. Lastly, in order to avoid slow convergence, EMODS is proposed. Unlike SAMODS and SAGAMODS, EMODS does not explore the neighborhood of a solution using Simulated Annealing, this step is done using Tabu Search. Thus, EMODS gets optimal solution faster than SAGAMODS and SAMODS. Lastly, the algorithms were tested using well known instances from TSPLIB and metrics from the specialized literature. The results shows that for instances of two, three and four objectives, the proposed algorithm has the best performance as the metrics values corroborate. For the last instance worked, quint-objective, the behavior of MODS, SAMODS and SAGAMODS tend to be the same, them have similar error rate but, EMODS has a the best performance. In all the cases, EMODS shows the best performance. However, for the last test, all the algorithms have different solutions sets of non-dominated solutions, and those form the optimal solution set.

8. Acknowledgment

First of all, I want to thank to God for being with me in my entire life, He made this possible. Secondly, I want to thank to my parents Elias Niño and Arely Ruiz and my sister Carmen Niño for their enormous love and support. Finally, and not less important, to thank to my beautiful wife Maria Padron and our baby for being my inspiration.

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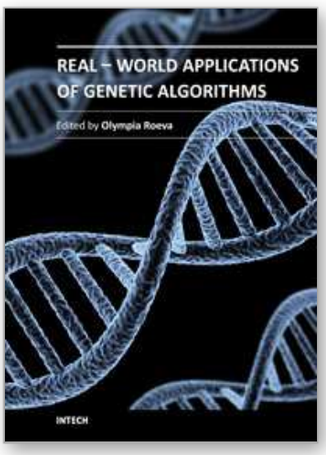

Real-World Applications of Genetic Algorithms
Edited by Dr. Olympia Roeva

ISBN 978-953-51-0146-8
Hard cover, 376 pages
Publisher InTech
Published online 07, March, 2012
Published in print edition March, 2012

The book addresses some of the most recent issues, with the theoretical and methodological aspects, of evolutionary multi-objective optimization problems and the various design challenges using different hybrid intelligent approaches. Multi-objective optimization has been available for about two decades, and its application in real-world problems is continuously increasing. Furthermore, many applications function more effectively using a hybrid systems approach. The book presents hybrid techniques based on Artificial Neural Network, Fuzzy Sets, Automata Theory, other metaheuristic or classical algorithms, etc. The book examines various examples of algorithms in different real-world application domains as graph growing problem, speech synthesis, traveling salesman problem, scheduling problems, antenna design, genes design, modeling of chemical and biochemical processes etc.

How to reference
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Elias D. Niño (2012). Evolutionary Algorithms Based on the Automata Theory for the Multi-Objective Optimization of Combinatorial Problems, Real-World Applications of Genetic Algorithms, Dr. Olympia Roeva (Ed.), ISBN: 978-953-51-0146-8, InTech, Available from: http://www.intechopen.com/books/real-world-applications-of-genetic-algorithms/evolutionary-algorithms-based-on-the-automata-theory-for-the-multi-objective-optimization-of-combina

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